

**Problem 1.** The forecasting system used by a firm uses the following seasonal indices for the 4 quarters:

S.I

Quarter 1:	0.90
Quarter 2:	1.10
Quarter 3:	1.05
Quarter 4:	0.95

The demands for the last 8 quarters are given below:

Year 1:	Quarter 1:	2244
	Quarter 2:	2482
	Quarter 3:	2310
	Quarter 4:	2120
Year 2:	Quarter 1:	2562
	Quarter 2:	2882
	Quarter 3:	2556
	Quarter 4:	2362

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- (a) Use the indices and the time series model to determine the forecasting equation (15 points).  
 (b) Forecast the demand for quarters 1 through 4 of Year 3 (5 points).

	demand	De-seasonalized demand	t	t <sup>2</sup>	ty
1Q1	2244	2493.3	1	1	2493.3
1Q2	2482	2256.3	2	4	4512
1Q3	2310	2200	3	9	6600
1Q4	2120	2231.6	4	16	
2Q1	2562	2846.6	5	25	
2Q2	2882	2620	6	36	
2Q3	2556	2434.3	7	49	17038
2Q4	2362	2486.3	8	64	19886
$\Sigma$		$\Sigma y = 19566$	$\Sigma t = 36$	$\Sigma t^2 = 204$	$\Sigma ty = 89449$

+ De-seasonalized = Actual demand / S.I  $\Rightarrow$  De-seasonalized 1Q1 =  $\frac{2244}{0.9} = 2493.3$   
 we have to de-seasonalized all actual data (shown in table)

$$\Sigma t = \frac{n(n+1)}{2} = \frac{8 \times 9}{2} = 36$$

$$\Sigma t^2 = \frac{n(n+1)(2n+1)}{6} = \frac{8 \times 9 \times 17}{6} = 204$$

$$b = \frac{N \Sigma ty - \Sigma y \Sigma t}{N \Sigma t^2 - (\Sigma t)^2} \Rightarrow b = \frac{8 \times 89449 - 19566 \times 36}{8(204) - 2(36)^2}$$

$$b = \frac{10864}{336} = 32.33$$

$$a = \frac{\Sigma y}{N} - b \frac{\Sigma t}{N} = \frac{19566}{8} - 32.33 \left( \frac{36}{8} \right) \Rightarrow$$

$$a = 2300.265$$

$$a = 2300.265$$

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$$b = 32.33$$

$$y = a + bt$$

$$\boxed{y = 2300.265 + 32.33t} \rightarrow \text{Forecasting equation}$$

$$(t=9) \text{ Year}_4 Q_1 \Rightarrow y = 2300.265 + 32.33(9) = 2591.235$$

$$\text{Forecast} = \underset{\text{Value}}{\text{deseasonalized}} \times \underset{Q_1}{S.I.} = 2591.235 \times 0.9 = \frac{2332.1}{Y4Q1} \checkmark$$

$$(t=10) \text{ Year}_4 Q_2 \Rightarrow y = 2300.265 + 32.33(10) = 2623.566$$

$$\text{Forecast} = 2623.56 \times 1.1 = \frac{2886}{Y4Q2} \checkmark$$

$$(t=11) \text{ Year}_4 Q_3 \Rightarrow y = 2300.265 + 32.33(11) = 2655.895$$

$$\text{Forecast} = \underset{\text{Value}}{\text{deseasonalized}} \times \underset{Q_3}{S.I.} = 2655.895 \times 1.05 = \frac{2789}{Y4Q3} \checkmark$$

$$(t=12) \rightarrow \text{Year}_4 Q_4 \rightarrow y = 2300.265 + 32.33(12) = 2688.225$$

$$\text{Forecast} = 2688 \times 0.95 = \frac{2554}{Y4Q4} \checkmark$$



**Problem 2.** A company produces a product that requires part X in its construction. The annual demand for part X is 5000 units. The cost of each part X is \$25 and each unit costs \$8 per year to carry in inventory (i.e.  $I=0$  and  $W=8$ ). (Assume multi-product case and that items are immediately available after manufacturing).

- (a) If the cost of placing an order for part X is \$15, what is the Economic Order Quantity? (5 points). 15
- (b) The company has the capability of producing part X internally. It estimates a setup cost of \$250 per production run. The production rate of part X would be 800 units per month. What is the Economic Manufacturing Quantity? (5 points).
- (c) Should the company produce part X internally or continue purchasing from outside sources? Why or why not? (10 points).

Given:  $D = 5000$  units/year

$I = 0$   
 $W = 8$  unit/year

a)  $P = 15 \$$   
 $C = 25 \$$

$$EOQ \quad Q_M^* = \sqrt{\frac{2PD}{I+W}} = \sqrt{\frac{2 \times 15 \times 5000}{0+8}} =$$

$$Q^* = 136.93$$

$$Q^* \approx 137$$

b)  $P = 250 \$$  / Production run

$A = 800$  unit/month  $\rightarrow A = 800 \times 12 = 9600$  unit/year

$$EMQ \rightarrow Q_M^* = \sqrt{\frac{2PD}{(I+W)(1+\frac{P}{A})}} = \sqrt{\frac{2 \times 250 \times 9600}{(0+8)(1+\frac{250}{800})}} = 453.29$$

(-1)

Scenario 2 EMQ (Batches)

the company should manufacture 453 units.

$$c) \quad TC_1 = C + \frac{P}{Q} + \frac{(I+W)Q}{2D} = 25 + \frac{15}{\frac{5000}{137}} + \frac{(0+8)137}{2 \times 5000} = 25.1396$$

$$TC_2 = C + \frac{P}{Q} + \frac{(I+W)(1+\frac{P}{A})Q}{2D} = 25 + \frac{250}{\frac{9600}{453.29}} + \frac{(0+8)(1+\frac{250}{800})453.29}{2 \times 5000}$$

$$= 0.55 + 2.62 = 3.17$$

$$25.619$$

(-2)

Should it purchase or produce?

(-2)

**Problem 3.** Find the cost of the following aggregate plan:

Reduce capacity to 2 units/day for the first 5 months and increase it to 6 units/day for the last 7 months. Regular time only is allowed for the first 7 months and regular plus overtime for the last 5 months. Stockout at the end of the year is acceptable. (20 points).

The following data is given:

✓ Initial inventory	100 units
(+) ✓ Cost to carry inventory	\$60/unit/month
✓ Cost to manufacture (regular time)	\$500/unit
✓ Cost to manufacture (overtime)	\$800/unit
(+) ✓ Cost of stock-out	\$120/unit/month
✓ Present capacity	5 units/day
Cost to increase capacity	\$900/unit $\times 2$ $\times 3$ $\times 1$
Cost to decrease capacity	\$700/unit $\times 3$

(20)

Month	J	F	M	A	M	J	J	A	S	O	N	D
Forecast	60	50	80	55	65	70	260	240	120	90	90	50
Prod. days												
Regular	22	18	23	21	22	21	20	20	22	22	21	20
Overtime	3	3	3	3	4	3	3	3	4	4	3	2

Month	Forecast	Production PT days	Production Overtime	Overall Production	Inventory change	Ending Inventory
Jan	60	22		44	-16	84
Feb	50	18		36	-14	70
Mar	80	23		46	-34	36
Apr	55	21		42	-13	23
May	65	22		44	-21	2
Jun	70	21		126	56	58
Jul	260	20		120	-140	-82
Aug	240	20 <sub>12</sub>	3 18	138	-102	-184
Sep	120	22 <sub>132</sub>	4 24	156	36	-148
Oct	90	22 <sub>12</sub>	4 24	156	66	-82
Nov	90	21 <sub>126</sub>	3 18	144	54	-28
Dec	50	20 <sub>124</sub>	2 12	132	82	+54

$$\Sigma \text{ production (Regular)} = 1088$$

$$\Sigma \text{ production (Overtime)} = 96$$

$$\text{Cost of manufacture (Regular)} = 1088 \times \frac{\text{units}}{500} = \underline{544000} \quad \checkmark$$

$$\text{Cost of manufacture (over time)} = 96 \times \frac{800}{\text{units}} = \underline{76800} \quad \checkmark$$

$$\text{Cost to Stock out} = 120 \text{ \$} \times \frac{524}{\text{units}} = \underline{62880} \quad \checkmark$$

$$\text{Cost of carry inventory} = 60 \times \frac{327}{\text{units}} = \underline{19620} \quad \checkmark$$

$$\text{Cost of changing capacity} = \underbrace{3 \times 700}_{\text{decrease}} + \underbrace{4 \times 900}_{\text{increase}} = 2100 + 3600 = \underline{5700} \quad \checkmark$$

$$\text{Total Cost} = \text{Carry Cost} + \text{Stock out Cost} + \text{changing plan Cost} + \text{Purchasing Cost}$$

$$\text{Total Cost} = 544000 + 76800 + 62880 + 19620 + 5700 =$$

$$\boxed{\text{TC} = 709000}$$

✓

**Problem 4.** The demand for a certain manufactured product during the next four months is expected to be 175, 235, 270 and 220 units respectively. The cost of manufacturing with regular, overtime and subcontracting options and the capacities are given below:

	Manufacturing Cost	Capacity/month
Regular Production	75	200
Overtime Production	85	50
Subcontract Operations	100	100

$$D_1 = 175$$

$$D_2 = 235$$

$$D_3 = 270$$

$$D_4 = 220$$

The carrying cost is 10% of manufacturing cost per month/unit. No backorders allowed.

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- (a) Set this up as a transportation problem and find the initial basic feasible solution using North-West Corner rule. Make sure that there are appropriate number of filled cells. (10 points)
- (b) How many filled cells are there? (5 points)
- (c) What is the total cost at this stage? (5 points)

	Month 1	Month 2	Month 3	Month 4	Slack	Capacity
Regular	175	25	82.5	97	100	200
Overtime	85	23.5	103	113	113	50
Sub Contract	100	100	100	100	100	100
Demand	175	235	270	220	550	350

Next pay = 1050

i ↓

	Month 1	Month 2	Month 3	Month 4	Stock	Capacity	
M1 Reg.	75 175	82.5 25	91	110	L	200	0
M1 over.	185	50	110	113	L	50	11
M1 Sub.	100	100	110	110	↑ 0	100	18.5
M2 Reg.		60	140	91	L	200	-7.5
M2 over.		165	50	103	L	50	3.5
M2 Sub.		110	80	20	L	100	10
M3 Reg.			175	200	0	200	-7.5
M3 over.			185	18.5	50	50	-7.5
M3 Sub.			110	110	100	100	-7.5
M4 Reg. <del>Demand</del>				175	200	200	-7.5
M4 over.				185	50	50	-7.5
M4 Sub.				110	100	100	-7.5
Demand	175	235	270	220	500		
→	75	82.5	90	90	7.5		

✓

b)  $a_i + b_j - 1 = 17 - 1 = 16$  ✓

c) 

**Problem 5.** The demand for a particular item for the next 12 weeks is 20 units in weeks 1, 2, 4, 5, 7, and 8; 25 units in weeks 3, 6, 10, 11 and 12; and 30 units in week 9. Initial on-hand inventory is 65 units. Another 70 units are scheduled for receipt in period 2. The lot size is 70 units and the lead-time is 4 weeks. Also, there is a safety stock requirement of 40 units. Develop the gross and net requirements report for the 12-week period (20 points).

Week	1	2	3	4	5	6	7	8	9	10	11	12
demand	20	20	25	20	20	25	20 +70	20	30	25	25	25

11.75

$$OHI = 65$$

$$LS = 70$$

$$LT = 4$$

$$SS = 40$$

Week	1	2	3	4	5	6	7	8	9	10	11	12
demand	20	20	25	20	20	25	20	20	30	25	25	25
Proj. Rec.		70			70		70	70			70	
Sched. Rec.												
OHI	45	95	70	50	30	75	125	105	75	50	95	75
Planned-order release	140			70			70					

-8.25