The cost of extraction increases with depth. At successive levels, the cost of extracting a block is as follows:

Level 1	£3000
Level 2	£6000
Level 3	£8000
Level 4	£10 000

The revenue obtained from a ‘100% value block’ would be £200 000. For each block here, the revenue is proportional to ore value.

Build a model to help decide the best blocks to extract. The objective is to maximise revenue−cost.

The larger version of this problem arose with open-cast iron mining in South Africa.

12.15 Tariff rates (power generation)

A number of power stations are committed to meeting the following electricity load demands over a day:

12 p.m. to 6 a.m.	15 000 MW
6 a.m. to 9 a.m.	30 000 MW
9 a.m. to 3 p.m.	25 000 MW
3 p.m. to 6 p.m.	40 000 MW
6 p.m. to 12 p.m.	27 000 MW

There are three types of generating unit available: 12 of type 1, 10 of type 2 and five of type 3. Each generator has to work between a minimum and a maximum level. There is an hourly cost of running each generator at minimum level. In addition, there is an extra hourly cost for each megawatt at which a unit is operated above the minimum level. Starting up a generator also involves a cost. All this information is given in Table 12.6 (with costs in £).

In addition to meeting the estimated load demands there must be sufficient generators working at any time to make it possible to meet an increase in load of up to 15%. This increase would have to be accomplished by adjusting the output of generators already operating within their permitted limits.

Table 12.6

	Minimum level	Maximum level	Cost per hour at minimum	Cost per hour per megawatt above minimum	Cost
Type 1	850 MW	2000 MW	1000	2	2000
Type 2	1250 MW	1750 MW	2600	1.30	1000
Type 3	1500 MW	4000 MW	3000	3	500

Which generators should be working in which periods of the day to minimise total cost?

What is the marginal cost of production of electricity in each period of the day; that is, what tariffs should be charged?

What would be the saving of lowering the 15% reserve output guarantee; that is, what does this security of supply guarantee cost?

12.16 Hydro power

This is an extension of the Tariff Rates (Power Generation) problem of Section 12.15. In addition to the thermal generators, a reservoir powers two hydro generators: one of type A and one of type B. When a hydro generator is running, it operates at a fixed level and the depth of the reservoir decreases. The costs associated with each hydro generator are a fixed start-up cost and a running cost per hour. The characteristics of each type of generator are shown in Table 12.7.

For environmental reasons, the reservoir must be maintained at a depth of between 15 and 20 m. Also, at midnight each night, the reservoir must be 16 m deep. Thermal generators can be used to pump water into the reservoir. To increase the level of the reservoir by 1 m, it requires 3000 MWh of electricity. You may assume that rainfall does not affect the reservoir level.

At any time, it must be possible to meet an increase in demand for electricity of up to 15%. This can be achieved by any combination of the following: switching on a hydro generator (even if this would cause the reservoir depth to fall below 15 m); using the output of a thermal generator, which is used for pumping water into the reservoir; and increasing the operating level of a thermal generator to its maximum. Thermal generators cannot be switched on instantaneously to meet increased demand (although hydro generators can be).

linear programming model, the optimal solution will be integer automatically. There is therefore no need to use, computationally much more costly, integer programming.

This property makes it possible to solve much larger versions of this problem in a reasonable amount of time. As described in Section 10.1, the dual of this linear programming model is a network flow problem and could be solved very efficiently by a specialized network flow algorithm, see Williams (1982).

The model has 56 constraints and 30 variables. Each variable has an upper bound of 1.

13.15 Tariff rates (power generation)

This problem is based on a model described by Garver (1963). The following formulation is suggested here.

13.15.1 Variables

n_{ij} = number of generating units of type i working in period j (where $j = 1, 2, 3, 4,$ and 5 are the five periods of the day listed in the question)
s_{ij} = number of generators of type i started up in period j
x_{ij} = total output rate from generators of type i in period j

x_{ij} are continuous variables; n_{ij} and s_{ij} are general integer variables.

13.15.2 Constraints

Demand must be met in each period:

$$\sum_i x_{ij} \geq D_j \quad \text{for all } j,$$

where D_j is demand given in period j.

Output must lie within the limits of the generators working:

$$x_{ij} \geq m_i n_{ij}, \qquad \text{for all } i \text{ and } j,$$

$$x_{ij} \leq M_i n_{ij}, \qquad \text{for all } i \text{ and } j,$$

where m_i and M_i are the given minimum and maximum output levels for generators of type i.

The extra guaranteed load requirement must be able to be met without starting up any more generators:

$$\sum_i M_i n_{ij} \geq \frac{115}{100} D_j \quad \text{for all } j.$$

The number of generators started in period j must equal the increase in number:

$$s_{ij} \geq n_{ij} - n_{ij-1}, \quad \text{for all } i \text{ and } j,$$

where n_{ij} is number of generators started in period j (when $j = 1$, period $j-1$ is taken as 5).

In addition, all the integer variables have simple upper bounds corresponding to the total number of generators of each type.

13.15.3 Objective function (to be minimized)

$$\text{Cost} = \sum_{i,j} C_{ij}\left(x_{ij} - m_i n_{ij}\right) + \sum_{i,j} E_{ij} n_{ij} + \sum_{i,j} F_i s_{ij},$$

where C_{ij} are costs per hour per megawatt above minimum level multiplied by the number of hours in the period; E_{ij} are costs per hour for operating at minimum level multiplied by the number of hours in the period and F_i are start-up costs.

This model has a total of 55 constraints and 30 simple upper bounds. There are 45 variables, of which 30 are general integer variables.

13.16 Hydro power

The model described in Section 13.15 can be extended by additional variables and constraints.

13.16.1 Variables

h_{ij} $= 1$ if hydro of type i working in period j
$= 0$ otherwise
where $i = 1, 2$
t_{ij} $= 1$ if hydro of type i started in period j
$= 0$ otherwise
l_j $=$ height of reservoir at beginning of period j
p_j $=$ number of megawatts of pumping in period j

13.16.2 Constraints

The demand constraint becomes

$$\sum_i x_{ij} + \sum_i L_i h_{ij} - p_j \geq D_j \quad \text{for all } j,$$

where L_i is the operating level of hydro i.

explored with a minimum sum of percentage deviations of 11.28 obtained at node 61. The maximum deviation associated with this solution was 2.77% (the SPIRITS goal). When the objective was to minimize the maximum deviation, 36 nodes were explored with no feasible solution being found.

The most successful formulation was the third one, where six single 'tighter' constraints, logically equivalent to the six constraints implied by the OIL goals in the three regions, were added. With the objective of minimizing the sum of percentage deviations, a feasible solution was obtained after 35 nodes. The sum of percentage deviations was 8.83, and the maximum deviation was 2.5% (in the GROWTH prospects goals). This run was terminated after 394 nodes with no other feasible solutions being found.

With the objective of minimizing the maximum deviation, a feasible solution was found after 44 nodes with a maximum deviation of 3.15% (in the SPIRITS goal). The associated sum of percentage deviations was 12.14. A better solution was found at node 200 with a maximum deviation of 2.5% (in the GROWTH prospects goals). The associated sum of percentage deviations was 9.7. No better solutions were found after 303 nodes.

With hindsight, it is possible to observe that the minimax objective cannot be reduced below 2.5% because the slack variable in the GROWTH goal cannot be less than 0.2, given a right-hand side of 3.2. Therefore, the slack in this constraint was *fixed* at 0.2 and the surplus at 0. The third formulation of the problem was then rerun in order to minimize the sum of percentage deviations. At node 681, the optimal integer solution was found where the sum of percentage deviations was 7.806. The solution was proved optimal after 889 nodes. This optimal solution allocates the following retailers to D1:

$$\text{M1, M2, M3, M4, M11, M16, M18, M21, M22.}$$

All other retailers are to be assigned to division D2.

This problem has proved of considerable interest in view of its computational difficulty. A remodelling approach using 'basis reduction' is described by Aardal *et al*. (1999).

14.14 Opencast mining

The optimal solution is to extract the shaded blocks shown in Figure 14.4. This results in a profit of £17 500.

This solution was obtained in 28 iterations.

14.15 Tariff rates (power generation)

The following generators should be working in each period giving the following outputs:

Period 1	12 of type 1, output 10 200 MW
	3 of type 2, output 4 800 MW
Period 2	12 of type 1, output 16 000 MW
	8 of type 2, output 14 000 MW
Period 3	12 of type 1, output 11 000 MW
	8 of type 2, output 14 000 MW
Period 4	12 of type 1, output 21 250 MW
	9 of type 2, output 15 750 MW
	2 of type 3, output 3 000 MW
Period 5	12 of type 1, output 11 250 MW
	9 of type 2, output 15 750 MW

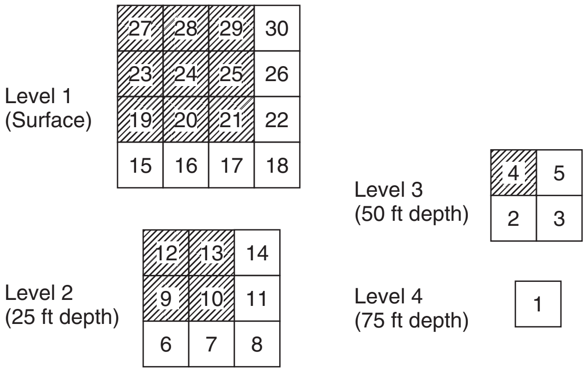

Figure 14.4

The total daily cost of this operating pattern is £988 540.

Deriving the marginal cost of production from a mixed integer programming model such as this encounters the difficulties discussed in Section 10.3. We could adopt the approach of fixing the integer variables at the optimal integer values and obtaining this economic information from the resulting linear programming model. The marginal costs then result from any changes within the optimal operating pattern, that is, without altering the numbers of different types of generator working in each period (although their levels of operation could vary).

The marginal costs of production per hour are then obtained from the shadow prices on the demand constraints (divided by the number of hours in the period). These give:

Period 1	£1.3	per megawatt hour
Period 2	£2	per megawatt hour
Period 3	£2	per megawatt hour
Period 4	£2	per megawatt hour
Period 5	£2	per megawatt hour