Homework for Computational Complexity — Created: 25/03/2020
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Homework 2

For a graph $G = (V, E)$ and an integer $k \geq 2$, a valid k-coloring of G is a map $f : V \to \{1, \ldots, k\}$ with the property that $f(u) \neq f(v)$ for every edge $\{u, v\} \in E$. Consider the decision and search versions of the associated computational problem:

k-COL: Given a graph $G = (V, E)$, does it have a valid k-coloring?

SEARCH k-COL: Given a graph $G = (V, E)$, find a valid 3-coloring, if there is one.

Exercise: Self-reducibility Show that if the decision version k-COL of the k-coloring problem can be solved in polynomial time, then its search version SEARCH-k-COL can also be solved in polynomial time.

Exercise: Zero-error Show that if k-COL is in BPP, then it is also in ZPP. In other words, show that if there is a probabilistic polynomial time algorithm that decides whether a graph has a valid k-coloring, and does so with bounded error, then there is also a probabilistic polynomial time algorithm that does that with zero error (in the sense of the definition of ZPP).

Exercise: Logarithmic space Show that 2-COL is in P. Show that it is even in NL. [Since it is known that 3-COL is NP-complete, make sure that you are using something special about 2-COL in your proof, and state what.]