

NOTES ON CONTEXT FREE GRAMMARS

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1. PALINDROMES OVER $\{a, b\}$

1.1. Grammar.

$$S \rightarrow aSa \mid bSb \mid \varepsilon$$

1.2. Leftmost derivation example.

$$S \rightarrow aSa \rightarrow abSba \rightarrow abbSbba \rightarrow abbbSbbba \rightarrow abbbaSabbba \rightarrow abbbaaSaabbba \rightarrow abbbaaaabbba$$

2. $\{b^n a^m b^{2n} \mid n \geq 0, m \geq 0\}$

2.1. Grammar.

$$\begin{aligned} S &\rightarrow bSbb \mid A \\ A &\rightarrow aA \mid \varepsilon \end{aligned}$$

2.2. Leftmost derivation examples.

$$S \rightarrow A \rightarrow \varepsilon$$
$$S \rightarrow bSbb \rightarrow bbAbbbb \rightarrow bbbbbb$$
$$S \rightarrow A \rightarrow aA \rightarrow aaA \rightarrow aaaA \rightarrow aaa$$
$$S \rightarrow bSbb \rightarrow bbSbbbb \rightarrow bbbAbbbbbb \rightarrow bbbbaAbbbbbb \rightarrow bbbbaaAbbbbbb \rightarrow bbbbaabbbbbb$$

3. WELL-FORMED NESTED PARENTHESES AND SQUARE BRACKETS

3.1. Grammar.

$$\begin{aligned} S &\rightarrow SS \\ S &\rightarrow () \\ S &\rightarrow (S) \\ S &\rightarrow [] \\ S &\rightarrow [S] \end{aligned}$$

3.2. Leftmost derivation example.

$$\begin{aligned} S &\rightarrow (S) \rightarrow ([S]) \rightarrow ([SS]) \rightarrow ([[S]S]) \rightarrow ([[[S]]S]) \rightarrow ([[[SS]]S]) \rightarrow ([[[SSS]]S]) \\ &\rightarrow ([[[SSSS]]S]) \rightarrow ([[[()SSS]]S]) \rightarrow ([[[()()SS]]S]) \rightarrow ([[[()()[]S]]S]) \\ &\rightarrow ([[[()()[] []]S]) \rightarrow ([[[()()[] []](S)]) \rightarrow ([[[()()[] []]([])]) \end{aligned}$$

4. STRINGS OVER $\{a, b\}$ WITH AN UNEQUAL NUMBER OF a 'S AND b 'S

4.1. Grammar.

$S \rightarrow U \mid V$
 $U \rightarrow TaU \mid TaT$
 $V \rightarrow TbV \mid TbT$
 $T \rightarrow aTbT \mid bTaT \mid \varepsilon$

Rule 1: a string with an unequal number of a 's and b 's is a string with more a 's or a string with more b 's.

Rule 2: a string with more a 's than b 's has a smallest initial segment with more a 's; this initial segment ends in a , has an equal number of a 's and b 's before that last occurrence of a , and is followed by a string with a number of occurrences of a 's at least equal to the number of occurrences of b 's.

Rule 3: a string with more b 's than a 's has a smallest initial segment with more b 's; this initial segment ends in b , has an equal number of a 's and b 's before that last occurrence of b , and is followed by a string with a number of occurrences of b 's at least equal to the number of occurrences of a 's.

Rule 4: a string with an equal number of a 's and b 's either starts with a or with b . If it starts with a , that initial a is followed by a smallest substring with a number of occurrences of b equal to 1 plus the number of occurrences of a ; such a substring ends in b , has an equal number of a 's and b 's before that last occurrence of b , and is followed by a string with a number of occurrences of a 's equal to the number of occurrences of b 's. If it starts with b , that initial b is followed by a smallest substring with a number of occurrences of a equal to 1 plus the number of occurrences of b ; such a substring ends in a , has an equal number of a 's and b 's before that last occurrence of a , and is followed by a string with a number of occurrences of a 's equal to the number of occurrences of b 's.

4.2. Leftmost derivation examples.

$a\ b\ b\ a\ b\ a\ a\ a\ b$
 S
 U
 T aT
 $aTbT$ aT
 $a\ bT$ aT
 $a\ b\ bTaT$ aT
 $a\ b\ b\ aT$ aT
 $a\ b\ b\ a\ bTaTaT$
 $a\ b\ b\ a\ b\ aTaT$
 $a\ b\ b\ a\ b\ a\ aT$
 $a\ b\ b\ a\ b\ a\ a\ aTbT$
 $a\ b\ b\ a\ b\ a\ a\ a\ bT$
 $a\ b\ b\ a\ b\ a\ a\ a\ b$

$S \rightarrow U \rightarrow TaT \rightarrow aTbTaT \rightarrow abTaT \rightarrow abbTaTaT \rightarrow abbaTaT \rightarrow abbabTaTaT \rightarrow abbabaTaT$
 $\rightarrow abbabaaT \rightarrow abbabaaaTbT \rightarrow abbabaaaabT \rightarrow abbabaaaab$

$b\ b\ b\ b\ a\ a$
 S
 V
 TbV
 bV
 $bTbT$
 $b\ bT$
 $b\ b\ bT$ aT
 $b\ b\ b\ bTa$ aT
 $b\ b\ b\ b\ a\ aT$
 $b\ b\ b\ b\ a\ a$

$S \rightarrow V \rightarrow TbV \rightarrow bV \rightarrow bTbT \rightarrow bbT \rightarrow bbbTaT \rightarrow bbbbaTaaT \rightarrow bbbbaaT \rightarrow bbbbaa$