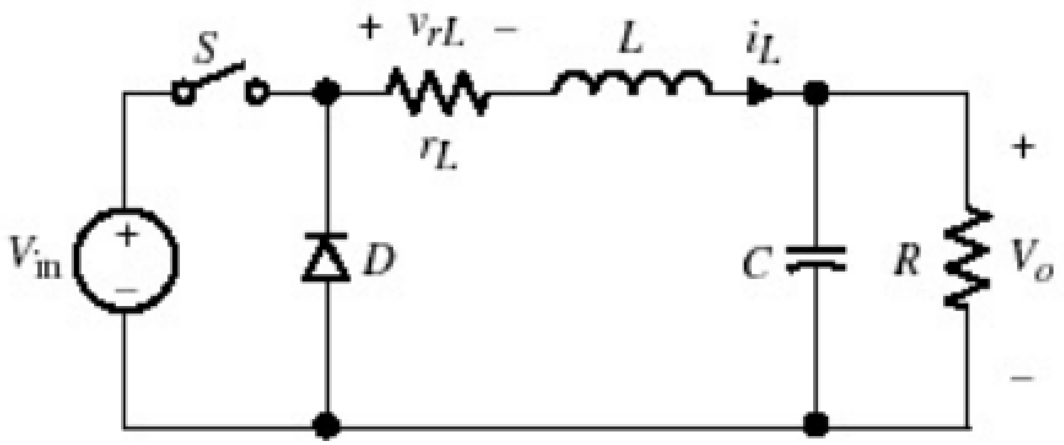
Linearized small signal model of buck converter with closed loop-controlled simulation model



Project Description:

Part 1: Derive linearized small signal model of buck converter in terms of $\frac{\hat{i}_L(s)}{\hat{d}(s)}$. For above models, if $v = 20V, v = 16V, fs = 20kHz$; load power = 100 W. Assume CCM operation and non-idea diode and inductor (inductor resistance = 0.025 ohm), decide appropriate L and C values and provide the pole-zero locations. Comment on the stability.

Part 2: Develop a closed loop-controlled simulation model of above converter. Use an appropriate PI and PID controllers (one by each student). Provide necessary simulation results for the steady-state and dynamic conditions (input voltage change and load change). Discuss the performance achieved with PI and PID controllers.

Part 1:

Derive the linearized small signal model of buck converter in terms of $\frac{\hat{i}_L(s)}{\hat{d}(s)}$.

For above models, if $v_{in} = 20V, v_o = 16V, f_s = 20kHz; load\ power = 100W$. Assume CCM operation and non-ideal diode and inductor (inductor resistance = 0.025 Ω), decide appropriate L and C values and provide the pole-zero locations. Comment on the stability.

Solution

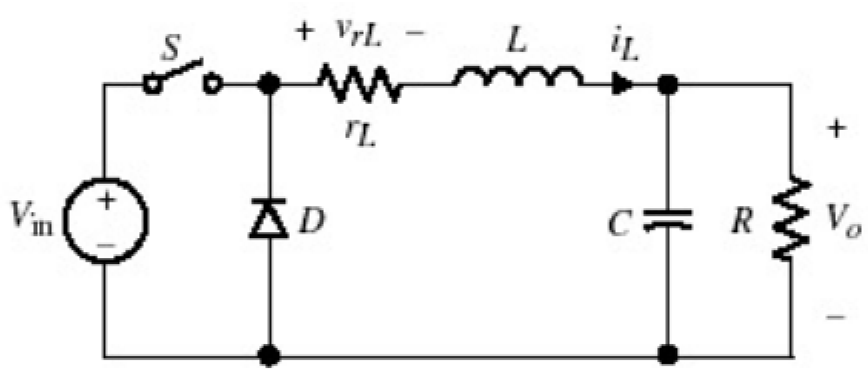


Figure 1: Buck Converter with non-idealities

Linearized Small Signal Model of Buck Converter:

Step 1: Average Model

(a) ON-state Variables (d(t))

Inductor Voltage

$$v_{in}(t) - v_o(t) - r_L i_L(t) = v_L(t) = L\frac{di_L(t)}{dt}$$

$$\frac{di_L(t)}{dt} = \frac{v_{in}(t)}{L} - \frac{v_o(t)}{L} - \frac{r_L i_L(t)}{L}$$

$$[\frac{di_L(t)}{dt} = \frac{v_{in}(t)}{L} - \frac{v_o(t)}{L} - \frac{r_L i_L(t)}{L}] \times d(t) \tag{1}$$

Capacitor Current

$$i_c(t) = i_L(t) - i_o(t)$$

$$\frac{Cdv_o(t)}{dt} = i_L(t) - \frac{v_o(t)}{R}$$

$$\frac{dv_o(t)}{dt} = \frac{i_L(t)}{C} - \frac{v_o(t)}{RC}$$

$$[\frac{dv_o(t)}{dt} = \frac{i_L(t)}{C} - \frac{v_o(t)}{RC}] \times d(t) \qquad (2)$$

(b) OFF-state Variables (1-d(t))

Inductor Voltage

$$-v_o(t) - v_D(t) - r_L i_L(t) = v_L(t) = L\frac{di_L(t)}{dt}$$

$$\frac{di_L(t)}{dt} = -\frac{v_o(t)}{L} - \frac{v_D(t)}{L} - \frac{r_L i_L(t)}{L}$$

$$[\frac{di_L(t)}{dt} = -\frac{v_o(t)}{L} - \frac{v_D(t)}{L} - \frac{r_L i_L(t)}{L}] \times (1 - d(t)) \qquad (3)$$

Capacitor Current

$$i_c(t) = i_L(t) - i_o(t)$$

$$\frac{Cdv_o(t)}{dt} = i_L(t) - \frac{v_o(t)}{R}$$

$$\frac{dv_o(t)}{dt} = \frac{i_L(t)}{C} - \frac{v_o(t)}{RC}$$

$$[\frac{dv_o(t)}{dt} = \frac{i_L(t)}{C} - \frac{v_o(t)}{RC}] \times (1 - d(t)) \qquad (4)$$

Avg Model: $\boldsymbol{d(t) \times ON\ state\ variable + \big(1 - d(t)\big) \times OFF\ state\ variable}$

- **From Eqns (1) and (3)**

$$\frac{di_L(t)}{dt}d(t) + \frac{di_L(t)}{dt}\big(1 - d(t)\big) = \frac{v_{in}(t)\,d(t)}{L} - \frac{v_o(t)\,d(t)}{L} - \frac{r_L i_L(t) d(t)}{L} - \frac{v_o(t)}{L}(1 - d(t)) - \frac{v_D(t)}{L}(1 - d(t)) - \frac{r_L i_L(t)}{L}(1 - d(t))$$

$$\frac{di_L(t)}{dt} = \frac{v_{in}(t)\, d(t)}{L} - \frac{v_o(t)}{L} - \frac{r_L i_L(t)}{L} - \frac{v_D(t)}{L} + \frac{v_D(t) d(t)}{L} \qquad (5)$$

In Frequency Domain:

$$i_L(s) = \frac{v_{in}(s)\, d(s)}{sL} - \frac{v_o(s)}{sL} - \frac{r_L i_L(s)}{sL} - \frac{v_D(s)}{sL} + \frac{v_D(s) d(s)}{sL}$$

- **From Eqns (2) and (4)**

$$\frac{dv_o(t)}{dt} d(t) + \frac{dv_o(t)}{dt}\left(1 - d(t)\right) = \frac{i_L(t) d(t)}{C} - \frac{v_o(t)\, d(t)}{RC} + [\frac{i_L(t)}{C} - \frac{v_o(t)}{RC}](1 - d(t))$$

$$\frac{dv_o(t)}{dt} = \frac{i_L(t)}{C} - \frac{v_o(t)}{RC} \qquad (6)$$

In Frequency Domain:

$$v_o(\mathrm{s}) = \frac{\mathrm{i_L(s)}}{\mathrm{sC}} - \frac{\mathrm{v_o(s)}}{\mathrm{sRC}}$$

<u>Step 2: Linearization</u>

$$\boxed{\begin{aligned} i_L(t) &= I_L + \tilde{\imath}_L \\ v_o(t) &= V_o + \tilde{v}_o \\ v_{in}(t) &= V_{in} + \tilde{v}_{in} \\ v_D(t) &= V_D + \tilde{v}_D \\ d(t) &= D + \tilde{d} \end{aligned}}$$

- **From Eqn (5)**

$$\frac{di_L(t)}{dt} = \frac{v_{in}(t)\, d(t)}{L} - \frac{v_o(t)}{L} - \frac{r_L i_L(t)}{L} - \frac{v_D(t)}{L} + \frac{v_D(t) d(t)}{L}$$

$$\frac{d(I_L + \tilde{\imath}_L)}{dt} = \frac{(V_{in} + \tilde{v}_{in})\,(D + \tilde{d})}{L} - \frac{V_o + \tilde{v}_o}{L} - \frac{r_L(I_L + \tilde{\imath}_L)}{L} - \frac{V_D + \tilde{v}_D}{L} + \frac{(V_D + \tilde{v}_D)(D + \tilde{d})}{L}$$

$$\frac{dI_L}{dt} + \frac{d\tilde{\imath}_L}{dt} = \frac{V_{in}D}{L} + \frac{V_{in}\tilde{d}}{L} + \frac{\tilde{v}_{in}\, D}{L} + \frac{\tilde{v}_{in}\tilde{d}}{L} - \frac{V_o}{L} - \frac{\tilde{v}_o}{L} - \frac{r_L I_L}{L} - \frac{r_L \tilde{\imath}_L}{L} - \frac{V_D}{L} - \frac{\tilde{v}_D}{L} + \frac{V_D D}{L} + \frac{V_D \tilde{d}}{L} + \frac{\tilde{v}_D D}{L} + \frac{\tilde{v}_D \tilde{d}}{L}$$

Removing the S.S and small values,

$$\frac{d\tilde{\imath}_L}{dt} = \frac{V_{in}\tilde{d}}{L} + \frac{\tilde{v}_{in}\, D}{L} - \frac{\tilde{v}_o}{L} - \frac{r_L\tilde{\imath}_L}{L} - \frac{\tilde{v}_D}{L} + \frac{V_D\tilde{d}}{L} + \frac{\tilde{v}_D D}{L} \tag{7}$$

- **From Eqn (6)**

$$\frac{\mathrm{dv_o(t)}}{\mathrm{dt}} = \frac{\mathrm{i_L(t)}}{\mathrm{C}} - \frac{\mathrm{v_o(t)}}{\mathrm{RC}}$$

$$\frac{\mathrm{d}(V_o + \tilde{v}_o)}{\mathrm{dt}} = \frac{I_L + \tilde{\imath}_L}{\mathrm{C}} - \frac{V_o + \tilde{v}_o}{\mathrm{RC}}$$

$$\frac{\mathrm{d}V_o}{\mathrm{dt}} + \frac{\mathrm{d}\tilde{v}_o}{\mathrm{dt}} = \frac{I_L}{\mathrm{C}} + \frac{\tilde{\imath}_L}{\mathrm{C}} - \frac{V_o}{\mathrm{RC}} - \frac{\tilde{v}_o}{\mathrm{RC}}$$

Removing the S.S values,

$$\frac{\mathrm{d}\tilde{v}_o}{\mathrm{dt}} = \frac{\tilde{\imath}_L}{\mathrm{C}} - \frac{\tilde{v}_o}{\mathrm{RC}} \tag{8}$$

<u>Step 3: Transfer Function</u>

The state space equations of the buck converter in the form $\dot{x} = Ax + Bu$ and $y = Cx$ are

$$\begin{bmatrix} \frac{d\tilde{\imath}_L}{dt} \\ \frac{\mathrm{d}\tilde{v}_o}{\mathrm{dt}} \end{bmatrix} = \begin{bmatrix} \frac{r_L}{L} & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} \tilde{\imath}_L \\ \tilde{v}_o \end{bmatrix} + \begin{bmatrix} \frac{V_{in} + V_D}{L} & \frac{D}{L} & \frac{D-1}{L} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{d} \\ \tilde{v}_{in} \\ \tilde{v}_D \end{bmatrix}$$

$$[i_L] = [1 \quad 0] \begin{bmatrix} \tilde{\imath}_L \\ \tilde{v}_o \end{bmatrix}$$

From the matrices, the transfer function (TF) can be obtained as follows.

$$TF = \frac{\mathcal{L}\ output}{\mathcal{L}\ input} = \frac{\tilde{\imath}_L}{[u]} = \begin{bmatrix} \frac{\tilde{\imath}_L}{\tilde{d}} & \frac{\tilde{\imath}_L}{\tilde{v}_{in}} & \frac{\tilde{\imath}_L}{\tilde{v}_D} \end{bmatrix} = C[sI - A]^{-1}B$$

$$TF = [1 \quad 0] \times [\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} \frac{r_L}{L} & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix}]^{-1} \times \begin{bmatrix} \frac{V_{in} + V_D}{L} & \frac{D}{L} & \frac{D-1}{L} \\ 0 & 0 & 0 \end{bmatrix}$$

$$= [1 \quad 0] \times \begin{bmatrix} s - \frac{r_L}{L} & \frac{1}{L} \\ \frac{-1}{C} & s + \frac{1}{RC} \end{bmatrix}^{-1} \times \begin{bmatrix} \frac{V_{in} + V_D}{L} & \frac{D}{L} & \frac{D-1}{L} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} s - \frac{r_L}{L} & \frac{1}{L} \\ \frac{-1}{C} & s + \frac{1}{RC} \end{bmatrix}^{-1} = \frac{1}{s^2 + s\left(\frac{1}{RC} - \frac{r_L}{L}\right) - \frac{r_L}{LRC} + \frac{1}{LC}} \begin{bmatrix} s + \frac{1}{RC} & \frac{-1}{L} \\ \frac{1}{C} & s - \frac{r_L}{L} \end{bmatrix}$$

$$TF = [1 \quad 0] \times \frac{1}{s^2 + s\left(\frac{1}{RC} - \frac{r_L}{L}\right) - \frac{r_L}{LRC} + \frac{1}{LC}} \begin{bmatrix} s + \frac{1}{RC} & \frac{-1}{L} \\ \frac{1}{C} & s - \frac{r_L}{L} \end{bmatrix} \times \begin{bmatrix} \frac{V_{in} + V_D}{L} & \frac{D}{L} & \frac{D-1}{L} \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \frac{1}{s^2 + s\left(\frac{1}{RC} - \frac{r_L}{L}\right) - \frac{r_L}{LRC} + \frac{1}{LC}} \times [1 \quad 0] \times \begin{bmatrix} s + \frac{1}{RC} & \frac{-1}{L} \\ \frac{1}{C} & s - \frac{r_L}{L} \end{bmatrix} \times \begin{bmatrix} \frac{V_{in} + V_D}{L} & \frac{D}{L} & \frac{D-1}{L} \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \frac{1}{s^2 + s\left(\frac{1}{RC} - \frac{r_L}{L}\right) - \frac{r_L}{LRC} + \frac{1}{LC}} \times \begin{bmatrix} s + \frac{1}{RC} & \frac{-1}{L} \end{bmatrix} \times \begin{bmatrix} \frac{V_{in} + V_D}{L} & \frac{D}{L} & \frac{D-1}{L} \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \frac{1}{s^2 + s\left(\frac{1}{RC} - \frac{r_L}{L}\right) - \frac{r_L}{LRC} + \frac{1}{LC}} \times \begin{bmatrix} (\frac{(V_{in} + V_D)s}{L} + \frac{V_{in} + V_D}{LRC}) & (\frac{Ds}{L} + \frac{D}{LRC}) & (\frac{s(D-1)}{L} + \frac{D-1}{LRC}) \end{bmatrix} = \begin{bmatrix} \frac{\tilde{\imath}_L}{\tilde{d}} & \frac{\tilde{\imath}_L}{\tilde{v}_{in}} & \frac{\tilde{\imath}_L}{\tilde{v}_D} \end{bmatrix}$$

Hence,

$$\left[\frac{\tilde{\imath}_L(s)}{\tilde{d}(s)}\right] = \frac{1}{s^2 + s\left(\frac{1}{RC} - \frac{r_L}{L}\right) - \frac{r_L}{LRC} + \frac{1}{LC}} \left[(\frac{(V_{in} + V_D)s}{L} + \frac{V_{in} + V_D}{LRC})\right]$$

Component Calculations

- **Duty Cycle**

While considering the non-idealities, the gain of the buck converter can be derived and found to be,

$$\frac{V_o}{V_{in}} = \frac{D\left(1+\frac{V_D}{V_{in}}\right)-\frac{V_D}{V_{in}}}{(1+\frac{r_L}{R})}$$

However, for simplicity, the ideal buck converter gain equation will be used to find the duty cycle.

$$\frac{V_o}{V_{in}} = D$$

Hence, the duty cycle,

$$D = \frac{16}{20} = 0.8$$

- **Resistor Value**

$$P = \frac{{V_o}^2}{R}$$

$$R = \frac{{V_o}^2}{P} = \frac{16^2}{100} = 2.56\Omega$$

- **Inductor Value**

Assuming the inductor current ripple to be 30% of the average inductor current ($I_L = I_O$),

$$\Delta i_L = 0.3 \times \frac{16}{2.56} = 1.875\ A$$

Hence,

$$L = \frac{V_o(1-D)}{\Delta i_L f_s}$$

$$= \frac{(16)(1-0.8)}{(1.875)(20k)} = 85.33\mu H$$

- **Capacitor Value**

$$C = \frac{1-D}{8L(\frac{\Delta V_o}{V_o}){f_s}^2}$$

Assuming a 5% voltage ripple,

$$C = \frac{1-0.8}{8(85.3\mu)(0.05)(20k)^2} = 14.65\mu F$$

<u>Poles and Zeros</u>

Assuming $V_D = 0.5V$, and substituting the calculated values in the obtained Transfer Function, the following function is obtained.

$$\frac{\tilde{\imath}_L(s)}{\tilde{d}(s)} = \frac{0.24e06\ s + 6.4057e09}{s^2 + 26.37e03\ s + 7.921e08}$$

$$= \frac{20.5\ s\ +\ 5.466e05}{8.533e-05\ s^2\ +\ 2.25\ s\ +\ 6.759e04}$$

Using MATLAB, the poles and zeros of the transfer function were obtained.

- **Poles:**

$$\mathbf{1.0e+04} * (\mathbf{-1.3185 \pm 2.4865}\boldsymbol{i})$$

- **Zero:**

  One zero at $s = -2.6664e + 04$.

Since the poles lie on the left hand side the system is stable.

Part 2:

Develop a closed loop-controlled simulation model of above converter. Use an appropriate PI and PID controllers. Provide necessary simulation results for the steady-state and dynamic conditions (input voltage change and load change). Discuss the performance achieved with PI and PID controllers.

Applying the same steps as in part 1, the transfer function of the plant was found in terms of $\frac{\tilde{v}_O(s)}{\tilde{d}(s)}$ as follows,

$$G_P(s) = \frac{\tilde{v}_O(s)}{\tilde{d}(s)} = \frac{1}{s^2 + s\left(\frac{1}{RC} - \frac{r_L}{L}\right) - \frac{r_L}{LRC} + \frac{1}{LC}} \times \frac{V_{in} + V_D}{LC}$$

After substituting the values,

$$G_P = \frac{1.64e10}{s^2 + 2.637e04\ s + 7.921e08}$$

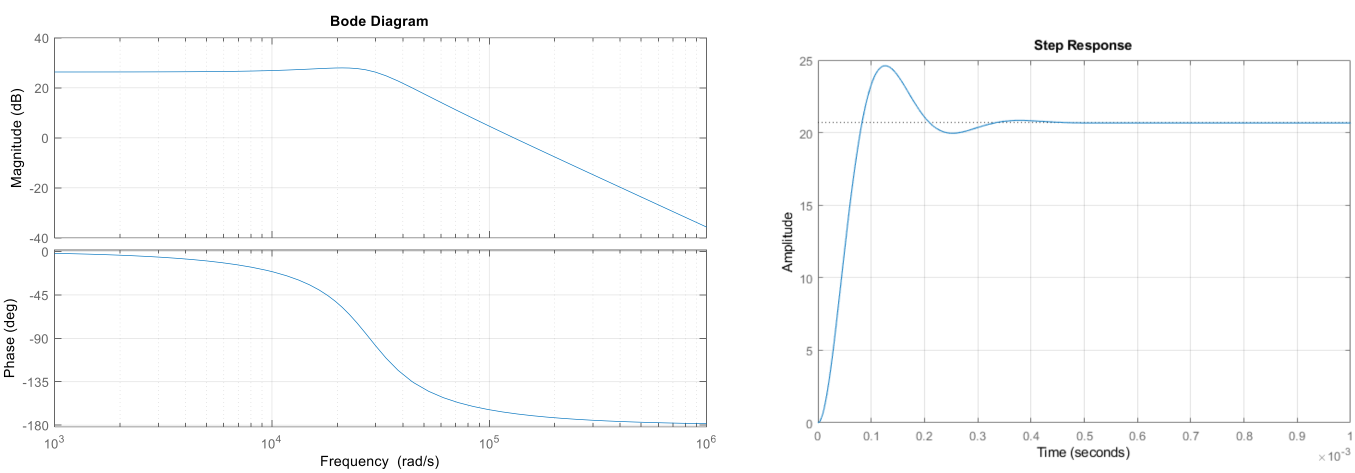


Figure 2: Bode Diagram and open loop step response of Gp

Open Loop Analysis

Using MATLAB/Simulink, the following model for a buck converter was built. The switching pulses were generated using simple PWM techniques.

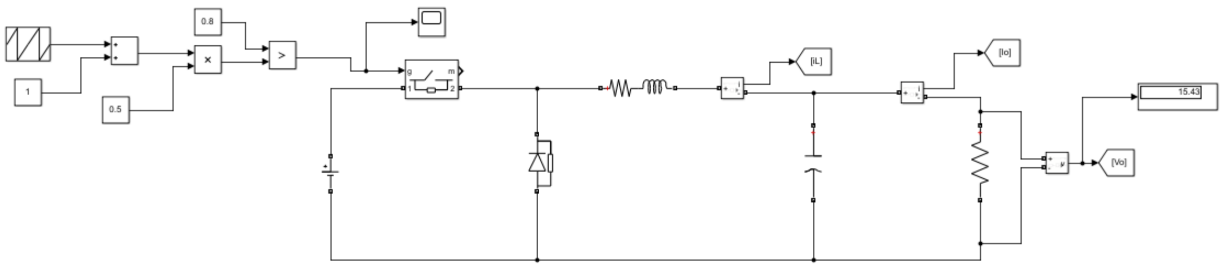


Figure 3: Open Loop Buck Converter

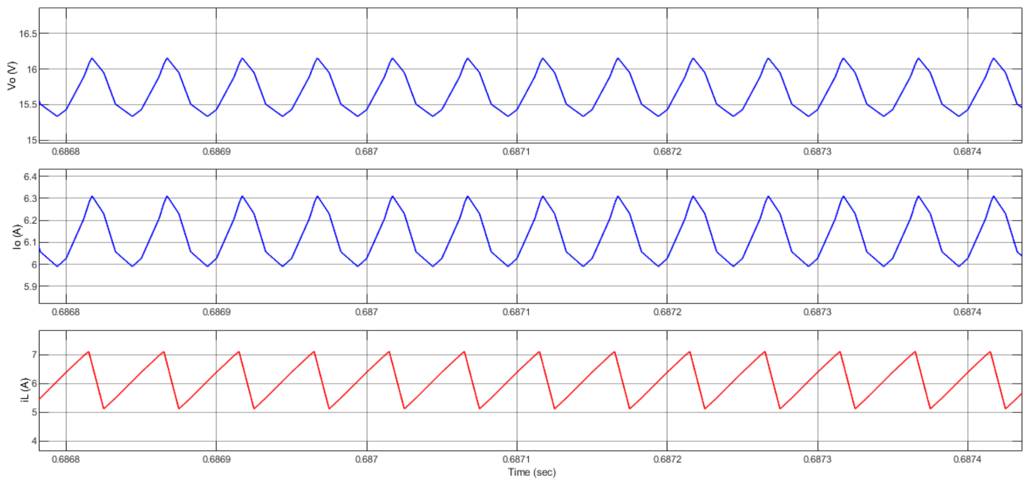


Figure 4: Output Voltage, Output Current and Inductor Current

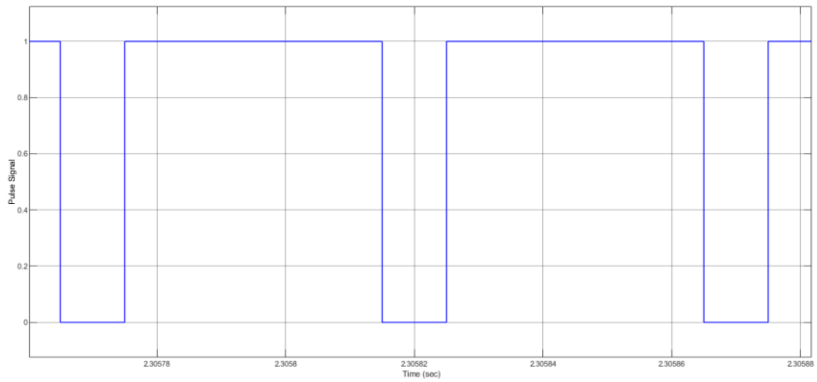


Figure 5: Generated Pulse Signal D=0.8

Next, the average model for the buck converter was built using Eqn (5). The non-idealities were considered while building the average model. The results from the average model were compared with that from the Simulink model.

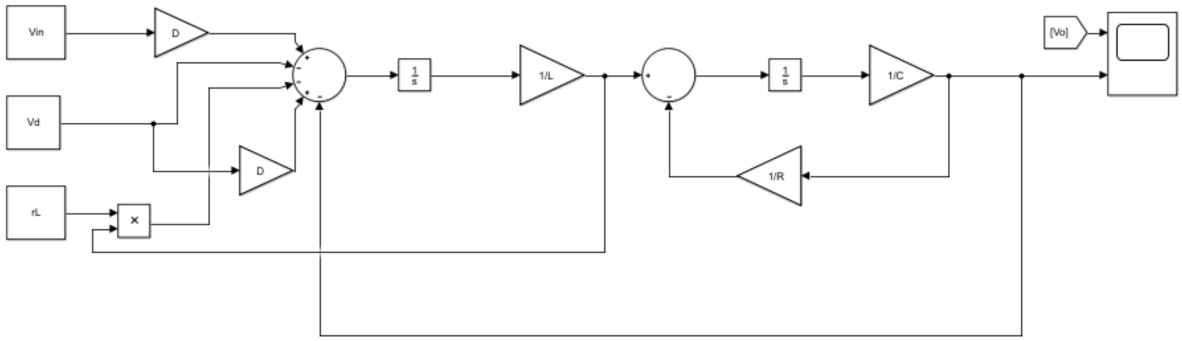


Figure 6: Average Model of Buck Converter

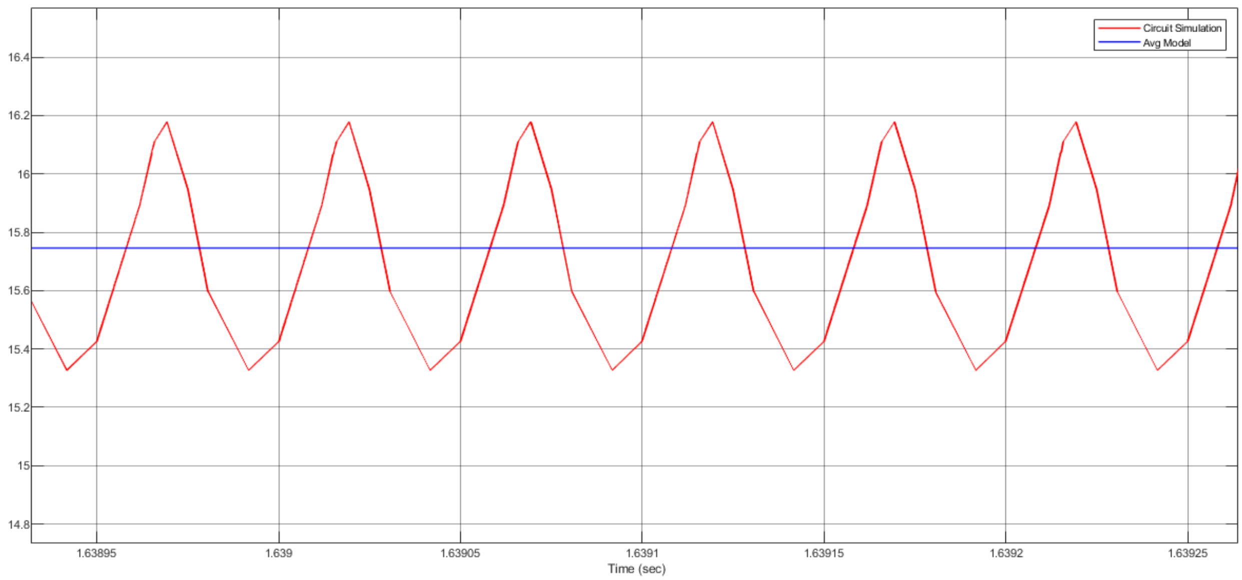


Figure 7: Output Voltage from Circuit and Avg Model

<u>Note:</u> If the value of 'D' was calculated while considering the non-idealities, the output voltage produced by the average model would be 16V as specified in the question. However, because a smaller value of 'D' is used, the output voltage is slightly less than 16V.

PI Controller Design

Using Sisotool

Initially, using the sisotool feature available in MATLAB, the PI controller was designed and the transfer function of the compensator was obtained. The compensator gain was modified and the step response of the system was observed until an acceptable response was obtained.

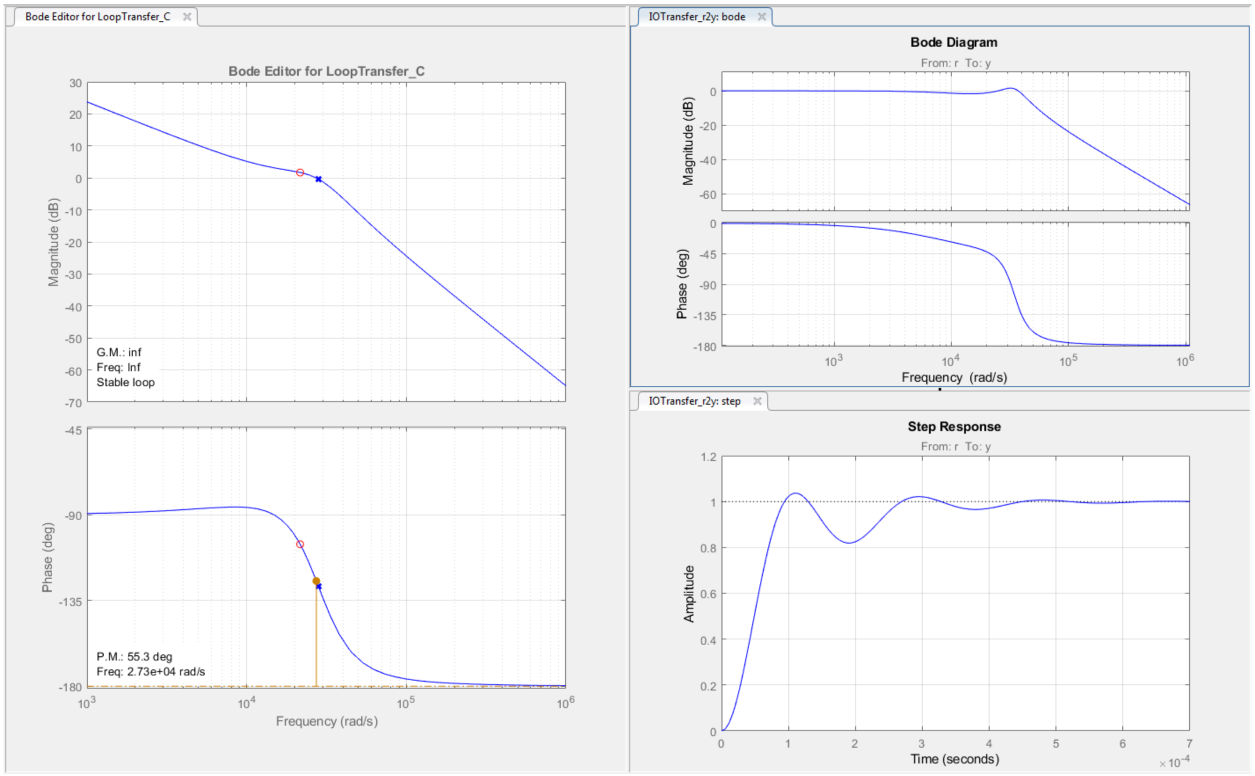


Figure 8: PI Controller Design

Hence, the transfer function of the PI controller was obtained after tuning the parameters as follows,

$$G_C = \frac{0.03451(s + 2.162e04)}{s}$$

However, the obtained integral gain was large ($K_i = 746$), and the system response was extremely fast, so instead, the student opted for a simple trial and error method for the PI controller tuning to achieve better observable results.

Using Trial and Error

Using trial and error, the PI controller gains were modified and the step response of the closed loop system was observed using MATLAB. Moreover, using the 'stepinfo' function, the parameters of the step response (overshoot, settling time, rise time, etc.) were analyzed. Hence, after multiple attempts, it was decided to go with a PI controller with $K_P = 0.03$ and $K_i = 10$.

Hence,

$$G_C = \frac{0.03s + 10}{s}$$

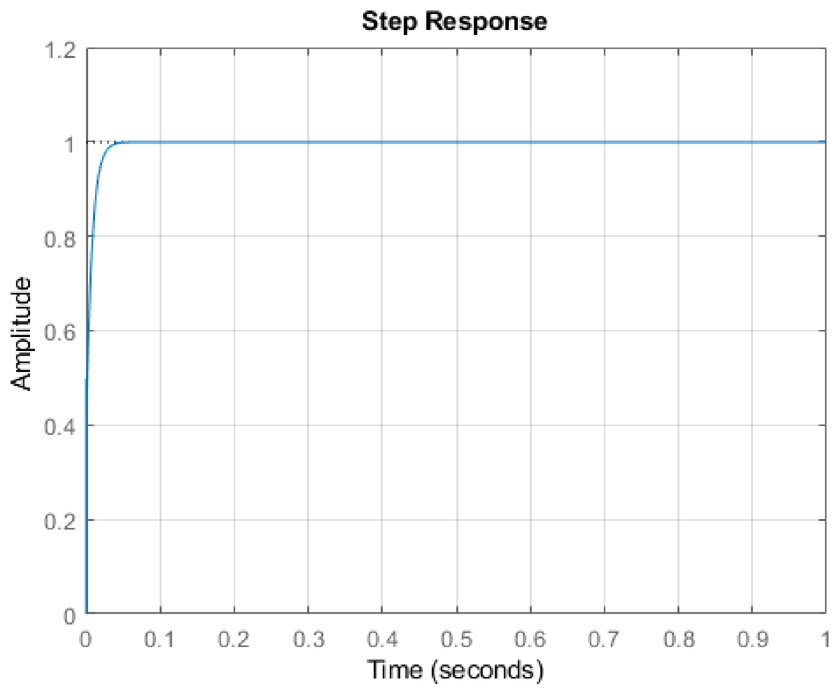


Figure 9: Closed Loop step response of system with controller

Closed Loop Analysis with PI controller

Voltage Change

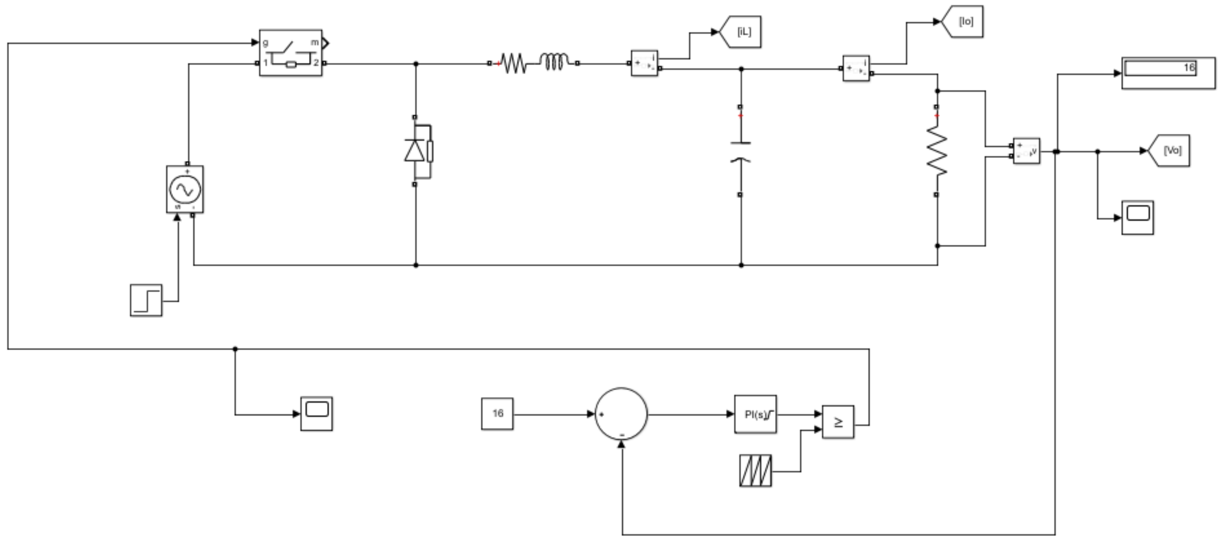


Figure 10: Closed Loop control of Buck Converter (PI)

Figure 10 shows the model built for the closed loop control of a buck converter. The output voltage was subtracted from the reference signal of 16V, to generate the error signal. The error signal was then fed to the PI controller (G_C) and compared with a sawtooth wave to generate the pulse signals for the switch. A controlled voltage source was used to vary the input voltage and observe the system response. The input voltage was changed from 20V to 30V after 1 sec.

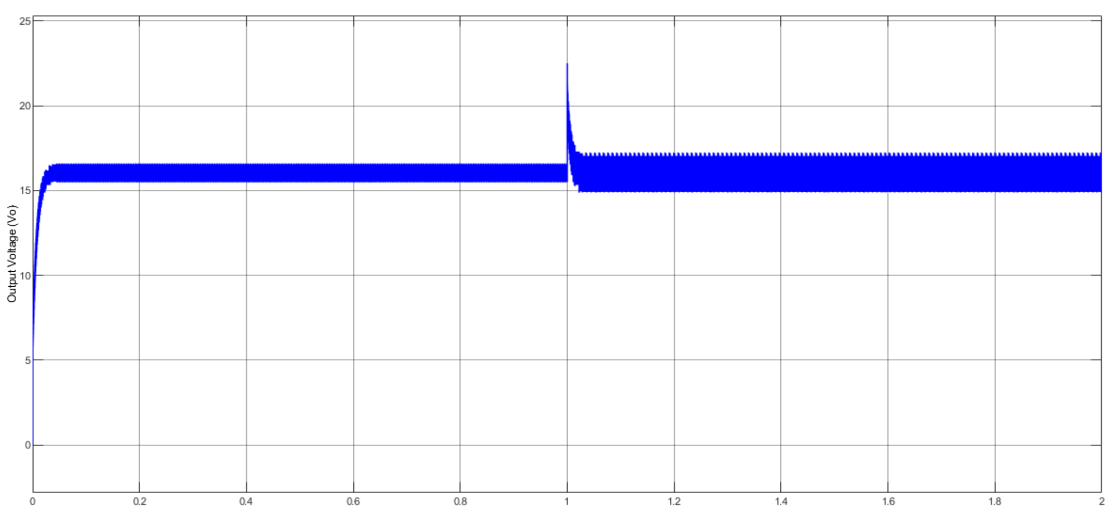


Figure 11: Output Voltage

As can be seen from Figure 11, the system follows the reference value of 16V. When the voltage was changed to 30V at t = 1 sec, an overshoot was observed, but the system was able to regulate the output voltage back to 16V. However, larger ripple can be observed when the input voltage is 30V when compared to 20V.

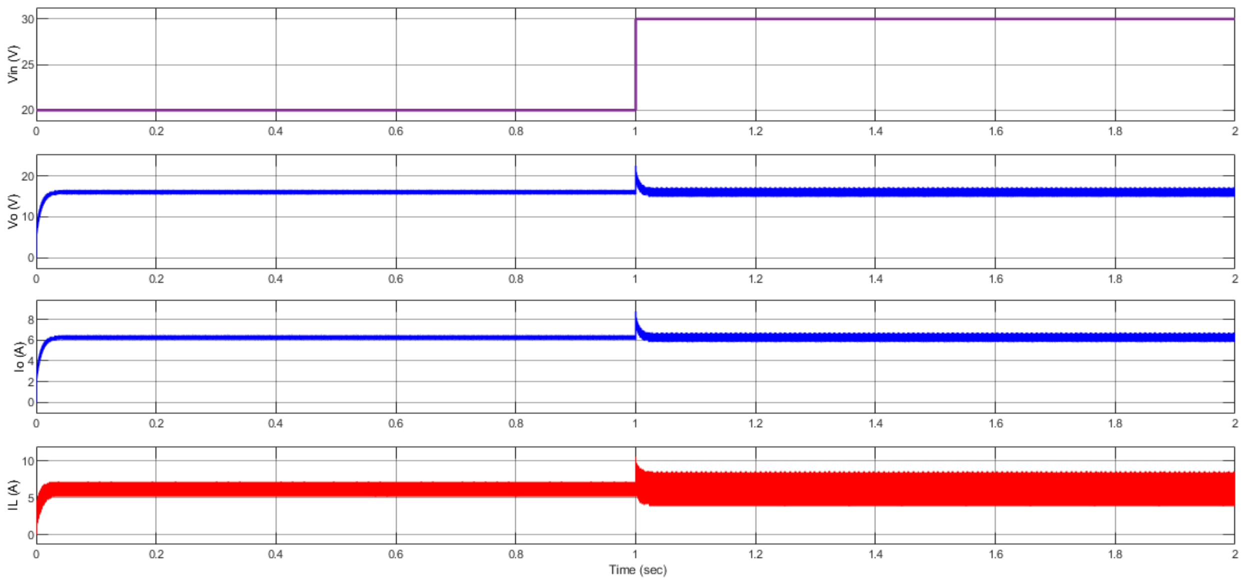


Figure 12: Voltage and Current Waveforms

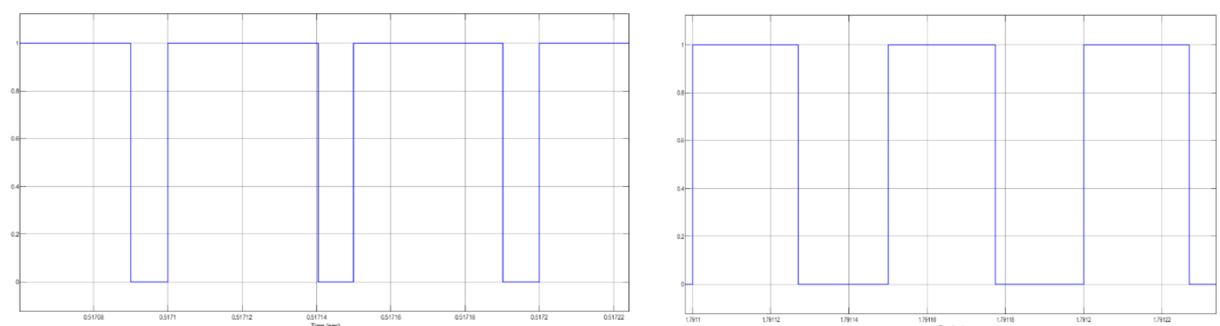


Figure 13: Pulse Signals before and after Vin change

As can be observed from Figure 13, before t = 1 sec (V_{in} = 20V), the pulse signals were generated for a duty cycle of D = 0.8. However, when V_{in} was increased to 30V, to maintain the output voltage at 16V, the duty cycle was changed to around, D = 0.53 by the operation of controller.

Next, to further verify the operation of the closed loop control of the converter. The effect of the load on the system performance was analyzed. In one case, the load value was increased, while in the other the load value was decreased.

Increasing the Load

In the first case, the value of the load was increased from 2.56Ω to 5Ω. The results can be seen below.

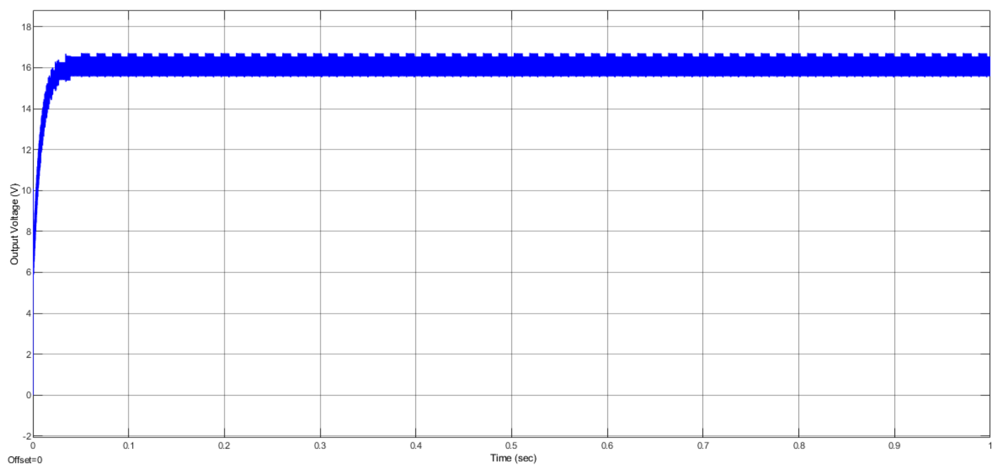


Figure 14: Output Voltage (R=5Ω)

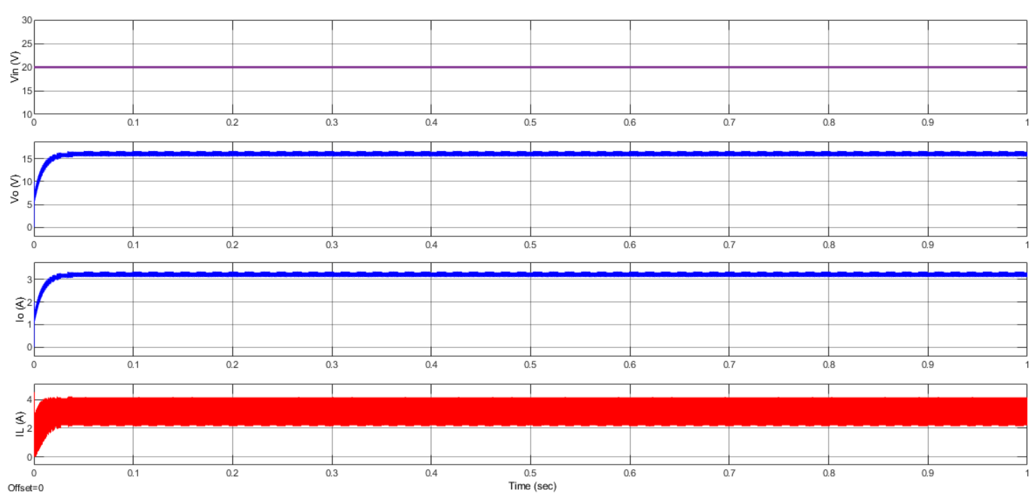


Figure 15: Voltage and Current waveforms (R=5Ω)

Decreasing the load

In this case, the load value was changed from 2.56Ω to 1Ω. The results obtained can be seen below.

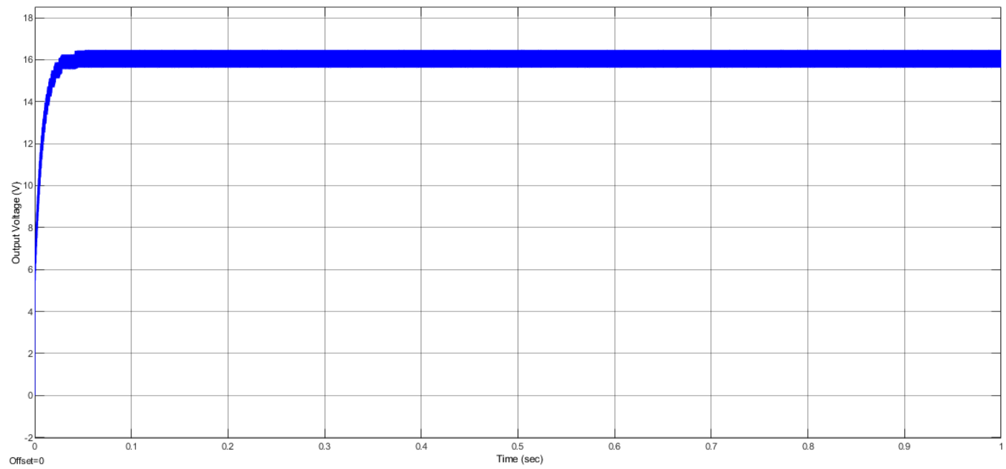


Figure 16: Output Voltage (R=1Ω)

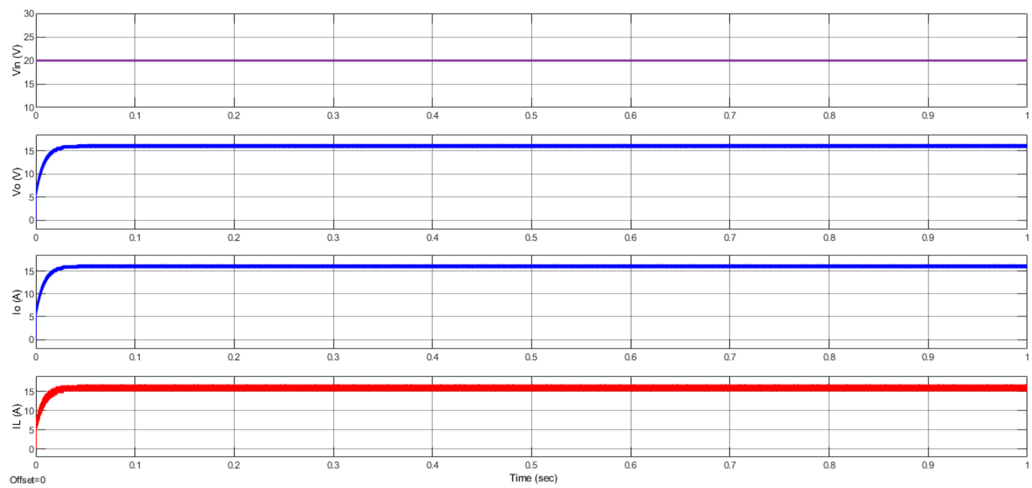


Figure 17: Voltage and Current Waveforms (R=1Ω)

It was observed that when the load value changes, the output current value changes. The change is governed by Ohm's law ($I = \frac{V_O}{R}$). Since the converter is voltage controlled, the duty cycle remains the same at D=0.8. If the converter was controlled based on the current (current mode control), the operation would be different.

Inductor Current Control (Results)

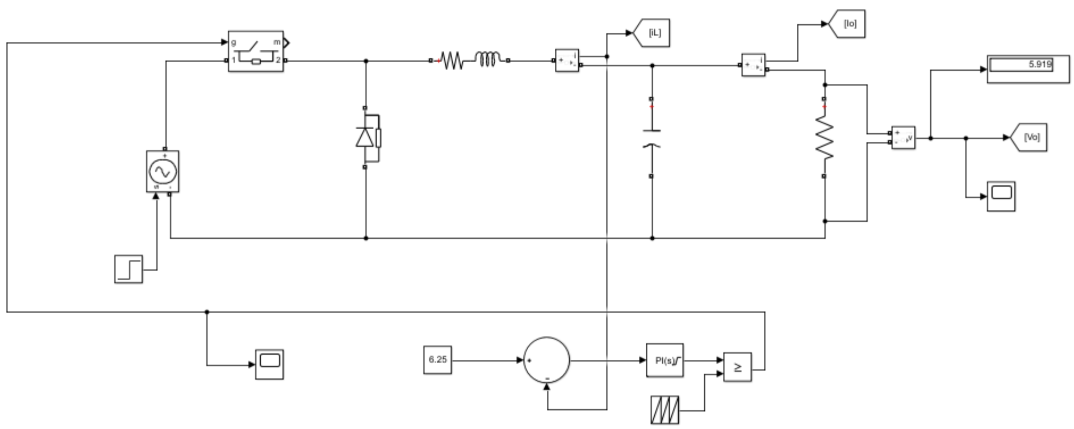


Figure 18: Current Control of PI controller

Input Voltage Change (20V to 30V)

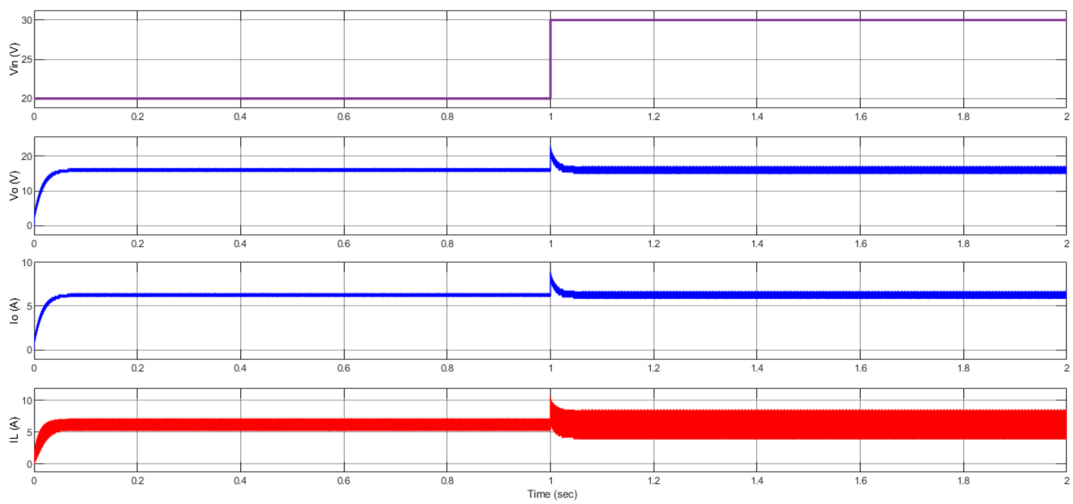


Figure 19: Voltage and Current Waveforms

From Figure 19, it can be seen that the system follows the reference value of 6.25. The operation is similar to the one explained in the voltage control. The duty cycle is reduced to maintain the inductor current.

Load Change

Increasing the load (R = 5 Ω)

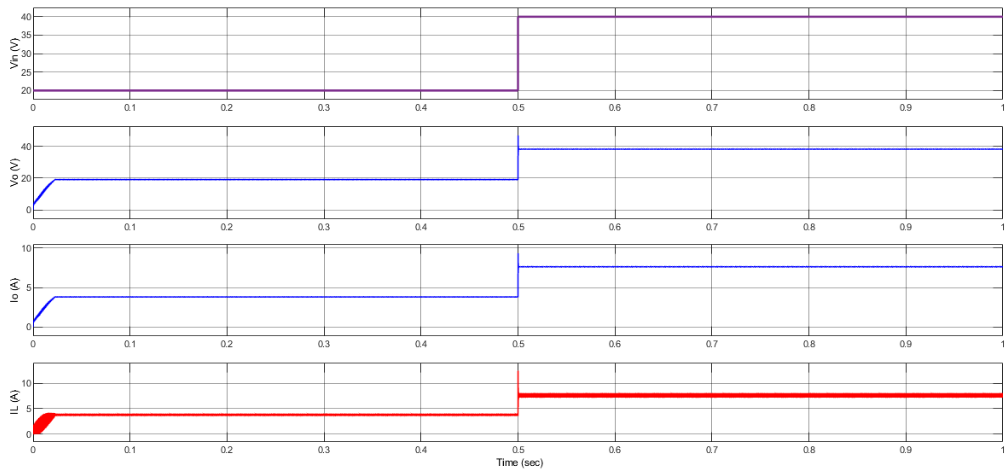


Figure 20: Voltage and Current waveforms (R=5Ω)

As can be seen in Figure 20, when the load is increased to 5Ω, the inductor current tries to follows the reference. However, for a buck converter, the output voltage cannot exceed the input voltage. Hence, the current is limited to 4 A, by the maximum output voltage (20V). Since, ($I = \frac{V_o}{R}$). After t = 0.5 sec, the input voltage was then increased to 40V, the input current increases and settles at the reference value.

Decreasing the load (R=1Ω)

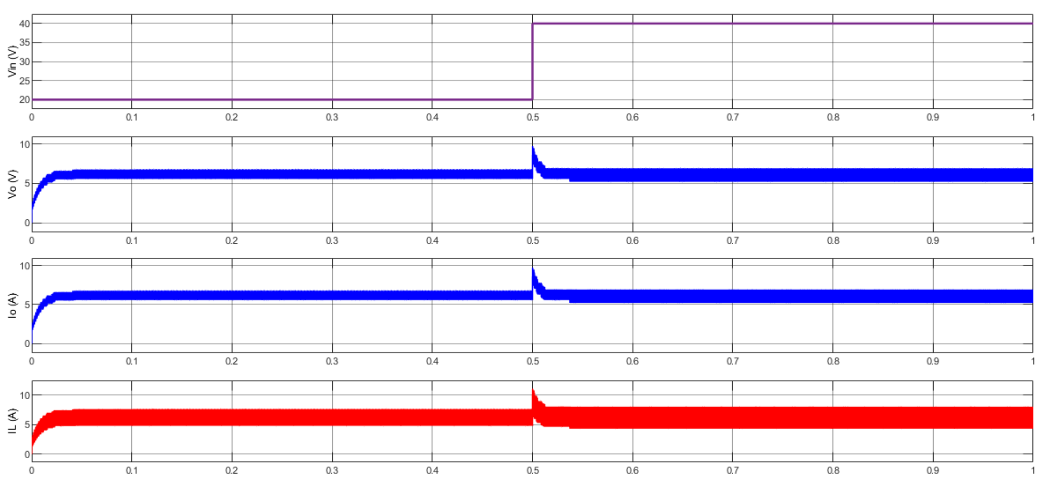


Figure 21: Voltage and Current waveforms (R=1Ω)

As can be seen in Figure 21, the inductor current follows the reference signal regardless of the input voltage value. The output voltage produced is 6.25V.

When comparing both Figures 20 and 21, it can be observed that a larger ripple can be observed when the load is smaller.

PID Controller Design

A PID controller was used for the buck converter closed loop control. The system performance with the controller was then compared with the one obtained using the PI controller discussed above. The PID was tuned using MATLAB sysotool as well as PID tuner app from Simulink. The MATLAB sysotool uses the system transfer function, which was obtained in part one, while the Simulink PID tuner uses the system Simulink model given in figure 10. After tuning the PID and obtaining its proportional(kp), integral(ki), and derivative (kd) gains, the converter closed loop step responses were generated and compared. The results showed that the PID constant gains obtained using the sisotool were better. The transfer function of the controller and the corresponding PID gains obtained using the sysotool are given below in table 1. Figure 22 shows the step response of the closed loop response of the buck converter using the tuned PID controller.

$$G_{C\_PID} = \frac{162.8(s + 4.179e4)\,(s + 1.499e4)}{s(s + 1.797e7)}$$

Table 1: PID parameters

Ki	Kp	Kd	Tf
0.514	5.67e+03	9.03e-06	5.57e-08

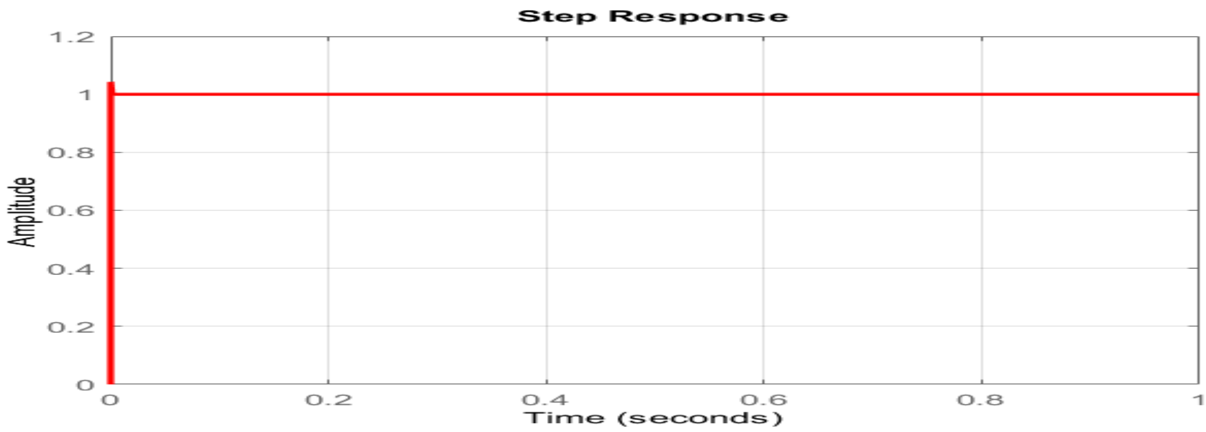


Figure 22:Closed Loop step response of system with controller tunned with sisotool

After tuning the PID controller, the closed loop response of the system was analyzed for varying input voltage and variable load resistor.

Closed Loop Analysis with PID controller

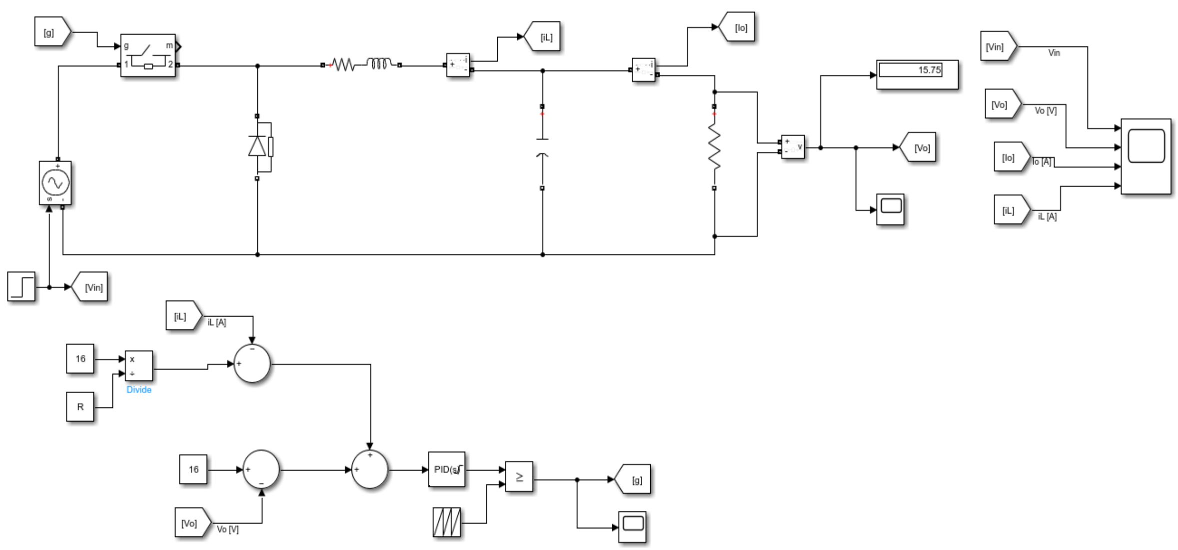


Figure 23: Closed Loop control of Buck Converter (PID)

The controller shown in figure 10 was modified as illustrated in Figure 23. In the modified controller, the output current was subtracted from the average current set point (Vo/Ro) to obtain the current error, which then was added with the voltage error and fed to the PID. This achieved better system response with very small voltage and current ripple as it will be discussed for the varying load and input voltage conditions.

Converter Response to Input Voltage Variation

As shown in figure 24, the input voltage was varied from 20V to 30V at t=1 s, and the controller regulated the output voltage at the set point (16 V) despite the change. A small current ripple can be seen in the inductor and output current, but it's very small compared to the one obtained with the PI controller.

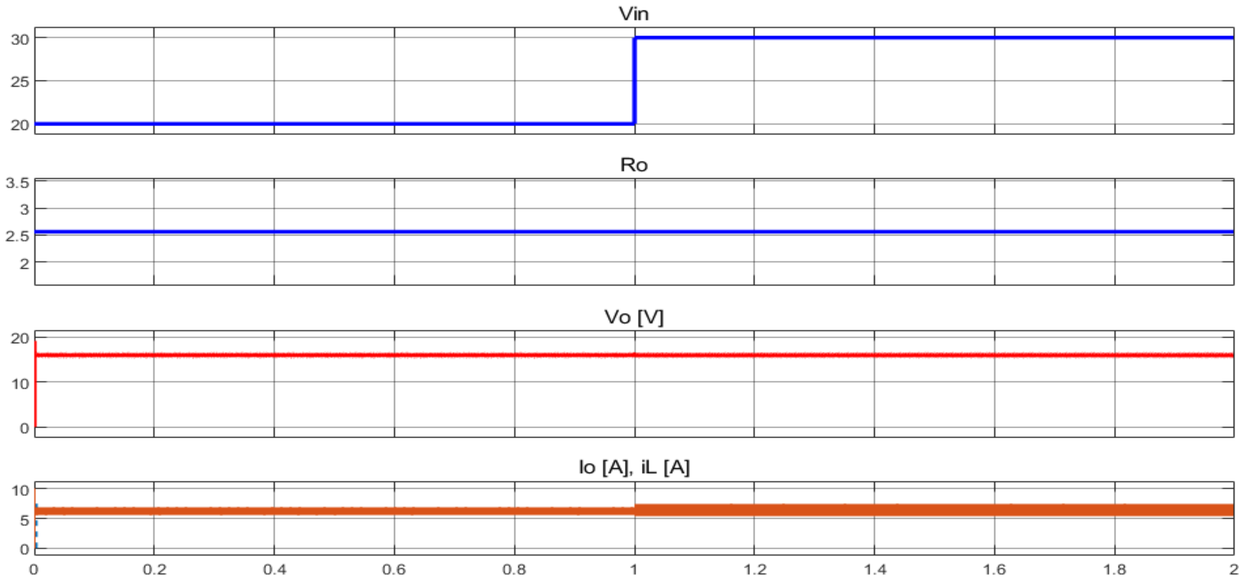


Figure 24: Voltage and current waveforms for varying input voltage

Converter Response to Load change

The system performance using the PID controller was also analyzed under varying load conditions by changing the resistor value from 2.56 Ω to 5 Ω at 0.745 s and from 5 Ω to 1.5 Ω at 1.255s to see the system response for increasing and decreasing load value. And as it can be seen from figure 25, the PID controller regulated the output voltage at the set point (16 V) very well, except for a small spike when the additional load was reduced from 5 Ω to 1.5 Ω at t = 1.255s.

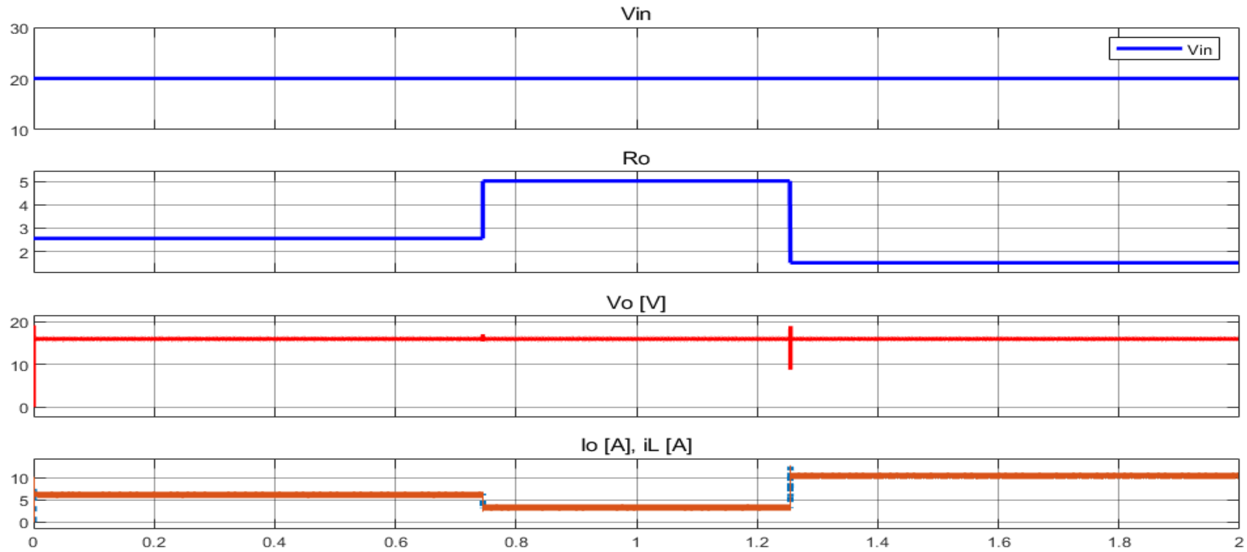


Figure 25: Voltage and current waveforms for varying load resistor

Discussions

The characteristics of the buck converter closed loop step responses shown in figures 9 and 22, using the PI and PID controller respectively, were compared as shown in table controller were compared based on the closed loop step response characteristics as shown in table 2 below. As it can be seen from the table, the PI controller took 0.0142 s to settle to the steady state, which is much slower compared to the PID, where the settling time was 0.0000153 s. However, the PID has higher overshoot (4.3 %) compared to the PI controller (0% overshoot). The highlighted difference are also noticeable in the voltage and current out put characteristics for the input change, as the PI takes some time to settle to the setpoint after the input voltage, while the PID PID returns to the set point voltage at unnoticeable speed as shown in figures 12 and 24.

Table 2: characteristics of buck converter closed loop step response using PI and PID controllers

Closed loop step response characteristics	PI controller	PID controller
Overshoot	0.0 %	4.3 %
Undershot	0.0 %	0.0%
Rise time	0.0142 s	1.53e-5 s
Settling time	0.0268 s	1.37e-4 s
Steady state value	1	1

Appendix

Buck converter closed loop step response using PI controller

```
clear

rL=0.025;
Vin=20;
Vo=16;
Vd=0.5;
L=85.33e-6;
C1=14.65e-6;
R=2.56;
D=0.8;
s=tf('s');
num=((Vin+Vd)*s)/L+(Vin+Vd)/(L*R*C1);
den1=s^2+((1/(R*C1))-(rL/L))*s+(1/(L*C1))*(1-rL/R);
Gp=num/den1;

num1=(Vin+Vd)/(L*C1);
den1=s^2+((1/(R*C1))-(rL/L))*s+(1/(L*C1))*(1-rL/R);
Gp2=num1/den1;

kp= 0.03;      % 0.003;
ki= 10;   % 28.31;
kd = 9.03e-06;  % 0.0001;
Tf=  5.57e-08;  % 100;
Gc_pi= 0.03 + 10/s;
step(feedback(Gp2*Gc_pi,1),1)
stepinfo(step(feedback(Gp2*Gc_pi,1),1))
```

Buck converter closed loop step response using PID controller

```
clear
rL=0.025;
Vin=20;
Vo=16;
Vd=0.5;
L=85.33e-6;
C1=14.65e-6;
R=2.56;
D=0.8;
s=tf('s');
num=((Vin+Vd)*s)/L+(Vin+Vd)/(L*R*C1);
den1=s^2+((1/(R*C1))-(rL/L))*s+(1/(L*C1))*(1-rL/R);
Gp=num/den1;

num1=(Vin+Vd)/(L*C1);
den1=s^2+((1/(R*C1))-(rL/L))*s+(1/(L*C1))*(1-rL/R);
Gp2=num1/den1;
```

```
kp= 0.514;       % 0.003;
ki= 5.67e+03;    % 28.31;
kd = 9.03e-06;  % 0.0001;
Tf=  5.57e-08;  % 100;
Gc_pid= kp + (ki/s) + (kd*s)/(Tf*s +1);

step(feedback(Gp2*Gc_pid,1),1)
stepinfo(step(feedback(Gp2*Gc_pid,1),1))
```