Inventory Models

Outline

- Inventory Management Overview
- Purpose of Inventory
- Decisions in Inventory
- Costs Associated with Inventory
- Types of Inventory
- Production Line and Buffer Inventory
- ABC Analysis
- Economic Order Quantity
- Economic Manufacturing Quantity

Inventory Management Overview

- For many manufacturing companies, inventory is one of the major assets
- In some cases it could be as high as 80% of all the assets of a company. In such cases, if something goes wrong with their inventory, they are doomed
- Then there are other companies which carry virtually no inventory
- Hence, inventory can represent very little to very large portion of assets of a company
- We are interested in inventory, because it is said that inventory can make or break a company if proper planning is not done

Inventory Management Overview

- Inventory is “deadwood” until it is used
- There are many reasons for carrying inventory
  - Companies do not want to disappoint their customers. That is, whenever the customer demands something, they want to be able to supply it. If there is insufficient inventory, they might loose a customer. Every time they loose a customer, they loose a chance to make a profit (opportunity loss)
  - In case of raw material, if we run out of some item it could hamper production. Even if we were assembling large products, running out of even a small item like a nut or a bolt could shut down an assembly line. In fact, there is more to inventory than just that. We are generally extra careful about not running out of inexpensive items. Thus we always carry ample supply of cheap items and monitor closely expensive items

Inventory Management Overview

- What if we carry too much inventory?
  - » The item in inventory could become obsolete
  - » Carrying inventory means that the money invested in it is blocked which means the company is loosing the opportunity to use that money somewhere else
  - » Then there are carrying costs, insurance costs, book-keeping costs, warehousing costs, etc.

- Thus, too much inventory is bad and too little inventory is bad.
- We should, therefore, strive to carry just the right amount of inventory!
- And what is the right amount?
  - It is the amount that will minimize the total inventory costs

Purpose of Inventory

- Inventory is an aid to continuous production. That means, because of inventory we are able to continue production
- If we were not allowed to carry inventory, there might be times when we might have to stop production due to some interruption in delivery of items

Purpose of Inventory

- Inventory has a sort of modular effect, i.e. it kind of separates different production functions if inventory is allowed to accumulate between production stages. On the other hand, if no inventory is allowed between production stages, like in an assembly line, interruption at any stage due to some breakdown or other reason will cause the entire operation to shut down. Having inventory between different stages allows activity to continue at all stages except the stage which is interrupted

Purpose of Inventory

- Inventory helps us to maintain level production. Since demand is generally not constant, inventory helps us in absorbing fluctuations in the demand curve. This leads to stable employment, job security, better labor-management relationships, etc.
- Inventory provides a hedge against future price increases. Sometimes, inventory helps reduce costs by storing items which are anticipated to increase in price due to inflation or other reasons

Purpose of Inventory

- Inventory is also carried to avoid the danger of strikes and other possible interruptions
- Sometimes one is forced to carry inventory because one cannot buy small quantities of certain items because of packaging or other reasons
- Quantity discounts might be another reason for carrying inventory
- Of course, inventory is also carried to satisfy customers with unanticipated demands

Decisions in Inventory

- The two basic decisions we need to make in any inventory planning are:
  - **How much to order?** - function of costs
    - » This depends on many factors, e.g. how much space do we have? Or how much money do we have? Or how much is the carrying cost? Or how much is the ordering cost?
  - **When to order?** – function of forecast or scheduled requirements
    - » That is, what is the order cycle?
- If these two questions are answered, then a lot of our inventory problems are solved
- The answers should be economically justified

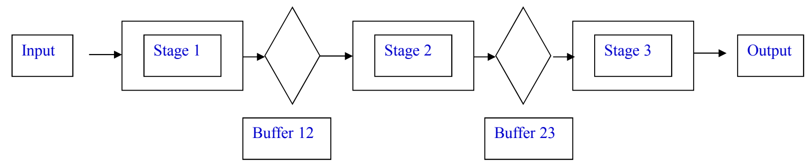
Costs Associated with Inventory

- There are three basic types of cost associated with inventory:
  - **Ordering and setup costs**
    - » This is the cost associated with placing orders or with setting up the machine (getting ready) to build inventory
  - **Carrying costs**
    - » Costs of carrying items in inventory which might include cost of the space used, interest charges on the money blocked in inventory etc.
  - **Purchase cost**
    - » Cost of the item itself

Types of Inventory

- There are several types of inventory. For example:
  - **Raw stock inventory**
  - **Work-in-process inventory (WIP)**
  - **Final product inventory**

Production Line and Buffer Inventory



di = downtime percentage for the ith stage
bij = buffer stock between ith and jth stages
Oi = output (in percentage of time) from ith stage

Production Line and Buffer Inventory

If $bij = 0$

$O1 = 1 - d1$
$O2 = 1 - d1 - d2 + d1d2$
$O3 = 1 - d1 - d2 - d3 + d1d2 + d1d3 + d2d3 - d1d2d3$

If $d1 = 5\%$, $d2 = 10\%$, $d3 = 2\%$,

$O1 = 1 - 0.5 = 0.95 \Rightarrow 95\%$
$O2 = 85.5\%$
$O3 = 83.8\%$

Production Line and Buffer Inventory

If $bij = \infty$

$O1 = 1 - d1$
$O2 = 1 - d2$
$O3 = 1 - d3$

If $d1 = 5\%$, $d2 = 10\%$, $d3 = 2\%$,

$O1 = 1 - 0.05 = 0.95 \Rightarrow 95\%$
$O2 = 90\%$
$\mathbf{O3 = 98\%}$

This is a lot of improvement over the "no buffer" case.
In reality, however, there is a limit on buffer inventory.

ABC Analysis

- Pareto's Principle
  - In the late 1800s, economist and avid gardener Vilfredo Pareto established that 80% of the land in Italy was owned by 20% of the population. The Pareto Principle or the 80:20 rule has proven its validity in a number of other areas.

- ABC analysis is also built on a similar idea. However, here we divide the inventory population into three categories, viz., **A**, **B** and **C**

ABC Classification

- Category **A items** include roughly **10-20%** of items that typically account for **70-80%** of total dollar value
- Category **B items** include roughly **30-40%** of items that typically account for **15-20%** of total dollar value
- Category **C items** include roughly **40-50%** of items that typically account for **5-10%** of total dollar value

Implications of ABC Classification

	Degree of Control	**Type of Records**	**Order of Priority**	**Ordering Procedure**
A items	Tight	Accurate and complete	High to reduce inventory	Careful, accurate and frequent review of EOQ and OP
B items	Normal	Good	Normal	Normal EOQ analysis
C items	Simple	Simple	Lowest	Long term supply reordered periodically

ABC Classification Example

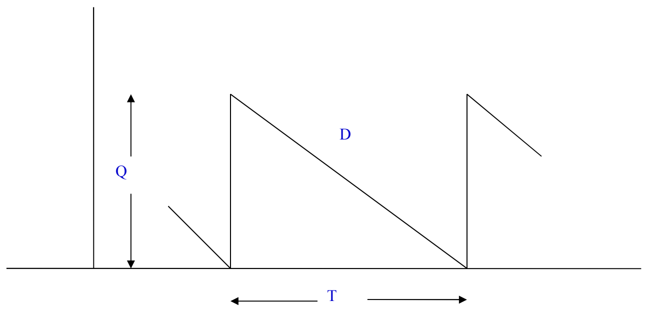
- In an attempt to establish an ABC classification, a firm wants to analyze its inventory of 13 items.
- Following the usual guidelines for ABC breakdown, indicate the items which will be classified in each category and find the percentage of value for each classification.

Item No.	Annual Investment
1	150
2	1050
3	950
4	100
5	1300
6	3000
7	250
8	22000
9	300
10	40
11	60
12	1100
13	500
Total	**30800**

ABC Classification

Number	Item No.	Annual Investment	Cumulative Investment	Value
C items 53.8%	10	40	40	4.6%
	11	60	100	
	4	100	200	
	1	150	350	
	7	250	600	
	9	300	900	
	13	500	1400	
B items 38.5%	3	950	2350	24%
	2	1050	3400	
	12	1100	4500	
	5	1300	5800	
	6	3000	8800	
A items 7.7%	8	22000	30800	71.4%

Economic Order Quantity (EOQ) Model



EOQ Model

- Nomenclature
  - Q = Order quantity
  - T = Time between orders (cycle time)
  - D = Demand rate or depletion rate
  - P = Ordering cost
  - I = Interest charges/unit of inventory/unit time
  - W = warehousing cost/unit of inventory/unit time
  - H = Overall inventory cost factor
  - C = Cost/item (capital investment cost)
  - TC = Total cost/unit

EOQ Model

- Multiple products case

$$TC_M = C + \frac{P}{Q} + \frac{(I+W)(T)(\text{Average Inventory})}{Q}$$

- Single product case

$$TC_S = C + \frac{P}{Q} + \frac{(I)(T)(\text{Average Inventory})}{Q} + \frac{(W)(T)(\text{Maximum Inventory})}{Q}$$

EOQ Model

- Using the following relationships (see figure)

$$T = \frac{Q}{D}$$

$$\text{Average Inventory} = \frac{Q}{2}$$

$$\text{Maximum Inventory} = Q$$

EOQ Model

- Multiple products case

$$TC_M = C + \frac{P}{Q} + \frac{(I+W)(Q)}{2D}$$

- Single product case

$$TC_S = C + \frac{P}{Q} + \frac{(I+2W)(Q)}{2D}$$

EOQ Model

- In order to minimize the total cost, we differentiate these total cost equations and put them equal to zeros. After simplification, we get the following:
- Multiple products case

$$Q_M = \sqrt{\frac{2DP}{I+W}}$$

- Single product case

$$Q_S = \sqrt{\frac{2DP}{I+2W}}$$

- Note that, for minimum, the second derivative must be positive.

EOQ Model

- Therefore

$$Q = \sqrt{\frac{P}{H}}$$

$$H_M = \frac{I+W}{2D}$$

$$H_S = \frac{I+2W}{2D}$$

$$TC = C + \frac{P}{Q} + (H)(Q)$$

EOQ Model Example

- Given the following

  D = 800 units/year
  P = $70 per order
  i = 20% per year
  C = $120 per unit
  I = $120 * 0.2 = $24 per unit-year
  W = $1.80 per unit-month
  = $1.80 * 12 = $21.60 per unit-year
  Assume the case of one product.

EOQ Model Example

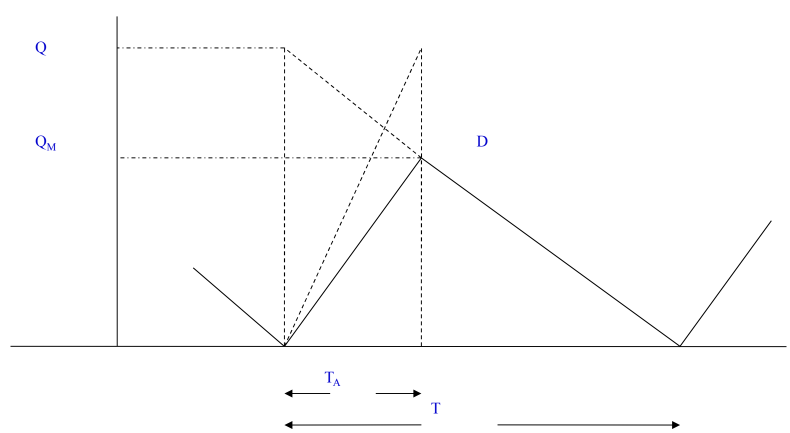
$$H_S = \frac{I + 2W}{2D} = \frac{24 + 2 \times 21.60}{2 \times 800} = 0.042$$

$$Q = \sqrt{\frac{P}{H}} = \sqrt{\frac{70}{0.042}} = 41 \text{ units}$$

$$\begin{aligned} TC &= C + \frac{P}{Q} + (H)(Q) \\ &= 120 + \frac{70}{41} + 0.042 \times 41 \\ &= \$123.42 \end{aligned}$$

$$T = \frac{Q}{D} = \frac{41}{800} = 0.05 \text{ years}$$

Economic Manufactured Quantity (EMQ) Model



EMQ Model

- Nomenclature

  A = Arrival rate

  T_A = Arrival time (manufacturing time)

- Assumptions

  $A \geq D$

  $T_A = 0; \quad A = \infty \quad \Rightarrow \quad$ EOQ Model

  ss = startup level (if item produced is not immediately available)

EMQ Model

- Note that the following relationships hold (see figure):

  $$D = \frac{Q}{T}$$

  $$A = \frac{Q}{T_A}$$

  Therefore $\quad Q = DT = AT_A$

  $$ss = DT_A = \frac{DQ}{A}$$

  Hence $\quad D = \frac{Q}{T} = \frac{ss}{T_A}$

  and $\quad Q\left(\frac{T_A}{T}\right) = Q\left(\frac{D}{A}\right)$

EMQ Model

- An item produced is **immediately available** (no safety stock):
  - Multiple products case

$$TC_M = C + \frac{P}{Q} + \frac{(I+W)(\text{Average Inventory})}{D}$$

  - Single product case

$$TC_S = C + \frac{P}{Q} + \frac{(I)(\text{Average Inventory})}{D} + \frac{(W)(\text{Maximum Inventory})}{D}$$

EMQ Model

- Note that the following relationships hold (see figure):

$$D = \frac{Q_M}{T - T_A}$$

$$D = \frac{Q}{T}$$

Therefore $Q_M = \left[\frac{T - T_A}{T}\right] Q = \left[1 - \frac{T_A}{T}\right] Q$

$$\text{Average Inventory} = \frac{Q_M}{2}$$

$$= \left[1 - \frac{T_A}{T}\right]\frac{Q}{2} \qquad \because Q_M = \left[1 - \frac{T_A}{T}\right] Q$$

$$= \left[1 - \frac{D}{A}\right]\frac{Q}{2} \qquad \because \frac{T_A}{T} = \frac{D}{A}$$

Maximum Inventory $Q_M = \left[1 - \frac{D}{A}\right] Q$

EMQ Model

- Substituting the above relationships in the total cost equations, we get
  - Multiple products case

$$TC_M = C + \frac{P}{Q} + \frac{(I+W)\left[1-\frac{D}{A}\right]Q}{2D}$$

  - Single product case

$$TC_S = C + \frac{P}{Q} + \frac{(I+2W)\left[1-\frac{D}{A}\right]Q}{2D}$$

EMQ Model

- Taking the derivative of the total cost equations and putting them equal to zero and solving for Q, we get
  - Multiple products case

$$Q_M = \sqrt{\frac{2PD}{(\mathrm{I}+\mathrm{W})\left(1-\frac{\mathrm{D}}{\mathrm{A}}\right)}}$$

  - Single product case

$$Q_S = \sqrt{\frac{2PD}{(\mathrm{I}+2\mathrm{W})\left(1-\frac{\mathrm{D}}{\mathrm{A}}\right)}}$$

EMQ Model

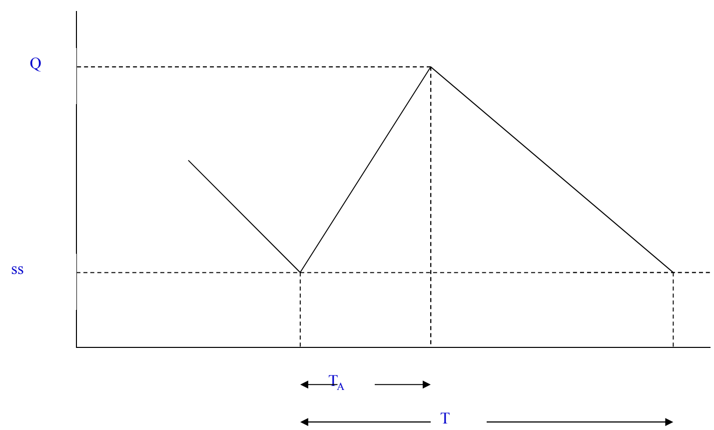
- An item produced is **not immediately available** (carry safety stock):
  - Multiple products case

$$TC_M = C + \frac{P}{Q} + \frac{(I+W)(\text{Average Inventory})}{D}$$

  - Single product case

$$TC_S = C + \frac{P}{Q} + \frac{(I)(\text{Average Inventory})}{D} + \frac{(W)(\text{Maximum Inventory})}{D}$$

EMQ Model



EMQ Model

- Note that the following relationships hold (see figure):

$$\text{Average Inventory} = \frac{Q - ss}{2} + ss$$
$$= \frac{Q + ss}{2}$$

$$\text{Maximum Inventory} = Q$$

$$ss = Q\left(\frac{D}{A}\right)$$

EMQ Model

- Substituting the above relationships in the total cost equations, we get

  – Multiple products case

$$TC_M = C + \frac{P}{Q} + \frac{(I + W)\left[1 + \frac{D}{A}\right]Q}{2D}$$

  – Single product case

$$TC_S = C + \frac{P}{Q} + \frac{I\left[1 + \frac{D}{A}\right]Q}{2D} + \frac{WQ}{D}$$

EMQ Model

- Taking the derivative of the total cost equations and putting them equal to zero and solving for Q, we get
  - Multiple products case

$$Q_M = \sqrt{\frac{2PD}{(\mathrm{I}+\mathrm{W})\left(1+\frac{\mathrm{D}}{\mathrm{A}}\right)}}$$

  - Single product case

$$Q_S = \sqrt{\frac{2PD}{I\left(1+\frac{\mathrm{D}}{\mathrm{A}}\right)+2\mathrm{W}}}$$

Summary of Learning Objectives

- How are the appropriate costs balanced to choose the optimal amount of cycle inventory in the supply chain?
- What are the effects of quantity discounts on lot size and cycle inventory?
- What are appropriate discounting schemes for the supply chain, taking into account cycle inventory?
- What are the effects of trade promotions on lot size and cycle inventory?
- What are managerial levers that can reduce lot size and cycle inventory without increasing costs?