COMP9334 - Assignment 1

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Question 1

(a) ***Determine the service demand for each device of the system.***

For CPU: Utilization U(j1) = B/T = 2929/3600 = 81.36%

X(0) = C(0)/T = 1267/3600 = 0.35 jobs/s

Service demand D(j1) = U(j1)/X(0) = 0.81/0.35 = 2311.8ms

For Disk: Utilization U(j2) = B/T = 2765/3600 = 76.81%

X(0) = C(0)/T = 1267/3600 = 0.35 jobs/s

Service demand D(j2) = U(j2)/X(0) = 0.77/0.35 = 2182.3ms

(b) ***Use bottleneck analysis to determine the asymptotic bound on the system throughput when there are 20 active terminals and the think time per job is 14 seconds.***

Considering think time(T) into calculating, the Bottleneck analysis will be:

$$\mathrm{X}(0) \leq min\left[\frac{1}{maxDi}, \frac{N}{\sum_{i=1}^{k} Di + T}\right]$$

The maximum service demand is D(cpu)=2.31ms, then $\frac{1}{maxDi} = \frac{1}{2.31} = 0.43$

$$\frac{N}{\sum_{i=1}^{k} Di + T} = \frac{20}{2.31 + 2.2 + 14} = 1.08$$

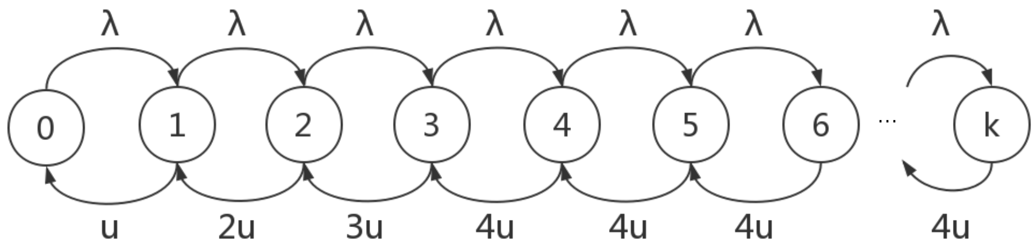
So, the asymptotic bound will be 0.43jobs/s.

Question 2

(a) ***Formulate a continuous-time Markov chain for a system similar to that described above with 4 staff and n waiting slots.***

$$\lambda = 15,\quad \mu = 3$$

There may have n+5 states for the Markov chain: 0-4 staffs busy while zero in waiting; 4 staffs busy while one call waiting; 4 staffs busy while two calls in queue; 4 staff busy while n calls waiting.



State 0: All staffs are idle;

State 1: Only 1 staffs are busy;

State 2: Only 2 staffs are busy;

State 3: 3 staffs are busy;

State 4: All staffs are busy;

State 5: All staffs are busy and one call in queue;

...

State k: All staffs are busy and n calls in queue. (k=n+4)

(b) ***Write down the balance equations for the continuous-time Markov chain that you have formulated.***

$$\lambda P_0 = \mu P_1$$
$$\lambda P_0 + 2\mu P_2 = (\mu+\lambda)P_1 \Longrightarrow 2\mu P_2 = \lambda P_1$$
$$\lambda P_1 + 3\mu P_3 = (2\mu+\lambda)P_2 \Longrightarrow 3\mu P_3 = \lambda P_2$$
$$\lambda P_2 + 4\mu P_4 = (3\mu+\lambda)P_3 \Longrightarrow 4\mu P_4 = \lambda P_3$$
$$\lambda P_3 + 4\mu P_5 = (4\mu+\lambda)P_4 \Longrightarrow 4\mu P_5 = \lambda P_4$$
$$\lambda P_4 + 4\mu P_6 = (4\mu+\lambda)P_5 \Longrightarrow 4\mu P_6 = \lambda P_5$$

...

$$4\mu P_k = \lambda P_{k-1} \text{ (k=n+4)}$$

(c) ***Derive expressions for the steady state probabilities of the continuous-time Markov chain that you have formulated.***

$$(1)\ \mathrm{P}_1 = \frac{\lambda \mathrm{P}_0}{\mu} = \rho \mathrm{P}_0$$

$$(2)\ \mathrm{P}_2 = \frac{\rho^2}{2}\mathrm{P}_0$$

$$(3)\ \mathrm{P}_3 = \frac{\rho^3}{6}\mathrm{P}_0$$

$$(4)\ \mathrm{P}_4 = \frac{\rho^4}{24}\mathrm{P}_0$$

$$(5)\ \mathrm{P}_5 = \frac{\rho^5}{96}\mathrm{P}_0$$

$$(6)\ \mathrm{P}_6 = \frac{\rho^6}{384}\mathrm{P}_0$$

...

$$\mathrm{P}_k = \frac{\rho^4}{24} * (\frac{\rho}{4})^{k-4}\mathrm{P}_0$$

$$\Rightarrow \mathbf{P}_k = \begin{cases} \mathbf{P}_0 \dfrac{1}{k!}\boldsymbol{\rho}^k, & k \le 4 \\ \mathbf{P}_0 \dfrac{1}{24 * 4^{k-4}}\boldsymbol{\rho}^k, & k > 4 \end{cases}$$

$$(7)\ \mathrm{P}_0 + \mathrm{P}_1 + \mathrm{P}_2 + \mathrm{P}_3 + \mathrm{P}_4 + \mathrm{P}_5 + \mathrm{P}_6 + \cdots + \mathrm{P}_k = 1$$

$$\Rightarrow \mathbf{P}_0 = \mathbf{1}/(\mathbf{1} + \boldsymbol{\rho} + \frac{\boldsymbol{\rho}^2}{\mathbf{2}} + \frac{\boldsymbol{\rho}^3}{\mathbf{6}} + \frac{\boldsymbol{\rho}^4}{\mathbf{24}} + \frac{\boldsymbol{\rho}^4}{\mathbf{24}} * \sum_{i=5}^{k} (\frac{\boldsymbol{\rho}}{\mathbf{4}})^{k-4})$$

(d) ***For the current configuration, i.e. for n = 2, determine:***

(i) The probability that an arriving query will be rejected. Let us denote the result of this by x.

When there are only 2 waiting slot, the final state will be state 6: 4 staffs are busy and 2 calls in queue. If the system is in this state, then any arriving query will be rejected.

We can combine (1)-(7) equations to get the probability of $\mathbf{P}_6$.

I use python program to compute result for questions, see attached file q2.py

The probability is that an arriving query will be rejected $\mathbf{P}_6 = \mathbf{0.29}$

(ii) The mean waiting time of an accepted query in the queue.

Using the Little' s law, we can know that $N = \sum_{k=0}^{6} k * \mathrm{P}_k = 6.295$

$$\mathrm{X}(0) = \lambda * (1 - \mathrm{P(reject)}) = 15 * (1 - 0.29) = 10.65/h$$

Hence R = N/X(0) = 6.295/10.65 = 0.59h = 2127.89 s

The mean waiting time will be T = R-T(s) = 2127.89 – 600 = 1527.89 s

(e) ***Determine the blocking probability if you add 5, 10, 15 and 20 waiting slots.***

I use python program to compute result for questions, see attached file q2.py

When adding 5 waiting slots, n=7, $P_{11} = 0.223$

When adding 5 waiting slots, n=12, $P_{16} = 0.207$

When adding 5 waiting slots, n=17, $P_{21} = 0.202$

When adding 5 waiting slots, n=22, $P_{26} = 0.201$

(f) ***Explain why there is little drop in blocking probability after adding 10 waiting slots. What should you do to reduce the blocking probability?***

According to Poisson Distribution, the probability drops abruptly in a specific segment, then it will drop little as increasing waiting slots.

This problem can be fixed if we increase the number of staff in the system.

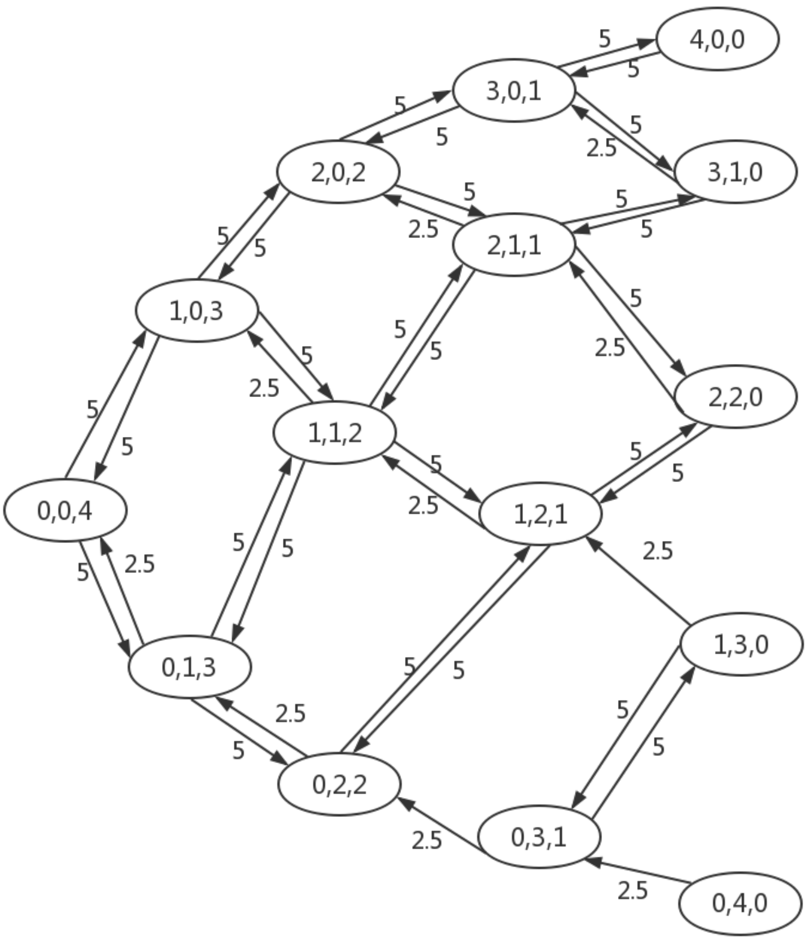
Question 3

(a) Formulate a continuous-time Markov chain for this computer system.

u(CPU1) = 1/0.2 = 5/s;

u(CPU2) = 1/0.4 = 2.5/s;

u(Disk) = 1/0.2 = 5/s;



(b) Write down the balance equations for the continuous-time Markov chain that you have formulated in Part (a).

10P(0,0,4)– 5P(1,0,3) - 2.5P(0,1,3) + 0P(2,0,2) + 0P(1,1,2) + 0P(0,2,2) + 0P(3,0,1) + 0P(2,1,1) + 0P(1,2,1) + 0P(0,3,1) + 0P(4,0,0) + 0P(3,1,0) + 0P(2,2,0) + 0P(1,3,0) + 0P(0,4,0) = 0

-5P(0,0,4)+**15P(1,0,3)** + 0P(0,1,3) - 5P(2,0,2)–2.5P(1,1,2) + 0P(0,2,2) + 0P(3,0,1) + 0P(2,1,1) + 0P(1,2,1) + 0P(0,3,1) + 0P(4,0,0) + 0P(3,1,0) + 0P(2,2,0) + 0P(1,3,0) + 0P(0,4,0) = 0

-5P(0,0,4)+0P(1,0,3)+**12.5P(0,1,3)** +0P(2,0,2) -5P(1,1,2) -2.5P(0,2,2) + 0P(3,0,1) + 0P(2,1,1) + 0P(1,2,1) + 0P(0,3,1) + 0P(4,0,0) + 0P(3,1,0) + 0P(2,2,0) + 0P(1,3,0) + 0P(0,4,0) = 0

0P(0,0,4)-5P(1,0,3)+0P(0,1,3) + **15P(2,0,2)** +0P(1,1,2) +0P(0,2,2) -5P(3,0,1) -2.5P(2,1,1) + 0P(1,2,1) + 0P(0,3,1) + 0P(4,0,0) + 0P(3,1,0) + 0P(2,2,0) + 0P(1,3,0) + 0P(0,4,0) = 0

0P(0,0,4)-5P(1,0,3)-5P(0,1,3) + 0P(2,0,2) +**17.5P(1,1,2)** +0P(0,2,2) + 0P(3,0,1) -5P(2,1,1) - 2.5P(1,2,1) + 0P(0,3,1) + 0P(4,0,0) + 0P(3,1,0) + 0P(2,2,0) + 0P(1,3,0) + 0P(0,4,0) = 0

0P(0,0,4)+0P(1,0,3)-5P(0,1,3) + 0P(2,0,2) -0P(1,1,2) +**7.5P(0,2,2)** + 0P(3,0,1) + 0P(2,1,1) - 5P(1,2,1) -2.5P(0,3,1) + 0P(4,0,0) + 0P(3,1,0) + 0P(2,2,0) + 0P(1,3,0) + 0P(0,4,0) = 0

0P(0,0,4)+0P(1,0,3)+0P(0,1,3) -5P(2,0,2) +0P(1,1,2) +0P(0,2,2) + **15P(3,0,1)** + 0P(2,1,1) + 0P(1,2,1) + 0P(0,3,1) -5P(4,0,0) -2.5P(3,1,0) + 0P(2,2,0) + 0P(1,3,0) + 0P(0,4,0) = 0

0P(0,0,4)+0P(1,0,3)+0P(0,1,3) -5P(2,0,2) -5P(1,1,2) +0P(0,2,2) + 0P(3,0,1) + **17.5P(2,1,1)** + 0P(1,2,1) + 0P(0,3,1) + 0P(4,0,0) -5P(3,1,0) -2.5P(2,2,0) + 0P(1,3,0) + 0P(0,4,0) = 0

0P(0,0,4)+0P(1,0,3)+0P(0,1,3) + 0P(2,0,2) -5P(1,1,2) -5P(0,2,2) + 0P(3,0,1) + 0P(2,1,1) + **12.5P(1,2,1)** + 0P(0,3,1) + 0P(4,0,0) + 0P(3,1,0) -5P(2,2,0) -2.5P(1,3,0) + 0P(0,4,0) = 0

0P(0,0,4)+0P(1,0,3)+0P(0,1,3) + 0P(2,0,2) +0P(1,1,2) +0P(0,2,2) + 0P(3,0,1) + 0P(2,1,1) + 0P(1,2,1) + **7.5P(0,3,1)** + 0P(4,0,0) + 0P(3,1,0) + 0P(2,2,0) -5P(1,3,0) -2.5P(0,4,0) = 0

0P(0,0,4)+0P(1,0,3)+0P(0,1,3) + 0P(2,0,2) +0P(1,1,2) +0P(0,2,2) -5P(3,0,1) + 0P(2,1,1) + 0P(1,2,1) + 0P(0,3,1) + **5P(4,0,0)** + 0P(3,1,0) + 0P(2,2,0) + 0P(1,3,0) + 0P(0,4,0) = 0

0P(0,0,4)+0P(1,0,3)+0P(0,1,3) + 0P(2,0,2) +0P(1,1,2) +0P(0,2,2) -5P(3,0,1) -5P(2,1,1) + 0P(1,2,1) + 0P(0,3,1) + 0P(4,0,0) + **7.5P(3,1,0)** + 0P(2,2,0) + 0P(1,3,0) + 0P(0,4,0) = 0

0P(0,0,4)+0P(1,0,3)+0P(0,1,3) + 0P(2,0,2) +0P(1,1,2) +0P(0,2,2) + 0P(3,0,1) -5P(2,1,1) - 5P(1,2,1) + 0P(0,3,1) + 0P(4,0,0) + 0P(3,1,0) + **7.5P(2,2,0)** + 0P(1,3,0) + 0P(0,4,0) = 0

0P(0,0,4)+0P(1,0,3)+0P(0,1,3) + 0P(2,0,2) +0P(1,1,2) +0P(0,2,2) + 0P(3,0,1) + 0P(2,1,1) + 0P(1,2,1) -5P(0,3,1) + 0P(4,0,0) +0P(3,1,0) + 0P(2,2,0) + **7.5P(1,3,0)** + 0P(0,4,0) = 0

0P(0,0,4)+0P(1,0,3)+0P(0,1,3) + 0P(2,0,2) +0P(1,1,2) +0P(0,2,2) + 0P(3,0,1) + 0P(2,1,1) + 0P(1,2,1) + 0P(0,3,1) + 0P(4,0,0) + 0P(3,1,0) + 0P(2,2,0) + 0P(1,3,0) + **2.5P(0,4,0)** = 0

P(0,0,4)+P(1,0,3)+P(0,1,3) + P(2,0,2) +P(1,1,2) +P(0,2,2) + P(3,0,1) + P(2,1,1) + P(1,2,1) + P(0,3,1) + P(4,0,0) + P(3,1,0) + P(2,2,0) + P(1,3,0) + P(0,4,0) = 1

(c) What are the steady state probabilities for each state?

I use python program to compute result for questions, see attached file q3.py

P(0,0,4) = 0.04

P(1,0,3) = 0.04

P(0,1,3) = 0.08

P(2,0,2) = 0.04

P(1,1,2) = 0.08

P(0,2,2) = 0.16

P(3,0,1) = 0.04

P(2,1,1) = 0.08

P(1,2,1) = 0.16

P(0,3,1) = 0.0

P(4,0,0) = 0.04

P(3,1,0) = 0.08

P(2,2,0) = 0.16

P(1,3,0) = 0.0

P(0,4,0) = 0.0

(d) What is the throughput of the system?

The throughput is decided by the disk, Throughput = U(disk)/S(disk)

U(disk)=1 – (P(4,0,0) + P(3,1,0) + P(2,2,0) + P(1,3,0) + P(0,4,0)) = 1-(0.04+0.08+0.16)=0.72

X(disk) = U(disk)/S(disk) = 0.72/0.2 = 3.6/s

(e) What is the mean number of jobs in CPU1?

$$N(cpu1) = \sum_{k=0}^{k} n * p(cpu1)$$

$$= \mathrm{P}(1,0,3) + 2\mathrm{P}(2,0,2) + \mathrm{P}(1,1,2) + 3\mathrm{P}(3,0,1) + 2\mathrm{P}(2,1,1) + \mathrm{P}(1,2,1) + 4\mathrm{P}(4,0,0) + 3\mathrm{P}(3,1,0) + 2\mathrm{P}(2,2,0) + \mathrm{P}(1,3,0) = 1.36$$

(f) What is the mean response time of CPU1?

$$X(cpu1) = \frac{U}{S}$$

$$= \frac{\mathrm{P}(1,0,3) + \mathrm{P}(2,0,2) + \mathrm{P}(1,1,2) + \mathrm{P}(3,0,1) + \mathrm{P}(2,1,1) + \mathrm{P}(1,2,1) + \mathrm{P}(4,0,0) + \mathrm{P}(3,1,0) + \mathrm{P}(2,2,0) + \mathrm{P}(1,3,0)}{0.2}$$

$$= \frac{0.72}{0.2} = 3.6/s$$

Mean response time is $\frac{N(cpu1)}{X(cpu1)} = \frac{1.36}{3.6} = 0.38s$