NORTHEASTERN UNIVERSITY
Department of Mechanical and Industrial Engineering

Supply Chain Engineering **Prof. Gupta**
IE 7200 **Spring 2014 (Mondays)**

Homework No. 5
(Solution)

Problem 1.

Annual cost of holding inventory is 25%
Therefore, holding cost, H = \$120 × 0.25 = \$30 per motor/year

Transit time
By truck = 3 days
By rail = 5 days

AM Rail Proposal

Minimum shipment is 20,000lbs or 2,000 motors
Replenishment lead time, L = 5+1 = 6 days
For Q = 2,000 motors,

Cycle inventory	=	$Q/2$ = 1,000 motors	
Safety inventory	=	$L/2$ days of demand	= (6/2)(120,000/365)
	=	986 motors	
In-transit inventory	=	120,000(5/365)	= 1,644 motors
Total average inventory	=	1,000 + 986 + 1,644	= 3,630 motors
Annual holding cost	=	3,630 × \$30	= \$108,900
Annual transportation cost	=	120,000 × 0.65	= \$78,000
Total annual cost	=	\$108,900 + \$78,000	= \$186,900

Northeast Trucking Proposal

Minimum shipment is 10,000lbs or 1,000 motors
Replenishment lead time, L = 3+1 = 4 days
For Q = 1,000 motors,

Cycle inventory	=	$Q/2$ = 500 motors	
Safety inventory	=	$L/2$ days of demand	= (4/2)(120,000/365)
	=	658 motors	
In-transit inventory	=	120,000(3/365)	= 986 motors
Total average inventory	=	500 + 658 + 986	= 2,144 motors
Annual holding cost	=	2,144 × \$30	= \$64,320
Annual transportation cost	=	120,000 × 0.75	= \$90,000
Total annual cost	=	\$64,320 + \$90,000	= \$154,320

Golden Freightways Proposal (50 cwt)

Minimum shipment is 5,000lbs or 500 motors
Replenishment lead time, L = 3+1 = 4 days
For Q = 500 motors,

Cycle inventory	=	*Q/2* = 250 motors	
Safety inventory	=	*L/2* days of demand	= (4/2)(120,000/365)
	=	658 motors	
In-transit inventory	=	120,000(3/365)	= 986 motors
Total average inventory	=	250 + 658 + 986	= 1,894 motors
Annual holding cost	=	1,894 × $30	= $56,820
Annual transportation cost	=	120,000 × 0.8	= $96,000
Total annual cost	=	$56,820 + $96,000	= $152,820

Golden Freightways Proposal (150 cwt)

Minimum shipment is 15,000lbs or 1,500 motors
Replenishment lead time, *L* = 3+1 = 4 days
For *Q* = 1,500 motors,

Cycle inventory	=	*Q/2* = 750 motors	
Safety inventory	=	*L/2* days of demand	= (4/2)(120,000/365)
	=	658 motors	
In-transit inventory	=	120,000(3/365)	= 986 motors
Total average inventory	=	750 + 658 + 986	= 2,394 motors
Annual holding cost	=	2,394 × $30	= $71,820
Annual transportation cost	=	120,000 × 0.8	= $96,000
Total annual cost	=	$71,820 + $96,000	= $167,820

Golden Freightways Proposal (250 cwt)

Minimum shipment is 25,000lbs or 2,500 motors
Replenishment lead time, *L* = 3+1 = 4 days
For *Q* = 2,500 motors,

Cycle inventory	=	*Q/2* = 1250 motors	
Safety inventory	=	*L/2* days of demand	= (4/2)(120,000/365)
	=	658 motors	
In-transit inventory	=	120,000(3/365)	= 986 motors
Total average inventory	=	1250 + 658 + 986	= 2,894 motors
Annual holding cost	=	2,894 × $30	= $86,820
Annual transportation cost	=	120,000 × **0.72***	= $86,400
Total annual cost	=	$86,820 + $86,400	= $173,220

***Note that 0.72 is derived from (150x0.80 + 100x0.60)/250**

Golden Freightways Proposal (300 cwt)

Minimum shipment is 30,000lbs or 3,000 motors
Replenishment lead time, *L* = 3+1 = 4 days
For *Q* = 3,000 motors,

Cycle inventory	=	*Q/2* = 1500 motors	
Safety inventory	=	*L/2* days of demand	= (4/2)(120,000/365)
	=	658 motors	
In-transit inventory	=	120,000(3/365)	= 986 motors
Total average inventory	=	1500 + 658 + 986	= 3,144 motors

Annual holding cost	=	3,144 × $30	= $94,320
Annual transportation cost	=	120,000 × **2/3***	= $80,000
Total annual cost	=	$94,320 + $80,000	= $174,320

***Note that 2/3 is derived from (150x0.80 + 100x0.60 + 50x0.40)/300**

Golden Freightways Proposal (Old Proposal)

Minimum shipment is 40,000lbs or 4,000 motors
Replenishment lead time, L = 3+1 = 4 days
For Q = 4,000 motors,

Cycle inventory	=	$Q/2$ = 2000 motors	
Safety inventory	=	$L/2$ days of demand	= (4/2)(120,000/365)
	=	658 motors	
In-transit inventory	=	120,000(3/365)	= 986 motors
Total average inventory	=	2000 + 658 + 986	= 3,644 motors
Annual holding cost	=	3,644 × $30	= $109,320
Annual transportation cost	=	120,000 × **0.60***	= $72,000
Total annual cost	=	$109,320 + $72,000	= $181,320

***Note that 0.60 is derived from (150x0.80 + 100x0.60 + 150x0.40)/400**

Golden Freightways Proposal (New Proposal)

Minimum shipment is 40,000lbs or 4,000 motors
Replenishment lead time, L = 3+1 = 4 days
For Q = 4,000 motors,

Cycle inventory	=	$Q/2$ = 2000 motors	
Safety inventory	=	$L/2$ days of demand	= (4/2)(120,000/365)
	=	658 motors	
In-transit inventory	=	120,000(3/365)	= 986 motors
Total average inventory	=	2000 + 658 + 986	= 3,644 motors
Annual holding cost	=	3,644 × $30	= $109,320
Annual transportation cost	=	120,000 × **0.5625***	= $67,500
Total annual cost	=	$109,320 + $67,500	= $176,820

***Note that 0.60 is derived from (150x0.80 + 100x0.60 + 150x0.30)/400**

From the results, the plant manager should sign a contract with Golden Freightways and order motors in lots of 500.

Problem 2.

Current Scenario

Replenishment lead time, L = 1 week
Reorder interval, T = 4 weeks
CSL = 0.997
$F^{-1}(0.997) = z$ = 2.75

1. HighMed Inventory Cost

For Highval
Average lot size, Q_H = expected demand during T weeks = $T \times \mu_H$
= 4 × 2 = 8 units
Safety inventory, SS_H = $F^{-1}(CSL) \times \sigma_{T+L} = F^{-1}(CSL) \times \sqrt{T+L} \times \sigma_H$
= $F^{-1}(0.997) \times \sqrt{4+1} \times 5$ = 30.7 units
Total Highval inventory = $Q_H/2 + SS_H$ = (8/2) + 30.7 = 34.7 units
Total across all 24 territories = 24 × 34.7 = 832.8 units

Average lot size, Q_L = expected demand during T weeks = $T \times \mu_L$
= 4 × 20 = 80 units
Safety inventory, SS_L = $F^{-1}(0.997) \times \sqrt{4+1} \times 5$ = 30.7 units
Total Lowval inventory = (80/2) + 30.7 = 70.7 units
Total across all 24 territories = 24 × 70.7 = 1696.8 units

Annual inventory holding cost for HighMed = (average HighVal inventory × \$200 + average LowVal inventory × \$30) × 0.25
= (832.8 × \$200 + 1696.8 × \$30) × 0.25
= \$54,366

2. HighMed Transportation Cost

Average weight of each replenishment order = $0.1Q_H + 0.04Q_L$ = 0.1 × 8 + 0.04 × 80
= 4 pounds
Shipping cost per replenishment order = \$0.66 + 0.26 × 4
= \$1.70
Each territory has 13 replenishment orders per year and there are 24 territories. Therefore,
Annual transportation cost = \$1.70 × 13 × 24
= \$530.40

3. HighMed Total Cost

HighMed Annual Total Cost = inventory cost + transportation cost
= \$54,366 + \$530.40
= \$54,896.40

Option A

Replenishment lead time, L = 1 week
Reorder interval, T = 1 week
CSL = 0.997

1. HighMed Inventory Cost

For Highval

Average lot size, Q_H = expected demand during T weeks
= $T \times \mu_H$
= 1 × 2 = 2 units

Safety inventory, SS_H = $F^{-1}(CSL) \times \sigma_{T+L} = F^{-1}(CSL) \times \sqrt{T+L} \times \sigma_H$

= $F^{-1}(0.997) \times \sqrt{1+1} \times 5$ = 19.4 units

Total Highval inventory = $Q_H/2 + SS_H$ = (2/2) + 19.4 = 20.4 units

Total across all 24 territories = 24 × 20.4 = 490 units

For Lowval

Average lot size, Q_L = expected demand during T weeks = $T \times \mu_L$
= 1 × 20 = 20 units

Safety inventory, SS_L = $F^{-1}(0.997) \times \sqrt{1+1} \times 5$ = 19.4 units

Total Lowval inventory = (20/2) + 19.4 = 29.4 units

Total across all 24 territories = 24 × 29.4 = 706 units

Annual inventory holding cost for HighMed = (average HighVal inventory × \$200 + average LowVal inventory × \$30) × 0.25
= (490 × \$200 + 706 × \$30) × 0.25
= \$29,795

2. HighMed Transportation Cost

Average weight of each replenishment order = $0.1Q_H + 0.04Q_L$ = 0.1 × 2 + 0.04 × 20
= 1 pound

Shipping cost per replenishment order = \$0.66 + 0.26 × 1
= \$0.92

Each territory has 52 replenishment orders per year and there are 24 territories. Therefore,

Annual transportation cost = \$0.92 × 52 × 24
= \$1,148.16

3. HighMed Total Cost

HighMed Annual Total Cost = inventory cost + transportation cost
= \$29,795 + \$1,148.16
= \$30,943.16

Option B

Replenishment lead time, L = 1 week
Reorder interval, T = 1 week
CSL = 0.997

1. HighMed Inventory Cost

For Highval

Average lot size, Q_H = expected demand during T weeks
= $T \times \mu_H$
= 1 × 48 = 48 units

Safety inventory, SS_H = $F^{-1}(0.997) \times \sqrt{1+1} \times \sqrt{24 \times 5^2}$ = 95.2 units

Total Highval inventory = $Q_H/2 + SS_H$ = (48/2) + 95.2 = 119.2 units

For Lowval

Average lot size, Q_L = expected demand during T weeks = $T \times \mu_L$
= 1 × 480 = 480 units

Safety inventory, SS_L = $F^{-1}(0.997) \times \sqrt{1+1} \times \sqrt{24 \times 5^2}$ = 95.2 units

Total Lowval inventory = (480/2) + 95.2 = 335.2 units

Annual inventory holding cost for HighMed = (average HighVal inventory × \$200 + average LowVal inventory × \$30) × 0.25
= (119.2 × \$200 + 355.2 × \$30) × 0.25
= \$8,474

2. HighMed Transportation Cost

Average weight of each shipment = 0.1 x 1 + 0.04 x 10
= 0.5 pound

Shipping cost per order = \$5.53 + 0.53 × 0.5
= \$5.80

Aggregate territory has 2 x 24 x 52 shipments per year. Therefore,

Annual transportation cost = \$5.80 × 2 × 24 × 52
= \$14,464.32

3. HighMed Total Cost

HighMed Annual Total Cost = inventory cost + transportation cost
= \$8,474 + \$14,464.32
= \$22,938

From the result, HighMed should apply Option B by aggregating all inventories and using FedEX transportation.

Comparison

	Current Scenario	***Option A***	***Option B***
No. of stocking locations	24	24	1
Reorder interval	4 weeks	1 week	1 week
HighVal cycle inventory	96 units	24 units	24 units
HighVal safety inventory	736.8 units	466 units	95.2 units
HighVal inventory	832.8 units	490 units	119.2 units
LowVal cycle inventory	960 units	240 units	240 units
LowVal safety inventory	736.8 units	466 units	95.2 units
LowVal inventory	1696.8 units	706 units	335.2 units
Annual inventory cost	\$54,366	\$29,795	\$8,474
Shipment type	Replenishment	Replenishment	Customer order
Shipment size	8 HighVal+80 LowVal	2 HighVal+20 LowVal	1 HighVal+10 LowVal
Shipment weight	4 lbs.	1 lbs.	0.5 lbs.
Annual transport cost	\$530	\$1,148	\$14,464
Total annual cost	**\$54,896**	**\$30,943**	**\$22,938**