Combinatorial Problem Solving (CPS)

Project: Box Wrapping.

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1 Description of the Problem

It is a common practice that banks reward their most loyal clients with presents, which are typically wrapped in costly corporative paper full of logos of the bank.

Imagine you work in one of such banks, which is going through a severe crisis. Since it is detected that the expenses on wrapping paper are excessive and therefore should be reduced, you (as one of the most competent computer engineers in the company) are designated responsible for solving the *Box Wrapping Problem (BWP)*: given a list of boxes to be wrapped, and given a huge roll of paper of a certain width, you have to decide how to cut off the pieces of paper for wrapping the boxes so that the length of roll that is used is minimized.

The following considerations are made:

- Each box is described by the dimensions of the rectangular piece of paper needed to wrap it.
- There may be several identical boxes.
- The rectangles of paper may be rotated when cutting off if needed.
- The roll has infinite length.

2 Input and Output Formats

This section describes the format in which instances of BWP are written (Section 2.1), as well as the expected format for the corresponding solutions (Section 2.2).

2.1 Input Format

An instance of BWP is a text file consisting of several lines of integer values. The first line contains W, the width of the roll of paper, followed by N, the total number of boxes. Each of the next lines has this format: a number n_i, followed by numbers x_i and y_i, meaning that there are n_i boxes that need a rectangular piece of paper of dimensions $x_i \times y_i$. It is ensured that $1 \leq W \leq 11$, $1 \leq n_i \leq 6$, $N \leq 13$, $1 \leq x_i \leq \min(y_i, W)$, $1 \leq y_i \leq 10$. Moreover, if $i \neq j$ then $(x_i, y_i) \neq (x_j, y_j)$. Note that $N = \sum n_i$.

For example, Figure 1 shows the data of an instance.

```
4 5
3    1 3
1    3 3
1    1 1
```

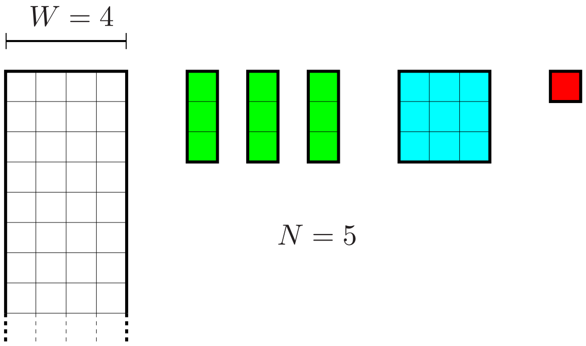

Figure 1: Input file of an instance and its graphical representation.

In this instance the roll of paper has width 4, and we have to cut off 5 rectangles of paper in total: 3 of dimensions 1 × 3, 1 of dimensions 3 × 3, and finally 1 of dimensions 1 × 1.

2.2 Output Format

Output files only contain integer values. The output starts with a copy of the input data. The rest of the lines should have the following format:

- The next line should be L, the length of roll of paper that needs to be consumed.
- For each of the N boxes, there should be a line, with the following format: first two numbers x_i^{tl} and y_i^{tl}, meaning the coordinates of the top left corner for the corresponding piece of paper; and then two numbers x_i^{br} and y_i^{br}, meaning the coordinates of the bottom right corner.

The roll of paper, which has width W and length L, has to be understood as a grid of cells. These cells are described with a Cartesian coordinate system, where the x-axis coordinates take values between 0 and $W - 1$, and the y-axis coordinates take values between 0 and $L - 1$.

For example, Figure 2 shows an optimal arrangement for the instance in Figure 1, and the corresponding output file.

```
4 5
3   1 3
1   3 3
1   1 1
5
0 0   2 2
3 0   3 2
0 3   2 3
0 4   2 4
3 3   3 3
```

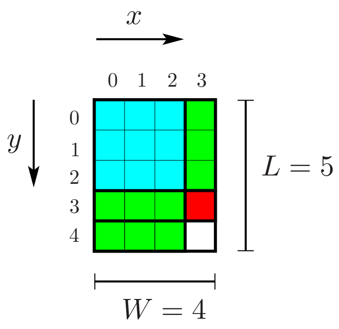

Figure 2: Output file of the solution and its graphical representation.

Note that, in this solution, one of the rectangles 1 × 3 is laid vertically, while the other two are laid horizontally.

3 Project

The purpose of this project is to model and solve BWP with the three problem solving technologies considered in the course: constraint programming (CP), linear programming (LP) and propositional satisfiability (SAT).

For the development of the project, in addition to this document, students will be provided with the following materials:

- a suite of *problem instances* (in the format specified in Section 2.1). Files are named `bwp_`W`_`N`_`k`.inp`, where W is the width of the roll, N is the number of boxes and k is an identifier.
- a *checker* that reads the output of solving a problem instance (following the format given in Sect. 2.2) from `stdin` and makes a sanity check. If possible, the solution is also plotted on the console.

Take into account that **the checker does not guarantee that the solution is optimal**. There may be better solutions with shorter paper rolls.

- a *result table* with the length of the optimal solution of some of the problem instances, to make debugging easier.

As a reference, solving processes that exceed a time limit of 60 seconds of wall clock time should be aborted (`Linux` command `timeout` may be useful). Take into account that, depending on the solving technology, on the machine, etc., some of the instances may be too difficult to solve within this time limit. For this reason, it is not strictly necessary to succeed in solving all instances to pass the project. However, you are encouraged to solve as many instances as possible.

The project has three deadlines, one for each problem solving technology:

- **CP:** 26 May.
- **LP:** 9 Jun.
- **SAT:** 23 Jun.

For each technology, a `tgz` or `zip` compressed archive should be delivered via Racó (`https://raco.fib.upc.edu`) with the following contents:

- a directory `out` with the output files of those instances that could be solved. Please **only** provide the outputs of those instances for which an optimal solution could be found. The output file corresponding to an instance `bwp_`W`_`N`_`k`.inp` should be named `bwp_`W`_`N`_`k`.out`.

- a directory `src` with all the source code (`C++`, scripts, `Makefile`, etc.) used to solve the problem, together with a `README` file with basic instructions for compiling and executing so that results can be reproduced.

  Programs should read the instance from `stdin` and write the solution to `stdout`. Other output (debugging information, etc.) should be written to `stderr`. In particular, suboptimal solutions should not be written to `stdout`, only the optimal solution (if found).

  If say your executable is named `p`, then your results will be recomputed by calling `p < bwp_`W`_`N`_`k`.inp`.

- a document in PDF describing the variables and constraints that were used in your model, as well as any remarks or comments you consider appropriate.

Please follow these indications when submitting your deliveries.