Determination of the magnetic field values cancelling D_1 line transitions of alkali-metal atoms

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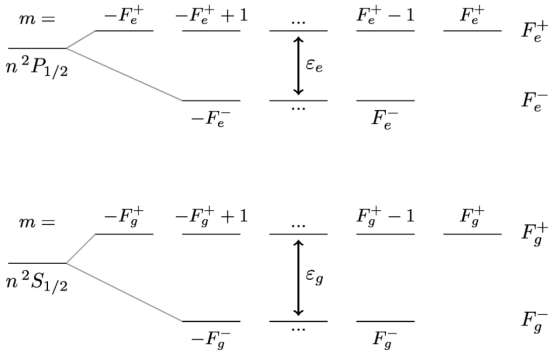
Generalities

For all alkali atoms, the D_1 line ($J_{g,e} = 1/2$) can be depicted as



where the quantum numbers are simply given by

$$F_{g,e}^{\pm} = I \pm 1/2$$
$$-F \leq m \leq F\,.$$

Theory

The diagonal elements of the Hamiltonian depict the linear Zeeman shifts of the states (in low fields)

$$\langle F, m| H |F, m\rangle = E_0(F) - \mu_B g_F(F) m B \,.$$

The non elements are given by

$$\langle F-1, m| H |F, m\rangle = \langle F, m| H |F-1, m\rangle \tag{1}$$

$$= -\frac{\mu_B}{2}(g_J - g_I) B \sqrt{1 - \left(\frac{2m}{1+2I}\right)^2} \,. \tag{2}$$

This last expression is notably simpler than in [Tremblay *et al.*], since many simplifications have been performed (only m and I are left as variables)

Theory

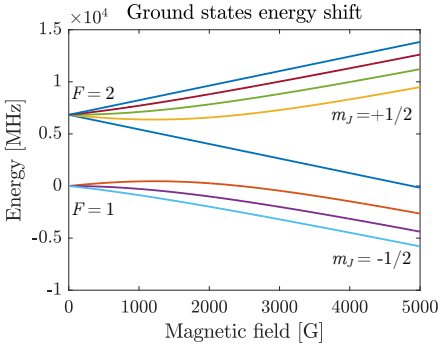
We can then express quite simple Hamiltonian matrices for the ground and excited states (for a given m)

$$H_{g,e} = \begin{array}{c} \\ \langle F^-_{g,e}, m_{g,e}| \\ \langle F^+_{g,e}, m_{g,e}| \end{array} \begin{array}{c} \begin{array}{cc} |F^-_{g,e}, m_{g,e}\rangle & \qquad |F^+_{g,e}, m_{g,e}\rangle \end{array} \\ \begin{pmatrix} -\mu_B \left(g_I + \dfrac{g_{g,e}}{1+2I} \right) m_{g,e} B & \dfrac{\mu_B}{2} g_{g,e} B \sqrt{1 - \left(\dfrac{2m_{g,e}}{1+2I} \right)^2} \\ \dfrac{\mu_B}{2} g_{g,e} B \sqrt{1 - \left(\dfrac{2m_{g,e}}{1+2I} \right)^2} & \varepsilon_{g,e} - \mu_B \dfrac{f_{g,e}}{1+2I} m_{g,e} B \end{pmatrix} \end{array}$$

with the following notations:

$$\begin{aligned} g_g &= g_I - g_S \\ g_e &= (3g_I - 4g_L + g_S)/3 \\ f_g &= g_S + 2g_I I \\ f_e &= (4g_L - g_S + 6g_I I)/3 \,. \end{aligned}$$

Energy shift



The eigenvalues (energy shifts) are given by:

$$\Lambda^{\pm}_{g,e} = \frac{\varepsilon_{g,e} - 2\mu_B g_I m_{g,e} B}{2} \pm \frac{1}{2}\sqrt{\varepsilon^2_{g,e} + \mu^2_B g^2_{g,e} B^2 + \frac{4\varepsilon_{g,e}\mu_B g_{g,e} m_{g,e} B}{1+2I}}$$

and the general form of the eigenvectors is:

$$|\psi(F^{\pm}_{g,e}, m_{g,e})\rangle = \frac{1}{\sqrt{1+\kappa^2_{g,e\pm}}} |F^{+}_{g,e}, m_{g,e}\rangle + \frac{\kappa_{g,e\pm}}{\sqrt{1+\kappa^2_{g,e\pm}}} |F^{-}_{g,e}, m_{g,e}\rangle \ .$$

Zeeman transitions

After diagonalization, we find that the general expression of the transition probability between two Zeeman sublevels is:

$$
\begin{aligned}
&a[\,|\psi(F_e^{\pm},m)\rangle\,,|\Psi(F_g^{\pm},m)\rangle\,,0] \\
&=\frac{\kappa_{e\pm}}{\sqrt{1+\kappa_{e\pm}^2}}a_m^{F_e^- F_g^-}\frac{\kappa_{g\pm}}{\sqrt{1+\kappa_{g\pm}^2}} \\
&+\frac{\kappa_{e\pm}}{\sqrt{1+\kappa_{e\pm}^2}}a_m^{F_e^- F_g^+}\frac{1}{\sqrt{1+\kappa_{g\pm}^2}} \\
&+\frac{1}{\sqrt{1+\kappa_{e\pm}^2}}a_m^{F_e^+ F_g^-}\frac{\kappa_{g\pm}}{\sqrt{1+\kappa_{g\pm}^2}} \\
&+\frac{1}{\sqrt{1+\kappa_{e\pm}^2}}a_m^{F_e^+ F_g^+}\frac{1}{\sqrt{1+\kappa_{g\pm}^2}}
\end{aligned}
$$

With the unperturbed transfer coefficients (simplified $3j$ and $6j$ symbols):

$$
a_m^{F_e^{\pm} F_g^{\pm}} = \pm\frac{1}{\sqrt{3}}\frac{2m}{1+2I}
$$

$$
a_m^{F_e^{\pm} F_g^{\mp}} = \pm\frac{1}{\sqrt{3}}\sqrt{1-\left(\frac{2m}{1+2I}\right)^2}
$$

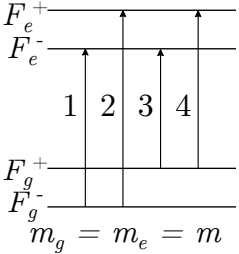
and the eigenvector factor

$$
\kappa_{g,e\pm} = \frac{2(1+2I)(\Lambda_{G,E}^{\pm}-\varepsilon_{g,e})+2\mu_B f_{g,e} m_{g,e} B}{\mu_B g_{g,e} B\sqrt{(1+2I)^2-4m_{g,e}^2}}
$$

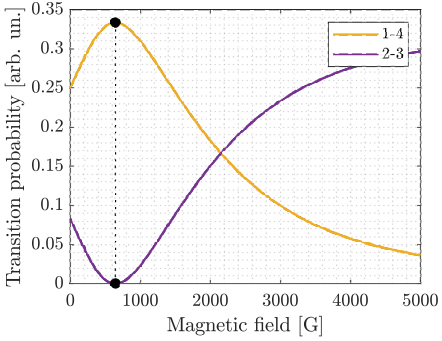
Zeeman transitions

For a single set of Hamiltonians $H_{g,e}$ (ie. for a given $m_g = m_e$), four π transitions are possible:

- 1: $|F_g^-, m\rangle \rightarrow |F_e^-, m\rangle$
- 2: $|F_g^-, m\rangle \rightarrow |F_e^+, m\rangle$
- 3: $|F_g^+, m\rangle \rightarrow |F_e^-, m\rangle$
- 4: $|F_g^+, m\rangle \rightarrow |F_e^+, m\rangle$

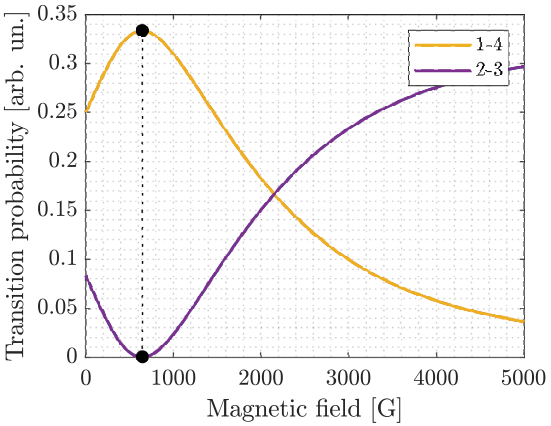


Here is an example with the $m = -1$ π transitions of ^{87}Rb D_1 line:



1 and 4 ($\Delta F = 0$) are identical
2 and 3 ($\Delta F = \pm 1$) are identical.

Zeeman transitions - Vanishing and maximization



Pair-transitions

While 1-4 reach their maximum, 2-3 vanish. Due to their symmetrical behavior, we call them pair-transitions (ensured by the calculation of the respective derivatives).

Solving $a[|\psi(F_e^{\pm}, m)\rangle, |\Psi(F_g^{\pm}, m)\rangle, 0] = 0$ yields

$$B = -\frac{1}{\mu_B}\frac{2m}{1+2I}\frac{2\varepsilon_g\varepsilon_e}{(g_I - g_S)\varepsilon_e + \frac{3g_I - 4g_L + g_S}{3}\varepsilon_g}$$

with the condition

$$0 \leq (-1)^{2I} m \leq I - \frac{1}{2}\,.$$

Results obtained

From the formula, which is valid for all alkali D_1 lines, we exhibited the following results:

Isotope	F	m	B (G)
^{23}Na	1	−1	153.2007(86)
^{23}Na	2	−1	153.2007(86)
^{39}K	1	−1	44.991(10)
^{39}K	2	−1	44.991(10)
^{40}K	9/2	7/2	190.20(33)
^{40}K	7/2	7/2	190.20(33)
^{40}K	9/2	5/2	135.85(24)
^{40}K	7/2	5/2	135.85(24)
^{40}K	9/2	3/2	81.51(15)
^{40}K	7/2	3/2	81.51(15)
^{40}K	9/2	1/2	27.171(48)
^{40}K	7/2	1/2	27.171(48)
^{41}K	1	−1	24.046(95)
^{41}K	2	−1	24.046(95)
^{87}Rb	1	−1	642.590(76)
^{87}Rb	2	−1	642.590(76)
^{133}Cs	3	−3	1359.237(26)
^{133}Cs	4	−3	1359.237(26)
^{133}Cs	3	−2	906.158(17)
^{133}Cs	4	−2	906.158(17)
^{133}Cs	3	−1	453.0790(84)
^{133}Cs	4	−1	453.0790(84)

Cancellation

Isotope	No.	F	m	B (G)
^{85}Rb	1	2	−2	380.73(13)
^{85}Rb	2	3	−2	380.73(13)
^{85}Rb	3	2	−1	190.368(66)
^{85}Rb	4	3	−1	190.368(66)

Maximization

Isotope	No.	F	m	B (G)
^{85}Rb	5	−1	−2	380.73(13)
^{85}Rb	6	1	−2	380.73(13)
^{85}Rb	7	−1	−1	190.368(66)
^{85}Rb	8	1	−1	190.368(66)

Discussion - Conclusion

Isotope	I	g_L	g_I [3]	ε_g (MHz)	ε_e (MHz)
^{23}Na	3/2	0.99997613 [19]	−0.00080461080(80)	1771.6261288(10) [3]	188.697(14) [20,21]
^{39}K	3/2	0.99997905339670(14)*	−0.00014193489(12)	461.73(14) [22]	57.696(10) [20]
^{40}K	4	0.99997974531640(14)*	0.00017649O(34)	−1285.87(35) [22]	−155.31(35) [22]
^{41}K	3/2	0.99998039390246(13)*	−0.00007790600(8)	253.99(12) [3,22,23]	30.50(16) [22]
^{85}Rb	5/2	0.99999354 [24]	−0.00029364000(60)	3035.7324390(60) [3]	361.58(17) [25,26]
^{87}Rb	3/2	0.99999369 [27]	−0.0009951414(10)	6834.682610904290(90) [28]	814.50(13) [3,25,26]
^{133}Cs	7/2	0.99999587 [29]	−0.00039885395(52)	9192.631770 (exact) [29]	1167.680(30) [30,31]

- We have determined a unique formula depicting the maximization or vanishing of D_1 π transitions. None of the σ transitions vanish.
- The main cause of uncertainty is due to the values of ε. All other parameters are known very precisely (Landé factors).
- Experimental measurements will be extremely difficult to perform since the variation of the transition intensities around the maxima/minima is far from being sharp. It would still be easier to record maxima rather than minima.
- These magnetic field values do not depend on any external condition nor parameter: possibly a good standard for magnetometer calibration, provided precise experimental measurements are performed.

References

P. Tremblay, A. Michaud, M. Levesque, S. Thériault, M. Breton, J. Beaubien, and N. Cyr.
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Phys. Rev. A **42**(5), 2766 (1990).

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