Solution to exercise 5

(Big-M method, with M symbolic)

Cost function:

$$-3\,x1 - 2\,x2 - x3 + M\,z$$

Equations:

$$\{4\,x1 + x2 + x3 + s1 = 30,\ -4\,x1 - x2 - x3 + s2 - z = -30,\ x1 + 2\,x2 + 3\,x3 + s4 = 40,\ 2\,x1 + 3\,x2 + x3 + s3 = 60\}$$

Basic variables in initial basis:

$$\{z, s1, s3, s4\}$$

Tableau:

$$\{s1 = -4\,x1 - x2 - x3 + 30,\ s4 = -x1 - 2\,x2 - 3\,x3 + 40,\ s3 = -2\,x1 - 3\,x2 - x3 + 60,\ z = -4\,x1 - x2 - x3 + s2 + 30\}$$

Basic solution:

$$\{z = 30,\ s1 = 30,\ s4 = 40,\ s3 = 60\}$$

Note that the initial basis is feasible.

PRICING. Expression of the cost function in terms on non-basic variables:

$$(-3 - 4\,M)\,x1 + (-2 - M)\,x2 + (-1 - M)\,x3 + M\,s2 + 30\,M$$

Reduced costs of non-basic variables x1, x2 and x3 are negative. Choose e.g. x1.

RATIO TEST. Freeze x2 = x3 = s2 = 0 :

$$[s1 = -4\,x1 + 30,\ s4 = -x1 + 40,\ s3 = -2\,x1 + 60,\ z = -4\,x1 + 30]$$

$$[0 <= -4\,x1 + 30,\ 0 <= -x1 + 40,\ 0 <= -2\,x1 + 60,\ 0 <= -4\,x1 + 30]$$

Best value for variable x1 is 7.5. Basic variables s1, z are tight.
Swap e.g. x1 and z.

Basic variables in current basis:

$$\{x1, s1, s3, s4\}$$

Tableau:

$$\{s4 = 65/2 - \frac{7\,x2}{4} - \frac{11\,x3}{4} - \frac{s2}{4} + z/4,\ s3 = 45 - \frac{5\,x2}{2} - \frac{x3}{2} - \frac{s2}{2} + z/2,\ x1 = 15/2 - \frac{x2}{4} - \frac{x3}{4} + \frac{s2}{4} - z/4,\ s1 = -s2 + z\}$$

Auxiliary variable z is non-basic. Drop it.
Tableau:

$$\{s4 = 65/2 - \frac{7\,x2}{4} - \frac{11\,x3}{4} - \frac{s2}{4},\ s3 = 45 - \frac{5\,x2}{2} - \frac{x3}{2} - \frac{s2}{2},\ x1 = 15/2 - \frac{x2}{4} - \frac{x3}{4} + \frac{s2}{4},\ s1 = -s2\}$$

Basic solution:

$$\{s1 = 0,\ x1 = 15/2,\ s4 = 65/2,\ s3 = 45\}$$

Note that the initial basis is feasible.

PRICING. Expression of the cost function in terms on non-basic variables:

$$- 45/2 - \frac{5\,x2}{4} - \frac{x3}{4} - \frac{3\,s2}{4}$$

Reduced cost of non-basic variables x2, x3, s2 is negative. Choose x2.

RATIO TEST. Freeze x3 = s2 = 0 :

$$[s4 = 65/2 - \frac{7\,x2}{4},\ s3 = 45 - \frac{5\,x2}{2},\ x1 = 15/2 - \frac{x2}{4},\ s1 = 0]$$

$$[0 \le 65/2 - \frac{7\,x2}{4},\ 0 \le 45 - \frac{5\,x2}{2},\ 0 \le 15/2 - \frac{x2}{4},\ 0 \le 0]$$

Best value for variable x2 is 18. Basic variable s3 is tight. Swap x2 and s3.

Basic variables in current basis:

$$\{x1, x2, s1, s4\}$$

Tableau:

$$\{s1 = -s2,\ x1 = \frac{s3}{10} + 3 - \frac{x3}{5} + \frac{3\,s2}{10},\ s4 = \frac{7\,s3}{10} + 1 - \frac{12\,x3}{5} + \frac{s2}{10},$$

$$x2 = - \frac{2\,s3}{5} + 18 - \frac{x3}{5} - \frac{s2}{5}\}$$

Basic solution:

$$\{s1 = 0,\ x1 = 3,\ s4 = 1,\ x2 = 18\}$$

PRICING. Expression of the cost function in terms on non-basic variables:

$$\frac{s3}{2} - 45 - \frac{s2}{2}$$

Reduced cost of non-basic variable s2 is negative. Choose s2.

RATIO TEST. Freeze s3 = x3 = 0 :

$$[s1 = -s2,\ x1 = 3 + \frac{3\,s2}{10},\ s4 = 1 + \frac{s2}{10},\ x2 = 18 - \frac{s2}{5}]$$

$$[0 \le -s2,\ 0 \le 3 + \frac{3\,s2}{10},\ 0 \le 1 + \frac{s2}{10},\ 0 \le 18 - \frac{s2}{5}]$$

Best value for variable s2 is 0. Basic variable s1 is tight. Swap s2 and s1.

Basic variables in current basis:

$$\{x1, x2, s2, s4\}$$

Tableau:

$$\left\{s2 = -s1,\ x1 = \frac{s3}{10} + 3 - \frac{x3}{5} - \frac{3\,s1}{10},\ s4 = \frac{7\,s3}{10} + 1 - \frac{12\,x3}{5} - \frac{s1}{10},\ x2 = -\frac{2\,s3}{5} + 18 - \frac{x3}{5} + \frac{s1}{5}\right\}$$

Basic solution:

$$\{s2 = 0,\ x1 = 3,\ s4 = 1,\ x2 = 18\}$$

PRICING. Expression of the cost function in terms on non-basic variables:

$$\frac{s3}{2} - 45 + \frac{s1}{2}$$

All reduced costs are non-negative. Current basis is optimal.