Algorithmic Methods for Mathematical Models (AMMM)

Intro to Heuristics Methods

Luis Velasco

(lvelasco @ ac.upc.edu)

Campus Nord D6-107

Mathematical programming typology

- Linear Programming (LP)

$$
\begin{array}{ll}
\min & c^T x \\
s.t. & Ax = b \\
& x \geq 0 \quad x \in \Re^n
\end{array}
$$

- Integer Linear Programming (ILP)

$$
\begin{array}{ll}
\min & c^T x \\
s.t. & Ax = b \\
& x \geq 0 \quad x \in Z^n
\end{array}
$$

- Mixed Integer Linear Programming (MILP)

$$
\begin{array}{ll}
\min & c^T x + d^T y \\
s.t. & Ax + By = b \\
& x \geq 0 \quad x \in Z^n \\
& y \geq 0 \quad y \in \Re^n
\end{array}
$$

- Nonlinear Programming (NLP)

$$
\begin{array}{ll}
\min & f(x) \\
s.t. & g_i(x) \leq b_i \quad \forall i \\
& x \geq 0
\end{array}
$$

Time to solve

$$\begin{array}{lll} \min & c^T x & \\ s.t. & Ax = b & \\ & x \geq 0 & x \in Z^n \end{array}$$

$A_{[n \times m]}$

Size = #vars × #constraints

Example

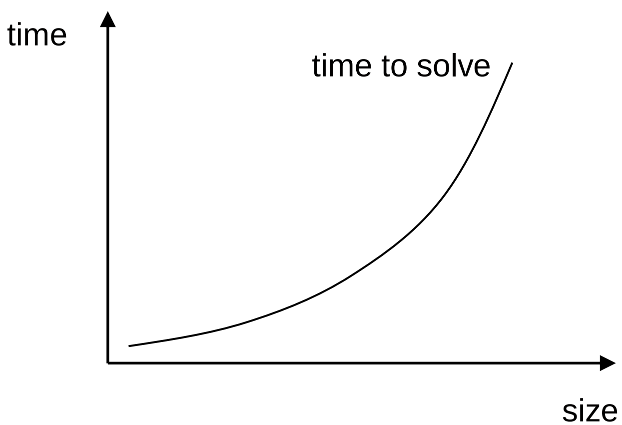
$$\text{minimize} \quad z$$

subject to:

$$\sum_{c \in C} x_{tc} = 1 \quad \forall t \in T$$

$$\sum_{t \in T} r_t \cdot x_{tc} \leq r_c \quad \forall c \in C$$

$$z \geq \frac{1}{r_c} \cdot \sum_{t \in T} r_t \cdot x_{tc} \quad \forall c \in C$$



What happens when

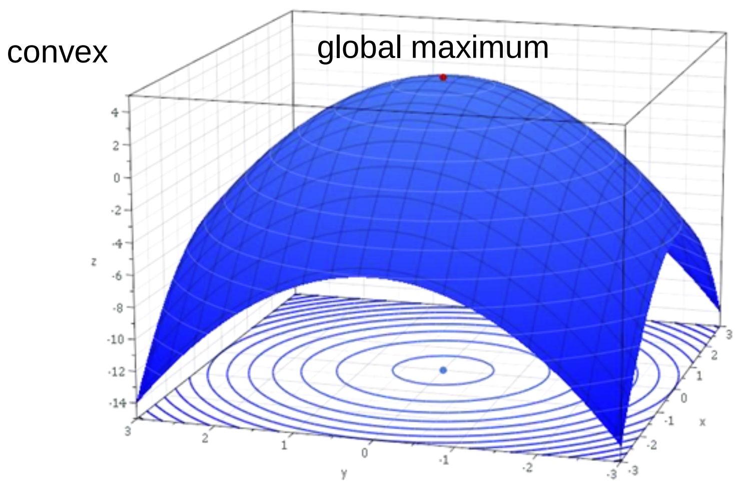
time to solve >>> time a solution is needed

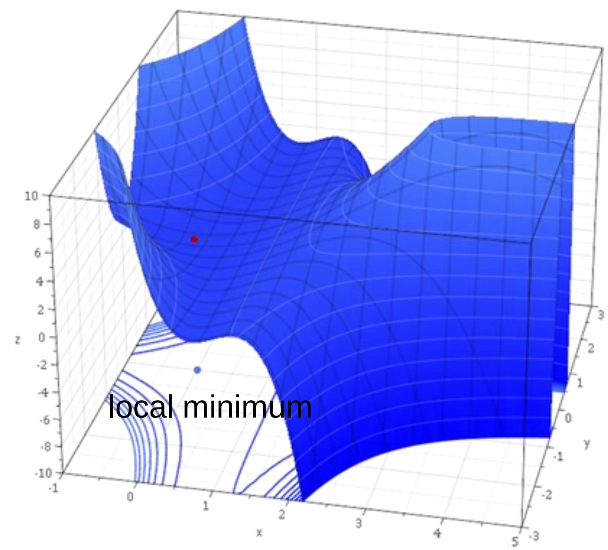
#constraints
$|C| + |C| + |T|$
$= O(|T| + |C|)$

vars
$\underbrace{|T| \times |C|}_{x_{Tc}} + \underbrace{1}_{z}$

How to find maxima and minima of a function

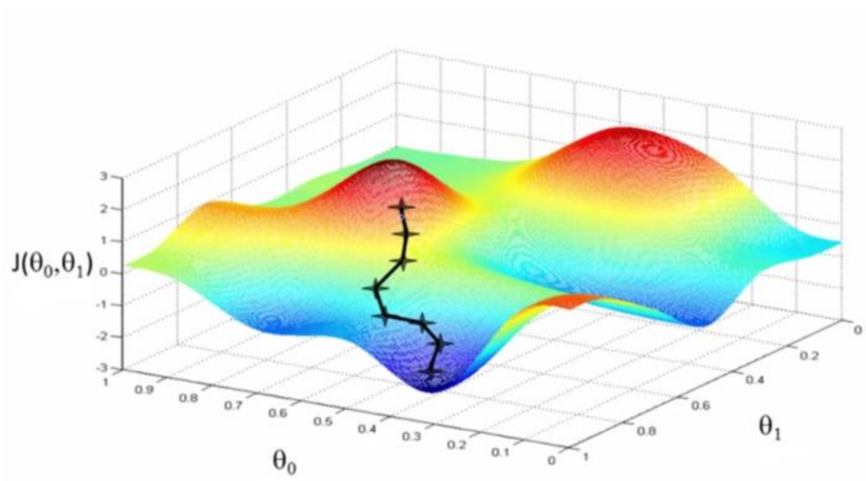
- Local extrema of **differentiable functions** can be found by Fermat's theorem, which states that they must occur at *critical points*.
- One can distinguish whether a critical point is a local maximum or local minimum by using the **first and second derivative tests**.
  - The **first derivative** finds the critical points.
  - The **second derivative** test uses the value of the second derivative at the critical points to determine whether they are a **local** maximum or minimum.

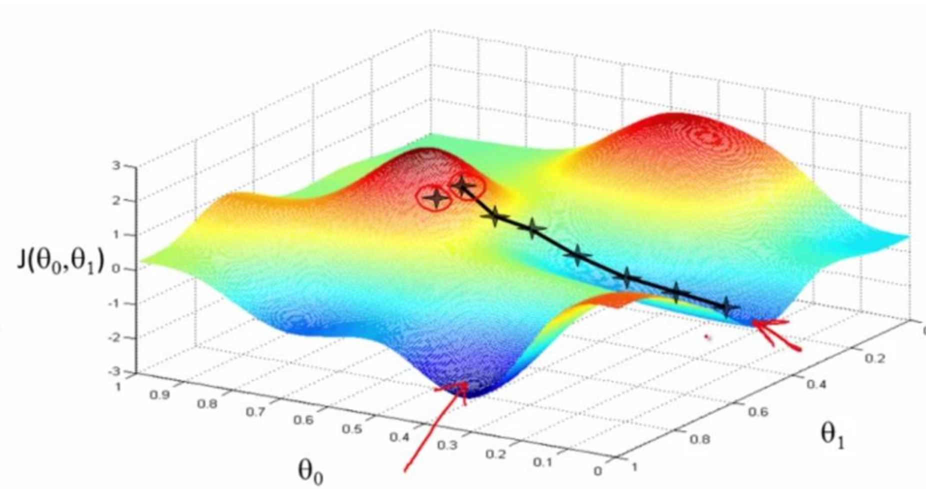




Gradient Descent

- A first-order iterative optimization algorithm for finding a local minimum of a differentiable function.
- We take steps proportional to the negative of the gradient (or approximate gradient) of the function at the current point.

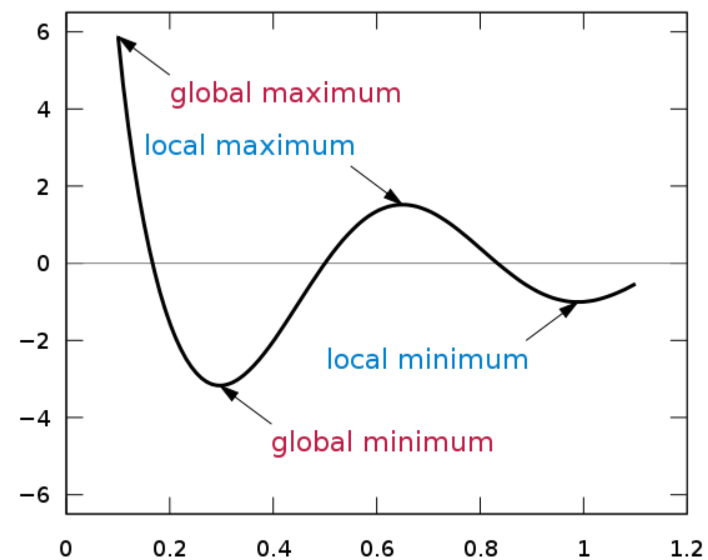




Can be susceptible to **local minima.**

Heuristics

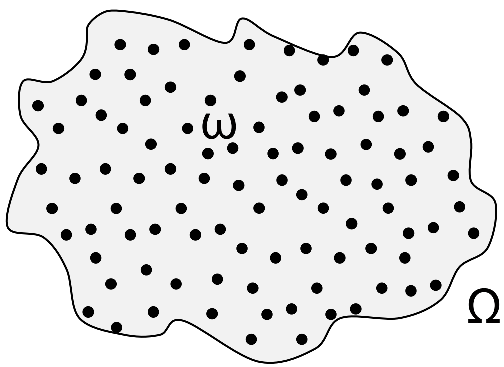
- Approximate solution techniques that have been used since the beginnings of operations research to tackle difficult **combinatorial problems**. ← ILP
- With the development of complexity theory in the early 70's, it became clear that, since most of these problems were indeed NP-hard,
  - there was **little hope** of ever finding efficient exact solution procedures for them.
- This emphasized the role of heuristics for solving the problems that were encountered in **real-life applications** and that needed to be tackled, whether or not they were NP-hard.
- Heuristics usually consists of two phases:
  - **Constructive Phase**, where a solution is built.
    - **Greedy algorithms** or any other method based on the **problem structure** can be used.
  - **Local search**, where the solution is improved.



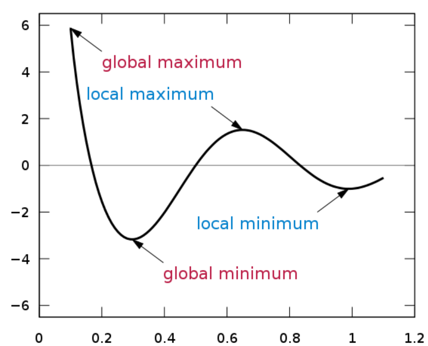
combinatorial problems !

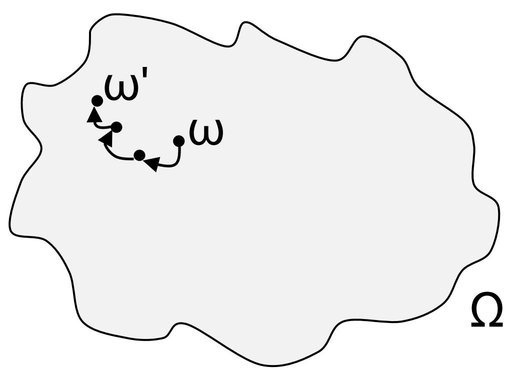
Heuristics

Ω	Solution space
$f: \Omega \rightarrow \mathbb{R}$	Objective function defined on the solution space
goal:	find $\omega^* \in \Omega$, $f(\omega) \geq f(\omega^*)\ \forall \omega \in \Omega$



$\omega^* \in \Omega$, $f(\omega) \geq f(\omega^*)\ \forall \omega \in \Omega$





ω' = is a local minimum

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