ECE M146, Spring 2018
Problem Set 0: Environment Setup

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1 Problem 1

Solution: Solution to problem 1
$y' = sin(z)e^{-x} - xsin(z)e^{-x}$

2 Problem 2

(a) problem 2(a)

Solution: Solution to problem 2a

$y^T z = \begin{pmatrix} 1 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = 2 + 9 = 11$

(b) problem 2(b)

Solution: Solution to problem 2b

$Xy = \begin{pmatrix} 2+12 \\ 1+9 \end{pmatrix} = \begin{pmatrix} 14 \\ 10 \end{pmatrix}$

(c) problem2(c)

Solution: Solution to problem 2c

X is invertible.

$X^{-1} = \frac{1}{2} \begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & -2 \\ \frac{-1}{2} & 1 \end{pmatrix}$

(d) problem 2(d)

Solution: Solution to problem 2d

rank(X)=2

3 Problem 3

(a) problem 3(a)

Solution: Solution to problem 3a

sample mean = 0.6

(b) problem 3(b)

Solution: Solution to problem 3b

sample variance $= \frac{1}{5}[3 \times (\frac{2}{5})^2 + 2 \times (\frac{3}{5})^2] = \frac{6}{25}$

(c) problem 3(c)

Solution: Solution to problem 3c

$P(S) = 0.5^5 = \frac{1}{32}$

(d) problem 3(d)

Solution: Solution to problem 3d Suppose that the probability of heads is p, then the probability of tails is (1-p).

$P(S) = p^2 \times (1-p)^3 = -p^5 + p^2$

$P'(S) = -5p^4 + 2p$

So $p = \sqrt[3]{\frac{5}{2}}$

(e) problem 3(e)

Solution: Solution to problem 3e

$P = \frac{P(X=T, Y=b)}{P(Y=b)} = \frac{0.1}{0.1+0.15} = \frac{1}{4}$

4 Problem 4

Solution: Solution to problem 4

(a) False

(b) True

(c) False

(d) False

(e) True

5 Problem 5

Solution: Solution to problem 5

(a) — (v)

(b) — (iv)

(c) — (ii)

(d) — (iii)

(e) — (i)

6 Problem 6

(a) problem 6(a)

Solution: Solution to problem 6(a)

mean: p

variance: p(1-p)

(b) problem 6(b)

Solution: Solution to problem 6(b)

variance of 2X: $4\delta^2$

variance of X+2: δ^2

7 Problem 7

(a) problem 7(a)

Solution: Solution to problem 7(a)

i. problem 7(a)(i)
Both. Since f(n) and g(n) grow in same speed as n becomes large.

ii. problem 7(a)(ii)
g(n)=O(f(n)). Since f(n) grows much rapidly as n becomes large.

iii. problem 7(a)(iii)
f(n)=O(g(n)). Since g(n) grows much rapidly as n becomes large.

(b) problem 7(b)

Solution: Solution to problem 7(b)

We can use binary search in this problem. Each time, we compare the number in the middle of the array with 1. If it is 0, then we search in its right area, if it is 1, then we search in the left area.

8 Problem 8

(a) problem 8(a)

Solution: Solution to problem 8(a)

$E[XY] = \int xyP(x,y)\,dxdy. = \int xP(x)yP(y)\,dxdy = E[x]E[Y]$

(b) problem 8(b)

Solution: Solution to problem 8(b)

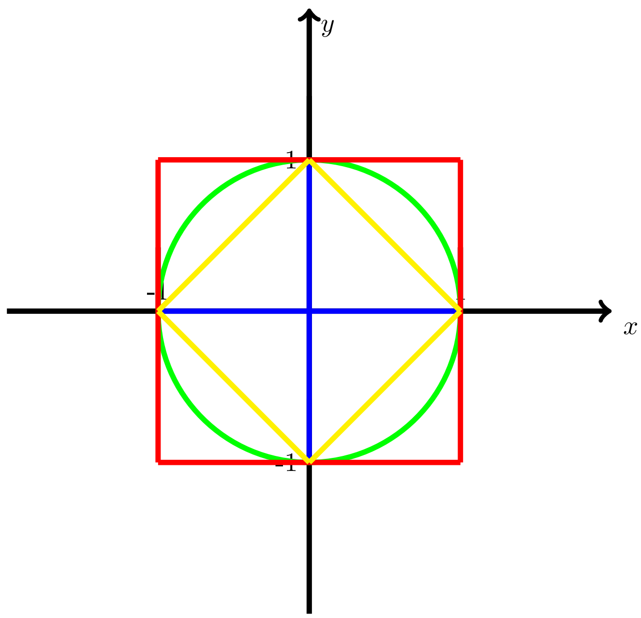
i. If a fair die is rolled 6000 times, the number of times 3 shows up is close to 1000 due to the law of large number.

ii. It is true due to the Central Limit Theorem

9 Problem 9

(a) problem 9(a)

Solution: Solution to problem 9(a)



i. shown in green line
ii. shown in blue line
iii. shown in yellow line
iv. shown in red line

(b) problem 9(b)

Solution: Solution to problem 7(a)

i. In linear algebra, an eigenvector of a linear transformation is a non-zero vector that only changes by a scalar factor when that linear transformation is applied to it. More formally, if T is a linear transformation from a vector space V over a field F into itself and v is a vector in V that is not the zero vector, then v is an eigenvector of T if T(v) is a scalar multiple of v. This condition can be written as the equation:

   $T(v) = \lambda v$

   where is a scalar in the field F, known as the eigenvalue associated with the eigenvector v.

ii. eigenvalue = 3 and -1

For eigenvalue=3, it's eigenvector = $\begin{pmatrix} 0.70710678 \\ -0.70710678 \end{pmatrix}$

For eigenvalue=-1, it's eigenvector = $\begin{pmatrix} 0.70710678 \\ 0.70710678 \end{pmatrix}$

iii. $Av = \lambda v$

$(Av)^k = (\lambda v)^k$

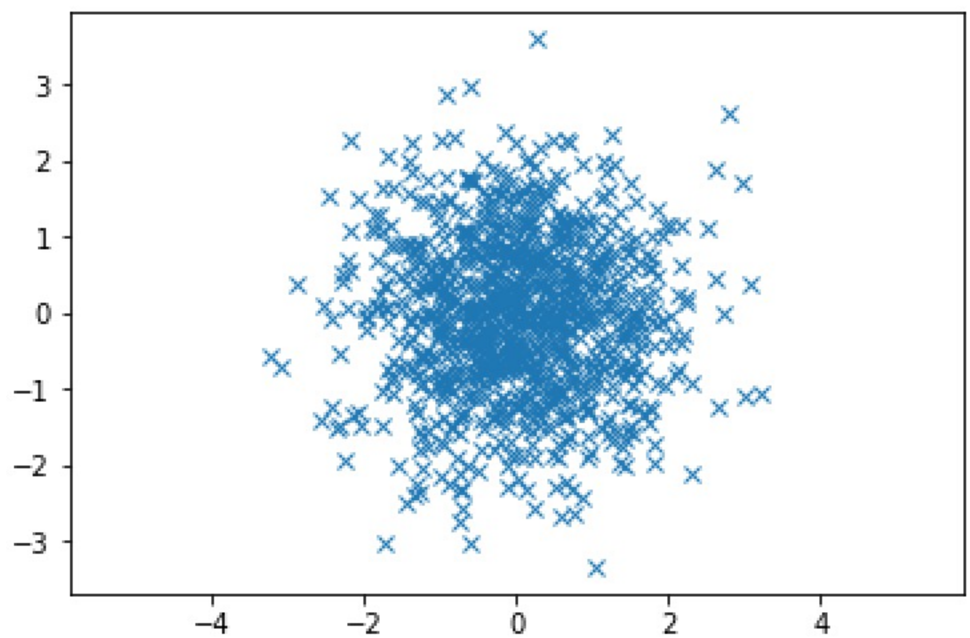
$A^k v = \lambda^k v$

So the eigenvalues of A^k are$\lambda 1^k, \lambda 2^k, ...\lambda n^k$the k^{th} powers of the eigenvalues of matrix A, and that each eigenvector of A is still an eigenvector of A^k.

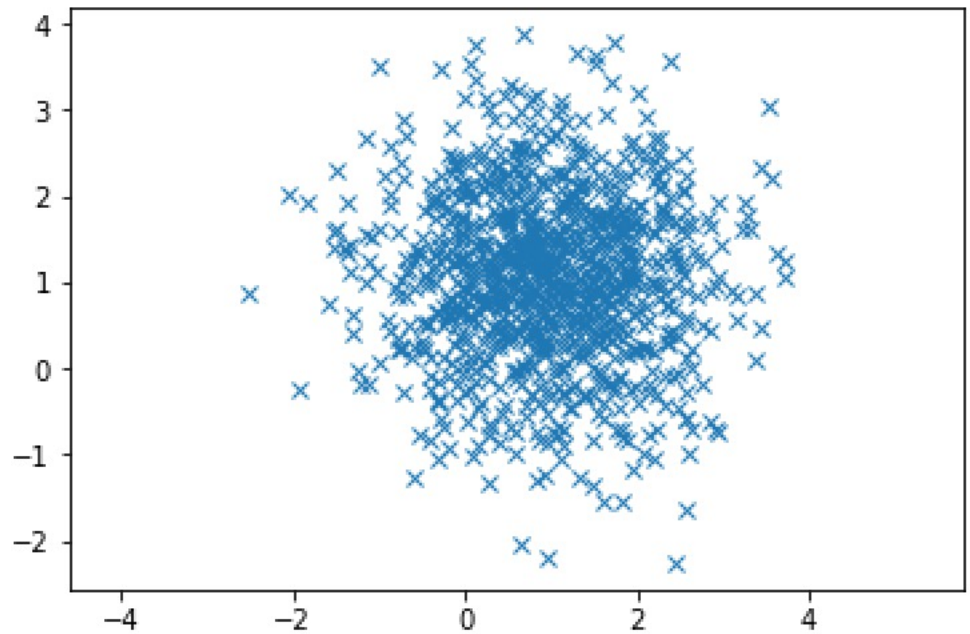
10 Problem 10

Solution: Solution to problem 10

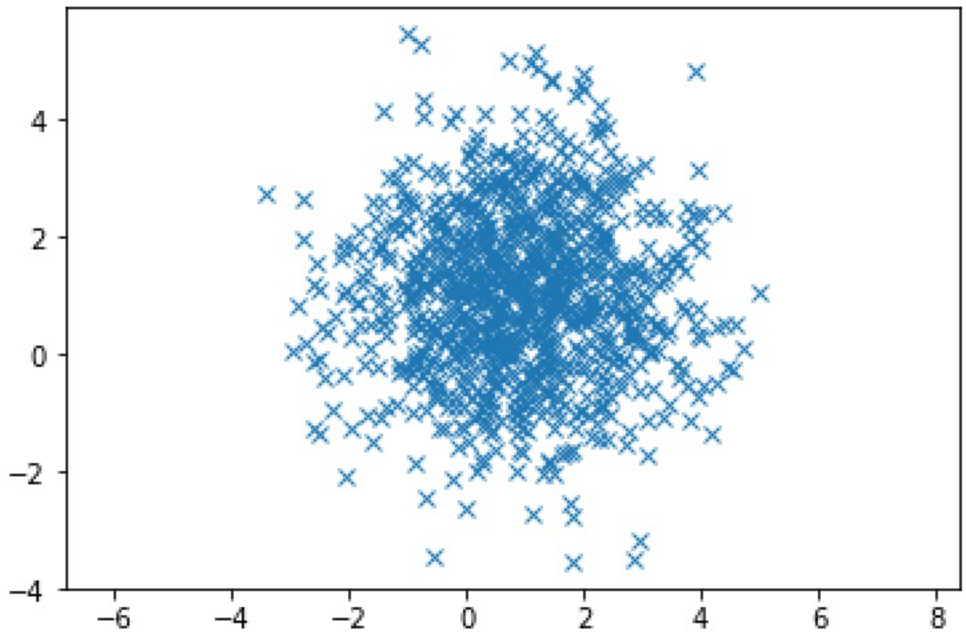
(a) problem 10(a)



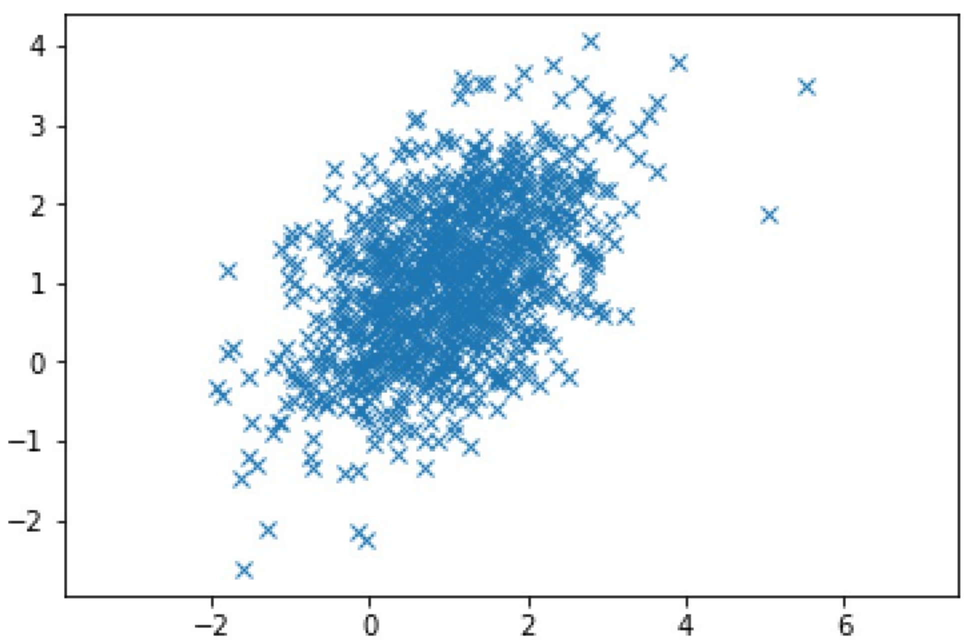
(b) problem 10(b)



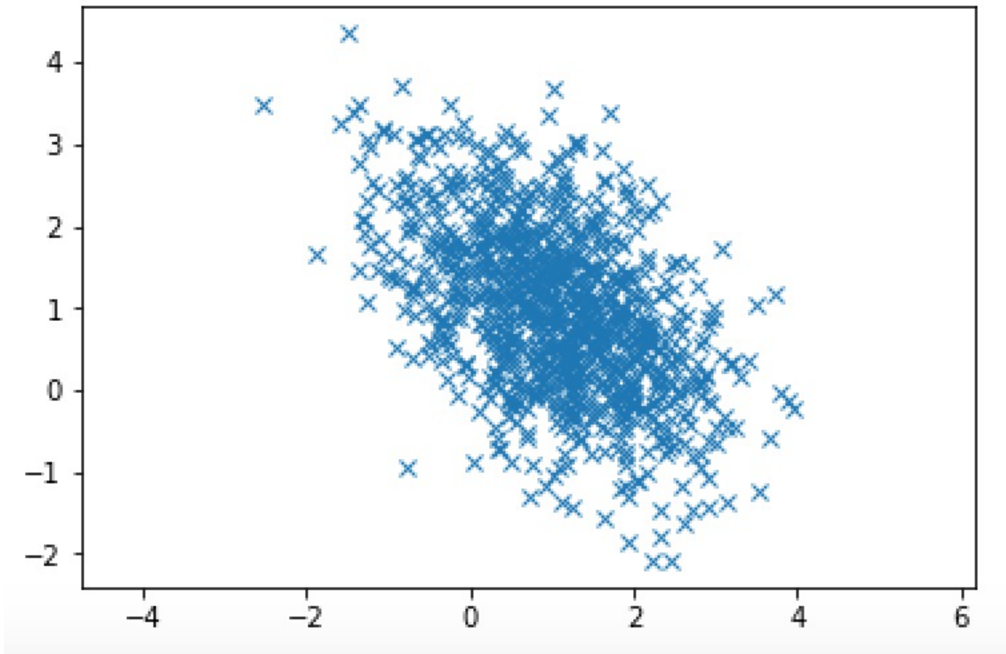
(c) problem 10(c)



(d) problem 10(d)



(e) problem 10(e)



11 Problem 11

Solution: Solution to problem 11

It's largest eigenvalue is 3. The computed eigenvector is $\begin{pmatrix} 0 \\ 0.89442719 \end{pmatrix}$

12 Problem 12

Solution: Solution to problem 12

(a) TV Human Interaction Dataset

(b) $http : //www.robots.ox.ac.uk/\ alonso/tv_human_interactions.html$

(c) TV Human Interaction Dataset consists of 300 video clips collected from over 20 different TV shows and containing 4 interactions: hand shakes, high fives, hugs and kisses, as well as clips that don't contain any of the interactions. It's used to predict the human interaction.

(d) 300

(e) There are 4 features: Hand Shakes, High Fives, Hugs, Kisses.