

1)

a	b	c	d	f	Ud
0	0	0	0	1	m0
0	0	0	1	x	m1
0	0	1	0	1	m2
0	0	1	1	1	m3
0	1	0	0	x	m4
0	1	0	1	1	m5
0	1	1	0	1	m6
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0	1	m10
1	0	1	1	1	m11
1	1	0	0		
1	1	0	1	x	m13
1	1	1	0	1	m14
1	1	1	1		

(m0)(m1)	0 0 0 -
(m0)(m4)	0 - 0 0
(m1)m3	0 0 - 1
(m1)m5	0 - 0 1
m2 m3	0 0 1 -
m2 m6	0 - 1 0
m3 m11	- 0 1 1
(m4)m5	0 1 0 -
(m4)m6	0 1 - 0
m5 (m13)	- 1 0 1 *
m6 m14	- 1 1 0
m10 m11	1 0 1 -
m10 m14	1 - 1 0

(m0)(m1) m2 m3	0 0 - -	$\bar{a}\bar{b}$
(m0)(m1)(m4) m5	0 - 0 -	$\bar{a}\bar{c}$
(m0)(m4) m2 m6	0 - - 0	$\bar{a}\bar{d}$
m2 m3 m10 m11	- 0 1 -	$\bar{b}c$
m2 m6 m10 m14	- - 1 0	$c\bar{d}$
m5 (m13)	- 1 0 1	$b\bar{c}$



	m2	m3	m5	m6	m10	m11	m14
$\bar{a}\bar{b}$	✓	✓					
$\bar{a}\bar{c}$			✓				
$\bar{a}\bar{d}$	✓			✓			
$\bar{b}c$	✓	✓			✓	✓	
$c\bar{d}$	✓			✓	✓		✓
$b\bar{c}\bar{d}$			✓				

↑ ↑  
Essential prime

- Essential prime implicants:  $\bar{b}c, c\bar{d}$
- Cover that minimizes # literals:  $\bar{b}c\bar{d} + \bar{a}\bar{c}$
- Cover that minimizes # variables:  $\bar{b}c + c\bar{d} + b\bar{c}\bar{d}$

2)

$$\begin{cases} x = abd + bc + be + abg + ae \\ y = ade + cd + ce + ef + h \end{cases}$$

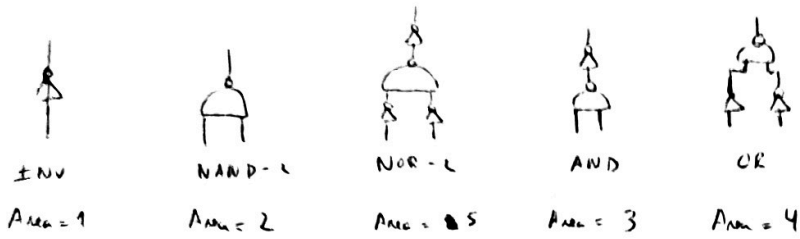
- $K(x) = \{(bd + bg + e), (ad + c + e + ag), (b + a), (abd + bc + be + abg + ae)\}$
- $\text{Cok}(x) = \{a, b, e, 1\}$
- $K(y) = \{(d + e), (ae + c), (ad + c + f), (ade + cd + ce + ef + h)\}$
- $\text{Cok}(y) = \{c, d, e, 1\}$

Best multiterm divisor =  $\{ad + c + e + ag\} \cap \{ad + c + f\} = \{ad + c\} = w$

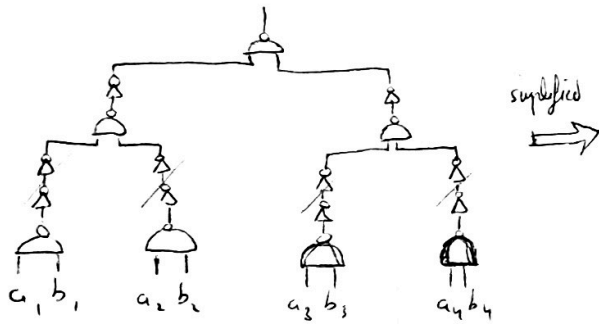
New node:  $w = ad + c$

$$\begin{cases} x = wb + be + abg + ae \\ y = w(e + d) + ce + ef + h \end{cases}$$

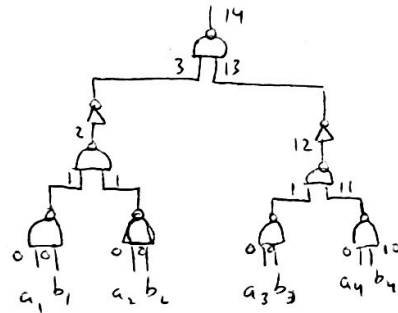
### 3) • Library patterns



$$f = a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4$$



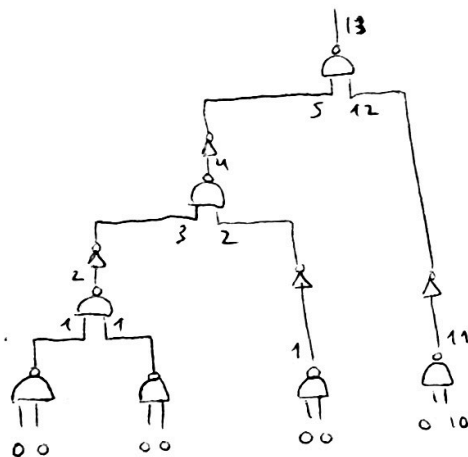
simplified  $\Rightarrow$



$$\text{Area} = 7 \text{ NAND-2} + 2 \text{ NOT} = 14 + 2 = 16 \text{ units}$$

$$\text{Delay} = 14 \text{ units of time}$$

• We can optimize in terms of delay with a new distribution

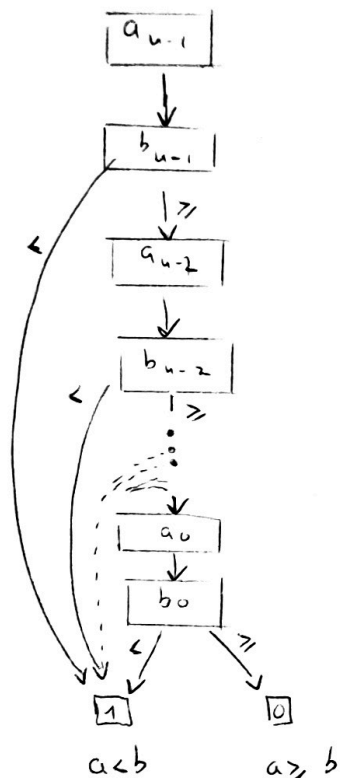


$$\text{Area} = 16 \text{ units}$$

$$\text{Delay} = 13 \text{ units}$$

4) • Best variable order to minimize BDD size?

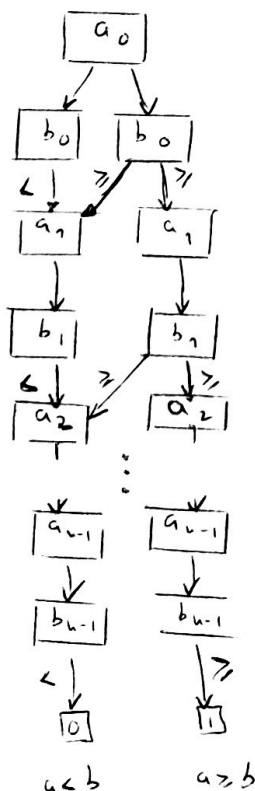
$$a_{u-1} < b_{u-1} < a_{u-2} < b_{u-2} < \dots < a_0 < b_0$$



- the size is  $O(2u)$  where  $u$  is the binary size of  $a, b$

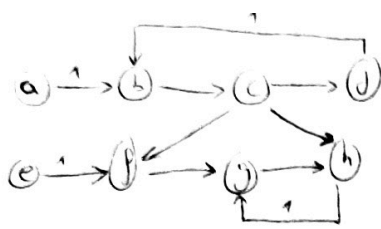
• Best variable order to maximize BDD size?

$$a_0 < b_0 < a_1 < b_1 < \dots < a_{u-1} < b_{u-1}$$



- the size is  $2(u-1) + 2u = O(4u-2)$  where  $u$  is the binary representation of  $a, b$ .

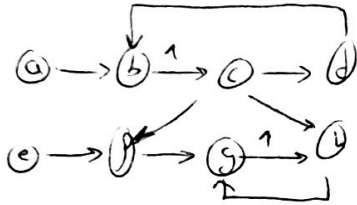
5)



• the minimum period is 5 and corresponds

to the critical path:  $b \rightarrow c \rightarrow f \rightarrow g \rightarrow h$

•  $P_{min}$  after retiming?



- the critical path after retiming is  $c \rightarrow d \rightarrow b$  with a period of 3.

• Is the min. number of registers  $R_{min}$  achievable after retiming?

- Each feedback loop has to be broken by at least one register and there are 2 loops in the combinational circuit.

The  $R_{min} = 2$  and it is achieved with the previous retimed sequential circuit.

• The previous retimed circuit achieves both  $P_{min}$  and  $R_{min}$ .