Algorithmic Methods for Mathematical Models

– COURSE PROJECT –

An Internet retail company wants to build several logistic centers in order to operate in a new country. Its goal is to spend the minimum amount of money, but making sure that customers receive their products quickly enough.

The company has a set of locations $\mathcal{L}$ where logistic centers could be installed, and a set of cities $\mathcal{C}$ that need to be served. For each location $\ell \in \mathcal{L}$ we know its coordinates (ℓ_x, ℓ_y), and for every city $c \in \mathcal{C}$ we know its coordinates (c_x, c_y) and its population p_c. We have available a set $\mathcal{T}$ of logistic center types. Each type $t \in \mathcal{T}$ represents a logistic center with capacity cap_t, working distance d_city_t, and installation cost $cost_t$.

Each city must be served by exactly one primary and one secondary center, which of course must be different. Logistic centers should be placed so that the distance[1] between any two of them is at least d_center. The capacity of a center of type t requires that the sum of the populations of the cities it serves as a primary center plus 10% the sum of the populations of the cities it serves as a secondary center cannot exceed cap_t. With respect to its working distance, a center of type t cannot be the primary center of any city at distance more than d_city_t, or the secondary center of any city at distance more than $3 * d\_city_t$.

The goal of this project is to decide where to install the logistic centers, determine of which type each center should be and to which primary and secondary center each city should be connected to in order to minimize the total installation cost.

1. **Work to be done:**

   (a) State the problem formally. Specify the given sets and parameters, the outputs and the objective.

   (b) Devise an integer linear programming model for the problem and implement it in OPL.

   (c) Because of the complexity of the problem, heuristic algorithms are applied. Here we will consider the following:

      i. a greedy constructive algorithm
      ii. a greedy constructive + a local search procedure
      iii. GRASP as a meta-heuristic algorithm.

      Design the three algorithms and implement them in the programming language you prefer.

   (d) Compare the performance of solving the model with CPLEX and applying the heuristic algorithms, both in terms of computation time and quality of the solutions as a function of the size of the instances. To that end, generate increasingly larger problem instances where solving each instance with CPLEX takes from 1 to 30 min.

   (e) Prepare a report and a presentation of your work on the project.

[1] Here we mean Euclidean distance. You can use functions `Math.sqrt(x)` and `Math.pow(x,y)` of OPL that compute $\sqrt{x}$ and x^y, respectively.

2. **Report:**

   Prepare a report (8-10 pages) in PDF format including:

   - Problem statement.
   - The integer linear programming model, with a definition and a short description of the variables, the objective function and the constraints. Do not include OPL code, but rather their mathematical formulation.
   - For the meta-heuristics, the pseudo-code of your constructive, local search, and GRASP algorithms, including equations for describing the greedy cost function(s) and the RCL.
   - Comparative results.
   - Together with the report, you should also give all sources and instructions on how to use them, so that results can be easily reproduced. If you implemented an instance generator, please provide it as well.

3. **Presentation:**

   You are expected to make an online presentation of your work (7-10 minutes long) at the end of the course. We will use the Google Meet room of the subject to that end:

   `https://meet.google.com/iwh-euho-taq`

   The slots of Wednesday 09/12/20 and 16/12/20 will we devoted to these presentations. The schedule will be announced in its due time.

   The slides of the presentation in PDF format should be delivered together with the report by **Tuesday 08/12/20**.

   The idea is that the presentation can contain figures, plots, equations and algorithms, etc. with a very short text that basically helps to understand them. It is expected that you give full explanation of those contents during your presentation. On the other hand, the report should contain that explanation in a well organized manner as a text.