1. (a) Step1: split RHS
   M->I, M->J, M->L, J->L, J->I, JN->K, JN->M, M->J(repeat, remove), KLN->M, K->I, K->J, K->L, IJ->K

   Step2: remove attributes from LHS

FD	Reason	Change or not
M->I	Only one attribute in LHS	No change
M->J	Only one attribute in LHS	No change
M->L	Only one attribute in LHS	No change
J->L	Only one attribute in LHS	No change
J->I	Only one attribute in LHS	No change
JN->K	J+ = {J, L, I, K} N+ = {N}, N is not in J+ and J is not in N+	No change
JN->M	J+ = {J, L, I, K} N+ = {N}, N is not in J+ and J is not in N+	No change
KLN->M	K+ = {K, I, J, L}, L is in K+ but N is not in K+	KN->M
K->I	Only one attribute in LHS	No change
K->J	Only one attribute in LHS	No change
K->L	Only one attribute in LHS	No change
IJ->K	J+ = {J, L, I, K}, I is in J	J->K

   Step3: remove unnecessary FDs

FD	Reason	Remove or Keep
M->I	M+ = {I, J, K, L, M}, M->I is implied by other FDs	Remove
M->J	M+ = {I, L, M}, M->J is not implied by other FDs	Keep
M->L	M+ = {I, J, K, L, M}, M->L is implied by other FDs	Remove
J->L	J+ = {I, J, K, L}, J->L is implied by other FDs	Remove
J->I	J+ = {I, J, K, L}, J->I is implied by other FDs	Remove

JN->K	JN+ = {I, J, K, L, M, N}, JN->K is implied by other FDs	Remove
JN->M	JN+ = {I, J, K, L, M, N}, JN->M is implied by other FDs	Remove
KN->M	KN+ = {I, J, K, L, N}, KN->M is not implied by other FDs since KN->M is removed	Keep
K->I	K+ = {J, K, L}, K->I is not implied by other FDs since J->I is removed	Keep
K->J	K+ = {I, K, L}, K->J is not implied by other FDs	Keep
K->L	K+ = {I, J, K}, K->L is not implied by other FDs since J->L is removed	Keep
J->K	J+ = {J}, J->K is not implied by other FDs	Keep

Therefore, the minimal basis for R is {J->K, K->I, K->J, K->L, KN->M, M->J}

1. (b) All attributes: IJKLMNOP
Since O and P do not appear in any FDs in the minimal basis in either LHS or RHS, O and P must be in every key.
Since N only appears in LHS of FD in the minimal basis, N must be in every key.
Since I and L only appear in RHS of FD in the minimal basis, I and L should not be in any key.
So we only consider NOP+combinations of {J, K, M} with a total of 2^3 = 8 possibilities.
    1. NOP+ = {N, O, P}, NOP is not a key
    2. JNOP+ = {I, J, K, L, M, N, O, P}, JNOP is a key
    3. KNOP+ = {I, J, K, L, M, N, O, P}, KNOP is a key
    4. MNOP+ = {I, J, K, L, M, N, O, P}, MNOP is a key
Since the rest possibilities are either superset of 2 or 3 or 4, we do not consider them.
Therefore, the keys for R are {J,N,O,P}, {K,N,O,P} and {M,N,O,P}

1. (c) 3NF, with minimal basis {J->K, K->IJL, KN->M, M->J}
By 3NF synthesis, we get {JK}, {KIJL}, {KMN}, and {MJ}, but {JK} is a subset of {KIJL}, so remove {JK}.
Also, no relation is a superkey of L, so we add relation {MNOP}.
Therefore, the 3NF decomposition result is {K,I,J,L}, {K,M,N}, {M,J}, and {M,N,O,P}
It is lossless and dependency preserving because 3NF guarantees preservation of dependencies and lossless join.

1. (d) The schema does not allow redundancy.

According to the worksheet solution, “Because we formed each relation from an FD, the LHS of those FDs are indeed superkeys for their relations. However, there may be other FDs that violate BCNF and therefore allow redundancy. The only way to find out is to project the FDs onto each relation.” Check FDs in minimal basis, J->K will project onto {K, I, J, L} and J+ = {K, I, J, L}, K->IJL will also project onto {K, I, J, L} and K+ = {K, I, J, L}, KN->M will project onto {K, M, N} and K+ = {K, M, N}, M->J will project onto {M, J} and M+ = {M, J}. Therefore our schema does not allow redundancy.

2. T: C, D, E, F, G, H, I, J
   ST = {C → EH, DEI → F, F → D, EH → CJ, J → FGI}

2(a).

FDs	Closure	Violate BCNF?
C → EH	C+ = {C, D, E, F, G, H, I, J}	No, does not violate (since C is a superkey)
DEI → F	DEI+ = {D, E, F, I}	Yes, violate (since DEI is not a superkey)
F → D	F+ = {D, F}	Yes, violate (since F is not a superkey)
EH → CJ	EH+ = {C, D, E, F, G, H, I, J}	No, does not violate (since EH is a superkey)
J → FGI	J+ = {D, F, G, I, J}	Yes, violate (since J is not a superkey)

2(b).

- Decompose T using FD: DEI → F. DEI+ = {D, E, F, I}, so this yields two relations:
  R1 = DEFI, R2 = CDEGHIJ.
- Project the FDs onto R1 = DEFI.

D	E	F	I	closure	FDs
√				D+ = {D}	nothing
	√			E+ = {E}	nothing
		√		F+ = {D, F}	F → D: violate BCNF; abort the projection

We must decompose R1 further.

- Decompose R1 using FD: F → D. F+ = {D, F}, so this yields two relations:
  R3 = DF, R4 = EFI.
- Project the FDs onto R3 = DF.

D	F	closure	FDs
√		D+ = {D}	nothing
	√	F+ = {D, F}	F → D; F is a superkey of R3
Superset of F		irrelevant	Can only generate weaker FDs than what we already have

This relation satisfies BCNF.

- Project the FDs onto R4 = EFI.

E	F	I	closure	FDs
√			E+ = E	nothing
	√		F+ = {D, F}	nothing
		√	I + = I	nothing
√	√		EF+ = {D, E, F}	nothing
√		√	EI + = {E, I}	nothing
	√	√	FI+ = {D, F, I}	nothing
√	√	√	EFI+ = {E, F, I}	nothing

This relation satisfies BCNF.

- Return to R2 = CDEGHIJ and project the FDs onto it.

C	D	E	G	H	I	J	closure	FDs
√							C+ = {C, D, E, F, G, H, I, J}	C → DEGHIJ; C is a superkey of R2
	√						D+ = {D}	nothing
		√					E+ = {E}	nothing
			√				G+ = {G}	nothing
				√			E+ = {E}	nothing
					√		I+ = {I}	nothing
						√	J+ = {D, F, G, I, J}	J → DGI: violate BCNF; abort the projection

We must decompose R2 further.

- Decompose R2 using FD: J → DGI. J+ = {D, G, I}, so this yields two relations: R5 = DGIJ, R6 = CEHJ.
- Project the FDs onto R5 = DGIJ.

D	G	I	J	closure	FDs
√				D+ = {D}	nothing
	√			G+ = {G}	nothing
		√		I+ = {I}	nothing
			√	J+ = {D, F, G, I, J}	J → DGIJ; J is a superkey of R5
√	√			DG+ = {D, G}	nothing
√		√		DI+ = {D, I}	nothing
	√	√		GI+ = {G, I}	nothing
√	√	√		DGI+ = {D, G, I}	nothing

This relation satisfies BCNF.

- Project the FDs onto R6 = CEHJ.

C	E	H	J	closure	FDs
√				C+ = {C, D, E, F, G, H, I, J}	C → EHJ; C is a superkey of R6
	√			E+ = {E}	nothing
		√		H+ = {H}	nothing
			√	J+ = {D, F, G, I, J}	nothing
	√	√		EH+ = {C, E, H, J}	EH → CJ; EH is a superkey of R6
	√		√	EJ+ = {E, J}	nothing
		√	√	HJ+ = {H, J}	nothing

This relation satisfies BCNF.

- Final decomposition:
    (a) R3 = DF with FD F → D.
    (b) R4 = EFI with no FDs.
    (c) R5 = DGIJ with FD J → DGI.
    (d) R6 = CEHJ with FD C → EHJ, EH → CJ.