Algorithmic Methods for Mathematical Models (AMMM)

Linear Programming Solving Exercises (I).

1. Consider the following linear program:

$$
\begin{aligned}
&\min\ -2x_2 + x_3 \\
&x_2 + 3x_3 \le 3 \\
&2x_1 - x_2 \le 1 \\
&x_1 + x_3 \le 4 \\
&x_1, x_2, x_3, \ge 0
\end{aligned}
$$

   Define the vectors x, c, b and the matrix A so that the previous linear program can be written in matrix notation as

$$
\begin{aligned}
&\min c^T x \\
&Ax \le b \\
&x \ge 0
\end{aligned}
$$

2. Does the following optimization problem have an optimal solution? (notice that one of the inequalities is *strict*). Give a geometric argument.

$$
\begin{aligned}
&\max x + 2y \\
&x + y < 3 \\
&x \le 2 \\
&y \le 2 \\
&x, y \ge 0
\end{aligned}
$$

3. Given a set S and a function $f : S \to \mathbb{R}$, prove that

$$\max\{\ f(x) \mid x \in S\ \} \quad = \quad -\min\{\ -f(x) \mid x \in S\ \}$$

   Assume that both $\max\{\ f(x) \mid x \in S\ \}$ and $\min\{\ -f(x) \mid x \in S\ \}$ exist.

   Note. This property makes the problems of solving minimization LP's and of solving maximization LP's equivalent.

4. Transform the following linear program into canonical form:

$$
\begin{aligned}
&\min\ -2x_2 + x_3 \\
&3x_2 - x_3 \le 3 \\
&x_1 + x_2 = 2 \\
&2x_1 - x_3 \ge 1
\end{aligned}
$$

5. Consider the following linear program:

$$\begin{aligned}
&\min -x-2y\\
&x+y+s_1=3\\
&x+y-s_2=1\\
&x+s_3=2\\
&y+s_4=2\\
&x,y,s_1,s_2,s_3,s_4\geq 0
\end{aligned}$$

Identify a basis that is feasible and another one which is not.

6. An *integer linear program* is an optimization problem of the form:

$$\begin{aligned}
&\min c^T x\\
&Ax=b\\
&x\geq 0\\
&x\in\mathbb{Z}^n
\end{aligned}$$

Could we extend the Fundamental Theorem of Linear Programming to this kind of optimization problems? Justify your answer.

7. Suppose that we consider optimization problems of the form:

$$\begin{aligned}
&\min f(x)\\
&Ax=b\\
&x\geq 0\\
&x\in\mathbb{R}^n
\end{aligned}$$

where f is a quadratic polynomial. Could we extend the Fundamental Theorem of Linear Programming to this kind of optimization problems? Justify your answer.