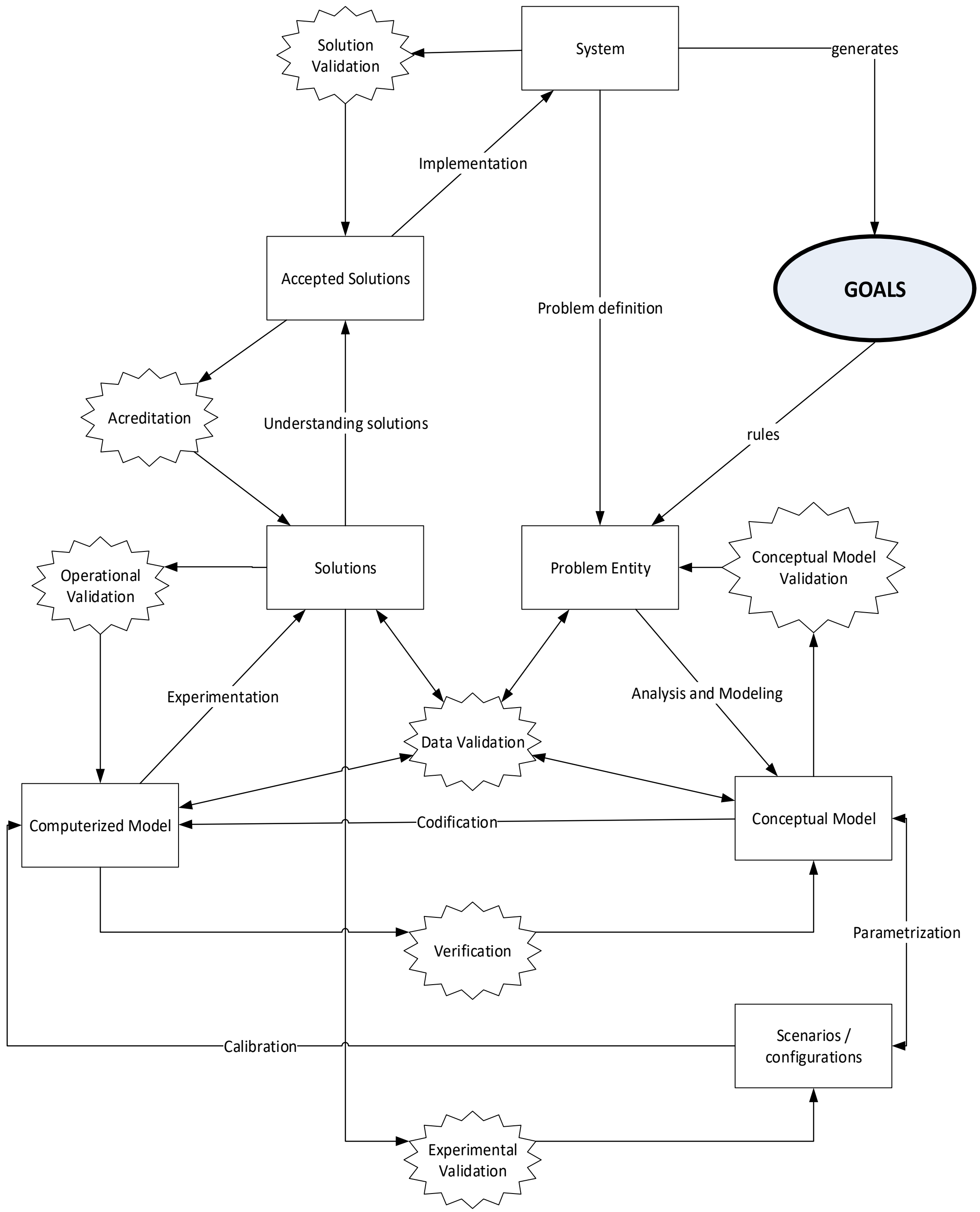

Solution Validation
System
generates
Implementation
Accepted Solutions
Problem definition
GOALS
Acreditation
Understanding solutions
rules
Operational Validation
Solutions
Problem Entity
Conceptual Model Validation
Experimentation
Analysis and Modeling
Data Validation
Computerized Model
Codification
Conceptual Model
Verification
Parametrization
Calibration
Scenarios / configurations
Experimental Validation

Experimental and Solution validation

Key elements on Industry 4.0

Experimentation validation

- **Experimentation** Validation: determining that the experimental procedures adopted are providing results that are sufficiently accurate.
- The important aspects to consider are:
  - the requirements for the load period.
  - the length of the executions.
  - the numbers of replications.
  - the experimental design.
  - the sensitivity analysis to assure the accuracy of the results.

Solution validation

- **Solution Validation**: determining that the results obtained from the model of the proposed solution are sufficiently accurate.
- This is similar to black-box validation in that it entails a comparison with the real world. It is different in that it only compares the final model of the proposed solution to the implemented solution.
  - The solution validation can only take place post-implementation.
  - Unlike the other forms of validation, it is not intrinsic to the simulation study itself.
  - It has no value in giving assurance to the user, but it does provide some feedback to the modeller.

Fonseca i Casas, P., Fonseca i Casas, A., Garrido-Soriano, N., Godoy, A., Pujols, W.C., Garcia, J.: Solution validation for a double façade prototype. Energies. 10, (2017). https://doi.org/10.3390/en10122013

Black-box validation example

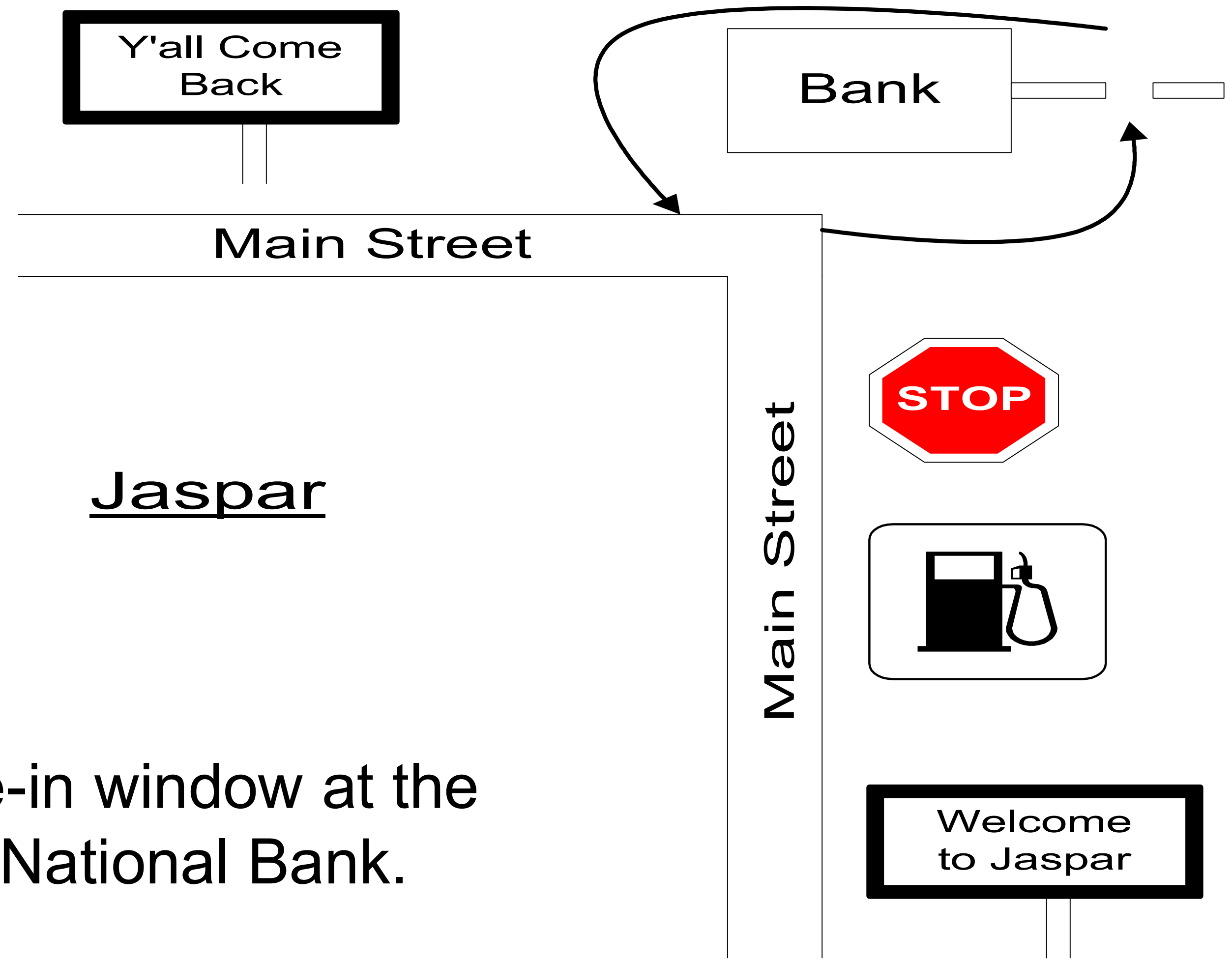
Example of objective test, using the output data.

The Fifth National Bank of Jaspar.

Black-box validation example

- The Fifth National Bank of Jaspar, is planning to expand its drive-in service at the corner of Main Street.
- Currently, there is one drive-in window serviced by one teller. Only one or two transactions are allowed at the drive-in window.
- It was assumed that each service time was a random sample from some underlying population.

Black-box validation example



Drive-in window at the Fifth National Bank.

Black-box validation example

- Service times $\{S_i, i = 1, 2, \ldots 90\}$ and interarrival times $\{A_i, i = 1, 2, \ldots 90\}$ were collected for the 90 customers who arrived between 11:00 A.M. and 1:00 P.M. on a Friday.
- This time slot was selected for data collection after consultation with management and the teller because it was felt to be representative of a typical rush hour.

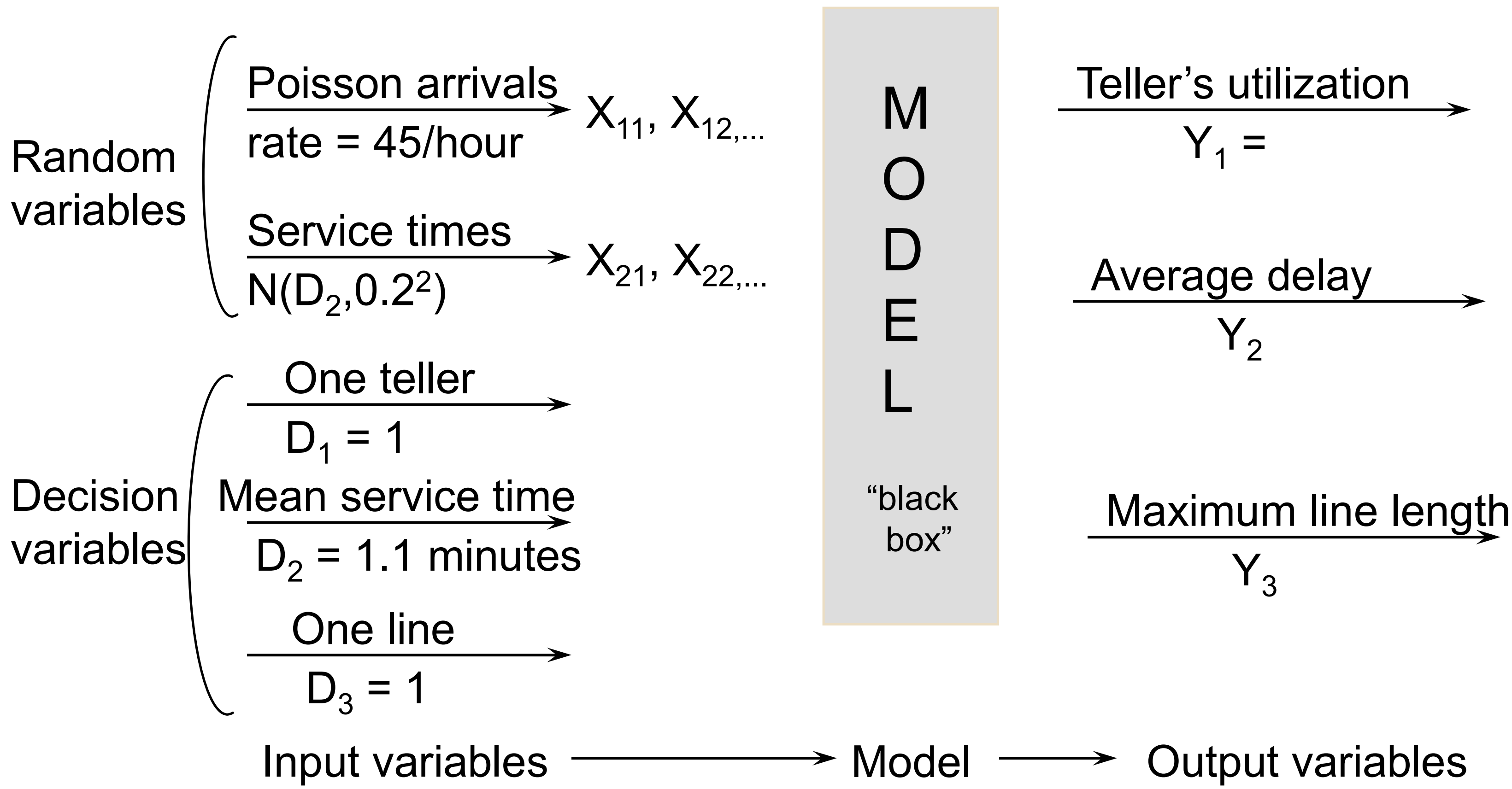
Black-box validation example

- Data analysis led to the conclusion that the arrival process could be modelled as a Poisson process with an arrival rate of 45 customers per hour; and that service times were approximately normally distributed with mean 1.1 minutes and standard deviation 0.2 minute.
- Thus, the model has two input variables:
  1. Interarrival times, exponentially distributed (i.e. a Poisson arrival process) at rate $\lambda = 45$ per hour.
  2. Service times, assumed to be $N(1.1, (0.2)^2)$.

Black-box validation example

- The uncontrollable input variables are denoted by X, the decision variables by D, and the output variables by Y.
- From the "black box" point of view, the model takes the inputs X and D and produces the outputs Y, namely
  - $(X, D) \xrightarrow{f} Y$
  - $f(X, D) = Y$

Black-box validation example



Model input-output transformation

Black-box validation example

Input variables	*Output variables, Y*
D = decision variables (interest) D_1 = 1 (a teller) D_2 = 1.1 min D_3 = 1 (a queue) X = Other variables Rate of arrivals Poisson= 45 / hour Service time: $N(D_2, 0.2^2)$	Primary variables of interest (Y_1, Y_2, Y_3) Y_1 = use of the teller Y_2 = average waiting time Y_3 = maximum length of queue Y_4 = observed rate of arrivals Y_5 = average time of service Y_6 = average time of service of sample Y_7 = mean size of the queue

Input and Output variables for model of current bank operation.

Black-box validation example

Statistical Terminology	Simulation Terminology	*Associated risk*
Type I : reject H_0 when H_0 is true.	Reject a valid model.	α
Type II : do not reject H_0 when H_0 is false.	Do not reject an invalid model.	β

Error type in the validation of a model

If the sample is fixed, the needs to reduce error of type II increases α and decreases β and inverse.
Once α has been determined, the only way to decrease β is increasing the sample.

Black-box validation example

Replicas	*Y_4 = Inputs (hour)*	*Y_5 (Minutes)*	*Y_2 =average delay (Minutes)*
1	51	1.07	2.79
2	40	1.12	1.12
3	45.5	1.06	2.24
4	50.5	1.10	3.45
5	53	1.09	3.13
6	49	1.07	2.38

Average: 2.51
Deviation : 0.82
Results of six replicas of the model bank

Black-box validation example

- Delay observed in the system $Z_2 = 4.3$ minutes.
- Delay of the model Y_2.
- We propose a statistical test of null hypothesis
  - $H_0 : E(Y_2) = 4.3$ minutes
  - $H_1 : E(Y_2) \neq 4.3$ minutes
- If H_0 is rejected following the rules of this test, there is no reason to consider the model invalid.
- If H_0 is rejected, the current version of the model can be rejected, and the modeler is forced to seek ways to improve the model.

Black-box validation example

- The appropriate statistical test is *t*, which is conducted as follows:
  - ***Step 1***. Select the level of significance a, sample e and size n. For the bank model:
    - a = 0.05, n = 6
  - ***Step 2***. Calculate the mean of Y2 and standard deviation S on these *n* replicas.

$$Y_2 = \frac{1}{n}\left(\sum_{i=1}^{n} Y_{2i}\right) = 2.37 \qquad S = \{(Y_{2i} - {}^{Y_2}/_{(n-1)}\}^{1/2} = 0.82$$

  - Where Y2i, i = 1, .., 6, are shown in the above table.

Black-box validation example

- ***Step 3***. Getting the critical value t of the table.
  - For a test of two queues, must use $t_{\alpha/2,\, n-1}$; for a test of one queue must use $t_{\alpha,\, n-1}$ or $-t_{\alpha,\, n-1}$.
  - n -1 are the degrees of freedom.
  - From the table $t_{0.025,5} = 2.571$ for a test of two tails.

Black-box validation example

- ***Step 4***. Calculate the statistic
  - $t_0 = (Y_2 - \mu_0) / \{S / \sqrt{n}\}$
  - on μ_0 is the specific value of the null hypothesis
  - H_0 . Where $\mu_0 = 4.3$ minutes, so
    - $t_0 = (2.51 - 4.3) / \{0.82 / \sqrt{6}\} = - 5.34$
- ***Step 5***. For a test of two queues:
  - if $|t_0| > t_{\alpha/2, n-1}$, reject H_0 .
  - Otherwise do not reject H_0.
  - [For a test of one queue with H_1: $E(Y_2) > \mu_0$,
    - reject H_0 if $t > t_{\alpha, n-1}$; with H_1 : $E(Y_2) < \mu_0$,
    - reject H_0 if $t < -t_{\alpha, n-1}$]

Black-box validation example

- Since $| t | = 5.34 > t_{0.025,5} = 2.571$, must reject H_0 and conclude that the model is not suitable in their prediction for the average delay for a client.
- Note that when you are making a hypothesis test, reject H_0 is a strong conclusion, so
  - $P(\text{reject } H_0 | H_0 \text{ is true}) = \alpha$

Black-box validation example

Replicas	Y_4 = *Inputs(hour)*	Y_5 *(Minutes)*	Y_2 =*average delay (Minutes)*
1	51	1.07	5.37
2	40	1.12	1.98
3	45.5	1.06	5.29
4	50.5	1.10	3.82
5	53	1.09	6.74
6	49	1.07	5.49

Average: 4.468
Deviation: 1.66
Results of six replicas of the model bank

Black-box validation example

- Step 1. Select $\alpha = 0.05$ and $n = 6$ (sample size).
- Step 2. Calculate $Y_2 = 4.468$ minutes, $S = 1.66$ minutes.
- Step 3. Calculate the critical value of t.
- $t_{0.025,5} = 2.571$.
- Step 4. Calculate the statistic

  $$t_0 = (Y_2 - \mu_0) / \{S / \sqrt{n}\} = 0.247$$
- Step 5. Since $| t | < t_{0.025,5} = 2.571$, cannot reject H_0, and can "tentatively" **accept the model as a valid.**