Zeeman transitions of ^{23}Na in an external magnetic field

A theoretical description

Rodolphe MOMIER

Laboratoire Interdisciplinaire Carnot de Bourgogne, UMR CNRS 6303, Université Bourgogne Franche-Comté, 21000 Dijon, France

Institute for Physical Research, National Academy of Sciences of Armenia, Ashtarak-2, 0203 Republic of Armenia

Theoretical model

Interaction with the magnetic field

Slide 1

Zero-field Hamiltonian

Magnetic contribution

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_m$$

$$\mathcal{H}_m = -\frac{\mu_B}{\hbar}(g_L L_z + g_S S_z + g_I I_z)$$

$$\downarrow$$

$$\langle F, m_F|\mathcal{H}|F, m_F\rangle = E_0(F) - \mu_B g_F m_F B$$

$$\begin{aligned}\langle F-1, m_F|\mathcal{H}|F, m_F\rangle &= -\frac{\mu_B B_z}{2}(g_J - g_I)\left(\frac{[(J+I+1)^2 - F^2][F^2 - (J-I)^2]}{F}\right)^{1/2}\left(\frac{F^2 - m_F^2}{F(2F+1)(2F-1)}\right)^{1/2} \\ &= \langle F, m_F|\mathcal{H}|F-1, m_F\rangle\end{aligned}$$

Interaction with the magnetic field

Slide 2

After diagonalisation, one obtains eigenvalues (energy of each Zeeman sub level) and eigenvectors (mixed states due to B) expressed as a function of the unperturbed atomic states

Eigenvectors of $\mathscr{H}$

$$\boxed{|\Psi(F', m_F)\rangle = \sum_F \alpha_{F',F} \, |\Psi(F, m_F)\rangle}$$

Transition intensity

$$A_{eg} \propto a_p^2[|\Psi(F'_e, m_{F_e})\rangle \,;\, |\Psi(F'_g, m_{F_g})\rangle \,;\, q]$$

Transfer coefficient modified by the presence of the magnetic field

$q = 0, \pm 1$ for $\pi, \sigma^{\pm}$ polarisation

$$a_p[|\Psi(F'_e, m_{F_e})\rangle \,;\, |\Psi(F'_g, m_{F_g})\rangle \,;\, q] = \sum_{F'_e} \sum_{F'_g} \alpha_{F_e,F'_e} a_{np}(F_e, m_{F_e}; F_g, m_{F_g}; q) \alpha_{F_g,F'_g}$$

Unperturbed transfer coefficients

$$a_{np}(\psi(F_e, m_{F_e}; \psi(F_g, m_{F_g}); q) = (-1)^{1+I+J_e+F_e+F_g-m_{F_e}}$$

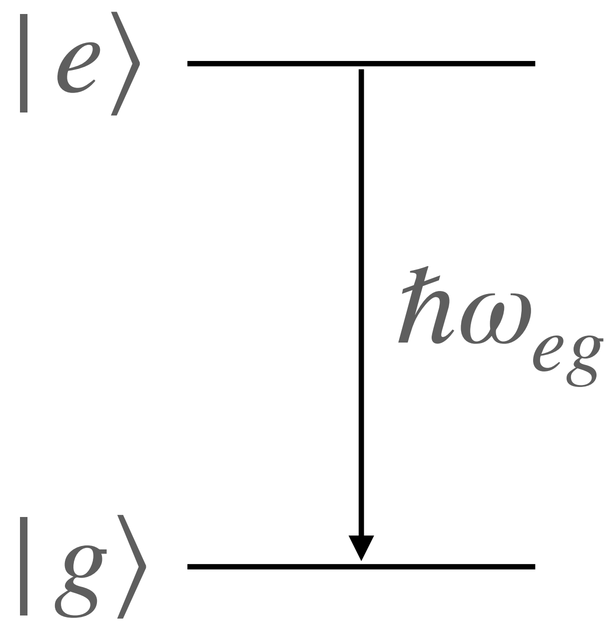
$$\sqrt{2J_e+1}\sqrt{2F_e+1}\sqrt{2F_g+1} \times \boxed{\begin{pmatrix} F_e & 1 & F_g \\ -m_{F_e} & q & m_{F_g} \end{pmatrix} \begin{Bmatrix} F_e & 1 & F_g \\ J_g & I & J_e \end{Bmatrix}}$$

Wigner symbols

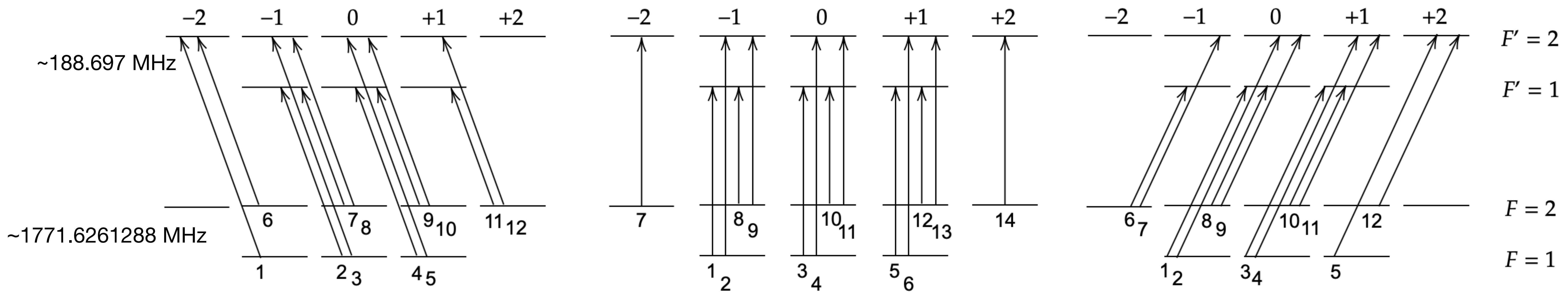
Interaction with the magnetic field

Slide 3

Transition frequency $\omega_{eg} = \dfrac{E_e(B) - E_g(B)}{\hbar}$ where E are the eigenvalues of the Hamiltonian

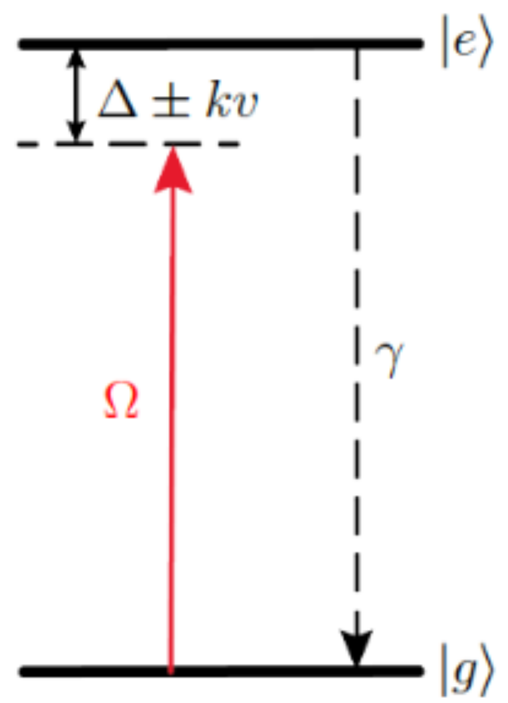


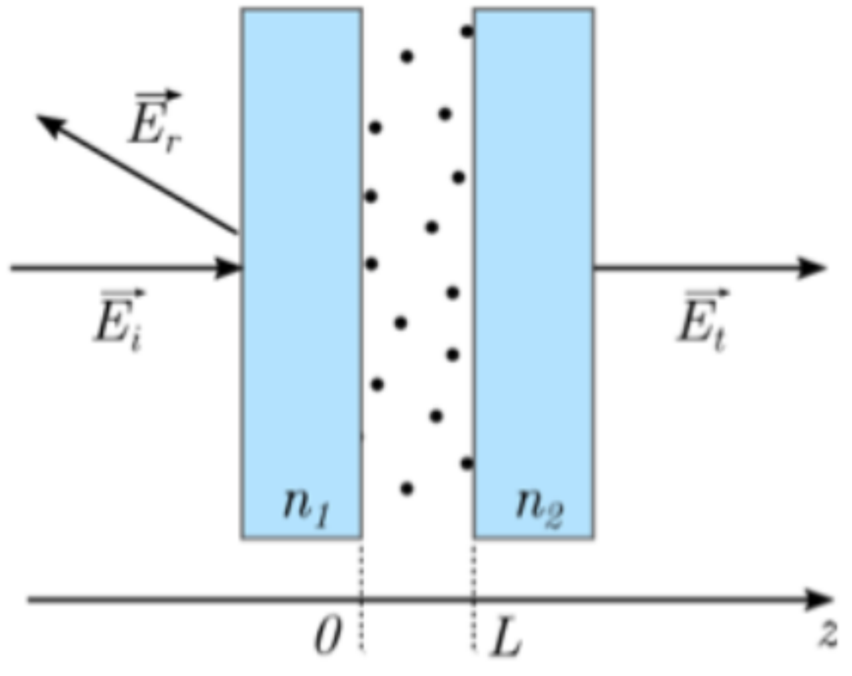
Numerical data	
μ_B (Bohr magneton)	-1.39962449361(42) MHz/G
A_{hf} (Magnetic dipole constant)	885.8130644(5) MHz
g_S	2.00231930436256(35)
g_L	0.99997613
g_I	-0.0008046108(8)



Laser interaction with the nano cell

Transmitted and reflected signal





$$\Lambda_{\pm} = \gamma - i\Delta \pm ikv$$

$$I_T^{lin} = \frac{C}{u\sqrt{\pi}} \int_{-\infty}^{+\infty} \exp\left(-\frac{v^2}{u^2}\right) g(\Delta, v, L) \mathrm{d}v$$

$$I_{SR}^{lin} = \frac{C}{u\sqrt{\pi}} \int_{-\infty}^{+\infty} \exp\left(-\frac{v^2}{u^2}\right) h_{\pm}(\Delta, v, L) \mathrm{d}v$$

$$\longrightarrow$$

$$I_f = I_T^{lin} - r_w I_{SR}^{lin}$$

$$I_b = I_{SR}^{lin} - r_w \exp(2ikL) I_T^{lin}$$

Where we denote $$g(\Delta, v, L) = -\frac{k}{\Lambda_+}\left\{L - \frac{|v|}{\Lambda_+}\left[1 - \exp\left(-\frac{-\Lambda_+ L}{|v|}\right)\right]\right\}$$

$$h_{\pm}(\Delta, v, L) = -\frac{1}{2i}\left[\frac{1}{\Lambda_{\mp}} - \frac{\exp(2ikL)}{\Lambda_p m}\right] - \frac{k|v|}{\Lambda_+\Lambda_-} \exp\left(-\frac{\Lambda_{\mp} L}{|v|}\right)$$

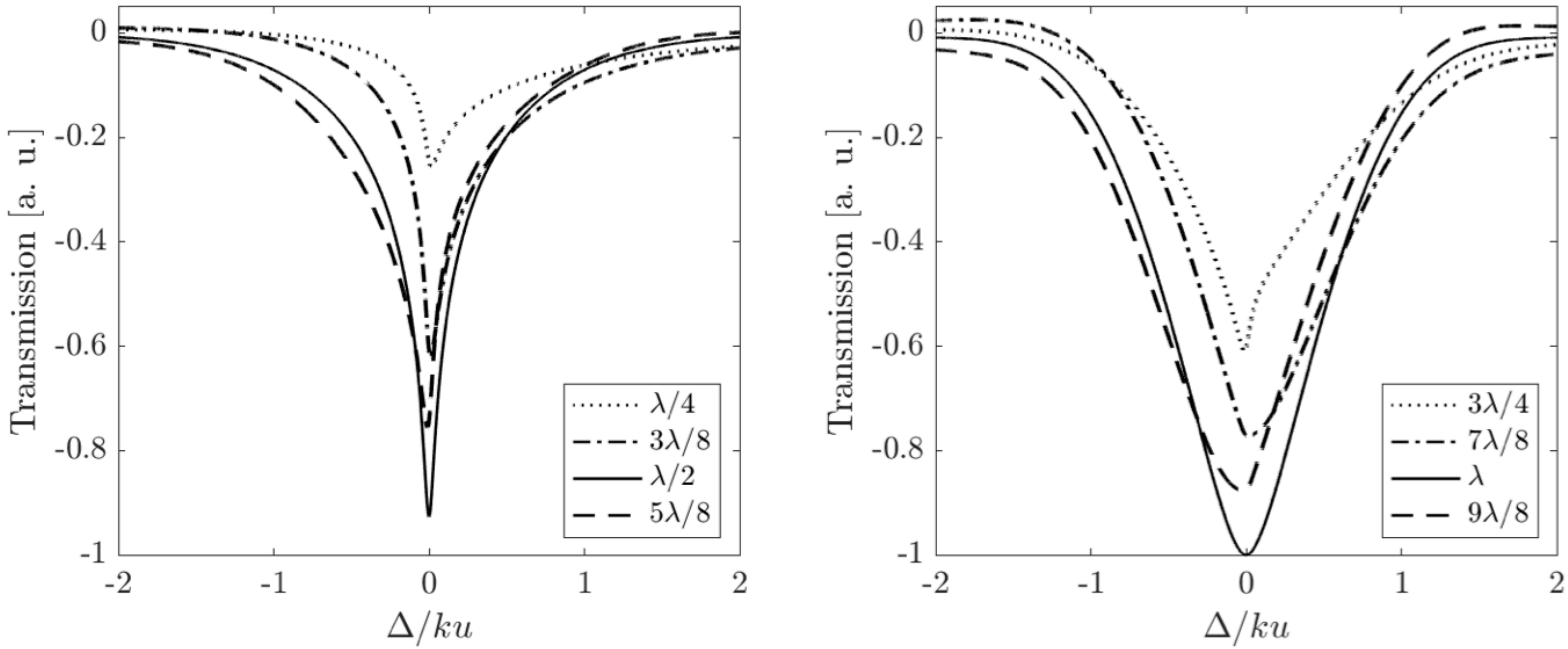
The transmitted and reflected signal have the form

$$S_t \approx 2t_{wc}t_{cw}^2 E_i \mathrm{Re}[I_f - r_w I_b]/|F|^2$$

$$S_r \approx 2t_{cw} E_i \mathrm{Re}[r_w(1 - \exp(2 - ikL)) \times (I_b - r_w I_f \exp(2ikL))]/|F|^2$$

Throughly described in Emmanuel's phD thesis.

Laser interaction with the nano cell

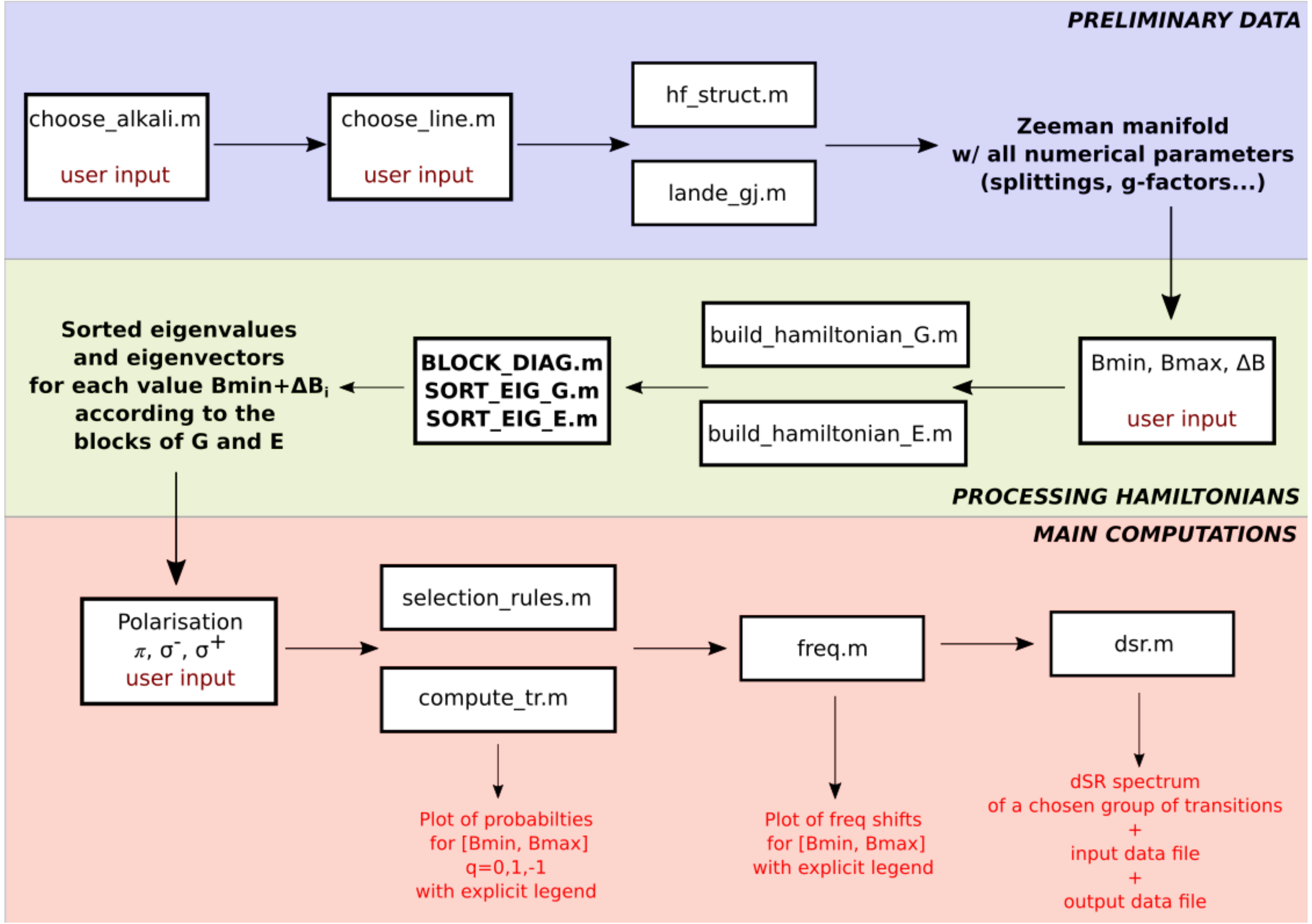


Transmission line shape for a cell thickness varying from $\lambda/4$ to $9\lambda/8$ with a step of $\lambda/8$
Two identical windows with $r_w \approx 0.28$, $\gamma/ku \approx 0.025$

Coherent Dicke Narrowing (CDN) is visible. All spectra will be computed for a cell length $\lambda/2$.
Reflected signal is not presented. However, it vanishes for thickness $(2p+1)\lambda/2,\ p \in \mathbb{N}$

Sodium D1 line in a magnetic field up to 1 Tesla

A complete tool for such computations

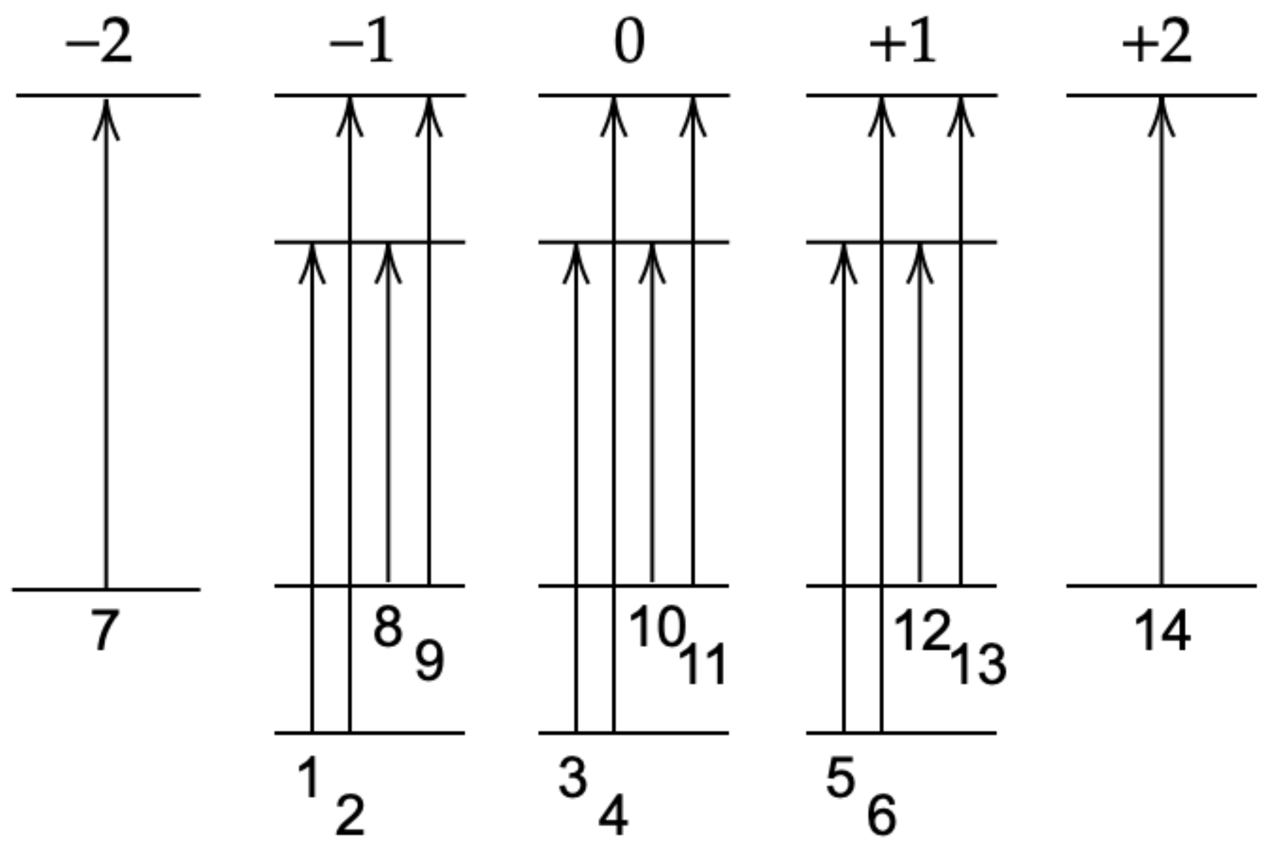


^{85}Rb, ^{87}Rb	$5^2S_{1/2} \to 5^2P_{1/2}$ (D1)
	$5^2S_{1/2} \to 5^2P_{3/2}$ (D2)
	$5^2S_{1/2} \to 6^2P_{1/2}$
	$5^2S_{1/2} \to 6^2P_{3/2}$
^{133}Cs	$6^2S_{1/2} \to 6^2P_{1/2}$ (D1)
	$6^2S_{1/2} \to 6^2P_{3/2}$ (D2)
^{39}K, ^{40}K, ^{41}K	$2^2S_{1/2} \to 2^2P_{1/2}$ (D1)
	$2^2S_{1/2} \to 2^2P_{3/2}$ (D2)
^{23}Na	$3^3S_{1/2} \to 2^3P_{1/2}$ (D1)
	$3^2S_{1/2} \to 2^3P_{3/2}$ (D2)

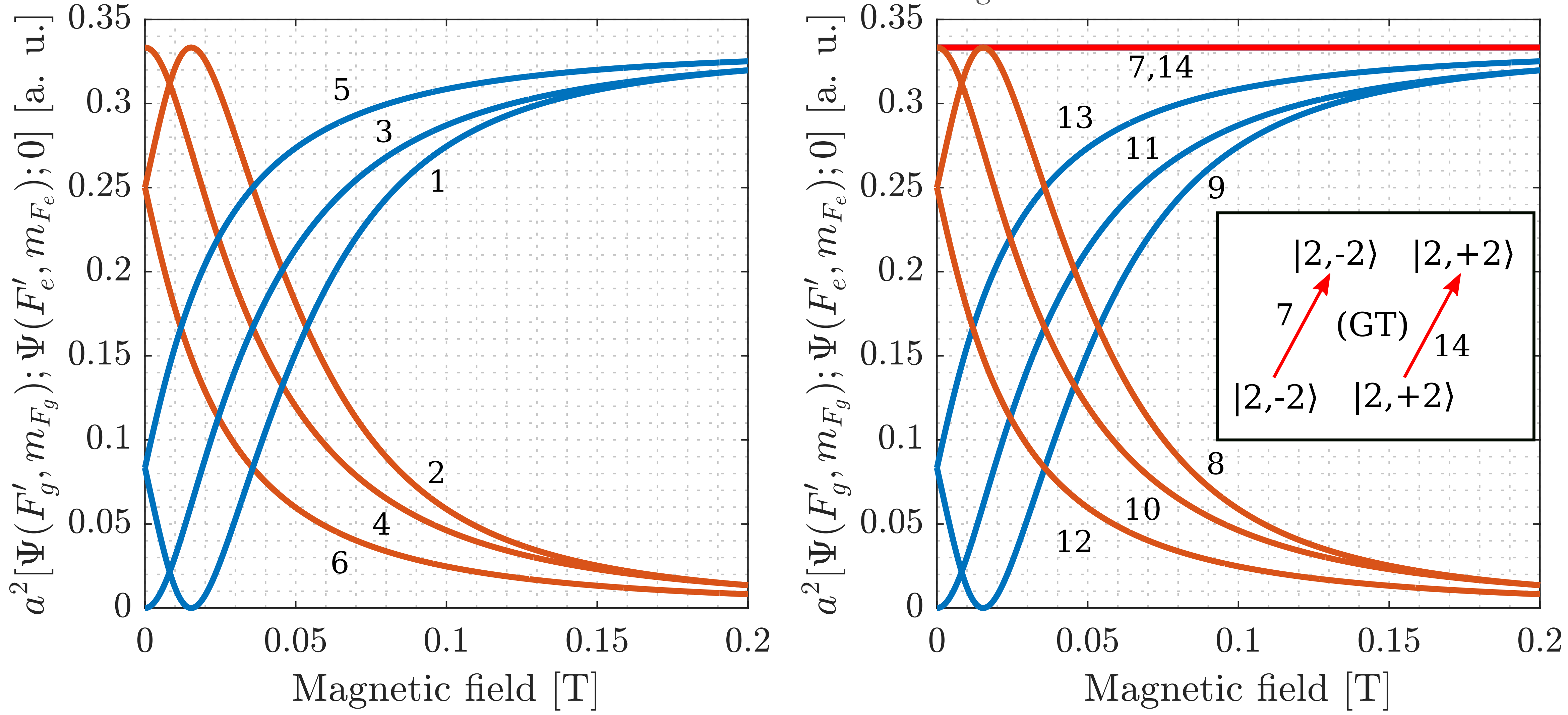
Allows to compute:
- Zeeman splitting
- Transition amplitudes
- Transition frequencies
- Reflection and transmission spectrum

of any Alkali (except Li for now)
For any of the three main polarisations
With or without magnetic field (up to 1T)

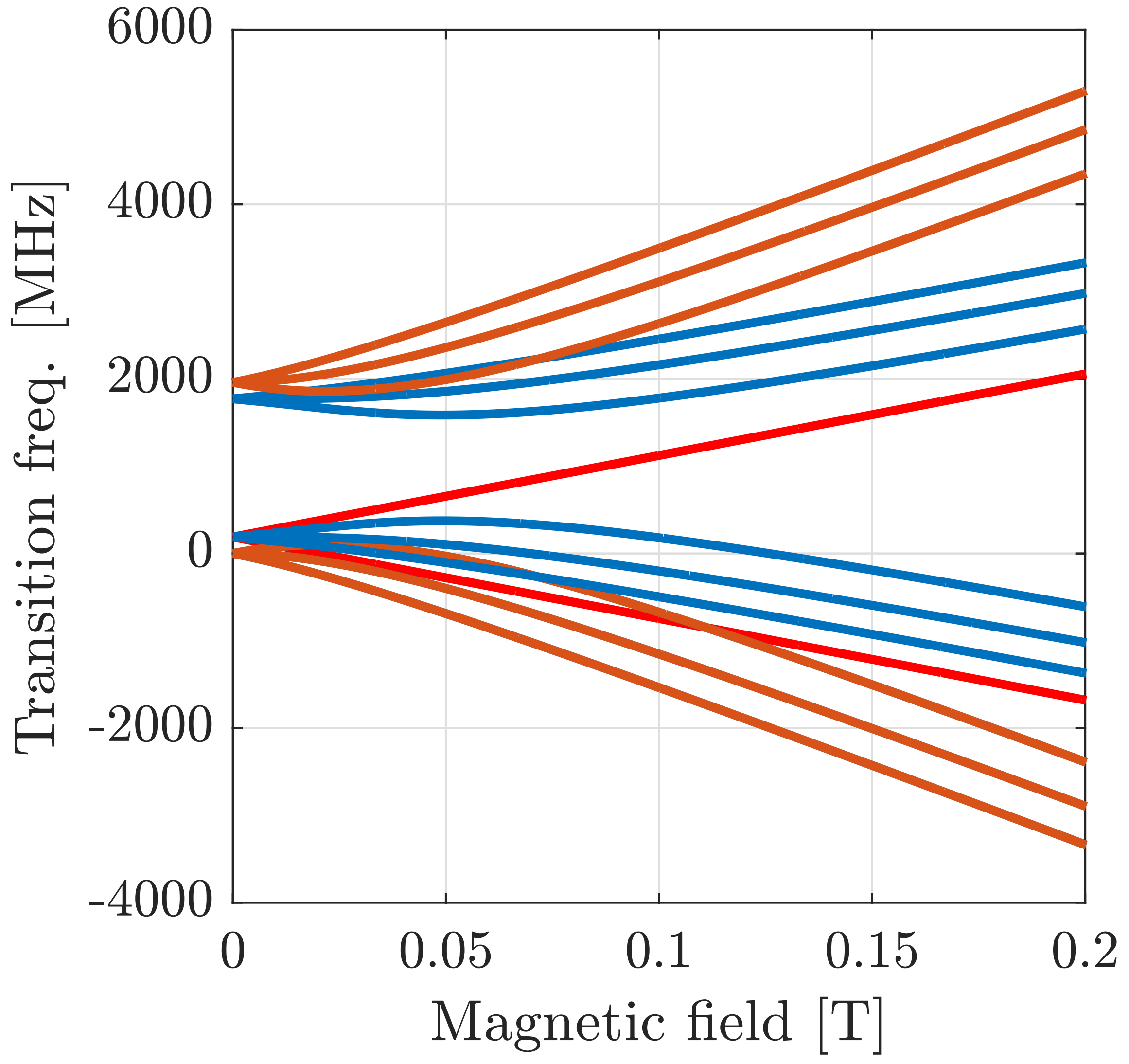
π-polarized incident laser radiation



Transition amplitudes $A_{eg}(B)$



Transitions 3 and 11 are called “Magnetically Induced” (MI)



Transition frequencies $\omega_{eg}(B)$

At low field, transition are grouped by value of F and described by the states $|F, m_F\rangle$

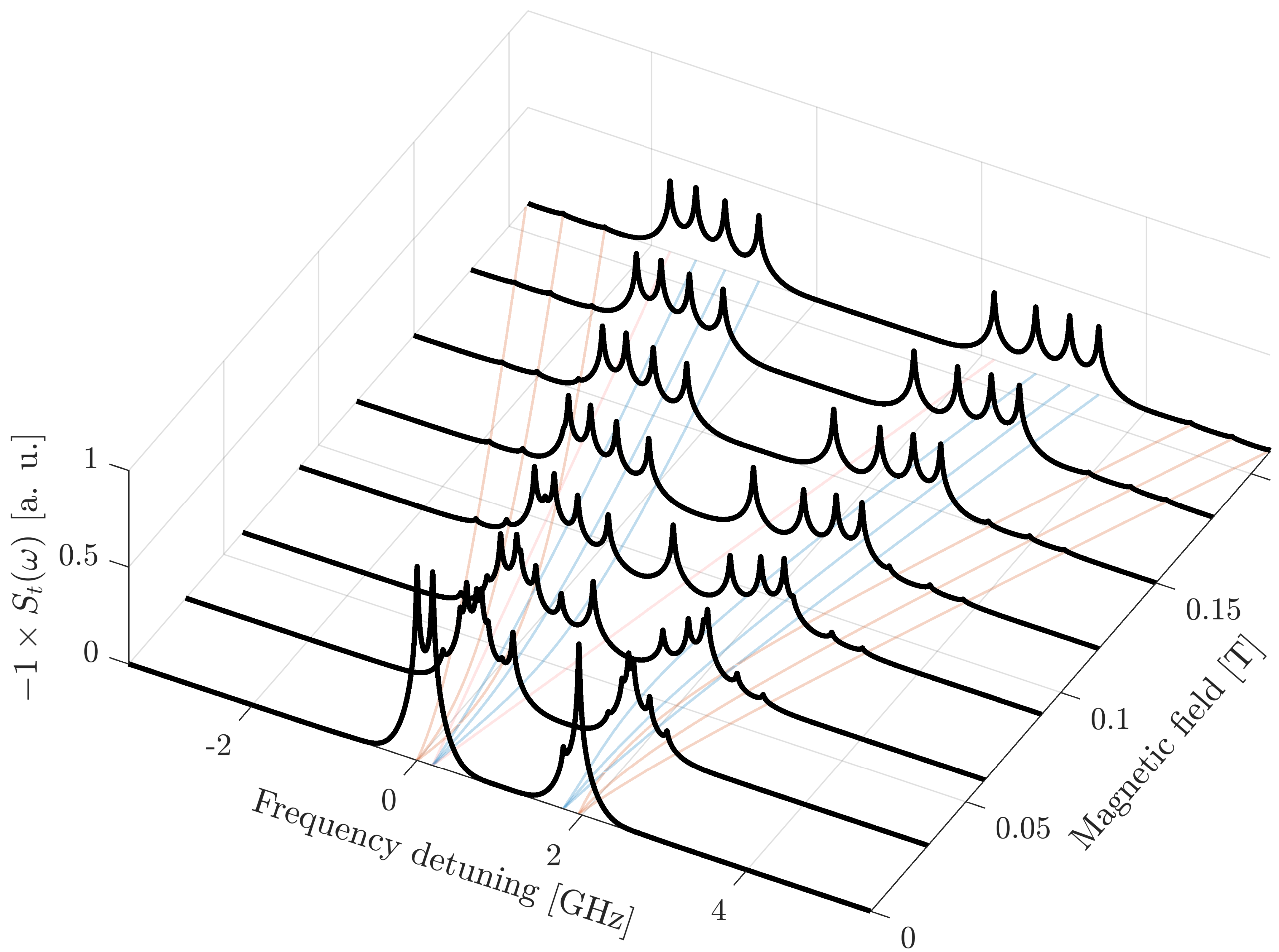
At high fields (with respect to $B_0 \approx 0.063T$), transitions are best described in the basis $|m_I, m_J\rangle$
Remaining transitions obey the selection rules $\Delta m_I = \Delta m_J = 0$ (here even $\Delta F = 0$)

$$\boxed{\begin{aligned} s_r^{\pm} &= s_{GT}^{\pm} \approx \pm\frac{2\mu}{3} \\ s_v^{\pm} &= 2s_{GT}^{\pm} \approx \pm\frac{4\mu}{3} \end{aligned}}$$

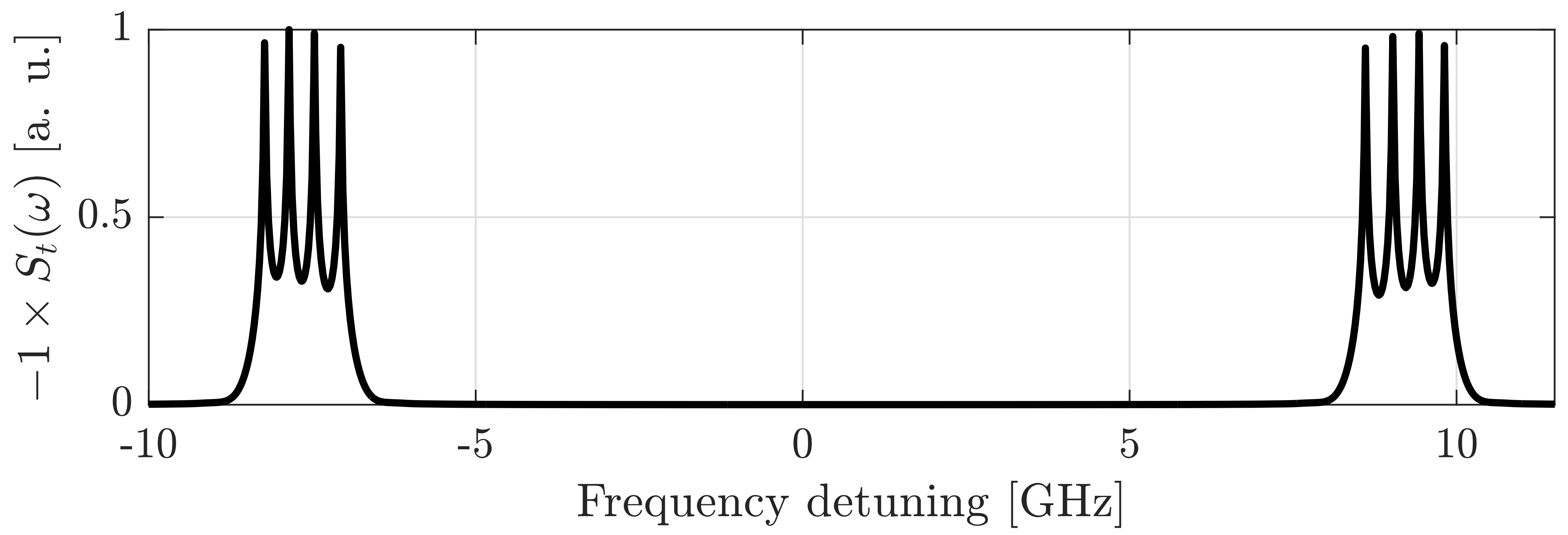
Assuming $g_S \approx 2,\ g_L \approx 1\ g_I \ll g_J$

Thorough analyses of dipole moment cancellation (here occurring at around 0.015T but it depends of the hyperfine splittings) has been published last year in J. Opt. Soc. Am. B and J. Quant. Spectrosc. Radiat. Transf.

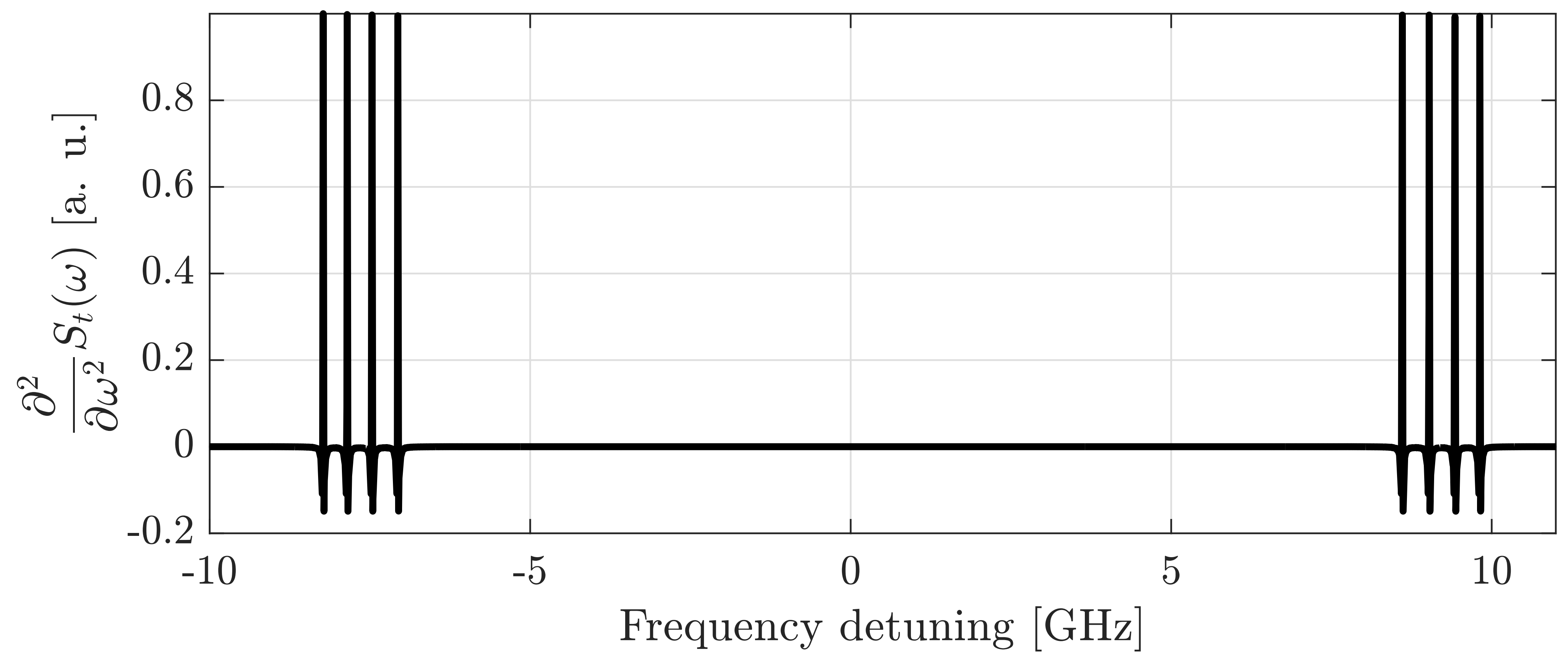
π-polarized incident laser radiation



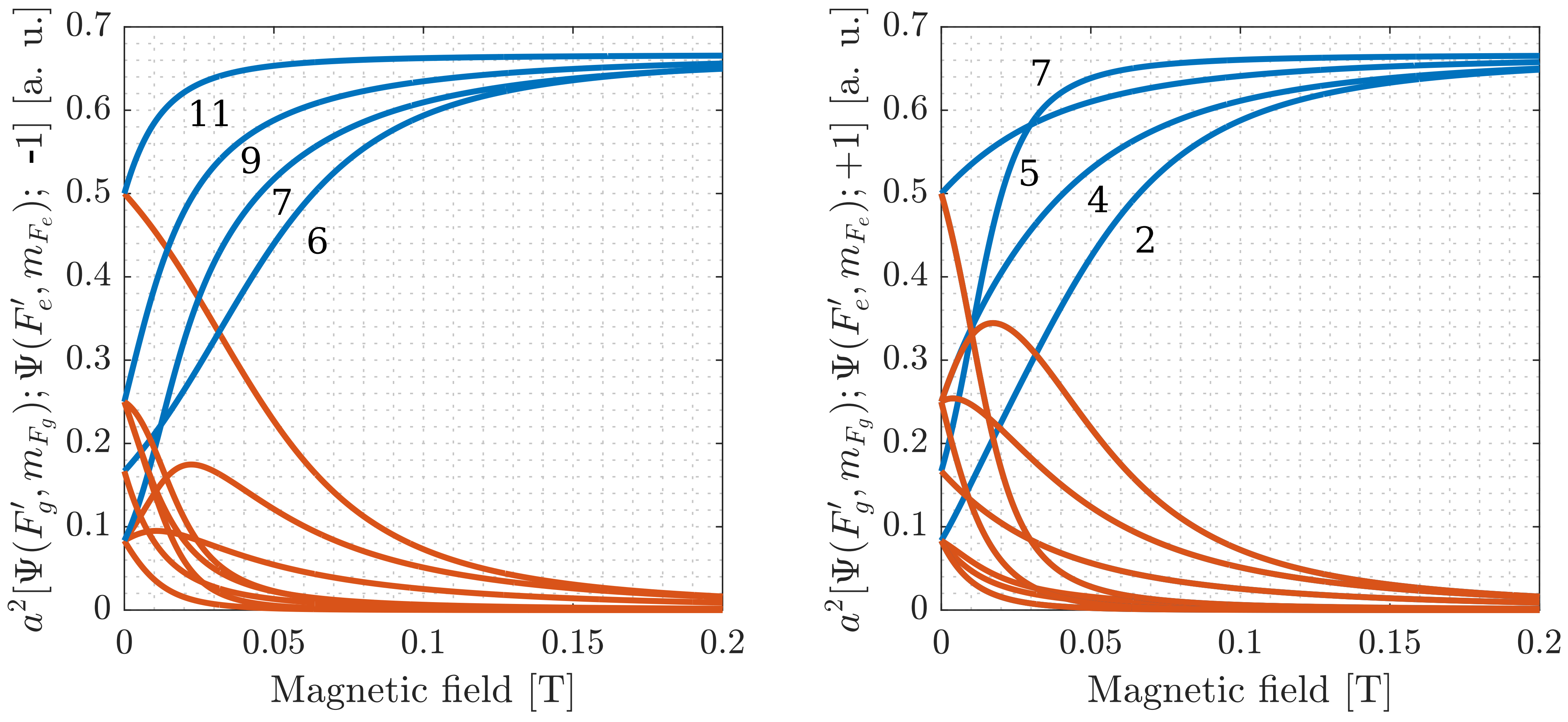
[23]Na D_1 line π-transition spectra for a magnetic field varying from 0 to 0.21T with a step of 0.030T. Shadow lines represent the transition frequencies as presented before. The color code is the same. Each components' FWHM is approx. 100 MHz (broadening is a free parameter). The line shape is exact. Cell thickness $L = \lambda/2$



[23]Na D_1 line π-transition spectra in the Hyperfine Paschen-Back regime for B = 0.9T. FWHM is the same. The peaks are not completely resolved SD or dSR would be convenient experimentally to obtain much narrower Resonances



σ-polarized incident laser radiation



Transition amplitudes A_{eg} for circularly polarized laser radiation.

Left panel = σ^- transitions. Right panel = σ^+ transitions.

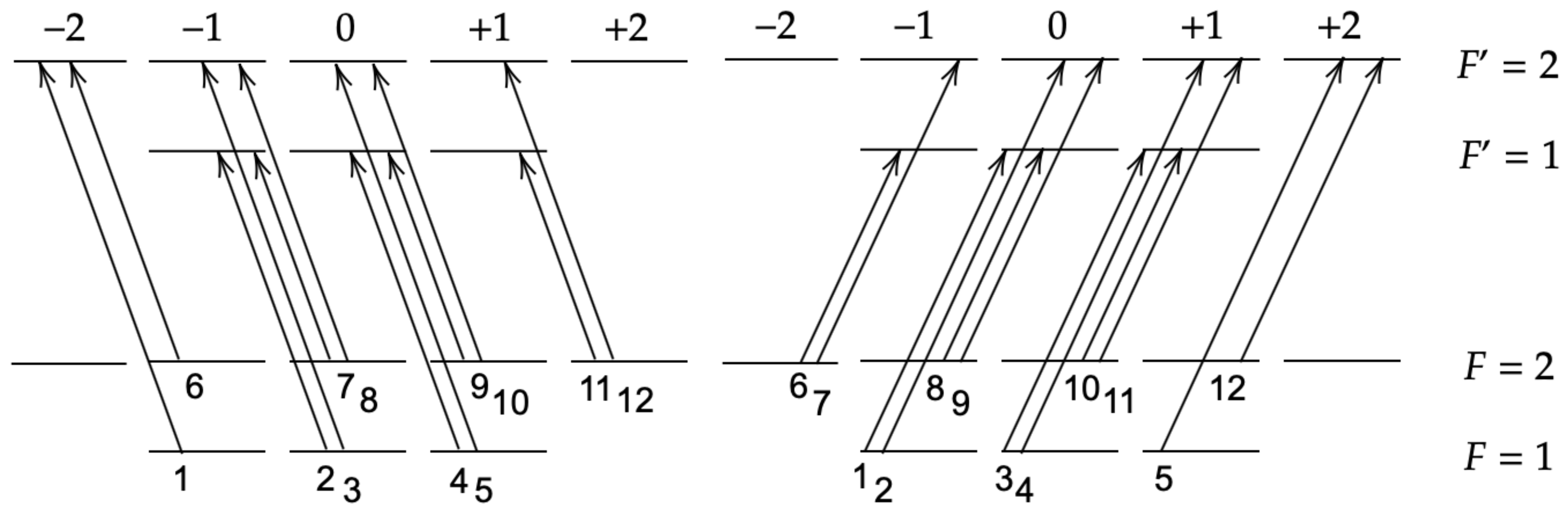
No MI nor guiding transitions for $\sigma^\pm$ polarisation.

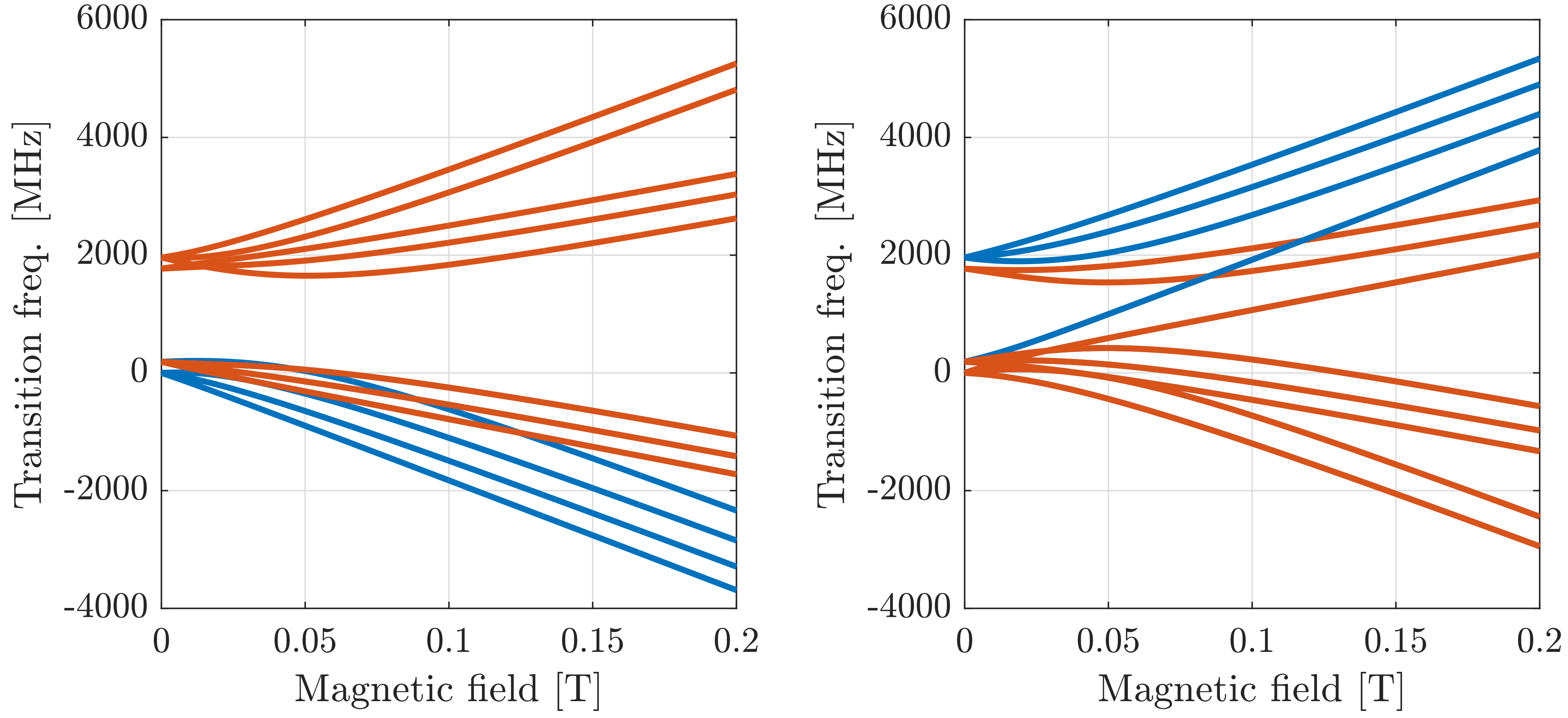
$$\boxed{\text{For } \sigma^\pm \quad s_r \approx \mp \frac{4\mu}{3}}$$

Assuming $g_S \approx 2,\ g_L \approx 1\ g_I \ll g_J$

⚠️ $\mu < 0$ for consistence (see Arimondo)

Reversed compared to pi transitions



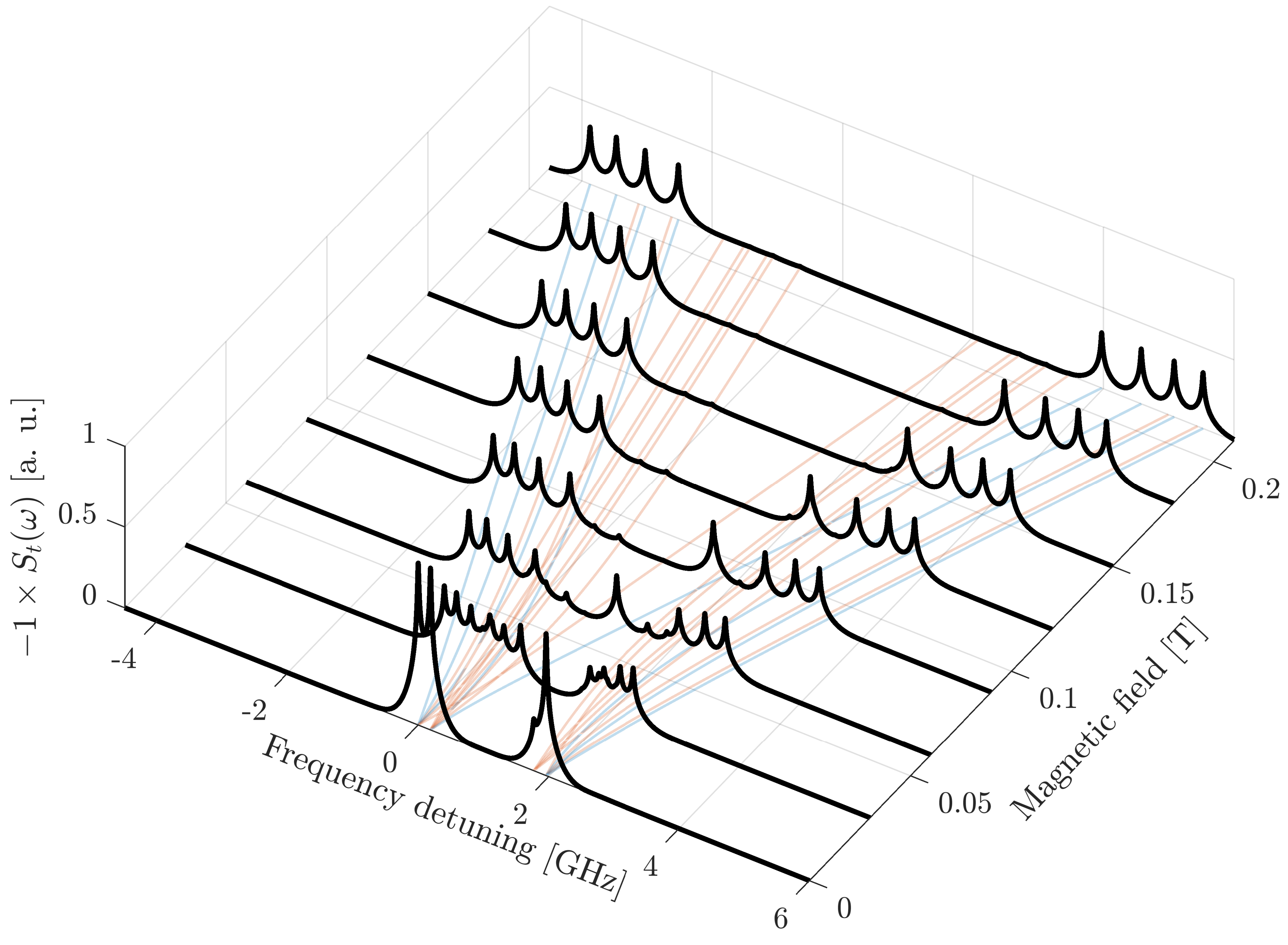


Transitions frequencies ω_{eg} for circularly polarized laser radiation.

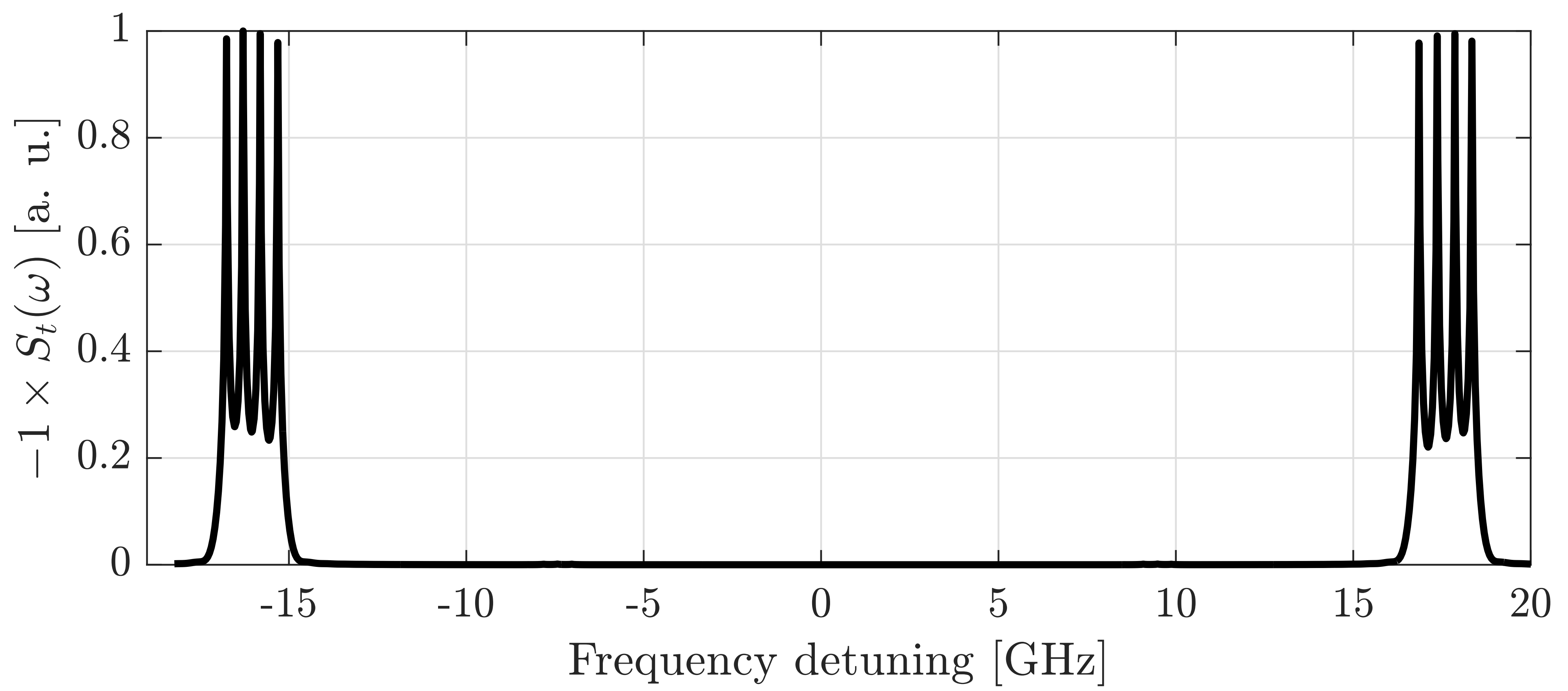
Left panel = σ^- transitions. Right panel = σ^+ transitions.

The zero is $F_g = 1 \rightarrow F_e = 1$

σ-polarized incident laser radiation



[23]Na D_1 line spectrum for a magnetic field varying from 0 to 0.21T with a step of 0.030T in case of simultaneous $\sigma^{\pm}$ excitation.
Shadow lines represent the transition frequencies as presented before. The color code is the same. Each components' FWHM is approx. 100 MHz (broadening is a free parameter). The lineshape is exact.
Cell thickness $L = \lambda/2$



[23]Na D_1 line spectrum in the Hyperfine Paschen-Back regime for B = 0.9T in case of $\sigma^{\pm}$ excitations.
FWHM is the same. The peaks are not completely resolved
SD or dSR would be convenient experimentally to obtain much narrower Resonances

Conclusion

Conclusion

- Although similar to the D1 line of Rubidium 87 and Potassium 39, such information has not been presented in the literature.
- The computations are performed on Matlab. Conversion to open-source language or standalone compilation would be more convenient.
- The behavior of the D1 line of Sodium in a magnetic field is completely depicted. Useful since Sodium is widely used in many areas.
- The spectra should be in perfect agreement with experiments (it worked perfectly for Cesium).
- Same work for the D2 line in on the way. In the HBP regime, peaks will be even overlapped.
- More magneto-optical processes will be observed such as Circular Dichroism.