NORTHEASTERN UNIVERSITY

Department of Mechanical and Industrial Engineering

Supply Chain Engineering **Prof. Gupta**

IE 7200 **Spring 2014 (Mondays)**

Homework No. 2
(Solution)

Problem 1.

$Y(t) = a(10)^{bt}$ is not linear. However, a logarithmic transformation will make it linear. Thus,

$$\log[Y(t)] = \log a + bt$$

This is a linear model with intercept log[*a*] and slope *b*. Hence the equation is of the form:

$$Y' = a' + bt \text{ where } Y' = \log[Y(t)] \text{ and } a' = \log a.$$

a' and *b* can be found from the following normal equations

$$\sum Y' = Na' + b\sum t$$

$$\sum tY' = a'\sum t + b\sum t^2$$

Problem 2.

	t	$Y(t)$	$Y' = \log[Y(t)]$	tY'	t^2
	1	8	0.90	0.90	1
	2	12	1.08	2.16	4
	3	20	1.30	3.90	9
	4	32	1.51	6.04	16
	5	48	1.68	8.40	25
	6	80	1.90	11.40	36
Total	21	200	8.37	32.80	91

Substituting the appropriate values in the normal equations and solving, we get

$a' = 0.695$ and $b = 0.20$

Hence, $a = 4.95$.

Therefore the forecasting equation is

$$Y(t) = 4.95(10)^{0.2t}$$

Forecast for period 8

$$Y(8) = 4.95(10)^{0.2*8}$$

$$= 197.06$$

Problem 3.

Year	**Quarter**				**Total**
	I	**II**	**III**	**IV**	
1	43	27	10	22	102
2	49	35	14	27	125
3	58	47	14	32	151
4	71	53	18	35	177
5	80	63	22	41	206
Total	301	225	78	157	761
S.I.	1.59	1.17	0.41	0.83	

Assuming linear growth and deseasonalizing the data, we get the following:

Year/Qtr	t	Y	Deseasonalized Y, Y_d	$Y_d t$	t^2
1/I	1	43	27.04	27.04	1
1/II	2	27	23.08	46.15	4
1/III	3	10	24.39	73.17	9
1/IV	4	22	26.51	106.02	16
2/I	5	49	30.82	154.09	25
2/II	6	35	29.91	179.49	36
2/III	7	14	34.15	239.02	49
2/IV	8	27	32.53	260.24	64
3/I	9	58	36.48	328.30	81
3/II	10	47	40.17	401.71	100
3/III	11	14	34.15	375.61	121
3/IV	12	32	38.55	462.65	144
4/I	13	71	44.65	580.50	169
4/II	14	53	45.30	634.19	196
4/III	15	18	43.90	658.54	225
4/IV	16	35	42.17	674.70	256
5/I	17	80	50.31	855.35	289
5/II	18	63	53.85	969.23	324
5/III	19	22	53.63	1019.51	361
5/IV	20	41	49.40	987.95	400
Total	210		761.02	9033.47	2870

Note: Deseasonalized values are calculated by dividing Y values by seasonal indices.

Substituting the above values in the normal equations and solving for a and b, we get
$a = 21.58$, $b = 1.57$.
Hence,

$$Y = 21.58 + 1.51t \quad \text{(origin at quarter 0)}$$

The forecast for year 6, quarter I
= (21.58 + 1.57 * 21) * 1.59 = 86.73

Similarly, the forecast for
Year 6, quarter II = (21.58 + 1.57 * 22) * 1.17 = 65.66
Year 6, quarter III = (21.58 + 1.57 * 23) * 0.41 = 23.65
Year 6, quarter IV = (21.58 + 1.57 * 24) * 0.83 = 49.19

Problem 4.
Given, $d_1 = 0.05$, $d_2 = 0.03$ and $d_3 = 0.07$, the following four configurations are possible:

b_{AB}*	b_{BC}	System Output	Inventory	Income**	Profit
0	0	$O_3 = 1 - d_1 - d_2 - d_3 + d_1d_2 + d_1d_3 + d_2d_3 - d_1d_2d_3$	0	4285	4285
0	1	$O_3 = 1 - d_3$	150	4650	4500***
1	0	$O_3 = 1 - d_2 - d_3 + d_2d_3$	100	4510	4410
1	1	$O_3 = 1 - d_3$	250	4650	4400

* 0 represents that there is no buffer whereas 1 signifies infinite buffer.
** Income is calculated by multiplying O_3 by 1000 and then by 5.
*** This is the optimal scenario.

Problem 5.
Given, P = \$85/order, i = 25%/year, I = 0.25 * 200 = \$50/unit-year, C = \$200/unit, D = 3000 units/year and W = \$2/unit-month = \$24/unit-year,

$$EOQ_S = \sqrt{\frac{2DP}{I+2W}}$$

Substituting and solving, we get EOQ_S = 72 units.

Problem 6.

Here, $TC = C + P/Q + H_S * Q$ and just the inventory cost per unit = $P/Q + H_S * Q$

Therefore, the annual inventory cost

$$= \left(P/Q + H_S Q\right)D$$

$$= \left(\frac{P}{Q} + \frac{I+2W}{2D}Q\right)D$$

By substituting, we get the annual inventory cost = $7068.95

Problem 7.

Ordering Cycle = Q/D = 72/3000 = 0.024 year.

Problem 8.

$$EOQ_M = \sqrt{\frac{2DP}{I+W}} = 83$$

Therefore, the annual inventory cost

$$= \left(P/Q + H_M Q\right)D$$

$$= \left(\frac{P}{Q} + \frac{I+W}{2D}Q\right)D$$

By substituting, we get the annual inventory cost = $6143.29

Ordering Cycle = Q/D = 83/3000 = 0.028 year.

Problem 9.

Rearrange the list in increasing order of annual usage and divide it into three parts to give the *ABC* classification as follows:

	Item No.	Annual Usage	Cumulative Usage	
C items	12	600	600	7.7%
	6	750	1350	
	9	800	2150	
	15	1200	3350	
	1	1500	4850	
	7	2000	6850	
	3	2200	9050	
	20	2900	11950	
	19	3500	15450	
	17	4000	19450	
B items	5	9600	29050	31.9%
	14	9900	38950	
	16	10200	49150	
	8	11000	60150	
	2	12000	72150	
	11	13000	85150	
	10	15000	100150	
A items	13	42000	142150	60.4%
	4	50000	192150	
	18	61000	253150	