Algorithmic Methods for Mathematical Models

Assignment: Greedy Algorithms

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(a) Specify the algorithm, including: i) the candidates; and ii) the greedy function $q()$. Specify $q()$ using mathematical notation and a short descriptive text.

Note: Local Search included.

Algorithm 1: Greedy Algorithm

Input:

- A set of offices O, each office o requires b_0 PetaBytes (10^6 GB) of data storage
- A set of storage providers D, each storage provider d has a capacity k_d in PBs, a fixed cost f_d and an additional cost s_d per stored PB

Output: A set w of assignments $\langle d, O' \rangle$ where $d \in D$, $O' \subseteq O$, $|O'| \leq 3$, $|w| \leq 3$, $\bigcup_{O'_i \in w} O'_i = O$ and $\bigcap_{O'_i \in w} O'_i = \emptyset$

$w \leftarrow \emptyset$

forall $o \in O$ **do**

    `/*` c_{min} `is the cost of the minimal assignment */`

    `/*` d_u `is the storage provider to upgrade */`

    $(c_{min}, d_{min}, d_u) \leftarrow argmin\{q(\langle o, d \rangle, w) |\ d \in D\}$

    if $c_{min} = \infty$ **then**

        return $INFEASIBLE$

    else if $d_{min} \in w$ **then** `/* update the assigned offices */`

        $O'' \leftarrow O'_{d_{min}} \cup \{o\}$

        $w \leftarrow w \cup \{\langle d_{min}, O'' \rangle\}$

    else if $|w| < 3 \land d_u = null$ **then** `/* add a new storage provider */`

        $w \leftarrow w \cup \{\langle d_{min}, \{o\} \rangle\}$

    else `/* replace` d_u `storage provider */`

        $w' \leftarrow w \setminus \{d_u\}$

        $O'' \leftarrow \{O'_{d_u} \cup \{o\}\}$

        $w \leftarrow w' \cup \{\langle d_{min}, O'' \rangle\}$

    end

end

return w

$$q(\langle o,d\rangle,w)=\begin{cases}\infty & d\in w\wedge(|O'_d|=3\vee b_o>(k_d-\sum_{o'\in O'}b_{o'}))\\ b_o s_d & d\in w\wedge|O'_d|<3\wedge b_o\leq(k_d-\sum_{o'\in O'}b_{o'})\\ q'(\langle o,d\rangle,w) & d\notin w\wedge|w|=3\\ min(f_d+b_0 s_d,\ q'(\langle o,d\rangle,w)) & d\notin w\wedge|w|<3\end{cases}$$

$$q'(o,d,w)=min\{q'(o,d,d',w)|d'\in w\}$$

$$q'(o,d,d',w)=\begin{cases}\infty & |O'_{d'}|=3\vee b_o>(k_d-\sum_{o'\in O'_{d'}}b_{o'})\\ (f_d+s_d(b_o+\sum_{o'\in O'_{d'}}b_{o'}))-(f_{d'}+s_{d'}(b_o+\sum_{o'\in O'_{d'}}b_{o'})) & |O'_{d'}|<3\wedge b_o\leq(k_d-\sum_{o'\in O'_{d'}}b_{o'})\end{cases}$$

Figure 1: Cost function

(b) Detail the decisions made in every iteration of the algorithm until a solution is obtained. For each iteration, compute the value of the proposed greedy function $q()$ for all the candidates.

Note: this execution does not include the Local Search.

Office	d_1	d_2	d_3	d_4	d_5
q(o_1)	500	380	380	890	700
q(o_2)	500	30	380	890	700
q(o_3)	500	∞	380	890	700
q(o_4)	500	∞	130	890	700
q(o_5)	500	∞	130	890	700
q(o_6)	500	∞	∞	890	700
q(o_7)	200	∞	∞	890	700