

Question 1:

$$\textcircled{1} L1 = \{a^n b^n a^m \mid n, m \geq 0\}$$

words = $\{\epsilon, ab, a, aba, aabba, abaa, aabbba, abaaa, adabbba, abaaaa \dots\}$

$$\begin{aligned} S &\rightarrow aSbB \mid B \mid \epsilon \\ B &\rightarrow aB \mid \epsilon \end{aligned}$$

$$\textcircled{2} L2 = \{w \in \Sigma^* \mid w \text{ is a palindrome}\}$$

words = $\{\epsilon, aa, bb, a, b, aba, bab, abba, baab, aaaa, bbbb, aaaa, bbbb \dots\}$

$$\begin{aligned} S &\rightarrow aSa \mid bSb \mid bB \mid aB \mid \epsilon \\ B &\rightarrow bB \mid aB \mid a \mid b \mid \epsilon \end{aligned}$$

$$\textcircled{3} L3 = \{a^i b^j a^k \mid j = i + k\}$$

words = $\{\epsilon, ab, ba, abba, aabbba, abbbba, aabbbbaa, aabb, bbaa \dots\}$

$$\begin{aligned} S &\rightarrow aSa \mid B \\ B &\rightarrow aBb \mid bBa \mid bbB \mid \epsilon \end{aligned}$$

Question 2

PDA

$$S \rightarrow S1 \mid S2$$

$$S1 \rightarrow aS1a \mid bS1b \mid a \mid b \mid \wedge$$

$$S2 \rightarrow aS2b \mid \wedge$$

First change it into CNF.

1. Remove \wedge -production.

$$S1 \rightarrow aS1a \mid bS1b \mid a \mid b \mid \wedge$$

↓

$$S1 \rightarrow aS1a \mid bS1b \mid a \mid b \mid aa \mid bb$$

$$S2 \rightarrow aS2b \mid \wedge$$

↓

$$S2 \rightarrow aS2b \mid ab$$

2. Remove unit-production

$$S_0 \rightarrow S$$

$$S \rightarrow S1 \mid S2$$

$$S1 \rightarrow aS1a \mid bS1b \mid a \mid b \mid aa \mid bb$$

$$S2 \rightarrow aS2b \mid ab$$

$$S_0 \rightarrow S$$

$$S \rightarrow aS1a \mid bS1b \mid a \mid b \mid aa \mid bb \mid aS2b \mid ab$$

$$S1 \rightarrow aS1a \mid bS1b \mid a \mid b \mid aa \mid bb$$

$$S2 \rightarrow aS2b \mid ab$$

$$S_0 \rightarrow aS_1a | bS_1b | a | b | aa | bb | aS_2b | ab$$

$$S \rightarrow aS_1a | bS_1b | a | b | aa | bb | aS_2b | ab$$

$$S_1 \rightarrow aS_1a | bS_1b | a | b | aa | bb$$

$$S_2 \rightarrow aS_2b | ab$$

↓

$$S_0 \rightarrow S_3S_1S_3 | S_4S_1S_4 | a | b | S_3S_3 | S_4S_4 | S_3S_2S_4 | S_3S_4$$

$$S \rightarrow S_3S_1S_3 | S_4S_1S_4 | a | b | S_3S_3 | S_4S_4 | S_3S_2S_4 | S_3S_4$$

~~$$S_1 \rightarrow aS_1a | bS_1b | a | b | aa | bb$$~~

~~$$S_2 \rightarrow aS_2b | ab$$~~

$$S_1 \rightarrow S_3S_1S_3 | S_4S_1S_4 | a | b | S_3S_3 | S_4S_4$$

$$S_2 \rightarrow S_3S_2S_4 | S_3S_4$$

$$S_3 \rightarrow a$$

$$S_4 \rightarrow b$$

↓

$$S_0 \rightarrow S_5S_3 | S_6S_4 | a | b | S_3S_3 | S_4S_4 | S_7S_4 | S_3S_4$$

$$S \rightarrow S_5S_3 | S_6S_4 | a | b | S_3S_3 | S_4S_4 | S_7S_4 | S_3S_4$$

$$S_1 \rightarrow S_5S_3 | S_6S_4 | a | b | S_3S_3 | S_4S_4$$

$$S_2 \rightarrow S_7S_4 | S_3S_4$$

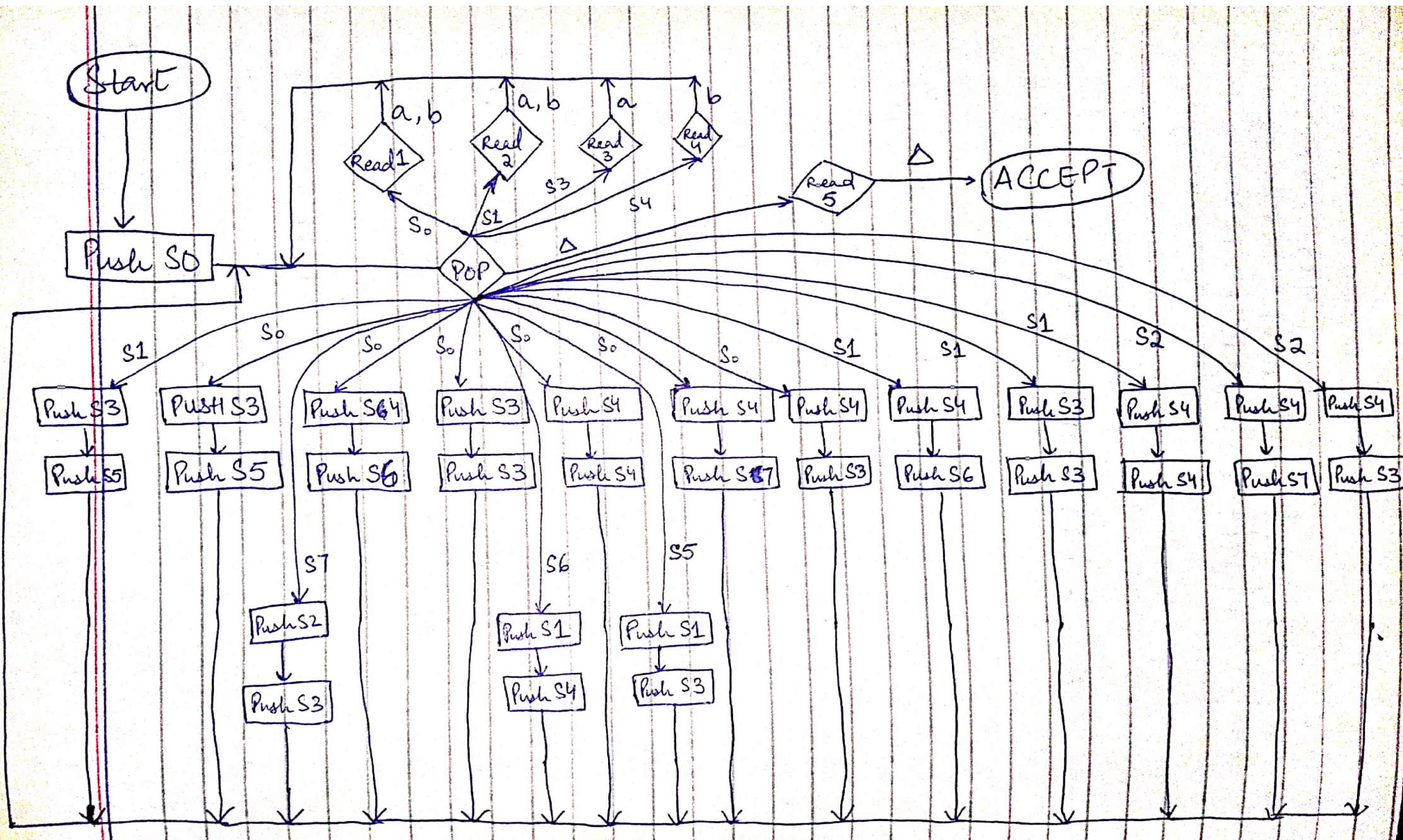
$$S_3 \rightarrow a$$

$$S_4 \rightarrow b$$

$$S_5 \rightarrow S_3S_1$$

$$S_6 \rightarrow S_4S_1$$

$$S_7 \rightarrow S_3S_2$$



2. CFG to PDA

equal a's and b's

$$S \rightarrow aSb \mid \Lambda$$

change to CNF by removing Λ -production

$$S \rightarrow aSb \mid \Lambda$$

↓

$$S \rightarrow aSb \mid ab$$

Remove unit-productions

$$S_0 \rightarrow aSb \mid ab$$

$$S \rightarrow aSb \mid ab$$

↓

$$S_0 \rightarrow S_1 S_2 \mid S_1 S_2$$

$$S \rightarrow S_1 S_2 \mid S_1 S_2$$

$$S_1 \rightarrow a$$

$$S_2 \rightarrow b$$

↓

$$S_0 \rightarrow S_1 S_2 \mid S_1 S_2$$

$$S \rightarrow S_1 S_2 \mid S_1 S_2$$

$$S_1 \rightarrow a$$

$$S_2 \rightarrow b$$

$$S_3 \rightarrow S_1 S_2$$

↓

$$S_0 \rightarrow \cancel{S_1 S_2} S_1 S_2 \mid S_3 S_2$$

$$S \rightarrow \cancel{S_1 S_2} S_1 S_2 \mid S_3 S_2$$

$$S_1 \rightarrow a$$

$$S_2 \rightarrow b$$

$$S_3 \rightarrow S_1 S_2$$

