

CS612

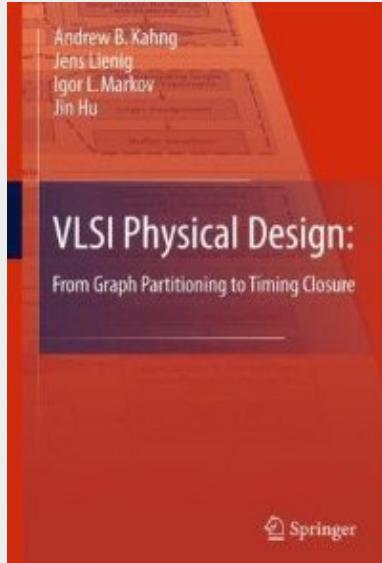
Algorithms for Electronic Design Automation

Placement

Mustafa Ozdal

MOST SLIDES ARE FROM THE BOOK:
VLSI Physical Design: From Graph Partitioning to Timing Closure
MODIFICATIONS WERE MADE ON THE ORIGINAL SLIDES

Chapter 2 – Netlist and System Partitioning



Original Authors:

Andrew B. Kahng, Jens Lienig, Igor L. Markov, Jin Hu

Chapter 4 – Global and Detailed Placement

4.1 Introduction

4.2 Optimization Objectives

4.3 Global Placement

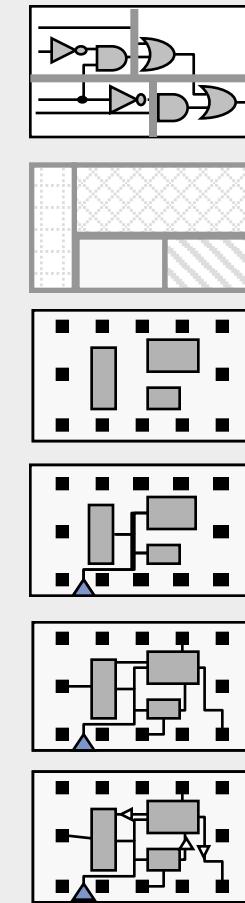
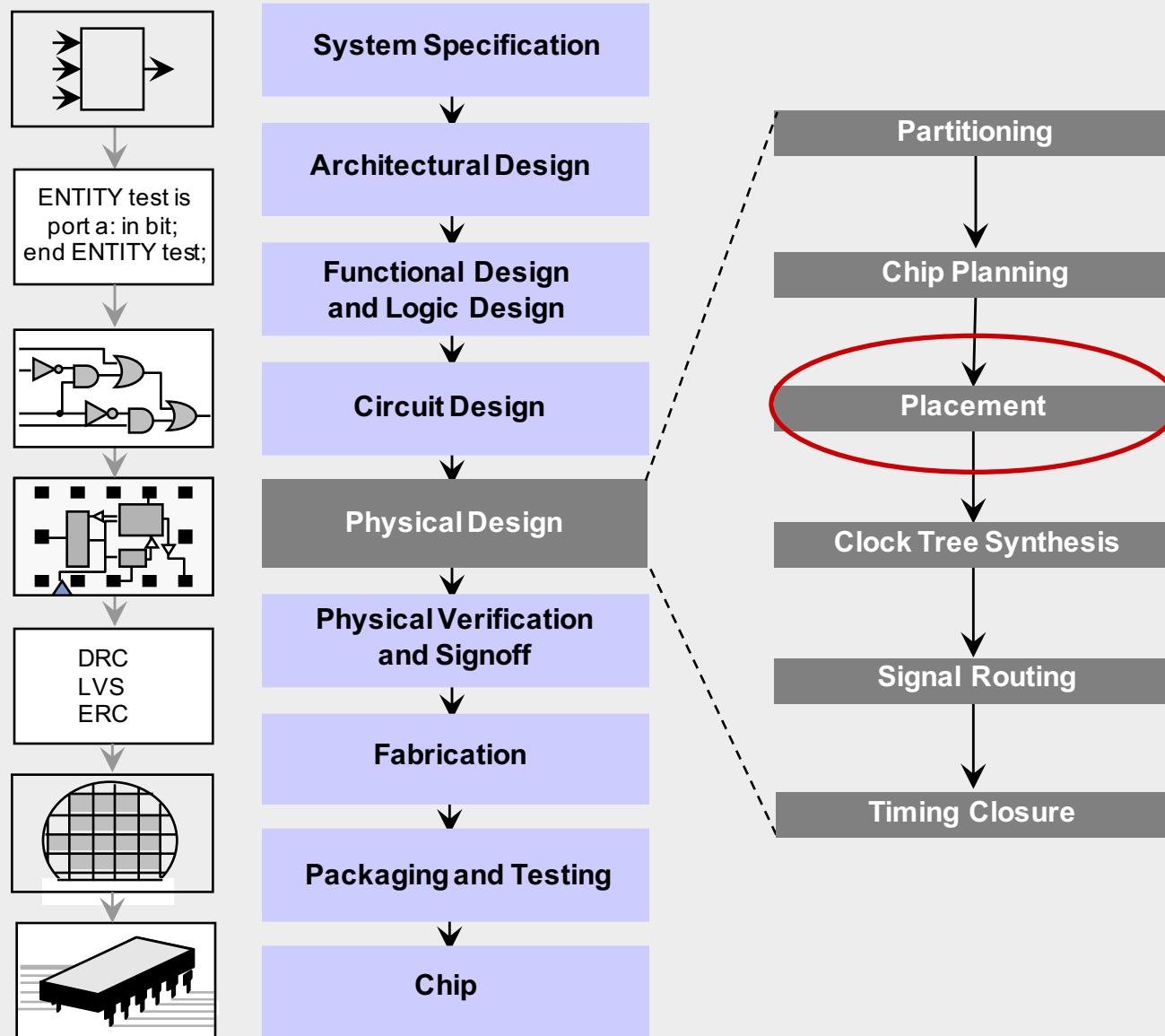
4.3.1 Min-Cut Placement

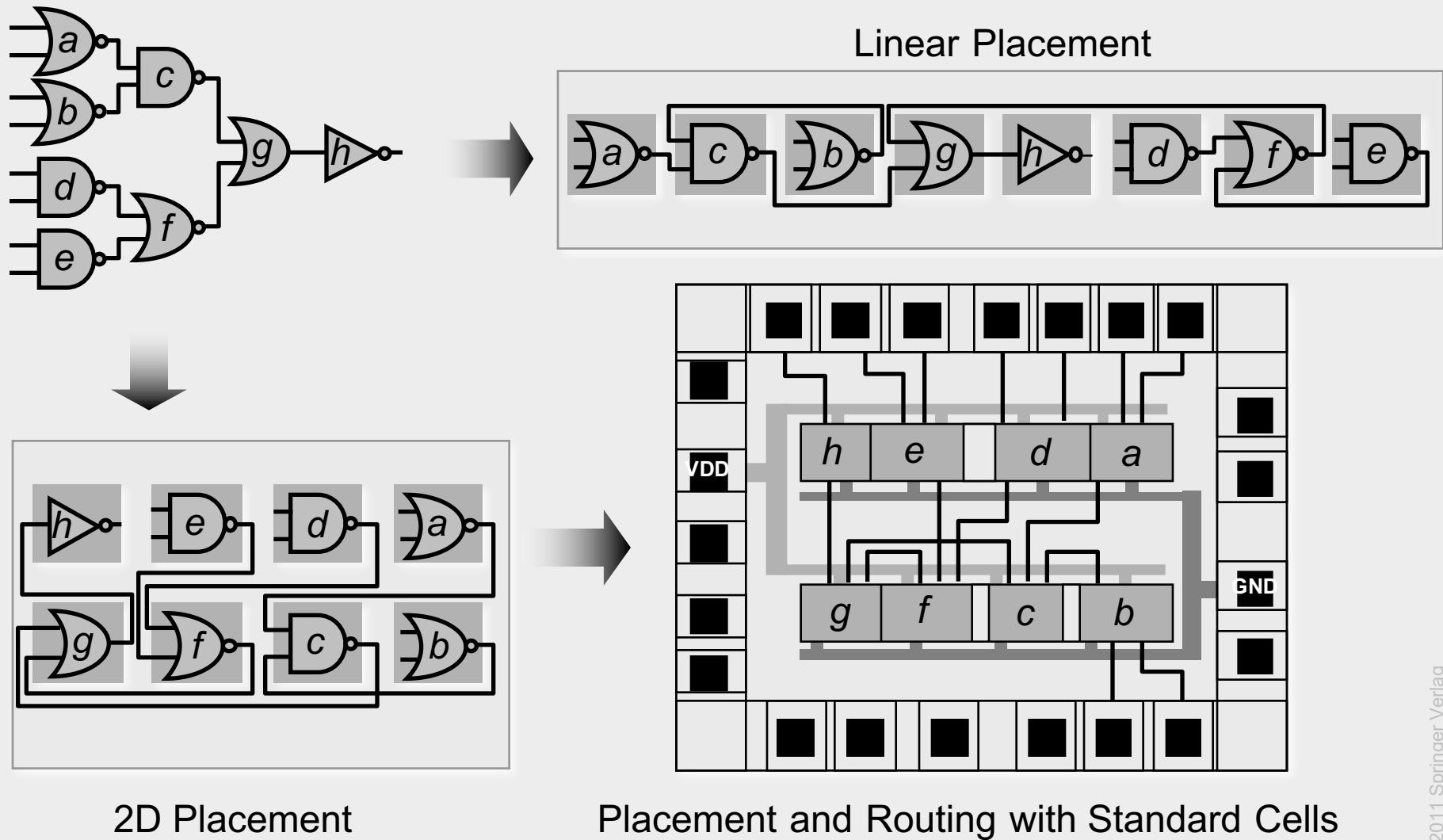
4.3.2 Analytic Placement

4.3.3 Simulated Annealing

4.3.4 Modern Placement Algorithms

4.4 Legalization and Detailed Placement

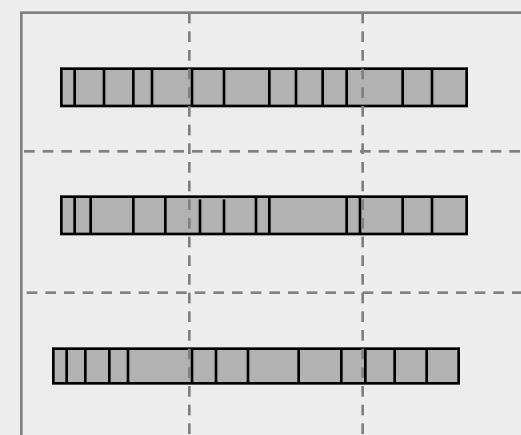




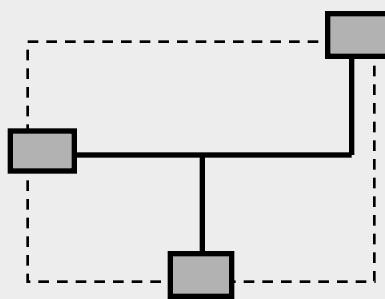
Global Placement



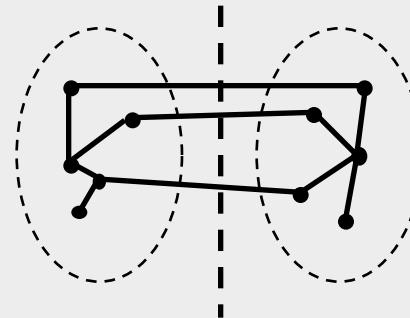
Detailed Placement



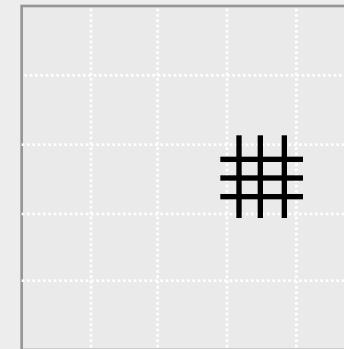
Total
Wirelength



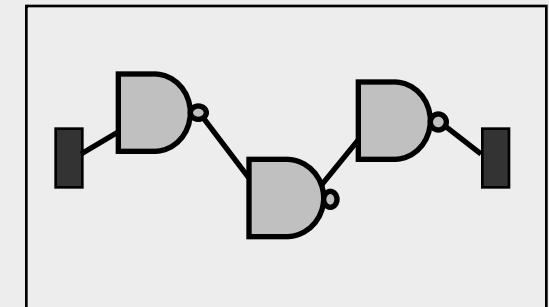
Number of
Cut Nets



Wire
Congestion



Signal
Delay



Floorplanning vs Placement

Floorplanning

Large blocks

Rectangles with arbitrary widths and heights

Rectangle packing

of blocks not very large

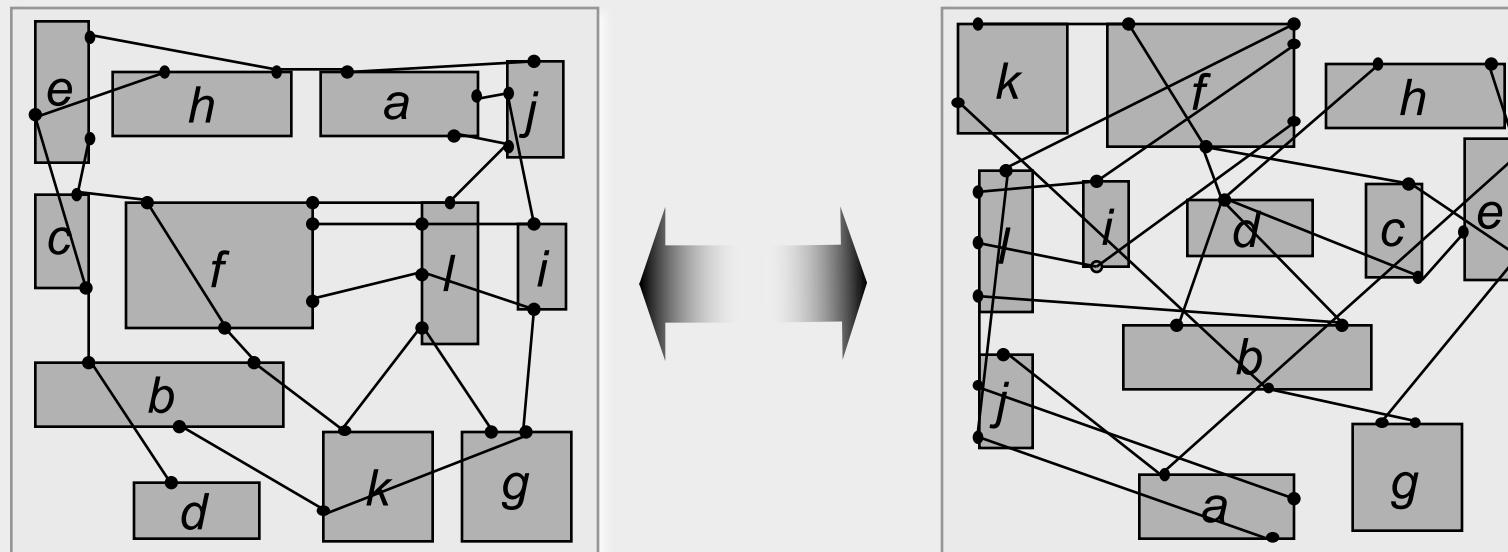
Placement

Much smaller cells

Cells with mostly identical heights

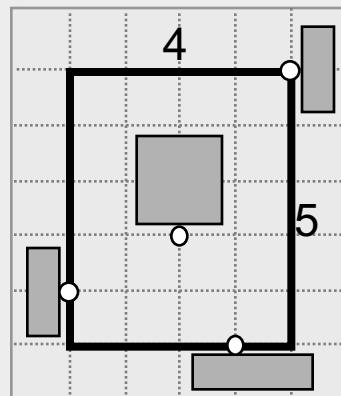
Placing cells on pre-defined rows

Up to a few million cells



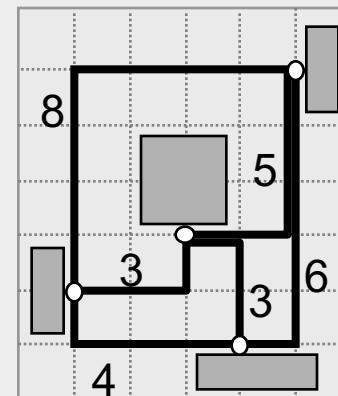
Wirelength estimation for a given placement

Half-perimeter
wirelength
(HPWL)



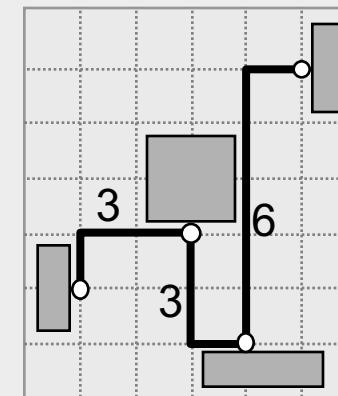
$$\text{HPWL} = 9$$

Complete
graph
(clique)



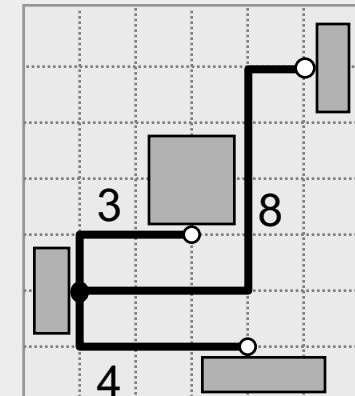
$$\text{Clique Length} = (2/p) \sum_{e \in \text{clique}} d_M(e) = 14.5$$

Monotone
chain



$$\text{Chain Length} = 12$$

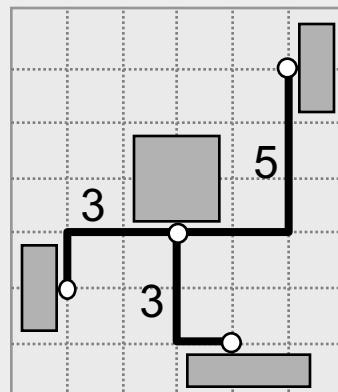
Star model



$$\text{Star Length} = 15$$

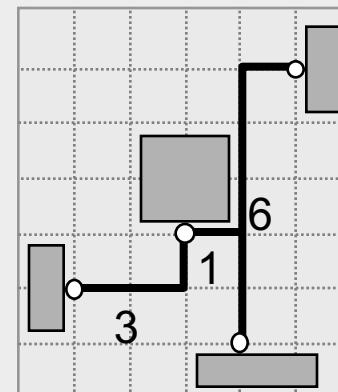
Wirelength estimation for a given placement (cont'd.)

Rectilinear
minimum
spanning
tree (RMST)



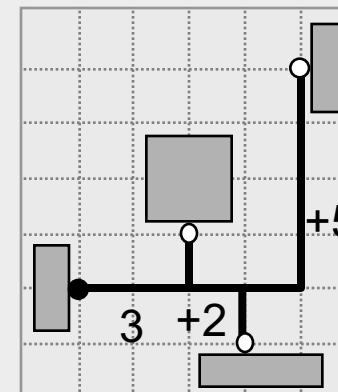
RMST Length = 11

Rectilinear
Steiner
minimum
tree (RSMT)



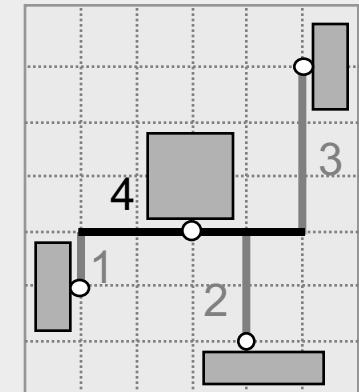
RSMT Length = 10

Rectilinear
Steiner
arborescence
model (RSA)



RSA Length = 10

Single-trunk
Steiner
tree (STST)

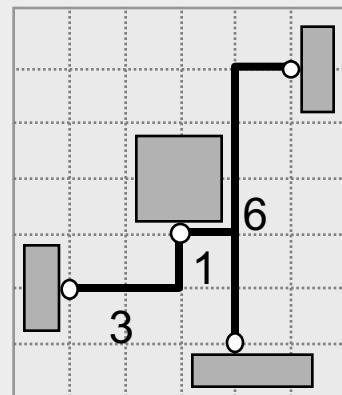


STST Length = 10

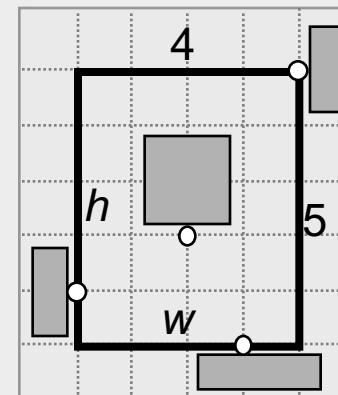
Wirelength estimation for a given placement (cont'd.)

Preferred method: Half-perimeter wirelength (HPWL)

- Fast (order of magnitude faster than RSMT)
- Equal to length of RSMT for 2- and 3-pin nets
- Margin of error for real circuits approx. 8% [Chu, ICCAD 04]



RSMT Length = 10



HPWL = 9

$$L_{\text{HPWL}} = w + h$$

Total wirelength with net weights (weighted wirelength)

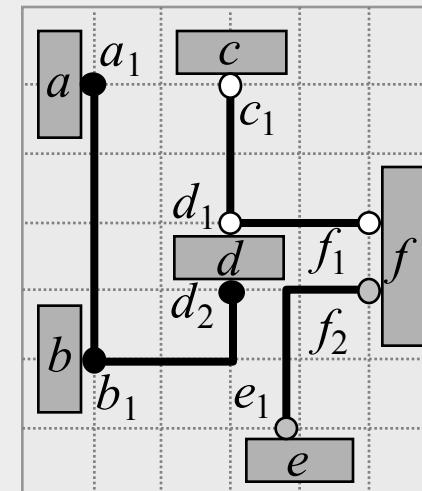
- For a placement P , an estimate of total weighted wirelength is

$$L(P) = \sum_{net \in P} w(net) \cdot L(net)$$

where $w(net)$ is the weight of net , and $L(net)$ is the estimated wirelength of net .

- Example:

Nets	Weights
$N_1 = (a_1, b_1, d_2)$	$w(N_1) = 2$
$N_2 = (c_1, d_1, f_1)$	$w(N_2) = 4$
$N_3 = (e_1, f_2)$	$w(N_3) = 1$



$$L(P) = \sum_{net \in P} w(net) \cdot L(net) = 2 \cdot 7 + 4 \cdot 4 + 1 \cdot 3 = 33$$

Cut sizes of a placement

- To improve total wirelength of a placement P , separately calculate the number of crossings of global vertical and horizontal cutlines, and minimize

$$L(P) = \sum_{v \in V_P} \psi_P(v) + \sum_{h \in H_P} \psi_P(h)$$

where $\Psi_P(cut)$ be the set of nets cut by a cutline cut

Cut sizes of a placement

- Example:

Nets

$$N_1 = (a_1, b_1, d_2)$$

$$N_2 = (c_1, d_1, f_1)$$

$$N_3 = (e_1, f_2)$$

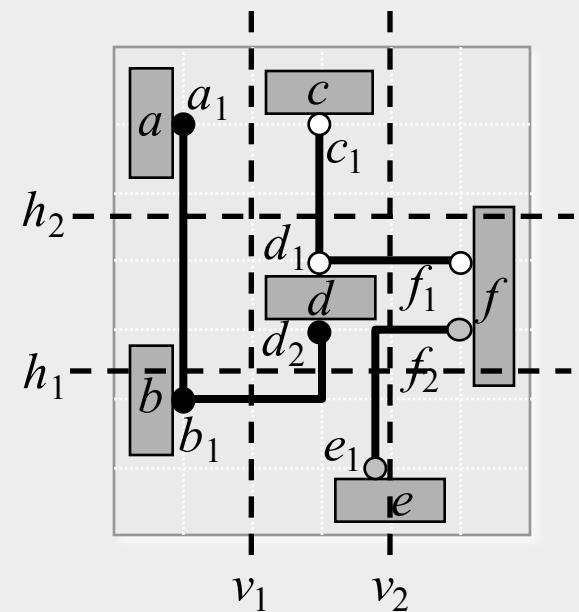
- Cut values for each global cutline

$$\Psi_P(v_1) = 1 \quad \Psi_P(v_2) = 2$$

$$\Psi_P(h_1) = 3 \quad \Psi_P(h_2) = 2$$

- Total number of crossings in P

$$\Psi_P(v_1) + \Psi_P(v_2) + \Psi_P(h_1) + \Psi_P(h_2) = 1 + 2 + 3 + 2 = 8$$



4.2 Optimization Objectives – Wire Congestion

Routing congestion of a placement

- Formally, the local wire density $\varphi_P(e)$ of an edge e between two neighboring grid cells is

$$\varphi_P(e) = \frac{\eta_P(e)}{\sigma_P(e)}$$

where $\eta_P(e)$ is the estimated number of nets that cross e and $\sigma_P(e)$ is the maximum number of nets that can cross e

- If $\varphi_P(e) > 1$, then too many nets are estimated to cross e , making P more likely to be unrouteable.
- The wire density of P is $\Phi(P) = \max_{e \in E}(\varphi_P(e))$

where E is the set of all edges

- If $\Phi(P) \leq 1$, then the design is estimated to be fully routable, otherwise routing will need to detour some nets through less-congested edges

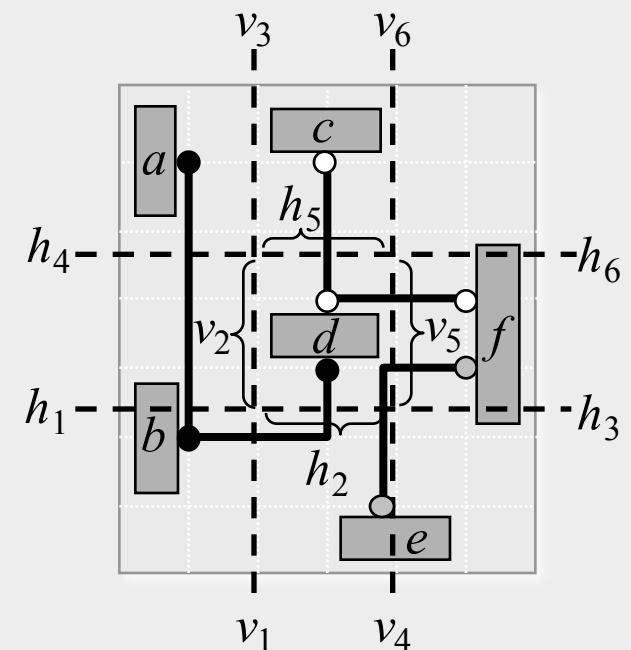
4.2 Optimization Objectives – Wire Congestion

Wire Density of a placement

Assume edge capacity is 3 for all edges

$$\begin{array}{ll}
 \eta_P(h_1) = 1 & \eta_P(v_1) = 1 \\
 \eta_P(h_2) = 2 & \eta_P(v_2) = 0 \\
 \eta_P(h_3) = 0 & \eta_P(v_3) = 0 \\
 \eta_P(h_4) = 1 & \eta_P(v_4) = 0 \\
 \eta_P(h_5) = 1 & \eta_P(v_5) = 2 \\
 \eta_P(h_6) = 0 & \eta_P(v_6) = 0
 \end{array}$$

Maximum: $\eta_P(e) = 2$



$$\Phi(P) = \frac{\eta_P(e)}{\sigma_P(e)} = \frac{2}{3}$$

→ Routable

Circuit timing of a placement

- Static timing analysis using actual arrival time (AAT) and required arrival time (RAT)
 - $AAT(v)$ represents the latest transition time at a given node v measured from the beginning of the clock cycle
 - $RAT(v)$ represents the time by which the latest transition at v must complete in order for the circuit to operate correctly within a given clock cycle.
- For correct operation of the chip with respect to setup (maximum path delay) constraints, it is required that $AAT(v) \leq RAT(v)$.

4.1 Introduction

4.2 Optimization Objectives

→ 4.3 Global Placement

- 4.3.1 Min-Cut Placement
- 4.3.2 Analytic Placement
- 4.3.3 Simulated Annealing
- 4.3.4 Modern Placement Algorithms

4.4 Legalization and Detailed Placement

- **Partitioning-based algorithms:**
 - The netlist and the layout are divided into smaller sub-netlists and sub-regions, respectively
 - Process is repeated until each sub-netlist and sub-region is small enough to be handled optimally
 - Detailed placement often performed by optimal solvers, facilitating a natural transition from global placement to detailed placement
 - Example: min-cut placement
- **Analytic techniques:**
 - Model the placement problem using an objective (cost) function, which can be optimized via numerical analysis
 - Examples: quadratic placement and force-directed placement
- **Stochastic algorithms:**
 - Randomized moves that allow hill-climbing are used to optimize the cost function
 - Example: simulated annealing

Global Placement

Partitioning-based

Analytic

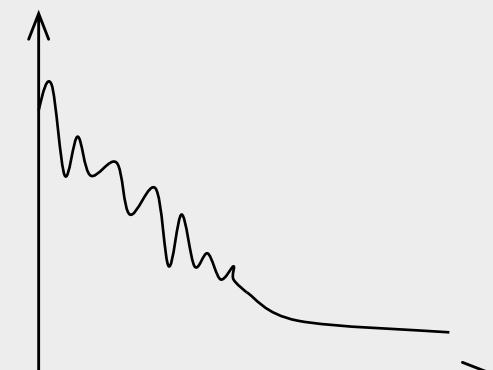
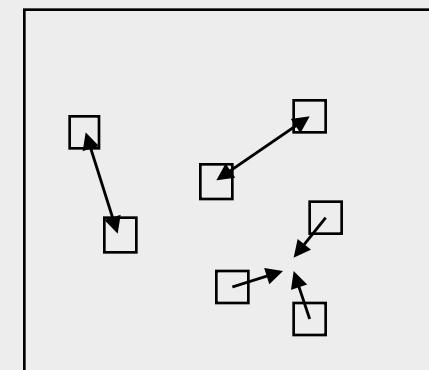
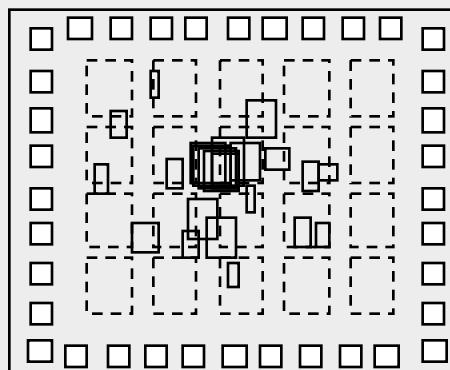
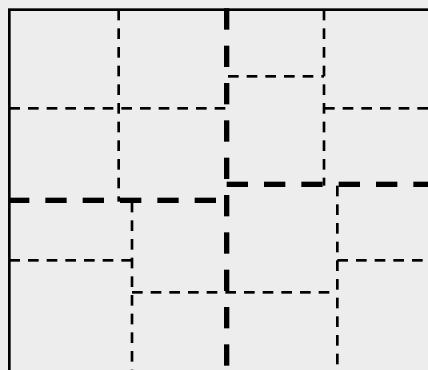
Stochastic

Min-cut
placement

Quadratic
placement

Force-directed
placement

Simulated annealing

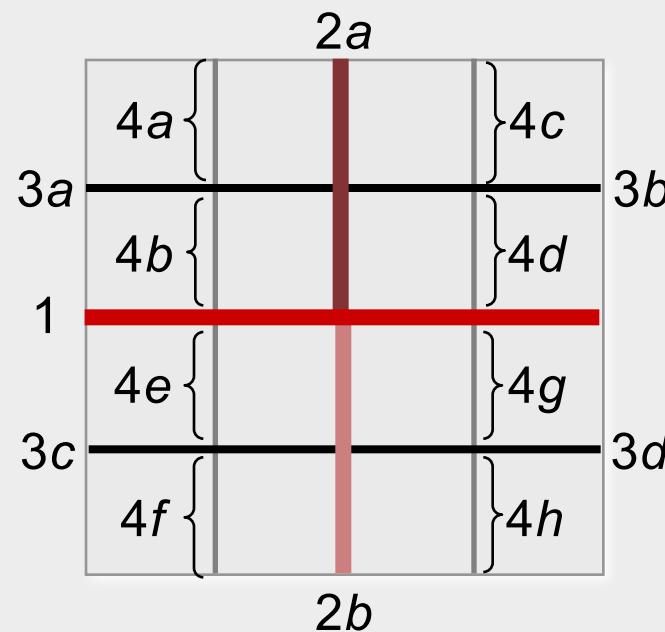


4.3.1 Min-Cut Placement

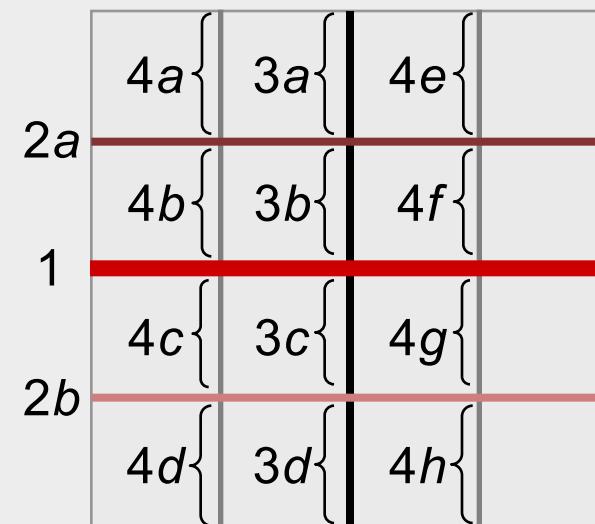
- Uses partitioning algorithms to divide (1) the netlist and (2) the layout region into smaller sub-netlists and sub-regions
- Conceptually, each sub-region is assigned a portion of the original netlist
- Each cut heuristically minimizes the number of cut nets using, for example,
 - Kernighan-Lin (KL) algorithm
 - Fiduccia-Mattheyses (FM) algorithm

4.3.1 Min-Cut Placement

Alternating cutline directions



Repeating cutline directions



4.3.1 Min-Cut Placement

Input: netlist *Netlist*, layout area *LA*, minimum number of cells per region *cells_min*

Output: placement *P*

P = \emptyset

regions = ASSIGN(*Netlist,LA*)

while (*regions* != \emptyset)

region = FIRST_ELEMENT(*regions*)

 REMOVE(*regions*, *region*)

if (*region* contains more than *cell_min* cells)

 (sr1,sr2)= BISECT(*region*)

 ADD_TO_END(*regions*,sr1)

 ADD_TO_END(*regions*,sr2)

else

 PLACE(*region*)

 ADD(*P*,*region*)

// assign netlist to layout area

// while regions still not placed

// first element in *regions*

// remove first element of *regions*

// divide *region* into two subregions

// sr1 and sr2, obtaining the sub-
// netlists and sub-areas

// add sr1 to the end of *regions*

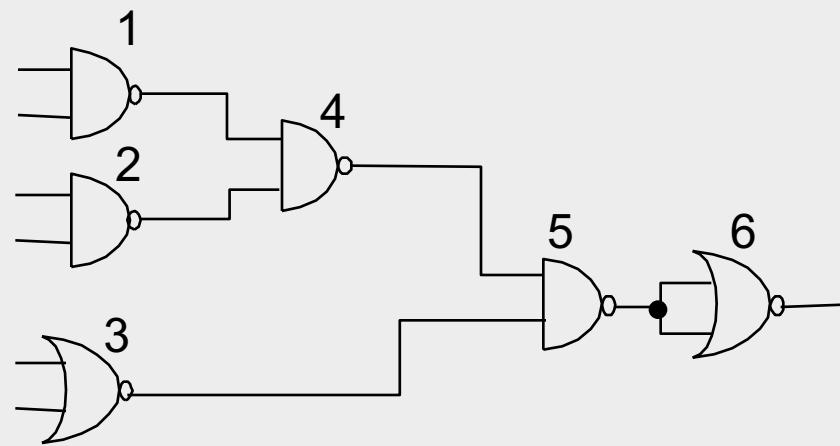
// add sr2 to the end of *regions*

// place *region*

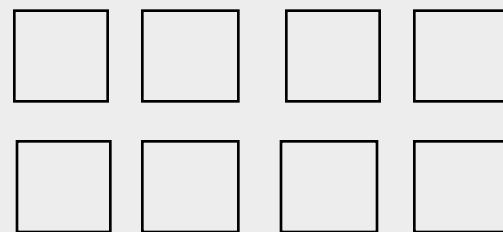
// add *region* to *P*

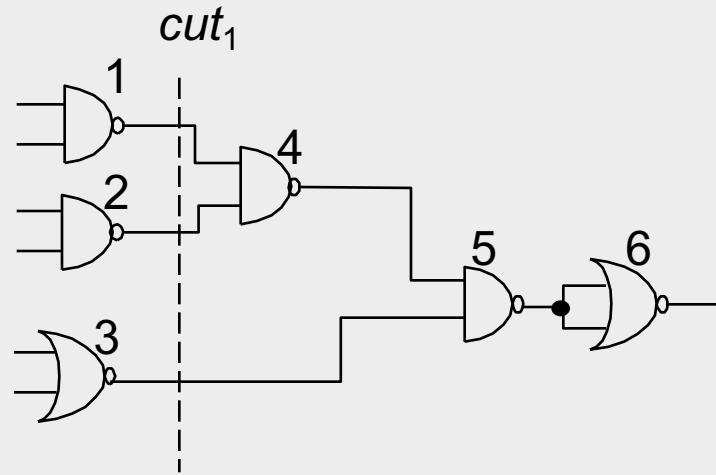
4.3.1 Min-Cut Placement – Example

Given:

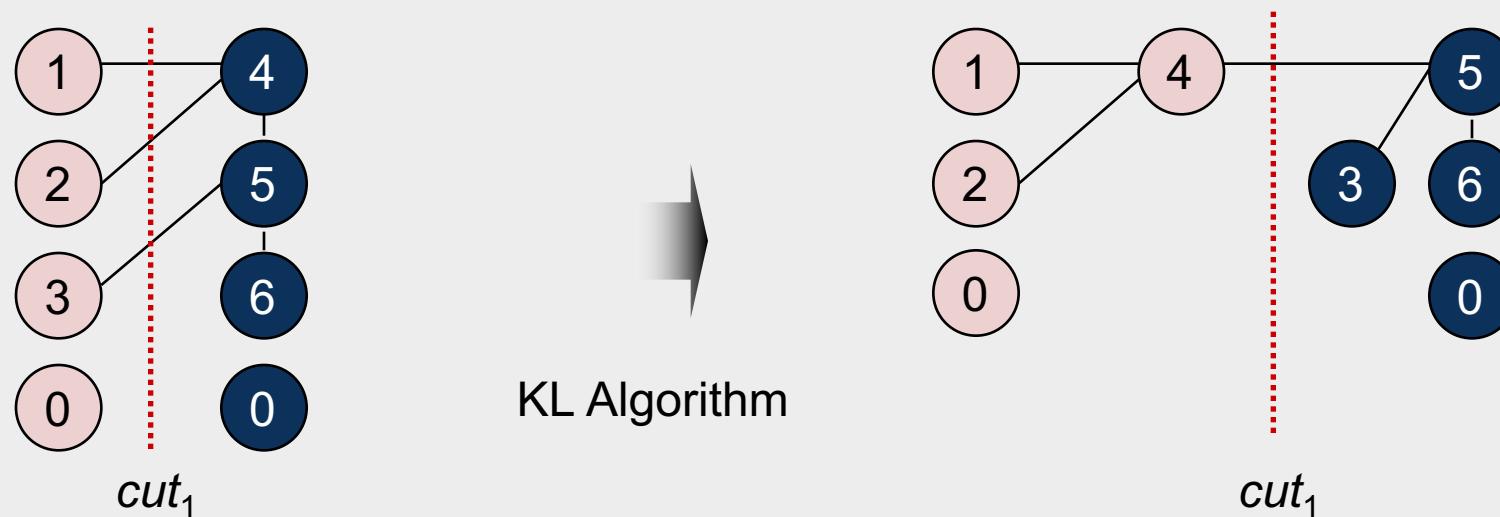


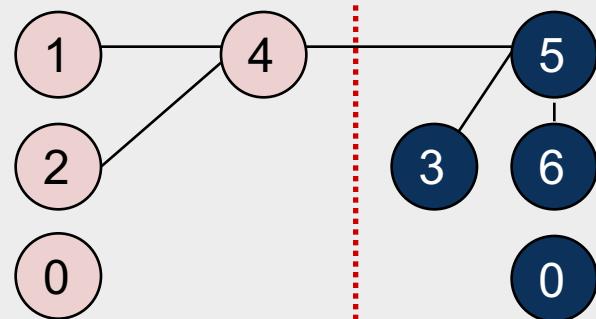
Task: 4 x 2 placement with minimum wirelength using alternative cutline directions and the KL algorithm





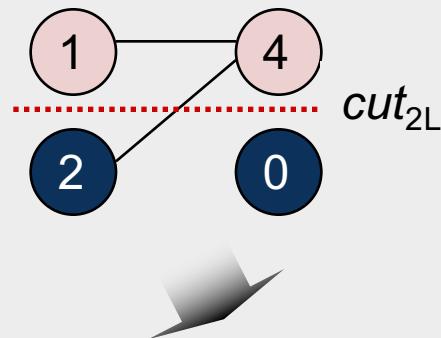
Vertical cut cut_1 : $L=\{1,2,3\}$, $R=\{4,5,6\}$



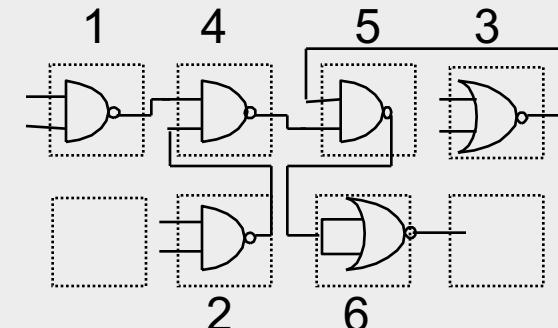
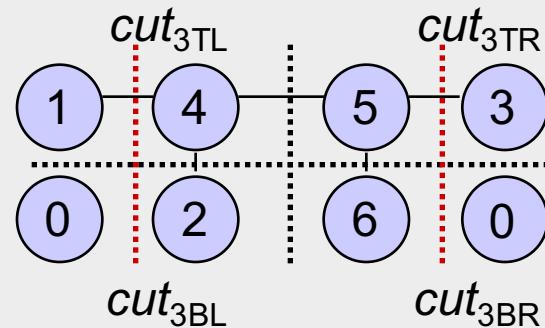
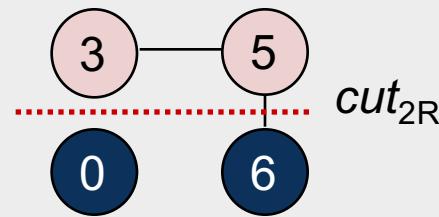


cut_1

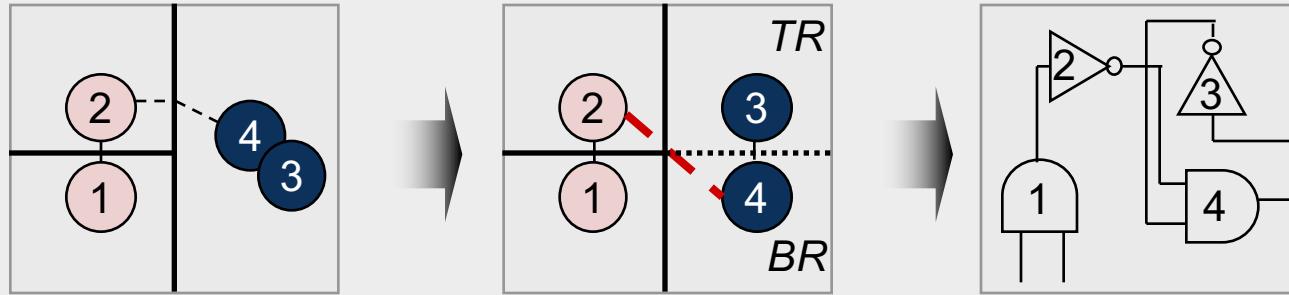
Horizontal cut cut_{2L} : $T=\{1,4\}$, $B=\{2,0\}$



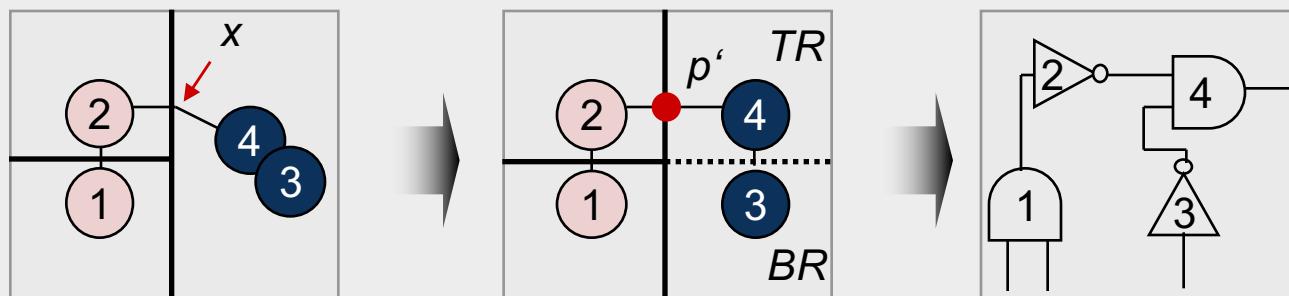
Horizontal cut cut_{2R} : $T=\{3,5\}$, $B=\{6,0\}$



4.3.1 Min-Cut Placement – Terminal Propagation



- Terminal Propagation
 - External connections are represented by artificial connection points on the cutline
 - Dummy nodes in hypergraphs



4.3.1 Min-Cut Placement

- Advantages:
 - Reasonable fast
 - Objective function can be adjusted, e.g., to perform timing-driven placement
 - Hierarchical strategy applicable to large circuits
- Disadvantages:
 - Randomized, chaotic algorithms – small changes in input lead to large changes in output
 - Optimizing one cutline at a time may result in routing congestion elsewhere

4.3.2 Analytic Placement – Quadratic Placement

- Objective function is quadratic; sum of (weighted) squared Euclidean distance represents placement objective function

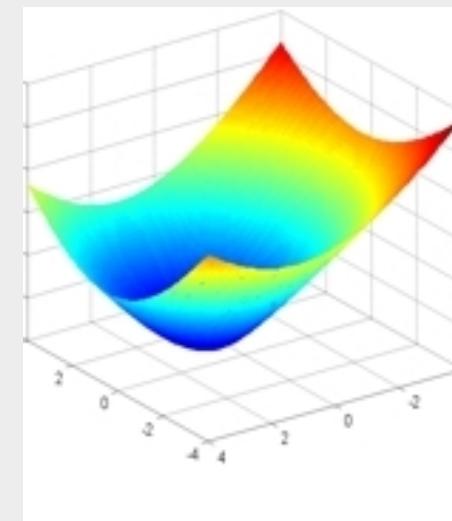
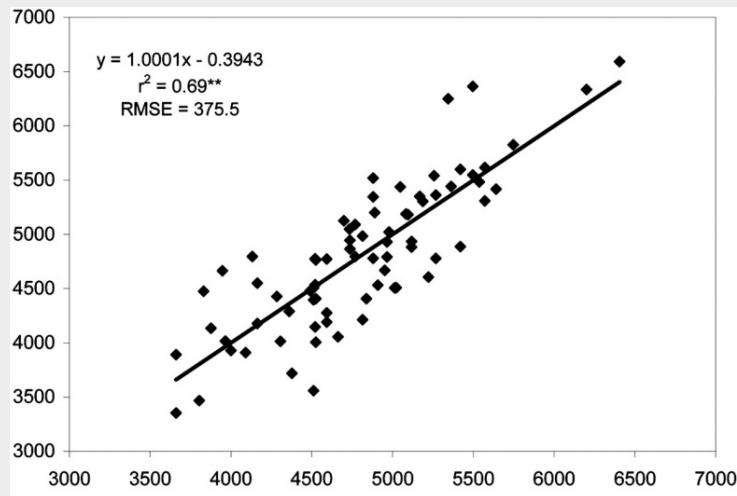
$$L(P) = \frac{1}{2} \sum_{i,j=1}^n c_{ij} \left((x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

where n is the total number of cells, and $c(i,j)$ is the connection cost between cells i and j .

- Only two-point-connections
- Minimize objective function by equating its derivative to zero which reduces to solving a system of linear equations

4.3.2 Analytic Placement – Quadratic Placement

- Similar to Least-Mean-Square Method (root mean square)
- Build error function with analytic form: $E(a,b) = \sum (a \cdot x_i + b - y_i)^2$



4.3.2 Analytic Placement – Quadratic Placement

$$L(P) = \frac{1}{2} \sum_{i,j=1}^n c_{ij} \left((x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

where n is the total number of cells, and $c(i,j)$ is the connection cost between cells i and j .

- Each dimension can be considered independently:

$$L_x(P) = \sum_{i=1, j=1}^n c(i, j)(x_i - x_j)^2 \quad L_y(P) = \sum_{i=1, j=1}^n c(i, j)(y_i - y_j)^2$$

- Convex quadratic optimization problem: any local minimum solution is also a global minimum
- Optimal x - and y -coordinates can be found by setting the partial derivatives of $L_x(P)$ and $L_y(P)$ to zero

4.3.2 Analytic Placement – Quadratic Placement

$$L(P) = \frac{1}{2} \sum_{i,j=1}^n c_{ij} \left((x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

where n is the total number of cells, and $c(i,j)$ is the connection cost between cells i and j .

- Each dimension can be considered independently:

$$L_x(P) = \sum_{i=1, j=1}^n c(i, j)(x_i - x_j)^2 \quad L_y(P) = \sum_{i=1, j=1}^n c(i, j)(y_i - y_j)^2$$

$$\frac{\partial L_x(P)}{\partial X} = AX - b_x = 0$$

$$\frac{\partial L_y(P)}{\partial Y} = AY - b_y = 0$$

where A is a matrix with $A[i][j] = -c(i,j)$ when $i \neq j$,
and $A[i][i] =$ the sum of incident connection weights of cell i .

X is a vector of all the x -coordinates of the non-fixed cells, and b_x is a vector with $b_x[i] =$ the sum of x -coordinates of all fixed cells attached to i .

Y is a vector of all the y -coordinates of the non-fixed cells, and b_y is a vector with $b_y[i] =$ the sum of y -coordinates of all fixed cells attached to i .

4.3.2 Analytic Placement – Quadratic Placement

$$L(P) = \frac{1}{2} \sum_{i,j=1}^n c_{ij} \left((x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

where n is the total number of cells, and $c(i,j)$ is the connection cost between cells i and j .

- Each dimension can be considered independently:

$$L_x(P) = \sum_{i=1, j=1}^n c(i, j)(x_i - x_j)^2 \quad L_y(P) = \sum_{i=1, j=1}^n c(i, j)(y_i - y_j)^2$$



$$\frac{\partial L_x(P)}{\partial X} = AX - b_x = 0$$

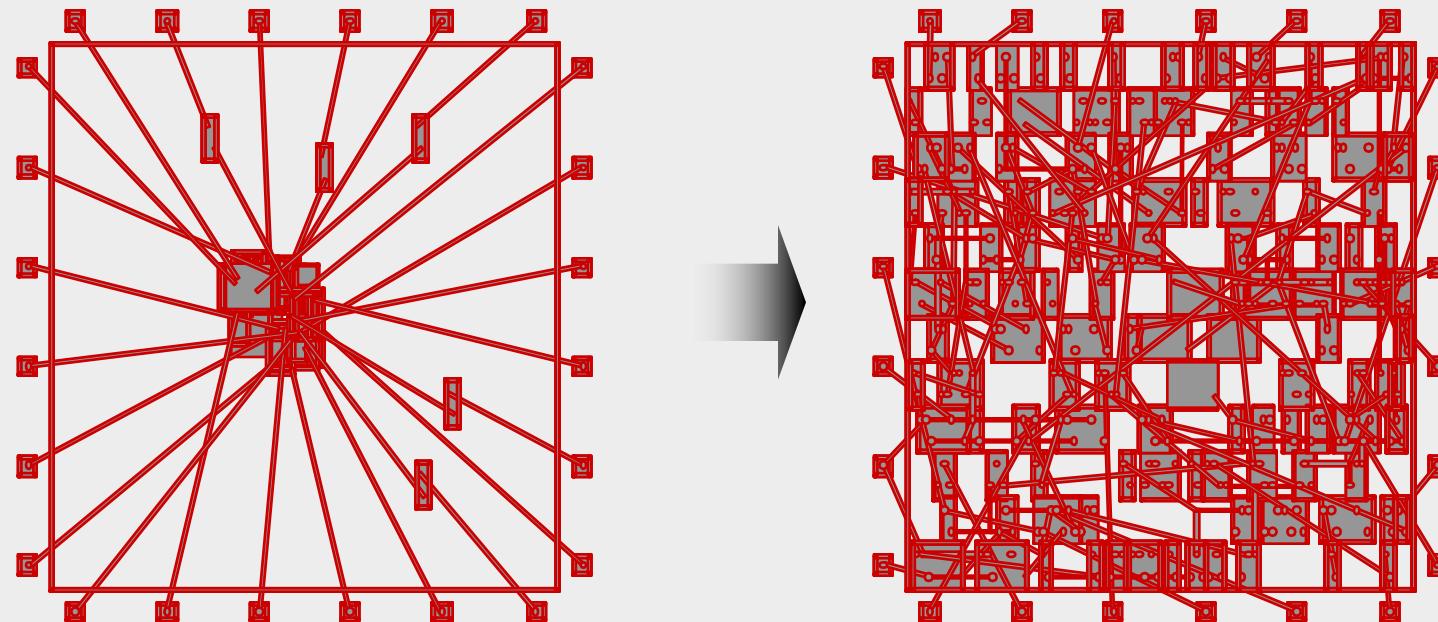


$$\frac{\partial L_y(P)}{\partial Y} = AY - b_y = 0$$

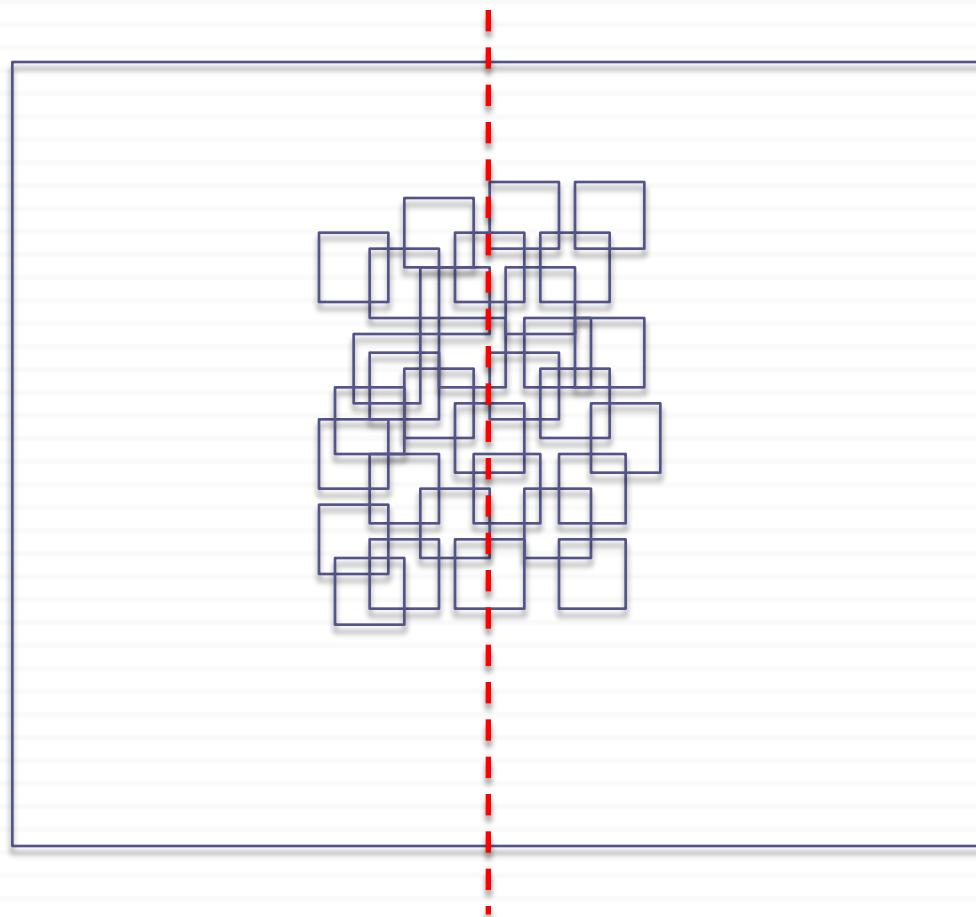
- System of linear equations for which iterative numerical methods can be used to find a solution

4.3.2 Analytic Placement – Quadratic Placement

- Second stage of quadratic placers: cells are spread out to remove overlaps
- Methods:
 - Adding fake nets that pull cells away from dense regions toward anchors
 - Geometric sorting and scaling
 - Partitioning, etc.



Cell Spreading Based on Partitioning



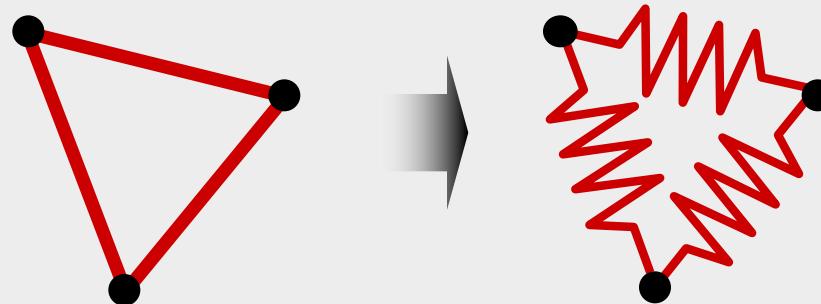
- Geometric partitioning:
 - Enforce partition constraints based on sizes of the regions
 - Try to respect the relative cell locations during partitioning
- Define center of gravity for each partition, and add it as a constraint to the quadratic placer.
- Terminal propagation

4.3.2 Analytic Placement – Quadratic Placement

- Advantages:
 - Captures the placement problem concisely in mathematical terms
 - Leverages efficient algorithms from numerical analysis and available software
 - Can be applied to large circuits without netlist clustering (flat)
 - Stability: small changes in the input do not lead to large changes in the output
- Disadvantages:
 - Connections to fixed objects are necessary: I/O pads, pins of fixed macros, etc.

4.3.2 Analytic Placement – Quadratic Placement

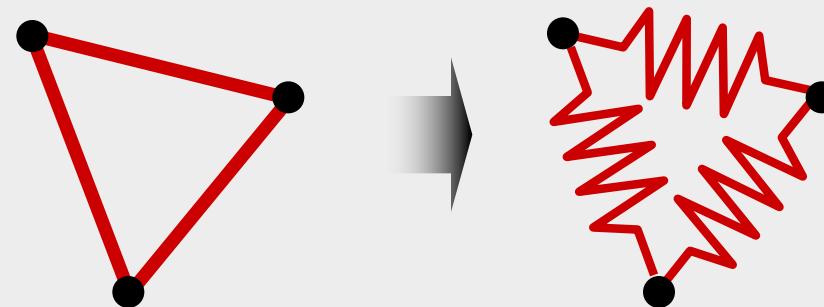
- Mechanical analogy: mass-spring system



- Squared Euclidean distance is proportional to the energy of a spring between these points
 - Quadratic objective function represents total energy of the spring system; for each movable object, the x (y) partial derivative represents the total force acting on that object
 - Setting the forces of the nets to zero, an equilibrium state is mathematically modeled that is characterized by zero forces acting on each movable object
 - At the end, all springs are in a force equilibrium with a minimal total spring energy; this equilibrium represents the minimal sum of squared wirelength
- Result: many cell overlaps

4.3.2 Analytic Placement – Force-directed Placement

- Cells and wires are modeled using the mechanical analogy of a mass-spring system, i.e., masses connected to Hooke's-Law springs



- Attraction force between cells is directly proportional to their distance
- Cells will eventually settle in a **force equilibrium** → minimized wirelength

4.3.2 Analytic Placement – Force-directed Placement

- Given two connected cells a and b , the attraction force \overrightarrow{F}_{ab} exerted on a by b is

$$\overrightarrow{F}_{ab} = c(a,b) \cdot (\vec{b} - \vec{a})$$

where

- $c(a,b)$ is the connection weight (priority) between cells a and b , and
- $(\vec{b} - \vec{a})$ is the vector difference of the positions of a and b in the Euclidean plane

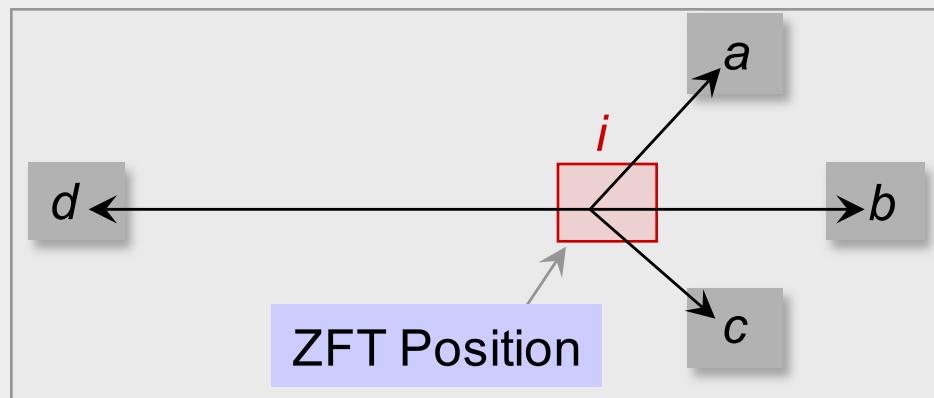
- The sum of forces exerted on a cell i connected to other cells $1 \dots j$ is

$$\overrightarrow{F}_i = \sum_{c(i,j) \neq 0} \overrightarrow{F}_{ij}$$

- Zero-force target (ZFT):** position that minimizes this sum of forces

4.3.2 Analytic Placement – Force-directed Placement

Zero-Force-Target (ZFT) position of cell i



$$\min \vec{F}_i = c(i,a) \cdot (\vec{a} - \vec{i}) + c(i,b) \cdot (\vec{b} - \vec{i}) + c(i,c) \cdot (\vec{c} - \vec{i}) + c(i,d) \cdot (\vec{d} - \vec{i})$$

4.3.2 Analytic Placement – Force-directed Placement

Basic force-directed placement

- Iteratively moves all cells to their respective ZFT positions
- x - and y -direction forces are set to zero:

$$\sum_{c(i,j) \neq 0} c(i,j) \cdot (x_j^0 - x_i^0) = 0 \quad \sum_{c(i,j) \neq 0} c(i,j) \cdot (y_j^0 - y_i^0) = 0$$

- Rearranging the variables to solve for x_i^0 and y_i^0 yields

$$x_i^0 = \frac{\sum_{c(i,j) \neq 0} c(i,j) \cdot x_j^0}{\sum_{c(i,j) \neq 0} c(i,j)}$$

$$y_i^0 = \frac{\sum_{c(i,j) \neq 0} c(i,j) \cdot y_j^0}{\sum_{c(i,j) \neq 0} c(i,j)}$$

Computation of
ZFT position of cell i
connected with
cells $1 \dots j$

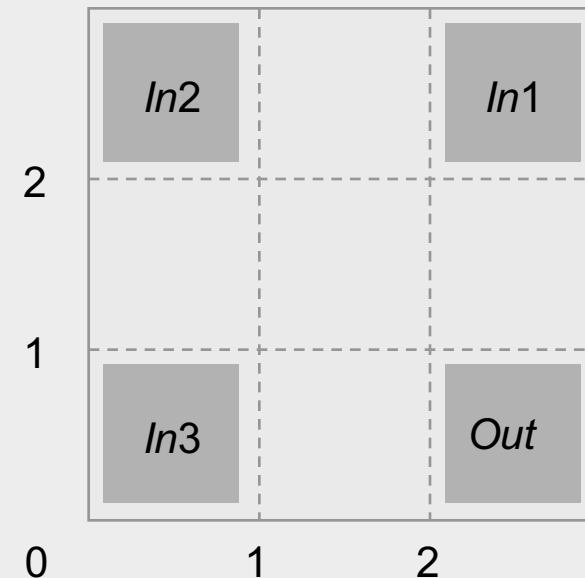
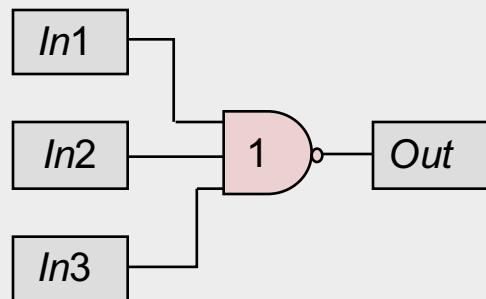
4.3.2 Analytic Placement – Force-directed Placement

Example: ZFT position

Given:

- Circuit with NAND gate 1 and four I/O pads on a 3×3 grid
- Pad positions: $In1(2,2)$, $In2(0,2)$, $In3(0,0)$, $Out(2,0)$
- Weighted connections: $c(a,In1) = 8$, $c(a,In2) = 10$, $c(a,In3) = 2$, $c(a,Out) = 2$

Task: find the ZFT position of cell a



4.3.2 Analytic Placement – Force-directed Placement

Example: ZFT position

Given:

- Circuit with NAND gate 1 and four I/O pads on a 3 x 3 grid
- Pad positions: *In1* (2,2), *In2* (0,2), *In3* (0,0), *Out* (2,0)

Solution:

$$x_a^0 = \frac{\sum_{c(i,j) \neq 0} c(a,j) \cdot x_j^0}{\sum_{c(i,j) \neq 0} c(a,j)} = \frac{c(a, In1) \cdot x_{In1} + c(a, In2) \cdot x_{In2} + c(a, In3) \cdot x_{In3} + c(a, Out) \cdot x_{Out}}{c(a, In1) + c(a, In2) + c(a, In3) + c(a, Out)} = \frac{8 \cdot 2 + 10 \cdot 0 + 2 \cdot 0 + 2 \cdot 2}{8 + 10 + 2 + 2} = \frac{20}{22} \approx 0.9$$

$$y_a^0 = \frac{\sum_{c(i,j) \neq 0} c(a,j) \cdot y_j^0}{\sum_{c(i,j) \neq 0} c(a,j)} = \frac{c(a, In1) \cdot y_{In1} + c(a, In2) \cdot y_{In2} + c(a, In3) \cdot y_{In3} + c(a, Out) \cdot y_{Out}}{c(a, In1) + c(a, In2) + c(a, In3) + c(a, Out)} = \frac{8 \cdot 2 + 10 \cdot 2 + 2 \cdot 0 + 2 \cdot 0}{8 + 10 + 2 + 2} = \frac{36}{22} \approx 1.6$$

ZFT position of cell *a* is (1,2)

4.3.2 Analytic Placement – Force-directed Placement

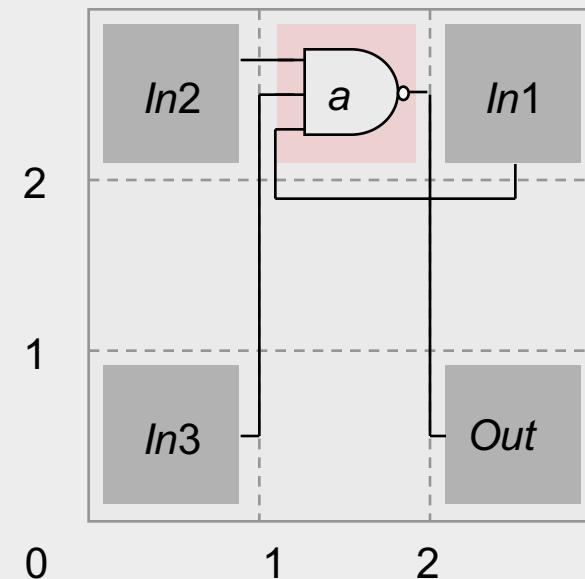
Example: ZFT position

Given:

- Circuit with NAND gate 1 and four I/O pads on a 3×3 grid
- Pad positions: $In1$ (2,2), $In2$ (0,2), $In3$ (0,0), Out (2,0)

Solution:

ZFT position of cell a is (1,2)



4.3.2 Analytic Placement – Force-directed Placement

Input: set of all cells V

Output: placement P

```
 $P = \text{PLACE}(V)$                                 // arbitrary initial placement
 $\text{loc} = \text{LOCATIONS}(P)$                       // set coordinates for each cell in  $P$ 
foreach (cell  $c \in V$ )
     $\text{status}[c] = \text{UNMOVED}$ 
while ( $\text{!ALL\_MOVED}(V) \parallel \text{!STOP}()$ )      // continue until all cells have been
                                                        // moved or some stopping
                                                        // criterion is reached
     $c = \text{MAX\_DEGREE}(V, \text{status})$                   // unmoved cell that has largest
                                                        // number of connections
     $ZFT\_pos = \text{ZFT\_POSITION}(c)$                     // ZFT position of  $c$ 
    if ( $\text{loc}[ZFT\_pos] == \emptyset$ )                 // if position is unoccupied,
         $\text{loc}[ZFT\_pos] = c$                             // move  $c$  to its ZFT position
    else
         $\text{RELOCATE}(c, \text{loc})$                       // use methods discussed next
         $\text{status}[c] = \text{MOVED}$                         // mark  $c$  as moved
```

4.3.2 Analytic Placement – Force-directed Placement

Finding a valid location for a cell with an occupied ZFT position
(p : incoming cell, q : cell in p 's ZFT position)

- If possible, move p to a cell position close to q .
- Chain move: cell p is moved to cells q 's location.
 - Cell q , in turn, is shifted to the next position. If a cell r is occupying this space, cell r is shifted to the next position.
 - This continues until all affected cells are placed.
- Compute the cost difference if p and q were to be swapped.
If the total cost reduces, i.e., the weighted connection length $L(P)$ is smaller, then swap p and q .

4.3.2 Analytic Placement – Force-directed Placement (Example)

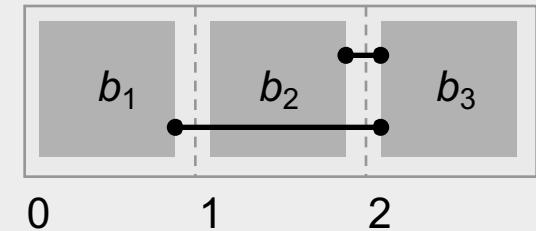
Given:

Nets

$$\begin{aligned}N_1 &= (b_1, b_3) \\N_2 &= (b_2, b_3)\end{aligned}$$

Weight

$$\begin{aligned}c(N_1) &= 2 \\c(N_2) &= 1\end{aligned}$$



4.3.2 Analytic Placement – Force-directed Placement (Example)

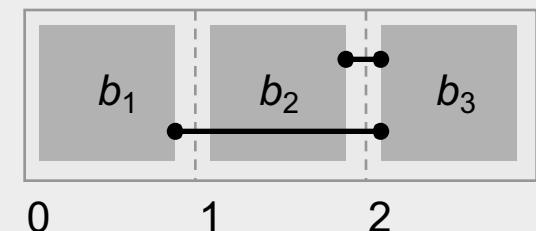
Given:

Nets

$$\begin{aligned} N_1 &= (b_1, b_3) \\ N_2 &= (b_2, b_3) \end{aligned}$$

Weight

$$\begin{aligned} c(N_1) &= 2 \\ c(N_2) &= 1 \end{aligned}$$



Incoming cell p	ZFT position of cell p	Cell q	$L(P)$ before move	$L(P)$ / placement after move
b_3	$x_{b_3}^0 = \frac{\sum_{c(b_3,j) \neq 0} c(b_3,j) \cdot x_j^0}{\sum_{c(b_3,j) \neq 0} c(b_3,j)} = \frac{2 \cdot 0 + 1 \cdot 1}{2 + 1} \approx 0$	b_1	$L(P) = 5$	$L(P) = 5$ <p>\Rightarrow No swapping of b_3 and b_1</p>

4.3.2 Analytic Placement – Force-directed Placement (Example)

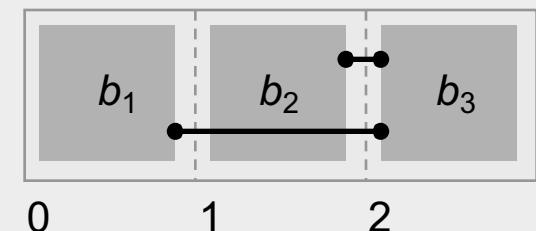
Given:

Nets

$$\begin{aligned} N_1 &= (b_1, b_3) \\ N_2 &= (b_2, b_3) \end{aligned}$$

Weight

$$\begin{aligned} c(N_1) &= 2 \\ c(N_2) &= 1 \end{aligned}$$



Incoming cell p	ZFT position of cell p	Cell q	$L(P)$ before move	$L(P)$ / placement after move
b_3	$x_{b_3}^0 = \frac{\sum_{c(b_3,j) \neq 0} c(b_3,j) \cdot x_j^0}{\sum_{c(b_3,j) \neq 0} c(b_3,j)} = \frac{2 \cdot 0 + 1 \cdot 1}{2 + 1} \approx 0$	b_1	$L(P) = 5$	$L(P) = 5$ <p>→ No swapping of b_3 and b_1</p>
b_2	$x_{b_2}^0 = \frac{\sum_{c(b_2,j) \neq 0} c(b_2,j) \cdot x_j^0}{\sum_{c(b_2,j) \neq 0} c(b_2,j)} = \frac{1 \cdot 2}{1} = 2$	b_3	$L(P) = 5$	$L(P) = 3$ <p>→ Swapping of b_2 and b_3</p>

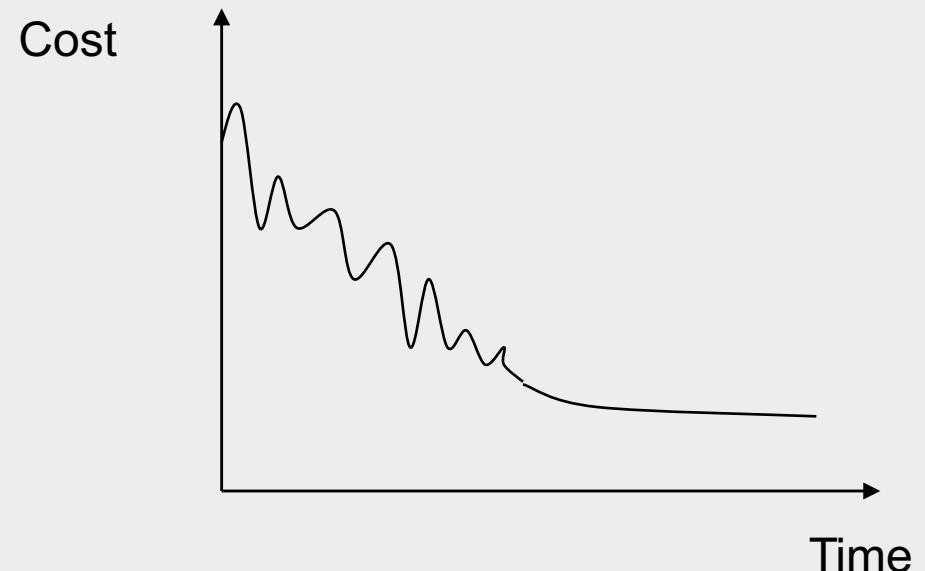
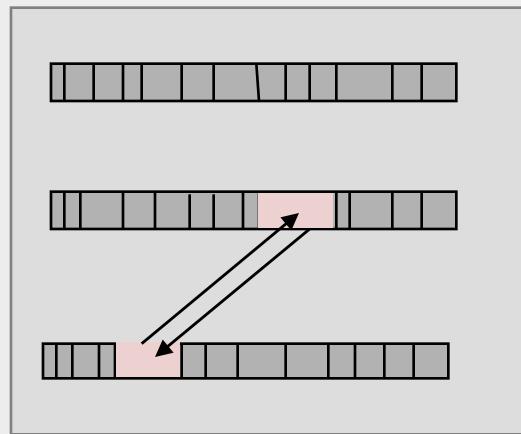
4.3.2 Analytic Placement – Force-directed Placement

- Advantages:
 - Conceptually simple, easy to implement
 - Primarily intended for global placement, but can also be adapted to detailed placement
- Disadvantages:
 - Does not scale to large placement instances
 - Is not very effective in spreading cells in densest regions
 - Poor trade-off between solution quality and runtime
- In practice, FDP is extended by specialized techniques for cell spreading
 - This facilitates scalability and makes FDP competitive

Modern Force-Directed Placement Algorithms

- Similar to the quadratic placement algorithms:
 - ▣ Cell locations are determined through quadratic optimization
- Cell overlaps are eliminated through repulsive forces
 - ▣ Repulsive forces: Perturbation to the quadratic formulation
 - ▣ Move cells from over-utilized regions to under-utilized regions
- Overlaps not resolved in a single iteration
 - ▣ Repulsive forces updated based on the cell distribution in every iteration
 - ▣ Accumulated over multiple iterations

4.3.3 Simulated Annealing



- Analogous to the physical **annealing process**
 - Melt metal and then slowly cool it
 - Result: energy-minimal crystal structure
- Modification of an initial configuration (placement) by moving/exchanging of randomly selected cells
 - Accept the new placement if it improves the objective function
 - If no improvement: Move/exchange is accepted with temperature-dependent (i.e., decreasing) probability

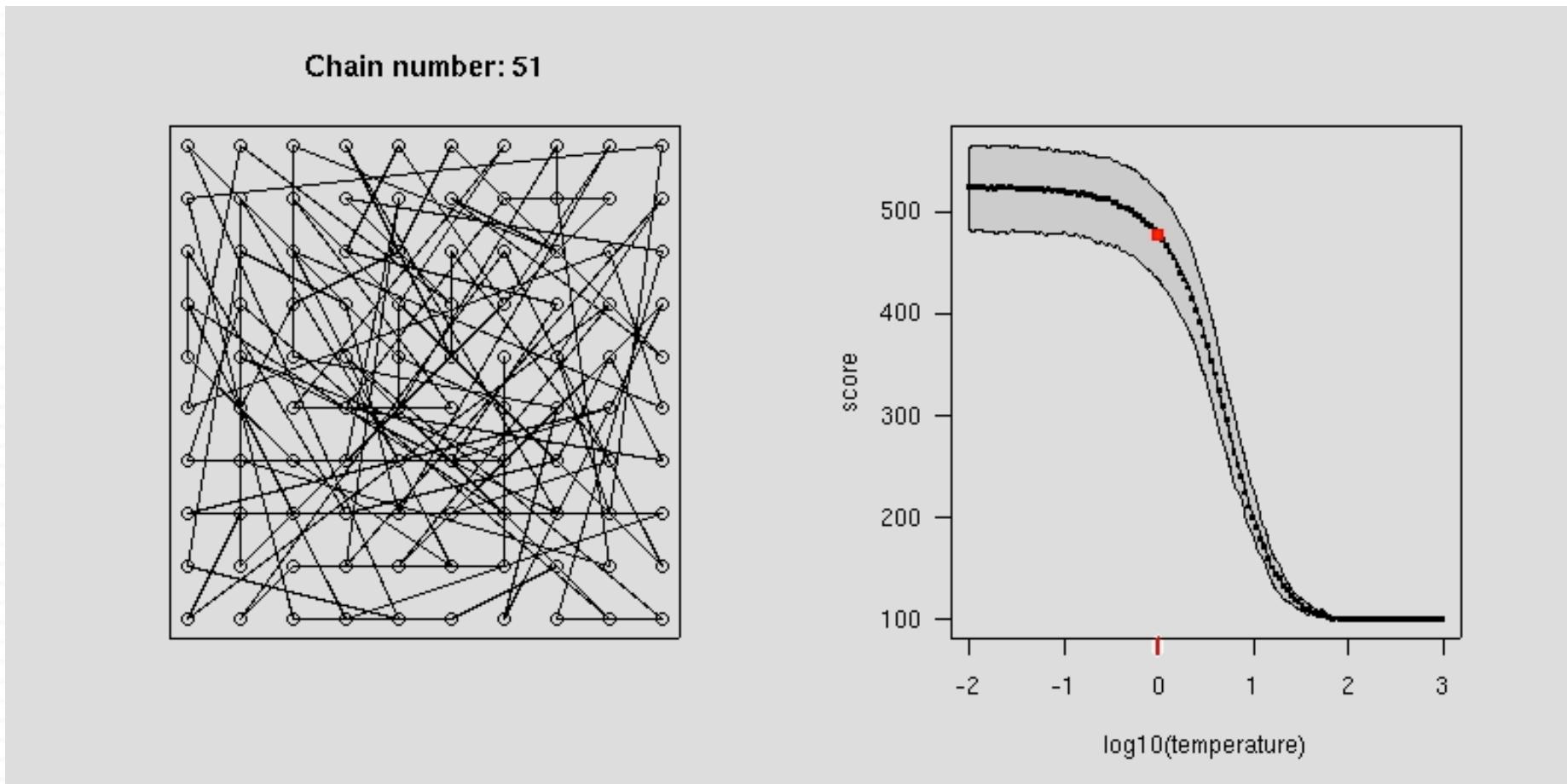
4.3.3 Simulated Annealing – Algorithm

Input: set of all cells V

Output: placement P

```
 $T = T_0$                                 // set initial temperature
 $P = \text{PLACE}(V)$                       // arbitrary initial placement
while ( $T > T_{min}$ )
    while (!STOP())
         $new\_P = \text{PERTURB}(P)$ 
         $\Delta\text{cost} = \text{COST}(new\_P) - \text{COST}(P)$ 
        if ( $\Delta\text{cost} < 0$ )
             $P = new\_P$ 
        else
             $r = \text{RANDOM}(0,1)$ 
            if ( $r < e^{-\Delta\text{cost}/T}$ )
                 $P = new\_P$ 
         $T = \alpha \cdot T$                          // reduce  $T$ ,  $0 < \alpha < 1$ 
```

Simulated Annealing – Animation



Source: <http://www.biostat.jhsph.edu/~iruczins/teaching/misc/annealing/animation.html>

4.3.3 Simulated Annealing

- Advantages:
 - Can find global optimum (given sufficient time)
 - Well-suited for detailed placement
- Disadvantages:
 - Very slow
 - To achieve high-quality implementation, laborious parameter tuning is necessary
 - Randomized, chaotic algorithms - small changes in the input lead to large changes in the output
- Practical applications of SA:
 - Very small placement instances with complicated constraints
 - Detailed placement, where SA can be applied in small windows (not common anymore)
 - FPGA layout, where complicated constraints are becoming a norm

4.3.4 Modern Placement Algorithms

- Predominantly analytic algorithms
- Solve two challenges: interconnect minimization and cell overlap removal (spreading)
- Two families:



Quadratic placers



Non-convex
optimization placers

4.3.4 Modern Placement Algorithms



Quadratic placers



Non-convex
optimization placers

- Solve large, sparse systems of linear equations (formulated using force-directed placement) by the Conjugate Gradient algorithm
- Perform cell spreading by adding fake nets that pull cells away from dense regions toward carefully placed anchors

4.3.4 Modern Placement Algorithms

Quadratic placers

Non-convex
optimization placers

- Model interconnect by sophisticated differentiable functions, e.g., log-sum-exp is the popular choice
- Model cell overlap and fixed obstacles by additional (non-convex) functional terms
- Optimize interconnect by the non-linear Conjugate Gradient algorithm
- Sophisticated, slow algorithms
- All leading placers in this category use netlist clustering to improve computational scalability (this further complicates the implementation)

4.3.4 Modern Placement Algorithms



Quadratic
Placement



Non-convex
optimization placers

Pros and cons:

- Quadratic placers are simpler and faster, easier to parallelize
- Non-convex optimizers tend to produce better solutions
- As of 2011, quadratic placers are catching up in solution quality while running 5-6 times faster [1]

4.4

Legalization and Detailed Placement

4.1 Introduction

4.2 Optimization Objectives

4.3 Global Placement

4.3.1 Min-Cut Placement

4.3.2 Analytic Placement

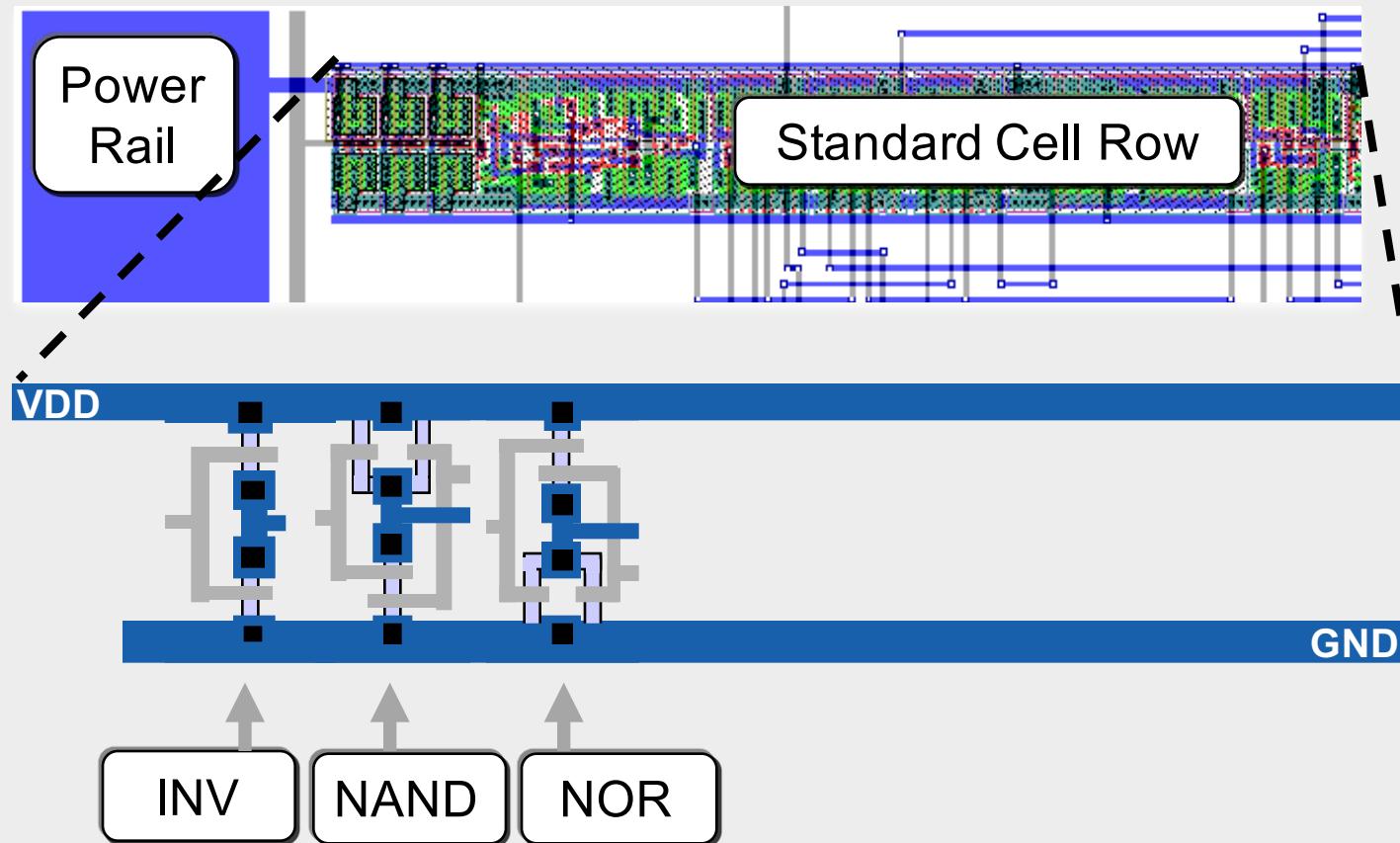
4.3.3 Simulated Annealing

4.3.4 Modern Placement Algorithms

→ 4.4 Legalization and Detailed Placement

- Global placement must be legalized
 - Cell locations typically do not align with power rails
 - Small cell overlaps due to incremental changes, such as cell resizing or buffer insertion
- Legalization seeks to find legal, non-overlapping placements for all placeable modules
- Legalization can be improved by detailed placement techniques, such as
 - Swapping neighboring cells to reduce wirelength
 - Sliding cells to unused space
- Software implementations of legalization and detailed placement are often bundled

Legal positions of standard cells between VDD and GND rails



Summary of Chapter 4 – Problem Formulation and Objectives

- Row-based standard-cell placement
 - Cell heights are typically fixed, to fit in rows (but some cells may have double and quadruple heights)
 - Legal cell sites facilitate the alignment of routing tracks, connection to power and ground rails
- Wirelength as a key metric of interconnect
 - Bounding box half-perimeter (HPWL)
 - Cliques and stars
 - RMSTs and RSMTs
- Objectives: wirelength, routing congestion, circuit delay
 - Algorithm development is usually driven by wirelength
 - The basic framework is implemented, evaluated and made competitive on standard benchmarks
 - Additional objectives are added to an operational framework

Summary of Chapter 4 – Global Placement

- Combinatorial optimization techniques: min-cut and simulated annealing
 - Can perform both global and detailed placement
 - Reasonably good at small to medium scales
 - SA is very slow, but can handle a greater variety of constraints
 - Randomized and chaotic algorithms – small changes at the input can lead to large changes at the output
- Analytic techniques: force-directed placement and non-convex optimization
 - Primarily used for global placement
 - Unrivaled for large netlists in speed and solution quality
 - Capture the placement problem by mathematical optimization
 - Use efficient numerical analysis algorithms
 - Ensure stability: small changes at the input can cause only small changes at the output
 - Example: a modern, competitive analytic global placer takes 20mins for global placement of a netlist with 2.1M cells (single thread, 3.2GHz Intel CPU) [1]

Summary of Chapter 4 – Legalization and Detailed Placement

- Legalization ensures that design rules & constraints are satisfied
 - All cells are in rows
 - Cells align with routing tracks
 - Cells connect to power & ground rails
 - Additional constraints are often considered, e.g., maximum cell density
- Detailed placement reduces interconnect, while preserving legality
 - Swapping neighboring cells, rotating groups of three
 - Optimal branch-and-bound on small groups of cells
 - Sliding cells along their rows
 - Other local changes
- Extensions to optimize routed wirelength, routing congestion and circuit timing
- Relatively straightforward algorithms, but high-quality, fast implementation is important
- Most relevant after analytic global placement, but are also used after min-cut placement
- Rule of thumb: 50% runtime is spent in global placement, 50% in detailed placement [1]