Algorithmic Methods for Mathematical Models (AMMM)

Linear Programming Modelling Exercises.

1. A company makes three products and has available four workstations. The production time (in minutes) per unit produced varies from workstation to workstation as shown below:

	Workstation 1	**Workstation 2**	**Workstation 3**	**Workstation 4**
Product 1	5	7	4	10
Product 2	6	12	8	15
Product 3	13	14	9	17

   Similarly the profit (€) contribution per unit varies from workstation to workstation as below:

	Workstation 1	**Workstation 2**	**Workstation 3**	**Workstation 4**
Product 1	10	8	6	9
Product 2	18	20	15	17
Product 3	15	16	13	17

   If next week there are 35 working hours available at each workstation, how much of each product should be produced given that we need at least 100 units of product 1, 150 units of product 2 and 100 units of product 3?

   Formulate this problem as a linear program.

2. A cargo plane has three compartments for storing cargo: front, centre and rear. These compartments have the following limits on both weight and space:

Compartment	**Weight capacity (tonnes)**	**Space capacity (cubic metres)**
Front	10	6800
Centre	16	8700
Rear	8	5300

   Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the plane.

   The following four cargoes are available for shipment on the next flight:

Cargo	**Weight (tonnes)**	**Volume (cubic metres/tonne)**	**Profit (€/tonne)**
C1	18	480	310
C2	15	650	380
C3	23	580	350
C4	12	390	285

   Any proportion of these cargoes can be accepted. The objective is to determine how much (if any) of each cargo C1, C2, C3 and C4 should be accepted and how to distribute each among the compartments so that the total profit for the flight is maximised.

   Formulate the above problem as a linear program.

3. A canning company operates two canning plants. The growers are willing to supply fresh fruits in the following amounts:

   - S1: 200 tonnes at 11 €/tonne
   - S2: 310 tonnes at 10 €/tonne
   - S3: 420 tonnes at 9 €/tonne

   Shipping costs in € per tonne are:

	To Plant A	**To Plant B**
From S1	3	3.5
From S2	2	2.5
From S3	6	4

   Plant capacities and labour costs are:

	Plant A	**Plant B**
Capacity	460 tonnes	560 tonnes
Labour cost	26 €/tonne	21 €/tonne

   The canned fruits are sold at 50 €/tonne to the distributors. The company can sell at this price all they can produce. The objective is to find the best mixture of the quantities supplied by the three growers to the two plants so that the company maximises its profits.

   Formulate the above problem as a linear program.

4. A company manufactures four products (1, 2, 3, 4) on two machines (X and Y). The time (in minutes) to process one unit of each product on each machine is shown below:

	Machine X	**Machine Y**
Product 1	10	27
Product 2	12	19
Product 3	13	33
Product 4	8	23

   The profit per unit for each product (1, 2, 3, 4) is 10 €, 12 €, 17 € and 8 € respectively.

   Each unit of product 1 needs to be processed by both machines X and Y (let's say first X and then Y, although this detail is not important). On the other hand, products 2, 3 and 4 can be produced with a single machine, X or Y.

   The factory is very small and this means that floor space is very limited. Only one week's production is stored in 50 square metres of floor space where the floor space taken up by each product is 0.1, 0.15, 0.5 and 0.05 (square metres) for products 1, 2, 3 and 4 respectively.

   Customer requirements mean that the amount of product 3 produced should be related to the amount of product 2 produced. Over a week approximately the number of units of product 2 to be produced should be twice as many units of product 3 ($\pm 5\%$).

   Machine X is out of action (for maintenance/because of breakdown) 5% of the time and machine Y 7% of the time.

   Assuming a working week 35 hours long formulate the problem of how to manufacture these products.

5. A company is planning its production schedule over the next six months (it is currently the end of month 2). The demand (in units) for its product to be met at the end of each month is as shown below:

Month	3	4	5	6	7	8
Demand	5000	6000	6500	7000	8000	9500

The company currently has in stock: 1000 units which were produced in month 2; 2000 units which were produced in month 1; 500 units which were produced in month 0.

The company can only produce up to 6000 units per month and the managing director has stated that stocks must be built up to help meet demand in months 5, 6, 7 and 8. Each unit produced costs 15 and the cost of holding stock is estimated to be 0.75 per unit per month (based upon the stock held at the beginning of each month).

The company has a major problem with deterioration of stock in that the stock inspection which takes place at the end of each month regularly identies ruined stock (costing the company 25 € per unit). It is estimated that, on average, the stock inspection at the end of month t will show that 11% of the units in stock which were produced in month t are ruined; 47% of the units in stock which were produced in month $t-1$ are ruined; 100% of the units in stock which were produced in month $t-2$ are ruined. The stock inspection for month 2 is just about to take place.

The company wants a production plan for the next six months that avoids stockouts. Formulate their problem as a linear program.

6. A company assembles four products (1, 2, 3, 4) from delivered components. The profit per unit for each product (1, 2, 3, 4) is 10 €, 15 €, 22 € and 17 € respectively. The maximum number of units that can be sold in the next week for each product (1, 2, 3, 4) is 50, 60, 85 and 70 units respectively. There are three stages (A, B, C) in the manual assembly of each product and the man-hours needed for each stage per unit of product are shown below:

	Product 1	**Product 2**	**Product 3**	**Product 4**
Stage A	1	2	3	4
Stage B	2	2	1	1
Stage C	2	4	1	2

The nominal time available in the next week for assembly at each stage (A, B, C) is 160, 200 and 80 man-hours respectively.

It is possible to vary the man-hours spent on assembly at each stage such that workers previously employed on stage B assembly could spend up to 20% of their time on stage A assembly and workers previously employed on stage C assembly could spend up to 30% of their time on stage A assembly.

Production constraints also require that the ratio (product 1 units assembled)/(product 4 units assembled) must lie between 0.9 and 1.15.

Formulate the problem of deciding how much to produce next week as a linear program.

7. A company makes a product which requires, at the final stage, three parts. To get one unit of the product, one unit of each of the three parts must be assembled. The parts can be produced by two different departments as detailed below:

	Prod. rate part 1	Prod. rate part 2	Prod. rate part 3	Cost
Dept. 1	7 units/h	6 units/h	9 units/h	25.0 €/h
Dept. 2	6 units/h	11 units/h	5 units/h	12.5 €/h

Because of the way production is organised, a department cannot produce only one or two parts; e.g., one hour of working in department 1 produces exactly 7 part 1 units, 6 part 2 units and 9 part 3 units, and this cannot be altered.

One week, 1050 finished (assembled) products are needed, but up to 1200 can be produced if necessary. If department 1 has 100 working hours available and department 2 has 110 working hours available, formulate the problem of minimising the cost of producing the finished (assembled) products needed this week as a linear program, subject to the constraint that storage space is limited and that at most 200 unassembled parts in total (counting units of part 1, 2 and 3 together) can be stored at the end of the week.

8. A quantity y is known to depend upon another quantity x. A set of corresponding values has been collected for x and y and is presented in the following table:

x	0.0	0.5	1.0	1.5	1.9	2.5	3.0	3.5	4.0	4.5
y	1.0	0.9	0.7	1.5	2.0	2.4	3.2	2.0	2.7	3.5

x	5.0	5.5	6.0	6.6	7.0	7.6	8.5	9.0	10.0
y	1.0	4.0	3.6	2.7	5.7	4.6	6.0	6.8	7.3

(a) Fit the best straight line $y = bx + a$ to this set of data points. The objective is to minimize the sum of absolute deviations of each observed value of y from the value predicted by the linear relationship.

(b) Fit the best straight line where the objective is to minimize the maximum deviation of all the observed values of y from the value predicted by the linear relationship.

(c) Fit the best quadratic curve $y = cx^2 + bx + a$ to this set of data points using the same objectives as in (1).

(d) Fit the best quadratic curve to this set of data points using the same objectives as in (2).

9. An iron foundry has a firm order to produce 1 tonne of castings containing at least 0.45 % manganese and between 3.25 % and 5.50 % silicon. The castings sell for 0.45 € per kg. The foundry has three types of pig iron available in essentially unlimited amounts, with the following properties:

	Pig A	Pig B	Pig C
Silicon	4 %	1 %	0.6 %
Manganese	0.45 %	0.5 %	0.4 %

Further, the production process is such that pure manganese can also be added directly to the melt. The costs of the various possible inputs are:

- Pig A: 21 € / tonne

- Pig B: 25 € / tonne
- Pig C: 15 € / tonne
- Manganese: 8 € / kg

It costs 0.005 € to melt down a kg. of pig iron. Out of what inputs should the foundry produce the castings in order to maximize profits?

Formulate the problem as a linear program.

10. A portfolio manager in charge of a bank portfolio has 10 million € to invest. The securities available for purchase, as well as their respective risks, maturities, and returns, are shown in the following table:

Bond name	**Bond type**	**Bank's risk scale**	**Years to maturity**	**Return**
A	Municipal	2	9	4.3 %
B	Agency	2	15	2.7 %
C	Government	1	4	2.5 %
D	Government	1	3	2.2 %
E	Municipal	5	2	4.5 %

The bank places the following policy limitations on the portfolio manager's actions:

- Government and agency bonds must total at least 4 million €.
- The average risk of the portfolio cannot exceed 1.4 on the bank's risk scale.
- The average years to maturity of the portfolio must not exceed 5 years.

Formulate as a linear program the following problems:

(a) Assume that the objective of the portfolio manager is to maximize earnings. What bonds should he purchase?

(b) Assume that the objective of the portfolio manager is to maximize earnings. If it became possible to borrow up to 1 million € at 5.5 %, what bonds should he purchase?

(c) Assume that the objective of the portfolio manager is to maximize the average annual return. What bonds should he purchase?

11. A division of a plastics company manufactures three basic products: sporks, packets, and school packs. A spork is a plastic utensil which purports to be a combination spoon, fork, and knife. The packets consist of a spork, a napkin, and a straw wrapped in cellophane. The school packs are boxes of 100 packets with an additional 10 loose sporks included.

Production of 1000 sporks requires 0.8 standard hours of molding machine capacity, 0.2 standard hours of supervisory time, and 2.50 € in direct costs. Production of 1000 packets, including 1 spork, 1 napkin, and 1 straw, requires 1.5 standard hours of the packaging-area capacity, 0.5 standard hours of supervisory time, and 4.00 € in direct costs. There is an unlimited supply of napkins and straws. Production of 1000 school packs requires 2.5 standard hours of packaging-area capacity, 0.5 standard hours of supervisory time, 10 sporks, 100 packets, and 8.00 € in direct costs.

Any of the three products may be sold in unlimited quantities at prices of 5.00 €, 15.00 €, and 300.00 € per thousand, respectively. If there are 200 hours of production time in the coming month, what products, and how much of each, should be manufactured to yield the most profit?

Formulate the problem as a linear program.