

STAT 560: Time Series Analysis Homework 6 (Chapter 5)

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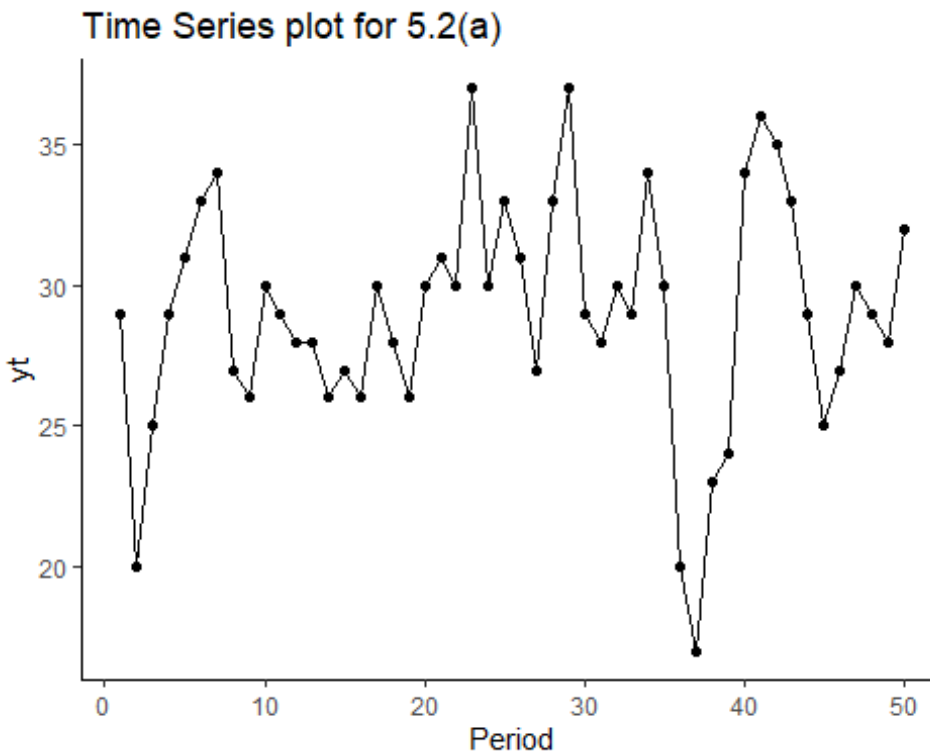
Exercise 5.2 (a) Consider the time series data shown in Table E5.1.

- a. Make a time series plot of the data.

Answer:

The following figure shows the time series plot of y_t over the time period. It seems that this time series is a constant process data and varies with a constant mean of 29 (or close to 30).

```
ggplot(data=my.data1)+  
  geom_line (aes(x=Period, y = yt))+  
  geom_point (aes(x=Period, y = yt))+  
  theme_classic()+  
  labs(x = "Period",  
       y = "yt",  
       title = "Time Series plot for 5.2(a)")
```



Exercise 5.2 (b)

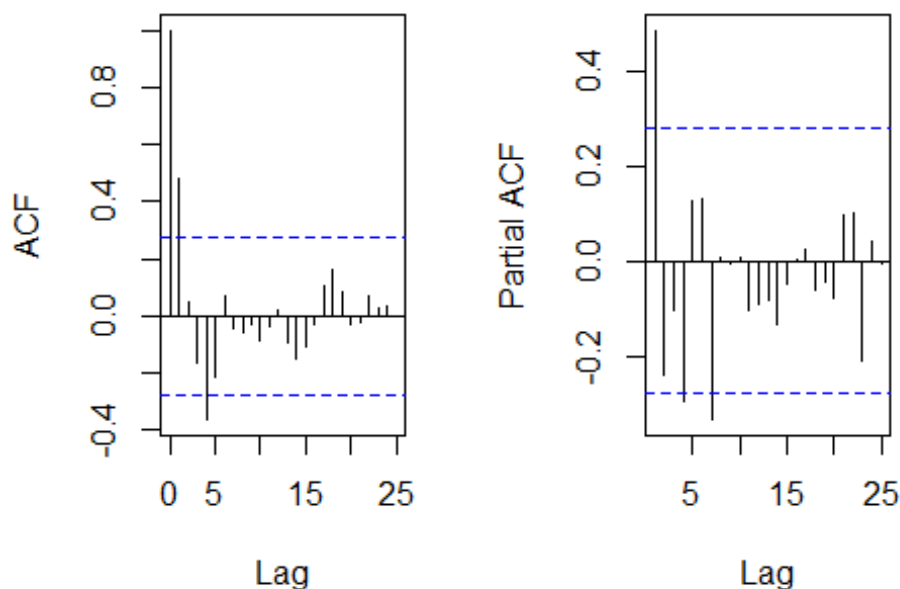
- b. Calculate and plot the sample autocorrelation and PACF. Is there significant autocorrelation in this time series?

Answer:

The following table show the calculated value of ACF and PACF values for this given time series data. The following two plots show the sample ACF and PACF for different lag k. Yes, there is a significant autocorrelation in this time series because the ACF values are quickly reduced to zero and later ACF values are almost varies within the boundary limit (except ACF for lag 4). So therefore, we can say this is a stationay time series with cut off at 1. Whereas PACF shows unstable autocorrelation as the valuse are several time going outside of the limit.

```
par(mfrow=c(1,2),oma=c(0,0,0,0))
my.data1.acf <- acf(my.data1[,2],lag.max=25,type="correlation",main="ACF for
the time series")
my.data1.pacf <- acf(my.data1[,2],lag.max=25,type="partial",main="PACF for th
e time series")
```

ACF for the time series PACF for the time serie



```
Table1 <- NULL
Table1$Lag <- seq(1,25,1)
Table1$ACF <- my.data1.acf$acf[2:26]
Table1$PACF <- my.data1.pacf$acf
Table1 <- as.data.frame(Table1)
Table1
```

##	Lag	ACF	PACF
## 1	1	0.48397075	0.483970746
## 2	2	0.05065940	-0.239716530
## 3	3	-0.16114672	-0.104640129
## 4	4	-0.36183934	-0.293598338
## 5	5	-0.21605363	0.126947995
## 6	6	0.07074210	0.128628429
## 7	7	-0.04003872	-0.332996134
## 8	8	-0.05901962	0.007295431
## 9	9	-0.03012882	-0.006531347
## 10	10	-0.08452953	0.007076603
## 11	11	-0.03757033	-0.103799840
## 12	12	0.01816018	-0.091727477
## 13	13	-0.09467604	-0.083205309
## 14	14	-0.14714084	-0.133401492
## 15	15	-0.10371884	-0.046816023
## 16	16	-0.03097154	0.002523423
## 17	17	0.10209938	0.023685228
## 18	18	0.16177338	-0.063157640
## 19	19	0.08753567	-0.042564399
## 20	20	-0.02838364	-0.076597566
## 21	21	-0.01868454	0.095339847
## 22	22	0.07158146	0.101169029
## 23	23	0.02702756	-0.210051169
## 24	24	0.03173156	0.041025313
## 25	25	-0.00369255	-0.005089582

Exercise 5.5 Consider the time series model

$$y_t = 150 - 0.5y_{t-1} + \varepsilon_t$$

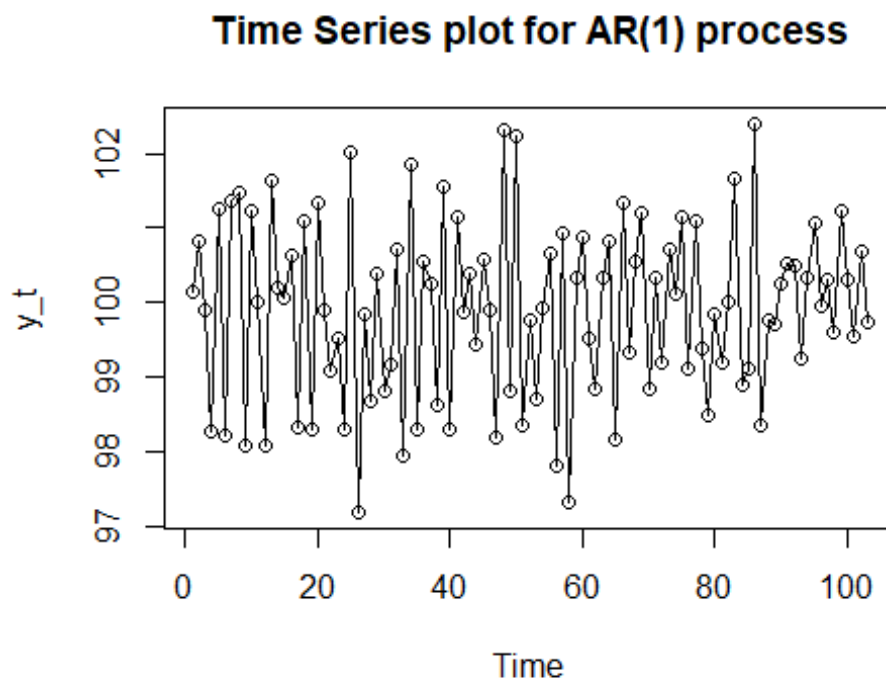
a. Is this a stationary time series process?

Answer:

Here, $y_t = 150 - 0.5y_{t-1} + \varepsilon_t$ is the given time series model. From this model's equation, we can say that this is First Order AR(1) process because it looks like general form of $y_t = \delta + \phi y_{t-1} + \varepsilon_t$ Where, $\delta = 150$, $\phi = -0.5$. As $|\phi| < 1$, So this time series is stationary.

```
# mean of the stationary AR(1) process 150/(1-(-0.5))=100
data_ar = arima.sim(list(order=c(1,0,0), ar = -0.5),n=103)+100

# Plot of my time series
ts.plot(data_ar, type="o", ylab = expression(y_t), main= "Time Series plot
for AR(1) process")
```



The above plot shows of given the time series. It also shows that y_t values are randomly plotted around the constant value (mean) of 100. It also indicates that time series is stationary.

b. What is the mean of the time series?

Answer:

The mean of the stationary AR(1) process is:

$$E(y_t) = \mu = \delta / (1 - \phi) = 150 / (1 - (-0.5)) = 100$$

- c. If the current observation $y_{100} = 85$, Would you expect the next observation to be above or below the mean?

Answer:

Given Equation,

$$y_t = 150 - 0.5y_{(t-1)} + \epsilon_t \dots \dots (1)$$

From, equation (1), it is observing that the time series value of next step is negatively correlated with current step (see that time series plot in 5.5a). That means, the next value will be high if the current values is small. And the next value will be small if the current values is high. See plot 5.5a, there is a lot of ups and downs in time series plot.

Therefore, we can expect that the next observation (y_{101}) will be above the mean value ($\mu = 100$) as the y_{100} is smaller than mean.

Lets try another way, $y_{100} = 85$, Putting the value in equation(1)

The observation will be $y_{100+1} = y_{101}$. We get: $y_{101} = 150 - 0.5 y_{100} + \epsilon_{101}$
or, $y_{101} = 150 - 0.5 * 85 + \epsilon_{101} = 107.5 + \epsilon_{101}$

From the above equation, we can expect the next (y_{101}) will be depends on the error of 101th value. But it is absence. However, we can expect that y_{101} value will more than 100.

Exercise 5.7 Consider the time series model

$$y_t = 20 + \epsilon_t + 0.2\epsilon_{t-1}$$

a. Is this a stationary time series process?

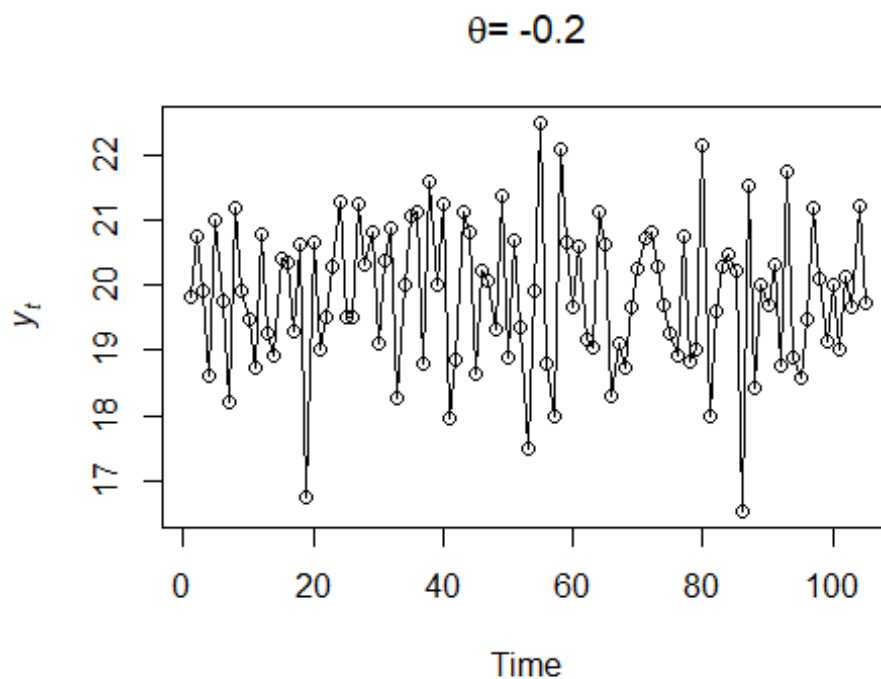
Answer:

Given time series model equation is $y_t = 20 + \epsilon_t + 0.2\epsilon_{t-1}$. From this equation, we can say that this is the first order MA(1) process because it follows the general equation of $y_t = \mu + \epsilon_t - \theta\epsilon_{t-1}$. Where, $\mu = 20, \theta = -0.2$. We know that, any finite order moving average process (MA(q)) is always stationary regardless of values of the weights. So, we can say that MA(1) process is stationary.

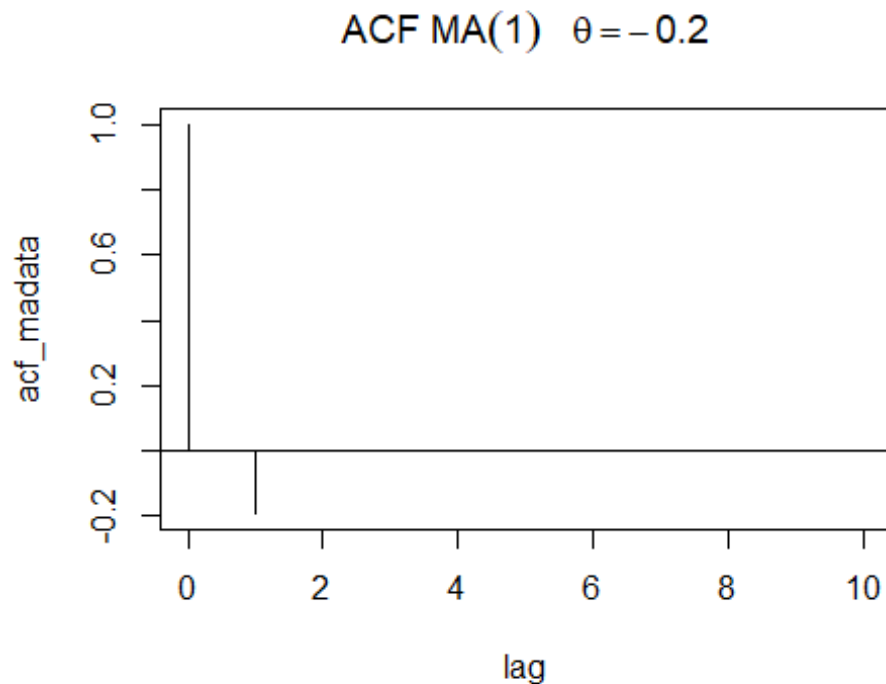
As $|\theta| < 1$, so, the time series is stationary.

```
#Model equation:  $y_t = 20 + \epsilon_t + 0.2\epsilon_{t-1}$ 
#List description for MA(1) model with weight
data_ma <- list(order=c(0,0,1), ma = -0.2, sd = 0.1)
#simulate MA(1)
MA1_process <- arima.sim(n= 105, model = data_ma) + 20

#plot the time series
ts.plot(MA1_process, type= "o", ylab = expression(italic(y)[italic(t)]), main
= expression(paste(theta, " = -0.2")))
```



```
nlag= 10
acf_madata <- ARMAacf(ma= -0.2, lag.max= nlag)
plot(0:nlag, acf_madata, type= "h", xlab= "lag", main= (expression(ACF~ MA(1)
~~~ theta== -0.2)))
abline(h= 0)
```



The above plot shows the time series for $y_t = 20 + \epsilon_t + 0.2\epsilon_{t-1}$ and its ACF. It also shows that y_t values are randomly plotted around the constant value (mean) of 20. It also indicates that this time series is stationary.

b. Is this an invertible time series?

Answer:

Yes, this is an invertible time series. From the model equation, we find that $|\theta| < 1$, So time series is invertible.

c. What is the mean of the time series?

Answer:

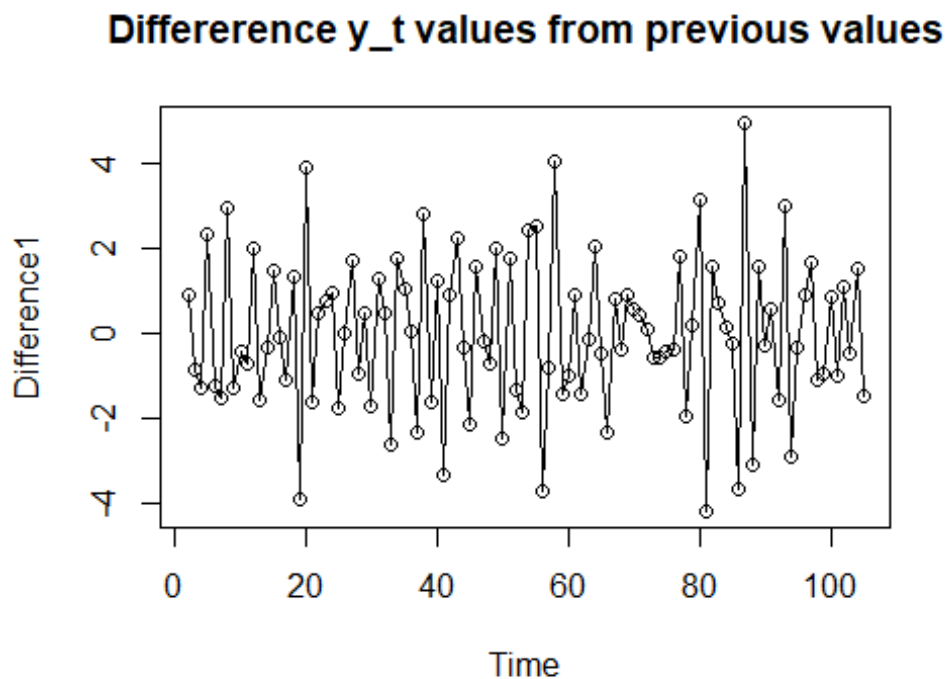
Mean value, $\mu = 20$

d. If the current observation is $y_{100} = 23$, would you expect the next observation to be above or below the mean? Explain your answer.

Answer:

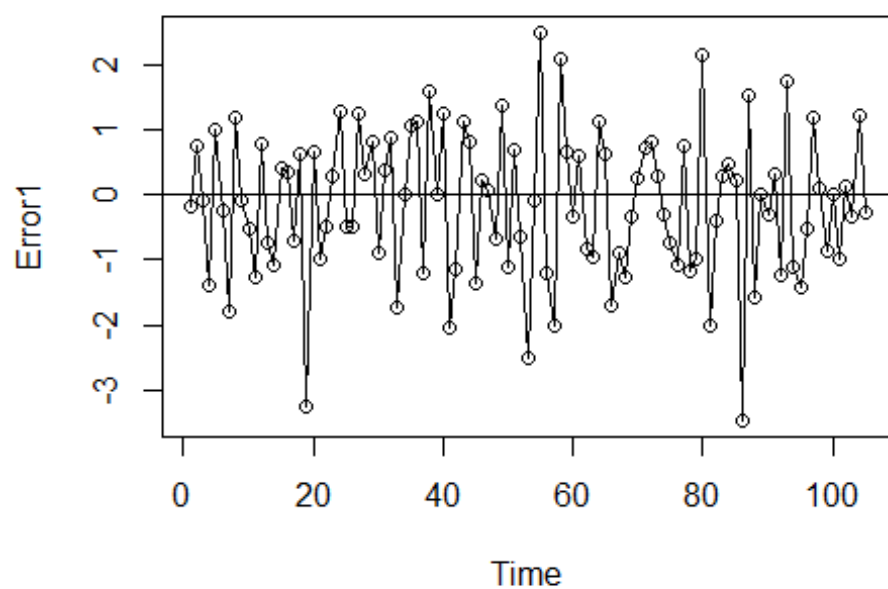
From the model equation and time series plot in 5.7a, we see that there are lots of ups and downs in data. Expecially for large and small values, having highest error from mean, gives more fluctuations. In this case, the given $y_{100} = 23$, which is larger value and have large deviation from mean. Therefore, next observation will fluctuate more (gives a down), so that we would expect the next observation will be below the value (23). Now, for more precise statement respect to mean, we have generates two plots. One is difference of time series and error or deviation from mean. We found that maximum deviation from consecutive y is 5.173763 and maximum deviation from mean is 2.954709. Now it becomes tough to conclue that the position of next obseration. It seems that the y_{101} would have value below the mean (20).

```
Difference1 <- diff(MA1_process)
plot (Difference1, type = "o", main = "Difference y_t values from previous values")
```



```
Error1 <- MA1_process - 20
plot (Error1, type = "o", main = "Deviation of y_t values from mean")
abline(h=0)
```


Deviation of y_t values from mean



```
max(Difference1)
```

```
## [1] 4.981412
```

```
max(Error1)
```

```
## [1] 2.489768
```

Exercise 5.8 Consider the time series model

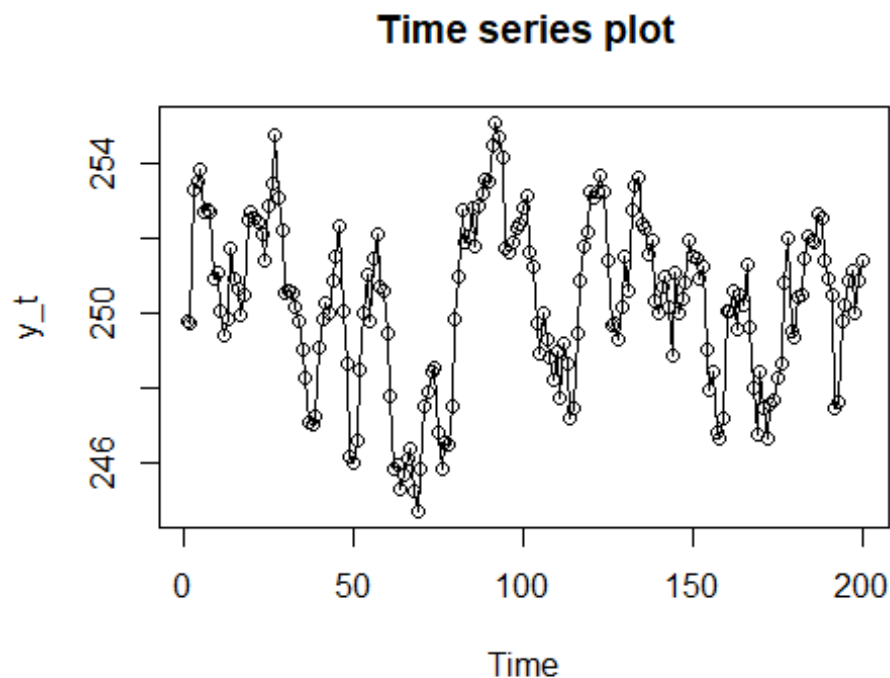
$$y_t = 50 + 0.8y_{t-1} + \epsilon_t - 0.2\epsilon_{t-1}$$

a. Is this a stationary time series process?

Answer:

This time series is a mixture of AR and MA process. This is basically ARMA(1,1) and follow the general equation of equation, $y_t - \phi_1 y_{t-1} = \delta + \epsilon_t - \theta_1 \epsilon_{t-1}$. Here, $\phi_1 = 0.8$. As $|\phi_1| < 1$, So the time series is stationary.

```
sim <- arima.sim(list(ar = 0.8,ma=0.2), n = 200)+ 250
ts.plot(sim, ylab="y_t", main="Time series plot",type="o")
```



For the visual inspection, we have plot the time series in above figure. It also shows that y_t values are randomly plotted around the constant value (mean) of 250. It also indicates that this time series is stationary.

b. What is the mean of the time series?

Answer:

The mean of the time series is $\mu = \delta / (1 - \phi_1) = 50 / (1 - 0.8) = 250$

c. If the current observation is $y_{100} = 270$, would you expect the next observation to be above or below the mean?

See the time series plot in 5.8a for the model $y_t = 50 + 0.8y_{t-1} + \varepsilon_t - 0.2\varepsilon_{t-1}$. It seems that y_t values are increasing gradually upto certain range and again decreasing up to certain range. But this ups and downs are not quick at all (see the below figure of deviation of y_t value from mean).

It also seems that y_t has the positive correlation with previous value and negative correlation with previous error.

Therefore, $y_{100} = 270$ and next (y_{101}) observation, we would expect above the mean (250). It will not reduce drastically and will not pass mean (250).

```
# Define an ARMA(1,1) function
# Error term
Error2 <- sim - 250
plot(Error2, type = "o", main = "Deviation of y_t values from mean")
abline(h=0)
```

