Homework for Computational Complexity — Created: 30/04/2020
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Instructor: Albert Atserias

Homework

Exercise: Oracles in BPP Prove that $\mathrm{BPP}^{\mathrm{BPP}} = \mathrm{BPP}$. In words: if a decision problem A can be solved by a probabilistic polynomial-time algorithm with bounded-error by making queries to an oracle for a decision problem that itself can be solved by a probabilistic polynomial-time algorithm with bounded-error, then A can also be solved by a probabilistic polynomial-time algorithm with bounded-error that does not make any query (here you can see why the notation for complexity classes is sometimes useful ...).

Exercise: Co-classes and the hierarchy Prove that if $\Sigma_i^{\mathrm{P}} = \Pi_i^{\mathrm{P}}$ for some $i \geq 1$, then the polynomial-time hierarchy collapses to its i-th level; i.e., $\Sigma_i^{\mathrm{P}} = \Pi_i^{\mathrm{P}} = \mathrm{PH}$.

Exercise: Circuits and the hierarchy Prove that if $\mathrm{NP} \subseteq \mathrm{P/poly}$, then $\mathrm{NP}^{\mathrm{NP}} \subseteq \mathrm{P/poly}$. It is also possible to prove that $\mathrm{NP} \subseteq \mathrm{P/poly}$ implies $\Sigma_2^{\mathrm{P}} = \Pi_2^{\mathrm{P}}$. This is called the "Karp-Lipton Theorem". Look up the original article by Richard Karp and Richard Lipton that contains this theorem (part of the homework is to find this) and give a high level description of their proof (checking other sources is also ok; simpler but essentially equivalent proofs are known).