Problem 3 - Heuristic Methods

A large company that produces huge amounts of data does not have the necessary resources to store monthly backups and decides to hire external backup providers. The company has O different offices and we know, for each office o, which is the amount of data b_o, in PetaBytes (10^6 GB), that it needs to store.

After studying several possibilities, a number of backup providers D are chosen. Using a backup provider d has a fixed cost f_d and an additional cost s_d for each PB it stores. Each backup provider d can store at most k_d PBs of data.

Each office can use just one backup providers, and each backup providers can store data store data from no more than 3 different offices. In addition, the total number of backup providers want to be kept limited to no more than 3.

The goal is to find out which backup centers will be used to serve all the demand of data and how many PBs of each office every will store backup provider, so that the total cost is minimized.

The data is as follows:

```
O = 1..7  // Offices
D = 1..5  // backup providers
b = [200 200 200 200 200 200 200] // Demand per office (in PB)
k = [1500 500 700 1000 1000]   // Capacity per backup provider (in PB)
f = [300 350 250 800 600]      // Fixed cost
s = [1.00 0.15 0.65 0.45 0.50] // Cost per PB
```

Because of the complexity of the optimization problem, we want to develop a heuristic algorithm. We are considering two options among the available in the literature, Greedy and GRASP.

(a) Specify the algorithm for the Greedy and GRASP *constructive phase*, including the candidates, the greedy function $q(\cdot)$, and the equation describing the RCL. Specify $q(\cdot)$ using mathematical notation and a short descriptive text.

Let S be a partial solution, $X(d|S)$ be 1 if provider d has been selected in S; 0 otherwise, $Y(d|S)$ the amount of already allocated capacity in d given S, and $Z(d|S)$ is the times that d has been selected given S .

$$q(o,d|S) = \big(1 - X(d|S)\big) * f_d + b_o * s_d$$

$$q(o|S) = min\{q(o,d|S) \quad \forall d \in D, \ \ k_d - Y(d|S) > b_o \ AND \ Z(d|S) < 3\}$$

C: Candidate set ⊆ O

RCL = $\{c, q(c) \leq q_{min} + \alpha(q_{max}\text{-}q_{min})\}$

(b) Let us assume that the algorithm specified in (a) for the Greedy is being executed. Complete the following table, where for each iteration, compute the value of the proposed greedy function $q(\cdot)$ for all the candidates.

The selection is done by iterating from left to right and selecting the assignment with the min greedy cost. (Selected assignment is in bold face).

	Office						
Iter #1	**1**	**2**	**3**	**4**	**5**	**6**	**7**
q(*o*)	**380**	380	380	380	380	380	380
d	**2**	2	2	2	2	2	2
	Office						
Iter #2	**1**	**2**	**3**	**4**	**5**	**6**	**7**
q(*o*)		**30**	30	30	30	30	30
d		**2**	2	2	2	2	2
	Office						
Iter #3	**1**	**2**	**3**	**4**	**5**	**6**	**7**
q(*o*)			**380**	380	380	380	380
d			**3**	3	3	3	3
	Office						
Iter #4	**1**	**2**	**3**	**4**	**5**	**6**	**7**
q(*o*)				**130**	130	130	130
d				**3**	3	3	3
	Office						
Iter #5	**1**	**2**	**3**	**4**	**5**	**6**	**7**
q(*o*)					**130**	130	130
d					**3**	3	3
	Office						
Iter #6	**1**	**2**	**3**	**4**	**5**	**6**	**7**
q(*o*)						**500**	500
d						**1**	1
	Office						
Iter #7	**1**	**2**	**3**	**4**	**5**	**6**	**7**
q(*o*)							**200**
d							**1**

Detail the obtained solution in the next table.

Backup Provider	Office						
	1	2	3	4	5	6	7
1						X	X
2	X	X					
3			X	X	X		
4							
5							

What is the total cost?

1,750 c.u.

Do you think this is solution is optimal? Why?

We cannot ensure that this is the optimal solution.

To ensure that the solution is optimal, we need the value of the linear relaxation of the ILP model.