Module 5 : Hypothesis Testing

Hypothesis Testing

- Defining Hypothesis
- Type I and Type II Error
- One Sample Z-Test
- One Sample T-Test
- Single Sample Test for Population Proportion
- Testing Equality of Variances
- Testing the Difference Between Two Population Means
- Chi-Squared Test for Independence

Defining Hypothesis

In the world we often need to make decisions based on population parameters.   Hypothesis Testing helps us make these decisions.

- Does a drug reduce blood pressure?
- Does reduced class size increase test scores?
- Is a person innocent or guilty of a crime?
- Does more money spent on education in low-income areas improve student performance?

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Defining Hypothesis (2)

- Null and Alternative Hypotheses
  - Overview of Hypotheses Testing
  - Null and Alternative Hypotheses
  - Hypothesis Testing and Null and Alternative Hypotheses
- Upper One-Sided Alternative
  - One and Two Tailed Tests
  - Upper One-Sided Alternative
- Lower One-Sided Alternative

Defining Hypothesis (3)

- Two-Tailed Alternative
- One-Tailed or Two-Tailed Test
  - Choosing between One-Tailed or Two-Tailed Test
  - One-Tailed or Two-Tailed Test

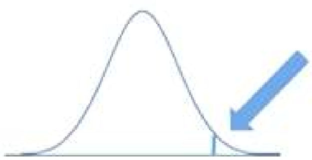
Hypothesis Testing

1. Defining Null (H_o)and Alternative Hypotheses (H_a or H_1)
2. Formulate Analysis Plan: level of significance (90%, 95%, 99%) & test statistics (t-test, z-test, f-test, etc.)
3. Conduct study & gather the data
4. Analyze the data
5. Interpret the result

One Tailed Test

- A test of a statistical hypothesis where the region of rejection is on only one side of the sampling distribution
- Example: Upper One Sided

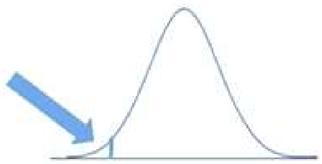
H_0: the mean is <= 500 (left side)



H_a: the mean is >500 (right side)

- Example: Lower One Sided

H_0: the mean is ≥ 500 (right side)



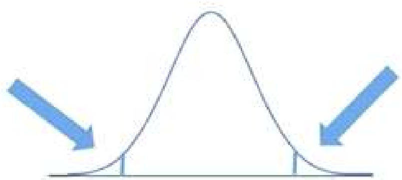
H_a: the mean is < 500 (left side)

Two Tailed Test

- A test of a statistical hypothesis where the region of rejection is on both side of the sampling distribution (left & right)
- Example:

H_0: the mean is = 500 (single line)

H_a: the mean is ≠ 500 (left & right side)



One Tailed Or Two Tailed Test ?

- Two-tailed test:

  A priori, use it if you have no idea of the direction in which deviations from the null hypothesis will occur.

- If a deviation from the null hypothesis is of interest in only one direction, then a one-tailed alternative hypothesis should be used

Type I and Type II Error

- Type I and Type II Error

  Type I Error: Rejects a null hypothesis (H_0) when it is TRUE

  Type II Error: Fail to reject a null hypothesis (H_0) when is it FALSE

		Test Conclusion	
		Do not reject H_0	Reject H_0
Truth	H_0 true	Correct	Type I error
	H_0 false	Type II error	correct

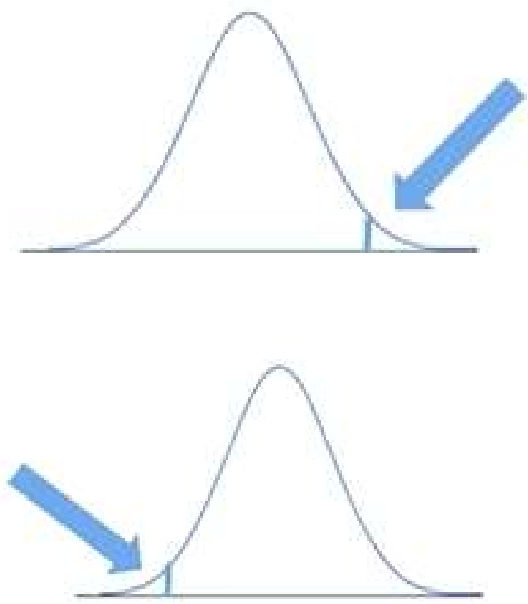
		Test Conclusion	
		Do not reject H_0	Reject H_0
Truth	No difference in quality	We can manufacture anywhere	Although there is no difference in quality, we only manufacture in county A
	Quality is better in country A	Quality is actually better in country A but we manufacture somewhere else	We manufacture in country A

Type I and Type II Error

- α (alpha): the probability of making type I Error
  or also known as Confidence Level
  Reject H_0, given it is TRUE
- β (Beta) : the probability of making Type II Error
  Fail to Reject H_0, given it is FALSE

Critical Region

- critical region is the range of values for a sample statistic that results in rejection of the null hypothesis
- Used to minimize the probability of making Type II error



Z-Test vs t-Test

- z-test is used for:
  - testing the mean of a population versus a standard
  - comparing the means of two populations
  - whether you know the population standard deviation or not
  - also used for testing the proportion of some characteristic versus a standard proportion,

    or comparing the proportions of two populations.

*needs large ($n \geq 30$) samples

Z-Test vs T-Test

- t-test is used for:

  - testing the mean of one population against a standard or

  - comparing the means of two populations if you do not know the populations' standard deviation

  if you know the populations' std. dev, you may use a z-test.

  Example:

  Measuring the average diameter of shafts from a certain machine when you have a small sample.

*can work with a limited sample (n < 30)

F-Test

- F-test is used for:

  - comparing statistical models that have been fitted to a data set, in order to identify the model that best fits the population from which the data were sampled (comparing the variances)

T-Test vs F-Test

- t-test is used for:

  - testing the mean of one population against a standard or

  - comparing the means of two populations if you do not know the populations’ standard deviation

   if you know the populations’ std. dev, you may use a z-test.

- F-test is used for:

  finding out whether there is any variance within the samples or comparing variances of two samples/populations

One Sample Z-Test

- When to use a One Sample Z-Test
  - One Sample Z-Test
- Critical Region
  - Critical Region of the One Sample Z-Test
  - One Sample Z Test Example
- P-Values

p-Values

- p-Values (probability values):

  the smallest value of alpha (α) for which the data indicates rejection of H_0
- Critical Region, P-Values, and T.INVERSE Function
- One Sample T-Test

  One Sample T-Test : One Tailed

  One Sample T-Test : Two Tailed

  One Sample T-Test in Excel

One Sample T-Test

- T Random Variable
  - T Random Variable
  - T Random Variable in Excel
- Critical Region, P-Values, and T.INVERSE Function
- One Sample T-Test
  - One Sample T-Test : One Tailed
  - One Sample T-Test : Two Tailed
  - One Sample T-Test in Excel

Single Sample Test for Population Proportion

- Single Sample Test for Population Proportion
  - Single Sample Test for Population Proportion
  - Single Sample Test for Population Proportion in Excel
  - Single Sample Test for Population Proportion Example
- Sample Size for Estimating a Population Proportion
  - Sample size for Estimating a Population Proportion

M5L5HW1- Question

- A cell phone chip vendor needs to prove that they are producing at most 1% defective chips.

  Design an appropriate null and alternative hypotheses.

  In a sample of 500 chips from a large population, suppose 10 were defective.

  For $\alpha = 0.05$, would you conclude that the new vendor's chips contain at most 1% defective chips?

  What do you conclude?

A. We fail to reject Ho; the p-value is 0.0311

B. We fail to reject Ho; the p-value is 0.0511

C. We reject Ho; the p-value is 0.0311

D. We reject Ho; the p-value is 0.0511

M5L5HW1- Answer

M5L5HW1		
H_0		$p \geq 0.01$
H_a		$p < 0.01$
a	0.05	
trial (n)	500	
probability (s)	0.01	1%
number_s1	10	
number_s2	500	
p	0.031102107	=BINOM.DIST.RANGE(B5,B6,B7,B8)
Answer	p < a	REJECT H_0 ; p=0.031102

Answer:

C. We reject Ho; the p-value is 0.0311

M5L5HW2- Question

- A basketball player has a long history of making 70% of his free throws.

  Early in the season, she has made 180 of 300 free throws.

  Does this provide sufficient evidence (for a=0.05) that the player's free throw shooting ability has changed?

  What do you conclude?

A. We fail to reject Ho; the p-value is 0.0003. The shooting ability has not changed.

B. We fail to reject Ho; the p-value is 0.03. The shooting ability has not changed.

C. We reject Ho, the p-value is 0.0003. The shooting ability has changed

D. We reject Ho, the p-value is 0.03. The shooting ability has changed

M5L5HW2- Answer

M5L5HW2			
H_0		$p = 0.7$	
H_a		$p \neq 0.7$	
a	0.05		
trial (n)	300		
probability (s)	0.7		70%
number_s1	180		
p	0.000285472	=2*BINOM.DIST(B18,B16,B17,TRUE)	
Answer	p < a	REJECT H_0 ; p=0.0003	

Answer:

C. We reject Ho, the p-value is 0.0003. The shooting ability has changed

Testing Equality of Variances & F-Test

- Testing Equality of Variances

  In statistics, an F-test of equality of variances is a test for the null hypothesis that two normal populations have the same variance.

  Testing Equality of Variances Example:

M5L6HW1- Question

- Use the data on annual stock and Tbill returns (provided in JulyVariancesHW.xlsx) to determine if Stocks and Tbills returns have equal variance. Use a = 0.05.

A. The variances are equal.
B. The variances are not equal.

M5L6HW1- Answer

H_0 : Stocks variance = Tbills variance

H_a: Stocks variance ≠ Tbills variance

p = 5.39908E-47 =F.TEST(H5:H92,I5:I92)

$p < a$

Answer:

B. The variances are not equal

M5L6HW3- Question

- You are given the total number of block shots and steals for 14 college basketball teams during 4 games. Determine if steals and block shots have equal variance. Use a = 0.01..

A. The variances are equal.
B. The variances are not equal.

M5L6HW3- Answer

H_0 : Block Shots variance = Steals variance

H_a: Block Shots variance ≠ Steals variance

p = 0.864097772 =F.TEST(O4:O17,P4:P17)

p > a

Answer:

A. The variances are equal

Testing the Difference Between Two Population Means (1)

- Four Types of Tests
  - Four Types of Tests
  - Which of the Four Types of Tests Should You Use and When
- Two Sample Z-Test
  - Two Sample Z-Test
  - Two Sample Z-Test in Excel
- Equal Variance T-Test
  - Equal Variance T-Test
  - Equal Variance T-Test in Excel

M5L7HW1- Question

- Use the given data on annual Tbills and 10 year bond returns (provided in Problem 1 of JulyTwoPopMeansHW.xlsx) to determine if Tbills and Bonds10 returns have equal means.

  Use $\alpha = 0.05$. The returns in each row are NOT from the same year.

A. Means are equal.

B. Means are not equal.

M5L7HW1- Answer

H_0 : Tbills mean = Bonds10 mean

H_a: Tbills mean ≠ Bonds10 mean

p value (two tails) = 0.0504

p > a

Answer:

A. The mean are equal

z-Test: Two Sample for Means		
	Tbills	Bonds10
Mean	0.034945455	0.052307955
Known Variance	0.000929	0.006002
Observations	88	88
Hypothesized Mean Difference	0	
z	-1.956390951	
P(Z<=z) one-tail	0.025209559	
z Critical one-tail	1.644853627	
P(Z<=z) two-tail	0.050419117	
z Critical two-tail	1.959963985	

M5L7HW2- Question

- You are given the amount of money spent at a supermarket by a sample of shoppers during the morning and afternoon hours (provided in Problem 2 of JulyTwoPopMeansHW.xlsx).

  For a = 0.05, does this data indicate that morning shoppers spend less than afternoon shoppers?.

A. The p-value is 0.01, so you conclude that the average spent by morning customers is less than the average spent by afternoon customers

B. The p-value is 0.10, so you conclude that the average spent by morning customers is greater than the average spent by afternoon customers

M5L7HW3- Question

- You are given the miles per gallon obtained for 20 cars using gasoline and ethanol (provided in Problem 3 of JulyTwoPopMeansHW.xlxs).

  For a = 0.01, does this data indicate that there is a significant difference in the MPG cars obtain when fueled by ethanol or gasoline?.

A. The p-value is 0.0082, so we reject that average mileage does not depend on whether gas or ethanol is used..

B. The p-value is 0.8200, so fail to reject that the average mileage does not depend on whether gas or ethanol is used

M5L7HW3- Answer

H_0 : MPG methanol mean = MPG gasoline mean

H_a: MPG methanol mean ≠ MPG gasoline mean

p value (two tails) = 0.82

p > a

Answer:

B. Fail to REJECT H0

t-Test: Paired Two Sample for Means		
	Variable 1	Variable 2
Mean	15.7	15.8
Variance	24.74736842	25.95789474
Observations	20	20
Pearson Correlation	0.929889284	
Hypothesized Mean Difference	0	
df	19	
t Stat	-0.236742894	
P(T<=t) one-tail	0.407694965	
t Critical one-tail	2.539483191	
P(T<=t) two-tail	0.815389929	
t Critical two-tail	2.860934606	

Testing the Difference Between Two Population Means (2)

- Unequal Variance T-Test
  - Unequal Variance T-Test
  - Unequal Variance T-Test in Excel
- Idea of Pairing Samples
  - Idea of Pairing
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- T-Test Paired Two Sample for Means
  - T-Test Paired Two Sample
  - Example of T-Test Paired Two Sample for Means

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Chi-Squared

- CHI-SQUARE TEST

  A chi-square independence test evaluates if two categorical variables are related in any way

  Example =>lab (eye color)

Chi-Squared Test for Independence

- Contingency Table and Hypothesis of Independence
  - Contingency Table and Hypothesis of Independence
  - Example of Contingency Table and Hypothesis of Independence
- Computation of Chi Squared Statistic
  - Chi-Squared Statistic
  - Computation of Chi Squared Statistic
- Conducting the Hypothesis Test and Computing the P-Value

Homework & Quiz