Algorithmic Methods for Mathematical Models (AMMM)

Linear Programming Solving Exercises (II).

1. Solve the following linear program using the simplex method:

$$
\begin{array}{rlrlrlrlrlcl}
\textbf{max} & 6x_1 &+& 14x_2 &+& 13x_3 & & & & & & \\
\textbf{subject to} & & & & & & & & & & & \\
& \frac{1}{2}x_1 &+& 2x_2 &+& x_3 &+& x_4 & & & =& 24 \\
& x_1 &+& 2x_2 &+& 4x_3 & & &+& x_5 &=& 60 \\
& x_1, & & x_2, & & x_3, & & x_4 & & x_5 &\geq& 0
\end{array}
$$

Solution: Assignment $x_1 = 36$, $x_3 = 6$, $x_2 = x_4 = x_5 = 0$ gives the optimal objective value 294

2. Solve the following linear program using the simplex method:

$$
\begin{array}{rlrlrlcr}
\textbf{min} & -3x_1 &-& 2x_2 &+& 5x_3 & & \\
\textbf{subject to} & & & & & & & \\
& 4x_1 &-& 2x_2 &+& 2x_3 &\leq& 4 \\
& -2x_1 &+& x_2 &-& x_3 &\geq& -1 \\
& x_1, & & x_2, & & x_3, &\geq& 0
\end{array}
$$

Solution: the linear program is unbounded

3. Solve the following linear program using the simplex method:

$$
\begin{array}{rlrlcr}
\textbf{min} & -4x_1 &-& 6x_2 & & \\
\textbf{subject to} & & & & & \\
& -x_1 &+& x_2 &\leq& 11 \\
& x_1 &+& x_2 &\leq& 27 \\
& 2x_1 &+& 5x_2 &\leq& 90 \\
& x_1, & & x_2 &\geq& 0
\end{array}
$$

Solution: Assignment $x_1 = 15$, $x_2 = 12$ gives the optimal objective value -132

4. Solve the following linear program using the simplex method:

$$
\begin{array}{rlrlrlcr}
\textbf{min} & -2x_1 &+& x_2 &-& 2x_3 & & \\
\textbf{subject to} & & & & & & & \\
& 2x_1 &+& x_2 & & &\leq& 10 \\
& x_1 &+& 2x_2 & & &\leq& 20 \\
& & & x_2 &+& 2x_3 &\leq& 5 \\
& x_1, & & x_2, & & x_3, &\geq& 0
\end{array}
$$

Solution: Assignment $x_1 = 5$, $x_2 = 0$, $x_3 = 2.5$ gives the optimal objective value -15

5. Solve the following linear program using the simplex method:

$$\begin{aligned}
\min \quad & -3x_1 \;-\; 2x_2 \;-\; x_3 \\
\text{subject to} \quad & \\
& 4x_1 \;+\; x_2 \;+\; x_3 \;=\; 30 \\
& 2x_1 \;+\; 3x_2 \;+\; x_3 \;\le\; 60 \\
& x_1 \;+\; 2x_2 \;+\; 3x_3 \;\le\; 40 \\
& x_1, \quad x_2, \quad x_3, \quad \ge \; 0
\end{aligned}$$

Solution: Assignment $x_1 = 3$, $x_2 = 18$, $x_3 = 0$ gives the optimal objective value -45

6. A manufacturer produces three types of plastic chairs. The profits and the times required for molding, trimming and packaging are given in the following table (times are given in hours per dozen chairs):

Process	Type A	Type B	Type C	Total time available
Molding	1	2	$\frac{3}{2}$	12000
Trimming	$\frac{2}{3}$	$\frac{2}{3}$	1	4600
Packaging	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	2400
Profit	11 €	16 €	15 €	—

How many dozen of each type of chair should be produced to obtain a maximum profit?

Solution: We should produce 600 dozen of type A, 5100 dozen of type A and 800 dozen of type C, which gives a total profit of 100200 €.

7. The advertising alternatives for a company include television, radio, and newspaper advertisements. The costs and estimates for audience coverage are given in the following table:

	Television	Newspaper	Radio
Cost per advertisement	2000 €	600 €	300 €
Audience per advertisement	100000	40000	18000

The local newspaper limits the number of weekly advertisements from a single company to ten. Moreover, in order to balance the advertising among the three types of media, no more than half of the total number of advertisements should occur on the radio, and at least 10 % should occur on television. The weekly advertising budget is 18200 €. How many advertisements should be run in each of the three types of media to maximize the total audience?

Solution: We should run 4 advertisements in television, 10 in the newspaper, and 14 in the radio, which gives a total audience coverage of 1052000.