B-field values cancelling 5S → 6P transitions of ^{85}Rb and ^{87}Rb

Rodolphe MOMIER (MSc. student)

Under the supervision of Pr. Claude LEROY [1] and Artur ALEKSANYAN [1,2] (Ph.D student)

[1] Laboratoire Interdisciplinaire Carnot de Bourgogne, Université Bourgogne Franche-Comté, 21000 Dijon, France

[2] Institute for Physical Research, NAS of Armenia, Ashtarak-2, 0203 Armenia

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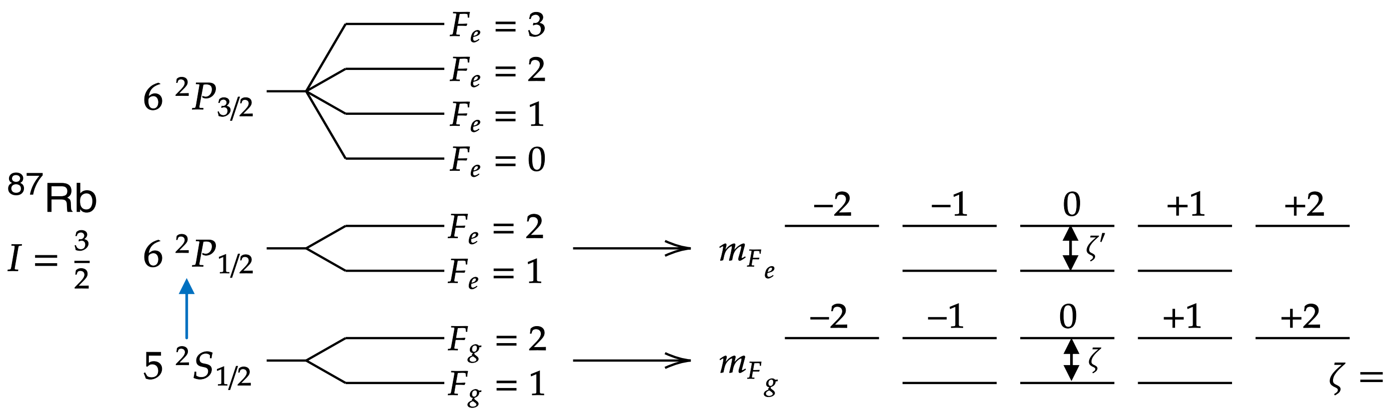
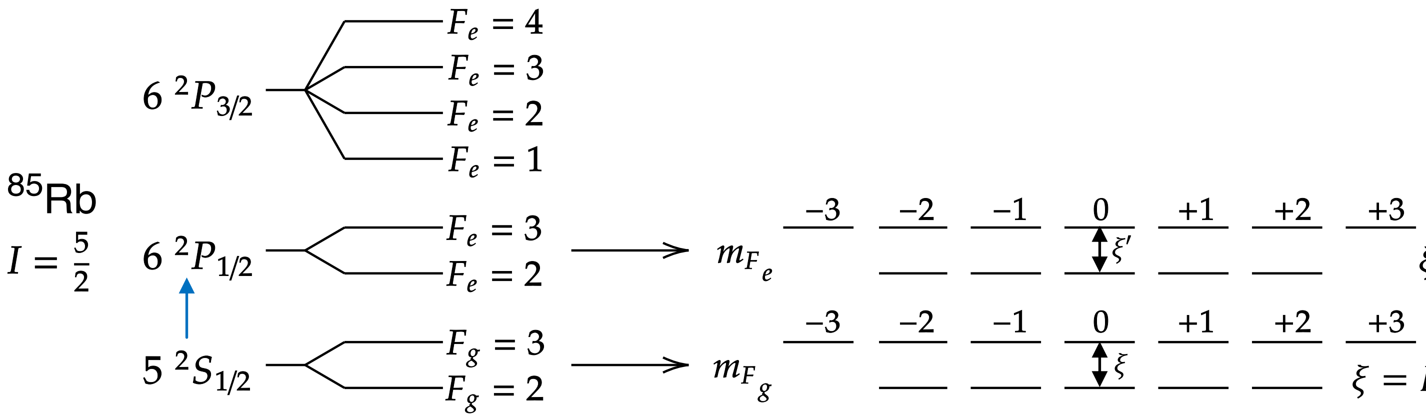
Hyperfine structure of ^{85}Rb and ^{87}Rb



$\zeta' = E_O(F_e = 2) - E_0(F_e = 1) \approx 265.196(371)$ MHz [1]

$\zeta = E_O(F_g = 2) - E_0(F_g = 1) \approx 6834.682610904290(90)$ MHz [2a]

Figure 1: Zeeman decomposition of the $5S$ and $6P$ states of ^{87}Rb



$\xi' = E_O(F_e = 3) - E_0(F_e = 2) \approx 118.402$ MHz [3]

$\xi = E_O(F_g = 3) - E_0(F_g = 2) \approx 3035.7324390(60)$ MHz [2b]

Figure 2: Zeeman decomposition of the $5S$ and $6P$ states of ^{85}Rb

[1] E. O. Nyakang'o, D. Shylla, V. Natarajan, and K. Pandey,
"Hyperfine measurement of the $6P_{1/2}$ state in ^{87}Rb using double resonance on blue and IR transition",
Journal of Physics B: Atomic, Molecular and Optical Physics 53, no. 9 (31 March 2020): 095001. https://doi.org/10.1088/1361-6455/ab7670.
[2a] D. A. Steck, "Rubidium 87 D Line Data", September 2001 (Latest revision November 2019) https://steck.us/alkalidata/rubidium87numbers.pdf
[2b] D. A. Steck, "Rubidium 85 D Line Data", April 2008 (Latest revision November 2019) https://steck.us/alkalidata/rubidium85numbers.pdf
[3] C. Glaser, F. Karlewski, J. Grimmel, M. Kaiser, A. Günther, H. Hattermann and J. Fortágh,
"Absolute frequency measurement of rubidium 5S-6P transitions", Physical Review A (accepted April 15th 2020)

Computation of the Hamiltonian $\mathcal{H}$

As stated in [4], we have for the diagonal elements:

$$\langle F, m_F | \mathcal{H} | F, m_F \rangle = E_0(F) - \mu_B g_F m_F B \tag{1}$$

For the off-diagonal elements:

$$\langle F-1, m_F | \mathcal{H} | F, m_F \rangle = \langle F, m_F | \mathcal{H} | F-1, m_F \rangle$$
$$= \frac{-\mu_B}{2} B\left(g_J - g_I\right) \sqrt{\frac{[(J+I+1)^2 - F^2][F^2 - (J-I)^2]}{F}} \sqrt{\frac{F^2 - m_F^2}{F(2F+1)(2F-1)}} \tag{2}$$

(Nonzero for $\Delta L = 0, \Delta J = 0, \Delta F = \pm 1, \Delta m_F = 0$)

[4] P. Tremblay, A. Michaud, M. Levesque, S. Thériault, M. Breton, J. Beaubien, and N. Cyr. ‘Absorption Profiles of Alkali-Metal D Lines in the Presence of a Static Magnetic Field’. *Physical Review A* 42, no. 5 (1 September 1990): 2766–73. https://doi.org/10.1103/PhysRevA.42.2766.

Structure of the Hamiltonian $\mathcal{H}$

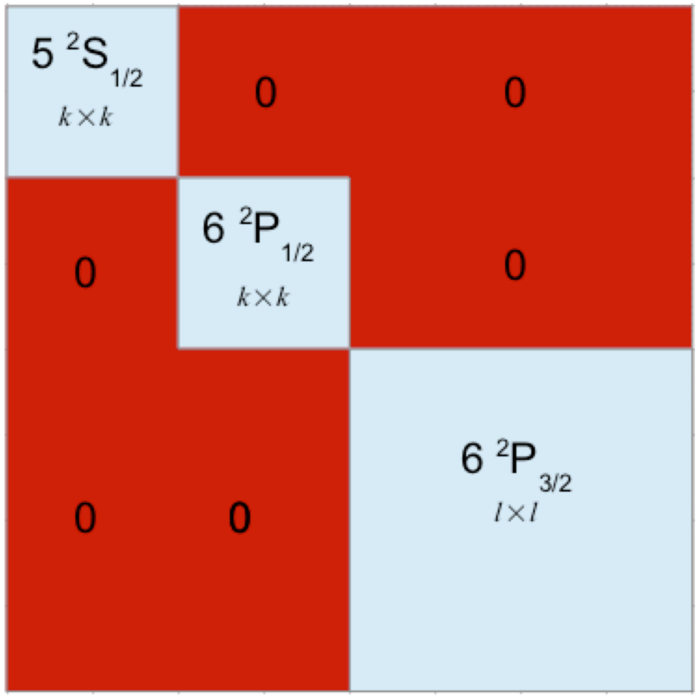


Figure 3: Structure of the Hamiltonian $\mathcal{H}$

$k \neq l$ since the value of J changes between 5 $^2S_{1/2}$ - 6 $^2P_{1/2}$ and 6 $^2P_{3/2}$ states. This structure is comparable to the one obtained in [5].

[5] H. Hakhumyan, Study of Optical and Magneto-optical processes in Rb atomic vapor layer of nanometric thickness, Ph.D Thesis, 2012

Hamiltonian of ^{87}Rb 5 $^2S_{1/2}$ and 6 $^2P_{1/2}$ states

By applying (1) and (2), we obtain in the basis $|F, m_F\rangle$:

$$\mathcal{H}_g^{^{87}\mathrm{Rb}} = \begin{array}{c} \begin{array}{cccccccc} |2,-2\rangle & |1,-1\rangle & |2,-1\rangle & |1,0\rangle & |2,0\rangle & |1,1\rangle & |2,1\rangle & |2,2\rangle \end{array} \\ \begin{pmatrix} \mu_B B + \zeta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\mu_B B}{2} & -\frac{\sqrt{3}}{2}\mu_B B & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2}\mu_B B & \frac{\mu_B B}{2} + \zeta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mu_B B & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mu_B B & \zeta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\mu_B B}{2} & -\frac{\sqrt{3}}{2}\mu_B B & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{2}\mu_B B & -\frac{\mu_B B}{2} + \zeta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_B B + \zeta \end{pmatrix} \end{array}$$

$$\mathcal{H}_e^{^{87}\mathrm{Rb}} = \begin{array}{c} \begin{array}{cccccccc} |2,-2\rangle & |1,-1\rangle & |2,-1\rangle & |1,0\rangle & |2,0\rangle & |1,1\rangle & |2,1\rangle & |2,2\rangle \end{array} \\ \begin{pmatrix} \frac{\mu_B B}{3} + \zeta' & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\mu_B B}{6} & -\frac{\mu_B B}{2\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\mu_B B}{2\sqrt{3}} & \frac{\mu_B B}{6} + \zeta' & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\mu_B B}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\mu_B B}{3} & \zeta' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\mu_B B}{6} & -\frac{\mu_B B}{2\sqrt{3}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\mu_B B}{2\sqrt{3}} & -\frac{\mu_B B}{6} + \zeta' & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu_B B}{3} + \zeta' \end{pmatrix} \end{array}$$

where $\mathcal{H}_g$ stands for the ground states and $\mathcal{H}_e$ for the excited states.

Hamiltonian of ^{87}Rb 5 $^2S_{1/2}$ and 6 $^2P_{1/2}$ states

We are interested in calculating the transitions, that is we only look at differences of energy levels. Thus, in order to simplify the expression, the energies $E_0(F)$ have been substracted from $\mathcal{H}_g$ and $\mathcal{H}_e$.
Each block corresponds to a given value of m_F (here from -2 to +2). For a better readability of the matrices, we used the following approximations to write them:

- $g_L = 0.99999369 \approx 1$
- $g_I = -0.0009951414(10) \approx 0$
- $g_S = 2.0023193043737(80) \approx 2$

However, the numerical computations have been done using the exact values of the Landé factors. These values are given in [2a], [2b] and [6].

[6] E. Arimondo, M. Inguscio and P. Violino, 'Experimental determinations of the hyperfine structure in the alkali atoms', Rev. Mod. Phys., 49:31-75, January 1977.

Transfer coefficients and transition intensities

After diagonalization, according to [1], we obtain the eigenvectors:

$$|\Psi(F_e, m_e)\rangle = \sum_{F_e'} c_{F_e F_e'} |F_e', m_e\rangle$$
$$|\Psi(F_g, m_g)\rangle = \sum_{F_g'} c_{F_g F_g'} |F_g', m_g\rangle$$

from which we obtain the transition intensities A_{eg} (proportional to the transfer coefficients $a[\Psi(F_e, m_e); \Psi(F_g, m_g); q]$)

$$A_{eg} \propto a^2[\Psi(F_e, m_e); \Psi(F_g, m_g); q]$$
$$a[\Psi(F_e, m_e); \Psi(F_g, m_g); q] = \sum_{F_e' F_g'} c_{F_e F_e'} a(\Psi(F_e, m_e); \Psi(F_g, m_g); q) c_{F_g F_g'}$$

where $a(\Psi(F_e, m_e); \Psi(F_g, m_g); q)$ depends on a 3j and a 6j symbol:

$$a(\Psi(F_e, m_e); \Psi(F_g, m_g); q) = (-1)^{1+I+J_e+F_e+F_g-m_e}(2J_e+1)^{\frac{1}{2}}(2F_e+1)^{\frac{1}{2}}(2F_g+1)^{\frac{1}{2}} \begin{pmatrix} F_e & 1 & F_g \\ -m_e & q & m_g \end{pmatrix} \begin{Bmatrix} F_e & 1 & F_g \\ J_g & I & J_e \end{Bmatrix}.$$

$q = \Delta m = m_e - m_g = 0, \pm 1$ depends on the polarization (0 for π, ± 1 for $\sigma^{\pm}$).

Eigenvalues of ^{87}Rb $5\ ^2S_{1/2}$ and $6\ ^2P_{1/2}$ states

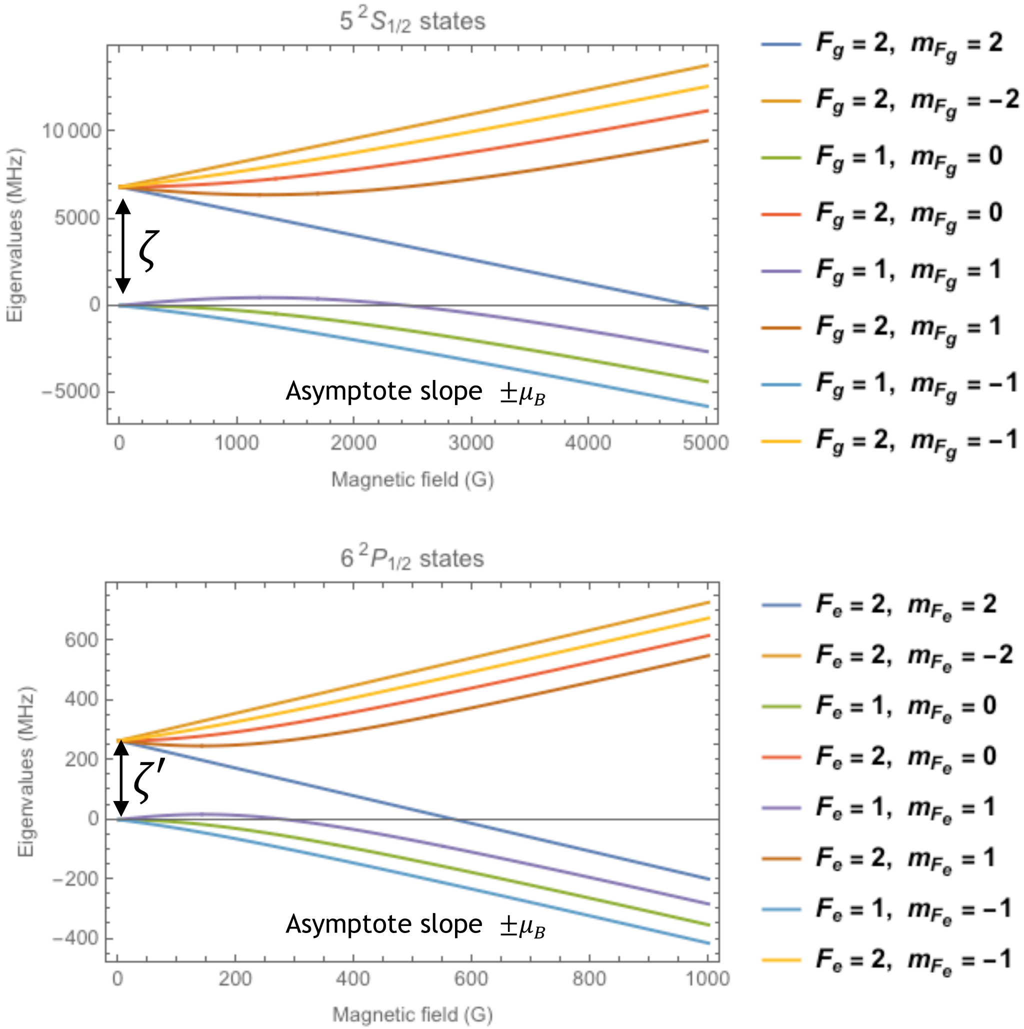


Figure 4: Eigenvalues (energy levels shifting) of the $5\ ^2S_{1/2}$ and $6\ ^2P_{1/2}$ states of ^{87}Rb versus magnetic field (G).

Eigenvectors of ^{87}Rb 5 $^2S_{1/2}$ and 6 $^2P_{1/2}$ states

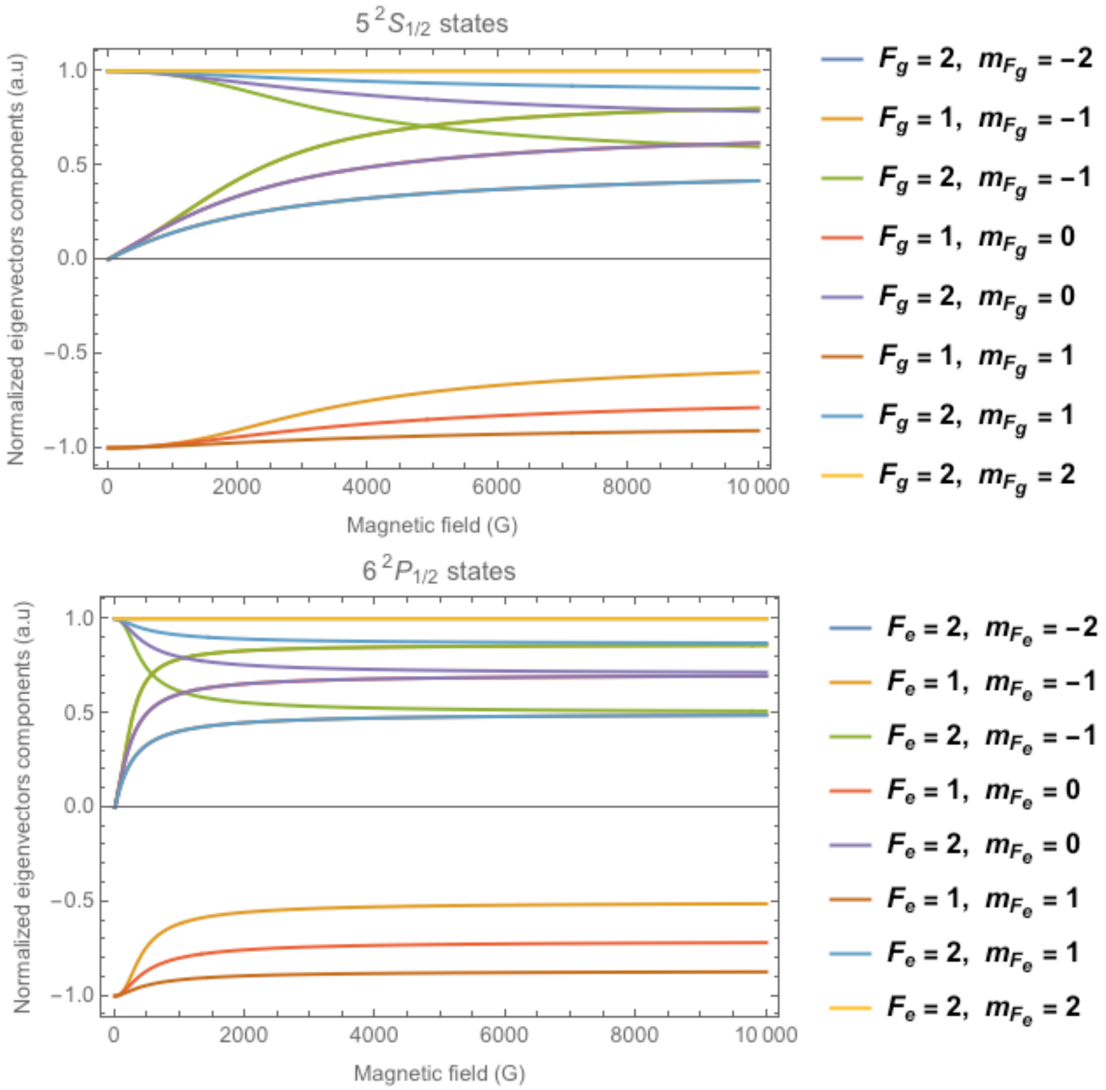


Figure 5: Eigenvectors (mixing coefficients) of the 5 $^2S_{1/2}$ and 6 $^2P_{1/2}$ states of ^{87}Rb versus magnetic field (G)

^{87}Rb π transitions from $5\,^2S_{1/2}$ to $6\,^2P_{1/2}$

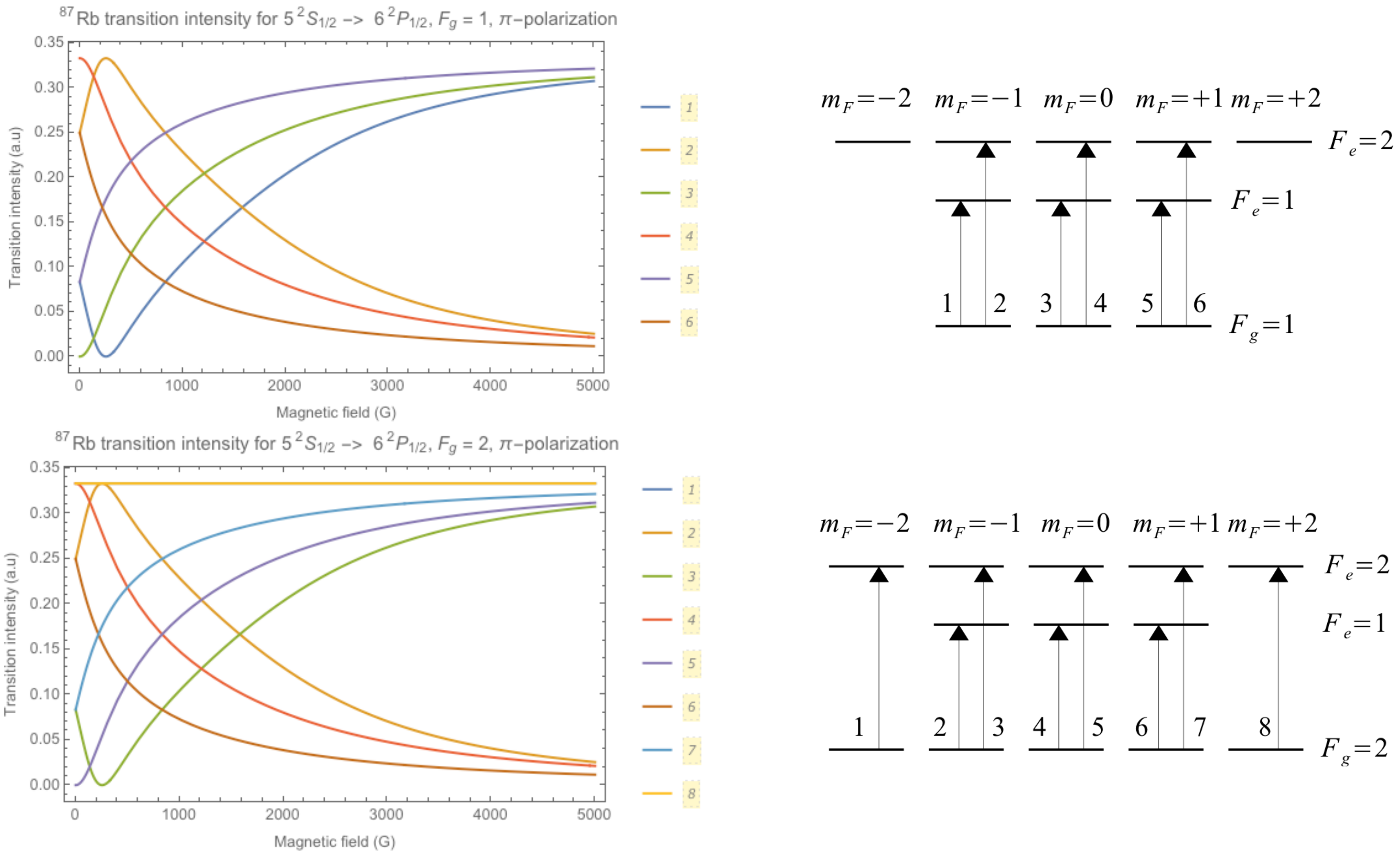


Figure 6: π transition intensities of ^{87}Rb from $5\,^2S_{1/2}$ to $6\,^2P_{1/2}$ states versus magnetic field (G)

A closer look at the transfer coefficients

We plot the transfer coefficients so that there exists at least one value of B which cancels them, thus we obtain:

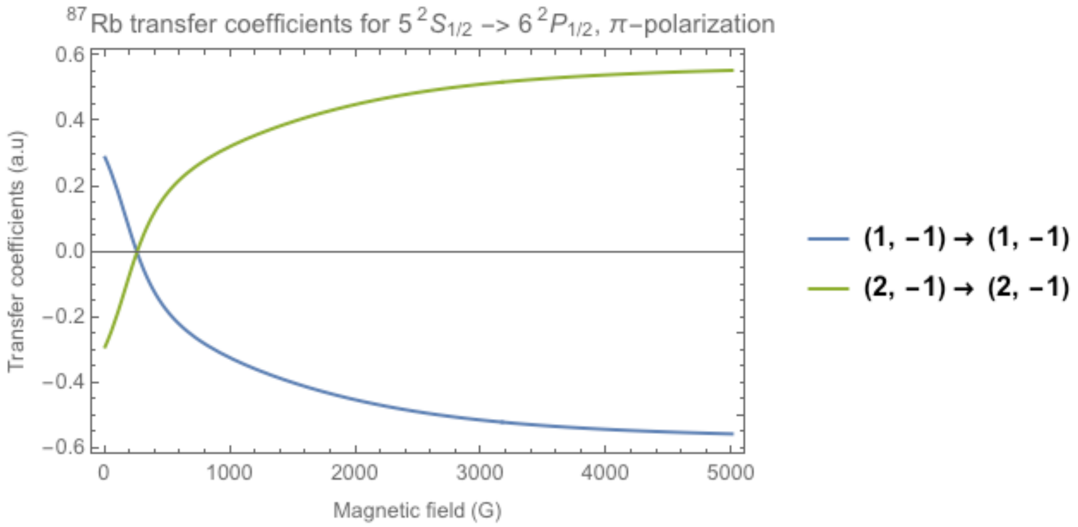


Figure 7: ^{87}Rb transfer coefficients which cross the x-axis, π-polarization

As observed before, some of the transfer coefficients a (and thus the transitions intensities a^2) cancel for a certain value of B, here for the transitions $|1,-1\rangle \rightarrow |1,-1\rangle$ (blue) and $|2,-1\rangle \rightarrow |2,-1\rangle$ (green), we obtain the same value of $B \approx 254.463\ \mathrm{G}$.

Hamiltonian of ^{85}Rb $5\ ^2S_{1/2}$ and $6\ ^2P_{1/2}$ states

By applying (1) and (2) we obtain in the basis $|F, m_F\rangle$:

$$
\mathcal{H}_g^{^{85}\mathrm{Rb}} =
\begin{array}{c}
\begin{matrix} |3,-3\rangle & |2,-2\rangle & |3,-2\rangle & |2,-1\rangle & |3,-1\rangle & |2,0\rangle & |3,0\rangle & |2,1\rangle & |3,1\rangle & |2,2\rangle & |3,2\rangle & |3,3\rangle \end{matrix} \\
\begin{pmatrix}
\mu_B B + \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-2\mu_B B}{3} & -\frac{\sqrt{5}\mu_B B}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\sqrt{5}\mu_B B}{3} & \frac{2\mu_B B}{3} + \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{\mu_B B}{3} & \frac{-2\sqrt{2}\mu_B B}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-2\sqrt{2}\mu_B B}{3} & \frac{\mu_B B}{3} + \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\mu_B B & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\mu_B B & \xi & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu_B B}{3} & \frac{-2\sqrt{2}\mu_B B}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-2\sqrt{2}\mu_B B}{3} & \frac{-\mu_B B}{3} + \xi & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2\mu_B B}{3} & -\frac{\sqrt{5}\mu_B B}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}\mu_B B}{3} & \frac{-2\mu_B B}{3} + \xi & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_B B + \xi
\end{pmatrix}
\end{array}
$$

$$
\mathcal{H}_e^{^{85}\mathrm{Rb}} =
\begin{array}{c}
\begin{matrix} |3,-3\rangle & |2,-2\rangle & |3,-2\rangle & |2,-1\rangle & |3,-1\rangle & |2,0\rangle & |3,0\rangle & |2,1\rangle & |3,1\rangle & |2,2\rangle & |3,2\rangle & |3,3\rangle \end{matrix} \\
\begin{pmatrix}
\frac{\mu_B B}{3} + \xi' & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-2\mu_B B}{9} & -\frac{\sqrt{5}\mu_B B}{9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\sqrt{5}\mu_B B}{9} & \frac{2\mu_B B}{9} + \xi' & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-\mu_B B}{9} & \frac{-2\sqrt{2}\mu_B B}{9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-2\sqrt{2}\mu_B B}{9} & \frac{\mu_B B}{9} + \xi' & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{\mu_B B}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{\mu_B B}{3} & \xi' & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\mu_B B}{9} & \frac{-2\sqrt{2}\mu_B B}{9} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-2\sqrt{2}\mu_B B}{9} & -\frac{-\mu_B B}{9} + \xi' & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2\mu_B B}{9} & -\frac{\sqrt{5}\mu_B B}{9} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{5}\mu_B B}{9} & \frac{-2\mu_B B}{9} + \xi' & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-\mu_B B}{3} + \xi'
\end{pmatrix}
\end{array}
$$

Hamiltonian of ^{85}Rb 5 $^2S_{1/2}$ and 6 $^2P_{1/2}$ states

For the same reason as before, the energies $E_0(F)$ have been substracted from $\mathcal{H}_g$ and $\mathcal{H}_e$. Each block corresponds to a given value of m_F (here from -3 to +3). For a better readability of the matrices, we used the following approximations to write them:

- $g_L = 0.99999354 \approx 1$
- $g_I = -0.0002936400(6) \approx 0$
- $g_S = 2.0023193043622(15) \approx 2$

However, the numerical computations have been done using the exact values of the Landé factors.

Eigenvalues of ^{85}Rb 5 $^2S_{1/2}$ and 6 $^2P_{1/2}$ states

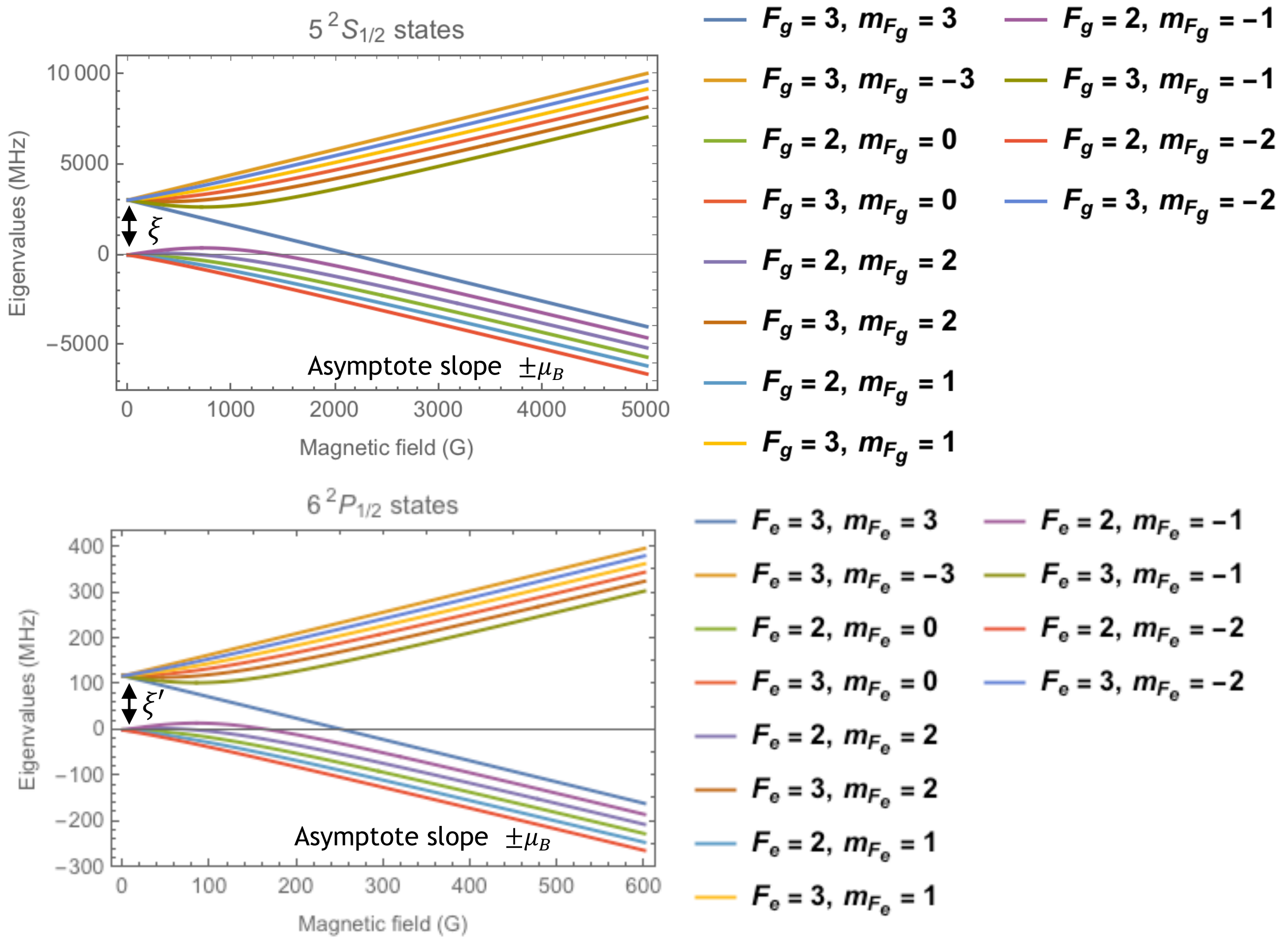


Figure 8: Eigenvalues (energy levels shifting) of the $5\,^2S_{1/2}$ and $6\,^2P_{1/2}$ states of ^{85}Rb versus magnetic field (G).

Eigenvectors of ^{85}Rb 5 $^2S_{1/2}$ and 6 $^2P_{1/2}$ states

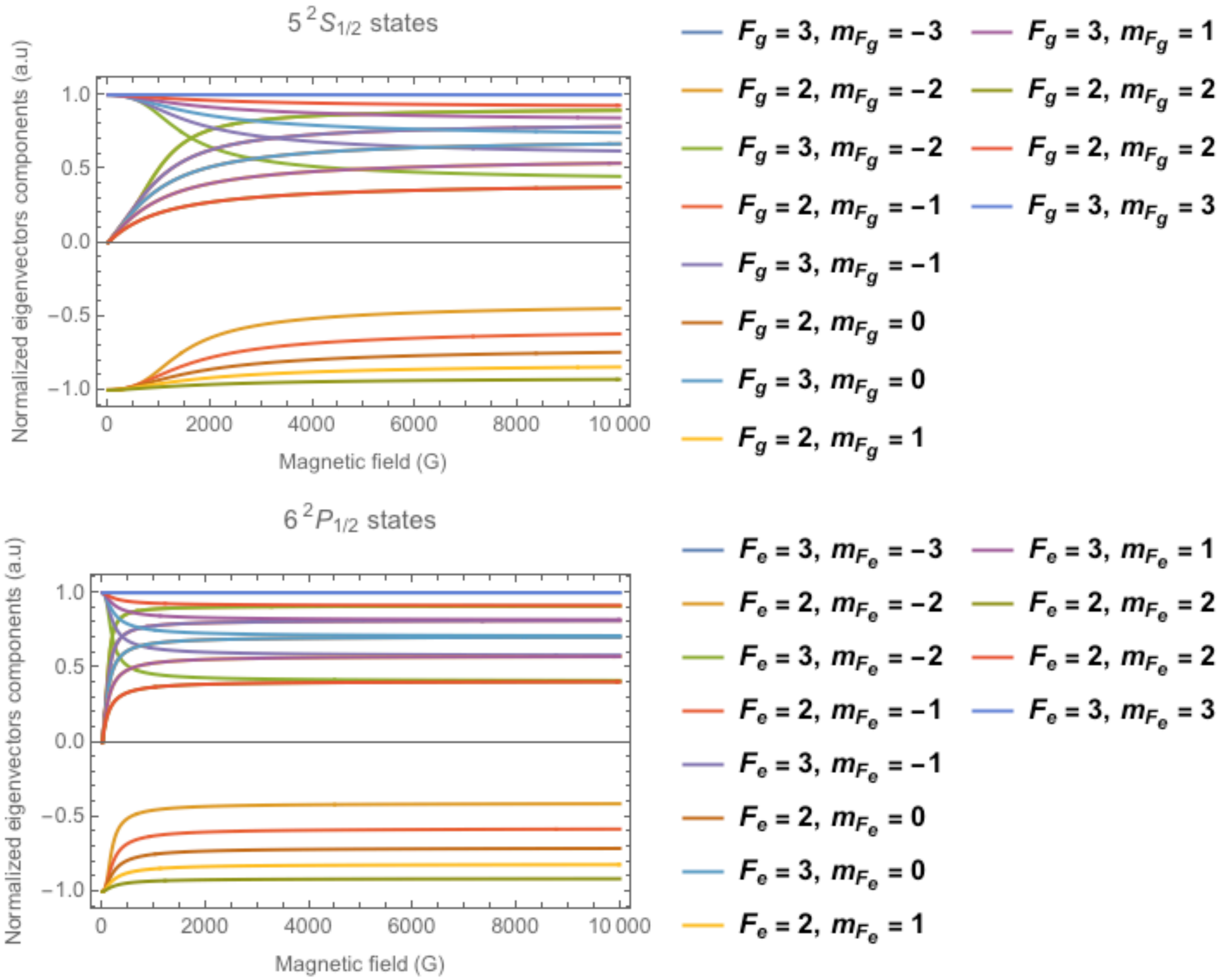


Figure 9: Eigenvectors (mixing coefficients) of the 5 $^2S_{1/2}$ and 6 $^2P_{1/2}$ states of ^{85}Rb versus magnetic field (G)

^{85}Rb π transitions from $5\ ^2S_{1/2}$ to $6\ ^2P_{1/2}$

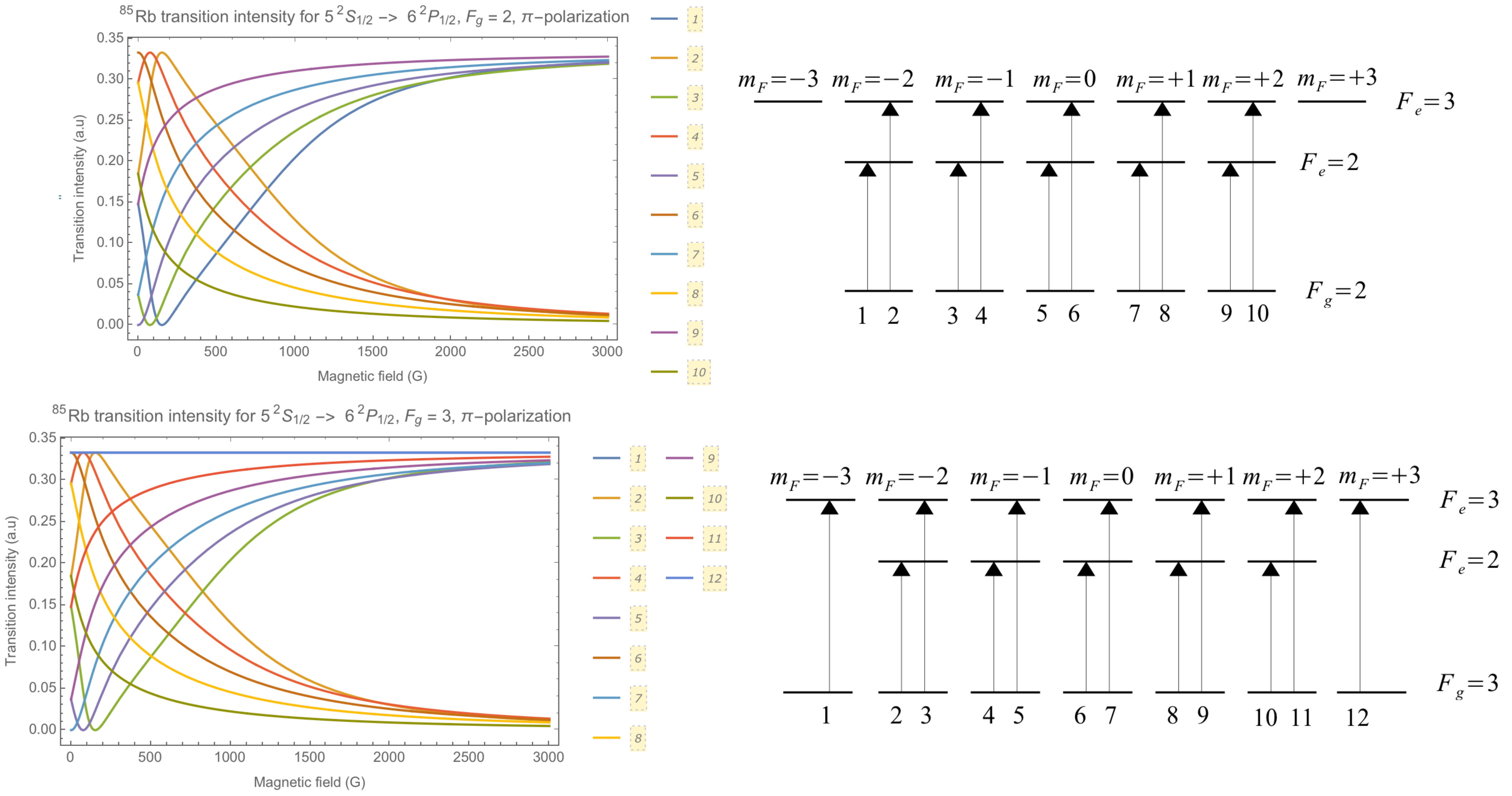
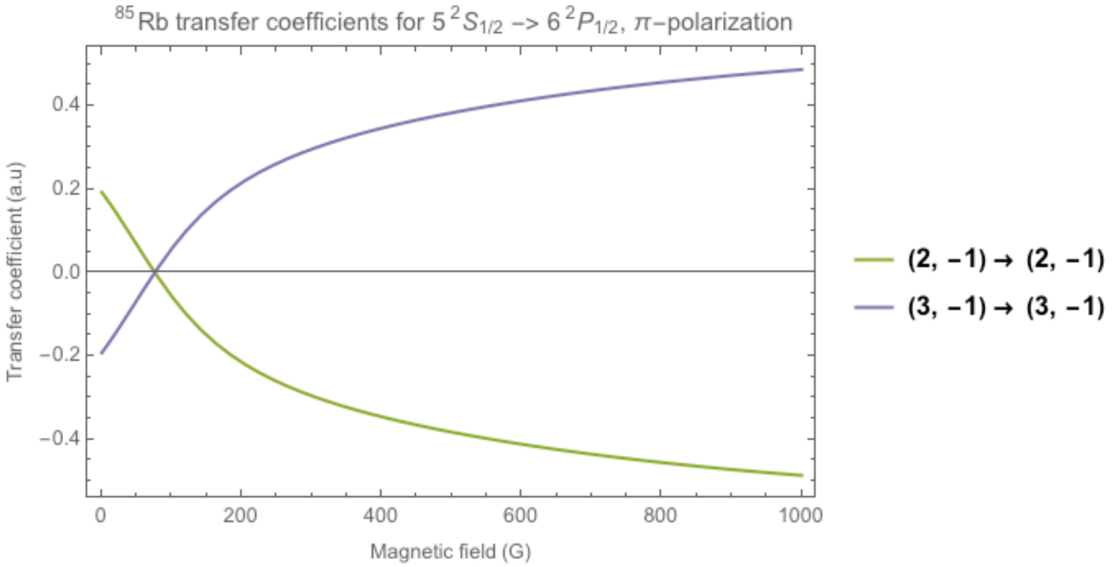


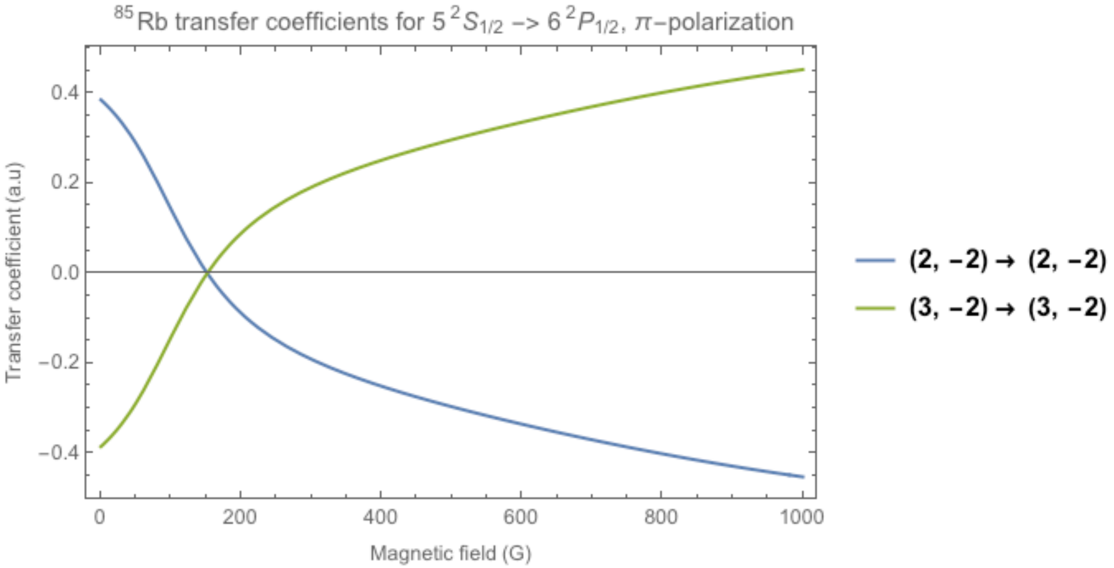
Figure 10: π transition intensities of ^{85}Rb from $5\ ^2S_{1/2}$ to $6\ ^2P_{1/2}$ states versus magnetic field (G)

A closer look at the transfer coefficients

We plot only the ones for which there exists at least one value of B which cancels them, thus we obtain:



(a)



(b)

Figure 11: ^{85}Rb transfer coefficients which cross the x-axis, π-polarization

As observed, some of the transfer coefficients cancel for certain values of B. Here, for the transitions $|2,-1\rangle \rightarrow |2,-1\rangle$ (green, Fig. 11a) and $|3,-1\rangle \rightarrow |3,-1\rangle$ (purple), we obtain $B \approx 75.773\,\mathrm{G}$. For the transitions $|2,-2\rangle \rightarrow |2,-2\rangle$ (blue) and $|3,-2\rangle \rightarrow |3,-2\rangle$ (green, Fig. 11b), we obtain $B \approx 151.547\,\mathrm{G}$.

Conclusion and perspectives

- It has been shown that certain transitions between the $5\ ^2S_{1/2}$ and $6\ ^2P_{1/2}$ states of both ^{85}Rb and ^{87}Rb cancel for certain values of B.

- For these states, no other cancellations can be observed (no cancellations for σ^+ nor σ^- polarizations)

- All the numerical data are known very precisely (10 digits) except the energy differences between the both atoms' excited states, thus the precision of the B-value obtained is reduced.

- To obtain more precise results, it would be great to refine experimentally the values of ζ' and ξ'.

- These results should be checked to see if the experiments provide B-values in good agreement with the theory. Differences would reflect the influence of the cell.

- We will also study the behavior of the transition intensities close to the cancellations for small variations of B.

- An article is being prepared in which we continue this study by calculating the transitions from $5\ ^2S_{1/2}$ to $6\ ^2P_{3/2}$ states.

Merci!

Thank you!