

Given:

- The set L of possible locations. Each location l consists of a coordinate (l_x, l_y) .
- The set C of cities. Each city c consists of a coordinate (c_x, c_y) and a population p_c .
- The set T of types. Each type t consist of a capacity cap_t , a working distance d_{city_t} and a installation cost $cost_t$
- The minimum distance between centres d_{centre} .

Find the locations to instal the logistic centres, the type for each centre and the assignment of primary and secondary centre to each city, subject to the following constraints:

- Each city has assigned exactly one primary centre.
- Each city has assigned exactly one secondary centre.
- For each city, its primary centre must be different of its secondary centre.
- The distance between each pair of centres must be at least the minimum.
- The distance between a city and its primary centre cannot exceed the working distance of that centre's type.
- The distance between a city and its secondary centre cannot exceed three times the working distance of that centre's type.
- For each centre, the capacity of the centre's type cannot be exceeded by the sum of the populations of the cities it serves as a primary centre and the 10% of the populations of the cities it serves as a secondary centre.

With *objective* to minimize the total installation cost.

L	Set of possible locations, index l .
C	Ser of cities, index c .
T	Set of types, index t .
d_centre	Minimum distance between centres.
cap_t	capacity of the type t .
d_city_t	Working distance of the type t .
$cost_t$	Installation cost of the type t .
p_c	Population of the city c .
p_{cl}	binary. Equals to 1 if a centre in location l serve as primary centre the city c ; 0 otherwise.
s_{cl}	binary. Equals to 1 if a centre in location l serve as secondary centre the city c ; 0 otherwise.
x_{lt}	binary. Equals to 1 if a centre of type t is emplaced in the location l ; 0 otherwise.

Minimize

$$\sum_{l \in L} \sum_{t \in T} x_{lt} \cdot cost_t$$

Subject to:

$$\sum_{l \in L} p_{cl} = 1 \quad \forall c \in C$$

$$\sum_{l \in L} s_{cl} = 1 \quad \forall c \in C$$

$$p_{cl} \cdot s_{cl} = 0 \quad \forall c \in C, l \in L$$

$$\sqrt{(l_{1x} - l_{2x})^2 + (l_{1y} - l_{2y})^2} \geq x_{l_1 t_1} \cdot x_{l_2 t_2} \cdot d_centre$$

$$\forall l_1, l_2 \in L, t_1, t_2 \in T \quad s.t. \quad l_1 \neq l_2 \text{ or } t_1 \neq t_2$$

$$p_{cl} \cdot \sqrt{(c_x - l_x)^2 + (c_y - l_y)^2} \leq \sum_{t \in T} x_{lt} \cdot d_city_t \quad \forall c \in C, l \in L$$

$$s_{cl} \cdot \sqrt{(c_x - l_x)^2 + (c_y - l_y)^2} \leq 3 \cdot \sum_{t \in T} x_{lt} \cdot d_city_t \quad \forall c \in C, l \in L$$

$$\sum_{c \in C} p_{cl} \cdot p_c + 0.1 \cdot \sum_c s_{cl} \cdot p_c \leq \sum_{t \in T} x_{lt} \cdot cap_t \quad \forall l \in L$$