Electric field due to ring & disc of charge

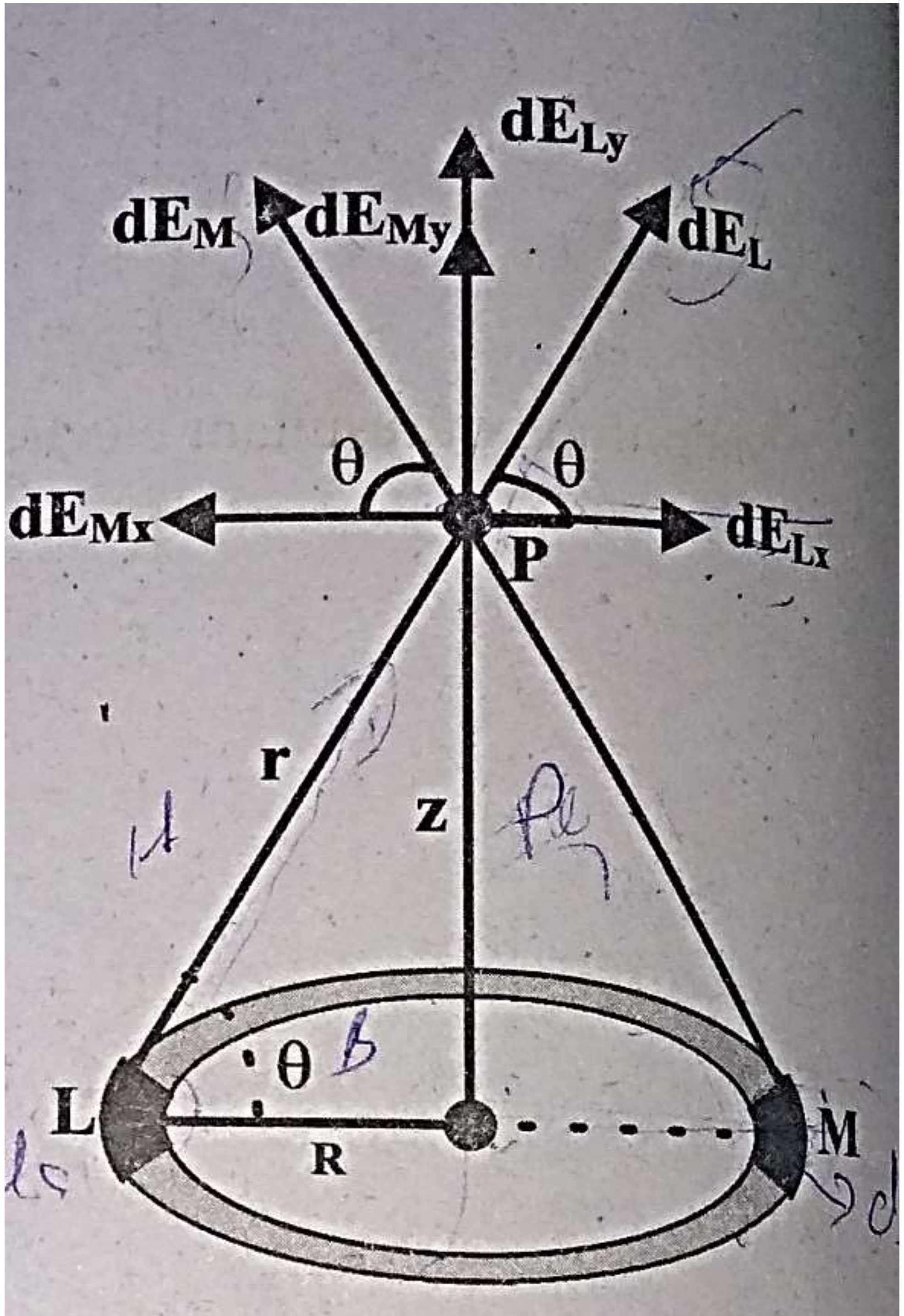
Electric Field due to Ring of Charge

Consider a positively charged ring having radius R on which positive charge q is distributed uniformly. This is called linear charge distribution. Take a small length element ds of ring having charge dq.

The linear charge density λ is defined as

$$\lambda = \frac{dq}{ds}$$

Now consider two length elements ds at opposite ends of a diameter of ring. We have to calculate electric field at P which is now perpendicular axis at distance z from the ring.


dELy
dEM
dEMy
dEL
θ
θ
dEMx
dELx
P
r
z
θ
L
R
M

Electric Field due to Ring of Charge

From figure:

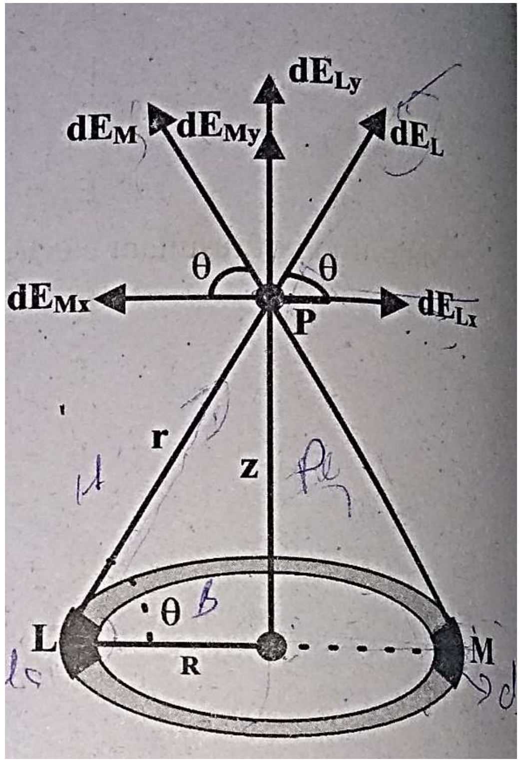
$$r^2 = z^2 + R^2$$

The magnitude of electric field at P due to charge element L is

$$dE_L = \frac{k\,dq}{r^2}$$

Similarly, the magnitude of electric field at P due to charge element M is

$$dE_M = \frac{k\,dq}{r^2}$$



Electric Field due to Ring of Charge

Comparing both equations, we get

$$dE_L = dE_M$$

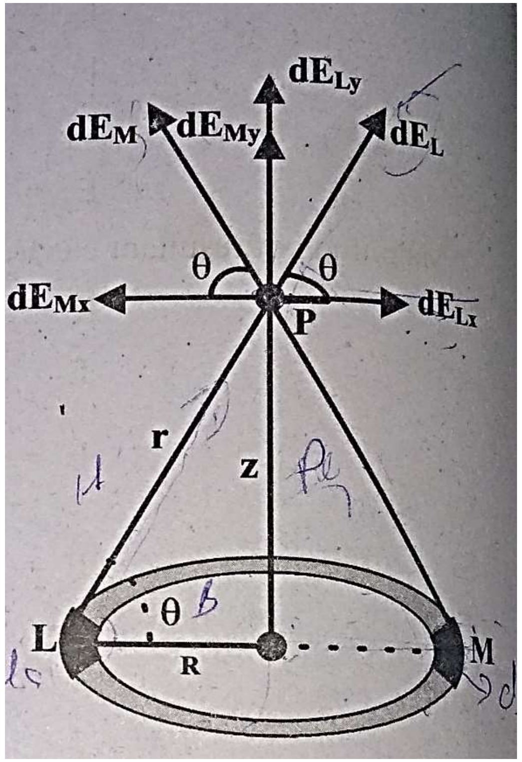
Now calculate the electric field at P due to both length elements, resolve the electric field dE_L and dE_M into components.

Rectangular components of dE_L are

$dE_{Lx} = dE_L \cos\theta$, $\quad$ $dE_{Ly} = dE_L \sin\theta$

Rectangular components of dE_M are

$dE_{Mx} = dE_M \cos\theta$, $\quad$ $dE_{My} = dE_M \sin\theta$



Electric Field due to Ring of Charge

Resultant x-component of electric field is

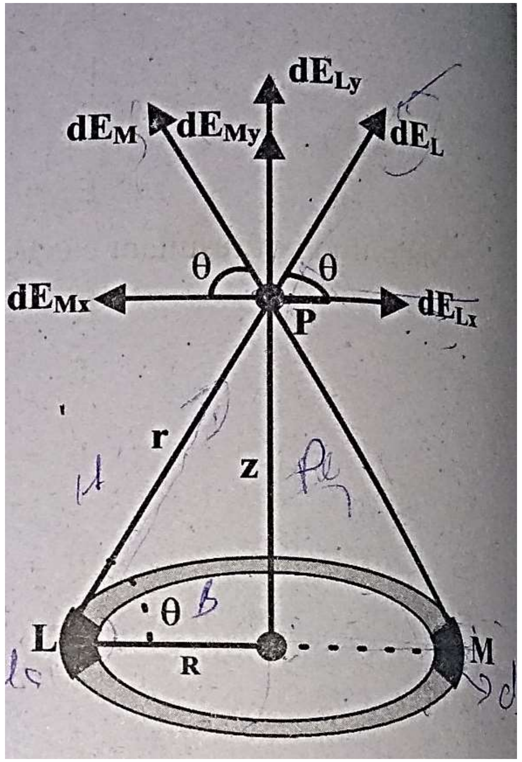
$$dE_x = (+dE_{Lx}) + (-dE_{Mx})$$

$$dE_x = dE_L \cos\theta - dE_M \cos\theta$$

$$dE_x = dE_L \cos\theta - dE_L \cos\theta$$

As $dE_L = dE_M$

$$dE_x = 0$$



Electric Field due to Ring of Charge

Resultant y-component of electric field is

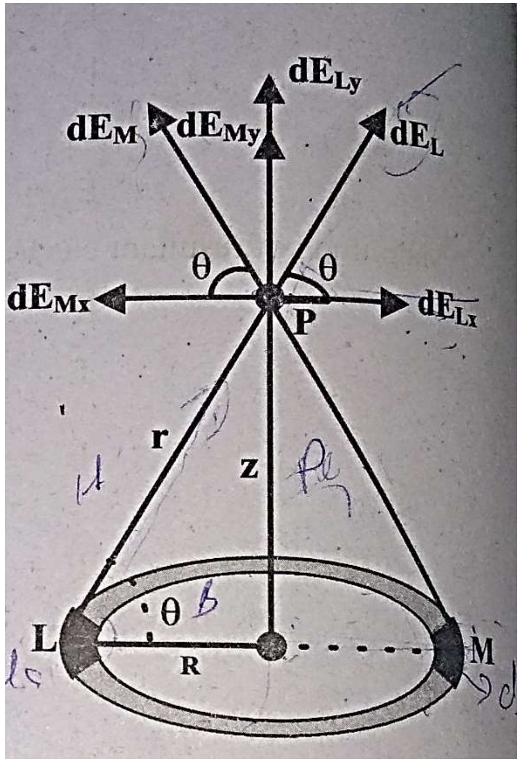
$$dE_y = (+dE_{Ly}) + (+dE_{My})$$

$$dE_y = dE_L \sin\theta + dE_M \sin\theta$$

As $dE_L = dE_M$

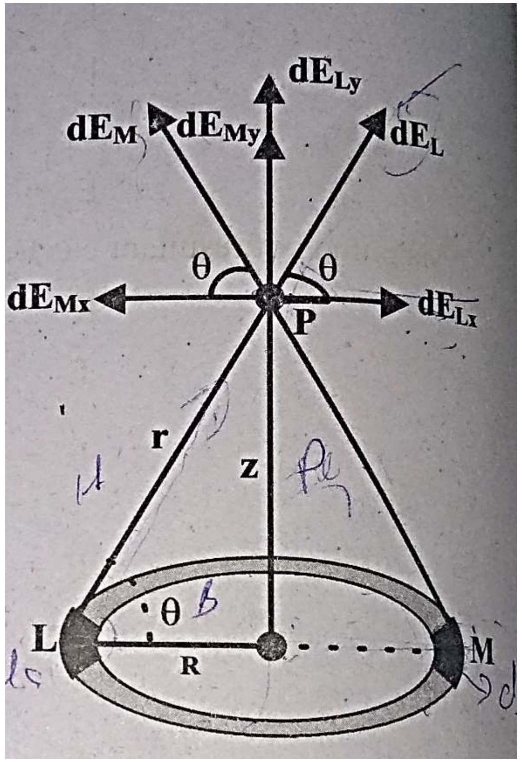
$$dE_y = dE_L \sin\theta + dE_L \sin\theta$$

$$dE_y = 2\, dE_L \sin\theta$$



Electric Field due to Ring of Charge

The magnitude of electric field at P due to whole ring of charge is



$$E = \int 2\, dE_L \sin\theta$$

$$E = \int 2\,\frac{k\,dq}{r^2}\,.\frac{z}{r}$$

As $\sin\theta = \frac{z}{r}$

$$E = \int 2\,\frac{z\,k\,dq}{r^3}$$

$$E = 2\,\frac{z\,k\,\lambda}{r^3}\int ds$$

From $\lambda = \frac{dq}{ds}$

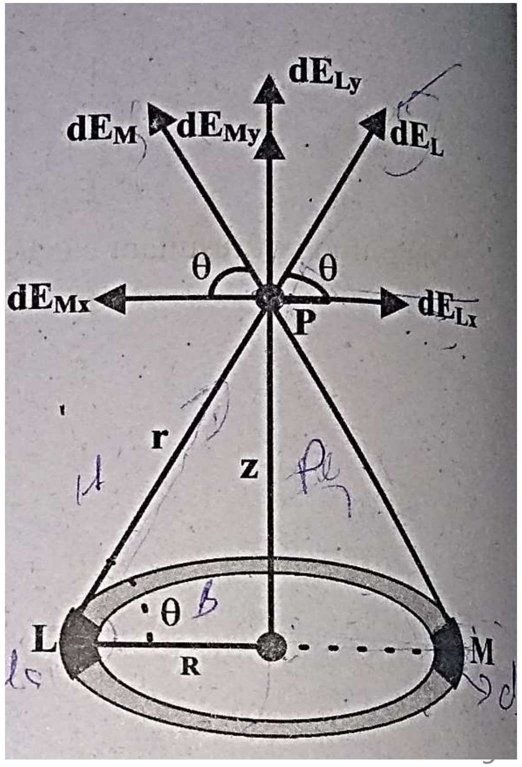
$$E = 2\,\frac{z\,k\,\lambda}{(z^2 + R^2)^{3/2}}\int ds$$

Electric Field due to Ring of Charge

As, $\int ds = \pi R$

Length of circumference of half ring because length elements are taken at both sides of diameter.

$$E = \frac{2\, z\, k\, \lambda}{(z^2 + R^2)^{3/2}} (\pi R)$$



As $\lambda = \frac{dq}{ds}$

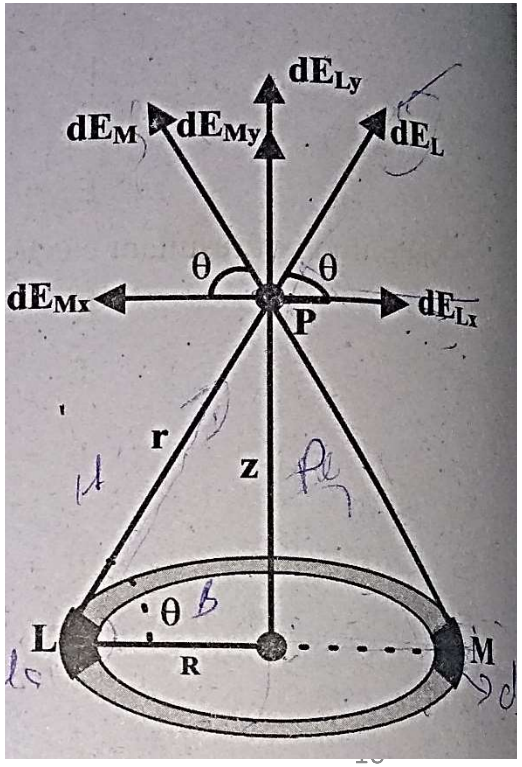
So q = $\lambda \int ds = \lambda(2\, \pi\, R)$

Electric Field due to Ring of Charge

$$E = \frac{2\, z\, k\, \lambda}{(z^2 + R^2)^{3/2}} (\pi\, R)$$

$$E = \frac{z\, k\, \lambda}{(z^2 + R^2)^{3/2}} (2\, \pi\, R)$$

$$E = \frac{z\, k\, q}{(z^2 + R^2)^{3/2}}$$



Electric Field due to Ring of Charge

This is the electric field at point P due to ring of charge. When point P lies at a large distance z. the term R^2 can be neglected as compared to z^2

$$E = \frac{z\,k\,q}{(z^2 + R^2)^{3/2}}$$

$$E = \frac{z\,k\,q}{(z^2 + 0)^{3/2}}$$

$$E = \frac{z\,k\,q}{(z^2)^{3/2}}$$

$$E = \frac{z\,k\,q}{z^3}$$

$$E = \frac{k\,q}{z^2}$$

It means that ring behaves as a point charge which is concentrated at center of ring when z>>R.

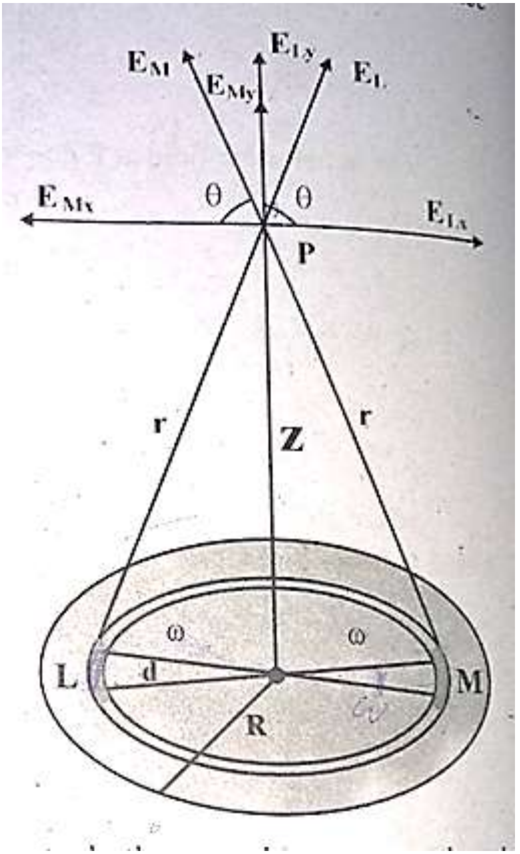
Electric Field due to Disc of Charge

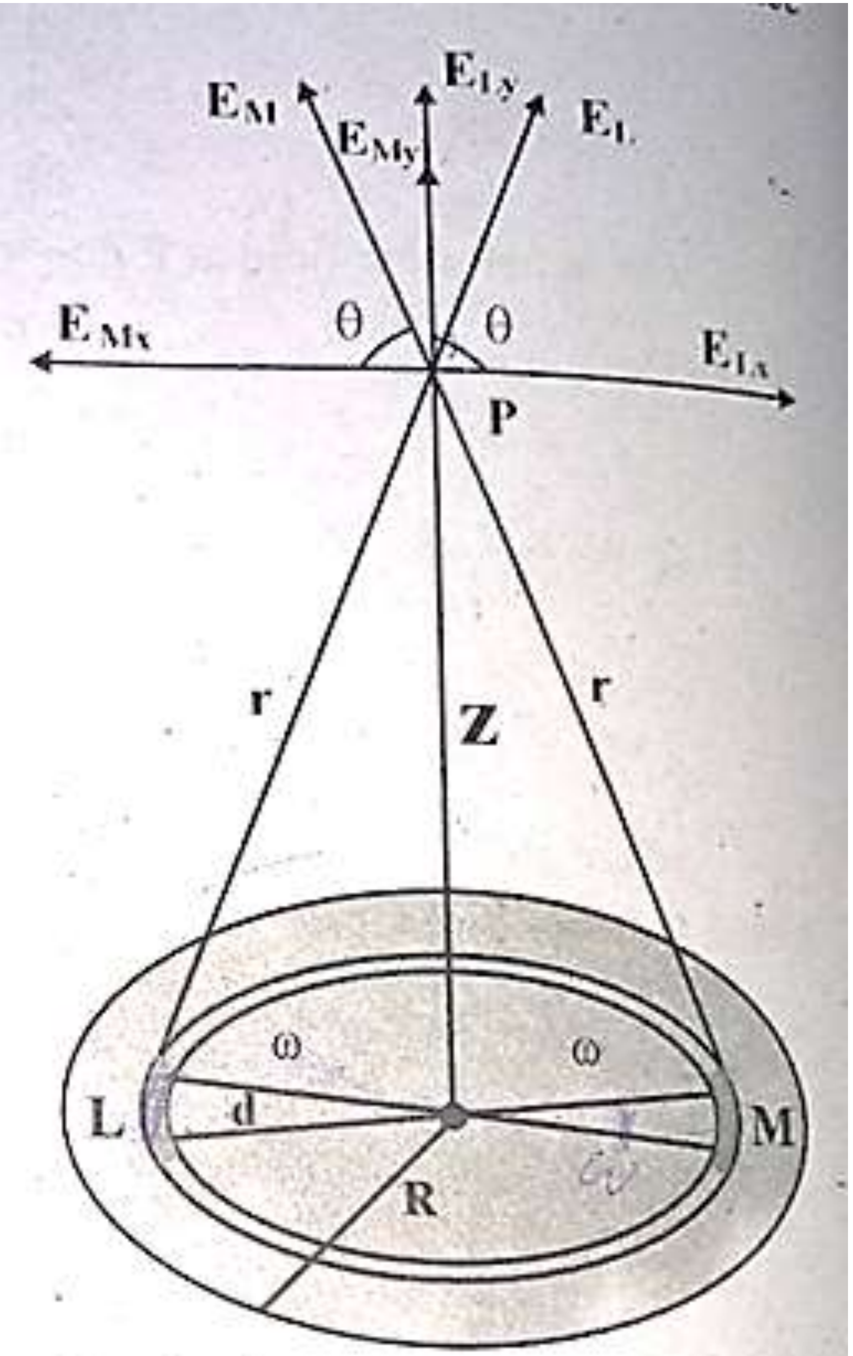
Consider a positively charged circular disc of radius R having surface charge density σ.

D*ivide the disc into small rings*. *now consider such a small ring* ha*ving radius* ω and dω

Take a pair of small area element da at opposite ends of diameter denoted by L and M. the area of the element having width dω and length S=ωdα is given as

dA= ω dα dω





E_M
E_{1y}
E_{My}
E_L
E_{Mx}
θ
θ
E_{1x}
P
r
Z
r
ω
ω
L
d
M
R

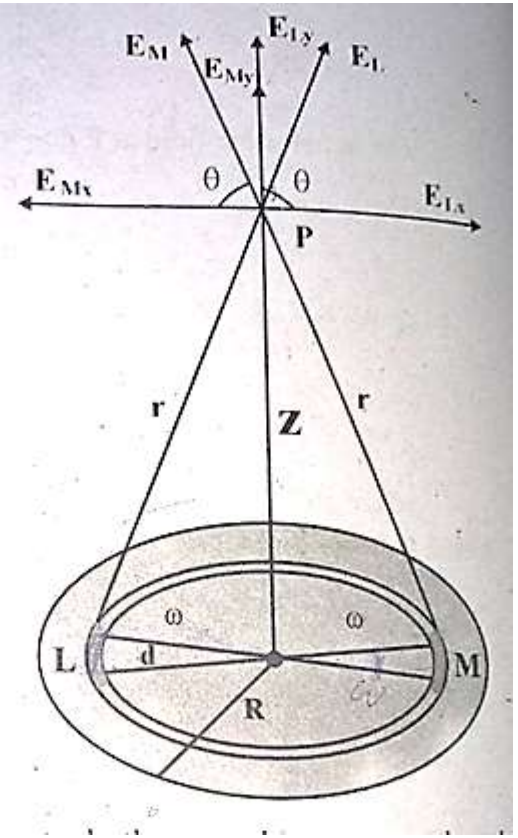
Electric Field due to Disc of Charge

The charge per unit area is called surface charge density

$$\sigma = \frac{dq}{dA}$$

$$dq = \sigma \, dA$$

$$dq = \sigma \, (\omega \, d\alpha \, d\omega)$$



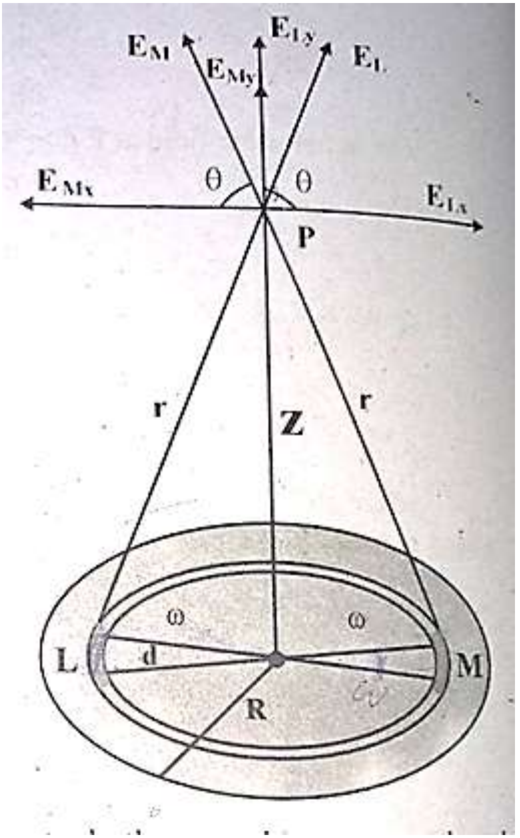
Electric Field due to Disc of Charge

The magnitude of electric field at P due to charge element L is

$$dE_L = \frac{k\,dq}{r^2}$$

Similarly, the magnitude of electric field at P due to charge element M is

$$dE_M = \frac{k\,dq}{r^2}$$



Electric Field due to Disc of Charge

Comparing both equations, we get

$$dE_L = dE_M$$

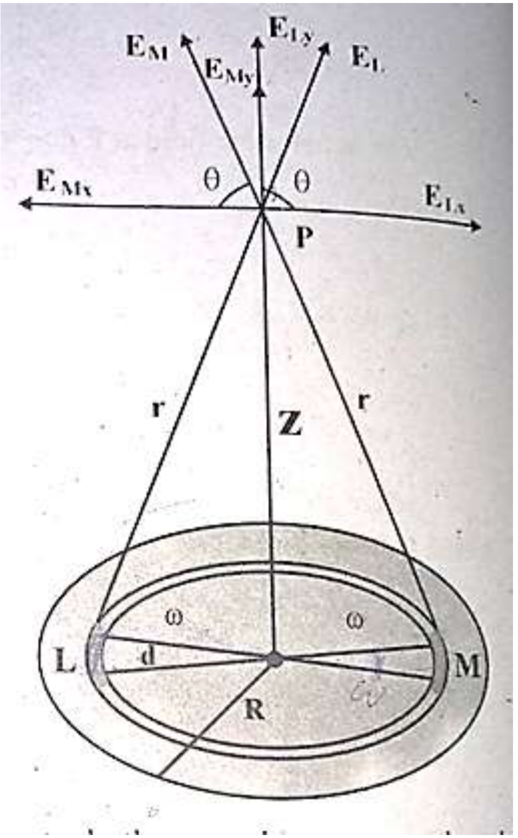
Now calculate the electric field at P due t both length elements, resolve the electric field dE_L and dE_M into components.

Rectangular components of dE_L are

$dE_{Lx} = dE_L \cos\theta$, $\quad$ $dE_{Ly} = dE_L \sin\theta$

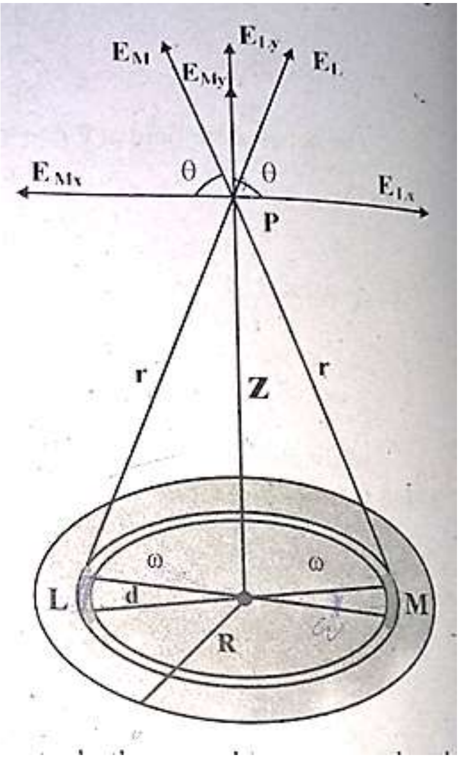
Rectangular components of dE_M are

$dE_{Mx} = dE_M \cos\theta$, $\quad$ $dE_{My} = dE_M \sin\theta$



Electric Field due to Disc of Charge

Resultant x-component of electric field is



$$dE_x = (+dE_{Lx}) + (-dE_{Mx})$$

$$dE_x = dE_L \cos\theta \; - dE_M \cos\theta$$

As $dE_L = dE_M$

$$dE_x = dE_L \cos\theta \; - dE_L \cos\theta$$

$$dE_x = 0$$

Electric Field due to Disc of Charge

Resultant y-component of electric field is

$$dE_y = (+dE_{Ly}) + (+dE_{My})$$

$$dE_y = dE_L \, sin\, \theta + dE_M \, sin\, \theta$$

As $dE_L = dE_M$

$$dE_y = dE_L \, sin\, \theta + dE_L \, sin\, \theta$$

$$dE_y = 2\, dE_L \, sin\, \theta$$

Electric Field due to Disc of Charge

The magnitude of electric field dE is

$$dE^2 = dE_x^2 + dE_y^2$$

$$dE = 2\, dE_L \sin\theta$$

$$\text{dE} = 2\ \frac{z\, k\, dq}{r^3}$$

dq = σ (ω dα dω) and $r^2 = (z^2 + \omega^2)$

Electric Field due to Disc of Charge

The magnitude of electric field at P due to whole ring of charge is

$$E = \int dE$$

$$E = \int 2\,\frac{z\,k\,dq}{r^3}$$

$$E = \int 2\,\frac{z\,k\,\sigma\,(\omega\,d\alpha\,d\omega)}{(z^2 + \omega^2)^{3/2}}$$

$$E = z\,k\,\sigma \int 2\,\frac{(\omega\,d\alpha\,d\omega)}{(z^2 + \omega^2)^{3/2}}$$

$$E = z\,k\,\sigma \int_0^R 2\,\frac{(\omega\,d\omega)}{(z^2 + \omega^2)^{3/2}} \int_0^\pi d\alpha$$

Electric Field due to Disc of Charge

$$E = z\, k\, \sigma \int_0^R 2\, \frac{(\omega\, d\omega)}{(z^2 + \omega^2)^{3/2}} \int_0^\pi d\alpha$$

$$E = z\, k\, \sigma \int_0^R (z^2 + \omega^2)^{-3/2} (2\, \omega\, d\omega)\, (\pi - 0)$$

$$E = z\, k\, \sigma\, \pi \int_0^R (z^2 + \omega^2)^{-3/2} (2\, \omega\, d\omega)$$

Electric Field due to Disc of Charge

Electric field disc of charge

$$E = z k \sigma \pi \left| \frac{(z^2+w^2)^{(-3/2)+1}}{(-3/2)+1} \right|_0^R$$

$$E = z k \sigma \pi \left| \frac{(z^2+w^2)^{-1/2}}{-1/2} \right|_0^R$$

$$E = z k \sigma \pi (-2) \left| \frac{1}{\sqrt{(z^2+w^2)}} \right|_0^R$$

$$E = (-2) z k \sigma \pi \left[\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{\sqrt{z^2+(0)^2}} \right]$$

$$E = (-2) z k \sigma \pi \left(\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{\sqrt{z^2}} \right)$$

$$E = (-2) z k \sigma \pi \left[- \left(\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}} \right) \right]$$

$$E = 2 z k \sigma \pi \left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}} \right]$$

$$E = 2 k \sigma \pi \left[\frac{\cancel{z}}{\cancel{z}} - \frac{z}{\sqrt{z^2+R^2}} \right]$$

$$E = 2 k \sigma \pi \left[1 - \frac{z}{\sqrt{z^2+R^2}} \right]$$

Electric Field due to Disc of Charge

$$E = 2 \cdot \frac{1}{4\pi\varepsilon_0} \sigma(\pi) \left[1 - \frac{z}{\sqrt{z^2+R^2}} \right]$$

$$E = \frac{\sigma}{2\varepsilon_0} \left[1 - \frac{z}{\sqrt{z^2+R^2}} \right]$$

If $R >> z$,

then

$$E = \frac{\sigma}{2\varepsilon_0} \left[1 - 0 \right]$$

$$E = \frac{\sigma}{2\varepsilon_0}$$