

Module 5 : Hypothesis Testing



Hypothesis Testing

- ▶ Defining Hypothesis
- ▶ Type I and Type II Error
- ▶ One Sample Z-Test
- ▶ One Sample T-Test
- ▶ Single Sample Test for Population Proportion
- ▶ Testing Equality of Variances
- ▶ Testing the Difference Between Two Population Means



Defining Hypothesis

In the world we often need to make decisions based on population parameters. Hypothesis Testing helps us make these decisions.

- ▶ Does a drug reduce blood pressure?
- ▶ Does reduced class size increase test scores?
- ▶ Is a person innocent or guilty of a crime?
- ▶ Does more money spent on education in low-income areas improve student performance?



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Defining Hypothesis (2)

- ▶ Null and Alternative Hypotheses

 - Overview of Hypotheses Testing

 - Null and Alternative Hypotheses

 - Hypothesis Testing and Null and Alternative Hypotheses

- ▶ Upper One-Sided Alternative

 - One and Two Tailed Tests

 - Upper One-Sided Alternative

- ▶ Lower One-Sided Alternative



Defining Hypothesis (3)

- ▶ Two-Tailed Alternative
- ▶ One-Tailed or Two-Tailed Test

Choosing between One-Tailed or Two-Tailed Test

One-Tailed or Two-Tailed Test



Hypothesis Testing

1. Defining Null (H_0) and Alternative Hypotheses (H_a or H_1)
2. Formulate Analysis Plan: level of significance (90%, 95%, 99%) & test statistics (t-test, z-test, f-test, etc.)
3. Conduct study & gather the data
4. Analyze the data
5. Interpret the result

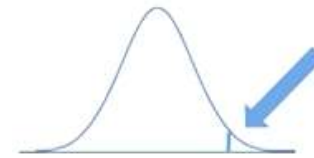
One Tailed Test

- ▶ A test of a statistical hypothesis where the region of rejection is on only one side of the sampling distribution

- ▶ Example: Upper One Sided

H_0 : the mean is ≤ 500 (left side)

H_a : the mean is > 500 (right side)



- ▶ Example: Lower One Sided

H_0 : the mean is ≥ 500 (right side)

H_a : the mean is < 500 (left side)



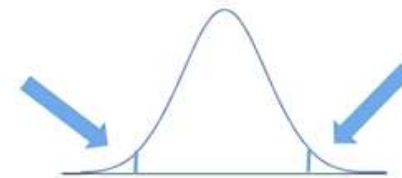
Two Tailed Test

- ▶ A test of a statistical hypothesis where the region of rejection is on both side of the sampling distribution (left & right)

- ▶ Example:

H_0 : the mean is = 500 (single line)

H_a : the mean is \neq 500 (left & right side)





One Tailed Or Two Tailed Test ?

- ▶ Two-tailed test:

A priori, use it if you have no idea of the direction in which deviations from the null hypothesis will occur.

- ▶ If a deviation from the null hypothesis is of interest in only one direction, then a one-tailed alternative hypothesis should be used

Type I and Type II Error

► Type I and Type II Error

Type I Error: Rejects a null hypothesis (H_0) when it is TRUE

Type II Error: Fail to reject a null hypothesis (H_0) when it is FALSE

		Test Conclusion	
		Do not reject H_0	Reject H_0
Truth	H_0 true	Correct	Type I error
	H_0 false	Type II error	correct

		Test Conclusion	
		Do not reject H_0	Reject H_0
Truth	No difference in quality	We can manufacture anywhere	Although there is no difference in quality, we only manufacture in county A
	Quality is better in country A	Quality is actually better in country A but we manufacture somewhere else	We manufacture in country A

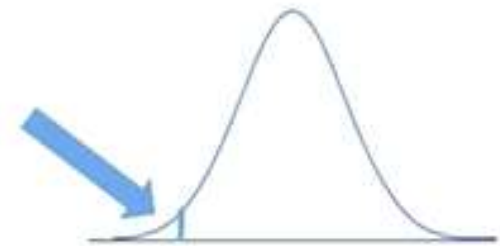
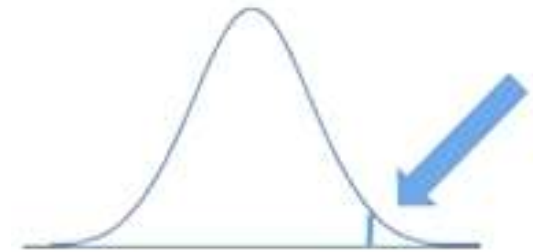


Type I and Type II Error

- ▶ α (alpha): the probability of making type I Error
or also known as Confidence Level
Reject H_0 , given it is TRUE
- ▶ β (Beta) : the probability of making Type II Error
Fail to Reject H_0 , given it is FALSE

Critical Region

- ▶ critical region is the range of values for a sample statistic that results in rejection of the null hypothesis
- ▶ Used to minimize the probability of making Type II error





Z-Test vs t-Test

► z-test is used for:

- testing the mean of a population versus a standard
- comparing the means of two populations
- whether you know the population standard deviation or not
- also used for testing the proportion of some characteristic versus a standard proportion,
or comparing the proportions of two populations.

*needs large ($n \geq 30$) samples

Z-Test vs T-Test

► t-test is used for:

- testing the mean of one population against a standard or
- comparing the means of two populations if you do not know the populations' standard deviation

if you know the populations' std. dev, you may use a z-test.

Example:

Measuring the average diameter of shafts from a certain machine when you have a small sample.

*can work with a limited sample ($n < 30$)



F-Test

► F-test is used for:

- comparing statistical models that have been fitted to a data set, in order to identify the model that best fits the population from which the data were sampled (comparing the variances)



T-Test vs F-Test

▶ t-test is used for:

- testing the mean of one population against a standard or
- comparing the means of two populations if you do not know the populations' standard deviation

if you know the populations' std. dev, you may use a z-test.

▶ F-test is used for:

finding out whether there is any variance within the samples or comparing variances of two samples/populations



One Sample Z-Test

- ▶ When to use a One Sample Z-Test
 - One Sample Z-Test
- ▶ Critical Region
 - Critical Region of the One Sample Z-Test
 - One Sample Z Test Example
- ▶ P-Values

p-Values

- ▶ p-Values (probability values):
the smallest value of alpha (α) for which the data indicates rejection of H_0
- ▶ Critical Region, P-Values, and T.INVERSE Function
- ▶ One Sample T-Test
 - One Sample T-Test : One Tailed
 - One Sample T-Test : Two Tailed
 - One Sample T-Test in Excel



One Sample T-Test

- ▶ T Random Variable

 - T Random Variable

 - T Random Variable in Excel

- ▶ Critical Region, P-Values, and T.INVERSE Function

- ▶ One Sample T-Test

 - One Sample T-Test : One Tailed

 - One Sample T-Test : Two Tailed

 - One Sample T-Test in Excel

Single Sample Test for Population Proportion

► Single Sample Test for Population Proportion

- Single Sample Test for Population Proportion

- Single Sample Test for Population Proportion in Excel

- Single Sample Test for Population Proportion Example

► Sample Size for Estimating a Population Proportion

- Sample size for Estimating a Population Proportion

Homework & Quiz