Transform invarient low-rank texture

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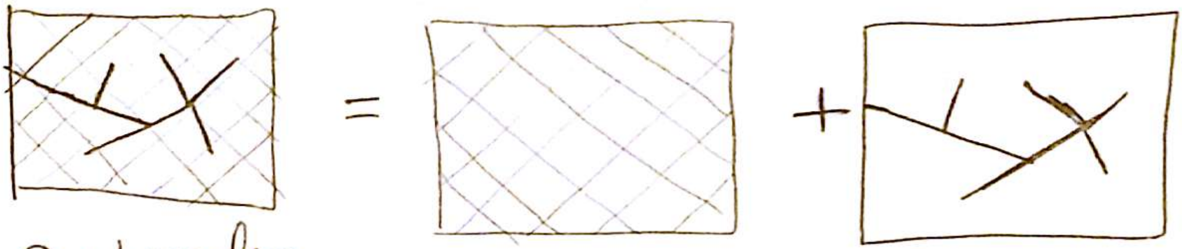
Low Rank Matrix Recovery:



D-observation A-low-rank E-sparse — occlusion

Problem: Given $D = A_0 + E_0$, recover A_0 and E_0

A_0 — Low rank Component; E_0 — Sparse Component, occlusion

So the objective function is as follows:

$$\begin{cases} \min \ \text{rank}(A) + \gamma \|E\|_0 & \text{~~Subject to~~} \\ \text{s.t. } A + E = D \end{cases}$$

we know E is sparse

NP-Hard - relax via Convex opt

$$\begin{cases} \|E_0\| \longrightarrow \|E_1\| = \sum_{ij} |E_{ij}| & L_1 \text{ norm} \\ \text{rank}(A) \longrightarrow \|A\|_* = \sum_i \sigma_i(A) & \text{nuclear norm} \end{cases}$$

as we know: $\underset{m\times n}{A} = \underset{m\times m}{U} \ \underset{m\times n}{\Sigma} \ \underset{n\times n}{V^T}$

$$\underset{m\times n}{\Sigma} = \begin{pmatrix} \sigma_1 & & & 0 \\ & \ddots & & \\ & & \sigma_r & \\ 0 & & & \ddots \end{pmatrix}$$

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$\Rightarrow$ Convex objective function :

$$\begin{cases} \min \ \|A\|_* + \lambda \|E\|_1 \\ \text{s.t.} \quad A + E = D \end{cases} \Rightarrow \text{Semidefinit polynomial time}$$

The question is when the solution of convex objective function is equivalent to the solution of original problem? Not always, but it maybe succeeds in cases we are about like survilances.

Low Rank Matrix Recovery + Deformation :

Problem setting :

n well-aligned grayscale images $I_1^0, I_2^0, \ldots, I_n^0 \in \mathbb{R}^{w \times h}$

well-aligned images are linearly corplated.

$$\text{vec} : \mathbb{R}^{w \times h} \longrightarrow \mathbb{R}^m \xrightarrow{\text{create } A} A = \left[\text{vec}(I_1^0) \middle| \cdots \middle| \text{vec}(I_n^0) \right] \in \mathbb{R}^{m \times n}$$

Modeling Corruption : $I_1 = I_1^0 + e_1 \ , \ \ldots \quad I_n = I_n^0 + e_n$

Modeling Misalignment:

original pic ? — well-aligned — occlusion — misallignment

$$I_1 = (I_1^0 + e_1) \circ \tau_1^{-1}$$

$\vdots$

$$I_n = (I_n^0 + e_n) \circ \tau_n^{-1}$$

where $\tau_1^{-1}, \dots, \tau_n^{-1} \in G$

Our goal is to : ① recover images $\{I_i^0\}$

② transformations $\{\tau_i\}$

Approach:

set of all well aligned no currupted images — Original Dataset

$$\min_{A,E,\tau} \operatorname{rank}(A) + \gamma \|E\|_0 \quad \text{s.t.} \quad D \circ \tau = A + E$$

NP-Hard

Convex form

(original set) or I — or I_0

$$\min_{A,E,\tau} \|A\|_* + \lambda \|E\|_1 \quad \text{s.t.} \quad D \circ \tau = A + E$$

$\circledast$

Definition :

$\Delta\tau \in \mathbb{R}^{p \times n}$

$$D \circ (\tau + \Delta\tau) \approx D \circ \tau + \overbrace{\sum_{i=1}^{n} J_i \Delta\tau_i \epsilon_i^T}^{D \circ \Delta\tau}$$

$$J_i \doteq \frac{\partial}{\partial \tau} \operatorname{vec}(I_{0i} \circ \tau)\Big|_{\tau = \tau_i} \Rightarrow \text{Jacobian } i\text{th}$$

Optimization Problem :

$$\begin{cases} \min\limits_{A, E, \Delta\tau} \ \|A\|_* + \lambda \|E\|_1 \quad (1) \\ \text{s.t. } D \circ \tau + \sum_{i=1}^{n} J_i \, \Delta\tau_i \, \varepsilon_i^T = A + E \end{cases}$$

Algorithm I : Solve Minimization Problem (*) (Prev. Page)

input : images $I_1, \ldots, I_n \in \mathbb{R}^{w \times h}$, initial transformation $\tau_1, \ldots, \tau_n$ in certain parametric group G, weight $\lambda > 0$

while not convergance Do:

1) Compute Jacobian matrices w.r.t transformation:

$$J_i \leftarrow \frac{\partial}{\partial \tau} \left(\frac{\mathrm{Vec}(I_i \circ \tau)}{\|\mathrm{Vec}(I_i \circ \tau)\|_2} \right)_{\tau = \tau_i} \quad i = 1, \ldots, n;$$

2) wrap and normalize images:

$$D \circ \tau \leftarrow \left[\frac{\mathrm{Vec}(I_1 \circ \tau_1)}{\|\mathrm{Vec}(I_1 \circ \tau_1)\|_2} \middle| \cdots \middle| \frac{\mathrm{Vec}(I_n \circ \tau_n)}{\|\mathrm{Vec}(I_n \circ \tau_n)\|_2} \right]$$

3) Solved Convex opts

$$(A^*, E^*, \Delta\tau^*) \leftarrow \arg\min_{A, E, \Delta\tau} \|A\|_* + \lambda \|E\|_1$$

$$\text{s.t. } D \circ \tau + \sum_{i=1}^{n} J_i \, \Delta\tau_i \, \varepsilon_i^T = A + E$$

4) update transformation: $\tau \leftarrow \tau + \Delta\tau^*$

End While

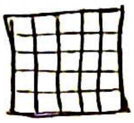
Output: A^*, E^*, τ ✓

~~Experimental~~

Experimental Results:

~~1) Synthetic~~

① Range of Convergence for affine transformation:

a checker board like pattern: 

is deformed by affine transforms of form $y = Ax + b$ where

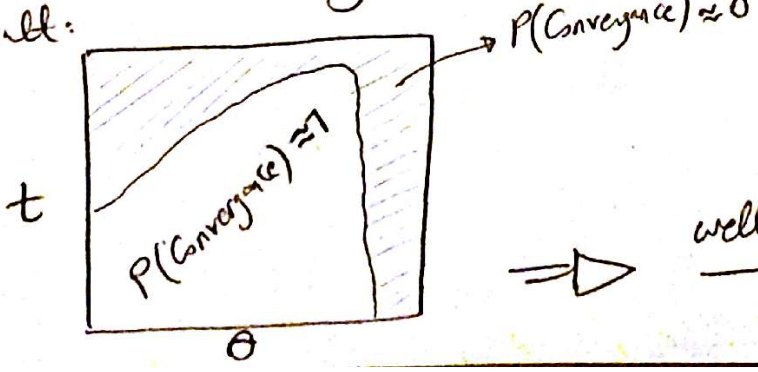
$$A(\theta, t) = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{\text{rotation}} \times \underbrace{\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}}_{\text{skew}}$$

Success of Convergance:

θ range $[0, \pi/6]$ stepsize $\pi/60$

t range $[0, 1]$ ~ 0.05

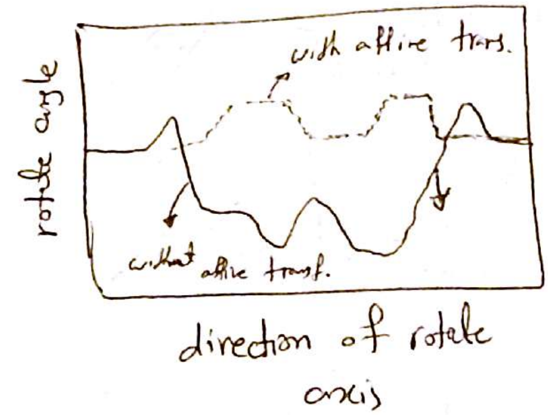
result:



$\Longrightarrow$ well performed for $\theta \le 20$ and $t \le 0.4$

② Range of Convergance for projective Transform:

we transform same checker board like pattern but with projective Transforms { 1) directio of rotate axis
2) rotate angle



~~under the line → 100% Convergance~~

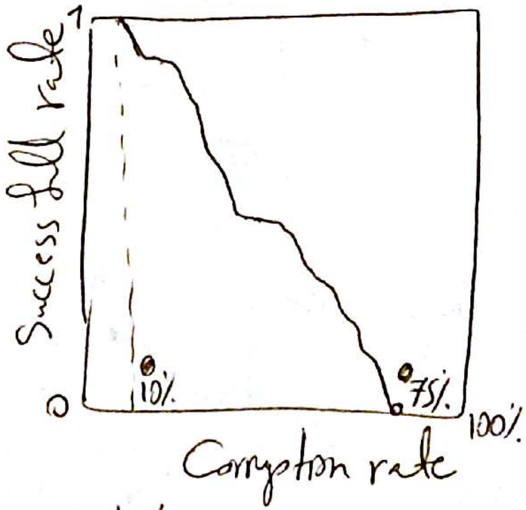
~~upper the line~~

below line ⇒ Convergance

③ Robustness Tests of TILT:

we experiment on some low rank patterns with deformation (rotation = 10°) and examine if TILT Convergens to correct Solution under different levels of random Corruption. (from 0% → 100% of pixels)

here is result:



* under 10% Corruption still 100% accuracy.

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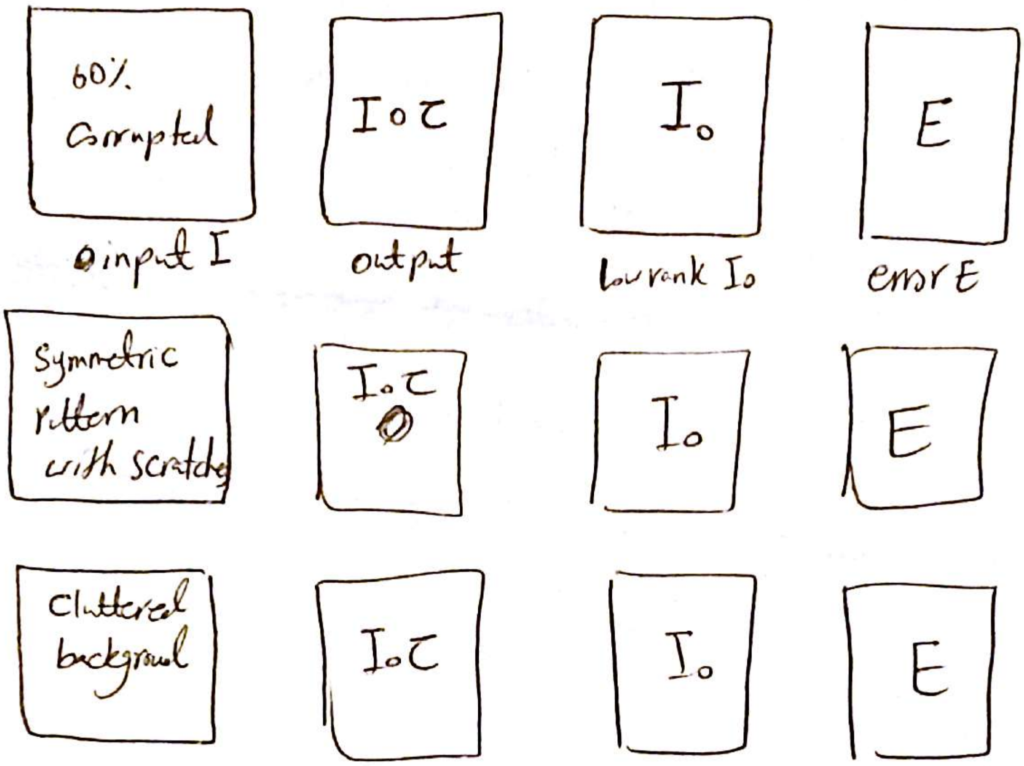
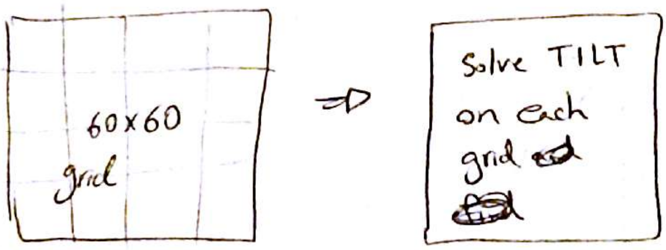
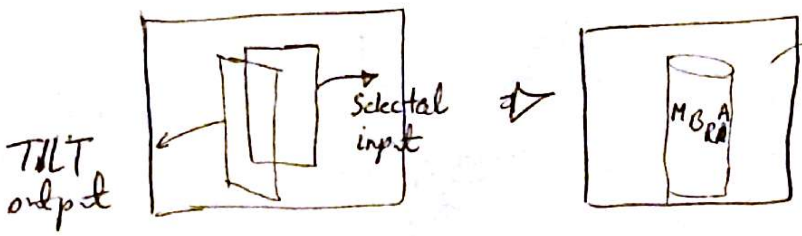
④ Robustness of TILT:



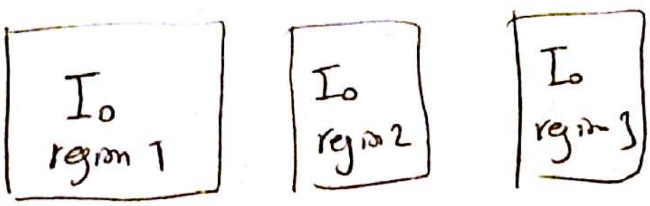
⑤ Shape from (low-rank) texture:

an image of urban scenery with tall buildings are given (image is deformed)





we can put object on it with the orientation of TILT algorithm



I_0 of different regions selected in 2nd row.

(6) Representative Results of TILT:

TILT is experimented on diff type of objects
- (1) regular patterns
- (2) signs, character, printed text
- (3) bar codes, ...

in all images input is the image part — red lines

~ ~ ~ output is IC — green lines

(7) Challenging Cases:

TILT was partially successful in the following challenging cases:

Cases:
- boundary problems
- not enough regular texture
- large perspective distortion
- too much random texture
- sparse low rank structure (binary image)

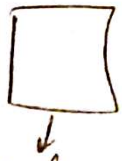
TILT ⟹ Partially Correct

(8) Failure Cases:

Cases
- high rank structure
- few low rank regions
- too much occlusion
- random texture

TILT ⟹ Fails X

9 Effect of initalization:

in part 7 if inalization was modified, 

real rectangle

output of TILT will be successful.

Conclusion:

1) Low rank minimization is a nice way to find regularities within the data
2) Nuclear norm is an efficiet and effective way for low rank minimization
3) impressive results for handling occlusion