

Robust video denoising

using low rank matrix

Completion

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- for each image consider one image patch $P_{j,k}$ Size $n \times n$
 $P_{j,k}$ ex: $n=5$
 patch number k^{th} image

- we search for patches that are similar to $P_{j,k}$ both in the same image and other existing frames. small P

- let's assume ~~more~~ m patches found similar to $P_{j,k}$: small P

$$\{P_{i,j,k}\}_{i=1}^m$$

~~Column~~

we present each $P_{i,j,k}$ as a vector with n^2 elements,

$$\Rightarrow \vec{P}_{j,k}^{\text{big } P} = (P_{1,j,k}, P_{2,j,k}, \dots, P_{m,j,k}) \Rightarrow P_{j,k} = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ m \end{bmatrix}_{n^2 \text{ vector}}$$

\Rightarrow we can write:

$$P_{j,k} = Q_{j,k} + N_{j,k}$$

- if the data is free of noise and patch matching is also perfect, all columns of $Q_{j,k}$ have similar underlying image structure, the rank $Q_{j,k}$ should be low, and the variance of each row vector in $P_{j,k}$ should be very small.

- Most video denoising algorithms assume a single statistical model of image noise which ~~is~~ does not work well in practice.
- This paper, presents a new patch-based video denoising algorithm able of removing serious mixed noise from video data.

Problem formulation

- Let $F = \{f_k\}_{k=1}^K$ be the image sequence with K frames (video)
- $F = \{f_1, f_2, \dots, f_K\}$
- each image f_k is a sum of its underlying clean image g_k and the noise n_k : $f_k = g_k + n_k$
- goal is to recover $\{g_1, g_2, \dots, g_M\}$ by removing n_k from f_k .

Some mathematics review:

$$\|X\|_F = \left(\sum_{i,j} |x_{i,j}|^2 \right)^{1/2}$$

↳ Frobenius norm

$$\|X\|_* = \sum_i (\sigma_i(X))$$

↳ nuclear norm

$\sigma_i(X)$ = i^{th} largest singular value of X .

- $X = U \sum V^T$ be SVD of X

- $\circledast D_\tau(X) = U \sum_\tau V^T$ where $\sum_\tau = \text{diag}(\max(\sigma_i - \tau, 0))$

↳ soft shrinkage

- \mathcal{R} be ~~row~~ index set

- $X|_{\mathcal{R}}$ denotes denotes the vector including elements in \mathcal{R} only.

① reliable elements in $P_{j,k}$ should not have large deviation from the mean of all elements in the same row.

② assume the reliable element indexes of $P_{j,k}$ is \mathcal{R} .

③ as we said, $Q_{j,k}$ (clean patches) should have low rank structure.

Therefore the mathematical formulation of the problem is as follows:

$$(*) \left\{ \begin{array}{l} \min_Q \|Q\|_* \\ \text{s.t. } \|\tilde{Q}|_{\mathcal{R}} - P|_{\mathcal{R}}\|_F^2 \leq \#(\mathcal{R}) \hat{\sigma}^2 \end{array} \right.$$

size of set \mathcal{R}
 $\#(\mathcal{R})$
 $\hat{\sigma}^2$
 estimate of standard deviation of noise from the noisy obs. in \mathcal{R}

Solve Lagrangian Version:

$$\left\{ \begin{array}{l} \min_Q \frac{1}{2} \|\tilde{Q}|_{\mathcal{R}} - P|_{\mathcal{R}}\|_F^2 + \mu \|Q\|_* \end{array} \right. \quad (***)$$

\downarrow
 equivalent to (*)

Parameter μ should be chosen in such a way

$$\text{ s.t. } \|\tilde{Q}|_{\mathcal{R}} - P|_{\mathcal{R}}\|_F^2 \approx \#(\mathcal{R}) \hat{\sigma}^2$$

$$\hookrightarrow \text{good heuristic for } \mu = (\sqrt{n_1} + \sqrt{n_2}) \sqrt{P} \hat{\sigma} \quad \left\{ \begin{array}{l} n_1 = n \times n \\ n_2 = m \end{array} \right.$$

Also the paper provides an algorithm to solve minimization problem $(***) \rightarrow \underline{\text{Algorithm I}}$

From denoising patches to denoising images



So far, we can effectively remove noises from all patches. Now we should synthesize the denoised image from these denoised patches.

denoised patches overlap in some pixels.

Therefore for those overlapping pixels, we simply take average.

Experiment

• $K = 50$ image Frames patch size = 8×8 pixels

range of image intensity = $[0, 255]$

for each reference patch, 5 most similar patches are used.

↳ based on L_1 norm distance.

⇒ totally 250 patches are stacked.

Input image with mixed noise

$$n_k = n_k^g + n_k^p + n_k^i$$

↗ Gaussian noise ↗ Poission noise ↗ impulsive noise

fixed gaussian noise $\sigma = 10$

varied poission noise level k in range $[5, 30]$

impulsive noise level S $[10\%, -40\%]$

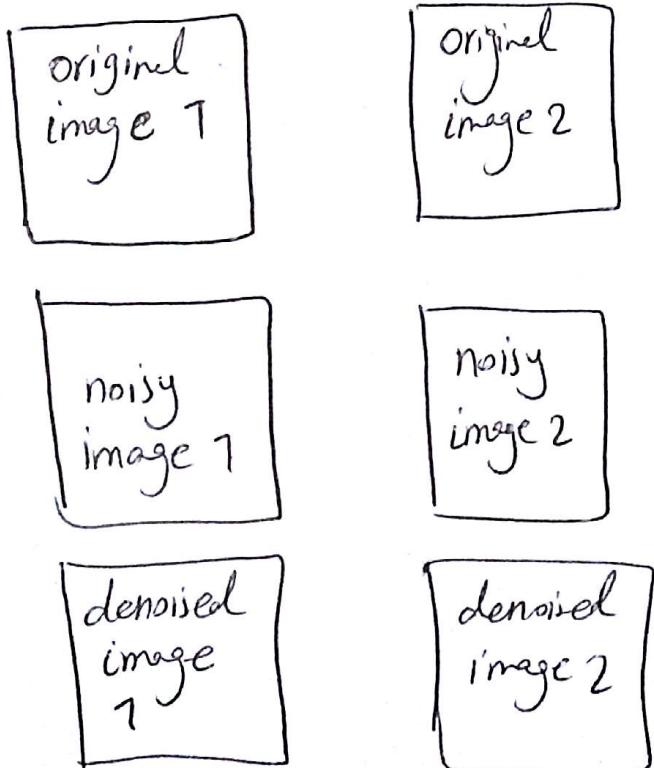
our measure of success

in denoising is measured

by PSNR standard:

$$\underline{PSNR(f)} = 10 \log_{10} \frac{2SS^2}{\|f - f^r\|_2^2}$$

↗ original image ↗ denoised image



S/k	5	10	20	30
10%				
20%				
30%				
40%				

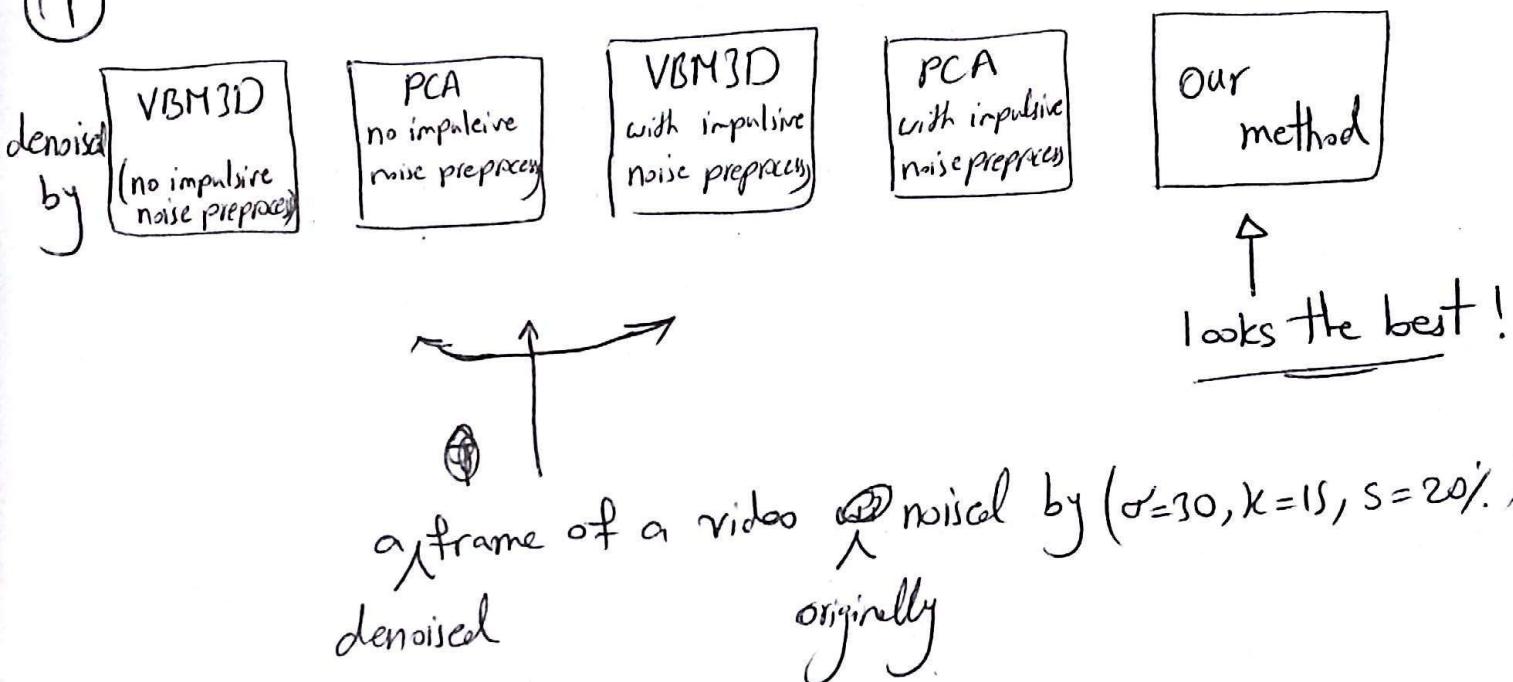
Corresponding PSNR values ($\sigma = 10$)

PSNR if k with fixed S

Comparison to other denoising approaches:

① Compare our denoising with existing video denoising methods such as: VBM3D method and PCA based method.

(1)



(2)

