Linear Programming: Box Wrappig

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1 Variables

The following are the variables appearing in the linear equations of the model:

n = total number of boxes.

w = width of the wrapping.

l = maximum length of the wrapping.

$length$ = current length of the wrapping (where $length \leq l$).

x_i^{tl} = x coordinate of the i-th box ($x^{tl} = \{0, \cdots, w-1\}$).

y_i^{tl} = y coordinate of the i-th box ($y^{tl} = \{0, \cdots, l-1\}$).

$width_i$ = assigned width of the i-th box ($width = \{0, \cdots, w-1\}$).

$height_i$ = assigned height of the i-th box ($height = \{0, \cdots, l-1\}$).

where $i = \{0, \cdots, n-1\}$.

2 Constraints

(Bounds) Boxes must be placed inside the width of the wrapping:

$$x_i^{tl} \leq w - witdth_i \quad \text{forall i} \tag{1}$$

(Rotation) The boxes must be placed in vertical or horizontal position:

$$width_i + height_i = b\_width_i + b\_heigth_i \tag{2}$$

where b_width_i and b_height_i are the given width and height of the i-th box.

(Overlapping) The boxes must not overlap:

$$\begin{aligned} &((x_i^{tl} \leq x_j^{tl} - width_i) \\ &+(x_i^{tl} \geq x_j^{tl} + width_j) \\ &+(y_i^{tl} \leq y_j^{tl} - height_i) \\ &+(y_i^{tl} \geq y_j^{tl} + heigth_j)) \geq 1 \quad \text{for all } i, j, i \neq j \end{aligned} \tag{3}$$

(Length) The current length of the wrapping must at least the position of the furthest box from the origin plus its height:

$$length \geq y_i^{tl} + height_i \quad \text{forall i} \tag{4}$$

3 Objective function (to be minimized)

$$Cost = length \tag{5}$$

4 Optimizations

- The first box, by symmetry, can be placed on the left-side of the wrapping without changing the final result.
- Repeating branches of identical boxes is redundant, imposing an arbitrary order will prevent this.
- Square boxes can be placed indistinct vertically or horizontally.