From DPLL to CDCL SAT solvers

Combinatorial Problem Solving (CPS)

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Overview of the session

- Conflict Analysis
  - Motivating example
  - Backjumping
  - Conflict graph
  - Lemma shortening
- Lemma removal
- Decision heuristics
- Restarts
- Efficient implementation of UnitProp:
  - Occur lists
  - Two-watched literals
- Final remarks

Motivating Example

$$\emptyset \Longrightarrow$$

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

Motivating Example

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$p_{25} \Longrightarrow$

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p_{25}

$\emptyset \Longrightarrow$
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$p_{25}\, \overline{p}_{21}^{d} \Longrightarrow$

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$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6 \Longrightarrow$

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$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6 \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17} \Longrightarrow$

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$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d} \Longrightarrow$

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$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13} \Longrightarrow$
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$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19} \Longrightarrow$

Motivating Example

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$$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19} \Longrightarrow$$
$$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19}\, \overline{p}_{20}^{d} \Longrightarrow$$

Motivating Example

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p_{25}

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$p_{25} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6 \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8 \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19}\, \overline{p}_{20}^{d} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19}\, \overline{p}_{20}^{d}\, p_{23} \Longrightarrow$

Motivating Example

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$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6 \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8 \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19}\, \overline{p}_{20}^{d} \Longrightarrow$

$p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19}\, \overline{p}_{20}^{d}\, p_{23} \Longrightarrow$

$\underbrace{p_{25}\, \overline{p}_{21}^{d}\, \overline{p}_6\, \overline{p}_{17}\, p_{22}^{d}\, \overline{p}_{13}\, p_8\, p_4^{d}\, p_{19}\, \overline{p}_{20}^{d}\, p_{23}}_{M}\, p_{11}^{d} \Longrightarrow$

Motivating Example

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p_{25}

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$p_{25}\, \overline{p}^d_{21} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6 \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8 \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4 \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20} \Longrightarrow$

$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23} \Longrightarrow$

$\underbrace{p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}}_{M}\; p^d_{11} \Longrightarrow$

$M\, p^d_{11} \Longrightarrow$

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

$\emptyset \Longrightarrow$
$p_{25} \Longrightarrow$
$p_{25}\, \overline{p}^d_{21} \Longrightarrow$
$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6 \Longrightarrow$
$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17} \Longrightarrow$
$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22} \Longrightarrow$
$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13} \Longrightarrow$
$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8 \Longrightarrow$
$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4 \Longrightarrow$
$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19} \Longrightarrow$
$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20} \Longrightarrow$
$p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23} \Longrightarrow$
$\underbrace{p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}}_{M}\, p^d_{11} \Longrightarrow$
$M\, p^d_{11} \Longrightarrow$

Before we continue, some notation:

- Literal p_{25} belongs to decision level (DL) 0
- Literals $\overline{p}^d_{21}$, $\overline{p}_6$, $\overline{p}_{17}$ belong to decision level 1
- ...

Motivating Example

M is $p_{25}\,\overline{p}_{21}^{d}\,\overline{p}_{6}\,\overline{p}_{17}\,p_{22}^{d}\,\overline{p}_{13}\,p_{8}\,p_{4}^{d}\,p_{19}\,\overline{p}_{20}^{d}\,p_{23}$

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$

Motivating Example

M is $p_{25}\,\overline{p}_{21}^{d}\,\overline{p}_{6}\,\overline{p}_{17}\,p_{22}^{d}\,\overline{p}_{13}\,p_{8}\,p_{4}^{d}\,p_{19}\,\overline{p}_{20}^{d}\,p_{23}$

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

$M\,p_{11}^{d} \Longrightarrow$
$M\,p_{11}^{d}\,\overline{p}_{12} \Longrightarrow$

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26} \Longrightarrow$

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5 \Longrightarrow$

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5\, p_{18} \Longrightarrow$

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5\, p_{18} \Longrightarrow$

conflict!

Motivating Example

$\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
$\overline{p}_{11} \vee p_{13} \vee p_{16}$
$p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
$p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
$p_{10} \vee \overline{p}_8 \vee p_1$
$p_{10} \vee p_3$
$\overline{p}_3 \vee p_{26}$
$p_{10} \vee \overline{p}_5$
$\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
$\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
$p_{21} \vee \overline{p}_6$
$p_{21} \vee \overline{p}_{17}$
$\overline{p}_{22} \vee \overline{p}_{13}$
$p_{13} \vee p_8$
$\overline{p}_4 \vee p_{19}$
$p_{20} \vee p_{23}$
$\overline{p}_{20} \vee p_{24}$
p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

$M\, p^d_{11} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26} \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5 \Longrightarrow$
$M\, p^d_{11}\, \overline{p}_{12}\, p_{16}\, \overline{p}_2\, \overline{p}_{10}\, p_1\, p_3\, p_{26}\, \overline{p}_5\, p_{18} \Longrightarrow$

conflict!

- Let's try to find out the causes of the conflict
- First of all we will compute,
  for each literal of the current decision level,
  the reason why it is true

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

Literal	p_{11}^d	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\perp$	1	2	3	4	5	6	7	8	9

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

Literal	p_{11}^d	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\perp$	1	2	3	4	5	6	7	8	9

Let us take the conflicting clause $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$.
p_{18} is true due to clause 9. Resolution gives:

$$\frac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}}$$

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

Literal	p_{11}^d	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\perp$	1	2	3	4	5	6	7	8	9

Let us take the conflicting clause $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$.
p_{18} is true due to clause 9. Resolution gives:

$$\frac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}}$$

Now the last assigned literal that is false in the new clause is p_5.
The reason why p_5 is false is clause 8.
Again, resolution:

$$\frac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

Literal	p_{11}^d	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\perp$	1	2	3	4	5	6	7	8	9

Let us take the conflicting clause $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$.
p_{18} is true due to clause 9. Resolution gives:

$$\frac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}}$$

Now the last assigned literal that is false in the new clause is p_5.
The reason why p_5 is false is clause 8.
Again, resolution:

$$\frac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

The process is now iterated...

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

Literal	p^d_{11}	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

Literal	p^d_{11}	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

Literal	p^d_{11}	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

Literal	p^d_{11}	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8}$$

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

Literal	p^d_{11}	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}}$$

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

M is $p_{25}\ \overline{p}^d_{21}\ \overline{p}_6\ \overline{p}_{17}\ p^d_{22}\ \overline{p}_{13}\ p_8\ p^d_4\ p_{19}\ \overline{p}^d_{20}\ p_{23}$

Literal	p^d_{11}	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}}$$

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

Literal	p^d_{11}	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}} \qquad \overline{p}_{11} \vee p_{13} \vee p_{16}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{11} \vee p_{13}}$$

Motivating Example

M is $p_{25}\ \overline{p}^d_{21}\ \overline{p}_6\ \overline{p}_{17}\ p^d_{22}\ \overline{p}_{13}\ p_8\ p^d_4\ p_{19}\ \overline{p}^d_{20}\ p_{23}$

Literal	p^d_{11}	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16} \qquad \overline{p}_{11} \vee p_{13} \vee p_{16}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{11} \vee p_{13}} \qquad \overline{p}_{11} \vee p_6 \vee \overline{p}_{12}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{11} \vee p_{13} \vee p_6}$$

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

Literal	p^d_{11}	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16} \qquad \overline{p}_{11} \vee p_{13} \vee p_{16}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{11} \vee p_{13}} \qquad \overline{p}_{11} \vee p_6 \vee \overline{p}_{12}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{11} \vee p_{13} \vee p_6}}$$

Now the process cannot continue any longer.

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

Literal	p^d_{11}	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16} \qquad \overline{p}_{11} \vee p_{13} \vee p_{16}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{11} \vee p_{13}} \qquad \overline{p}_{11} \vee p_6 \vee \overline{p}_{12}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{11} \vee p_{13} \vee p_6}$$

All obtained clauses are false in the assignment.

Motivating Example

1. $\overline{p}_{11} \vee p_6 \vee \overline{p}_{12}$
2. $\overline{p}_{11} \vee p_{13} \vee p_{16}$
3. $p_{12} \vee \overline{p}_{16} \vee \overline{p}_2$
4. $p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}$
5. $p_{10} \vee \overline{p}_8 \vee p_1$
6. $p_{10} \vee p_3$
7. $\overline{p}_3 \vee p_{26}$
8. $p_{10} \vee \overline{p}_5$
9. $\overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}$
10. $\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18}$
11. $p_{21} \vee \overline{p}_6$
12. $p_{21} \vee \overline{p}_{17}$
13. $\overline{p}_{22} \vee \overline{p}_{13}$
14. $p_{13} \vee p_8$
15. $\overline{p}_4 \vee p_{19}$
16. $p_{20} \vee p_{23}$
17. $\overline{p}_{20} \vee p_{24}$
18. p_{25}

M is $p_{25}\, \overline{p}^d_{21}\, \overline{p}_6\, \overline{p}_{17}\, p^d_{22}\, \overline{p}_{13}\, p_8\, p^d_4\, p_{19}\, \overline{p}^d_{20}\, p_{23}$

Literal	p^d_{11}	$\overline{p}_{12}$	p_{16}	$\overline{p}_2$	$\overline{p}_{10}$	p_1	p_3	p_{26}	$\overline{p}_5$	p_{18}
Reason	$\emptyset$	1	2	3	4	5	6	7	8	9

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_{18} \qquad \overline{p}_1 \vee \overline{p}_3 \vee p_5 \vee p_{17} \vee p_{18}}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_5 \vee p_{17}} \qquad p_{10} \vee \overline{p}_5}{\overline{p}_3 \vee \overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}} \qquad p_{10} \vee p_3}{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10}}$$

$$\dfrac{\dfrac{\dfrac{\overline{p}_{19} \vee \overline{p}_1 \vee p_{17} \vee p_{10} \qquad p_{10} \vee \overline{p}_8 \vee p_1}{\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_8} \qquad p_2 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{10}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}} \qquad p_{12} \vee \overline{p}_{16} \vee \overline{p}_2}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16}}$$

$$\dfrac{\dfrac{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{16} \qquad \overline{p}_{11} \vee p_{13} \vee p_{16}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee p_{12} \vee \overline{p}_{11} \vee p_{13}} \qquad \overline{p}_{11} \vee p_6 \vee \overline{p}_{12}}{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee \overline{p}_4 \vee p_{20} \vee \overline{p}_{11} \vee p_{13} \vee p_6}$$

Lits assigned at last decision level 5 in blue

Motivating Example

- Three clauses with only one literal assigned at the last DL (5):
  - $\overline{p}_{19} \vee p_{17} \vee p_{10} \vee \overline{p}_{8}$ (max DL of others: 3)
  - $\overline{p}_{19} \vee p_{17} \vee \overline{p}_{8} \vee p_{2} \vee \overline{p}_{4} \vee p_{20}$ (max DL of others: 4)
  - $\overline{p}_{19} \vee p_{17} \vee \overline{p}_{8} \vee \overline{p}_{4} \vee p_{20} \vee \overline{p}_{11} \vee p_{13} \vee p_{6}$ (max DL of others: 4)
- If we had had those clauses:
  - At DL. 3 we could've propagated p_{10}
  - At DL. 4 we could've propagated p_2
  - At DL. 4 we could've propagated $\overline{p}_{11}$
- In practice procedure stops when we find the first such clause, because:
  - It is the cheapest one to find
  - It can propagate lits at a lower DL
    (literals of previous decision levels are never removed)

Backjump rule

This example motivates us to introduce the rule:

$$\text{Backjump} \quad M\, l^{\mathsf{d}}\, N \parallel F \Longrightarrow M\, l' \parallel F \quad \textbf{if} \begin{cases} \text{for some clause } C \vee l' : \\ \quad F \models C \vee l' \text{ and } M \models \neg C \\ \quad l' \text{ is undefined in } M \\ \quad l' \text{ or } \neg l' \text{ occurs in } F \end{cases}$$

The only thing we need is a backjump clause $C \vee l'$ such that:

1. It is a logical consequence of the rest of the clauses
2. All its literals are false at some previous decision level d, except one which was undefined at d

Conflict Analysis

- The procedure shown in the example is called conflict analysis
- Why is the obtained clause a logical consequence of the input?
  - Because resolution is correct

Conflict Analysis

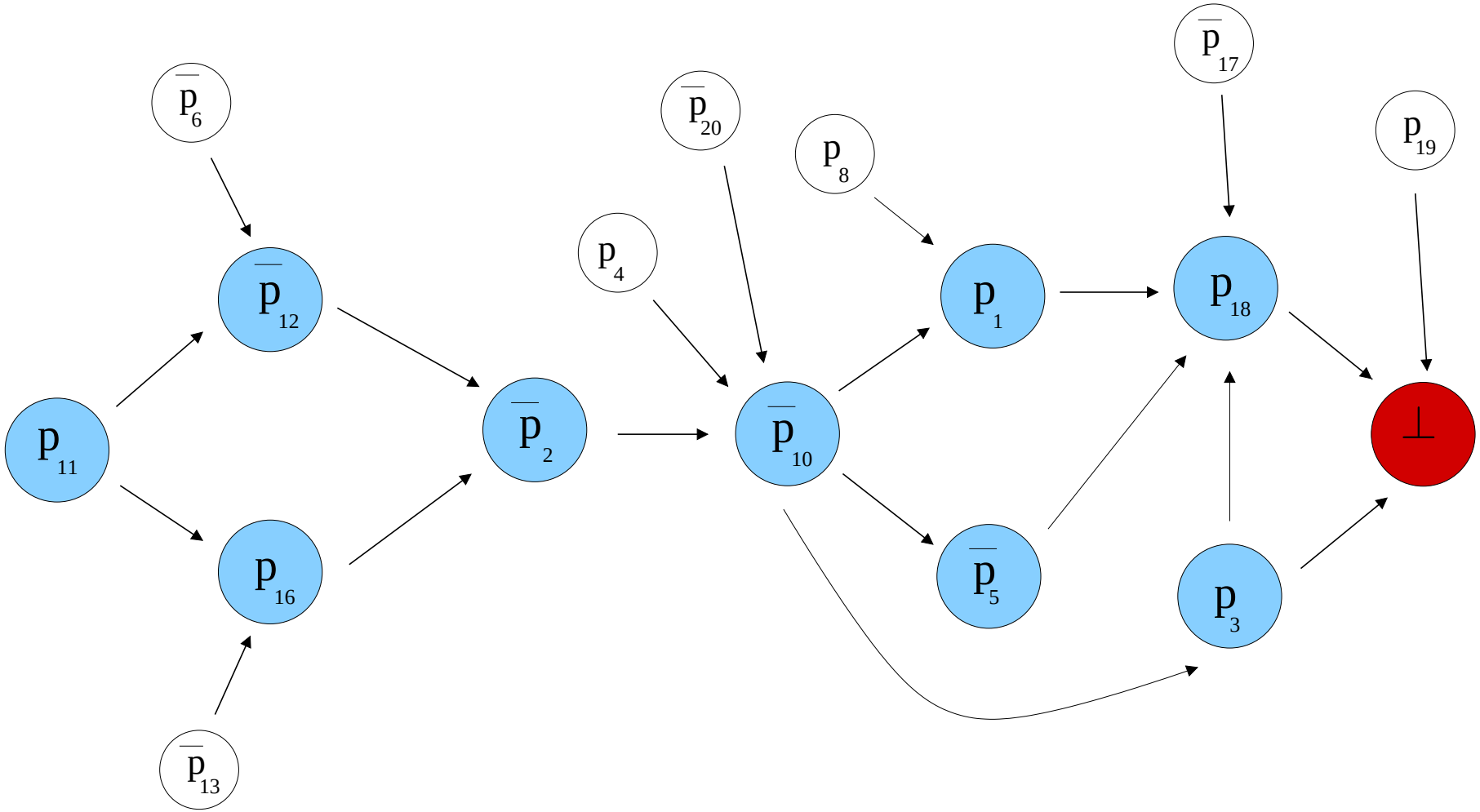
- The procedure shown in the example is called conflict analysis
- Why is the obtained clause false under the current assignment?
  - Conflicting clause is false under the current assignment
  - Each non-decision lit l false at the last decision level (dl) can be resolved away with a reason clause of the form $l_1 \vee \cdots \vee l_n \vee \neg l$
  - l is replaced by lits $l_1, \ldots, l_n$ such that all of them are false

Conflict Analysis

- The procedure shown in the example is called conflict analysis
- Why a clause with only one lit at last decision level (dl) can be obtained?
  - Conflicting clause has at least two lits at decision level dl
    (provided unit propagation applied before any decision)
  - Each non-decision lit l at decision level dl is replaced by lits $l_1, \ldots, l_n$ such that
    1. All of them have been added to the assignment before l
       (hence their decision level is $\leq dl$)
    2. At least one was set at decision level dl (again, provided ....)
  - In the worst case, we will terminate with
    a clause with the last decision lit being the only set at decision level dl

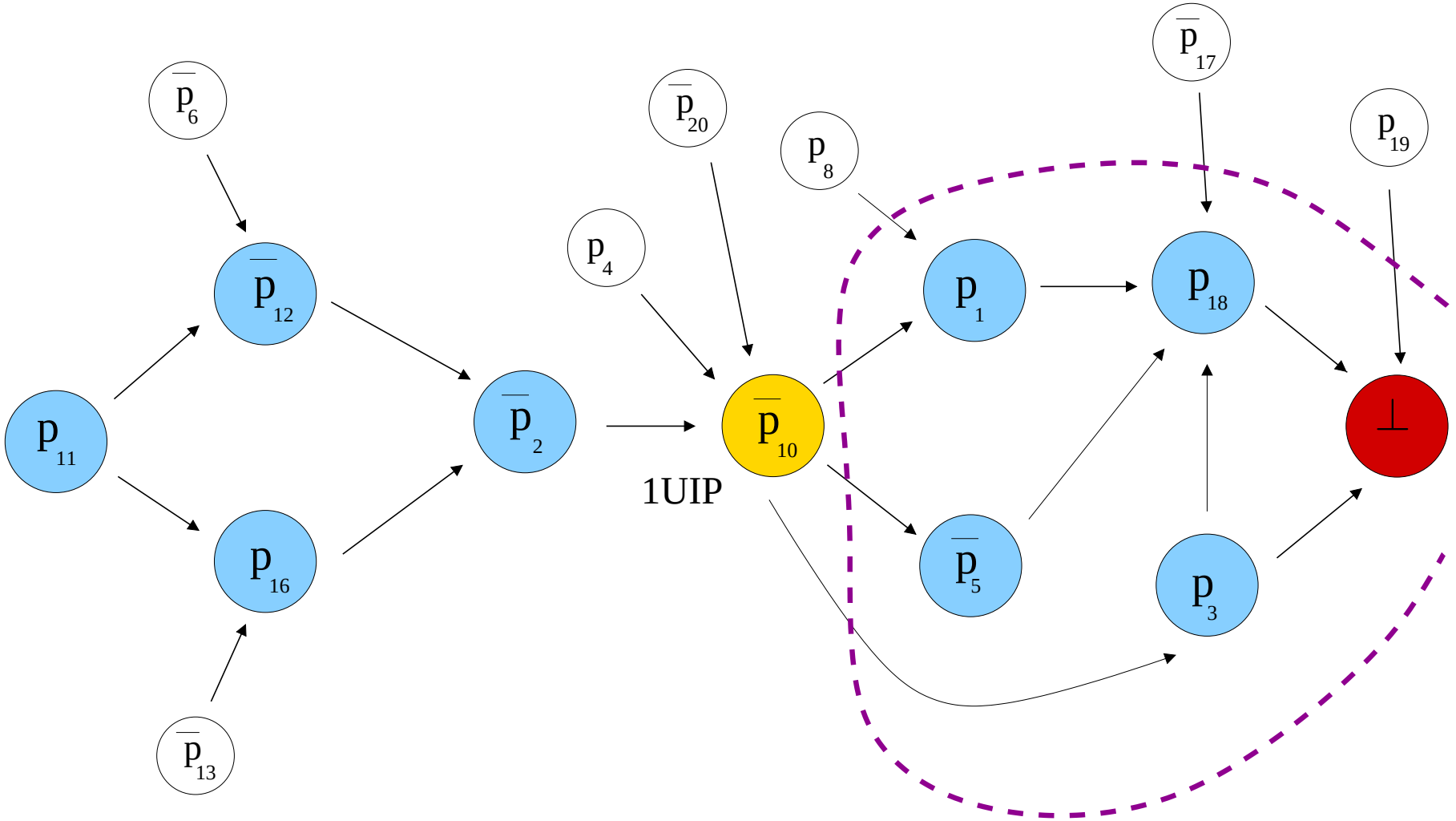
Conflict Analysis - Conflict Graph

- Situation can be represented with the conflict graph:



Conflict Analysis - Conflict Graph

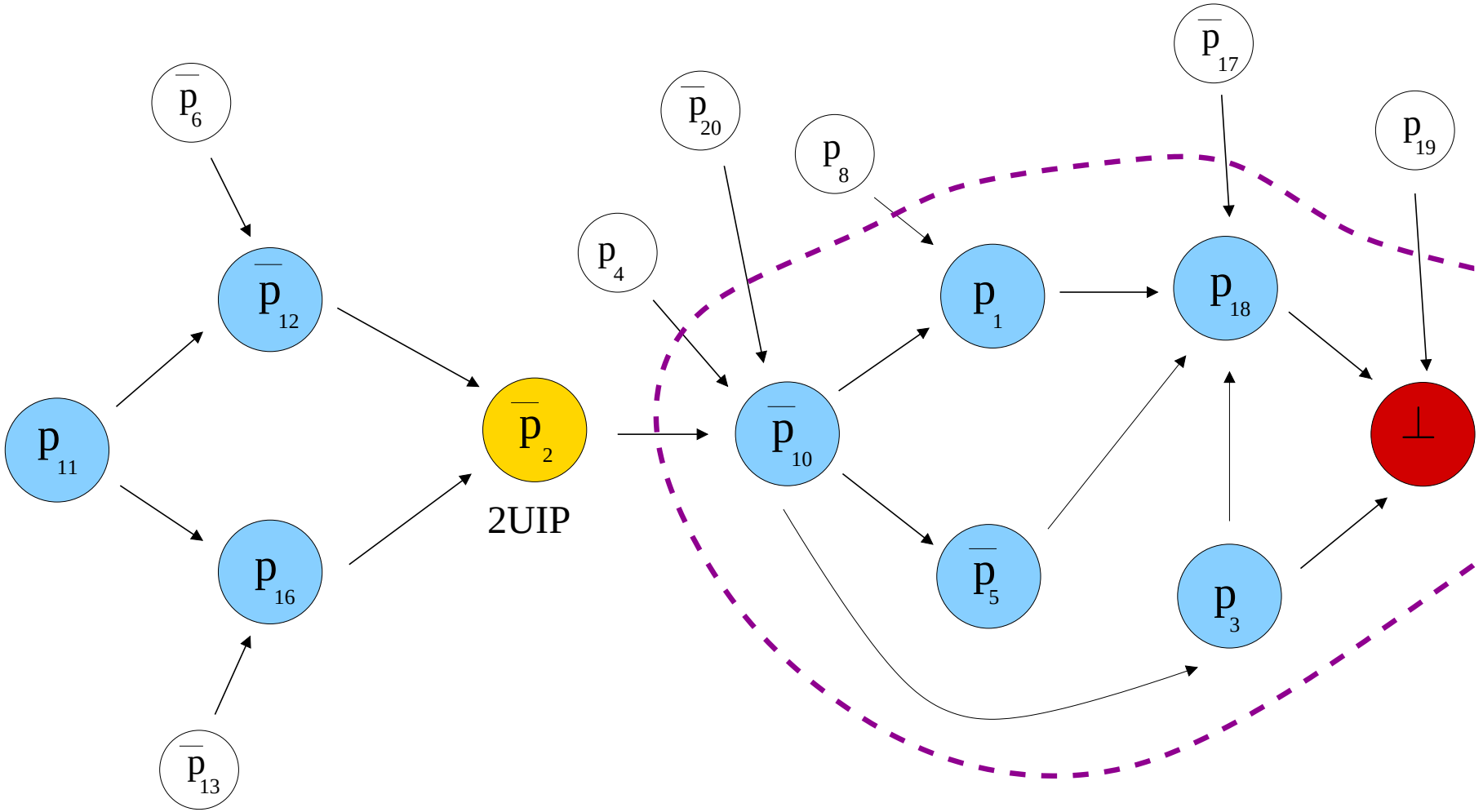
- The cut represents 1-UIP learning scheme (UIP ≡ Unique Implication Point)



- Backjump clause is $\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_{10}$

Conflict Analysis - Conflict Graph

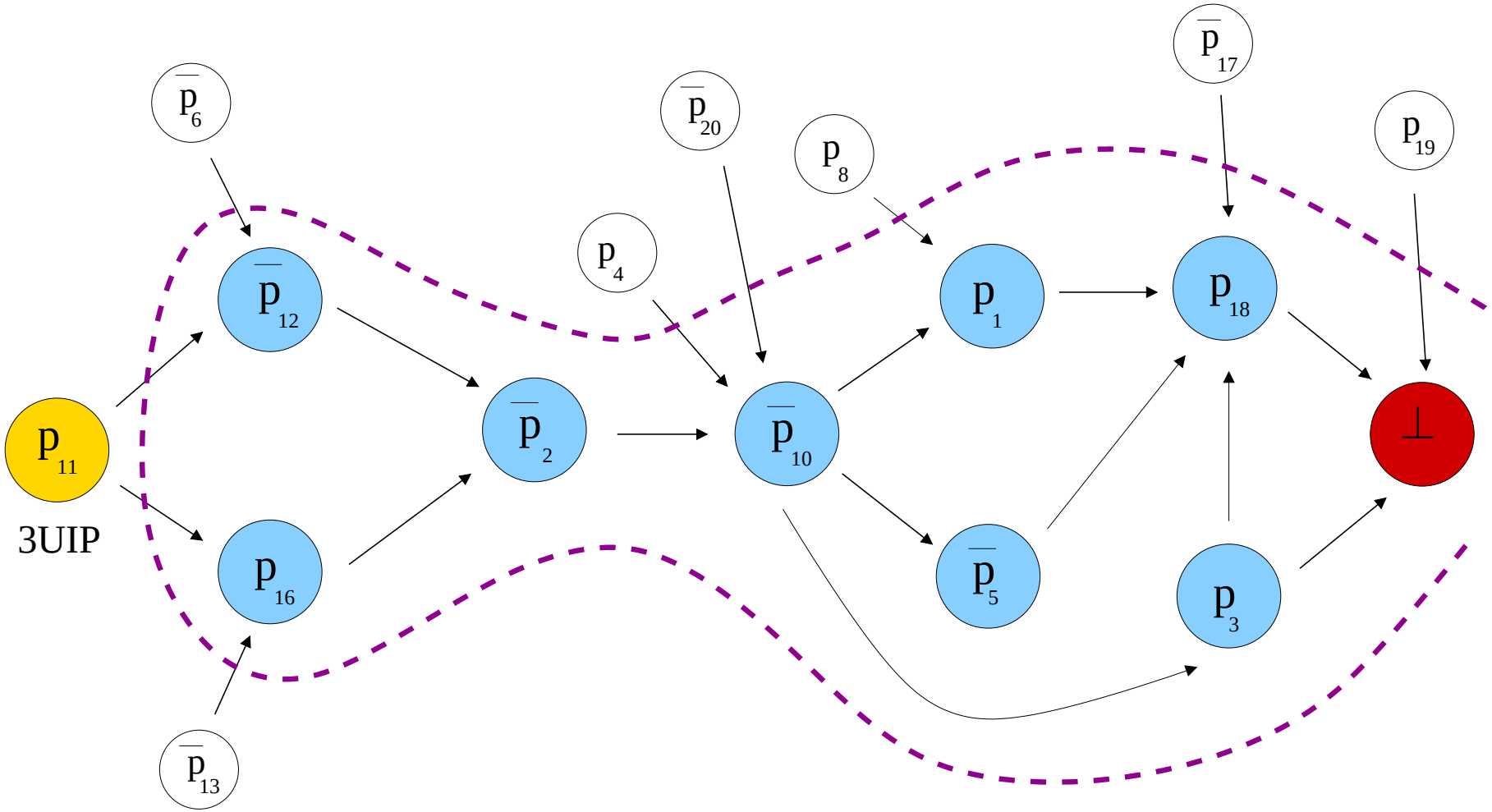
- This cut represents the 2-UIP learning scheme



- Backjump clause is $\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_{20} \vee \overline{p}_4 \vee p_2$

Conflict Analysis - Conflict Graph

- This cut represents the 3-UIP learning scheme



- Backjump clause is $\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_{20} \vee \overline{p}_4 \vee p_6 \vee \overline{p}_{11} \vee p_{13}$

What is a good lemma?

- Every time a conflict is found, conflict analysis is started
- Backjump clause is added to the clause database (we say it is learned):

  Learn
  $$M \parallel F \implies M \parallel F, C \ \textbf{if} \ \begin{cases} \text{all atoms of } C \text{ occur in } F \\ F \models C \end{cases}$$
- Backjump clauses once they are learned are referred to as lemmas
- Learning them helps to prevent future similar conflicts
- The set of literals of previous DL in the 2UIP
  contains the literals of previous DL in the 1UIP
- So 1UIP gives shorter clauses than 2UIP
- Also 1UIP allows one to backjump to a lower or equal DL
- However, in general difficult to assess in advance the quality of a lemma

Lemma Shortening

- But, given a lemma L, any lemma $L' \subseteq L$ is clearly better.
- Given L, how to obtain a possible L'?
- **LOCAL MINIMIZATION:**
  - Generate lemma L and mark its literals
  - Remove non-decision literals $l \in L$ such that $reason(\bar{l}) \setminus \{\bar{l}\}$ contains only marked literals

EXAMPLE: our 2-UIP clause was

$$\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}$$

with $reason(p_{19}) = \overline{p}_4 \vee p_{19}$. Hence $\overline{p}_{19}$ can be removed. Why?

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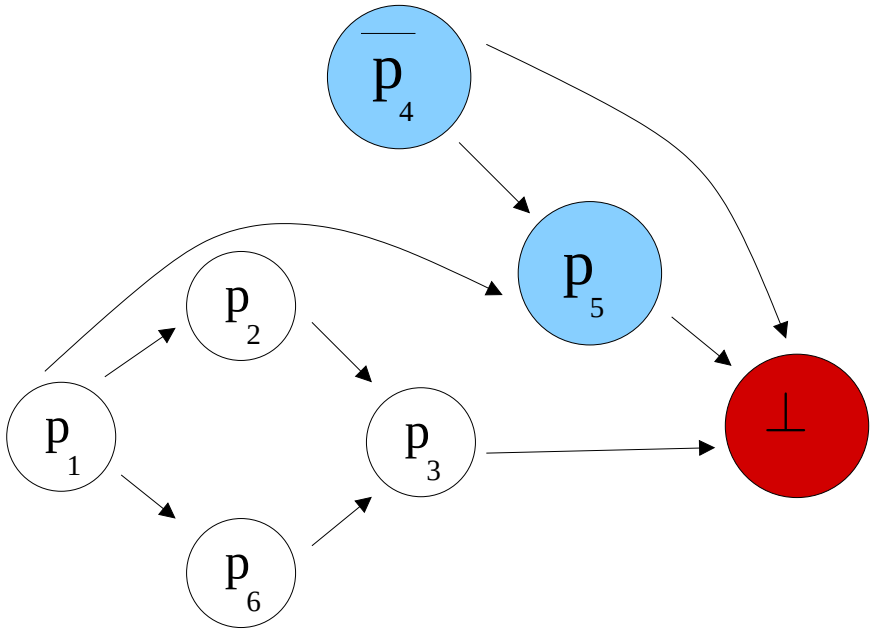
$$\frac{\overline{p}_{19} \vee p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20} \qquad \overline{p}_4 \vee p_{19}}{p_{17} \vee \overline{p}_8 \vee p_2 \vee \overline{p}_4 \vee p_{20}}$$

Lemma Shortening

- **RECURSIVE MINIMIZATION:**
  - Generate lemma L and mark its literals
  - Remove non-decision literals $l \in L$ such that search backwards from $\overline{l}$ in the implication graph reaches only negations of marked literals

1. $\overline{p}_1 \vee p_2$
2. $\overline{p}_1 \vee p_6$
3. $\overline{p}_2 \vee \overline{p}_6 \vee p_3$
4. $\overline{p}_1 \vee p_4 \vee p_5$
5. $\overline{p}_3 \vee p_4 \vee \overline{p}_5$

$\emptyset \Longrightarrow \ldots \Longrightarrow p_1{}^{\mathsf{d}}\, p_2\, p_6\, p_3\, \overline{p}_4{}^{\mathsf{d}}\, p_5$



- 1UIP lemma is $\overline{p}_3 \vee p_4 \vee \overline{p}_1$
- $\overline{p}_3$ is clearly removable

Overview of the session

- Conflict Analysis
  - Motivating example
  - Backjumping
  - Conflict graph
  - Lemma shortening
- Lemma removal
- Decision heuristics
- Restarts
- Efficient implementation of UnitProp:
  - Occur lists
  - Two-watched literals
- Final remarks

Lemma Removal

- Effects of adding lemmas:
  - \+ Reduces the search space
  - \- Space traversal slower since unit propagation becomes expensive
- Hence we cannot keep all generated lemmas. We need:

  Forget
  $M \parallel F, C \implies M \parallel F \ \textbf{if} \ \ F \models C$
- Which lemmas to keep and which ones to forget?
  - Each lemma has a number called activity
  - Activity is incremented when lemma is used in conflict analysis
  - From time to time, lemmas with low activity are removed
  - Mixed policies: keep
    - short lemmas
    - recent lemmas
    - lemmas with low Literals Blocks Distance (LDB): no. of different decision levels involved in a clause

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Decision Heuristic

- SAT instances may have (tens, hundreds of ?) thousands of variables
- Most SAT instances have clusters of variables:
  sets of variables that are semantically linked

GOAL: to force the SAT solver to work on one cluster at a time

- Each variable has an associated activity
- Each time it appears in a conflict analysis, its activity is incremented
- Recent activity should be given more importance:
  - Divide all activities by integer K from time to time, or
  - Keep increasing the activity increment
- Decide chooses unassigned var with highest activity
- Note that heuristic does not depend on clauses: CHEAP!
- Value selection: take $false$, or last tried value for that variable (last phase)

Overview of the session

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Restarts

- Early mistakes in the search tree have dramatic effects in running time

HOW TO AVOID THIS BEHAVIOUR?

- Introduce restarts:

  Restart

  $$M \parallel F \implies \emptyset \parallel F$$

- Why should a new run behave differently? And why could it be better?
- In a new run, the solver may behave better among others thanks to:
  - the activities of the variables
  - the learned lemmas

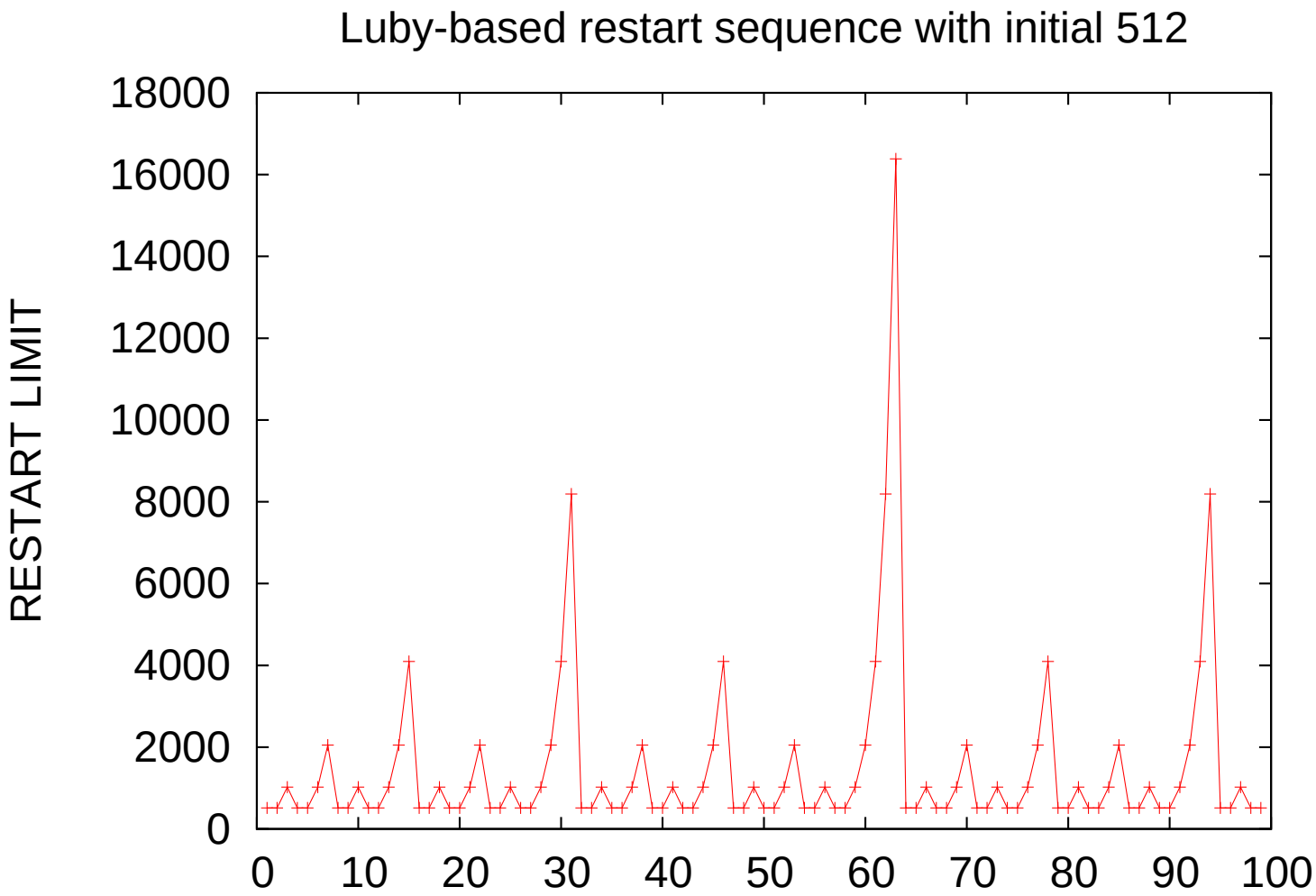
  Only the assignment is reset!

Restarts - Strategies

- Unrestricted application of Restart leads to incompleteness
- What is done in practice?
- A restart strategy tells after how many conflicts a restart should be made
  - Set initial RESTART_LIMIT
  - After RESTART_LIMIT conflicts:
    - Update RESTART_LIMIT according to the restart strategy
    - Apply Restart
- For example:
  - Let RESTART_LIMIT follow the Luby sequence, defined as:
    - $r_0 := N$; $r_i := N \cdot l_i$, where

$$l_i = \begin{cases} 2^{k-1} & \text{if } \exists k \text{ with } i = 2^k - 1 \\ l_{i-2^{k-1}+1} & \text{if } \exists k \text{ with } 2^{k-1} \leq i < 2^k - 1 \end{cases}$$

Restarts - Strategies



Restarts - Strategies

Another possibility is an inner-outer geometric sequence:

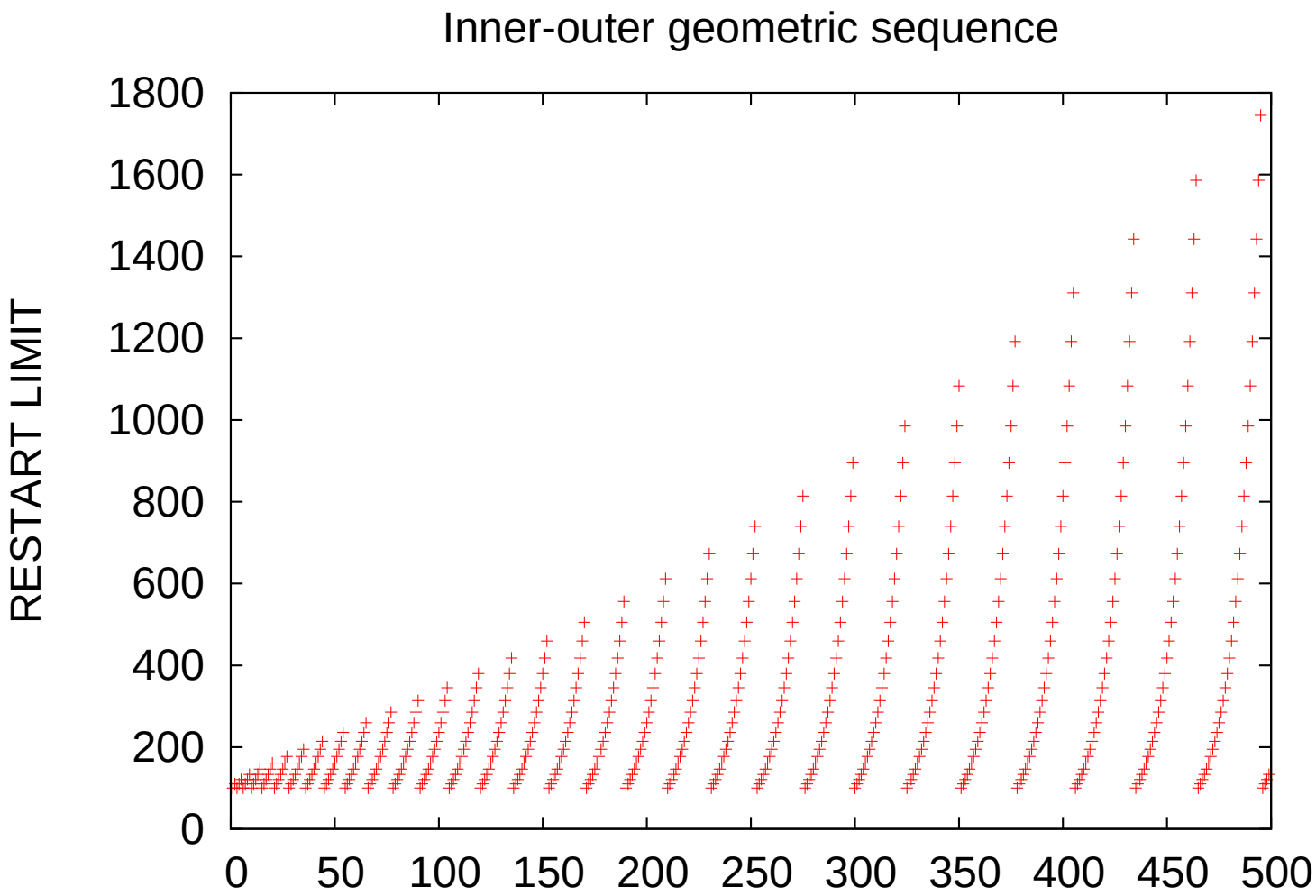
```
 for (int inner = 100, outer = 100;;){

  // Run SAT-solver for 'inner' conflicts

  if (inner >= outer){
    outer *= 1.1;
    inner = 100;
  }
  else
    inner *= 1.1

}
```

Restarts - Strategies



Overall CDCL algorithm

```
while(true){

    while (propagate_gives_conflict()){
        if (decision_level==0) return UNSAT;
        else analyze_conflict();
    }

    restart_if_applicable();
    remove_lemmas_if_applicable();

    if (!decide()) returns SAT; // All vars assigned
}
```

Overview of the session

- Conflict Analysis
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Performance of SAT Solvers

- The most important tasks that a SAT solver performs are:
  - Choose which variable to Decide on
  - Apply unit propagation exhaustively
  - Analyze conflicts
- When profiling a state-of-the art SAT solver we get:
  - Variable selection ($\approx 10\%$)
  - Unit propagation application ($\approx 80\%$)
  - Conflict analysis ($\approx 10\%$)
- Hence, the most important thing to optimize is unit propagation, aka BCP (Boolean Constraint Propagation)

BCP with Occur Lists

- BCP only has to detect propagating or conflicting clauses
- There is no need to detect that all clauses are true
- Instead of traversing the whole clause set again and again:
  - For each literal, store the clauses where it appears in occur lists
  - Every time a new lit l is added to the assignment, only clauses containing $\bar{l}$ need to be visited
- Let's see how it would work with an example

BCP with Occur Lists

1. p_1 $\overline{p}_2$ $\overline{p}_6$
2. p_2 $\overline{p}_3$ p_5 $\overline{p}_1$ $\overline{p}_6$
3. p_6 p_2 p_4
4. p_1 p_2
5. $\overline{p}_6$ $\overline{p}_1$ p_3
6. $\overline{p}_5$ p_4 p_2

Model

p_1	p_2	p_3	p_4	p_5	p_6
U	U	U	U	U	U

ToPropagate

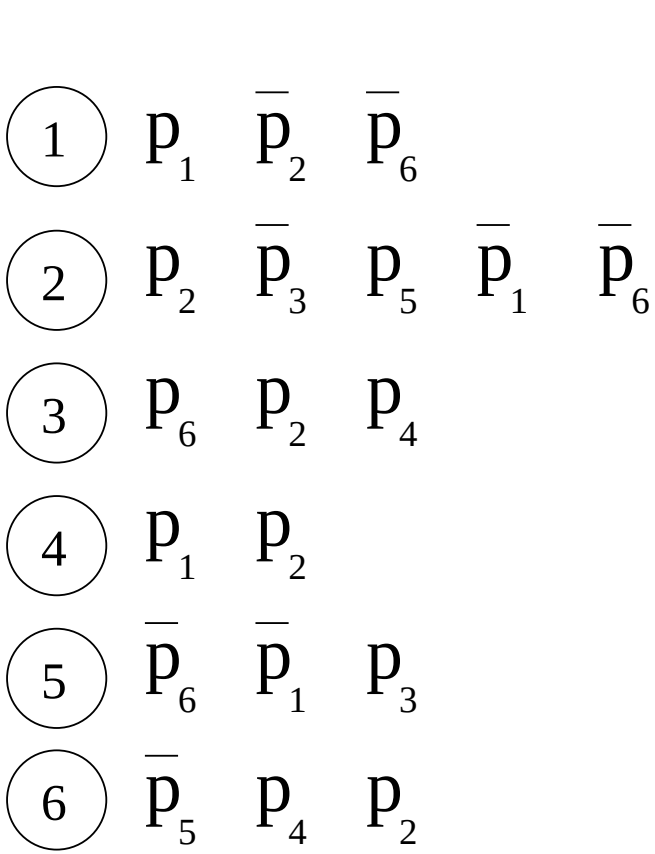
ClausesWith

p_1	1	4			
p_2	2	3	4	6	
p_3	5				
p_4	3	6			
p_5	2				
p_6	3				

$\overline{p}_1$	2	5	
$\overline{p}_2$	1		
$\overline{p}_3$	2		
$\overline{p}_4$			
$\overline{p}_5$	6		
$\overline{p}_6$	1	2	5

Current assignment: $\emptyset$

BCP with Occur Lists



Model

p_1	p_2	p_3	p_4	p_5	p_6
U	F	U	U	U	U

ToPropagate

$\overline{p}_2$

ClausesWith

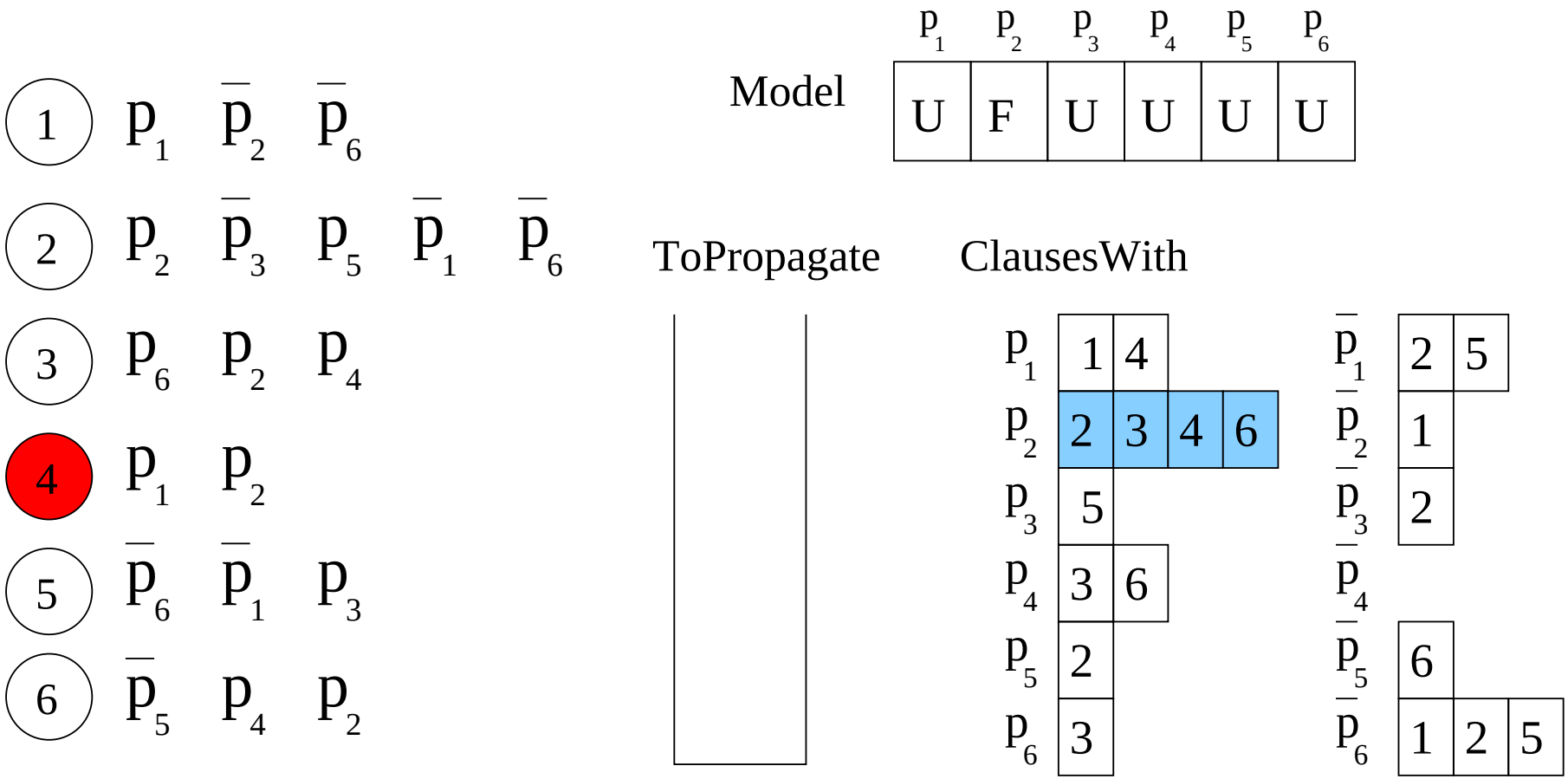
p_1	1	4		
p_2	2	3	4	6
p_3	5			
p_4	3	6		
p_5	2			
p_6	3			

$\overline{p}_1$	2	5	
$\overline{p}_2$	1		
$\overline{p}_3$	2		
$\overline{p}_4$			
$\overline{p}_5$	6		
$\overline{p}_6$	1	2	5

Current assignment: $\overline{p}_2{}^{\mathsf{d}}$

Now, we propagate $\overline{p}_2$ visiting ClausesWith$[p_2]$

BCP with Occur Lists



Current assignment: $\overline{p}_2{}^{\mathsf{d}}$

Literal p_1 has to be added to the assignment

BCP with Occur Lists

1. p_1 $\overline{p}_2$ $\overline{p}_6$
2. p_2 $\overline{p}_3$ p_5 $\overline{p}_1$ $\overline{p}_6$
3. p_6 p_2 p_4
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6. $\overline{p}_5$ p_4 p_2

Model

p_1	p_2	p_3	p_4	p_5	p_6
T	F	U	U	U	U

ToPropagate

p_1

ClausesWith

p_1	1	4		
p_2	2	3	4	6
p_3	5			
p_4	3	6		
p_5	2			
p_6	3			

$\overline{p}_1$	2	5	
$\overline{p}_2$	1		
$\overline{p}_3$	2		
$\overline{p}_4$			
$\overline{p}_5$	6		
$\overline{p}_6$	1	2	5

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1$

Now, we propagate p_1 visiting ClausesWith$[\overline{p}_1]$

BCP with Occur Lists

1. p_1 $\overline{p}_2$ $\overline{p}_6$
2. p_2 $\overline{p}_3$ p_5 $\overline{p}_1$ $\overline{p}_6$
3. p_6 p_2 p_4
4. p_1 p_2
5. $\overline{p}_6$ $\overline{p}_1$ p_3
6. $\overline{p}_5$ p_4 p_2

Model

p_1	p_2	p_3	p_4	p_5	p_6
T	F	U	U	U	U

ToPropagate

ClausesWith

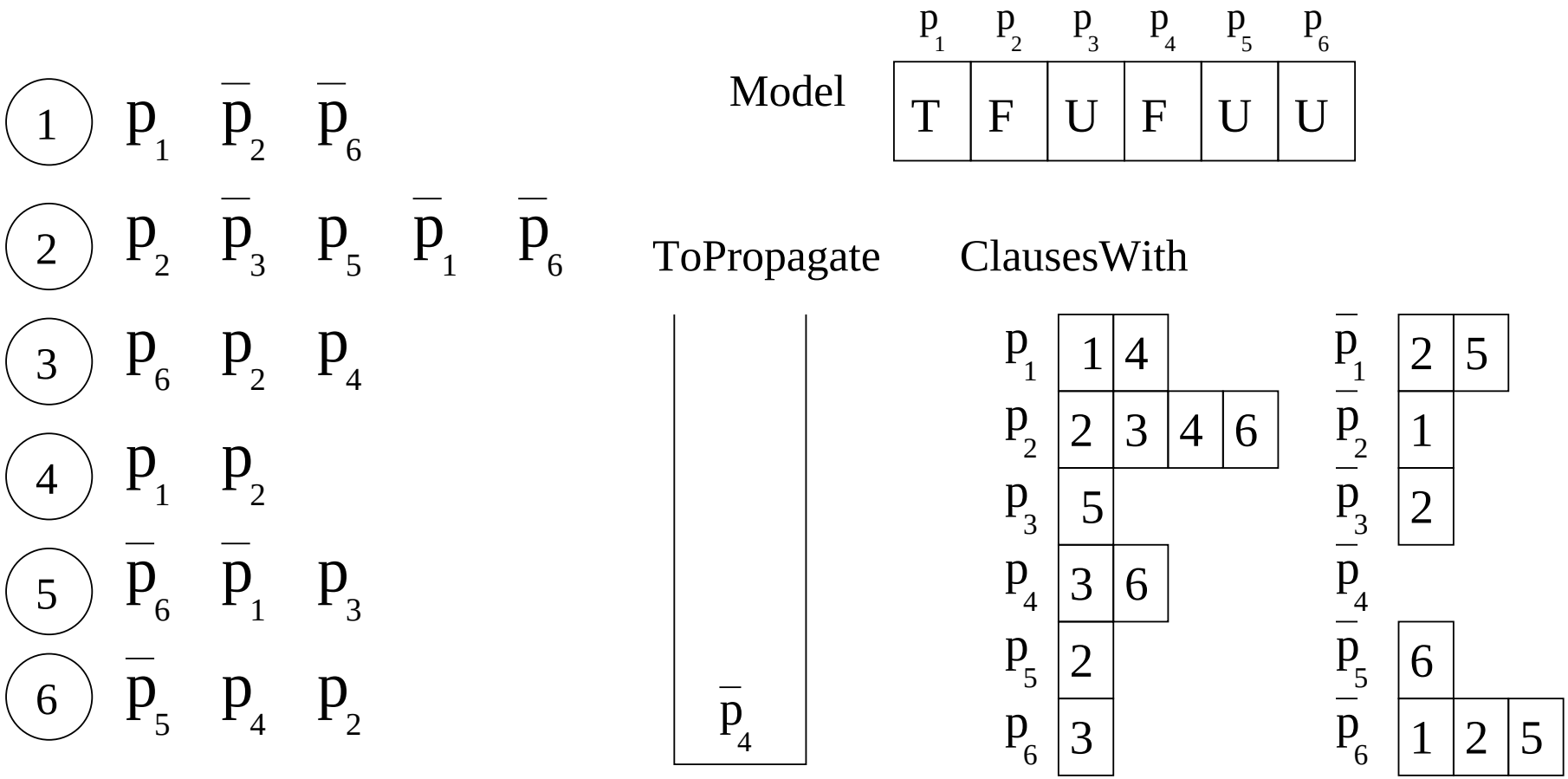
p_1	1	4
p_2	2	3	4	6
p_3	5			
p_4	3	6		
p_5	2			
p_6	3			

$\overline{p}_1$	2	5
$\overline{p}_2$	1	
$\overline{p}_3$	2	
$\overline{p}_4$		
$\overline{p}_5$	6	
$\overline{p}_6$	1	2	5

Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1$

No lit is propagated, we have to decide

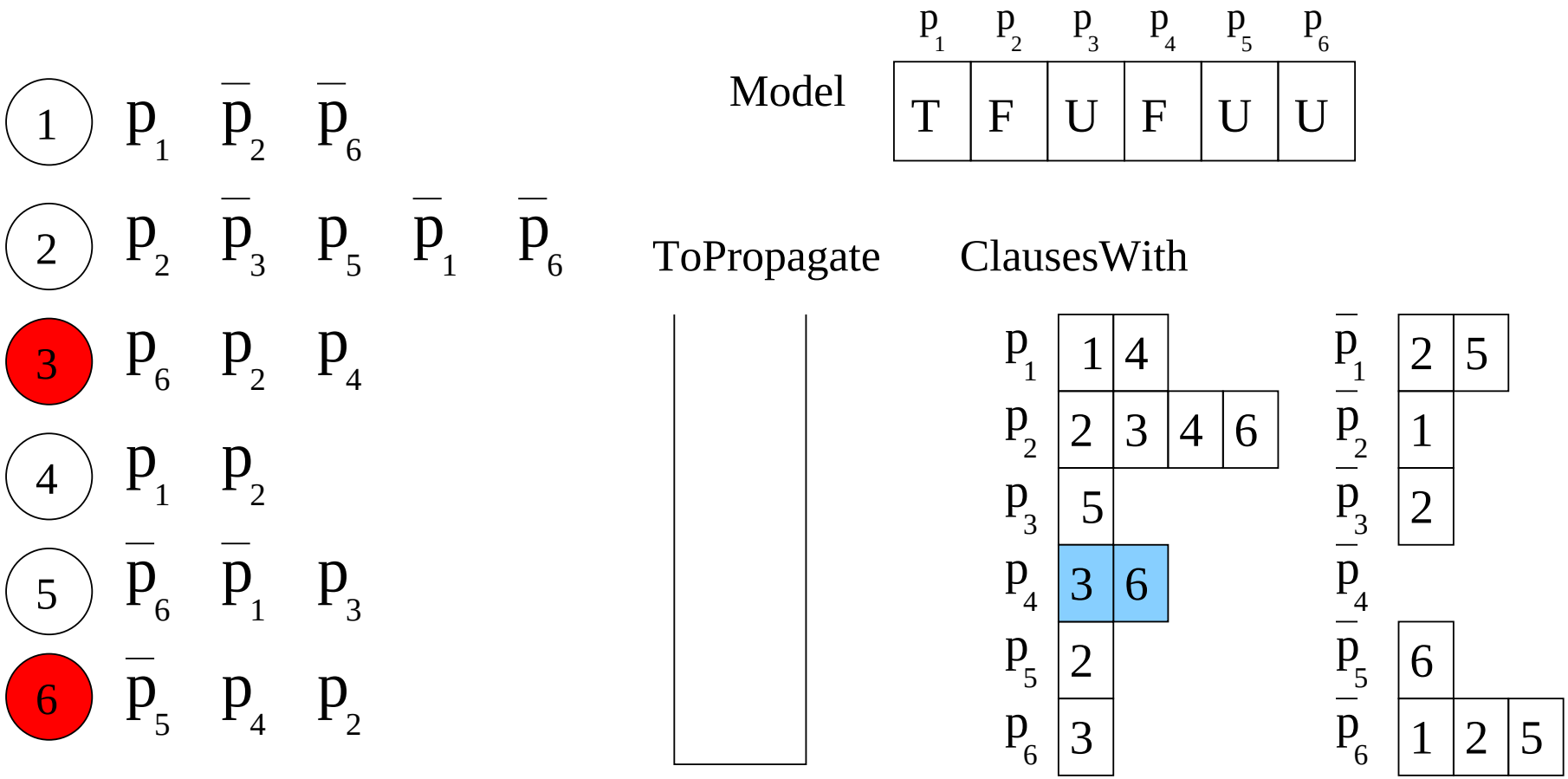
BCP with Occur Lists



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}$

Now, we propagate $\overline{p}_4$ visiting ClausesWith$[p_4]$

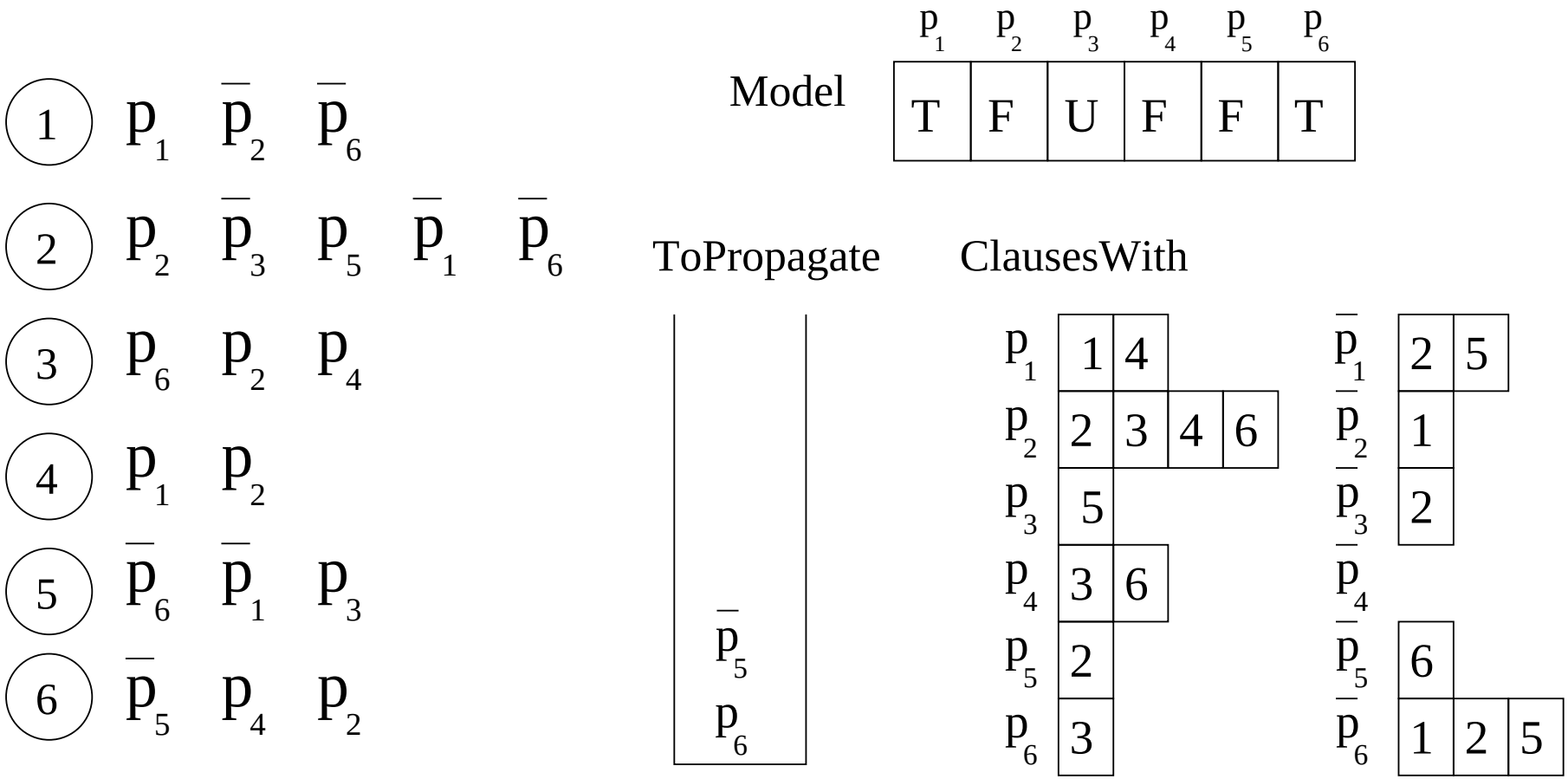
BCP with Occur Lists



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}$

Literals p_6, $\overline{p}_5$ have to be added to the assignment

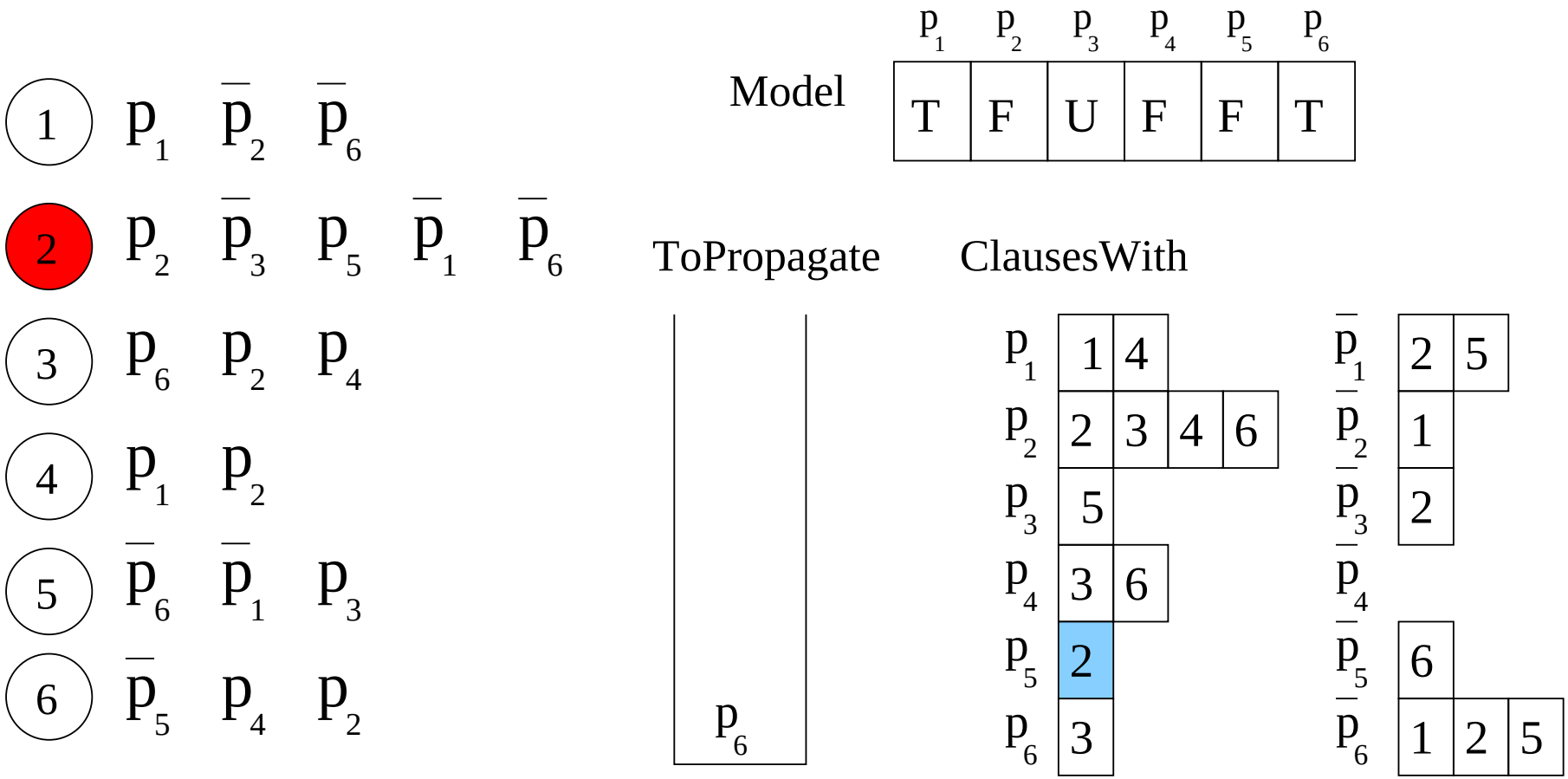
BCP with Occur Lists



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, p_6\, \overline{p}_5$

Now, we propagate $\overline{p}_5$ visiting ClausesWith$[p_5]$

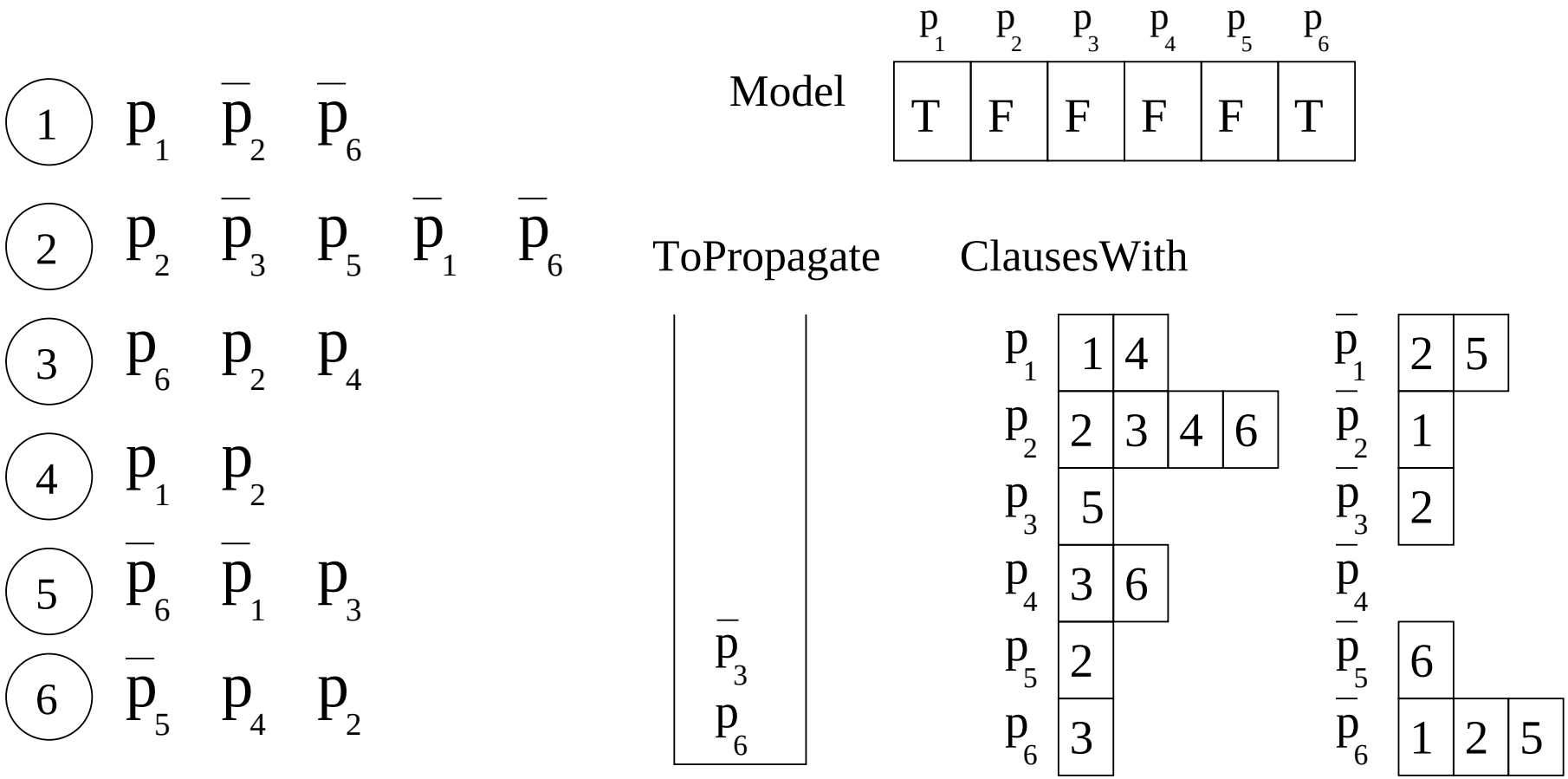
BCP with Occur Lists



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, p_6\, \overline{p}_5$

Literal $\overline{p}_3$ has to be added to the assignment

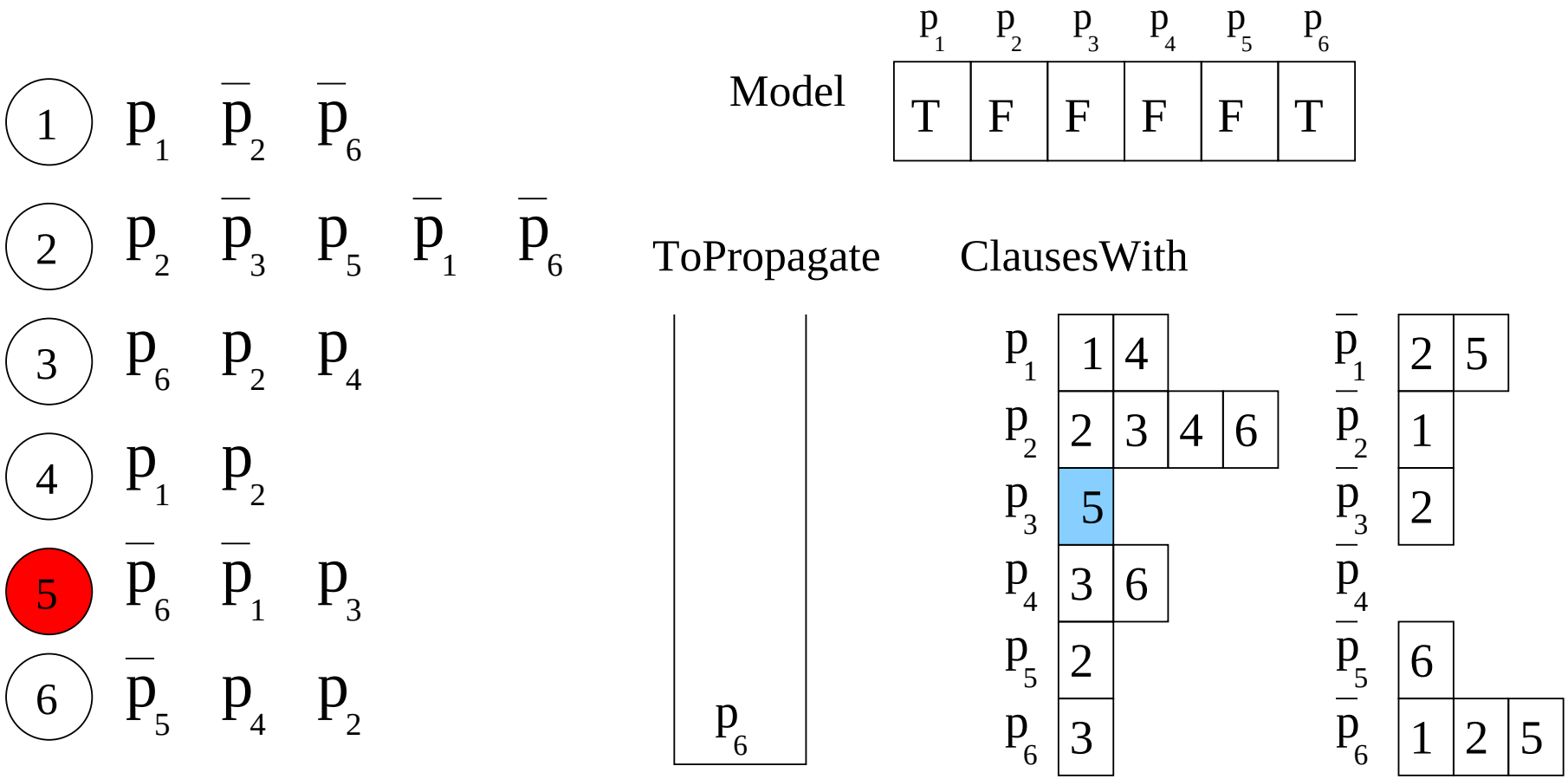
BCP with Occur Lists



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, p_6\, \overline{p}_5\, \overline{p}_3$

Now, we propagate $\overline{p}_3$ visiting ClausesWith[p_3]

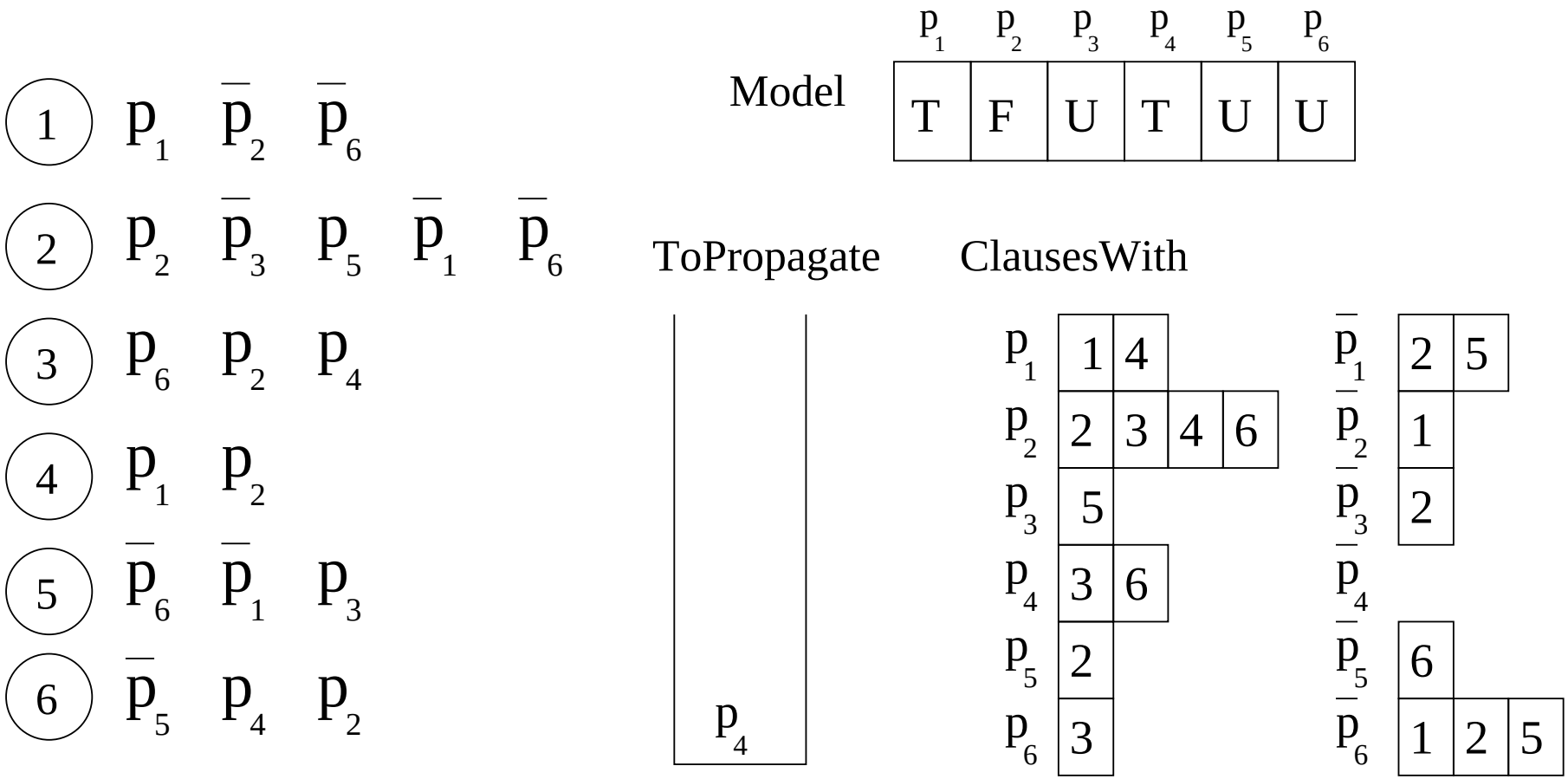
BCP with Occur Lists



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, p_6\, \overline{p}_5\, \overline{p}_3$

Clause 5 indicates a conflict. Backtrack/backjump is called.

BCP with Occur Lists



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, p_4$

Procedure continues propagating p_4

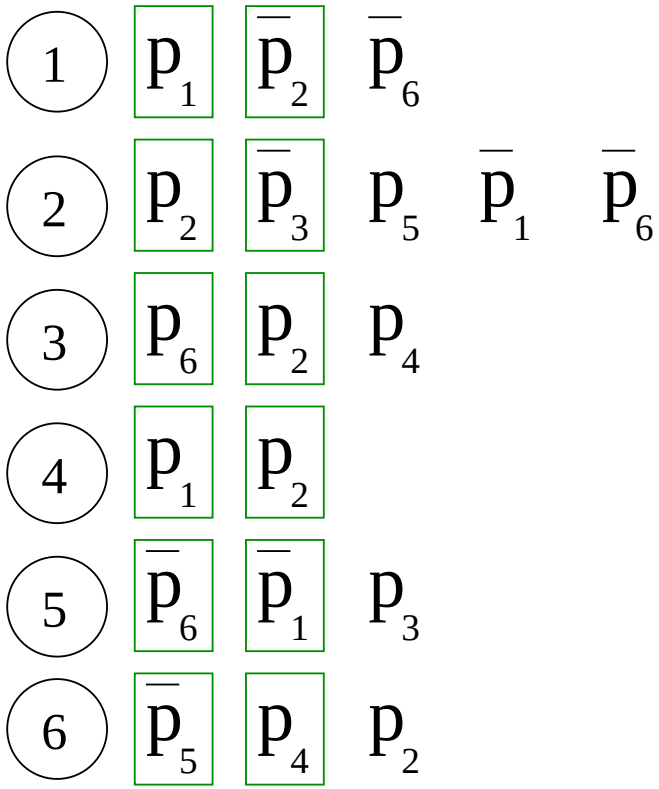
BCP - How to improve it?

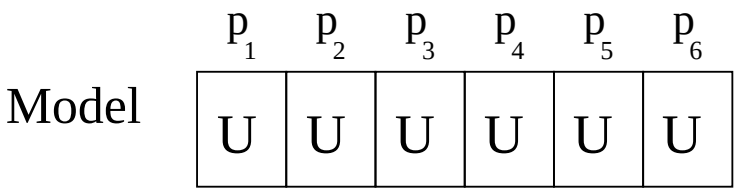
- The key observation is the following:

  A clause with 2 non-false literals can't be propagating or conflicting

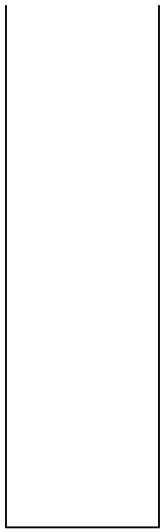
- For each clause we will try to watch two non-false literals
- Enough to visit a clause when a watched literal becomes false
- If 2 non-false literals do not exist, this is because:
  - All the lits are false (then backtrack)
  - All the lits are false but one, which is undef (then propagate)
  - All the lits are false but one, which is true
- This is called the two watched literals scheme

BCP - Two Watched Literals

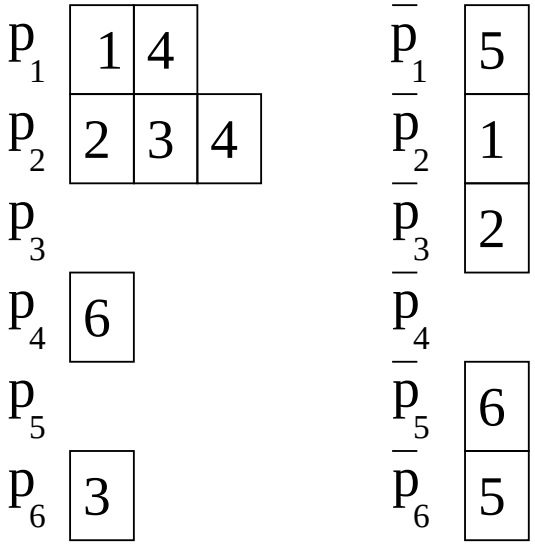




ToPropagate

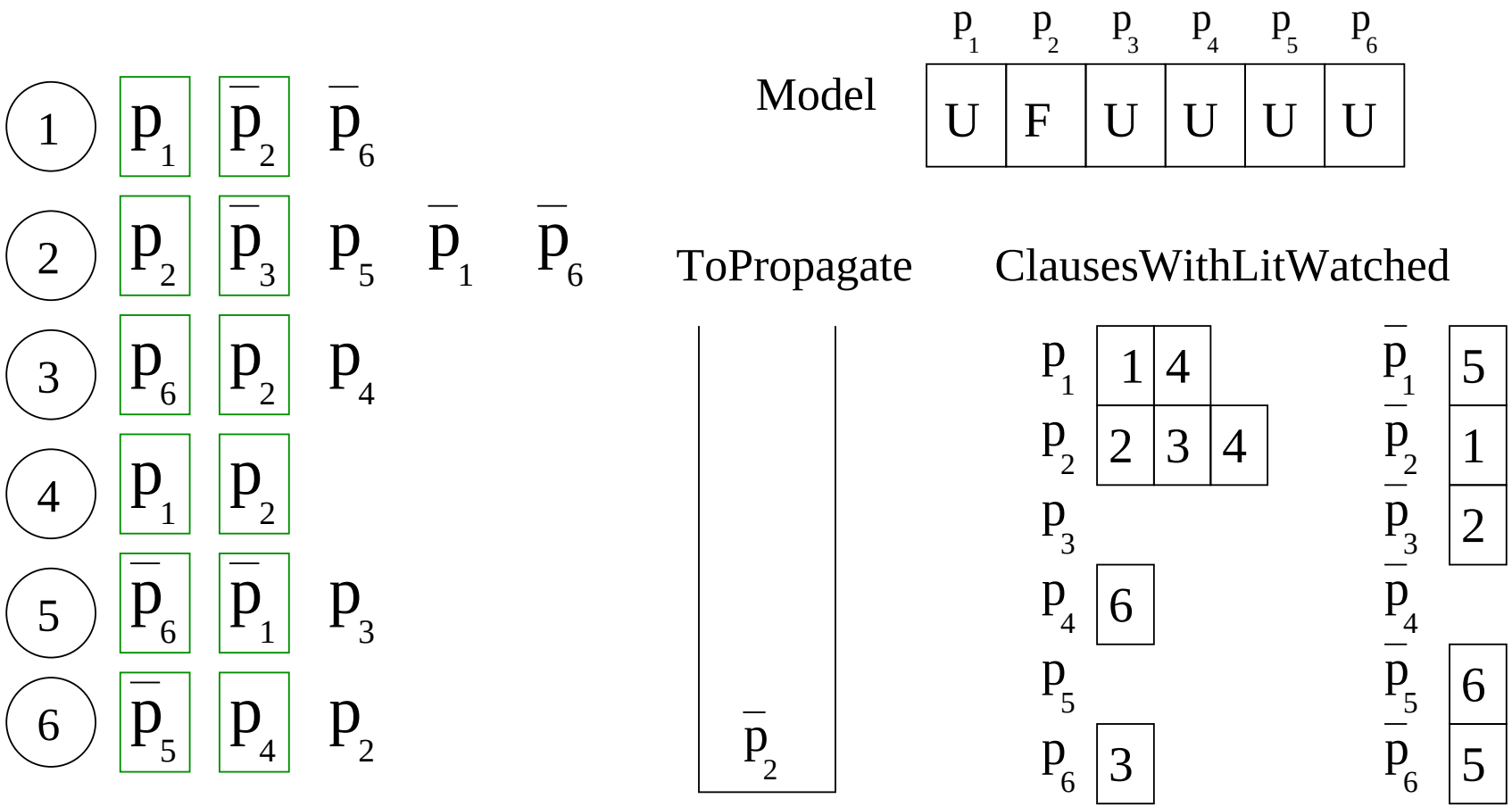


ClausesWithLitWatched



Current assignment: $\emptyset$

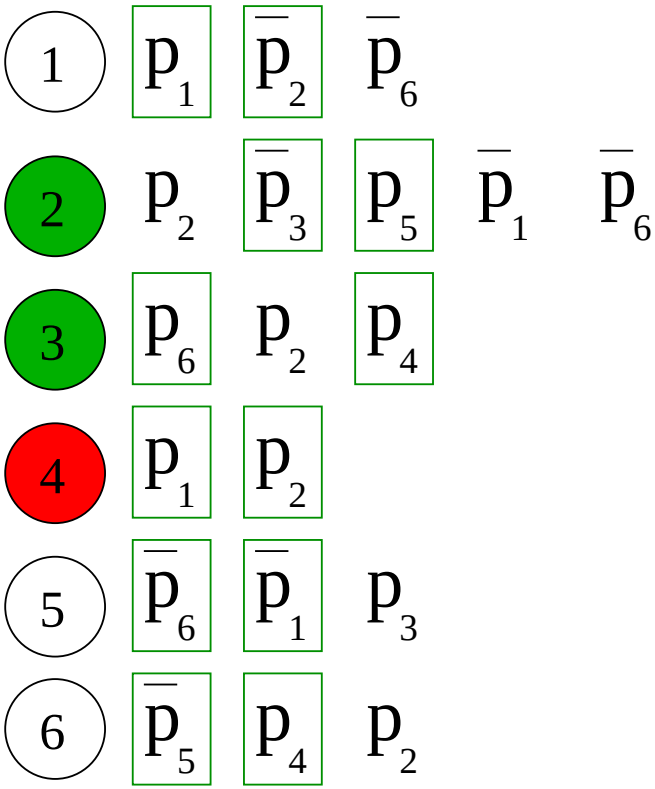
BCP - Two Watched Literals



Current assignment: $\overline{p}_2{}^{\mathsf{d}}$

Now, we propagate $\overline{p}_2$ visiting ClausesWithLitWatched[p_2]

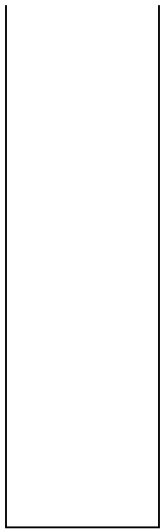
BCP - Two Watched Literals



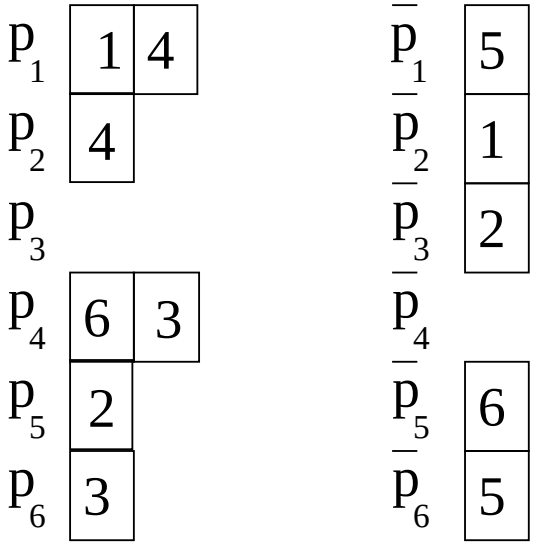
Model

p_1	p_2	p_3	p_4	p_5	p_6
U	F	U	U	U	U

ToPropagate



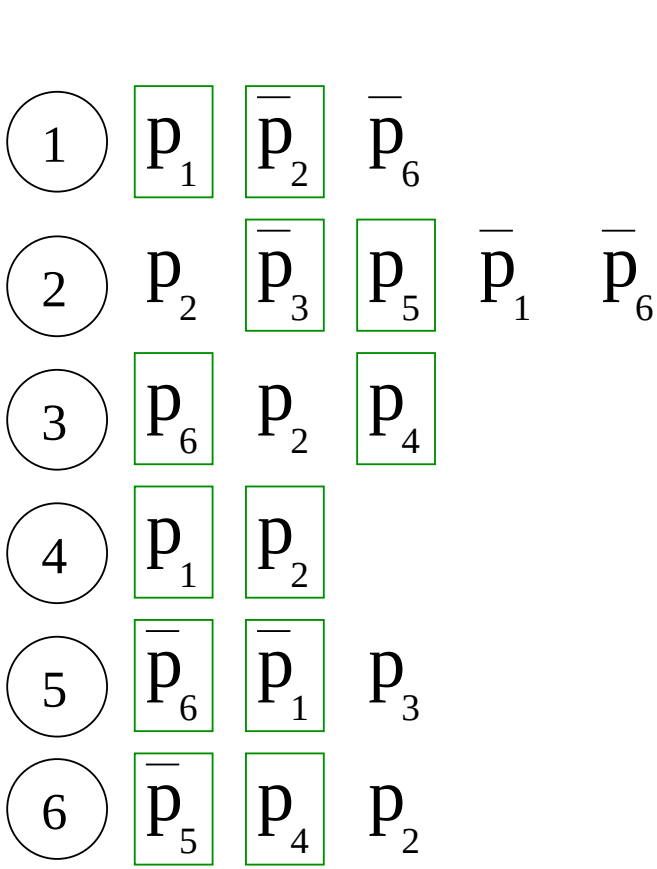
ClausesWithLitWatched

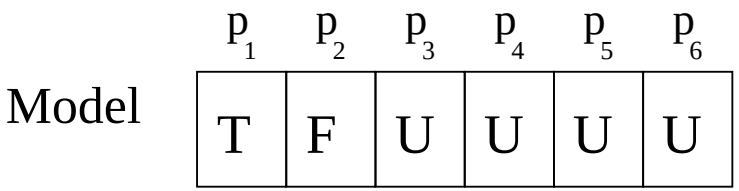


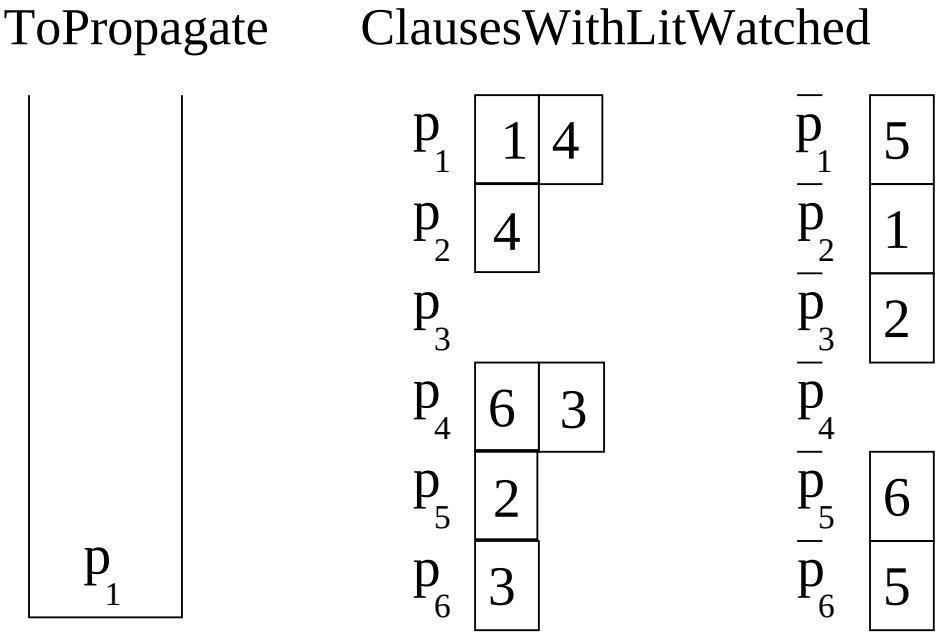
Current assignment: $\overline{p}_2{}^{\mathsf{d}}$

Clauses 2 and 3 are rewatched. Clause 4 can't because it is unit (p_1)

BCP - Two Watched Literals



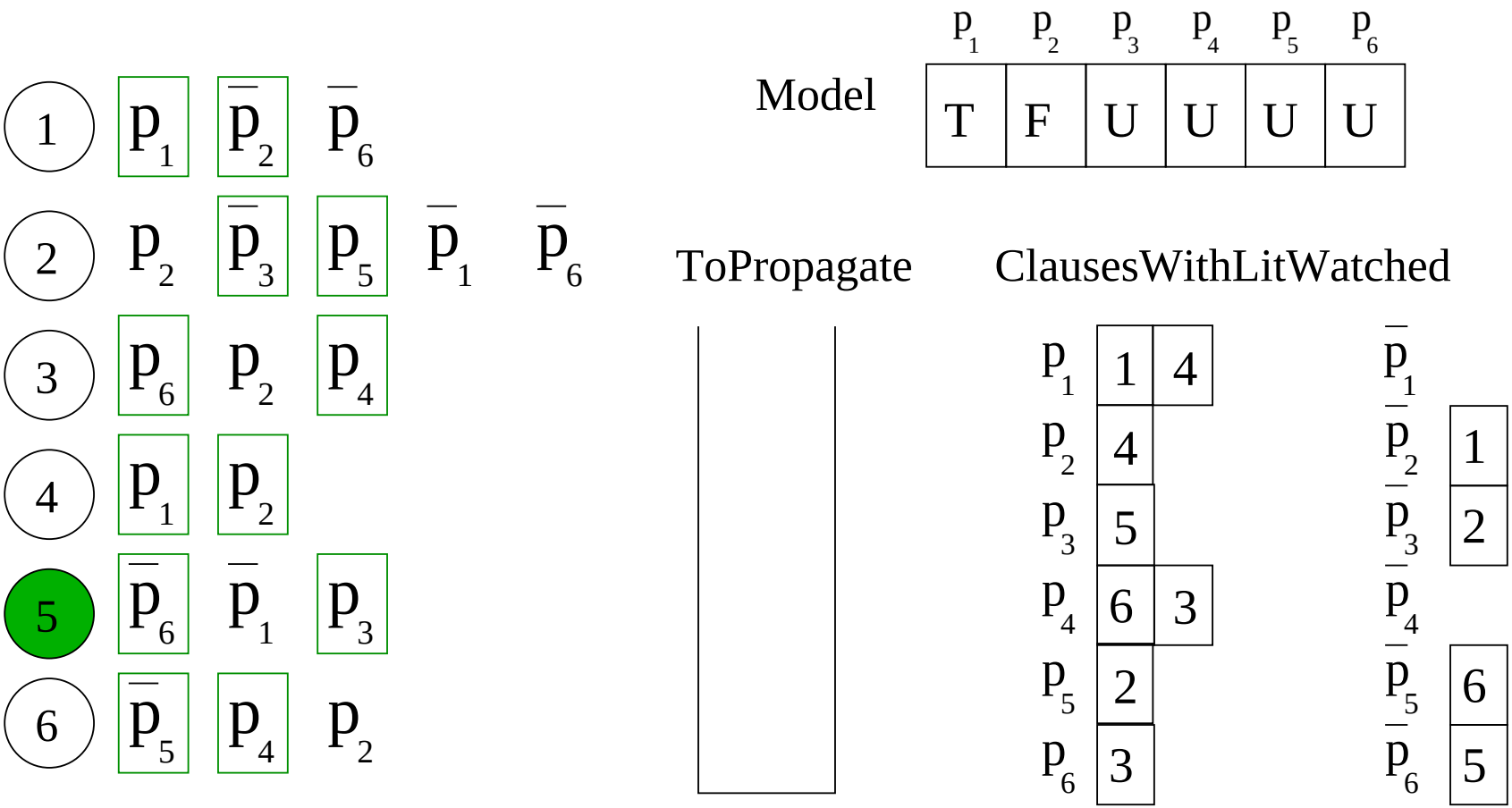




Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1$

Now, we propagate p_1 visiting ClausesWithLitWatched$[\overline{p}_1]$

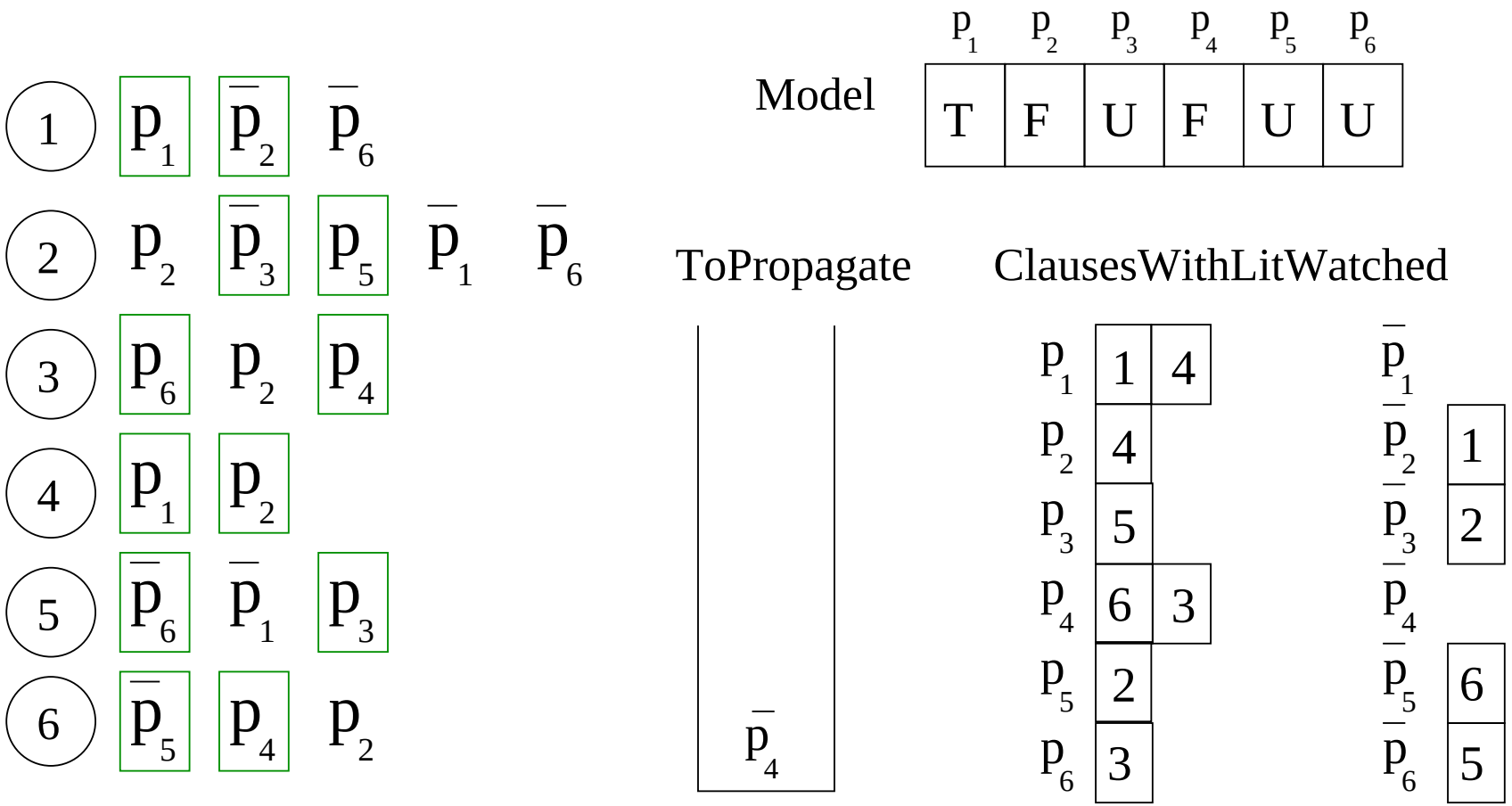
BCP - Two Watched Literals



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1$

Clauses 5 is reselected. No lit is unit propagated. We have to decide.

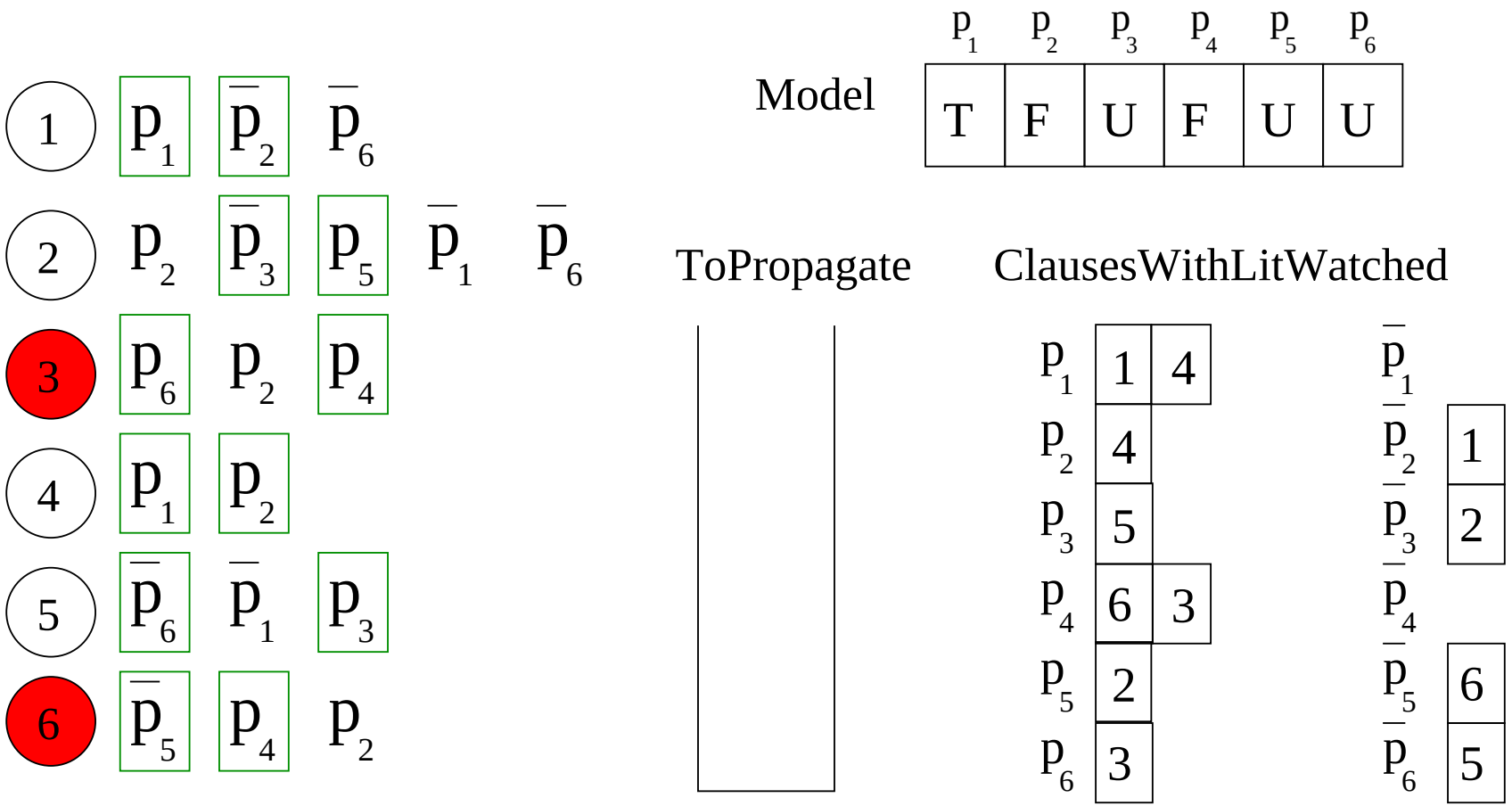
BCP - Two Watched Literals



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}$

Now, we propagate $\overline{p}_4$ visiting ClausesWithLitWatched[p_4]

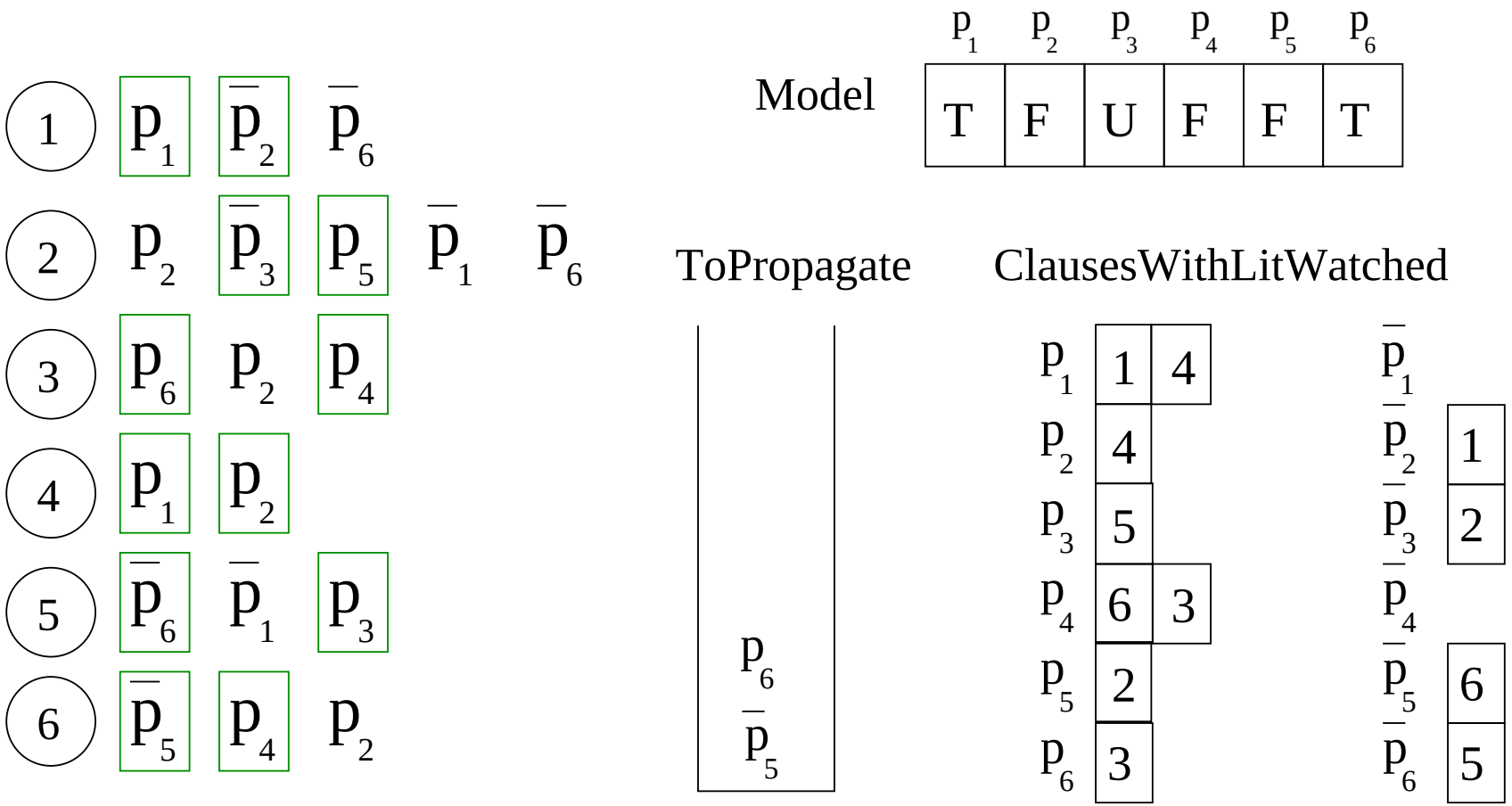
BCP - Two Watched Literals



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}$

Clause 6 unit propagates $\overline{p}_5$ and clause 3 propagates p_6.

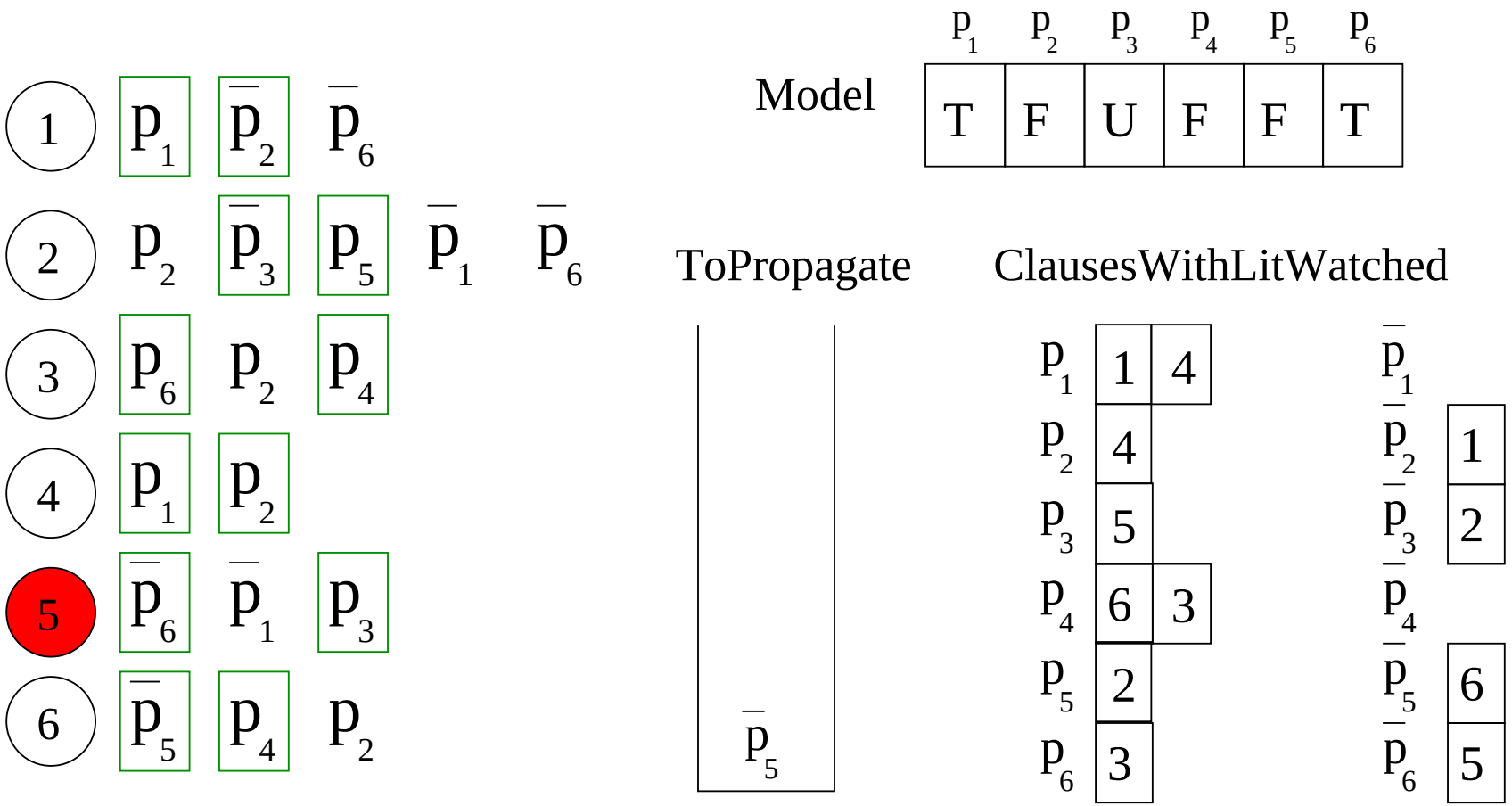
BCP - Two Watched Literals



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, \overline{p}_5\, p_6$

Now, we propagate p_6 visiting ClausesWithLitWatched$[\overline{p}_6]$

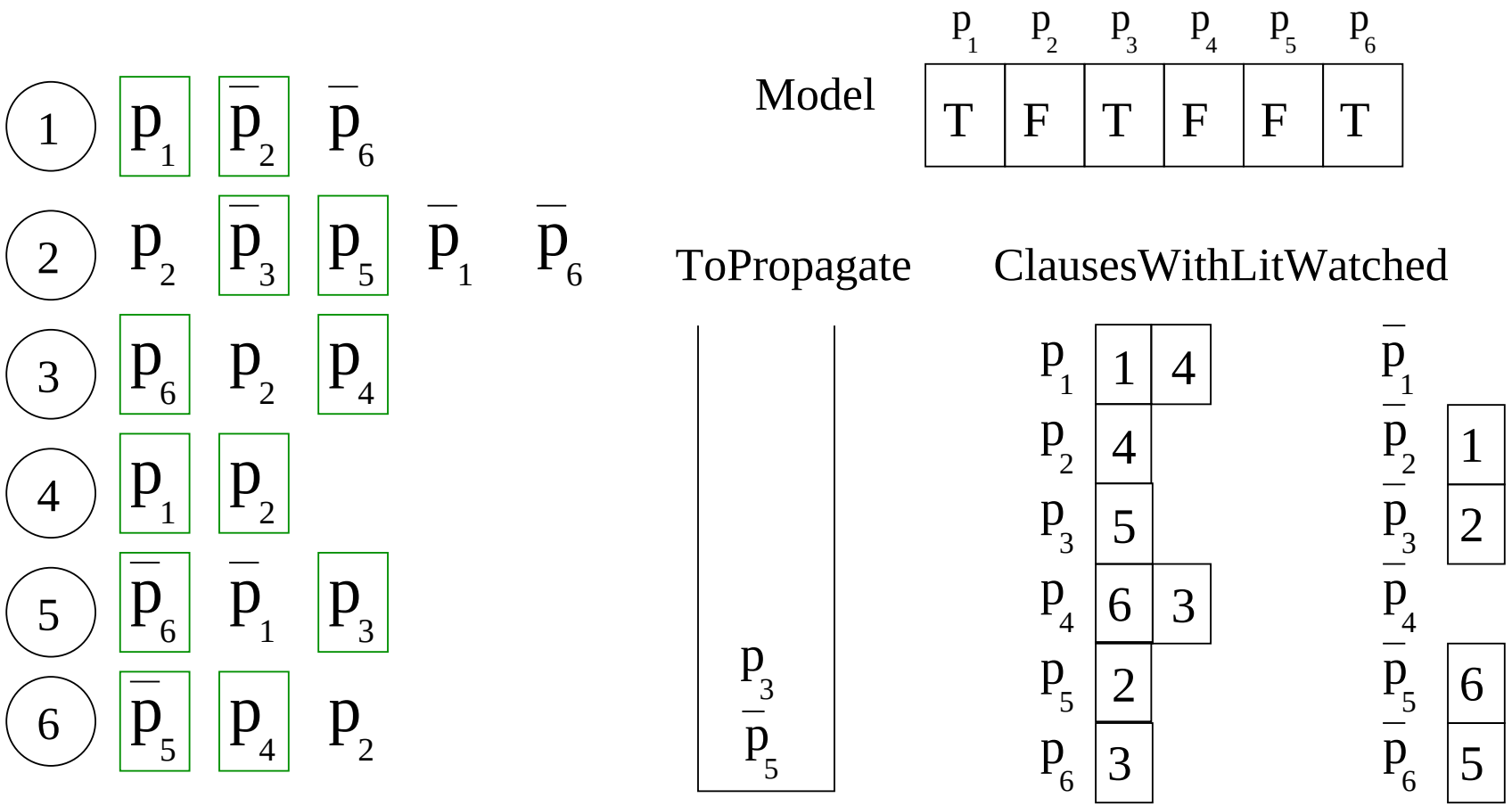
BCP - Two Watched Literals



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, \overline{p}_5\, p_6$

Clause 5 can't be reselected because it is unit (p_3).

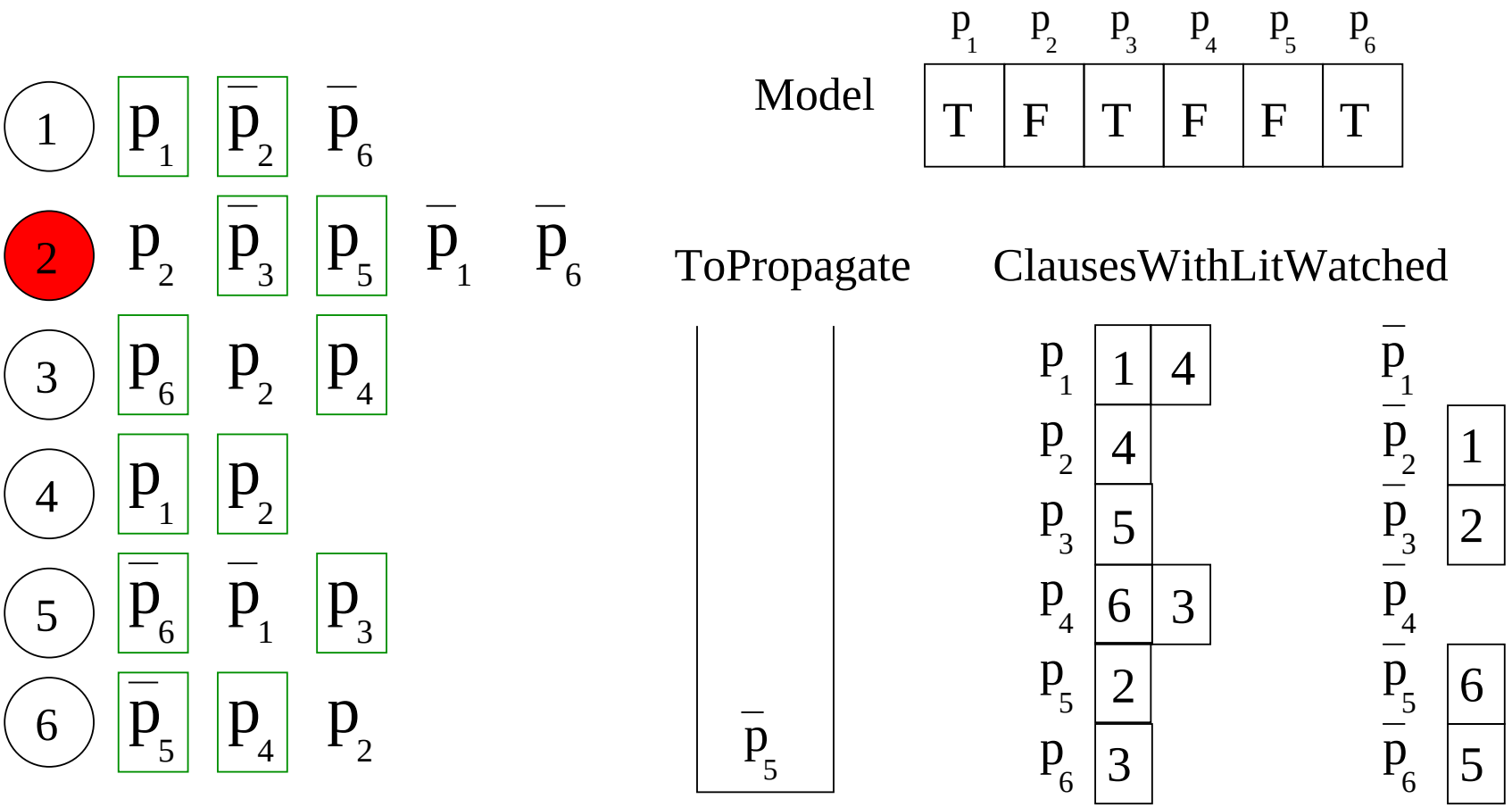
BCP - Two Watched Literals



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, \overline{p}_5\, p_6\, p_3$

Now, we propagate p_3 visiting ClausesWithLitWatched$[\overline{p}_3]$

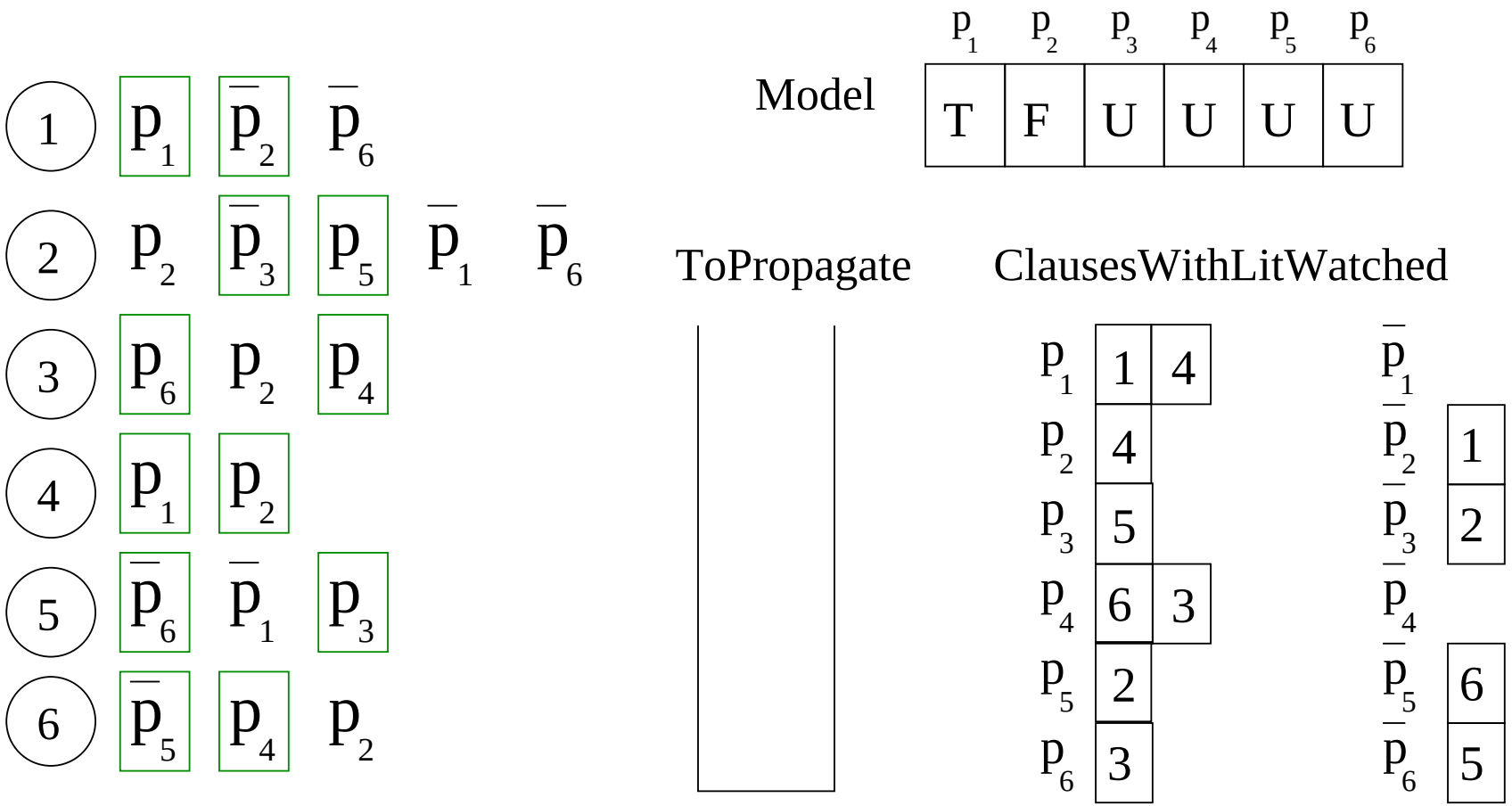
BCP - Two Watched Literals



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1\, \overline{p}_4{}^{\mathsf{d}}\, \overline{p}_5\, p_6\, p_3$

Clause 2 indicates a conflict. Backjump/backtrack is called.

BCP - Two Watched Literals



Current assignment: $\overline{p}_2{}^{\mathsf{d}}\, p_1$ (lit p_4 not yet added)

After backtrack watches are properly placed!

Two watched literals - Analysis

- Each clause is visited far less often
- Upon backtrack, nothing has to be done
- Inactive literals tend to be watched, hence further reducing the number of clauses to be visited
- Very effective for long clauses (e.g. lemmas)

Overview of the session

- Conflict Analysis
  - Motivating example
  - Backjumping
  - Conflict graph
  - Lemma shortening
- Lemma removal
- Decision heuristics
- Restarts
- Efficient implementation of UnitProp:
  - Occur lists
  - Two-watched literals
- Final remarks

Why SAT solvers are really good?

Three key ingredients that only work if used TOGETHER:

- Learn at each conflict the backjump clause as a lemma:
  - makes UnitProp more powerful
  - prevents future similar conflicts
- Decide on the variable with most occurrences in recent conflicts:
  - so-called activity-based heuristics
  - idea: work off clusters of tightly related (by many clauses) vars
- Forget from time to time low-activity lemmas:
  - crucial to keep UnitProp fast and afford memory usage
  - idea: lemmas from worked off clusters no longer needed!

These are the most important features of
CDCL (Conflict-Driven Clause Learning) SAT solvers

Bibliography - Further reading

- Matthew W. Moskewicz, Conor F. Madigan, Ying Zhao, Lintao Zhang, Sharad Malik. *Chaff: Engineering an Efficient SAT Solver.* DAC 2001: 530-535

- Lintao Zhang, Conor F. Madigan, Matthew W. Moskewicz, Sharad Malik. *Efficient Conflict Driven Learning in Boolean Satisfiability Solver*. ICCAD 2001: 279-285

- Niklas En, Niklas Srensson. *An Extensible SAT-solver*. SAT 2003: 502-518

- Robert Nieuwenhuis, Albert Oliveras, Cesare Tinelli. *Solving SAT and SAT Modulo Theories: From an abstract Davis–Putnam–Logemann–Loveland procedure to DPLL(T)*. J. ACM 53(6): 937-977 (2006)