AMMM Final Exam

June 4th, 2020

- *This part contains two exercises and 2 pages*
- *Handwrite your solutions on white paper, scan with app, and upload at racó as **one** pdf file*
- *You are allowed to use a basic calculator*
- *You are **not** allowed to use anything else or communicate with anyone during the exam*

Part 1: Linear Programming **(3 pts)**

Exercise 1 (1.5 pt). Consider a linear program that includes n variables $(x_1, \ldots, x_n)$. These variables are unconstrained: their range is the set of all real numbers. One of the steps in the transformation that takes a general linear program and converts it into one in canonical form is to replace each unconstrained variable x_i by the difference of two new non-negative variables; i.e., x_i gets replaced by $y_i - z_i$. This transforms the program in n unconstrained variables into an equivalent one that has $2n$ non-negative variables. Suggest a method to achieve the same effect using only $n + 1$ many non-negative variables $(y_1, \ldots, y_n, z)$ instead of $2n$ many $(y_1, z_1, \ldots, y_n, z_n)$.

For concreteness, explain your solution by answering the following questions:

1. Apply your transformation to the linear program below. Your new program should have non-negative variables (y_1, y_2, y_3, y_4, z) and be equivalent to the given one in the sense that there should be a way to convert every feasible solution for one program into a feasible solution for the other program, with the same value of the objective function.

$$\begin{aligned}
&\min \quad 3x_1 - 2x_2 + x_3 + x_4 \\
&\text{subject to:} \\
&\quad x_1 + x_2 + x_3 \geq 3 \\
&\quad 2x_1 - x_3 - x_4 \leq -2 \\
&\quad x_2 + x_3 = 4 \\
&\quad x_1, x_2, x_3, x_4 \in \mathbb{R}.
\end{aligned}$$

2. Give the feasible solution of your program on the variables (y_1, y_2, y_3, y_4, z) that corresponds to the feasible solution $(x_1, x_2, x_3, x_4) = (-1, -3, 7, 3)$ of the original program.
3. In one paragraph of text, explain how your generic transformation works: how do you obtain a feasible solution of the new program on the variables $(y_1, \ldots, y_n, z)$ from a feasible solution of the original program on the variables $(x_1, \ldots, x_n)$? How do you obtain a feasible solution on the original program on the variables $(x_1, \ldots, x_n)$ from a feasible solution of the new program on the variables $(y_1, \ldots, y_n, z)$?

Exercise 2 (1.5 pt). Solve the following linear program by using the simplex method:

$$\begin{array}{l} \max \quad x_1 + 2x_2 - x_3 \\ \text{subject to:} \\ \quad 2x_1 + x_2 + x_3 \leq 22 \\ \quad 4x_1 + 2x_2 + 3x_3 \leq 28 \\ \quad 2x_1 + 5x_2 + 5x_3 \leq 30 \\ \quad x_1, x_2, x_3 \geq 0. \end{array}$$

For concreteness, answer the following questions:

1. Write the result of transforming the given linear program into canonical form.
2. In transforming the program into canonical form, did you need to introduce slack variables? surplus variables? artificial variables?
3. Did you need to apply Phase I to obtain an initial feasible solution? Why? What is your initial feasible solution for the canonical form and how did you obtain it?
4. Write the sequence of tableaux that starts at the canonical form and leads to the optimal solution.
5. Write down the optimal solution.