Bartolomeo Ryan 10208
Jeux et Sports

Modélisation de solides déformables

Plan

I. Présentation et première approche

A. Inspiration

B. Étude d'un mouvement

C. Première solution : les systèmes masse-ressort

II. Réalisation

A. Première méthode d'intégration : Euler

B. Deuxième Méthode : procédé de Runge et Kutta

C. Problème de la réaction du support

III. Deuxième approche

A. Modèle du gaz parfait

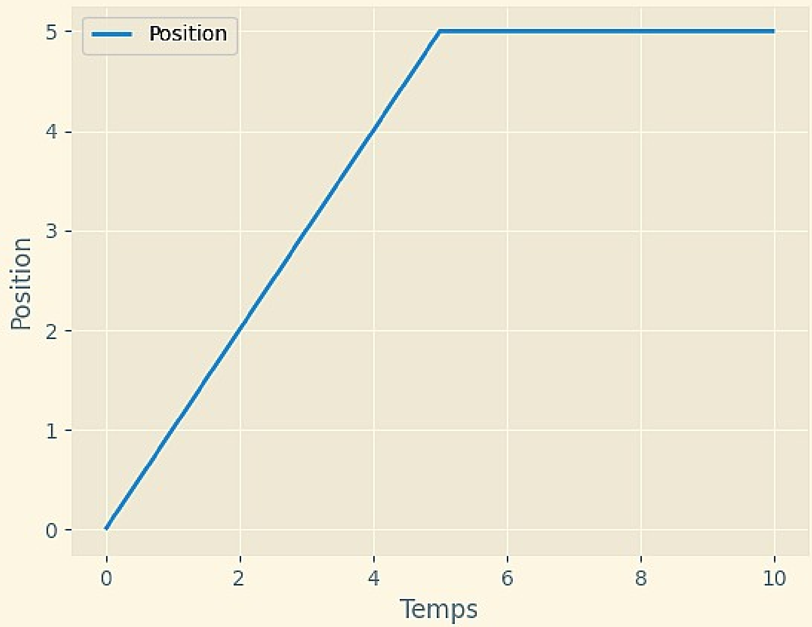
B. Théorème de Stokes

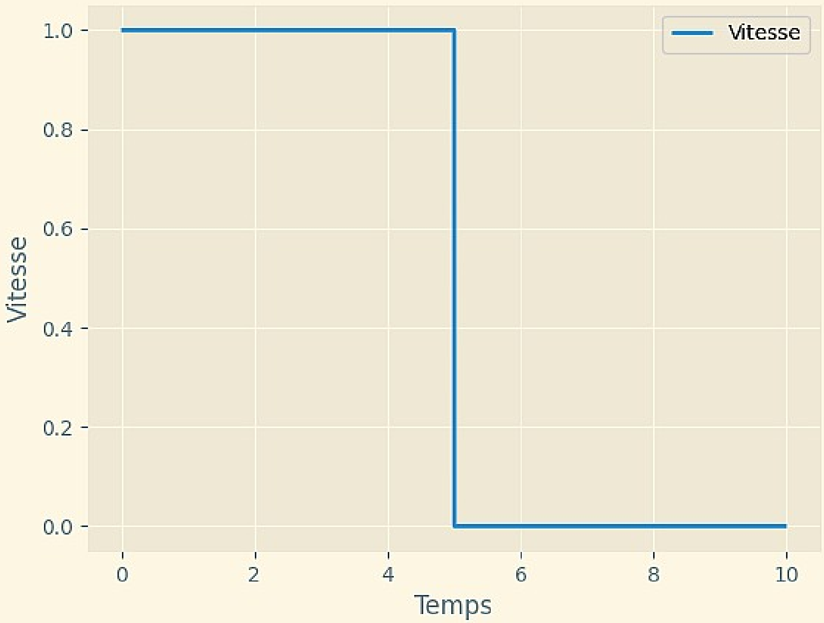
Présentation du problème et première approche

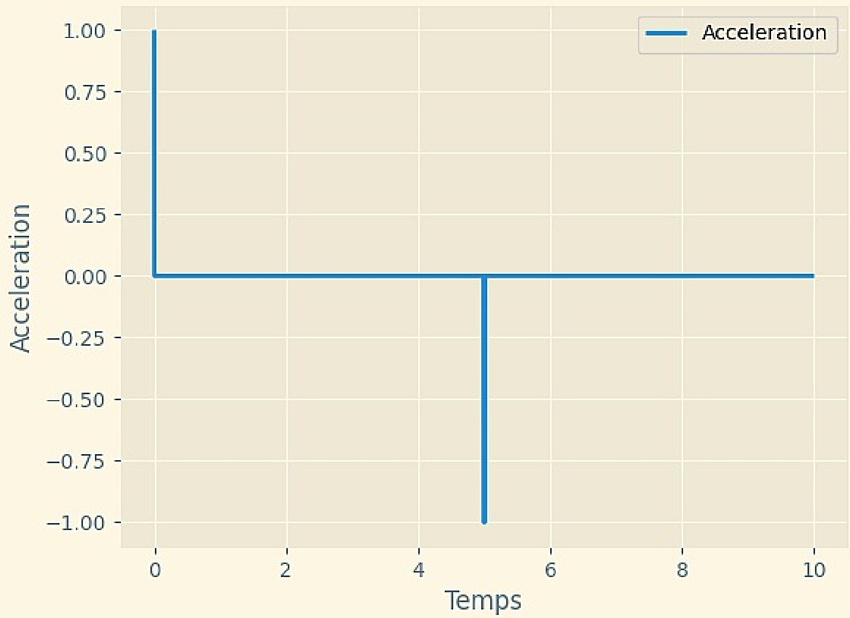
Inspiration

Modélisation réaliste du mouvement

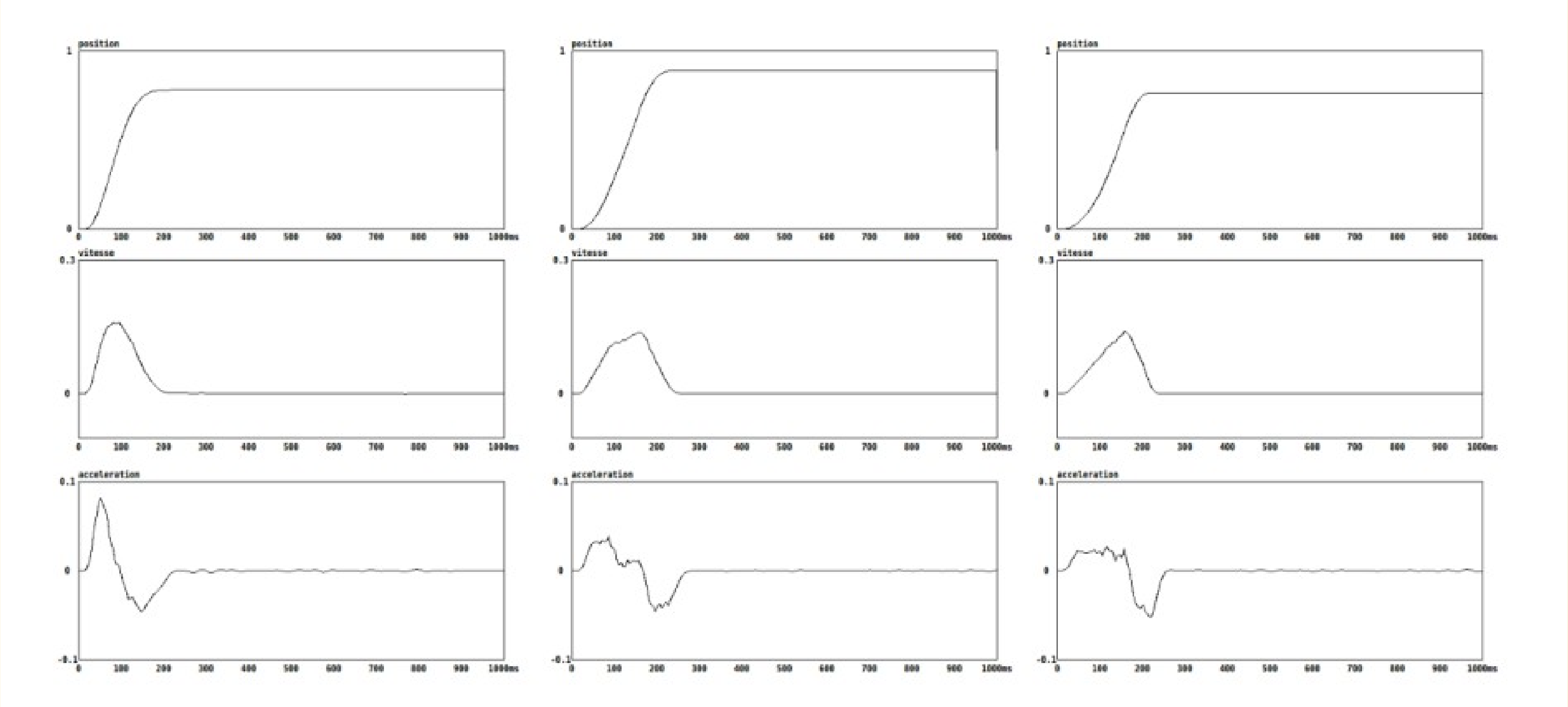
Modèle informatique simple





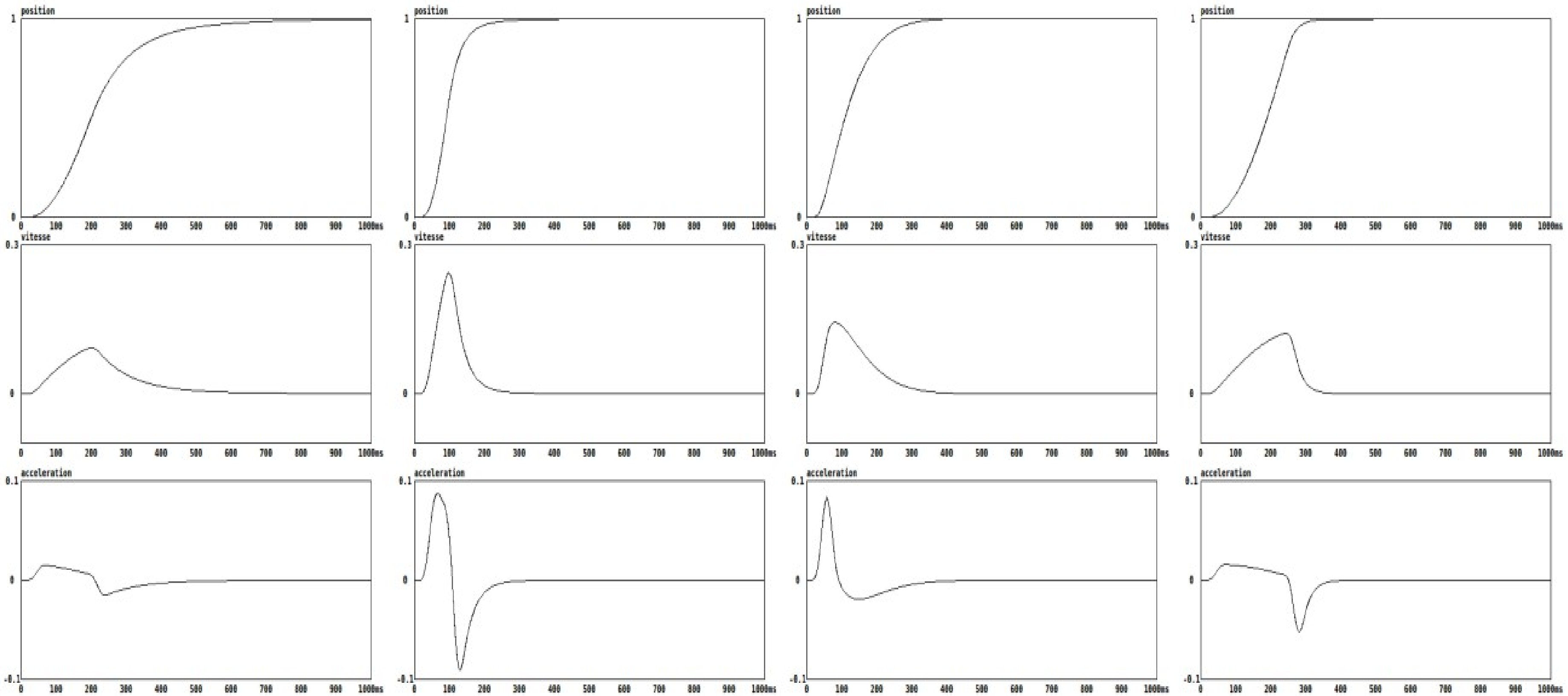


Mouvements naturels



Mass-Spring-System model for real time expressive behaviour synthesis ~ Cyrille Henry

Mouvements type Système masse-ressort



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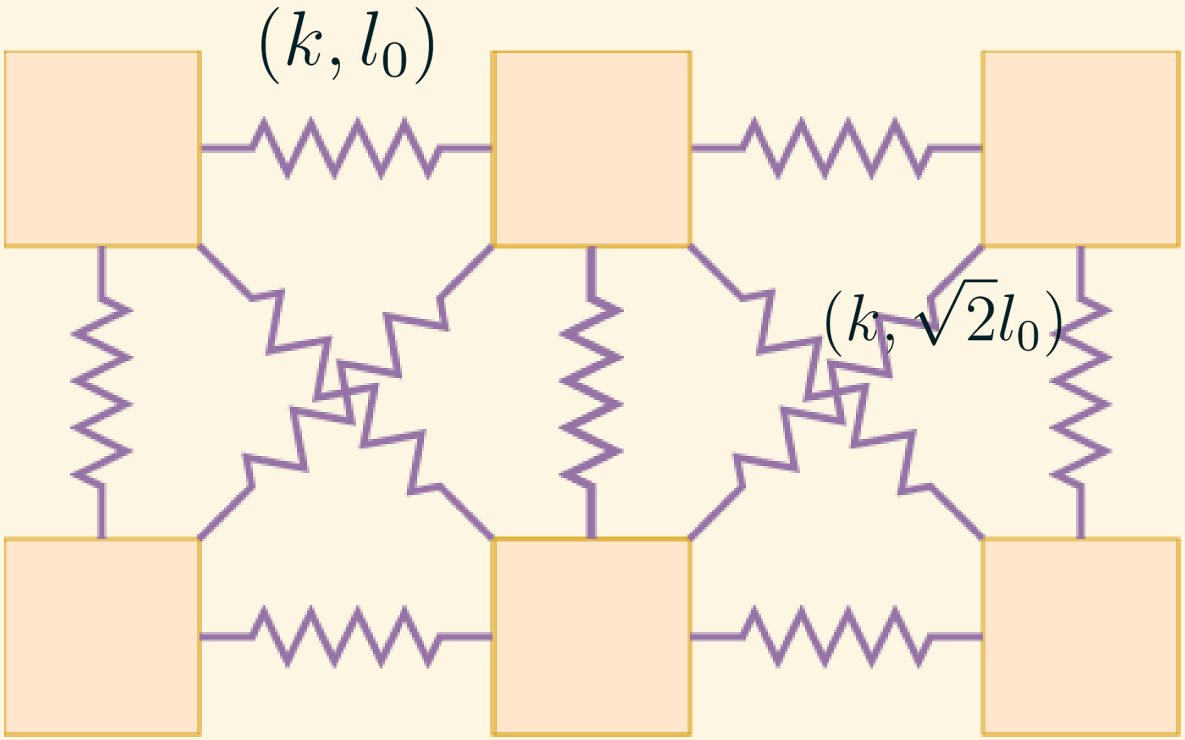
Simplification

II. Réalisation

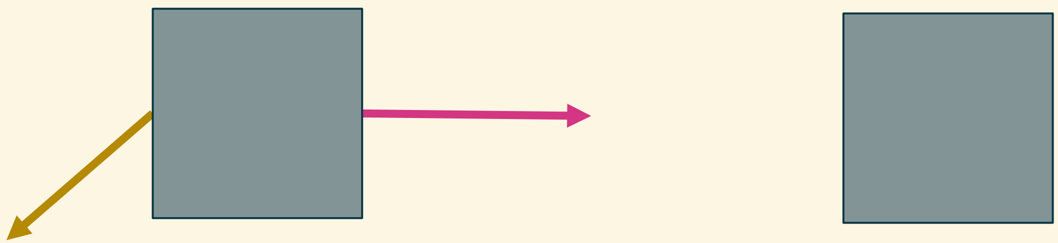
Structure du système



Méthode d’Euler

$$\vec{R} = k * (l - l_0)\vec{e_x}$$

$$\vec{f} = -\alpha\vec{v}$$



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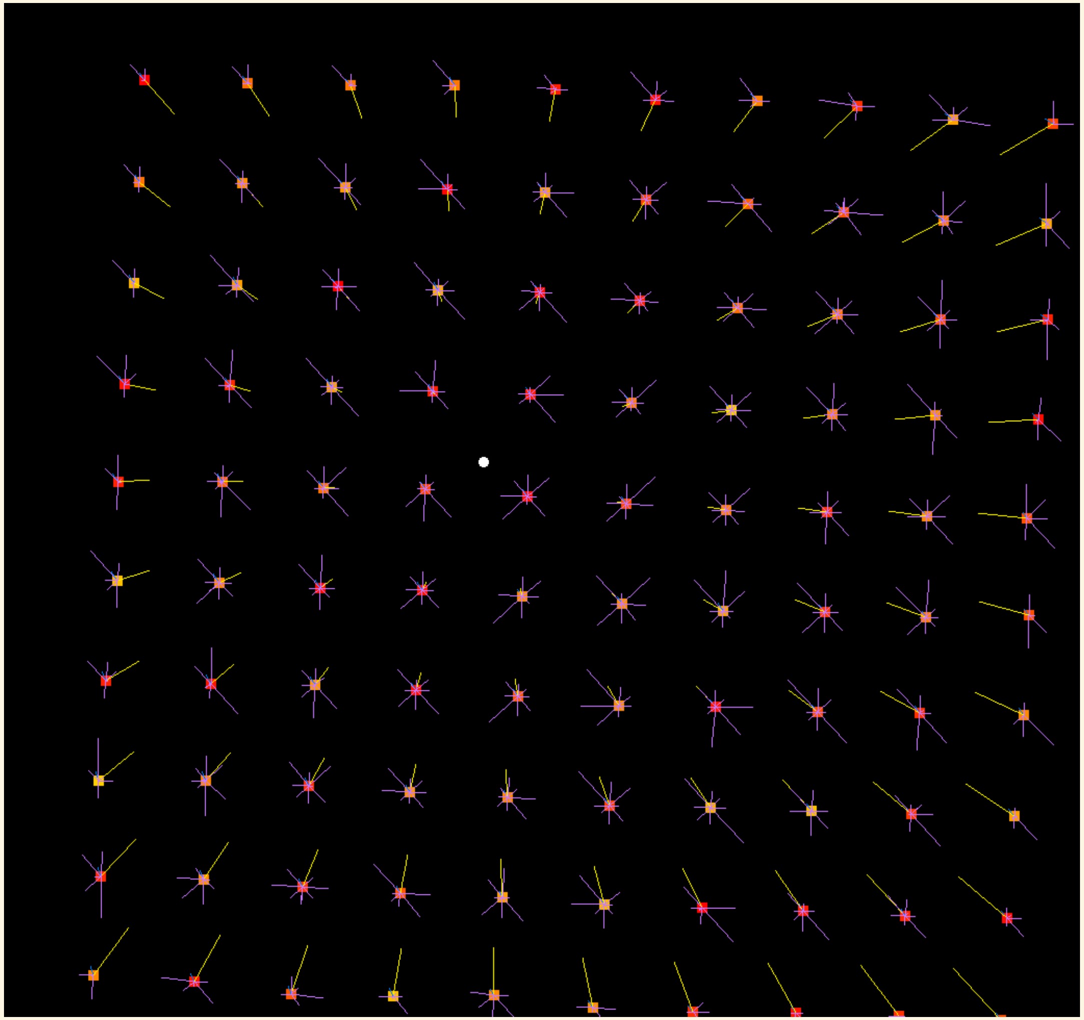
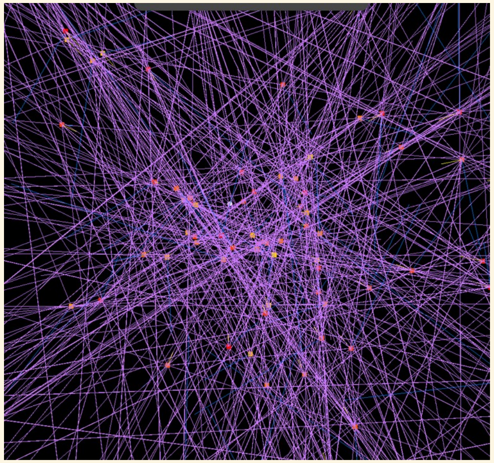
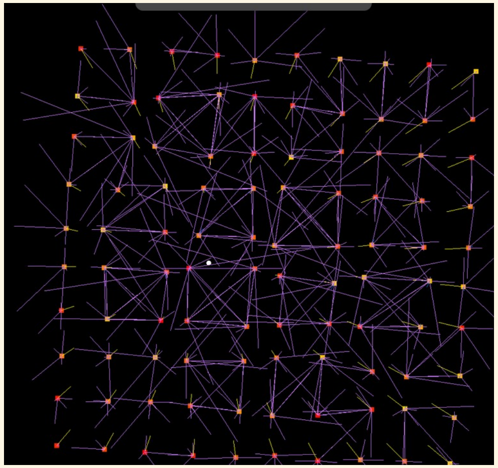
x[t + 1] = x[t] + v[t] * dt
v[t + 1] = v[t] + acceleration(t) * dt
```

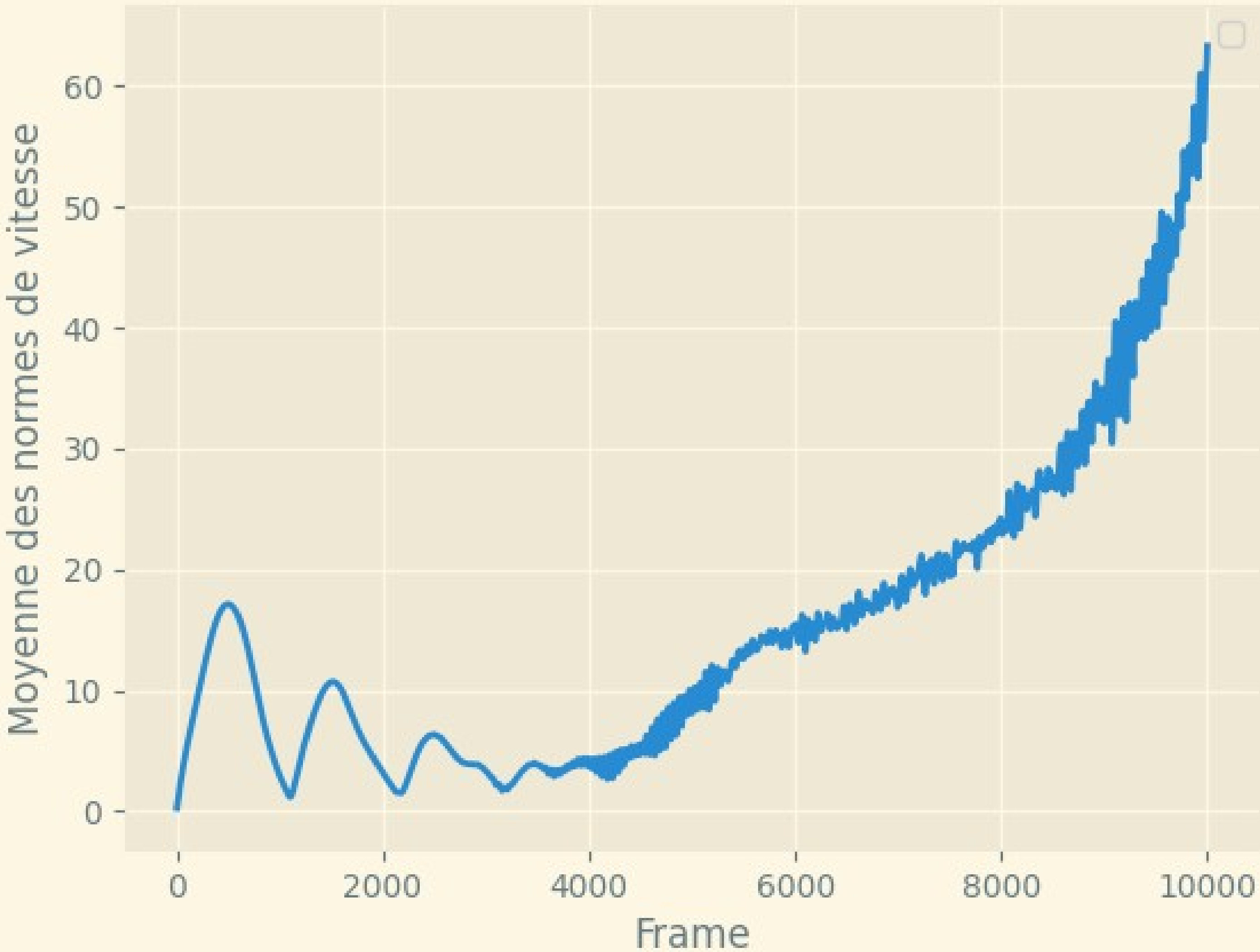
Premier algorithme







Premier résultat



Méthode de Runge-Kutta

Approximation de Runge-Kutta

Problème de Cauchy à résoudre

$$\begin{cases} y'(t) = f(t, y) \\ y(t_0) = y_0 \end{cases}$$

Tableau des coefficients

$$\begin{array}{c|cccc} (c_i) & & & & \\ 0 & 0 & 0 & 0 & 0\,(a_{i,j}) \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ \hline & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6}\ (b_j) \end{array}$$

$$\mathrm{t}_{n,i} = t_n + c_i \times h$$

$$z(t_{n,i}) = z(t_n) + h \sum_{j<i} a_{i,j} f(t_{n,j}, z(t_{n,j}))$$

$$z(t_{n+1}) = z(t_n) + h \sum_{j \leq 4} b_j f(t_{n,j}, z(t_{n,j}))$$

Approximation de Runge-Kutta

Problème différentiel physique

$$\begin{cases} y'' = \mathrm{acc}(t, y, y') \\ y'(t_0) = y'_0 \\ y(t_0) = y_0 \end{cases}$$

On pose alors :

$$\mathrm{Y(t)} = \begin{pmatrix} y'(t) \\ y(t) \end{pmatrix}$$

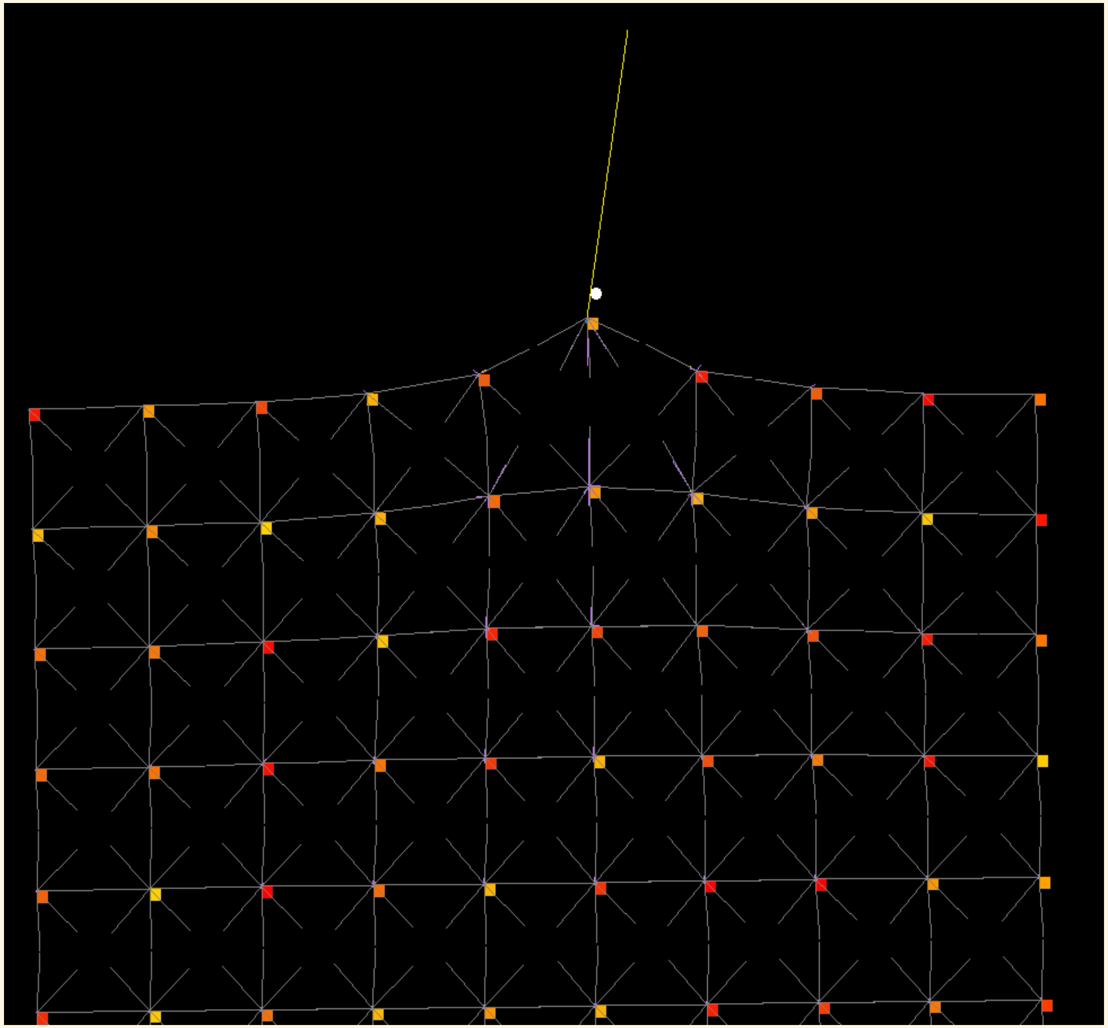
$$\mathrm{Y(t_0)} = \begin{pmatrix} y'_0 \\ y_0 \end{pmatrix}$$

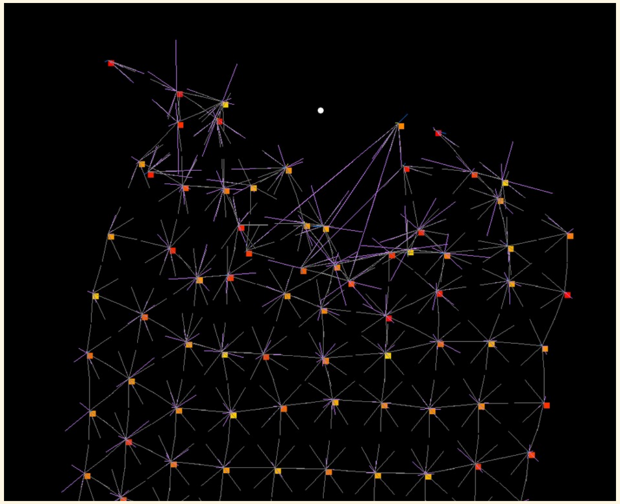
$$\text{d'où } \mathrm{Y'(t)} = \begin{pmatrix} y''(t) \\ y'(t) \end{pmatrix} = \mathrm{F(t,Y)} = \begin{pmatrix} \mathrm{acc}(t, y, y') \\ y'(t) \end{pmatrix}$$

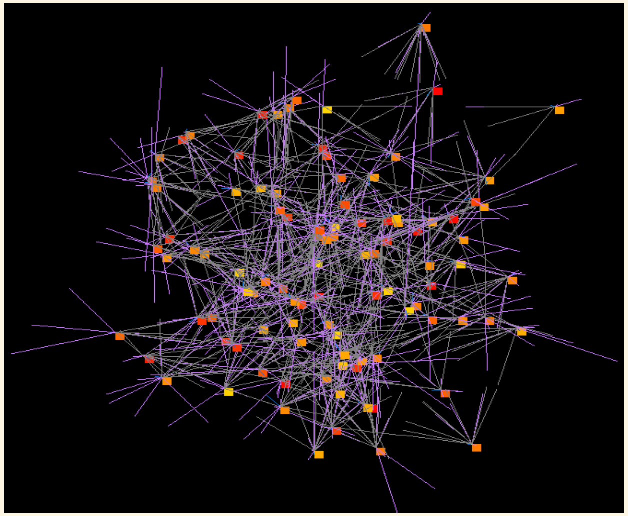
On a alors le problème de Cauchy d'ordre 1 :

$$\begin{cases} Y'(t) = F(t, Y) \\ Y(t_0) = Y_0 \end{cases}$$

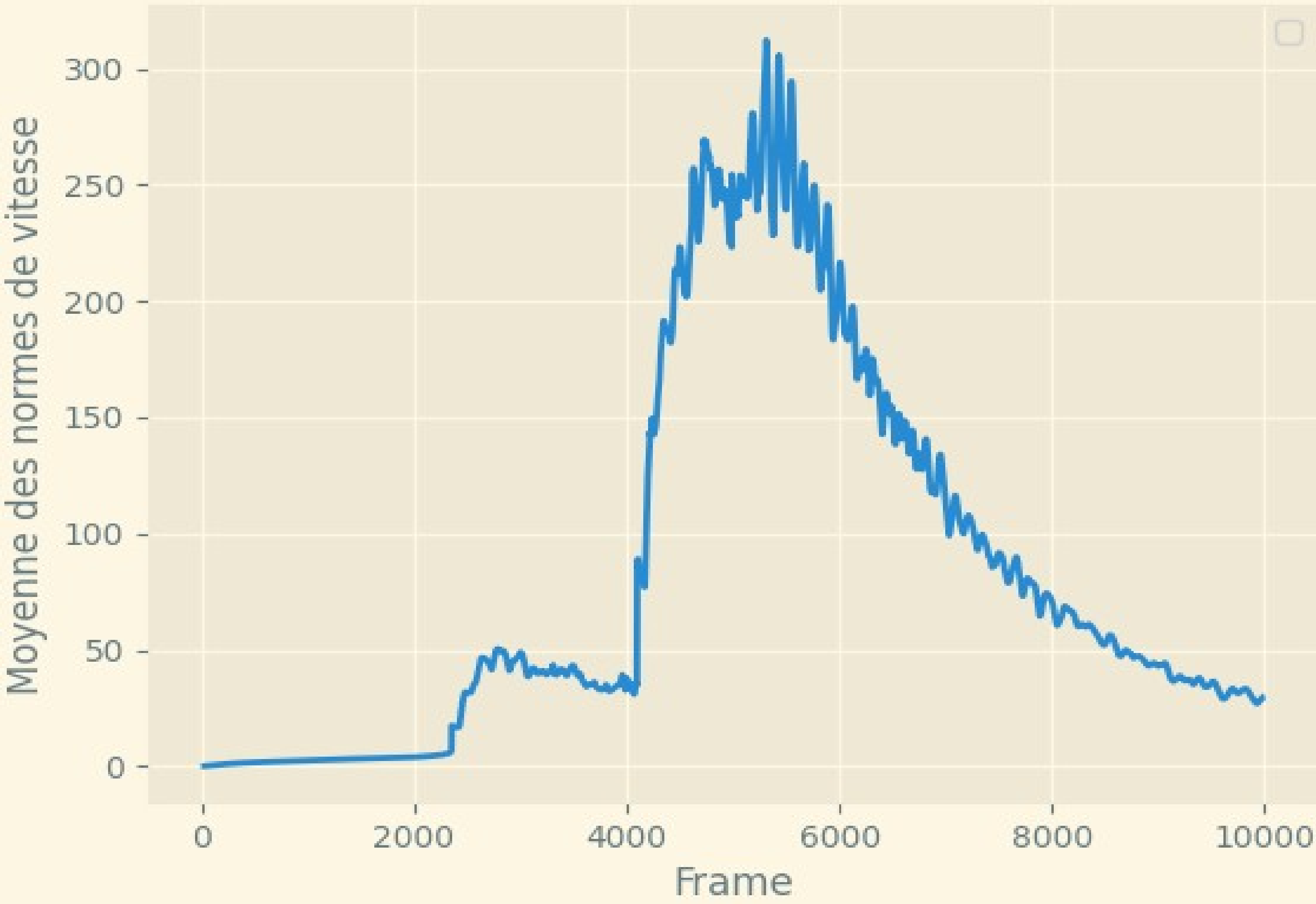
Expérimentation



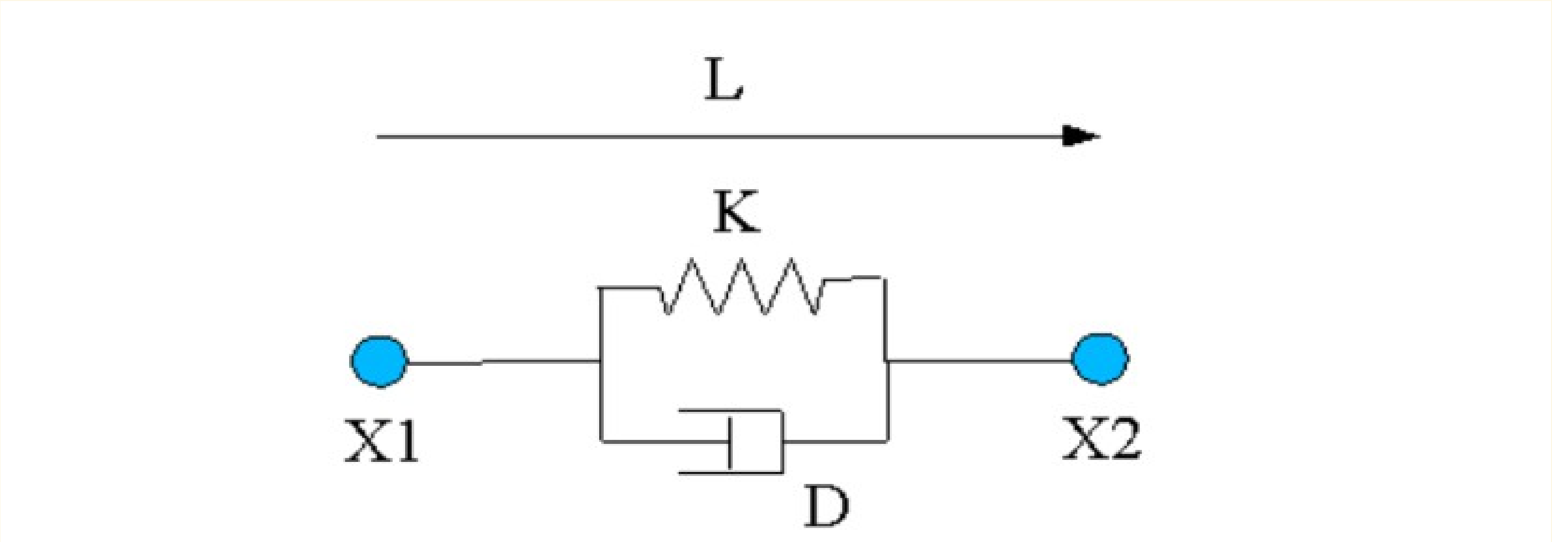




Résultat de la simulation avec RK4



Nouvelle description du système



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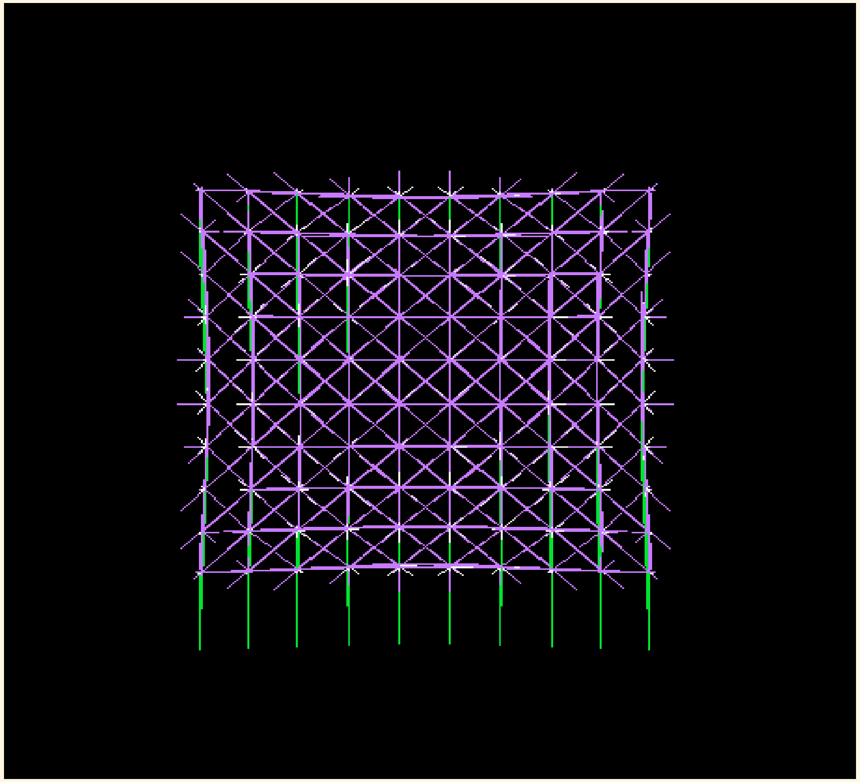
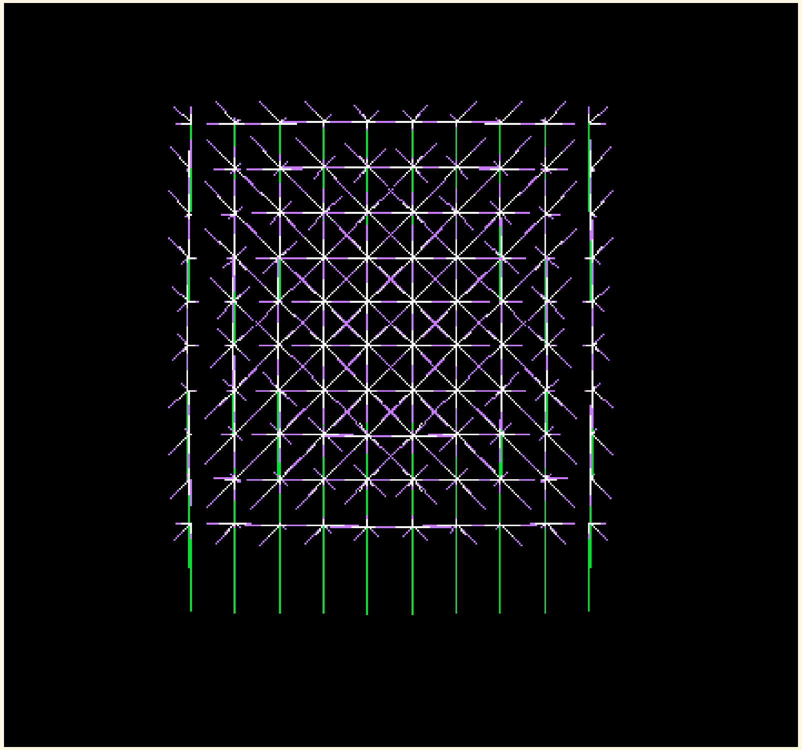
Effet de respiration





gravité

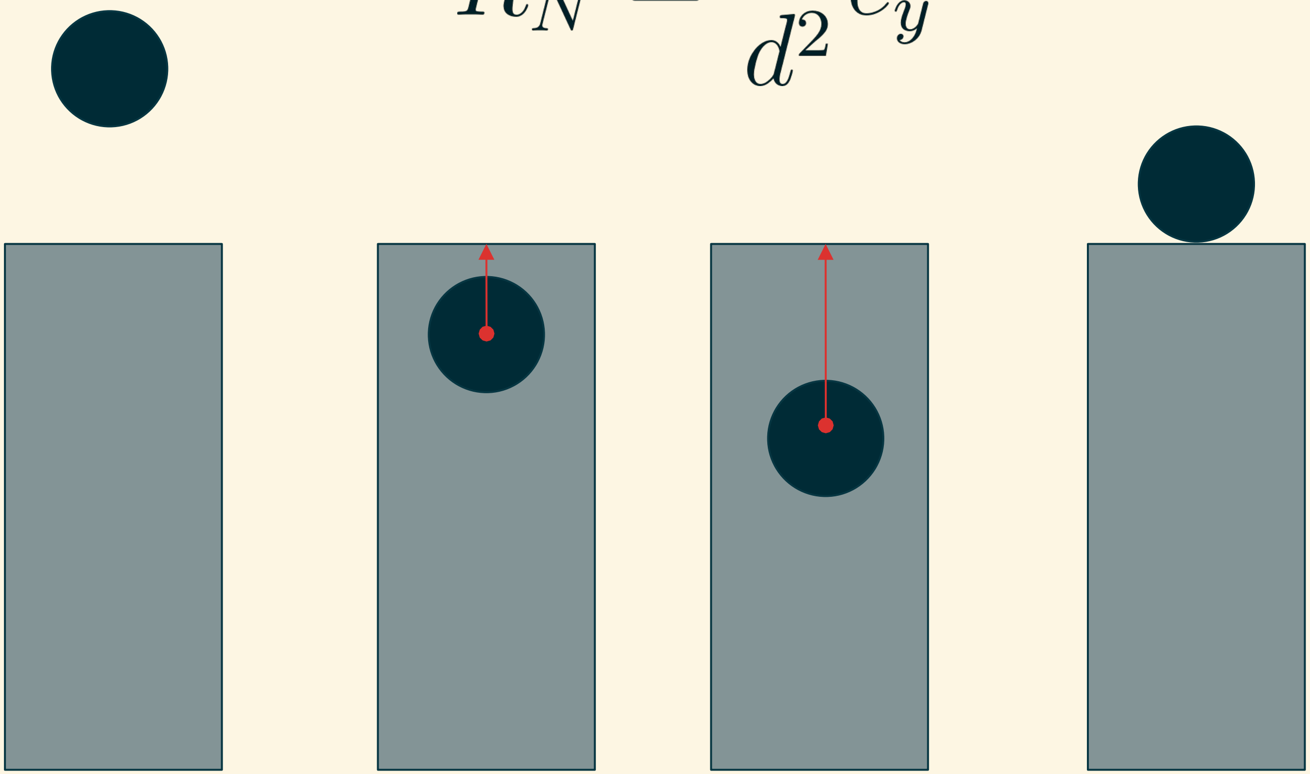
frottement
amortisseur

force de ressort
élastique

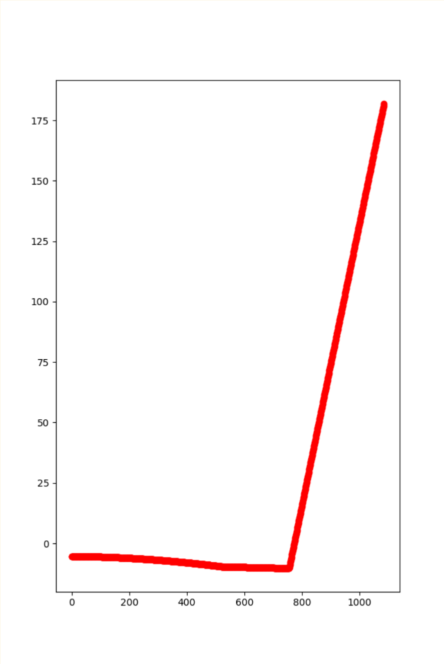
Collisions

Collision “magnétique”

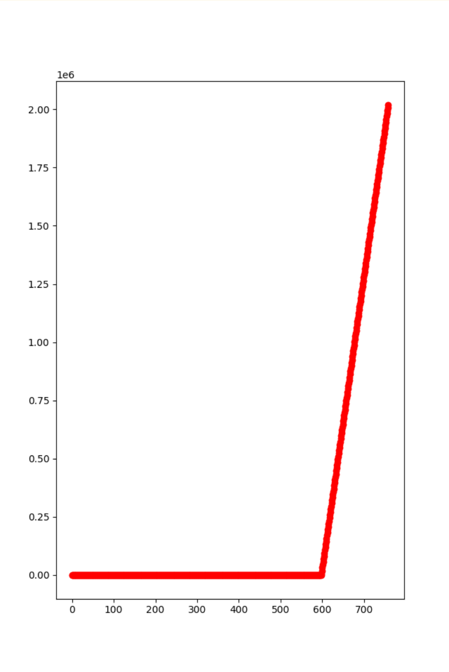
$$\vec{R_N} = \frac{K}{d^2}\vec{e_y}$$



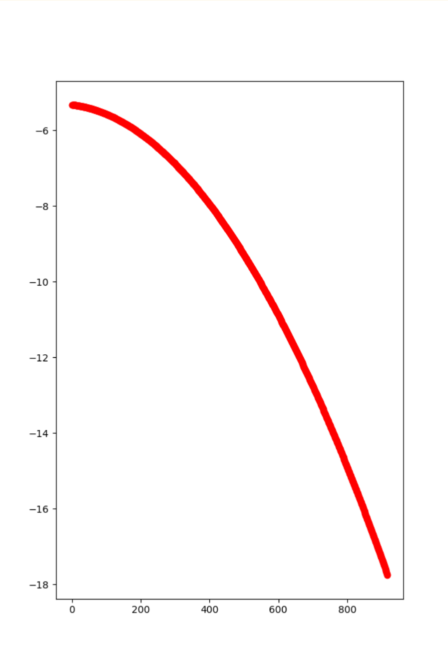
Résultats pour différentes valeurs de K



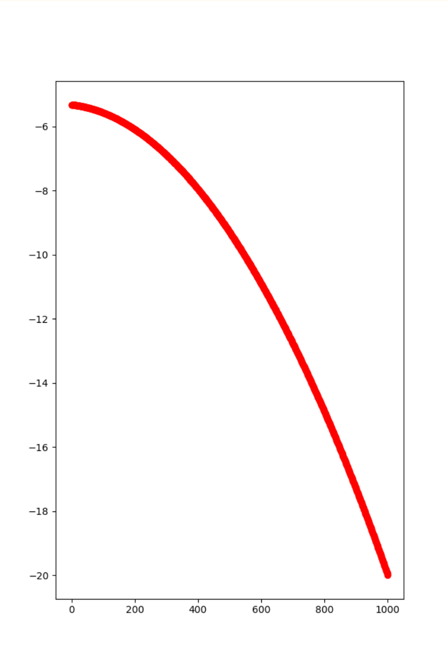
$K = 1.0$



$K = 1.0e-1$



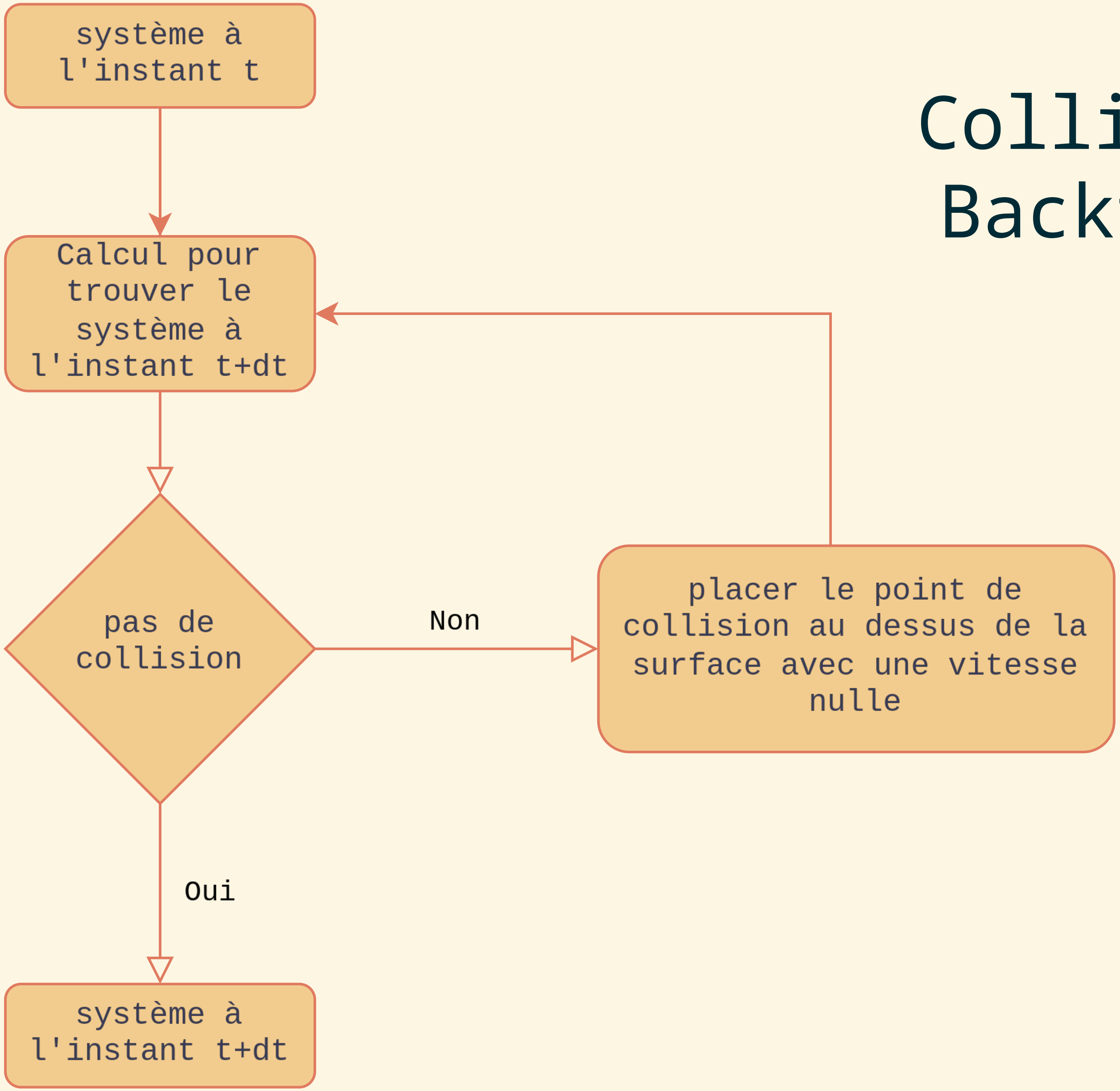
$K = 1.0e-2$



$K = 1.0e-4$

Position moyenne verticale en fonction du temps

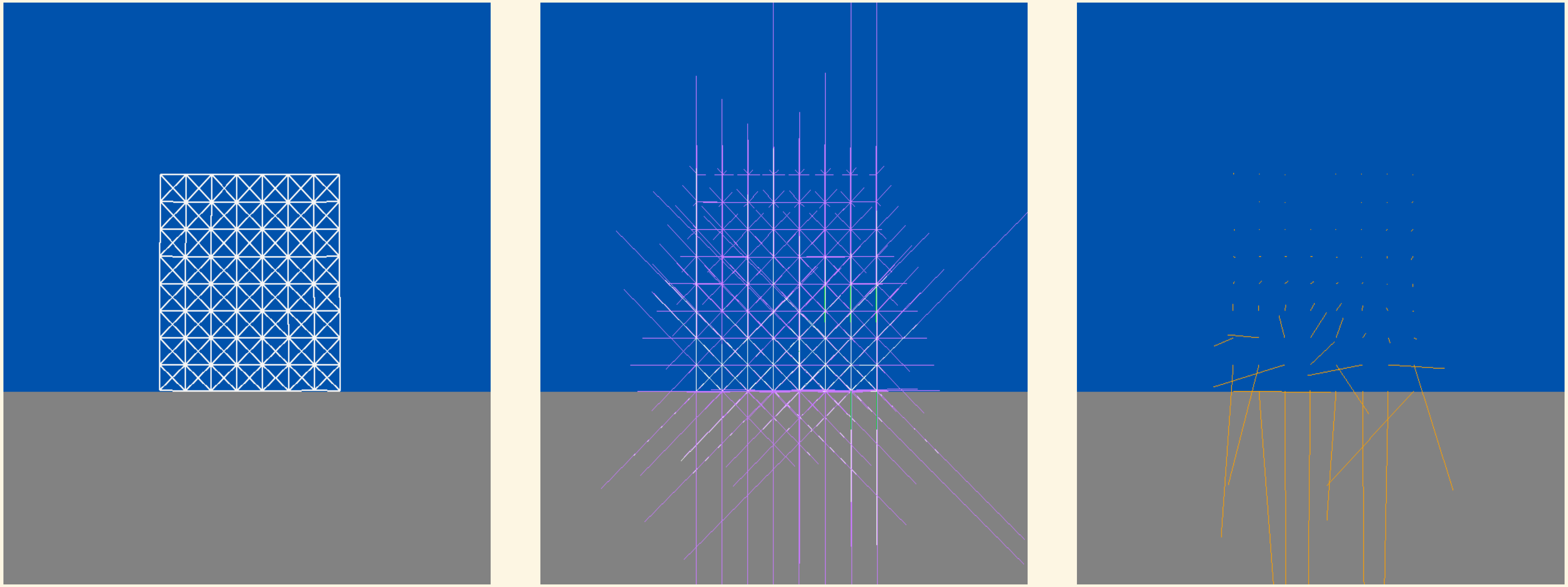
Collision via Backtracking



Résultats

forces

accélération



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frottement de l’amortisseur

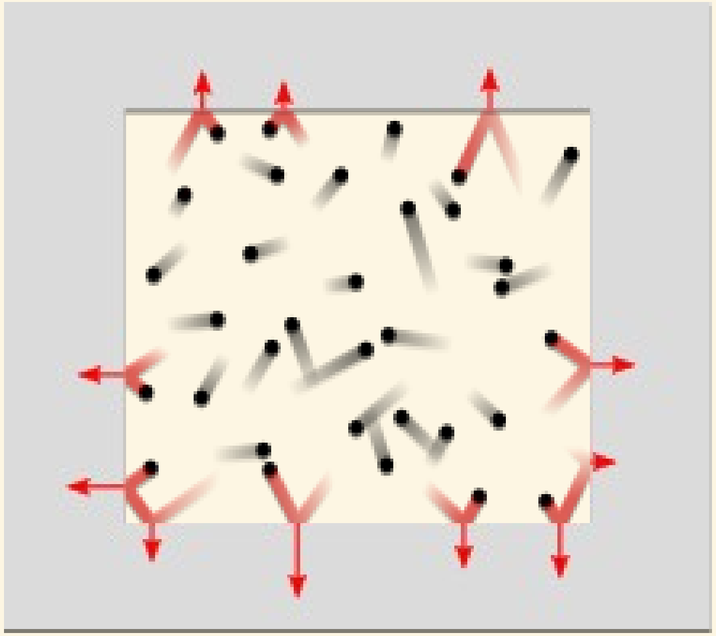
force de ressort élastique

Problèmes :
- Nombreux calculs de replacement
- Imprécision sur les points à la surface

→ Diminuer le nombre de points à la surface

III. Deuxième approche

Modèle du gaz parfait

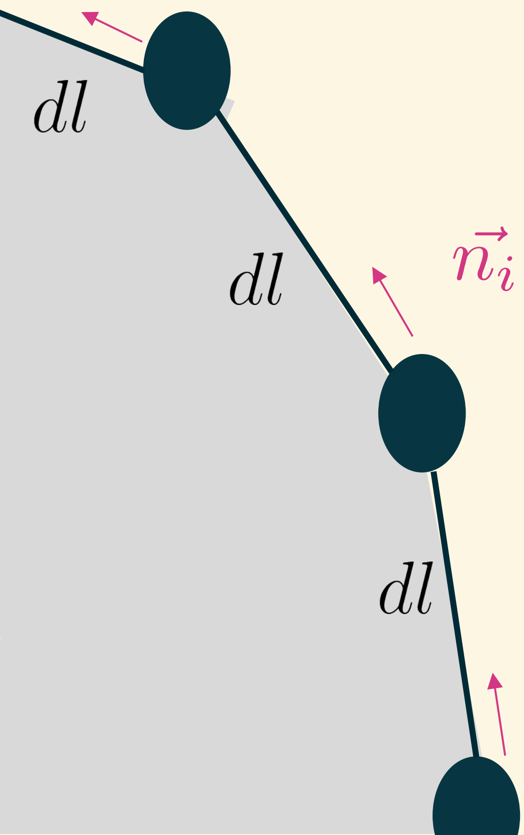


$$\vec{F} = P d\vec{S}$$

$$P = \frac{nRT}{V}$$

$$\vec{F} = K_{nRT} \frac{1}{V} d\vec{S}$$

Calcul du volume



Théorème de Stokes :

$$\boxed{\iint_S \mathrm{div}\vec{F}\cdot\vec{dS} = \oint_C \vec{F}\cdot\vec{dl}}$$

$$\vec{F} = x\vec{e_x}$$

$$\mathrm{div}\vec{F} = 1$$

$$\iint_S \mathrm{div}\vec{F}\cdot\vec{dS} = S$$

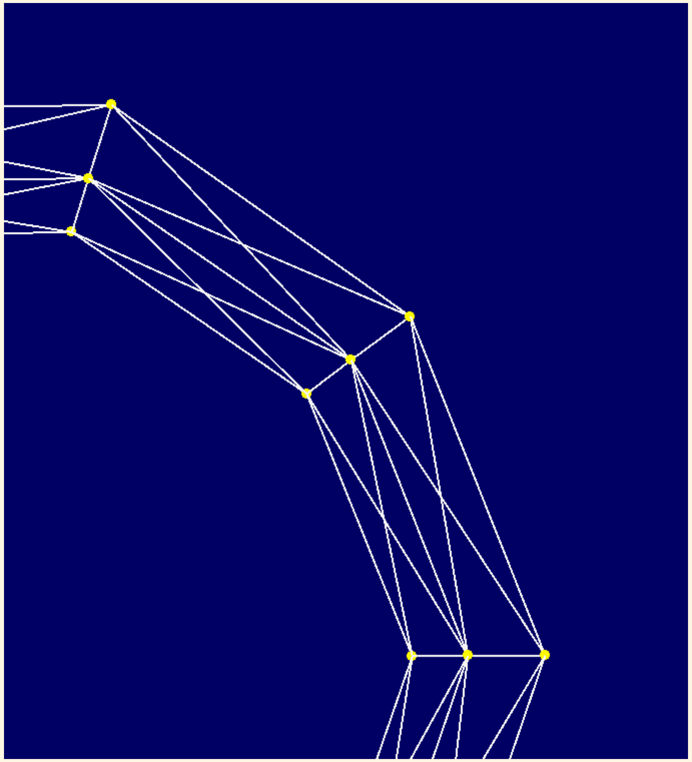
$$\vec{F}\cdot\vec{dl}$$

$$= \vec{F}\cdot\vec{n}dl$$

$$= x\cdot n_x\cdot dl$$

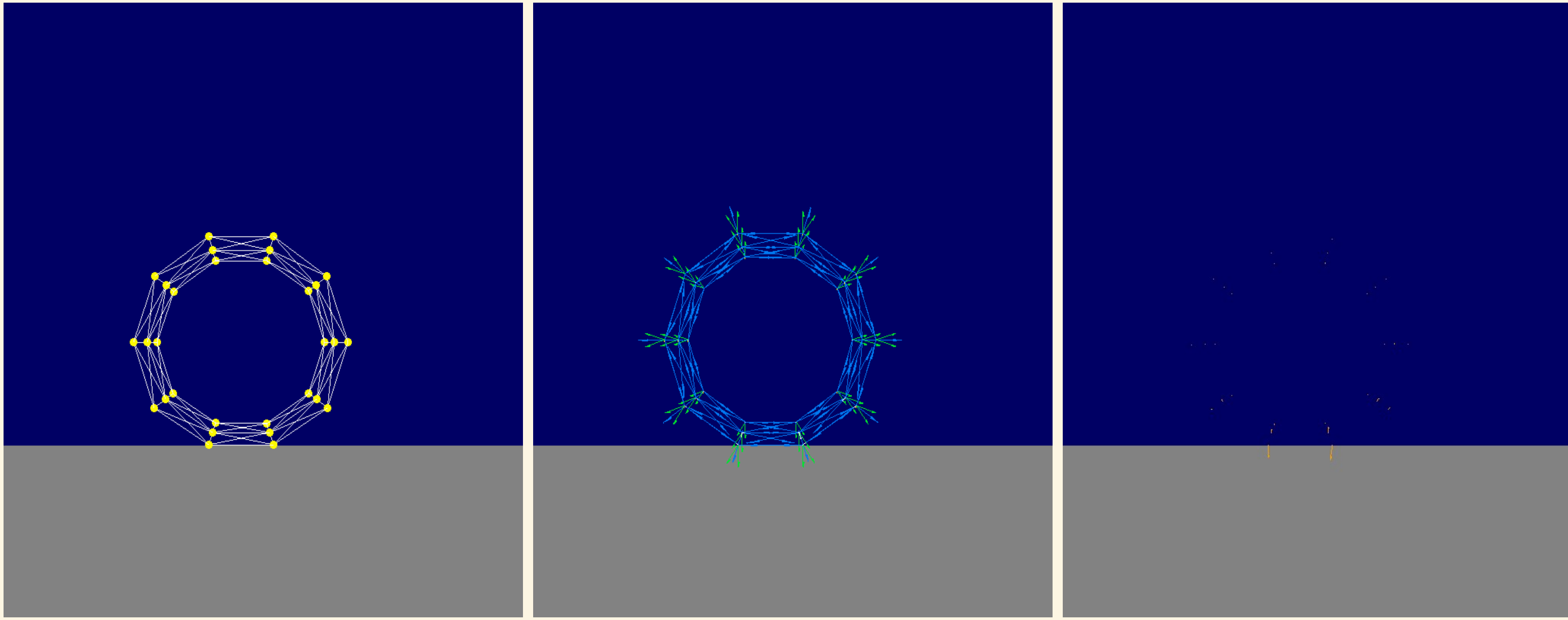
$$S \approx \sum x_i\cdot n_i\cdot dl$$

Compromis ressort/gaz



Résultat

forces accélération

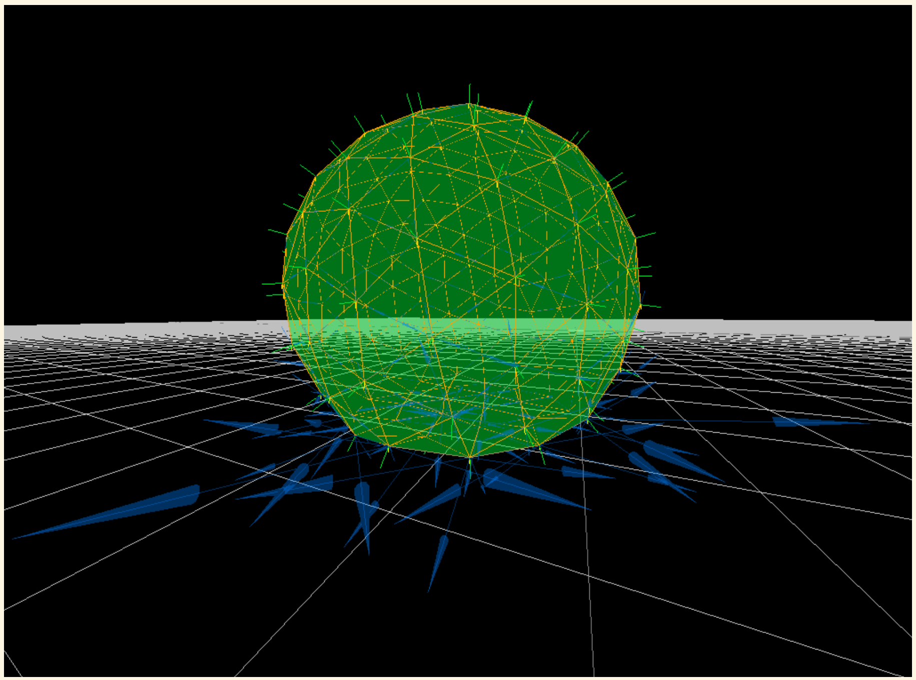


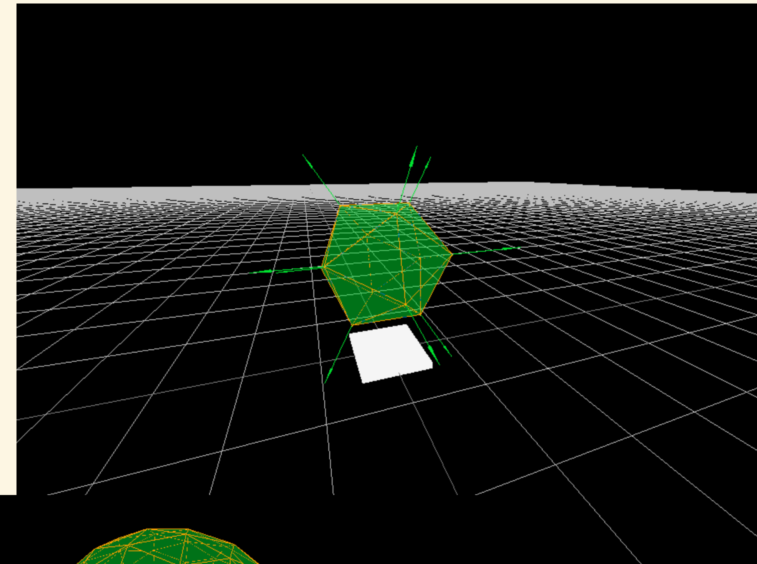
gaz

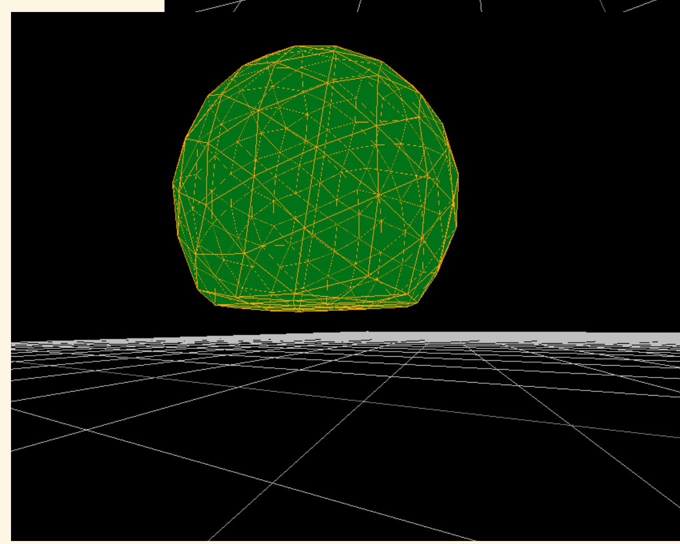
gravité

force de ressort élastique

Extrapolation en 3 dimensions







Annexe 1