Robust video denoising using low rank matrix Completion

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- for each image f_k Consider one image patch p_{jk} size $n \times n$ ex: $n=5$

j: patch number j, k: k^{th} image

- we search for patches that are similar to $p_{j,k}$ (small p) both in the same image and other existing frames.

- lets assum m patches found similar to $p_{j,k}$ (small p):

$$\{P_{i,j,k}\}_{i=1}^{m}$$

we present each $P_{i,j,k}$ as a column vector with n^2 elements.

$$\Rightarrow P_{j,k} = (P_{1,j,k}, P_{2,j,k}, \dots, P_{m,j,k}) \Rightarrow P_{j,k} = \begin{bmatrix} 1 & 2 & \cdots & m \end{bmatrix}_{n^2 m}$$

big P

size n^2 vector

$\Rightarrow$ we can write:

$$P_{j,k} = Q_{j,k} + N_{j,k}$$

- if the data is free of noise and patch matching is also perfect, all columns of $Q_{j,k}$ have similar underlying image structure, the rank $Q_{j,k}$ should be low, and the varience of each row vector in $P_{j,k}$ should be very small.

Scanned by CamScanner

- Most video denoising algorithms assume a single statistical model of image noise which does not work well in practice.

- This paper, presents a new patch-based video denoising algorithim capble of removing seriouse mixed noise from video data.

Problem formulation

- Let $F = \{f_k\}_{k=1}^{K}$ be the image sequence with K frames. (Video)

$$F = \{f_1, f_2, \dots, f_K\}$$

- each image f_k is a sum of its underlying clean image g_k and the noise n_k:

$$\underline{f_k = g_k + n_k}$$

- goal is to recover $\{g_1, g_2, \dots, g_M\}$ by removing n_k from f_k.

Some mathematics review:

$$\|X\|_F = \left(\sum_{i,j} |x_{i,j}|^2\right)^{1/2}$$

Forbenious norm

$$\|X\|_* = \sum_i (\sigma_i(X))$$

$\sigma_i(X)$ = ith largest singular value of X.

nuclear norm

- $X = U \Sigma V^T$ be SVD of X
- $D_\tau(X) = U \Sigma_\tau V^T$ where $\Sigma_\tau = \operatorname{diag}(\max(\sigma_i - \tau, 0))$

soft shrinkage

- Ω be and index set
- $X|_\Omega$ denotes denots the vector including elements in Ω only.

(1) reliable elements in $P_{j,k}$ should not have large deviation from the mean of all elements in the same row.

(2) assume the reliable element indexs of $P_{j,k}$ is Ω.

(3) as we said, $Q_{j,k}$ (clean patches) should have low rank structure.

Therefore the mathematical formulation of the problem is as follows:

size of set Ω

$$(*)\begin{cases} \min_{Q} \|Q\|_* \\ \text{s.t.} \ \|Q|_\Omega - P|_\Omega\|_F^2 \leq \#(\Omega)\,\hat{\sigma}^2 \end{cases}$$

estimate of standard deviation of noise from the noisy obs. in Ω

Solve Lagrangian Version:

$$\left\{ \min_{Q} \frac{1}{2}\|Q|_\Omega - P|_\Omega\|_F^2 + \mu\|Q\|_* \right.$$

$(**)$ equivalent to $(*)$

Parameter μ should be chosen in such a way that: $\|Q|_\Omega - P|_\Omega\|_F^2 \approx \#(\Omega)\hat{\sigma}^2$

good heuristic for $\mu = (\sqrt{n_1} + \sqrt{n_2})\sqrt{p}\,\hat{\sigma}$

$$\begin{cases} n_1 = n \times n \\ n_2 = m \end{cases}$$

Also the paper provides an algorithm to solve minimization problem $(**)$ $\rightarrow$ Algorithm I

From denoising patches to denoising images

So far, we can effectively remove noises from all patches. No we should synthetize the denoised image from these denoised patches.

denoised patches overlap in some pixels.

Therefore for those overlapping pixels, we simply take average.

Experiment

- $K = 50$ image Frames   patchsize $= 8 \times 8$ pixels

range of image intensity $= [0, 255]$

for each reference patch, 5 most similar patches are used.

↳ based on l_1 norm distance.

⇒ totally 250 patches are stacked.

Input image with mixed noise

$$n_k = n_k^g + n_k^p + n_k^i$$

n_k^i → impulsive noise

n_k^g → gaussian noise

n_k^p → Poission noise

fixed gaussian noise $\sigma = 10$

varied poission noise level k in range [5, 30]

impulsive noise level s [10% - 40%]

our measure of success in denoising is measured by PSNR standard:

$$\underline{PSNR(f^r)} = 10 \log_{10} \frac{255^2}{\|f - f^r\|_2^2}$$

f → original image

f^r → denoised image

original image 1	original image 2
noisy image 1	noisy image 2
denoised image 1	denoised image 2

S\K	5	10	20	30
10%	Corresponding PSNR values ($\sigma = 10$)			
20%				
30%				
40%				

PSNR ↓ if k ↑ with fixed s

Comparison to other denoising approaches:

Ⓟ

Compare our denoising with existing video denoising methods

Such as: VBM3D method and PCA based method.

(1)

denoised by

VBM3D (no impulsive noise preprocess)	PCA no impulsive noise preprocess	VBM3D with impulsive noise preprocess	PCA with impulsive noise preprocess	our method

looks the best!

a denoised frame of a video originally noised by ($\sigma=30, k=15, s=20\%$)

(2)

original frame	noised frame with $\sigma=50, k=5, s=10\%$	denoised by VBM3D	denoised by PCA-based	denoised by our method

again looks better than other methods.

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