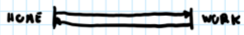
24. I have 4 umbrellas, some at home, some in the office. I keep moving between home and office. I take an umbrella with me only if it rains. If it does not rain I leave the umbrella behind (at home or in the office). It may happen that all umbrellas are in one place, I am at the other, it starts raining and must leave, so I get wet.

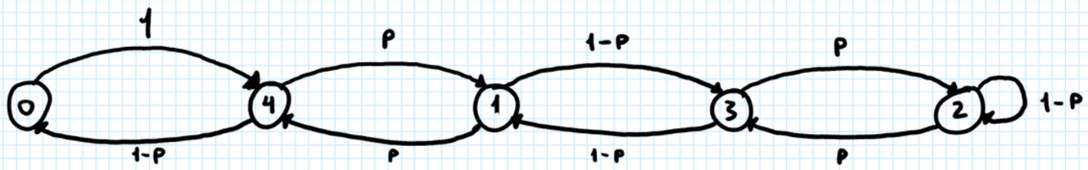
(a) If the probability of rain is p, what is the probability that I get wet?

24)



a) 4 UMBRELAS

WE CAN REPRESENT THIS PROBLEM IN A DIAGRAM WHERE EACH STATE REPRESENTS THE NUMBER OF UMBRELAS IN OUR CURRENT LOCATION
EACH STATE HAS 2 TRANSITIONS TO ANOTHER STATE, ONE WHERE IT IS RAINING AND WE TAKE AN UMBRELA, AND ANOTHER WHERE IT IS NOT RAINING AND WE DON'T TAKE AN UMBRELA. EXCEPT FOR STATE 0 WHERE WE CAN'T TAKE AN UMBRELA.



$$P_r(\text{WET}) = \pi_0 \cdot p$$

$$\pi_0 = ?$$

BECAUSE EVERY STATE, EXCEPT FOR STATE 0, HAS EQUIVELENT IN AND OUT FLUXES, WE KNOW THAT:

$$\pi_1 = \pi_2 = \pi_3 = \pi_4$$

AND ALSO:

$$\pi_0 = \pi_4 \cdot (1-p)$$

SO:

$$\pi_4 \cdot (1-p) + 4 \cdot \pi_4 = 1$$

$$\Leftrightarrow \pi_4 \cdot ((1-p) + 4) = 1$$

$$\Leftrightarrow \pi_4 = \frac{1}{(1-p)+4}$$

$$\pi_0 = \frac{1}{(1-p)+4} \cdot (1-p)$$

$$\Leftrightarrow \pi_0 = \frac{(1-p)}{(1-p)+4}$$

$$\Pr(\text{WET}) = \pi_0 \cdot p$$

$$= \frac{(1-p)}{(1-p)+4} \cdot p$$

$$= \frac{p \cdot (1-p)}{(1-p)+4}$$

(b) If the current forecast shows a $p = 0.6$, how many umbrellas should I have so that, if I follow the strategy above, the probability I get wet is less than 0.1?

b) N = NUMBER OF UMBRELAS

WE ALREADY KNOW THAT THE TOTAL NUMBER OF STATES IS $N+1$, AND THE NUMBER OF STATES THAT HAVE EQUIVELENT IN AND OUT FLUXES IS N. SO WE KNOW THAT:

$$\pi_1 = \dots = \pi_N$$

$$\pi_0 = \pi_1 \cdot (1-p)$$

$$\pi_1 \cdot (1-p) + N \cdot \pi_1 = 1$$

SO:

$$\pi_1 \cdot (1-p) + N \cdot \pi_1 = 1$$

$$\Leftrightarrow \pi_1 \cdot ((1-p) + N) = 1$$

$$\Leftrightarrow \pi_1 = \frac{1}{(1-p)+N}$$

$$\pi_0 = \pi_1 \cdot (1-p)$$

$$\Leftrightarrow \pi_0 = \frac{1}{(1-p)+N} \cdot (1-p)$$

$$\Leftrightarrow \pi_0 = \frac{1-p}{(1-p)+N}$$

$$Pr(WET) = \pi_0 \cdot p$$

$$= \frac{1-p}{(1-p)+N} \cdot p$$

$$= \frac{p \cdot (1-p)}{(1-p) + N}$$

FOR A PROBABILITY OF GETTING WET OF LESS THAN 0,1

$$Pr(WET) = \frac{p \cdot (1-p)}{(1-p) + N}$$

$$0,1 > \frac{0,6 \cdot (1-0,6)}{(1-0,6) + N}$$

$$\Leftrightarrow 0,1 > \frac{0,24}{0,4 + N}$$

$$\Leftrightarrow 0,1 \cdot (0,4 + N) > 0,24$$

$$\Leftrightarrow 0,04 + 0,1 \cdot N > 0,24$$

$$\Leftrightarrow 0,1 N > 0,24 - 0,04$$

$$\Leftrightarrow N > \frac{0,2}{0,1}$$

$$\Leftrightarrow N > 2$$