

Algorithmic Methods for Mathematical Models (AMMM)

Genetic Algorithms

Luis Velasco

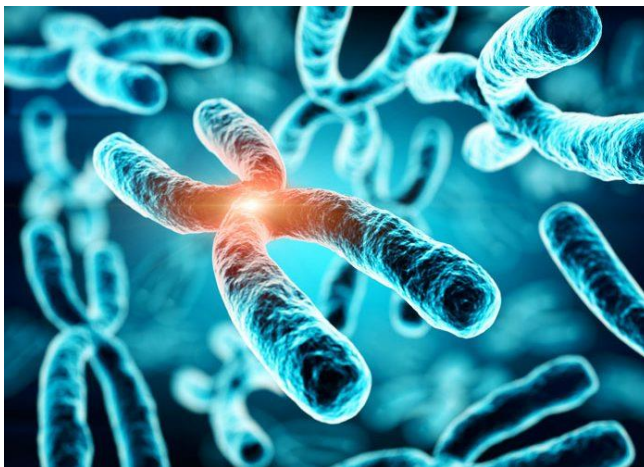
(lvelasco @ ac.upc.edu)

Campus Nord D6-107

Genetic Algorithms (GA)

- One of the distinctive features of GA is to separate the **representation** of the problem from the **actual variables** in which it was originally formulated.
- In biological terms:
 - **genotype**: set of genes in our DNA which is responsible for a particular trait.
 - **phenotype**: the physical expression (characteristics) of that trait.

Genotype

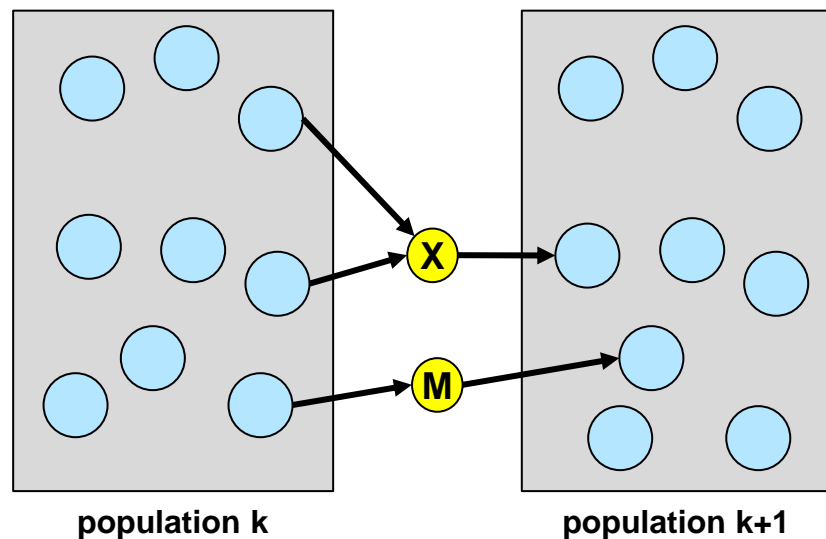


Phenotype



Genetic Algorithms

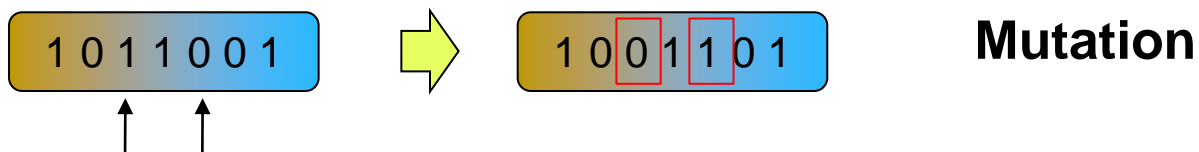
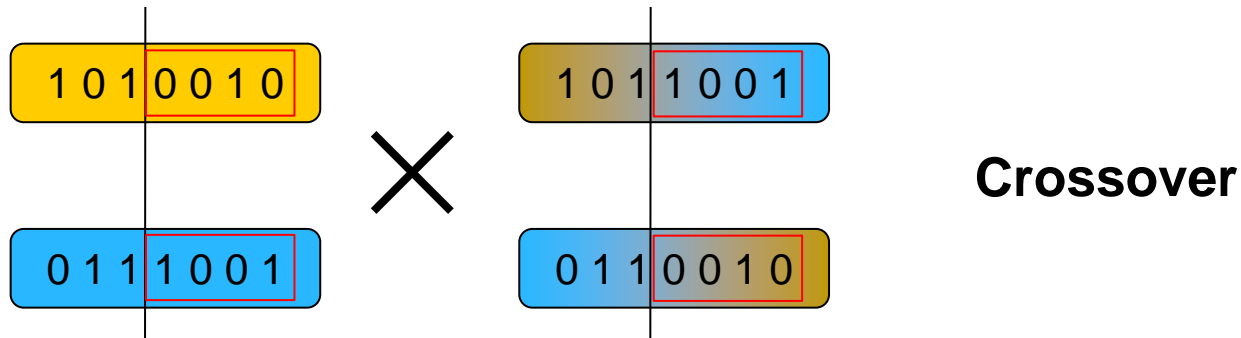
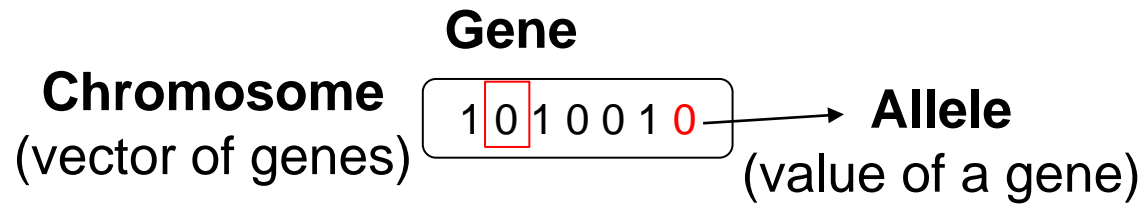
- A population of fixed size is used.
 - Individual solutions are represented by a string named **chromosome**.
 - The recombination of chromosomes is carried out using simple analogies of genetic **crossover** and **mutation**.
 - The search is guided by the results of evaluating the objective function for each individual in the population.
 - Chromosomes that have higher fitness (i.e., represent better solutions) can be identified, and these are given more opportunity to *breed*.



population k

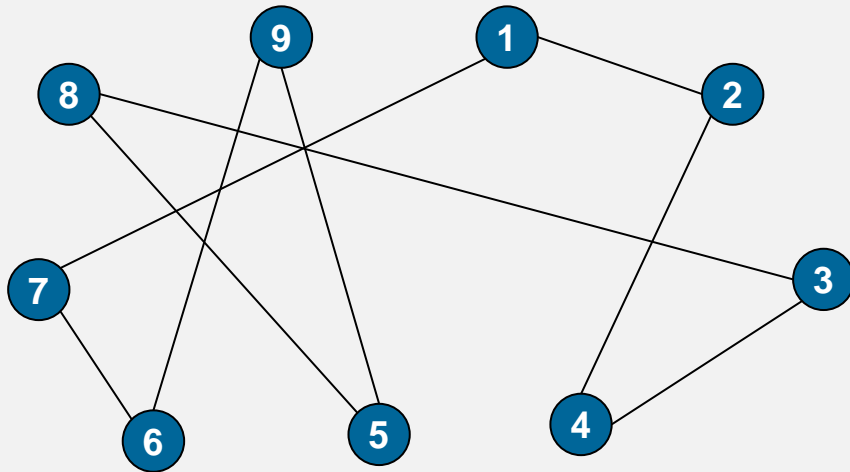
population k+1

Operators



Example TSP

Adjacency Representation



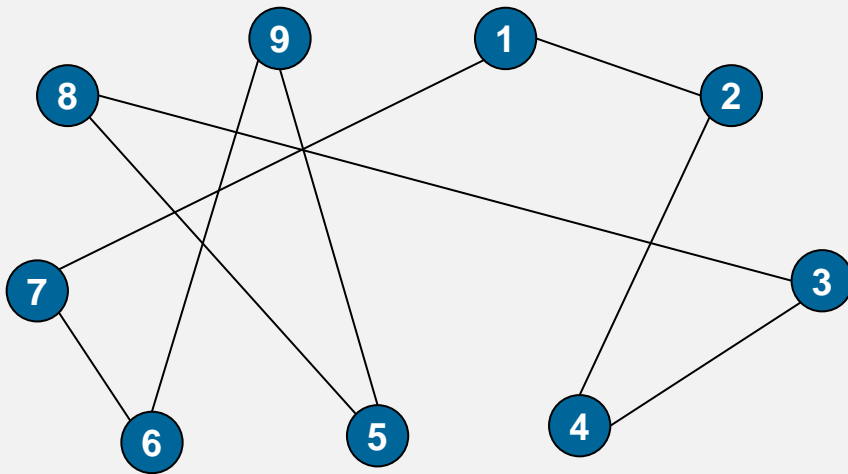
value= next node

2 4 8 3 9 7 1 5 6
1 2 3 ..
pos=node

- Mutations are **not supported**
- Cross over: **difficult** to maintain feasibility

Example TSP: Crossover

Path Representation

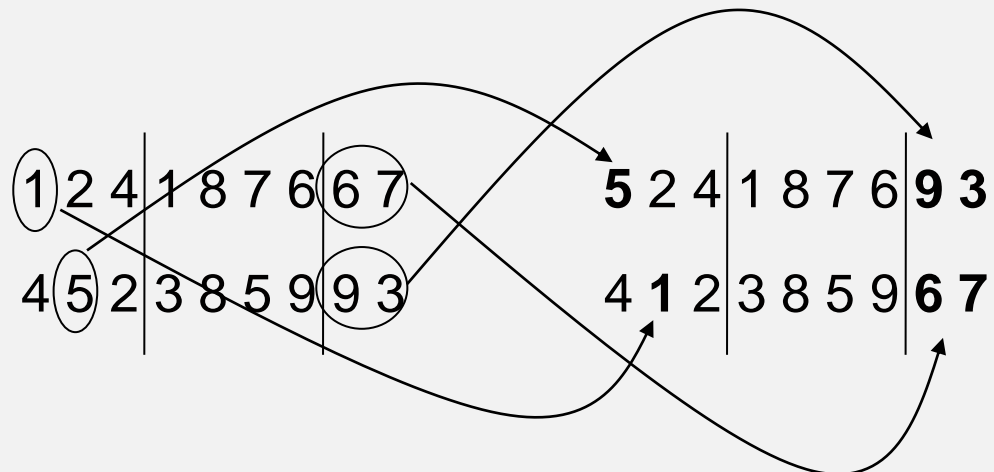


After doing crossover
some **arrangement** is
needed to reach
feasibility.

1	2	4	3	8	5	9	6	7
4	5	2	1	8	7	6	9	3

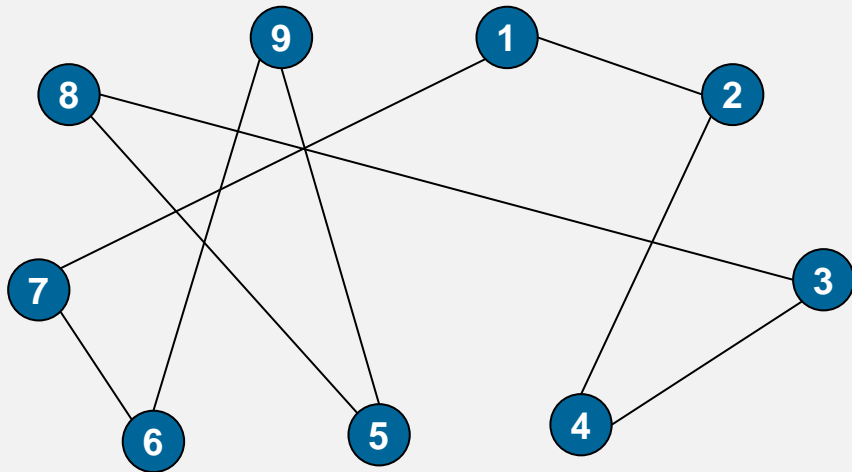
1	2	4	1	8	7	6	6	7
4	5	2	3	8	5	9	9	3

5	2	4	1	8	7	6	9	3
4	1	2	3	8	5	9	6	7

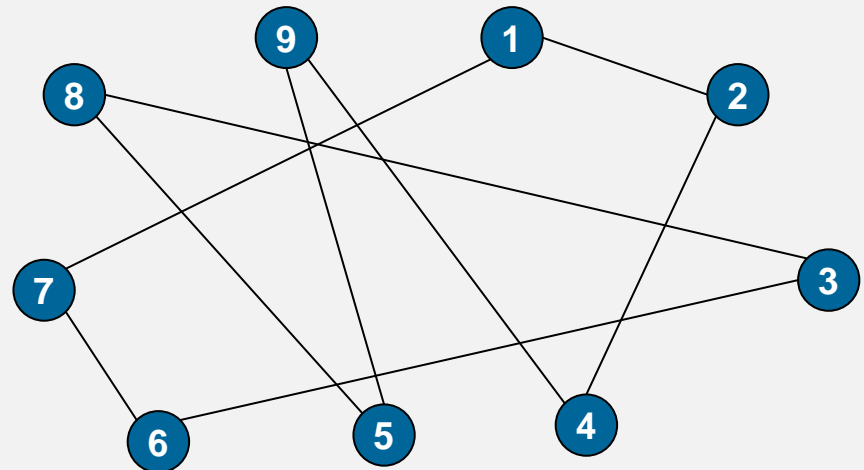


Example TSP: Mutation

Path Representation



Mutation can be done through inversion.



Biased Random Key GA (BRKGA)

- Individuals are vectors of real-valued numbers (random keys) in the interval $[0,1]$.
- Mating is done using parametrized uniform crossover
- For each gene, flip a **biased** coin to choose which parent passes the allele to the child.

parents $a = (0.25, 0.19, 0.67, 0.05, 0.89)$
 $b = (0.63, 0.90, 0.76, 0.93, 0.08)$



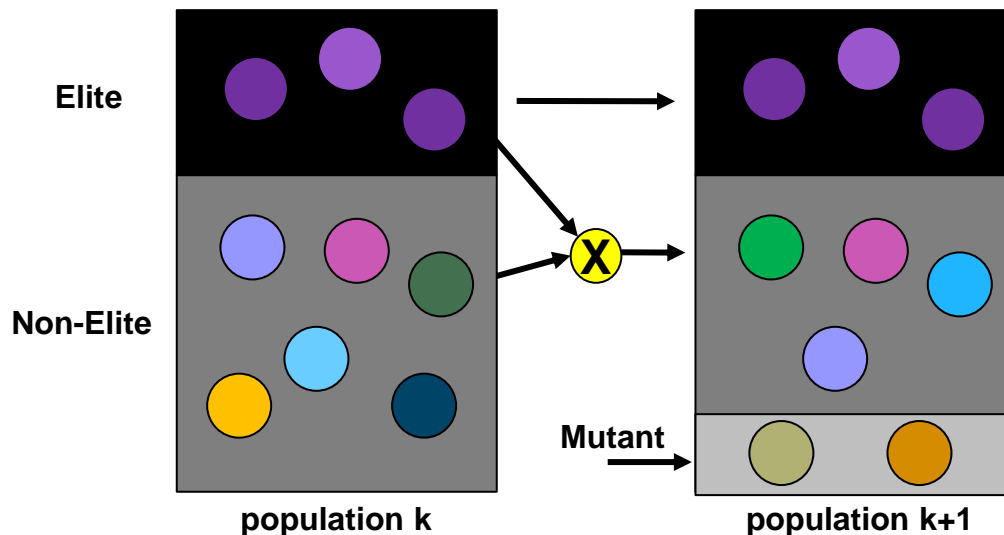
child $c = (0.25, 0.90, 0.76, 0.05, 0.89)$

If every random-key vector corresponds to a feasible solution: Mating always produces feasible offspring.

M. Resende (<http://mauricio.resende.info/>)

BRKGA

- Initial population is made up of P chromosomes, each with n genes, each having a value (allele) generated uniformly at random in the interval $[0,1]$.
- At the k -th generation, compute the cost of each solution and partition the solutions into two sets: **elite** and **non-elite** solutions.
- The elite set should be small and contain the best solutions.

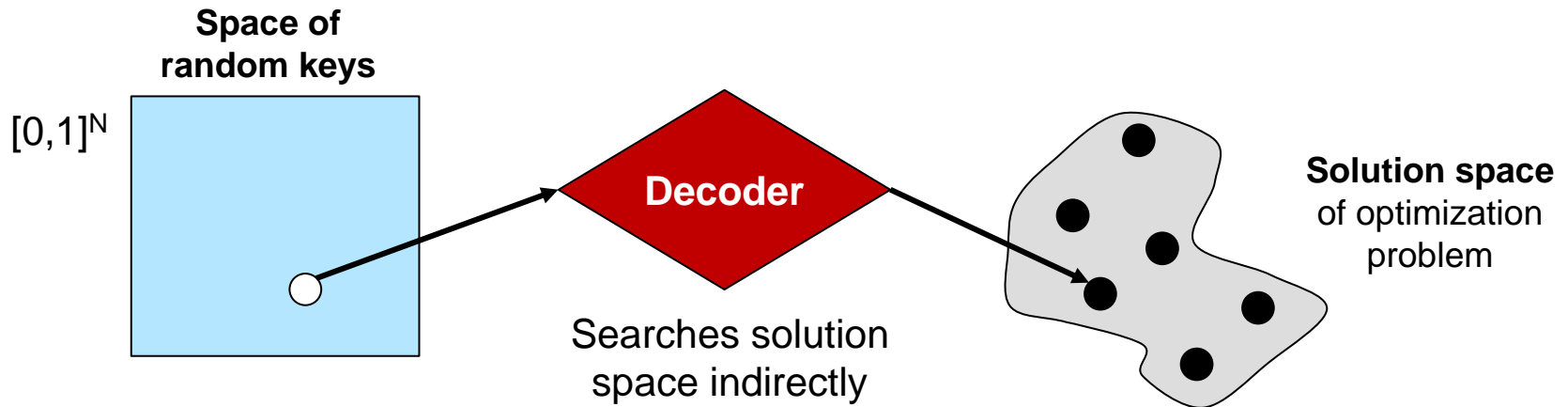
**BIASED RANDOM KEY GA**

Mate elite solution with non-elite to produce offspring in population $k+1$.
Mates are chosen at random.

Child inherits more characteristics of elite parent; the probability that child inherits keys of the elite parent > 0.5

Decoders

- Decoder: a **deterministic** algorithm that takes as input a random-key vector and returns a feasible solution of the optimization problem.
- BRKGA searches the solution space indirectly by searching the space of random keys and using the decoder to evaluate fitness of the random key.



Transforming a Greedy Algorithm into a Decoder

Greedy Algorithm

```
Initialize  $C$   
 $S \leftarrow \{\}$   
while  $S$  is not a solution do  
    evaluate  $q(c) \forall c \text{ in } C$   
     $c_{best} \leftarrow \operatorname{argmax}\{q(c) \mid c \text{ in } C\}$   
     $S \leftarrow S \cup \{c_{best}\}$   
    update  $C$ , e.g.,  $C \leftarrow C \setminus \{c_{best}\}$   
return  $\langle f(S), S \rangle$ 
```

Decoder

Given ch a vector of random keys (chromosome),
where $|ch| \equiv |C|$

```
Initialize  $C$   
for each  $c \text{ in } C$  do  $c.weight = c.weight * ch[c]$   
 $S \leftarrow \{\}$   
while  $S$  is not a solution do  
    evaluate  $q(c) \forall c \text{ in } C$   
     $c_{best} \leftarrow \operatorname{argmax}\{q(c) \mid c \text{ in } C\}$   
     $S \leftarrow S \cup \{c_{best}\}$   
    update  $C$ , e.g.,  $C \leftarrow C \setminus \{c_{best}\}$   
return  $\langle f(S), S \rangle$ 
```

Distances

	1	2	3	4	5
1	-	10	12	7	9
2		-	11	17	4
3			-	5	14
4				-	9
5					-

Chromosome 1

	1	2	3	4	5
1	-	1	1	1	1
2		-	1	1	1
3			-	1	1
4				-	1
5					-

Weighted Distances

	1	2	3	4	5
1	-	10	12	7	9
2		-	11	17	4
3			-	5	14
4				-	9
5					-

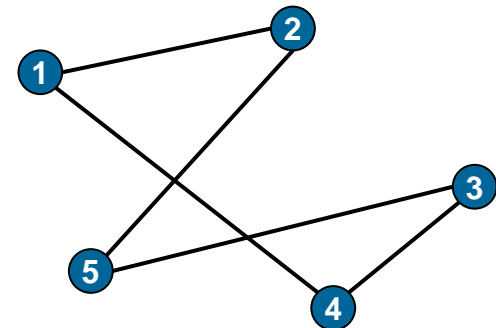
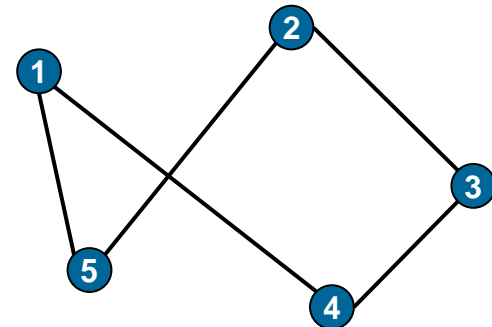
Chromosome 2

	1	2	3	4	5
1	-	0.3	0.7	0.6	0.4
2		-	0.4	0.7	0.3
3			-	0.9	0.3
4				-	0.65
5					-

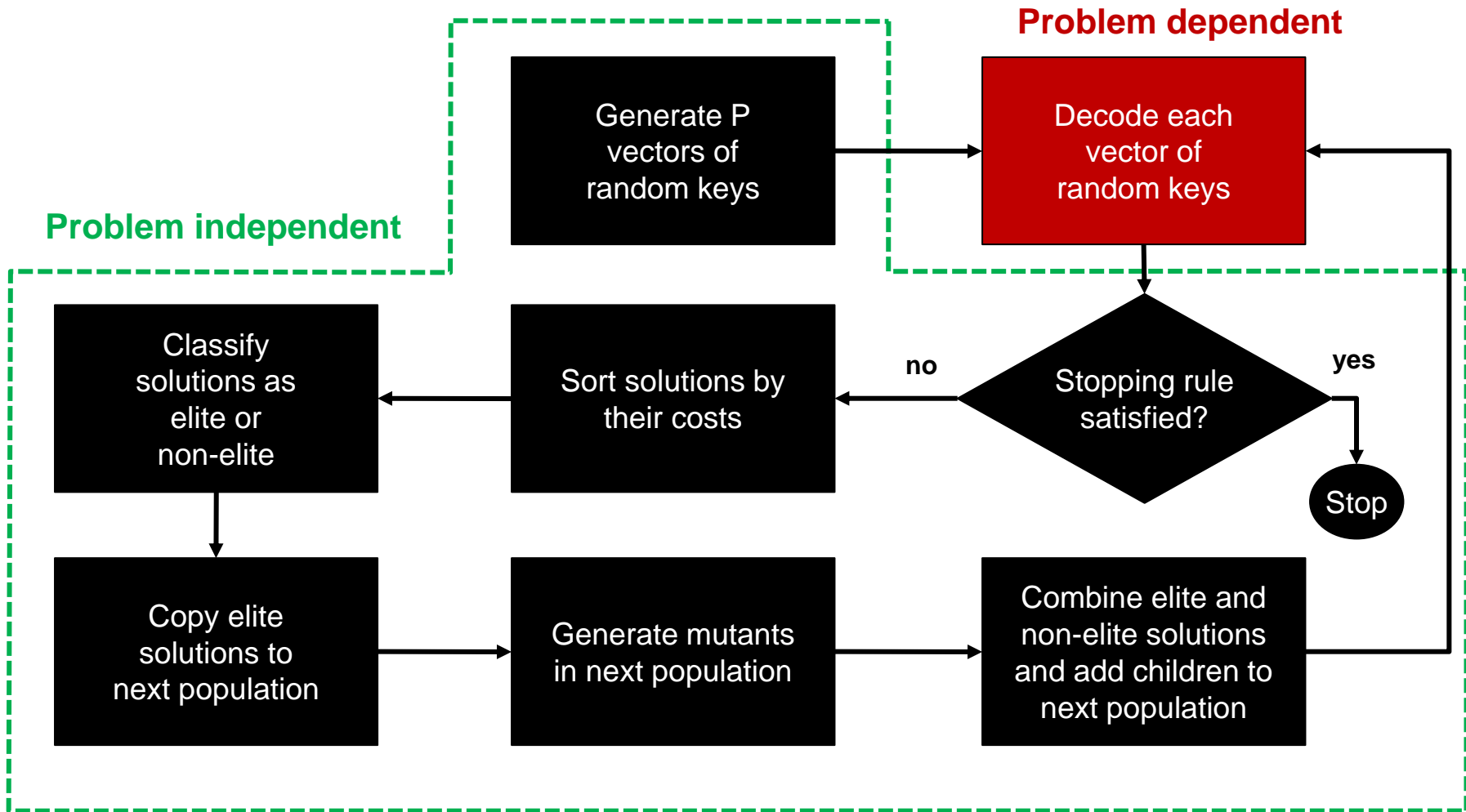
Weighted Distances

	1	2	3	4	5
1	-	3	8.4	4.2	3.6
2		-	4.4	11.9	1.2
3			-	4.5	4.2
4				-	5.85
5					-

Example: TSP



Framework for BRKGA algorithms



Specifying a BRKGA algorithm

- Encoding is always done the same way, i.e. with a vector of n random-keys (parameter n must be specified)
- Decoder that takes as input a vector of N random-keys and outputs the corresponding solution of the combinatorial optimization problem and its cost (this is usually a heuristic)
- Parameters:
 - Size of population: a function of N , say N or $2N$
 - Size of elite partition: 15-25% of population
 - Size of mutant set: 5-15% of population
 - Child inheritance probability: > 0.5 , say 0.7
 - Stopping criterion: e.g. time, # generations, solution quality, # generations without improvement

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