Stochastic Gradient Push for Distributed Deep Learning + HHHFL: Hierarchical Heterogeneous Horizontal Federated Learning for Electroencephalography

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Stochastic Gradient Push for Distributed Deep Learning

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Distributed Optimization

Problem formulation

$$\min_{\boldsymbol{x}_i \in \mathbb{R}^d, i=1,\ldots,n} \quad \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}_{\xi_i \sim D_i} F_i\left(\boldsymbol{x}_i; \xi_i\right)$$

$$\text{subject to} \quad \boldsymbol{x}_i = \boldsymbol{x}_j, \forall i, j = 1, \ldots, n$$

Two Principle

1. Fit Local Model and Data (Training)
2. Fit Local Model and Other Model (Consensus)

Consensus

Approximate distributed averaging

$$\frac{1}{n}\sum_{i=1}^{n}\boldsymbol{y}_i^{(0)} \qquad \boldsymbol{y}_i^{(0)} \in \mathbb{R}^d \qquad \boldsymbol{Y}^{(0)} \in \mathbb{R}^{n\times d}$$

$$\boldsymbol{y}_i^{(k+1)} = \sum_{j=1}^{n} p_{i,j}^{(k)}\boldsymbol{y}_j^{(k)}$$

$$\boldsymbol{Y}^{(k+1)} = \boldsymbol{P}^{(k)}\boldsymbol{Y}^{(k)} \qquad \boldsymbol{P}^{(k)} \in \mathbb{R}^{n\times n}$$

Doubly-stochastic Matrix

$$P = \begin{pmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{pmatrix}, \begin{pmatrix} 0.3 & 0.7 \\ 0.7 & 0.3 \end{pmatrix}, \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}$$

$$\lim_{K\to\infty}\prod_{k=0}^{K}\boldsymbol{P}^{(k)} = \{p(i,j) = \frac{1}{n}\} \qquad \boldsymbol{y}_i^{(\infty)} = \frac{1}{n}\sum_{i=1}^{n}\boldsymbol{y}_i^{(0)}$$

Consensus

Column-stochastic Matrix

$$\boldsymbol{Y}^{(k+1)} = \boldsymbol{P}^{(k)}\boldsymbol{Y}^{(k)} \qquad \boldsymbol{P}^{(k)} \in \mathbb{R}^{n\times n}$$

$$P = \begin{pmatrix} 0.2 & 0.3 \\ 0.8 & 0.7 \end{pmatrix}, \begin{pmatrix} 0.3 & 0.4 \\ 0.7 & 0.6 \end{pmatrix}, \begin{pmatrix} 0.23 & 0.23 \\ 0.76 & 0.76 \end{pmatrix}$$

$$\lim_{K\to\infty} \prod_{k=0}^{K} \boldsymbol{P}^{(k)} = \boldsymbol{\pi}\mathbf{1}^\top$$

$$\boldsymbol{Y}^{(\infty)} = \boldsymbol{\pi}\left(\mathbf{1}^\top \boldsymbol{Y}^{(0)}\right)$$

$$\boldsymbol{y}_i^{(\infty)} = \pi_i \sum_{j=1}^{n} \boldsymbol{y}_j^{(0)}$$

Add a scalar parameter $w_i^{(k)}$

Consensus

Column-stochastic Matrix

$$\lim_{K\to\infty} \prod_{k=0}^{K} \boldsymbol{P}^{(k)} = \boldsymbol{\pi}\mathbf{1}^\top$$

$$\boldsymbol{w}^{(k+1)} = \boldsymbol{P}^{(k)}\boldsymbol{w}^{(k)} \qquad w_i^{(0)} = 1$$

$$\boldsymbol{w}^{(\infty)} = \boldsymbol{\pi}\left(\mathbf{1}^\top\boldsymbol{w}^{(0)}\right)$$

$$w_i^{(\infty)} = \pi_i n$$

De-biased ratio

$$\frac{\boldsymbol{y}_i^{(\infty)}}{w_i^{(\infty)}} = \frac{1}{n}\sum_{i=1}^{n}\boldsymbol{y}_i^{(0)}$$

Stochastic Gradient Push

Algorithm 1 Stochastic Gradient Push (SGP)

Require: Initialize $\gamma > 0$, $\boldsymbol{x}_i^{(0)} = \boldsymbol{z}_i^{(0)} \in \mathbb{R}^d$ and $w_i^{(0)} = 1$ for all nodes $i \in \{1, 2, \ldots, n\}$

for $k = 0, 1, 2, \cdots, K$, at node i, **do**

  Sample new mini-batch $\xi_i^{(k)} \sim \mathcal{D}_i$ from local distribution

  Compute mini-batch gradient at $\boldsymbol{z}_i^{(k)}$: $\nabla \boldsymbol{F}_i(\boldsymbol{z}_i^{(k)}; \xi_i^{(k)})$

  $\boldsymbol{x}_i^{(k+\frac{1}{2})} = \boldsymbol{x}_i^{(k)} - \gamma \nabla \boldsymbol{F}_i(\boldsymbol{z}_i^{(k)}; \xi_i^{(k)})$

  Send $\left(p_{j,i}^{(k)} \boldsymbol{x}_i^{(k+\frac{1}{2})}, p_{j,i}^{(k)} w_i^{(k)}\right)$ to out-neighbors;
  receive $\left(p_{i,j}^{(k)} \boldsymbol{x}_j^{(k+\frac{1}{2})}, p_{i,j}^{(k)} w_j^{(k)}\right)$ from in-neighbors

  $\boldsymbol{x}_i^{(k+1)} = \sum_j p_{i,j}^{(k)} \boldsymbol{x}_j^{(k+\frac{1}{2})}$

  $w_i^{(k+1)} = \sum_j p_{i,j}^{(k)} w_j^{(k)}$

  $\boldsymbol{z}_i^{(k+1)} = \boldsymbol{x}_i^{(k+1)} / w_i^{(k+1)}$

end for

HHHFL: Hierarchical Heterogeneous Horizontal Federated Learning for Electroencephalography

DashanGao, CeJu, Xiguang Wei, Yang Liu, Tianjian Chen, Qiang Yang

Heterogeneous Data

Electroencephalography (EEG)

Heterogeneous & Privacy

Data Heterogeneity

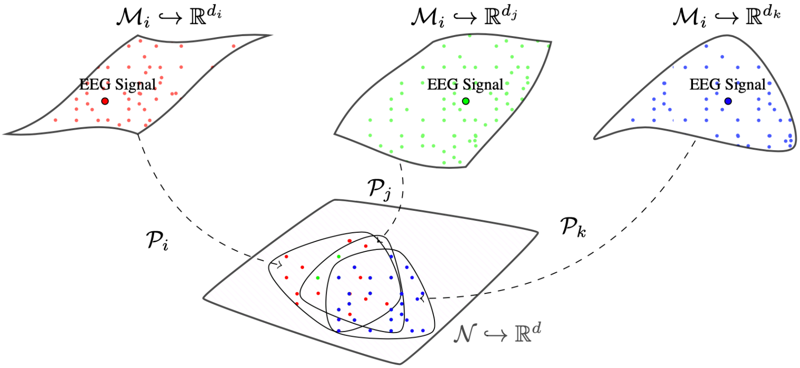
Heterogeneous Domain Adaptation

Privacy-Preserving

Federated Learning

Heterogeneous Domain Adaptation

Heterogeneous Domain Adaptation



$\mathcal{P}_i$?

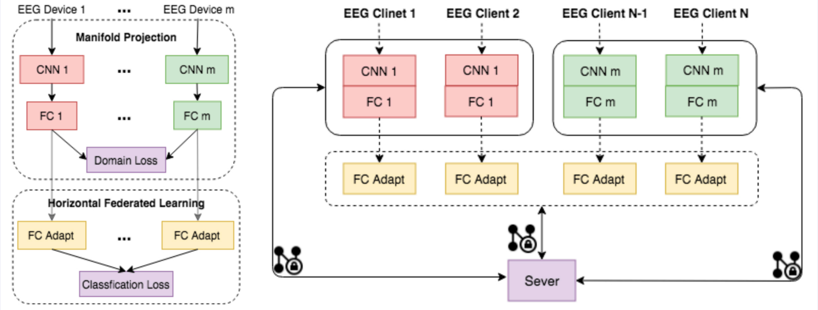
Neural Network Approach

Heterogeneous Domain Adaptation

Loss Function

Classification Loss & Domain Loss

Heterogeneous Domain Adaptation & Federated Learning



Maximum Mean Discrepancy (MMD)

Maximum Mean Miscrepancy (MMD)

$$\mathcal{L} := \mathcal{L}_C \left(X_{EEG}, Y \right) + \sum_{1 \leq i < j \leq m} \lambda_{i,j} \cdot \mathrm{MMD}^2 \left(\mathcal{Q}_i, \mathcal{Q}_j \right)$$

$$\mathrm{MMD} \left(\mathcal{Q}_i, \mathcal{Q}_j \right) := \left\| \mathbb{E}_{\mathcal{P}_i(X_i) \sim \mathcal{Q}_i} \psi \left(\mathcal{P}_i \left(X_i \right) \right) - \mathbb{E}_{\mathcal{P}_j(X_j) \sim \mathcal{Q}_j} \psi \left(\mathcal{P}_j \left(X_j \right) \right) \right\|_{\mathcal{H}}$$

$$\mathcal{P}_i \left(\{ x_1, \cdots, x_{N_i} \} \right) \sim \mathcal{Q}_i, \quad \psi : \mathcal{N} \longrightarrow \mathcal{H}$$

Reproducing Kernel Hilbert Space (RKHS)

Maximum Mean Discrepancy (MMD)

MMD in Euclidean Space

$$MMD[F, p, q] := \sup_{f \in F} \left(E_{x \sim p}[f(x)] - E_{y \sim q}[f(y)] \right)$$

$$MMD[F, X, Y] := \sup_{f \in F} \left(\frac{1}{m} \sum_{i=1}^{m} f(x_i) - \frac{1}{n} \sum_{j=1}^{n} f(x_j) \right)$$

F is rich enough but not too much

Maximum Mean Discrepancy (MMD)

Reproducing Kernel Hilbert Space (RKHS)

$$f(x) = \langle f, \phi(x) \rangle_{\mathcal{H}} \qquad f \to f(x)$$

$$E_p[f(x)] = \langle f, E_p[\phi(x)] \rangle_{\mathcal{H}}$$

MMD in RKHS

$$\begin{aligned} MMD[F, p, q] &= \sup_{\|f\|_{\mathcal{H}} \leq 1} E_p[f(x)] - E_q[f(y)] \\ &= \sup_{\|f\|_{\mathcal{H}} \leq 1} E_p\left[\langle \phi(x), f \rangle_{\mathcal{H}}\right] - E_p\left[\langle \phi(y), f \rangle_{\mathcal{H}}\right] \\ &= \sup_{\|f\|_{\mathcal{H}} \leq 1} \langle \mu_p - \mu_q, f \rangle_{\mathcal{H}} \\ &= \|\mu_p - \mu_q\|_{\mathcal{H}} \end{aligned}$$

Maximum Mean Discrepancy (MMD)

MMD in RKHS

$$
\begin{aligned}
MMD^2[F,p,q] &:= \langle \mu_p - \mu_q, \mu_p - \mu_q \rangle_{\mathcal{H}} \\
&= \langle \mu_p, \mu_q \rangle_{\mathcal{H}} + \langle \mu_q, \mu_p \rangle_{\mathcal{H}} - 2 \langle \mu_p, \mu_q \rangle_{\mathcal{H}} \\
&= E_p \left\langle \phi(x), \phi\left(x'\right) \right\rangle_{\mathcal{H}} + E_q \left\langle \phi(y), \phi\left(y'\right) \right\rangle_{\mathcal{H}} \\
&- 2E_{p,q} \langle \phi(x), \phi(y) \rangle_{\mathcal{H}}
\end{aligned}
$$

Universal Kernel

$$
k\left(x, x'\right) = \exp\left(-\frac{\left\|x - x'\right\|^2}{2\sigma^2}\right)
$$

MMD in Euclidean Space

$$
MMD^2[F,p,q] = \frac{1}{m(m-1)} \sum_{i \neq j}^{m} k\left(x_i, x_j\right) + \frac{1}{n(n-1)} \sum_{i \neq j}^{n} k\left(y_i, y_j\right) - \frac{2}{mn} \sum_{i,j=1}^{m,n} k\left(x_i, y_j\right)
$$

The End