- Let $a_1, a_2, a_3, \dots, a_n$ be the n items that we have seen. We want to proof that the probability of any of them being in memory is $\frac{1}{n}$.
- Let's define a r.v. X_n which will take the value of the item in memory after n steps. So, we need to proof that $Pr[X_n = a_i] = \frac{1}{n}$ $\forall 1 \leq i \leq n$.
- Induction proof:
  - Base case: $n = 1 \rightarrow X_n = a_1 = 1$ (only one item at this point) ✓
  - I.H.: Assuming that $Pr[X_n = a_i] = \frac{1}{n}$ works $\forall 1 \leq i \leq n$, we will proof that it holds for $n+1$.
  - $n+1$: After $n+1$ ~~items~~ steps, we have that $X_{n+1} = a_{n+1}$ with a probability of $\frac{1}{n+1}$. Therefore, $Pr[X_{n+1} = a_{n+1}] = \frac{1}{n+1}$. $\forall 1 \leq i \leq n$ we have:

    - $Pr[X_{n+1} = a_i] = Pr[\text{no replacement after } n \text{ steps } \& \ X_n = a_i] =$

$$= Pr[\text{no replacement after } n \text{ steps}] \cdot Pr[X_n = a_i] =$$

$$\stackrel{\text{I.H.}}{=} \frac{n}{n+1} \cdot \frac{1}{n} = \frac{1}{n+1} \checkmark$$