

**Exercise 1: Simplex Method (2.5 pts).**

**Solution:**

Answer to Q1:

	x1	x2	x3	x4	x5	
x2	0.50	1.00	-1.50	0.50	0.00	2.50
x5	1.50	0.00	-2.50	-0.50	1.00	4.50
obj	-4.25	0.00	-0.25	-0.25	0.00	-1.25

The optimal solution is

$$\begin{aligned}x_1 &= 0, \\x_2 &= 2.5, \\x_3 &= 0, \\x_4 &= 0, \\x_5 &= 4.5.\end{aligned}$$

Answer to Q2:

	x1	x2	x3	x4	x5	
x2	0.50	1.00	-1.50	0.50	0.00	2.50
x5	1.50	0.00	-2.50	-0.50	1.00	4.50
obj	-5.50	0.00	0.50	-0.50	0.00	-2.50

The infinite family of feasible solutions with unbounded objective value is given by

$$\begin{aligned}x_1 &= 0.0t + 0.0 \\x_2 &= 1.5t + 2.5 \\x_3 &= 1.0t + 0.0 \\x_4 &= 0.0t + 0.0 \\x_5 &= 2.5t + 4.5.\end{aligned}$$

Answer to Q3:

$$\alpha = 11/3 = 3.66\dots$$

Justification: If we repeat the calculations with the generic initial last row  $[3 - 2\alpha, \alpha - 3, -1, 0, 0, 0]$  we see that, in the interval  $[3.5, 4.0]$ , the only positive entry is the second. After pivoting on it, the last row becomes  $[-2.5\alpha + 4.5, 0, 1.5\alpha - 5.5, -0.5\alpha + 1.5, 0, -2.5\alpha + 7.5]$ . In the interval  $[3.5, 4.0]$ , all these quantities are negative except  $1.5\alpha - 5.5$ , which is positive if and only if  $\alpha > 5.5/1.5 = 11/3$ .