ABELLA GASSOL, ARNAU

AVLSI

29th Oct 2020

7.5

1)

a	b	c	d	f	U d	
0	0	0	0	1		m0
0	0	0	1	x		m1
0	0	1	0	1		m2
0	0	1	1	1		m3
0	1	0	0	x		m4
0	1	0	1	1		m5
0	1	1	0	1		m6
0	1	1	1			
1	0	0	0			
1	0	0	1			
1	0	1	0	1		m10
1	0	1	1	1		m11
1	1	0	0			
1	1	0	1	x		m13
1	1	1	0	1		m14
1	1	1	1			

(m0)(m1)	0 0 0 -
(m0)(m4)	0 - 00
(m1) m3	00 - 1
(m1) m5	0 - 01
m2 m3	0 0 1 -
m2 m6	0 - 10
m3 m11	- 0 11
(m4) m5	0 1 0 -
(m4) m6	0 1 - 0
m5 (m13)	- 1 0 1 *
m6 m14	- 1 1 0
m10 m11	1 0 1 -
m10 m14	1 - 1 0

(m0)(m1) m2 m3	0 0 - -	$\bar{a}\bar{b}$
(m0)(m1)(m4) m5	0 - 0 -	$\bar{a}\bar{c}$
(m0)(m4) m2 m6	0 - - 0	$\bar{a}\bar{d}$
m2 m3 m10 m11	- 0 1 -	$\bar{b}c$
m2 m6 m10 m14	- - 1 0	$c\bar{d}$
m5 (m13)	- 1 0 1	$b\bar{c}d$

	m2	m3	m5	m6	m10	m11	m14
$\bar{a}\bar{b}$	✓	✓					
$\bar{a}\bar{c}$			✓				
$\bar{a}\bar{d}$	✓			✓			
$\bar{b}c$	✓	✓			✓	✓	
$c\bar{d}$	✓			✓	✓		✓
$b\bar{c}d$			✓				

↑ ↑ Essential prime

2

- Essential prime implicants: $\bar{b}c$, $c\bar{d}$
- Cover that minimizes #literals: $\bar{b}c + c\bar{d} + \bar{a}\bar{c}$
- Cover that minimizes # variables: $\bar{b}c + c\bar{d} + b\bar{c}d$

---

2) $\begin{cases} x = abd + bc + be + abg + ae \\ y = ade + cd + ce + ef + h \end{cases}$

d+g

- $K(x) = \{(bd + bg + e), (ad + c + e + ag), (b + a), (abd + bc + be + abg + ae)\}$
- $CoK(x) = \{a, b, e, 1\}$
- $K(y) = \{(d + c), (ae + c), (ad + c + f), (ade + cd + ce + ef + h)\}$
- $CoK(y) = \{c, d, e, 1\}$

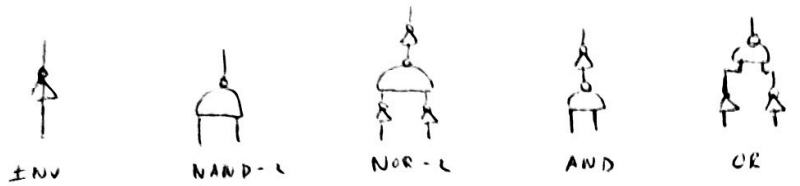
1.5

Best multicube divisor $= \{ad + c + e + ag\} \cap \{ad + c + f\} = \{ad + c\} = w$

New node: $w = ad + c$

$$\begin{cases} x = wb + be + abg + ae \\ y = w(e + d) + ce + ef + h \end{cases}$$

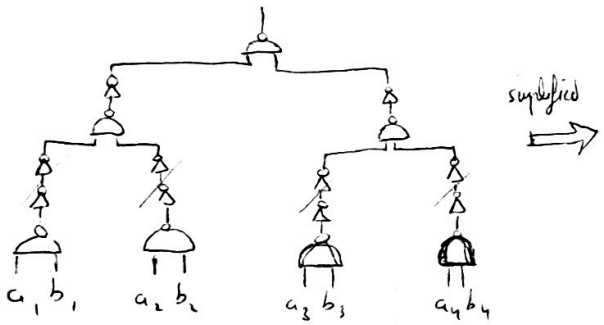
3) • Library patterns

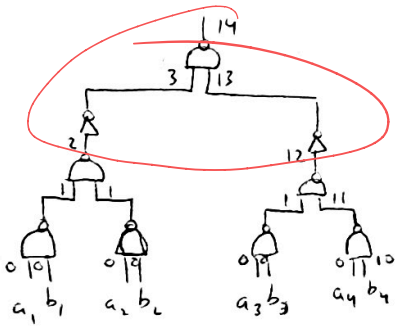


Area = 1 | Area = 2 | Area = 5 | Area = 3 | Area = 4

1

• $f = a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4$



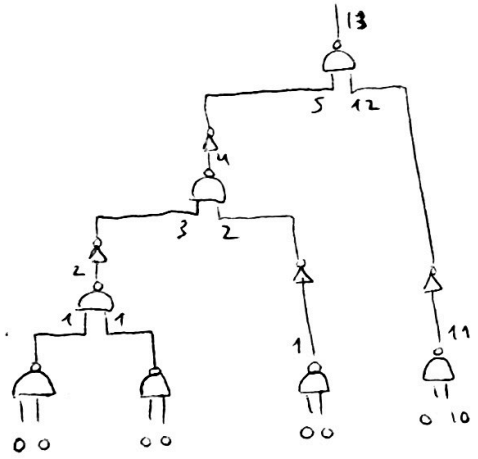


cost = 15

Area = 7 NAND-2 + 2 INV = 14 + 2 = 16 units

Delay = 14 units of time

• We can optimize in terms of delay with a new distribution



Area = 16 units

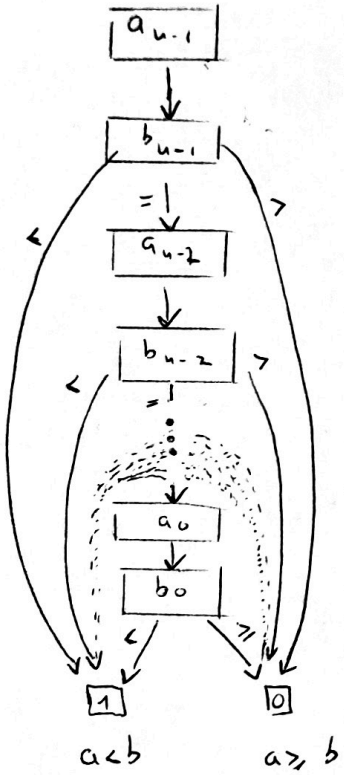
Delay = 13 units

Incorrect mapping (wrong polarity)

Every path should have an even number of bubbles

4) • Best variable order to minimize BDD size ?

$a_{n-1} < b_{n-1} < a_{n-2} < b_{n-2} < \dots < a_0 < b_0$

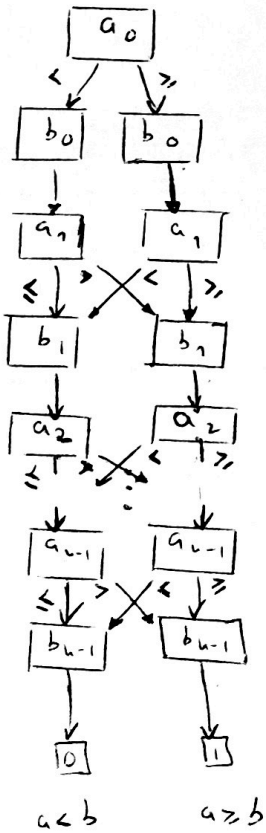


- the size is $O(2n)$ where n is the binary size of a, b.

This does not look like a BDD

• Best variable order to maximize BDD size ?

$a_0 < b_0 < a_1 < b_1 < \dots < a_{n-1} < b_{n-1}$

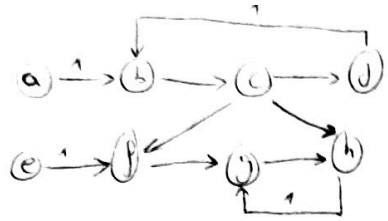


- the size is $2(n-1) + 2n = O(4n-2)$ where n is the binary representation of a, b

Worst case is exponential

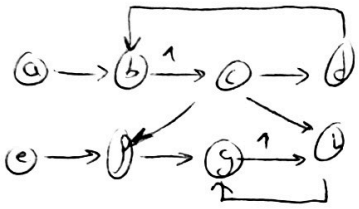
$a_0 < b_0 < a_1 < b_1 \dots$

5)



- the minimum period is 5 and corresponds to the critical path : b → c → f → g → h

- P_{min} after retiming ?



- the critical path after retiming is c → d → b with a period of 3.

2

- Is the min. number of registers R_{min} achievable after retiming ?

  - Each feedback loop has to be broken by at least one register and there are 2 loops in the combinational circuit.

  the $R_{min} = 2$ and it is achieved with the previous retimed sequential circuit.

- the previous retimed circuit achieves both P_{min} and R_{min}.