Demand Forecasting in a Supply Chain

Forecasting

- What is a forecast?
- Why forecast?
- What is the cost of forecasting?

Types of Forecasting

There are many types of forecasts. For example:

- **Demand forecasts**
  - Important input to operations manager for decision making and scheduling production activities
- **Environmental forecasts**
  - Concerned with the social, political and economic state of the environment, e.g., pollution control requirements
- **Economic forecasts**
  - Valuable because they highlight current and expected economic fluctuations which may effect production
- **Technological forecasts**
  - Concerned with new developments in existing technologies as well as the development of new techniques

Role of Forecasting in a Supply Chain

- Demand forecasting forms the basis for all planning decisions in a supply chain
- Used for both push and pull processes
  - For push processes - must plan level of activity of production, transportation and other planned activity
  - For pull processes – must plan level of capacity and inventory but not the actual amount to be executed
- In both cases, the first step is demand forecasting
- All stages must collaborate to get better forecast

Role of Forecasting in a Supply Chain

- Once demand forecasting is done, it triggers rest of the processes, for example:
  - Production: scheduling, inventory, aggregate planning
  - Marketing: sales force allocation, promotions, new production introduction
  - Finance: plant/equipment investment, budgetary planning
  - Personnel: workforce planning, hiring, layoffs
- All of these decisions are interrelated

Timing of Forecasts

- Short range forecasting – up to one year
- Medium range forecasting – one to three years
- Long range forecasting – more than three years

Characteristics of Forecasts

- Forecasts are always wrong! Should include expected value and measure of error.
- Long-term forecasts are less accurate than short-term forecasts (forecast horizon is important)
- Aggregate forecasts are more accurate than disaggregate forecasts

Classification of Forecasting Methods

- **Statistical** – its basis is that the future looks the same as the past
  - Time Series Models
    - Trend projection (good for long range forecasts)
    - Classical decomposition (good for intermediate range forecasting, particularly when there are seasonal effects present)
    - Smoothing (for short range forecasting; can also be combined with seasonal effects to do intermediate range forecasting)
  - Causal or regression models (good for long range forecasts)

Classification of Forecasting Methods

- **Judgmental** – its basis is that no historical data is available (e.g., forecasting for new technology or launching a new product)
  - Expert Opinion (gathering judgment and opinion of key personnel; based on this knowledge and experience of market situation; low cost)
  - Market Survey (use of telephone, interviews, internet; high cost)
  - Delphi (group technique using a panel of experts; useful for long range forecast)

Time Series

- Time series is a set of observations of some variable over time

Components of a Time Series

- **Trend (T)** – Long term secular movement
- **Seasonal (S)** – Similar patterns occurring during corresponding months of successive years (or days of a week or hours of a day)
- **Cyclical (C)** – Long term swings about the trend line associated with business cycles
- **Random (R)** – Sporadic effects due to chance and unusual occurrence (noise).

Y = TSCR

Forecasting Procedure

1. Obtain historical data and plot it to confirm the type of relationship (e.g. linear, quadratic, etc.)
2. Develop a trend equation to describe the data
3. Develop seasonal index (if desired)
4. Project trend into the future (T)
5. Multiply monthly trend values by seasonal index (S)
6. Modify projected values by knowledge of:
   a) Cyclical business conditions (C)
   b) Anticipated irregular effects (R)

Methods of Estimating Trend

Freehand

- Plot the data and draw a smooth curve through the data points
- A simple extension can give a rough forecast
- However, this type of forecast is subjective

Methods of Estimating Trend

Moving Average

- A moving average is obtained by summing and averaging the values from a given number of periods repetitively, each time deleting the oldest value and adding a new value
- $$N \text{ period moving average} = \frac{\text{Summation of } N \text{ data points}}{\text{Number of periods, } N}$$
- If N is large, it will not react heavily to swift changes in the data whereas small values of N, tend to be more sensitive to quick changes in data behavior. Thus, the smaller the N, the faster it reacts and conversely, the larger the N, the smaller the effect of sudden changes in the data transmitted to the moving average

Methods of Estimating Trend

Moving Average

- Used when demand has no observable trend or seasonality
- Systematic component of demand = level
- The level in period t is the average demand over the last N periods (the N-period moving average)
- Current forecast for all future periods is the same and is based on the current estimate of the level

$L_t = (D_t + D_{t-1} + \ldots + D_{t-N+1}) / N$

$F_{t+1} = L_t$ and $F_{t+n} = L_t$

After observing the demand for period t+1, revise the estimates as follows:

$L_{t+1} = (D_{t+1} + D_t + \ldots + D_{t-N+2}) / N$

$F_{t+2} = L_{t+1}$ and $F_{t+n} = L_{t+1}$ where $n > 1$

Methods of Estimating Trend

Moving Average Example

- A manager wishes to determine which of the following moving average method would be better. She is trying to decide between 3, 5, or 7 week moving average. She decides to compare the precision of the 3, 5, and 7 week average period forecasts for the most recent weeks for which data is available. The available data is given.

Methods of Estimating Trend

Moving Average Example Data

Week	Demand (in thousands of unit)
1	100
2	125
3	90
4	110
5	105
6	130
7	85
8	102
9	110
10	90
11	105
12	95
13	115
14	120
15	80
16	95
17	100

Methods of Estimating Trend

Week	Demand	Forecasts		
	(in thousands of unit)	3 week MA	5 week MA	7 week MA
1	100			
2	125			
3	90			
4	110	105.0		
5	105	108.3		
6	130	101.7	106.0	
7	85	115.0	112.0	
8	102	106.7	104.0	106.4
9	110	105.7	106.4	106.7
10	90	99.0	106.4	104.6
11	105	100.7	103.4	104.6
12	95	101.7	98.4	103.9
13	115	96.7	100.4	102.4
14	120	105.0	103.0	100.3
15	80	110.0	105.0	105.3
16	95	105.0	103.0	102.1
17	100	98.3	101.0	100.0

Methods of Estimating Trend

Week	Demand	Forecasts					
	(in thousands of unit)	3 week MA	3 wk Error	5 week MA	5 wk error	7 week MA	7 wk error
1	100						
2	125						
3	90						
4	110	105.0					
5	105	108.3					
6	130	101.7		106.0			
7	85	115.0		112.0			
8	102	106.7	4.7	104.0	2.0	106.4	4.4
9	110	105.7	4.3	106.4	3.6	106.7	3.3
10	90	99.0	9.0	106.4	16.4	104.6	14.6
11	105	100.7	4.3	103.4	1.6	104.6	0.4
12	95	101.7	6.7	98.4	3.4	103.9	8.9
13	115	96.7	18.3	100.4	14.6	102.4	12.6
14	120	105.0	15.0	103.0	17.0	100.3	19.7
15	80	110.0	30.0	105.0	25.0	105.3	25.3
16	95	105.0	10.0	103.0	8.0	102.1	7.1
17	100	98.3	1.7	101.0	1.0	100.0	0.0

Methods of Estimating Trend

Week	Demand	Forecasts					
	(in thousands of unit)	3 week MA	3 wk Error	5 week MA	5 wk error	7 week MA	7 wk error
1	100						
2	125						
3	90						
4	110	105.0					
5	105	108.3					
6	130	101.7		106.0			
7	85	115.0		112.0			
8	102	106.7	4.7	104.0	2.0	106.4	4.4
9	110	105.7	4.3	106.4	3.6	106.7	3.3
10	90	99.0	9.0	106.4	16.4	104.6	14.6
11	105	100.7	4.3	103.4	1.6	104.6	0.4
12	95	101.7	6.7	98.4	3.4	103.9	8.9
13	115	96.7	18.3	100.4	14.6	102.4	12.6
14	120	105.0	15.0	103.0	17.0	100.3	19.7
15	80	110.0	30.0	105.0	25.0	105.3	25.3
16	95	105.0	10.0	103.0	8.0	102.1	7.1
17	100	98.3	1.7	101.0	1.0	100.0	0.0
	Total errors		104.0		92.6		96.3
	Forecast for week 18			102.0			

Methods of Estimating Trend

Moving Average Example

- So which model should we select?
- This is a qualitative decision. Consider the following

	Noise Dampening Ability	Impulse Response	Precision
MA = 3 wk	Low	High	Low
MA = 5 wk	Mid	Mid	High
MA = 7 wk	High	Low	Mid

Methods of Estimating Trend

Moving Average

- There are a few disadvantages of using this method:
  - There is no equation
  - It looses data and can be strongly effected by extreme values because all past data is weighed equally
- If one feels that the recent data has more effect on the forecast as opposed to the more distant data, higher weights could be given to the recent data. This modified method of moving average is known as the Weighted Moving Average

Methods of Estimating Trend

Exponential Smoothing

- This is a type of moving average forecasting technique which weighs past data in a exponential manner so that the most recent data carries more weight in the moving average
- Simple exponential smoothing (First order)

$$F(t) = F(t-1) + \alpha\left[D(t-1) - F(t-1)\right]$$
$$= \text{old forecast} + \alpha \text{ (error in old forecast)}$$

where $F(t)$ = current period forecast
$F(t\text{-}1)$ = last period forecast
α = smoothing constant (a number between 0 and 1)
$D(t\text{-}1)$ = last period demand

Methods of Estimating Trend

Exponential Smoothing

- Notice that this is a recursive formula. It thus, can be written as follows:

$$F(t) = A_0 D(t-1) + A_1 D(t-2) + A_2 D(t-3) + \cdots\cdots + A_{n-1} D(t-n)$$

where $A_i = \alpha(1 - \alpha)^i$ and all A_i's add up to 1.

- This shows that exponential smoothing is a weighted moving average.
- If α is low, more weight is given to past data whereas high values weigh recent data more heavily. The value of α is generally chosen between 0.005 and 0.4.

Methods of Estimating Trend

Exponential Smoothing

	A0	A1	A2	A3	A4
Coefficient	**α**	**α(1-α)**	**α(1-α)^2**	**α(1-α)^3**	**α(1-α)^4**
α = 0.1	0.1	0.09	0.081	0.0729	0.06561
α = 0.9	0.9	0.09	0.009	0.0009	0.00009

- **When *α* = 0.9**
  - 99.99% of the forecast value is determined by the four most recent demands
- **When *α* = 0.1**
  - Only 34.39% of the forecast value is determined by the four most recent demands

Methods of Estimating Trend

Exponential Smoothing Example

- The following data is given:
  - January trend forecast = 400 units
  - Actual sales in January = 344 units
  - Actual sales in February = 414 units
  - Seasonal index for January = 0.8
  - Seasonal index for February = 0.9
  - Seasonal index for March = 1.2
  - $\alpha = 0.1$
- Use exponential smoothing to find the forecast (seasonalized) for March

Methods of Estimating Trend

Exponential Smoothing

- Note the following
  - Data could be Deseasonalized or Seasonalized
  - Deseasonalized is synonymous with Trend
  - Seasonalized is synonymous with Actual

Methods of Estimating Trend

Exponential Smoothing Example Solution

- Deseasonalize January demand
  - 344/0.8 = 430 units
- Compute February deseasonalized (trend) forecast
  - $$F(t) = F(t-1) + \alpha[D(t-1) - F(t-1)]$$
    $$= 400 + 0.1(430 - 400)$$
    $$= 403 \text{ units}$$
- Although not required, one can calculate the seasonalized (actual) forecast of February
  - 403 x 0.9 = 363 units

Methods of Estimating Trend

Exponential Smoothing Example Solution

- Deseasonalize February demand
  - 414/0.9 = 460 units
- Compute March deseasonalized (trend) forecast
  - $F(t) = F(t-1) + \alpha\left[D(t-1) - F(t-1)\right]$
    $= 403 + 0.1(460 - 403)$
    $= 409$ units
- Calculate seasonalized (actual) forecast for March
  - 409 x 1.2 = 491 units

Methods of Estimating Trend

Method of Least Squares

- This is one of the most widely used methods
- It yields a "line of best fit". It could be linear or nonlinear. If it is linear, it is of the following form:

$$Y = a + bX$$

- Associated with this equation are the following Normal Equations, which help in calculating the coefficients *a* and *b*.

$$\sum Y = Na + b\sum X$$
$$\sum XY = a\sum X + b\sum X^2$$

Methods of Estimating Trend

Method of Least Squares

- The line of best fit has the following properties
  - the summation of all vertical deviations about it is zero
  - the summation of all vertical deviations squared in minimum
  - the line goes through the means of *X* and *Y*
- The coefficients of the line can be calculated by simultaneously solving the Normal equations or directly from the following equations:

$$b = \frac{N\sum XY - \sum X \sum Y}{N\sum X^2 - \left(\sum X\right)^2}$$

$$a = \frac{\sum Y}{N} - b\frac{\sum X}{N}$$

Methods of Estimating Trend

Linear Regression: Time Series Example

- The annual sales is given for the next ten years. The manager wishes to determine a forecasting equation to determine the forecast for the following three years.

Methods of Estimating Trend

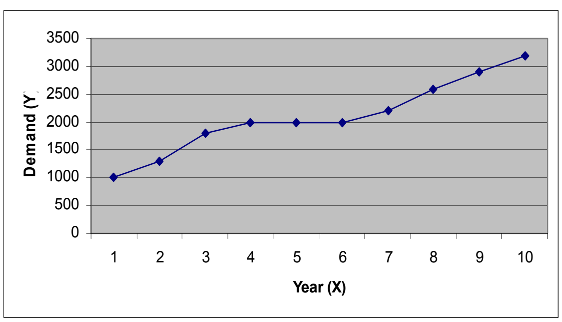
Linear Regression: Time Series Example Data

Year	Demand (in thousands of unit)
1	1000
2	1300
3	1800
4	2000
5	2000
6	2000
7	2200
8	2600
9	2900
10	3200

Methods of Estimating Trend

Linear Regression Example Solution

- If one examines the demand data, one can see that the trend is towards a linear increase.



Methods of Estimating Trend

Linear Regression Example Solution

- Thus we will try to fit the following equation:

$$Y = a + bX$$

- For that, we will solve the Normal Equations in order to calculate *a* and *b* using the following formulae

$$b = \frac{N\sum XY - \sum X \sum Y}{N\sum X^2 - \left(\sum X\right)^2}$$

$$a = \frac{\sum Y}{N} - b\frac{\sum X}{N}$$

Methods of Estimating Trend

	Year (X)	Demand (Y)	X^2	XY
	1	1000	1	1000
	2	1300	4	2600
	3	1800	9	5400
	4	2000	16	8000
	5	2000	25	10000
	6	2000	36	12000
	7	2200	49	15400
	8	2600	64	20800
	9	2900	81	26100
	10	3200	100	32000
Totals	55	21000	385	133300

Methods of Estimating Trend

Linear Regression Example Solution

- Now let us solve for *a* and *b*

$$b = \frac{N\sum XY - \sum X \sum Y}{N\sum X^2 - \left(\sum X\right)^2} = \frac{10 * 133300 - 55 * 21000}{10 * 385 - (55)^2} = 215.758$$

$$a = \frac{\sum Y}{N} - b\frac{\sum X}{N} = \frac{21000}{10} - 215.758 * \frac{55}{10} = 913.333$$

Methods of Estimating Trend

Linear Regression Example Solution

- Now that we know the values of *a* and *b*, the equation can be used to forecast future demand:

$$Y = a + bX$$

$$Y = 913.333 + 215.758X$$

Methods of Estimating Trend

Linear Regression Example Solution

- Therefore the forecast for next three years is as follows:

$$Y_{11} = 913.333 + 215.758(11) = 3286.670 \text{ thousand units}$$
$$Y_{12} = 913.333 + 215.758(12) = 3502.430 \text{ thousand units}$$
$$Y_{13} = 913.333 + 215.758(13) = 3718.190 \text{ thousand units}$$

Methods of Estimating Trend

Standard Error of Forecast

$$S_{YX} = \sqrt{\frac{\sum Y^2 - a\sum Y - b\sum XY}{N-2}}$$

Methods of Estimating Trend

	Year (X)	Demand (Y)	X^2	XY	Y^2
	1	1000	1	1000	1000000
	2	1300	4	2600	1690000
	3	1800	9	5400	3240000
	4	2000	16	8000	4000000
	5	2000	25	10000	4000000
	6	2000	36	12000	4000000
	7	2200	49	15400	4840000
	8	2600	64	20800	6760000
	9	2900	81	26100	8410000
	10	3200	100	32000	10240000
Totals	55	21000	385	133300	48180000

Methods of Estimating Trend

Linear Regression Example Solution

- We can now compute the value of S_{YX}:

$$S_{YX} = \sqrt{\frac{\sum Y^2 - a\sum Y - b\sum XY}{N-2}}$$

$$= \sqrt{\frac{48180000 - 913.333 * 21000 - 215.758 * 133300}{10-2}}$$

$= 173$ thousand units

Methods of Estimating Trend

Linear Regression Example Solution

- We can now compute the lower and upper limits of the forecast. For example, for year 11:

$$\text{Lower Limit} = Y_{11} - t * S_{YX}$$

$$\text{Upper Limit} = Y_{11} + t * S_{YX}$$

where *t* is the number of standard deviations out from the mean of the distribution to provide a given probability of exceeding these lower and upper limits through chance (from student's *t* distribution)

Methods of Estimating Trend

Linear Regression Example Solution

- If we wish to set the limits so that there is only a 10% probability of being outside the limits, we look at the *t* values in the tables for *N*-2 degrees of freedom. The value of *t* = 1.860. Therefore, for our example:

$$\text{Lower Limit} = 3286.670 - 1.860 * 173 = 2964.890 \text{ thousand units}$$

$$\text{Upper Limit} = 3286.670 + 1.860 * 173 = 3608.450 \text{ thousand units}$$

where *t* is the number of standard deviations out from the mean of the distribution to provide a given probability of exceeding these lower and upper limits. 3286.670 thousand units is our best estimate.

Methods of Estimating Trend

Seasonalized Time Series Example

- Given the following data, determine the seasonal indices for each quarter. Use the indices and the time series model to forecast the demand for every quarter of year 4.

Methods of Estimating Trend

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
1	520	730	820	530
2	590	810	900	600
3	650	900	1000	650

Methods of Estimating Trend

Seasonalized Time Series Example Solution

- First compute the seasonal indices

Year	Q1	Q2	Q3	Q4	Totals
1	520	730	820	530	2600
2	590	810	900	600	2900
3	650	900	1000	650	3200
Totals	1760	2440	2720	1780	**8700**
Qtr Ave	586.667	813.333	906.667	593.333	725
SI	0.809	1.122	1.251	0.818	

Methods of Estimating Trend

Seasonalized Time Series Example Solution

- Next, deseasonalize the data by dividing each quarterly data by its SI

Year	Q1	Q2	Q3	Q4
1	643	651	656	648
2	729	722	720	733
3	803	802	800	794

Methods of Estimating Trend

Seasonalized Time Series Example Solution

- Now, perform the time series analysis on the deseasonalized data (12 quarters) and forecast for the next four quarters

Methods of Estimating Trend

Year Qtr	X	Y	X^2	XY
Y1Q1	1	643	1	643
Y1Q2	2	651	4	1302
Y1Q3	3	656	9	1968
Y1Q4	4	648	16	2592
Y2Q1	5	729	25	3645
Y2Q2	6	722	36	4332
Y2Q3	7	720	49	5040
Y2Q4	8	733	64	5864
Y3Q1	9	803	81	7227
Y3Q2	10	802	100	8020
Y3Q3	11	800	121	8800
Y3Q3	12	794	144	9528
Totals	78	8701	650	58961

Methods of Estimating Trend

Seasonalized Time Series Example Solution

- Substituting the appropriate values in the formulae, we get

$$b = \frac{N\sum XY - \sum X \sum Y}{N\sum X^2 - \left(\sum X\right)^2} = \frac{12 * 58961 - 78 * 8701}{12 * 650 - (78)^2} = 16.843$$

$$a = \frac{\sum Y}{N} - b\frac{\sum X}{N} = \frac{8701}{12} - 16.843 * \frac{78}{12} = 615.439$$

Methods of Estimating Trend

Seasonalized Time Series Example Solution

- Now that we know the values of *a* and *b*, the equation can be used to forecast future demand:

$$Y = a + bX$$

$$Y = 615.439 + 16.843X$$

Methods of Estimating Trend

Seasonalized Time Series Example Solution

- Therefore the forecast for next four quarters (Year 4, Q1, Q2, Q3, Q4) are as follows:

$$Y_{13} = 615.439 + 16.843(13) = 834.398 \text{ units}$$
$$Y_{14} = 615.439 + 16.843(14) = 851.241 \text{ units}$$
$$Y_{15} = 615.439 + 16.843(15) = 868.084 \text{ units}$$
$$Y_{16} = 615.439 + 16.843(16) = 884.927 \text{ units}$$

Methods of Estimating Trend

Seasonalized Time Series Example Solution

- Using the seasonal indices, we can get the actual forecast as follows:

$$\hat{Y}_{13} = 834.398 * 0.809 = 675.028 \text{ units}$$
$$\hat{Y}_{14} = 851.241 * 1.122 = 955.092 \text{ units}$$
$$\hat{Y}_{15} = 868.084 * 1.251 = 1085.973 \text{ units}$$
$$\hat{Y}_{16} = 884.927 * 0.818 = 723.870 \text{ units}$$

Forecasting in Practice

- Collaborate in building forecasts
- The value of data depends on where you are in the supply chain
- Be sure to distinguish between demand and sales

Summary of Learning Objectives

- What are the roles of forecasting for an enterprise and a supply chain?
- What are the components of a demand forecast?
- How is demand forecast given historical data using time series methodologies?
- How is a demand forecast analyzed to estimate forecast error?