

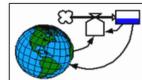
SYSTEM DYNAMICS, AN APPROXIMATION TO CONTINUOUS SIMULATION

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In this part of the course we are going to learn System Dynamics, a good approximation to continuous Simulation.

History 1

- The Bolshevik Belarusian **Alexander Bogdanov**, physician and Economist, already developed between 1913 and 1922 a systems theory of certain sophistication which it termed 'tektology', or science of structures. Bogdanov came to anticipate that in the 1960s was called 'catastrophe theory', concept that later took the nickname of 'fork'.
- 1948 *Cybernetics*, of **Norbert Wiener**, father of cybernetics (the study of the communications and the control of animals and machines).
 - Feedback appear.
- **Ludwig von Bertallanfy**, *General system theory* 1968, is a mathematical description of the systems defined over nature.
 - Is not useful to predict but allows a deeper understanding of the system.



Let's go to start this System Dynamics part, with a little review of its history to do a contextualization of the method. Remember also that exist a clear connection between system dynamics and Activity Scanning approach we see in the initial part of the course.

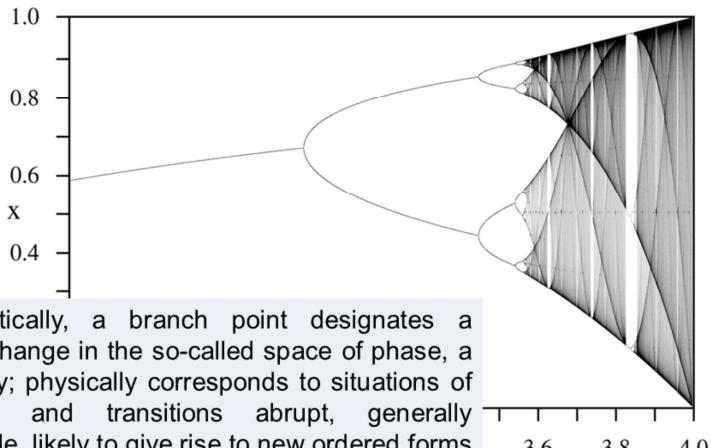
We can start with the work done by the Bolshevik Belarusian **Alexander Bogdanov**, it was a physician and Economist, that developed between 1913 and 1922 a systems theory of certain sophistication which it termed 'tektology', or science of structures.

Is interesting because Bogdanov came to anticipate what in the 1960s is going to be called 'catastrophe theory', concept that later took the nickname of 'fork'.

On 1948 the work *Cybernetics*, from **Norbert Wiener**, that is considered the father of cybernetics, that is the study of the communications and the control of animals and machines, represents an important step to understand how to model complex behaviours. In this work, for the first time a central element appears, the Feedback loop.

Later, **Ludwig von Bertallanfy**, on his work, *General system theory*, of 1968, defines a mathematical description of the systems defined over nature. This work is not yet useful to predict but allows a deeper understanding of the system.

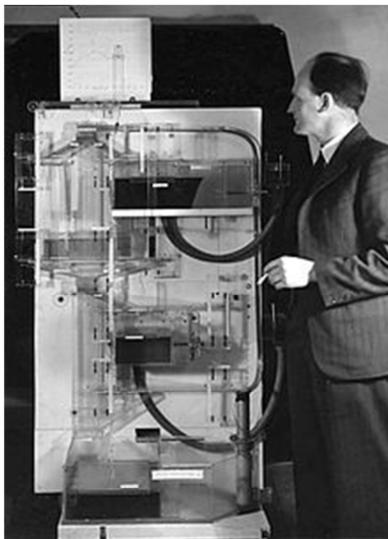
Fork



On this slide is presented a bifurcation, a central element in chaos theory or in catastrophe theory.

As you can read on the slide, Mathematically, a branch point designates a sudden change in the so-called space of phase, a singularity; physically corresponds to situations of instability and transitions abrupt, generally irreversible, likely to give rise to new ordered forms that appear suddenly.

Applications on economy



Professor A.W.H (Bill) Phillips with Phillip's Machine. Phillips was an LSE economist known for the Phillips curve and he developed MONIAC, the **analog** computer, shown here, that modeled economic theory with **water flows**.

The first known application of system dynamics equations to the economic system took place in 1933, although they were quite limited and unhelpful. A little later the sociologist and control engineer, and later economist at the London School of Economics, William Phillips, much more successfully established the (nonlinear) equations that were at the base of the Newlyn / Phillips machine, described in the next slide. Phillips established in 1954 the conditions under which an economy can be stabilized by PID93 regulators. Seeing that the matter was more complicated than he had imagined, especially because of the greater influence of delays, Phillips corrected his conclusions in 1957. Then came World3 and The Limits to Growth in 1972, and a regionalized successor in 1974.

And then a lot of silence.

The Moniac (1949)



By Kaihsu Tai - Own work, Public Domain, <https://commons.wikimedia.org/w/index.php?curid=3956307>

The MONIAC (Monetary National Income Analogue Computer) also known as the Phillips Hydraulic Computer and the Financephalograph, was created in 1949 by the New Zealand economist Bill Phillips (William Phillips) to model the national economic processes of the United Kingdom, while Phillips was a student at the London School of Economics (LSE).

The MONIAC was an analogue computer which used fluidic logic to model the workings of an economy. The MONIAC name may have been suggested by an association of money and ENIAC, an early electronic digital computer.

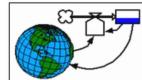
References

- (320) A.W. Phillips (1957) - Stabilisation policy and the time-forms of lagged responses - The Economic Journal 67:265–77 doi:10.1017/CBO9780511521980.019 - London School of Economics - <http://ebooks.cambridge.org/chapter.jsf?bid=CBO9780511521980&cid=CBO9780511521980A027> "A study, using frequency-response analysis and electronic simulators, of the properties of models in which the lags are given more realistic time-forms has shown that the problem of stabilisation is more complex than appeared to be the case when attention was confined to the simpler lag forms used in my earlier article."
- 321. Ragnar Frisch (1933) - Propagation Problems and Impulse Problems in Dynamic Economics - En: Economic essays in honour of Gustav Cassel - University of Oslo - <http://www.sv.uio.no/econ/om/tall-og-fakta/nobelprisvinnerere/ragnar-frisch/publishedscientific-work/PPIP%5B1%5D.pdf> "If a cyclical variation is analysed from the point of view of a free oscillation, we have to distinguish between two fundamental problems: first, the propagation problem; second, the impulse problem."
- 322. W.T. Newlyn (1950) - The Phillips/Newlyn Hydraulic Model - Yorkshire Bulletin of Economic and Social Research 2:111-127 doi:10.1111/j.1467-8586.1950.tb00370.x - The University, Leeds
- 323. A.W. Phillips (1954) - Stabilisation policy in a closed economy - The Economic Journal 64:290-323 - London School of Economics - <http://xmservices.unisi.it/depfid/joomla/iscrizione/materiali/16888/Phillips%20EJ%201954.pdf> "if any stabilisation policy is to be successful it must be made up of a suitable combination of proportional, integral and derivative elements ... If the system itself has a considerable tendency to oscillate [...], the integral element in the policy should be made very weak or avoided entirely, unless it can be accompanied by sufficient derivative correction to offset the destabilising effects [...]."
- 324. Dennis L. Meadows et al (1974) - Dynamics of Growth in a Finite World - Productivity Press Inc. - Massachusetts Institute of Technology - ISBN-13: 978-0262131421 - 637 Págs.

Here you have some references that are interesting to consult.

History 2

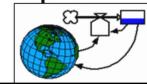
- **Forrester**, system engineer of the **Massachusetts Institute of Technology (MIT)** develops this methodology during the decade of the 50's. The first application was the analysis of the structure of an USA enterprise and the study of the oscillations, strongly defined, that present in the sells. Published as **Industrial Dynamics** on 1961. On 1969 publish **Urban Dynamics**, on it is shown how the "**DS modeling**" can be applied to a systems that represents cities. On 1970 appears the "world model", work that represents the base to develop the **I Inform of the Club of Rome** by Meadows and Meadows, published next within the book "**The limits to growth**". This works and the posterior discussion popularizes the System Dynamics in the world.



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History 3

- Forrester establishes a **parallelism** between **dynamics systems** (or in evolution) and **hydrodynamics systems**, constituted by tanks, communicated by channels with or without delays, modifying with the flows its level, maybe with the help of exogenous elements.
- System dynamics now allows to analyze systems deeper, understanding more than with the simple case study or descriptive theories.. System dynamics is not constricted to linear systems, can use the **non linear characteristics of the systems**.
- **Using computers**, system dynamics models allows a effective simulation of complex systems. This simulation represents sometimes the only way to analyze the behavior of complex systems.



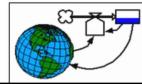
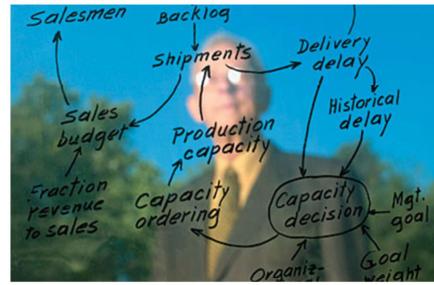
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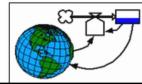
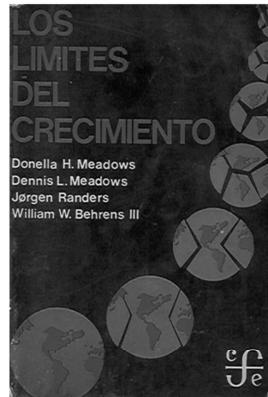
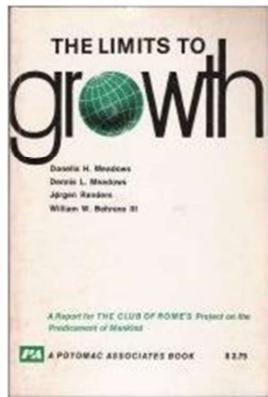
History 4

JAY FORRESTER



Here a Picture of Jay Forrester.

History 5



It is recommended to read the book, *Limits To Growth: The 30-Year Update published in 2004*.

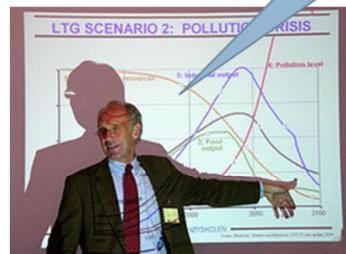
History 6

Jørgen
Randers

Dennis L.
Meadows

Donella H.
Meadows

William W.
Behrens III

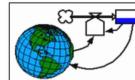


The Club of Rome is a global think tank that deals with a variety of international political issues. It was founded in April 1968 and raised considerable public attention in 1972 with its report The Limits to Growth. From 1970 to 1972 at MIT Meadows was director of the "Club of Rome Project on Predicament of mankind at MIT which constructed the world model underlying that publication

Definitions 1



- Forrester, Jay W. "Dinámica industrial". Editorial Ateneo, Buenos Aires, 1981.
- Study the **feedback** features of the **industrial activity** with the main objective of show how the structure of the organization, the amplification (of policies) and the delays (between the decision and the actions) interacts and influences the success of the enterprise.

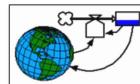


Now we have three definitions of what is system dynamics. This first definition is historically the first one, still strongly focuses on the initial work of Jay Forrester, describing the capabilities of the method to expose the feedback loops. Please stop the video and read carefully the definition.

Definitions 2



- Aracil Javier y Gordillo Francisco. "Dinámica de sistemas", Alianza Editorial, Madrid, 1997.
- Is a method where **analysis** and **synthesis** lives' together, giving an example of the systemic methodology. System dynamics is a **language that allow to express the relations** that take place in the nucleus of a system and explain how its behavior appears.

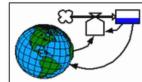


This second definition is the newest definition of all the three definitions we are going to present. Important elements of this definition is that it remarks that is a method that allows the analysis and the syntheses of the system, it means, obtaining prevision from the model that can be executed. Also, it is interesting that represents quite clear the relations that exist on the model in a graphical way. Similar to the way we can represent the processes in GPSS Process Interaction schema. Stop the video and read carefully this definition.

Definitions 3

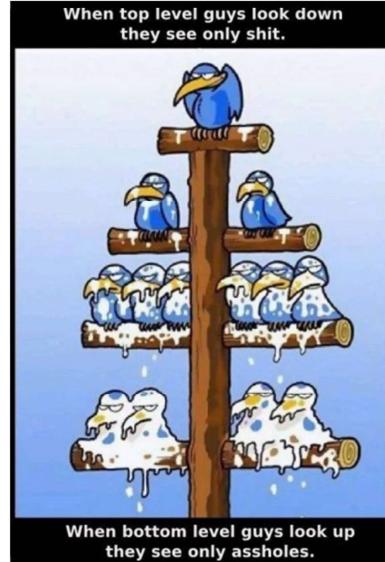


- Martínez Silvio y Requema Alberto. "Simulación dinámica por ordenador" Alianza Editorial, Madrid, 1988.
- Is a methodology that can be used to model and study the behavior of any system through the time, if presents **delays** and **feedbacks**.



This is the last definition and, from my point of view, the one that represents better what is System Dynamics. It states that can be used for any kind of system, that is true considering some limitations, and that makes sense to use this techniques if the system presents delays and feedbacks. Stop the video and read carefully the definition.

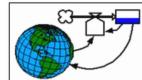
An organization, two visions



It is worth to mention that the way we are going to analyze the systems, is going to follow and approach that is slightly different to the way that we will analyze usually a system. On this slide, on the right side you can see the usual approach to see an organization (it is a joke but have some elements of the reality). On the left side, one can see that systemic approach is going to be more heterogeneous and ductile, because we will be focused on the definition of the process that rules an specific behaviors and the understanding of the causality that rules this behavior.

Systemic thinking

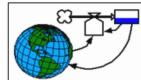
- In front of an organization or system:
 - One can think that the structure of the organization is its organization chart.
 - Or one can think that the structure is the information flow, the work flow and the business processes.
- In the **systemic thinking**, the structure of the organization, or system, is the configuration of the interrelations that exists between the different key components of the system.



In this slide we have just a review of the main important differences between systemic thinking and traditional one. Notice that this kind of approach is the one you already use in the definition of a simulation model using the discrete approach.

Static models

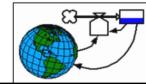
- Static models describes a system, in terms of mathematical equations, where the potential effect of every alternative is evaluated through the equations.
- The global behavior is defined through sum of the effects of every individual effect.
- Static models ignore the time variations.
- Tends to REDUCTIONISM: try to analyze all due to separate component.



At this point it is also interesting to remember that a static model is a model that do not represents the time. This models represents the global behavior through the sum of the effects of all the equations that defines the model. This models, applied to a dynamic system, tends to reductionism because they tend to analyze the different parts that constitutes the system individually.

Dynamic models

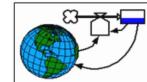
- Dynamic models are a representation of the dynamic conduct of the system.
- An static model usually implies the application of a single equation, a dynamic model implies an iteration.
- Dynamic models continuously apply their equations considering modifications in time.



In the other side, the dynamic models, analyzes the time, and to be able to understand the implications of time, usually it is needed to apply the equations iteratively. This kind of models are the models we will be focused on, since time will be a key element to understand the evolution of the system.

Susceptible systems

- Ecology.
 - Ecosystems.
 - Populations.
- Society.
 - Sustainable growing.
- Economy.



The kind of systems that can be analyzed with this approach are multiple, but because the constraints of system dynamics, usually are focused on systems that include a large number of individuals. Some examples of this kind of problems are presented on this slide. As one can see, are big problems, from the sense of the huge number of entities they will include.

Applied to economy

- Microeconomic Theory (1995) Andreu Mas Colell
- “A characteristic fact that differentiates the economy of other scientific fields is that... the balance equations are the center of our discipline. Other sciences, such as physics or ecology, have comparatively more emphasis in the determination of the dynamic laws of change.”

Just to mention that applied to economy there is an interesting work, due to one well known politician of Catalonia, that uses this techniques. Maybe is time to change and generate wealth without grow.

First attempts



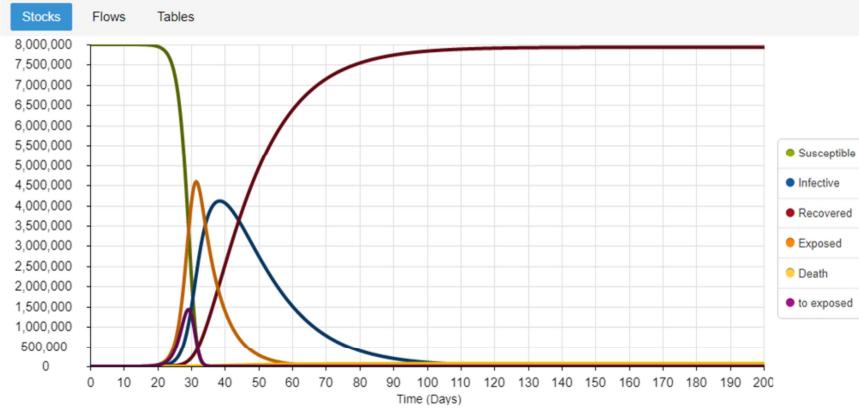
https://www.youtube.com/watch?time_continue=18&v=6UqHcIIXfwA&feature=emb_logo

And first attempts to degrow, or do a redefinition of what will be the economy are starting.

You can see the video on Youtube

https://www.youtube.com/watch?time_continue=18&v=6UqHcIIXfwA&feature=emb_logo

Applied to pandemics



<https://insightmaker.com/insight/185902/COVID-19-spread>

Sadly, now everyone is aware of this kind of models. This is a first model I developed to understand what happens to Catalonia if no containment measures will be applied and considering that some kind of immunity is generated.

You can enter in Insignmaker to tune the model. We will learn and understand how to make and read this shapes at the end of this course.

Prisoner's dilemma

Systemic Risks in Networks

A simple simulation model showing that increasing the connection density in a network can lead to less cooperation.

www.soms.ethz.ch

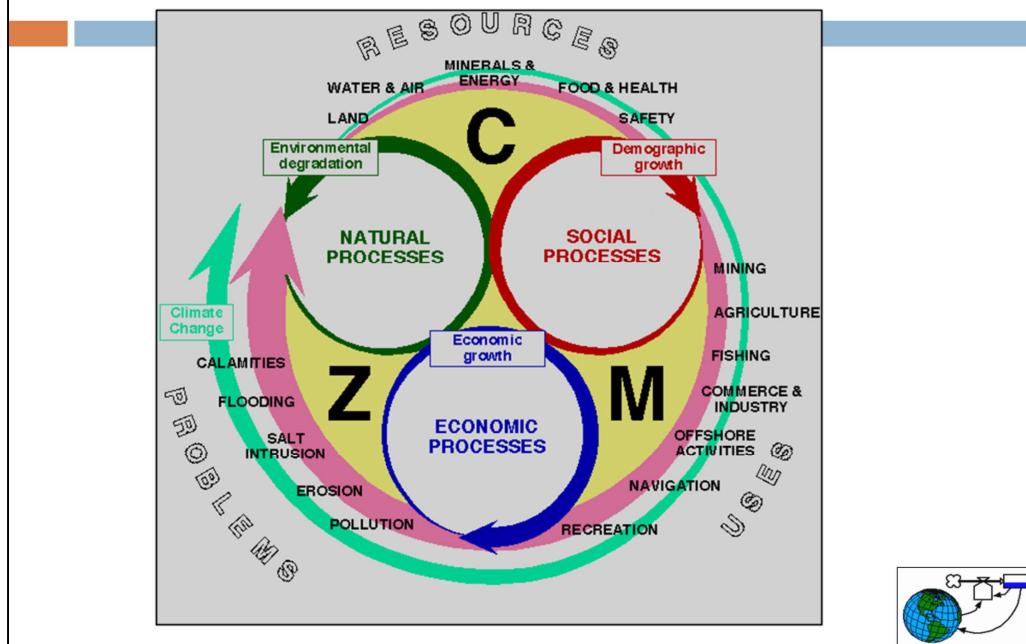
ETH RISK CENTER

Spreading and erosion of cooperation in a prisoner's dilemma game. The computer simulations assume the payoff parameters T57, R56, P52, and S51 and include success-driven migration³². Although cooperation would be profitable to everyone, non-cooperators can achieve a higher payoff than cooperators, which may destabilize cooperation. The graph shows the fraction of cooperative agents, averaged over 100 simulations, as a function of the connection density (actual number of network links divided by the maximum number of links when all nodes are connected to all others). Initially, an increasing link density enhances cooperation, but as it passes a certain threshold, cooperation erodes. The computer simulations are based on a circular network with 100 nodes, each connected with the four nearest neighbors. n links are added randomly. 50 nodes are occupied by agents. The inset shows a 'snapshot' of the system: blue circles represent cooperation, red circles non-cooperative behavior, and black dots empty sites. Initially, all agents are non-cooperative. Their network locations and behaviors (cooperation or defection) are updated in a random sequential way in 4 steps: (1) The agent plays two-person prisoner's dilemma games with its direct neighbors in the network. (2) After the interaction, the agent moves with probability 0.5 up to 4 steps along existing links to the empty node that gives the highest payoff in a fictitious play step, assuming that no one changes the behavior. (3) The agent imitates the behavior of the neighbor who got the highest payoff in step 1 (if higher than the agent's own payoff). (4) The behavior is spontaneously changed with a mutation rate of 0.1.

This kind of Agent based simulation will be connected to System dynamics at the

end of the course.

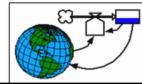
Susceptible models



One important point that is needed to remember is, that in this type of models, all the variables are related between them. This implies that often a modification in one area of the model implies a strong variation of the answers we obtain in other part of the model. This makes that the verification of this models is little more complex than in discrete simulation.

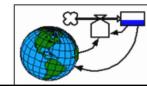
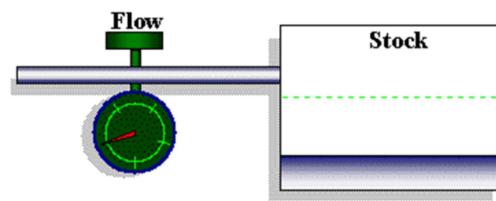
Continuous event

- Continuous simulation is analogous to a tank where the fluid cross the pipe continuously.
- A change in the values is based in the change on the time variable.



But what is the approach we will use, and how it differs form discrete Simulation? The main difference that exists, is that the event is not a discrete element, but a continuous element that evolve continuously. This continuous event can be represented as a fluid that moves from one tank of fluid to other tank of fluid due to the time evolution. The modifications on the quantity of “liquid” that we have in each tank, allows us to understand the evolution of the model. Remember the MONIAC machine?

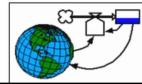
Continuous event



On this slide one can see intuitively the evolution of a continuous event. You can notice that there is a Stock, an accumulation of the fluid. It contains the liquid and depending on the amount of liquid it have is going to provide us an answer or another. The amount of liquid that enters in the Stock through the pipe is controlled by a Flow, this flow also is connected to the Stock, if we have more liquid on the Stock, maybe we are going to increase o reduce the Flow, hence a feedback loop appears, the genesis of the exponential distribution.

Diagrams (i)

- Causal diagram.
 - Represents the system variables.
 - Represents its relations.
 - Represents the sense of its feedbacks.



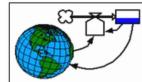
To define this models we will use some graphical tools. We will start with the causal diagrams that will represent the system variables, its relations and really interesting to understand the nature of the models, the sense of its feedbacks.

Causal diagrams are not only used on System Dynamics, hence can be a good tool to understand the main elements to consider in any simulation project. Notice however that this kind of diagrams are not complete and also owns an inherent ambiguity, hence does not represent a conceptual model for a simulation.

Diagrams (ii)

□ Flow diagrams:

- Also named Forrester diagrams.
- Characteristic of the system dynamics.
- Simplify the definition of the equations.
- In brief: classification of the elements of the system.



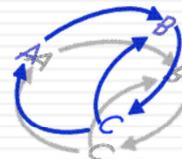
From the causal diagrams, we will be able to define the Flow (or Forrester diagrams, named in honor to Jay Forrester). These diagrams are specific of System Dynamics, and with the definition of the inner expressions are complete and not ambiguous.

These diagrams simplify the understanding and the definition of the models that will be composed by a set of differential equations.

Also these diagrams do a classification of the system elements, defined previously on the causal diagram, a classification that depends on the kind of answers we want to obtain from the model.

Causal diagrams

Representing the concepts and its relations.



Now we will review the causal diagrams, that will be used as an starting point to understand the main elements and concepts on the system, and how this different elements are going to be connected. More interesting, we will see that from the loops that appears we can anticipate the behavior of the systems prior to any execution of the simulation model.

Elements of a causal diagram

- Elements to be related:
 - Population
 - Hunger
- Relations
 - **Positives:** an increment of A causes an increment on B
 - **Negatives:** an increment of A causes a decrement on B
- Cycles
 - **Positives:** Contains a pair number or negatives relations
 - **Negatives:** Contains an odd number or negative relations



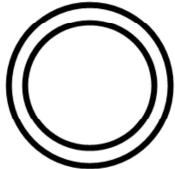
The main elements of the causal diagram are the circles and the arrows. The circles are going to represent the concepts, or elements, that we can consider as relevant for the simulation model we are going to build. As an example we can consider, for a populational model that tries to describe the hunger in a specific population the two main elements to consider will be the population and the hunger level. Hence this two elements are going to be connected, surely with other elements with arrows. These arrows are the relations. The relations can be positive or negative. A positive relation implies that an increment of A, the first concept that is related, the origin of the arrow, causes an increment on B, the arrow destination. A negative relation implies that an increment of A causes a decrement on B.

With this we can analyze the cycles that will be generated. Cycles can be positives, if they contain a pair number or negatives relations, and negatives, if they contain an odd number or negative relations.

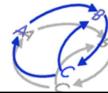
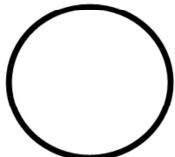
We will add, at the side of the arrows a sign, plus or minus, to represent the type of relation also, in the middle of the loop, we will add a plus or minus sign, describing the nature of the loop.

Variable typologies

- Exogenous: Affects the system but are not modified by it.



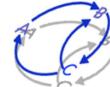
- Endogenous: Affects the system and can be modified by it.



Regarding the type of variables that can be in the model, we can see exogenous variables, that has no input arrows, that means that its value is not going to be modified, or endogenous, with at least one incident arrow, implying that its value is going to be modified.

Relations

- **Causal** relation: an element A determines an element B with a cause/effect relation.
- **Correlative** relation: exists a relation between A and B but is not a cause/effect relation.



Remember also that the relations that we are going to represent on the diagrams are the causal relations, not the correlative relations. Adding this kind of correlative relation to the diagrams, will represent a problem for the correct interpretation of the causality that drives the simulation model.

Causal diagrams

- Shows the behavior of the system
- Allows to know the structure of a dynamic system, given by the specification of the variables and the relations of each pair of variables.

$A \rightarrow B$	A influences to B
$A \rightarrow B +$	an increment on A implies an increment on B (positive relation)
$A \rightarrow B -$	an increment on A implies a decrement on B" (negative relation)



To summarize this concepts, on this slide we show the relations we have in a causal diagram, by default, a relation without any sign is just a “influence” relation. This kind of relations can be useful at the beginning of the definition of the model, but at the end, when we will use this diagram to define a Forrester, or other kind of conceptual model, we need to define if the relation is positive or negative.

Cycles

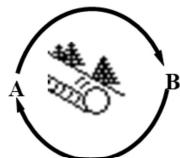
- Only two cases:
 - Negative feedback
 - Positive feedback



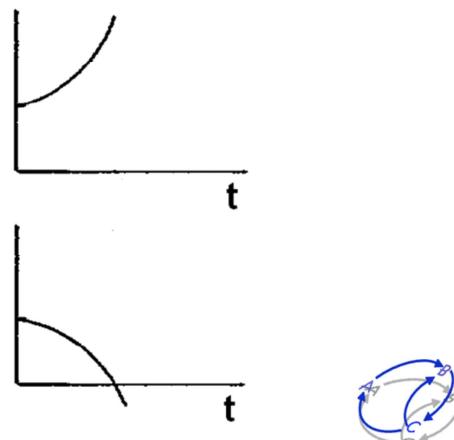
Regarding the cycles remember that we have two cycles, positives and negatives, and quite interestingly, depending on how these cycles are going to be combined, we can predict the behavior of the model, hence the behavior of the system, prior to any execution. We will see this on the next slides.

Positive feedback

- Are those feedbacks who the variations of an element makes the initial variation stronger.
- Snowball effect.
- Creates instability.



- Reinforcing feedback



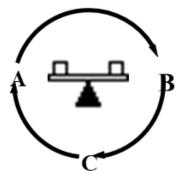
The first loop we will analyze if the positive feedback. In this case the variables that are in the loop are going to grow, or to degrow, to the infinite, or the minus infinite. Is only matter of time to achieve this values.

Is like a snowball effect also, because since the values are increased, the effect of the loop is reinforced, making that at each iteration the values increases more than in the previous iteration.

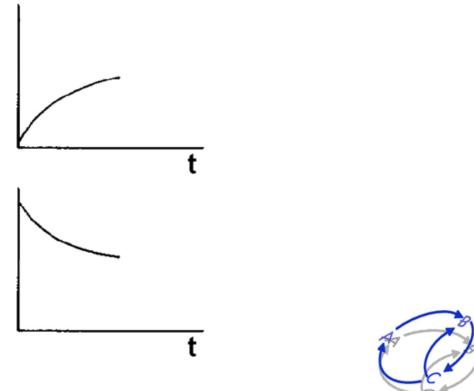
On the slide, on the right side you can see a reinforcement where the values are increased until the infinite value or to the minus infinite values, depending if the values are positive or negative.

Negative feedback

- Negative feedbacks are those who the variation of an element of the loop is propagated in a way that compensates the initial variation
- Tendency to equilibrium



- Stabilizing feedback

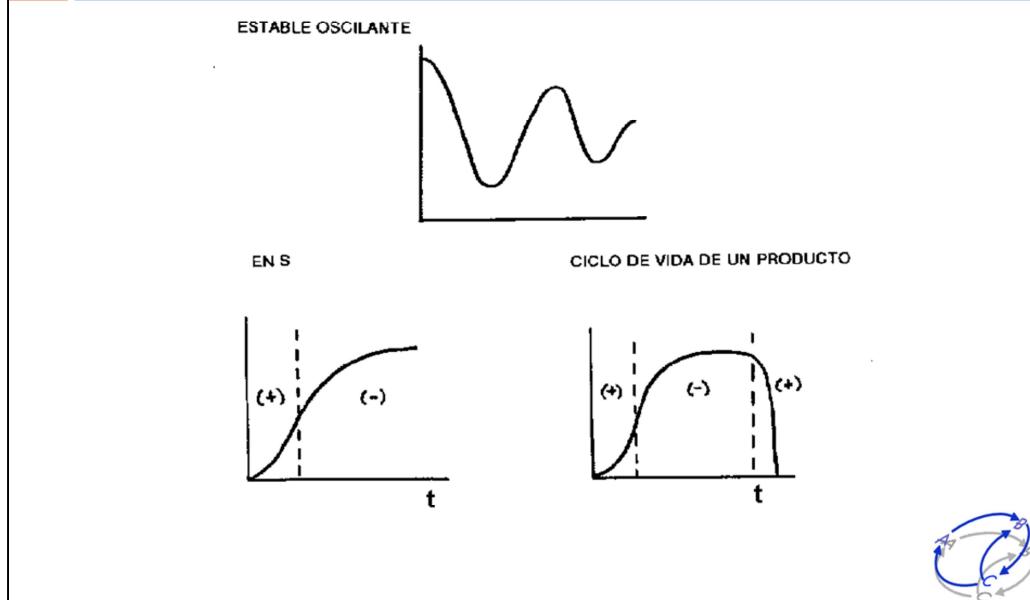


The negative loops allows are those where the number of arrows that owns a negative sign are odd, (1, 3, etc.). This kind of loops generates stability, hence represents an important element to assure that a system is going to become stable during the time we perform the analysis.

On the right side you can see the two possible outcomes for this kind of loop, in case the values in the loop are positive or negative.

Notice that without this kind of loop any system is not going to be stable, hence, detect this kind of loops is crucial to perform the analysis of the possible results we will obtain with the posterior simulation.

Feedback combinations



The problem appears when there is a combination of loops. Think in a variable, that belongs to two, or more, different loops, one that is positive and other that is negative.

In that case the outcome of the chart will depend on the variables that will reinforce one loop or other loop. Also this reinforcement can change, due to the dynamics of the model, during the time.

On the picture are show three classical configurations of combining loops, the stable oscillation, caused by tree loops, the growth in S , caused by two loops, a positive loop that causes the growth and a negative, that causes the stabilization. We will detail latter this schema, since is one of the main combination to describe the evolution of several different phenomena, like a pandemic, a rumor, or information spread.

The last chart represents the cycle of live of a product, similar to the growth in S but with a reinforcement at the end of the positive loop, that causes the decrement of the answer. This last schema is of a crucial interest to understand when a product is interesting for a company or not, and to justify the resources invested on an specific product.

Causal diagram construction

- Define the elements of the causal diagram as quantities that can be increased or decreased.
 - Clear definitions of the elements using names, the verbs are the actions.
 - No flair to the crime, but tolerance to the crime.
- Define the time units for the elements
- Define the element positively
- Disaggregate the relations if its signification is complex.



To build a causal diagram we must follows this simple instructions.

First, we will define the elements of the causal diagram as quantities, this quantitates can be increased or decreased.

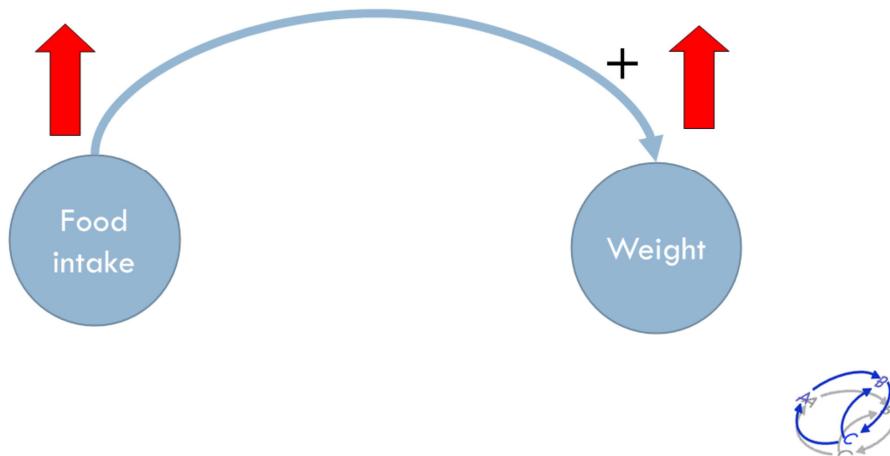
Here is important to use clear definitions of the elements using names, the verbs are the actions and will be used to define the semantics that rules the behavior of the model. As an example, we will not use the expression, flair to the crime, but we will use, tolerance to the crime. This last expression can be quantified positively or negatively.

We will define the time units for the elements, remember that this approach have a lot of similarities with an Activity scanning approach, hence it is needed to define the delta t, that rules the model evolution, at least define this and explain what is your main temporal unit.

Since it is needed to define the sign of the arrows, define the element positively, this will reduce the chance of error due to the semantics.

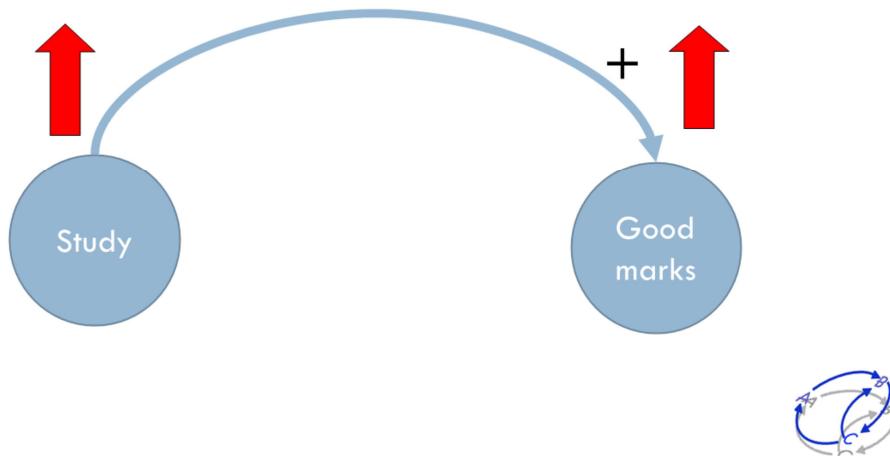
Finally, disaggregate the relations if its signification is complex. Do it as simple as possible.

Example 1



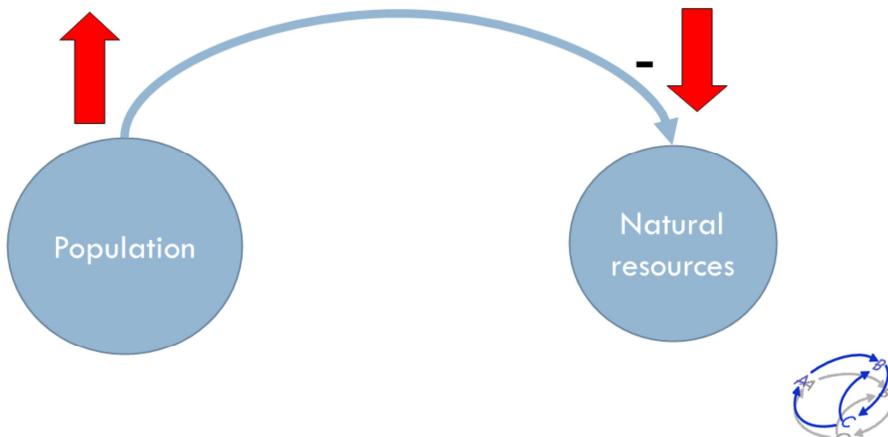
Here you have the first example. We can agree that, for a huge amount of people, if we increase the food intake, we increase the weight, hence the relation between these two concepts are positive.

Example 2



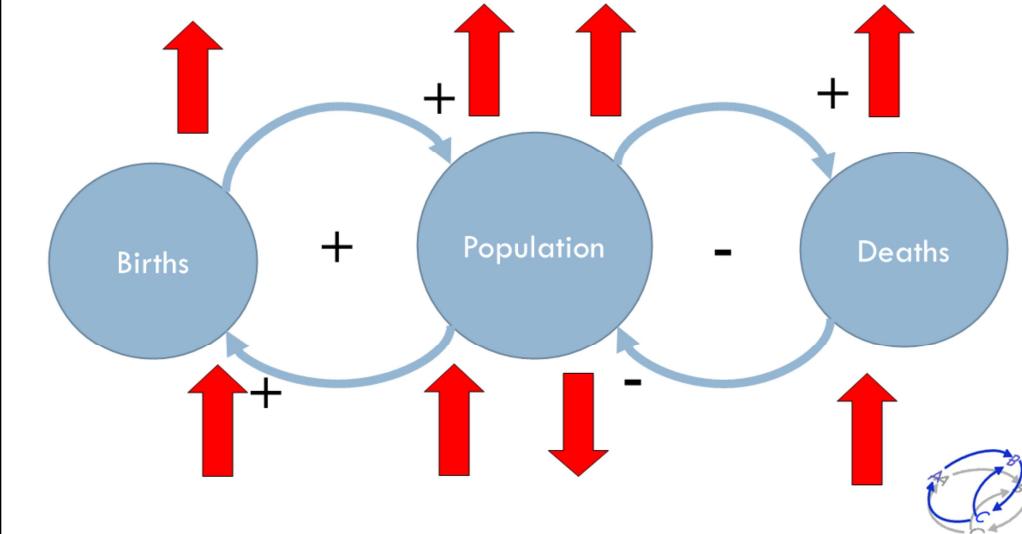
Here we have the same model, changing the semantics of the elements. Do you agree with it?

Example 3



On this third example, we want to relate two concepts, population and natural resources. If we increase population, we decrease the natural resources, hence the relation is negative.

Example 4

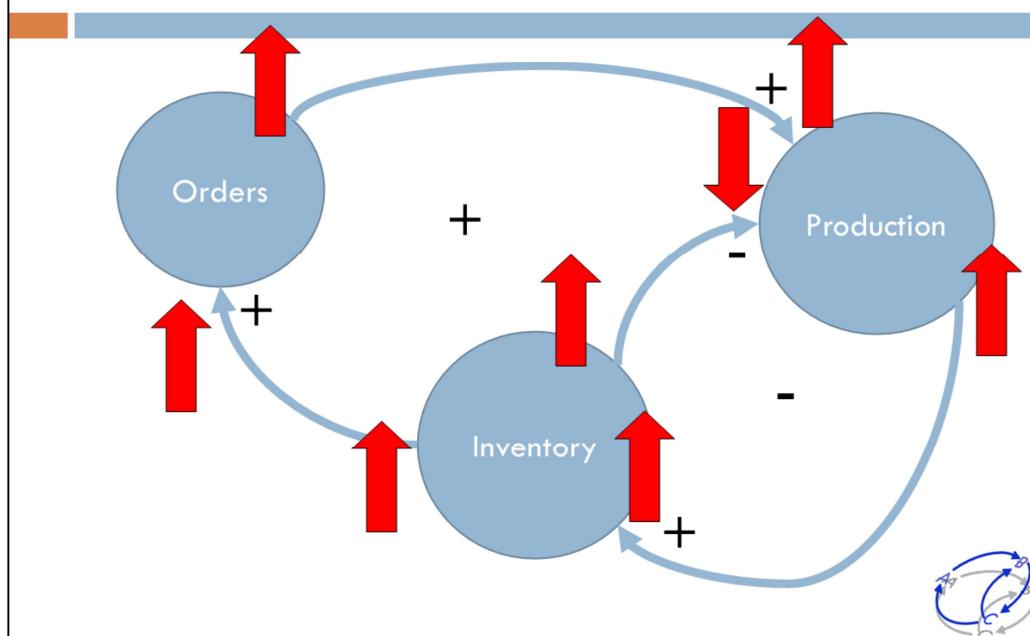


This is a little more complex, but not difficult, model. The concepts we have are population death a births, and is the basis for tons of System Dynamics models that wants to analyze the dynamic of an specific population.

If Births is increased, the Population grows, a positive relation. If Population grows, the Deaths grow, a positive relation. If the death grows the population decreases, a negative relation. Since one of the two arrows of the loop is negative, the number of negative arrows are odd, just one, the loop is negative.

Finally, if the population grows the Births grows, creating a positive relation. The in that case is loop is positive. This is an example where the final outcome of Population cannot be predicted directly without understanding what is stronger, the Death rate or the Births rate.

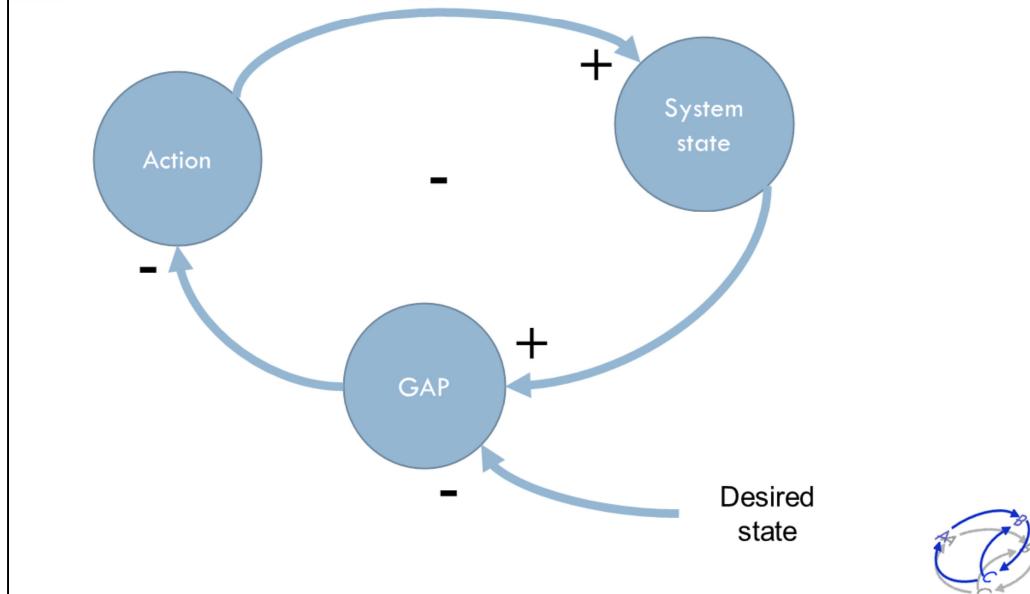
Example 5



In this other example we can see an industrial example. We want to relate the concepts Orders, Production and Inventory. If we increase the inventory, we can increase the orders, suppose that the marketing campaigns increases this orders. This implies that this relation is positive. If we increase the order we increase the Production, hence this is a positive relation. If we increase the production, we increase the inventory, hence this is also a positive relation. Notice however that if we increase the inventory, we decrease the Production, hence this is a negative relation.

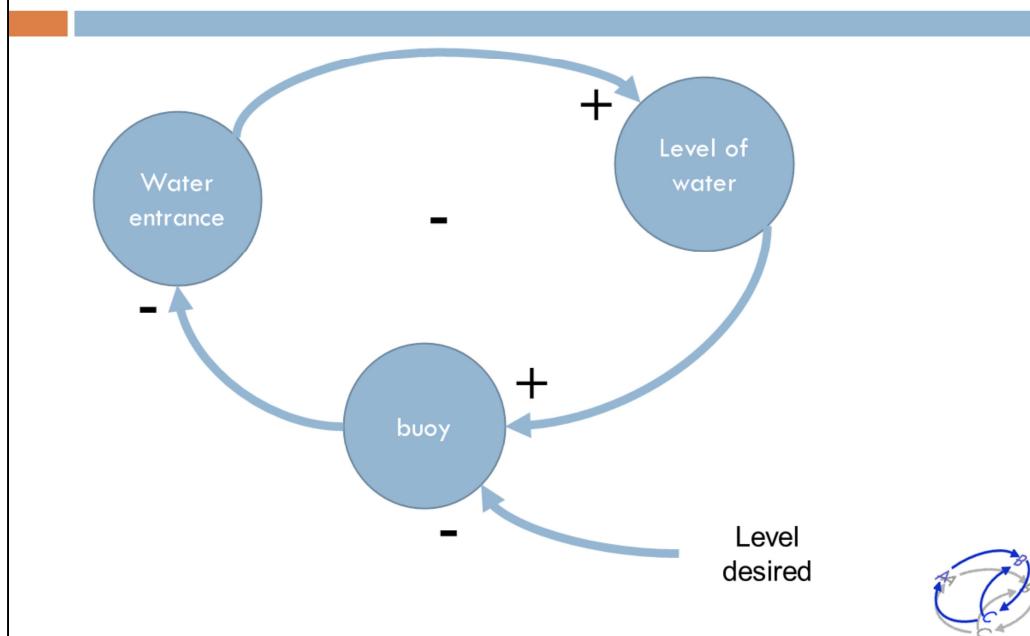
Having all the relations defined on the Causal diagram, we can now define the loops, a negative one that includes Production and inventory and a positive one that includes the tree concepts, Orders, Production and Inventory.

Causal diagram (Finding the stability)



Sometimes the definition of the causal diagram can be guided by the results we expect to obtain, because we want to validate some assumptions or to understand the value of a specific parameter. In this diagram is shown a classic definition of a causal diagram with a negative reinforcement loop, we are looking for stability, for a system that looks for a specific desired state using an element, a GAP, that allows to execute an action to correct the system internal state.

The cistern

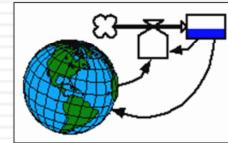


As an example, think on a cistern. In this case the water entrance depends on a buoy that increases or decreases its value depending on a comparison between the level of water of the cistern and the desired level. The buoy measures this level of water, if this level grows the water entrance decreases, if the level is low, the water entrance increases, hence this relation is negative. If the water entrance grows, the level of water grows, hence this relation is positive. If the level of water grows, the buoy grows, hence the relation is positive. In this case the loop is negative, but to assure the sign of the loop is negative we can use the GAP element, the buoy in that case and make that the measurement works in the opposite.

Flow diagrams

Defining the differential equations of the model.

Defined through the causal diagrams.

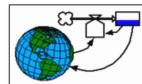


Flow diagrams or Forrester diagrams, defines the differential equations of the model, hence this diagrams, including the equations are a complete and unambiguous representation of the model.

The definition of this diagrams is done using the Causal diagrams as a starting point. As we will see next, no new elements that the one existing on the causal diagrams, are added.

Equations examples

- $\text{LEVEL}(t+\Delta t) = \text{LEVEL}(t) + \Delta t * (\text{F_IN} - \text{F_OUT})$
- $\text{LEVEL}(t+\Delta t)$: Level on time $t+\Delta t$.
- $\text{LEVEL}(t)$: Level on time t .
- F_IN : Input function.
- F_OUT : Output function.

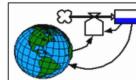


Here you have an example of the equation you will see on this type of models. Each level of the model represents one variable of interest, the modification of the levels, dynamically due to the time evolution, depends on the input and output flows, represented on the equations as F_IN , for the input flow, and F_OUT for the output flow.

Notice that this equations bases the time evolution in a delta t that will be defined at the beginning of the simulation execution. Did you notice the relation with Activity Scanning approach?.

Flow diagram elements

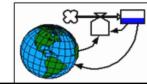
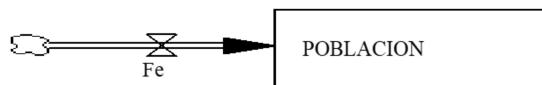
- **Levels:** Describes on each time instant the situation of the system.
- **Flows:** Temporal functions, defines the variations on the levels.
- **Auxiliary variables and constants:** parameters that allow a better visualization of some aspects that determines the behavior of the flows.
- **Material channels:** Allows the transmission of physical magnitudes between flows and levels
- **Information channels:** Allows the transmission of information. Information is not stored.
 - DELAY1, DELAY3: Delays in the transmission of material
 - SMOOTH, DLINF: Delays in the transmission of information



On this slide are presented the main flow diagram elements, the levels, the flows, the auxiliary variables, the material and the information channels.

Levels

- Also named accumulations or state variables.
- Levels modifies its values through a time period depending on the flows and the auxiliary variables.
- Mathematical representation: $N(t+dt) = N_0 + dt[(F_e(t))]$

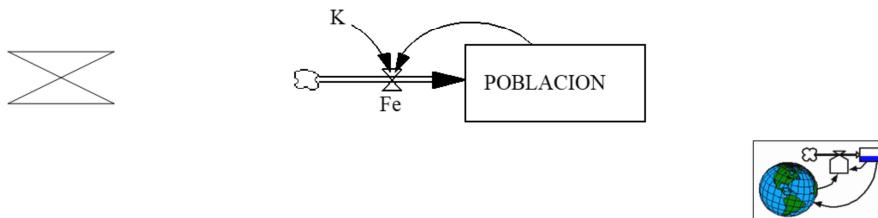


As we said previously, the Levels represents the accumulation points, and from the semantic point of view, the state variables of the model. They changes its value depending on the evolution of the model.

Notice that the mathematical representation of the equation that rules this behavior can use an infinite small delta t. However, from a practical point of view, and since we will use an approximate method to calculate the equations, we can use, as is presented on the previous slides, delta t.

Flows

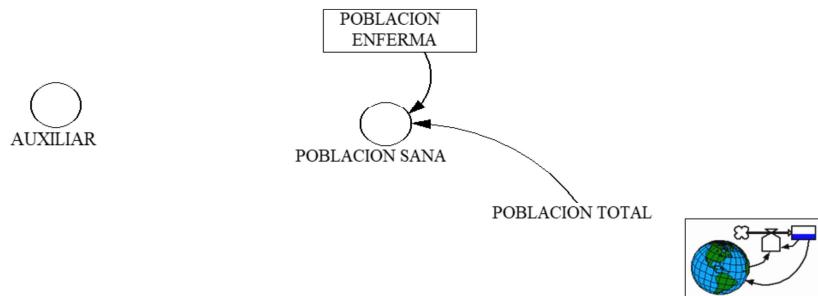
- Flows or valves are variables connected to one pipe and defines how the level grows or decreases its value.
- Flows carry “material”.
- Mathematical representation:
 - $Fe(t) = N(t) * K$



The flow represents the elements that regulates the amount of liquid that navigate through the pipes when it moves from a level to other level, or from the environment to the first level or from the last level to the environment.

Auxiliary variables

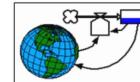
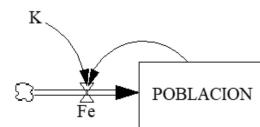
- An auxiliary variable allows to perform the calculus and explain better (clearly) the model.
- To modify a level always is better to use flows.
- Mathematical representation: $PS(t) = PT - PE(t)$



On this diagrams we can also use auxiliary variables. This variables can be just a simple value, but also can be a table or a database, there is no constrain in the nature of the data we will use. The only constrain is semantic, the auxiliary variables are not elements that are in our main analysis interest and are used to be able to calculate correctly the movement of the liquid from one level to other through the flows.

Constants

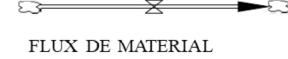
- Constants are values that are not modified during the execution of the model.
- Mathematical representation:
 - $K = 20\%/\text{any}$
 - $F_e(t) = N(t) * K$



We can define also constants, that are not going to be modified during the execution of the Simulation.

Arrows

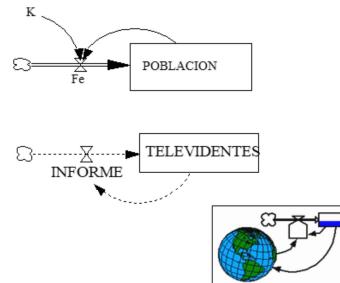
- Arrows represents the relations between variables and represents the causal relations.
- Represents the transmission:
 - Information between variables
 - Material between levels



FLUX DE MATERIAL



FLUX D'INFORMACIÓ



Finally, just to mention that we have several arrows to join all the elements, these arrows represents the pipes contains the liquid that is regulated by the flows.

Flow diagram elements



System not studied



Level



Flow



Material channel



Information channel



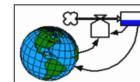
Auxiliary variable



Exogenous variable



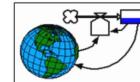
No lineal relation



On this slide we summarize the different elements we can find in a Forrester diagram.

Flow diagram construction

- Is needed to define what elements are levels:
 - System variables are used to be represented with levels.
- Flows modify the levels:
 - Birth rate, deaths, etc...

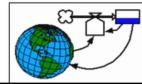


To build a Forrester diagram we will start from a causal diagram, and from it we must select what elements will be the levels. Mainly we select, from all the elements of the causal diagram, those concepts that represents the system variables, the main elements to analyze of our analysis.

Then we must define the flows and the variables. Notice that all these elements, flows, variables, also must be detailed previously on the causal diagram.

Random

- We can use random parameters in the equations.
 - Increases the time calculus.
 - Now the computer allows to perform more complex equations.



Just to mention that this kind of models often do not use randomness. This is because, often this kind of models represents systems that are huge, an ecosystem, a society, and the modeler can consider that the variability is low. However, to fit better the models one must use randomness.

The discussion regarding the use of randomness in this kind of models is the same as in any other kind of models, please refer to the Design of Experiments course to review how to deal with randomness in simulation.

Phases to construct a model

□ CONCEPTUALIZATION

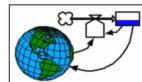
1. “Informal” description of the system.
2. Precise definition of time model.
3. Causal diagram.

□ FORMULATION

1. Forrester diagrams construction.
2. Simulation equations definition.

□ ANALYSIS AND EVALUATION

1. Model analysis (comparison, sensibility analysis, policy analysis)
2. Evaluation, communication and implementation of the results.



The phases to build a simulation model based on system dynamics is summarized on this slide.

First, we do the CONCEPTUALIZATION. We can start with an Informal description of the system, later we will detail the time units we will use, in a simulation model we will use the smallest time units possible. Finally we will finish with the Causal diagram

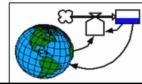
Next, we will do the FORMULATION, that starts with the definition of the charts that compose the Forrester diagrams, to later detail the equation that rules the model behavior.

Finally we will do an ANALYSIS AND EVALUATION of the model, the Model analysis based on an experimental design, comparison, sensibility analysis, policy analysis, etc.

On this part we can also do the Evaluation of the several alternatives we have and the communication and implementation of the results.

Recommendations

- Aggregation level.
 - Start with a high level of aggregation.
 - Aggregation level depends on the experience.
- Classification of the models.
- Estimation of the parameters.
- Use the expert's opinions.



Finally, some recommendations to build a good model.

First, be careful with the aggregation level, we can start with a high level of aggregation, and go down detailing further and further the different elements that constitutes the system. This is a top-down approach.

The models can be classified; hence, the model's classification can help to understand the behavior we have.

The estimation of the parameters also is an important aspect to consider. This kind of models often lacks on precise parameters that can be used on the simulation. In that case, the comparison of the model with the reality and the understanding of the model nature can help to do this estimation.

Examples and model typology

Simplify the model's construction.



On this section we will review some examples and typologies of models that help us to understand and anticipate the possible solutions to our models. This will be extremely helpful to do the models Verification, Validation and Accreditation.

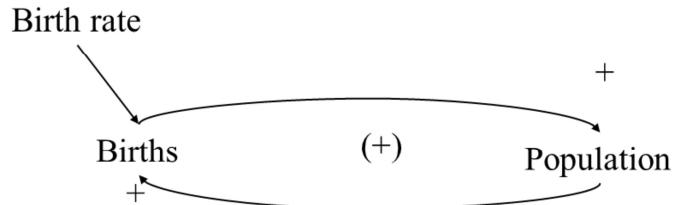
Example

- We want to study a population N.
- With a specific birth rate.
- We want study the evolution of a population depending on these parameters.



Think this common problem in Population dynamics, we want to analyze the evolution of an specific population and we have an specific birth rate to describe it.

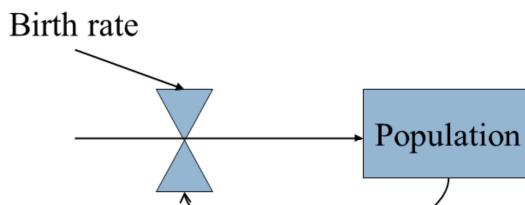
Example(I): Causal diagram



The first thing to do is the Causal Diagram. Since we have only three concepts, the Birth rate , hence the Births and the Population, it is clear that the Causal diagram must look like the one presented on this slide. Birth affects Population, positively, and Population also affects the Births, positively. Birth rate is just a constant that affects the Births, used to calculate the number of newborns at every iteration.

Example (I): Forrester diagram

- Forrester diagram is:



- Differential equations are:

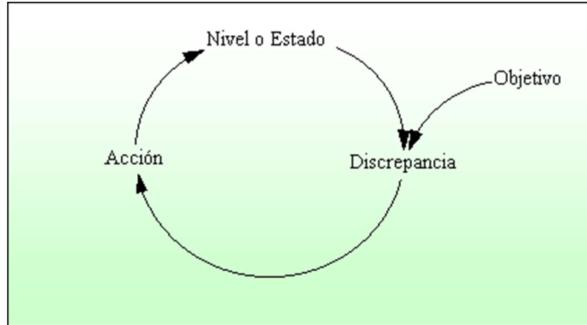
- $\text{Population} = \text{Population} + N dt$



From this we can define the Forrester diagram, that is like the previous Causal diagram, but understanding that now the Population will be the level we analyze.

Negative feedback system (causal)

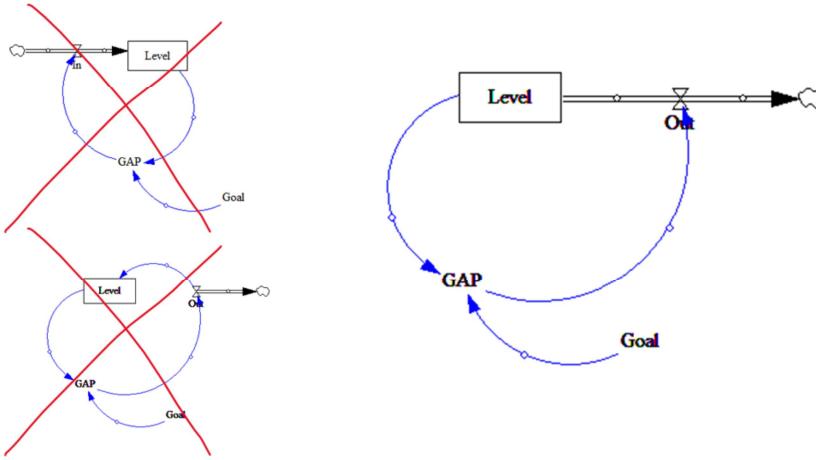
- Systems that have self regulation mechanisms
- Homeostatic systems.
- A goal is defined: exogenous variable



Let's go further to analyze more complex systems. In this slide we present a negative loop, that represents an stable system. Here is clear that the results tends to stability at some time.

Negative feedback (Forrester)

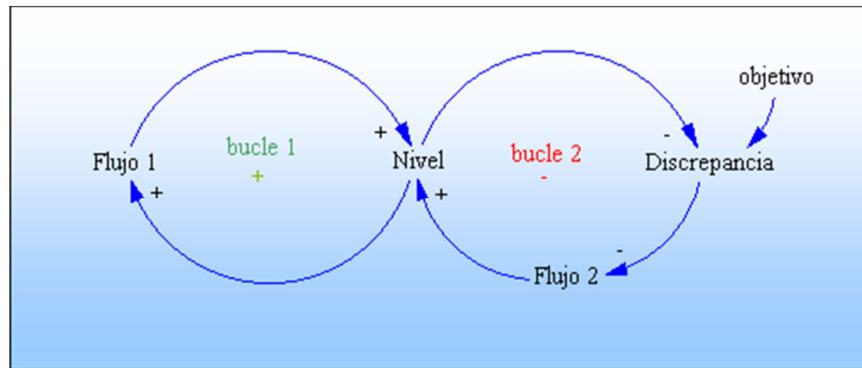
- Is needed an auxiliary variable to define the Forrester diagram



To define this type of Causal diagram, we can define a Forrester diagram like the one presented on this slide. However, this diagram presents a subtle problem. Notice that the Forrester diagram expresses the Flows, and in that case the Flow must be a flow that decreases the number of elements that exists in the Level. With the image we are presented here is not fully correct, since it will be better to represent the flow as an Output Flow.

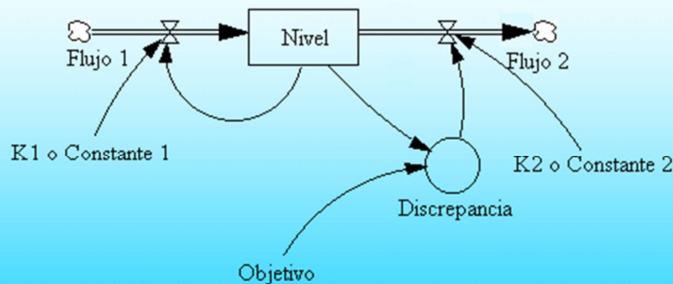
Also notice that the Flows are always connected with the Level, from the Flow to the Level. This is clear if you use some tools where if you try to connect with an arrow the Flow and the level the resulting diagram will be this, that is wrong.

System with double feedback: Causal diagram



With this considerations we can go further and define the Forrester diagram for a Causal diagram like the one presented on this slide. Notice that the names of the elements. Flujo, that means Flow, or Nivel, that means Level, are defined like this to simplify the posterior classification in the Forrester diagram.

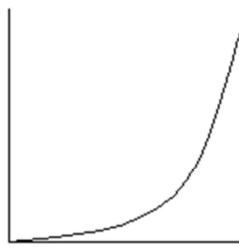
System with double feedback: Forrester diagram



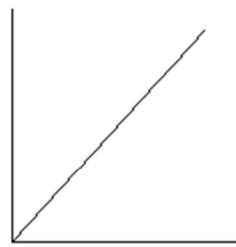
As we know from previous slides, the Forrester diagram is this. Notice that the input flow reflects the positive loop, and the output flow represents the negative loop.

Looking this diagram we are not able to predict what will be the shape for the Level, that is affected by the two flows. Obviously it depends on the constants.

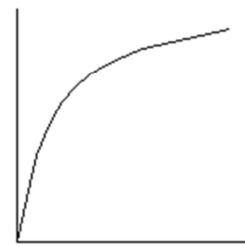
System with double feedback



Exponential
 $K_1 > K_2$



Crescent
 $K_1 = K_2$



Asymptotic
 $K_1 < K_2$



These can be the results that we obtain depending on the values of the constants. If $K_1 > K_2$ we will obtain an exponential growth, if K_1 is equal to K_2 , a crescent ascent, but if we are in the case where K_1 is small than K_2 we find an asymptotic shape.

Growing systems on S

- Transitory regime with two phases
 - ▢ Exponential growing
 - ▣ Caused by the positive feedback.
 - ▢ Asymptotic negative growing.
 - ▣ Caused by the negative feedback.
- At the end stabilizes (exponential sharp growing do not exist in the real world).



At this point we can analyze a model that is widely used for the analysis of several systems. This model is defined by the existence of a transitory regime with two phases.

First, we have an exponential growth, that is caused by the positive feedback of the model, then we have an asymptotic negative growth, caused by the negative feedback of the model. This type of model, at the end stabilizes, that is one of the more interesting lessons of this type of models, since exponential sharp growing do not exist in the real world.

Growing systems on S

- Examples:

- Ecological studies
- Social areas
- Rumors
- Epidemics
- Cellular growth of a plant
- Market saturation
- Religion
- Mode diffusion
- Physical and mental development of a child
- Urbanization in a specific area



But what kind of systems can be represented by this model?, on this slide is presented some examples of systems that can be modeled with this, Ecological studies, Social areas that encompasses behavioral sciences, the propagation of a rumor, epidemics, the analysis and description of the cellular growth of a plant, the market saturation, the propagation of a religion, the diffusion of a mode, how physical and mental development of a child evolves, the urbanization in a specific area, among many, many other areas where this model can be applied.

Growing systems on S

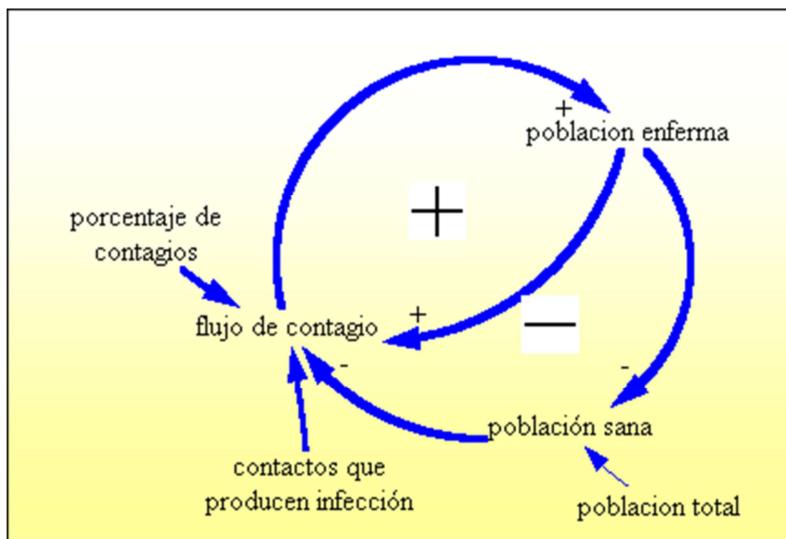
□ Example of epidemics; hypothesis

1. Population is constant.
2. The epidemics is enough *smoothly* to allow the infected people to live as usual. During the period of the epidemics nobody is healed, this avoid the re infection.
3. Health and ill population are merged homogeneity.



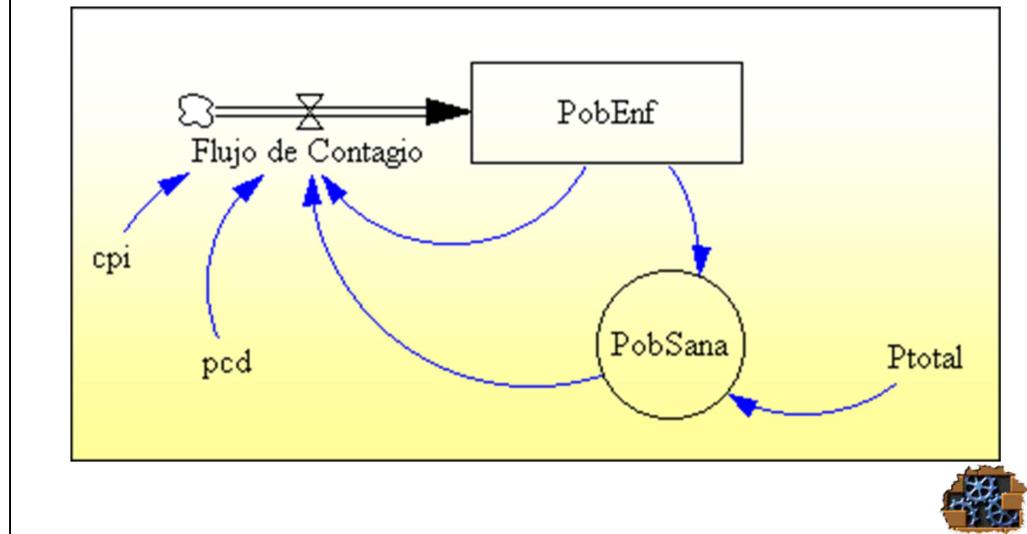
Let's go to review an example for an epidemics. With some constrains, we suppose that the population is constant, and that the epidemics is enough smoothly to allow the infected people to live as usual. During the period of the epidemics nobody is healed, this avoid the re infection. We also suppose no confinement measures, that is health and ill population are merged homogeneity.

Growing systems on S



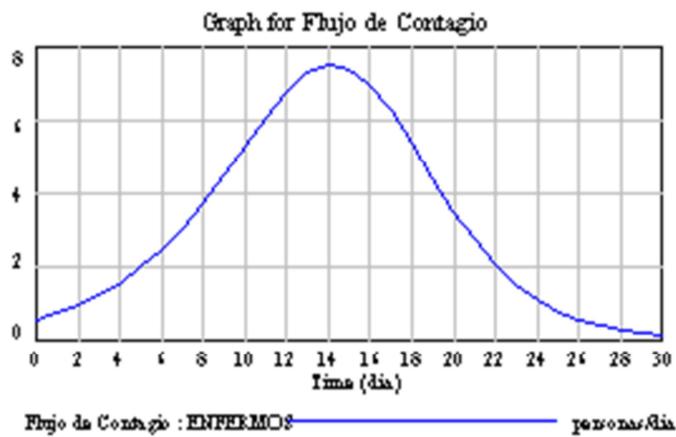
The structure of this model can be shown on this slide. As we already know, this model is composed by two loops, the first positive and external loop is the one that causes the growing, while the internal negative loop causes the stabilization.

Growing systems on S



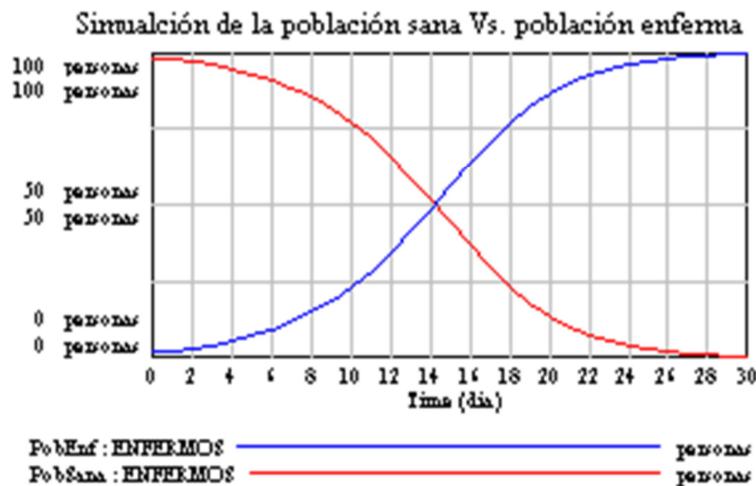
The definition of this model in a Forrester diagram can be something as simple as this. Where the level will be the ill population, that may cause some saturation in the healthcare system if grows so fast.

Growing systems on S



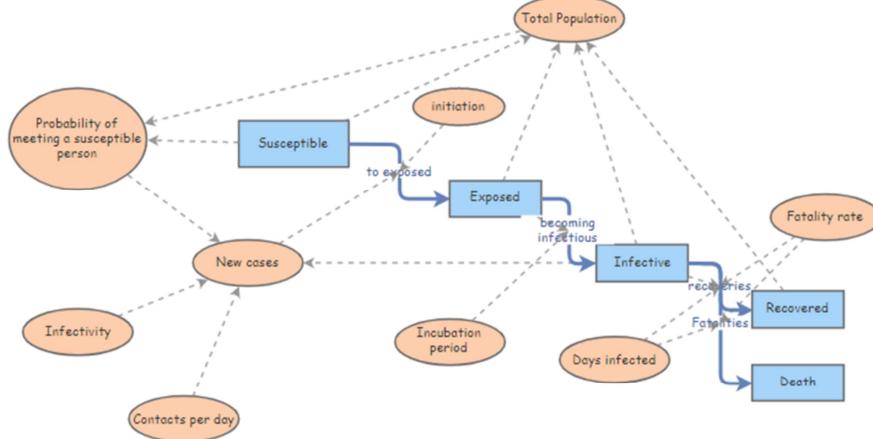
Here we have the results of the model analyzing the infective flow. Obviously, the flow grows exponentially until the half of the population is infected, then degrows, since becomes more and more complex to find Health population to infect.

Growing systems on S



If we analyze the populations, the levels, we see that the ill population is going to growth until be all the population. In the same way, the health population will decrease to zero.

SEIR model



<https://insightmaker.com/insight/185902/COVID-19-spread>

We can expand this kind of models to what is named a SEIR model, Susceptible, Exposed, population that is ill but is not yet infective, Infective and Removed, in the slide divided on Death and Recovered.

In the link you will see the SEIR model for Catalonia for the 2020 Pandemic without containment measures.

Dynamics systems of second order

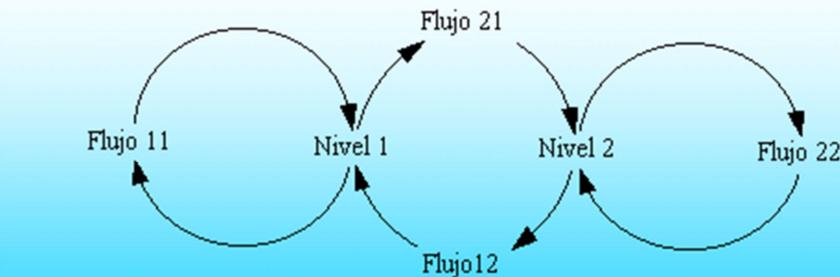
- Two levels in its structure.
- Levels in at least three feedback loops.
 - Main loop: connecting two levels.
 - Two secondary loops: connecting the levels with himself.
- Present oscillations.



We can continue with more advanced models, defining a second order system where we have at least two levels on the structure, nested by at least three feedback loops. The main loop will connect two levels, while the two secondary loops will be used to connect the levels with himself.

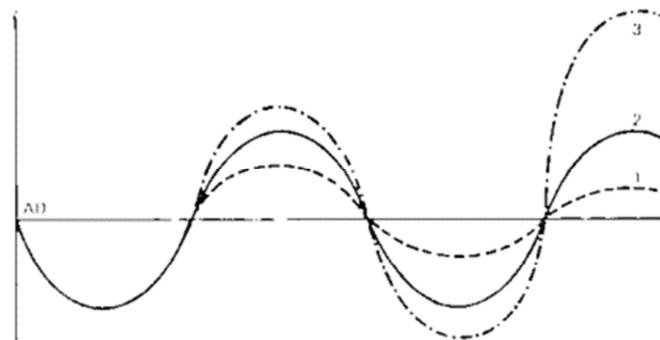
This kind of models are interesting because may present oscillations.

Dynamics systems of second order



Here you have the Causal diagram that represents this kind of models. Notice that the levels are nested in a retro-alimentation.

Dynamics systems of second order



- (1) Soften
- (2) Maintained
- (3) Crescent



With this, the solutions that we can obtain will be similar to the one that we present on this slide. Notice that depending on the parameters we can find a Crescent scenario, that is not stable, or Maintained and Soften scenario, that is stable.

Example: Minerals

- We want to analyze the evolution of the minerals that are dissolved in the ground, and the evolution of the vegetables in an specific area.
 - Minerals depends directly in the amount of rocs.
 - The amount of vegetables depends on the richness of the ground, the sun and the water.
 - The vegetables increases the deeper of the ground, this increases the richness of the ground.
 - Water depends on the rain.



Let's go to review an example of this type of systems, in that case an example of soil, plants and minerals.

We want to analyze the evolution of the minerals that are dissolved in the ground, don't care regarding the type of minerals, we suppose that are the good ones for the plant's growth. Also we want to analyze how this minerals and the water affects the evolution of the vegetables in a specific area.

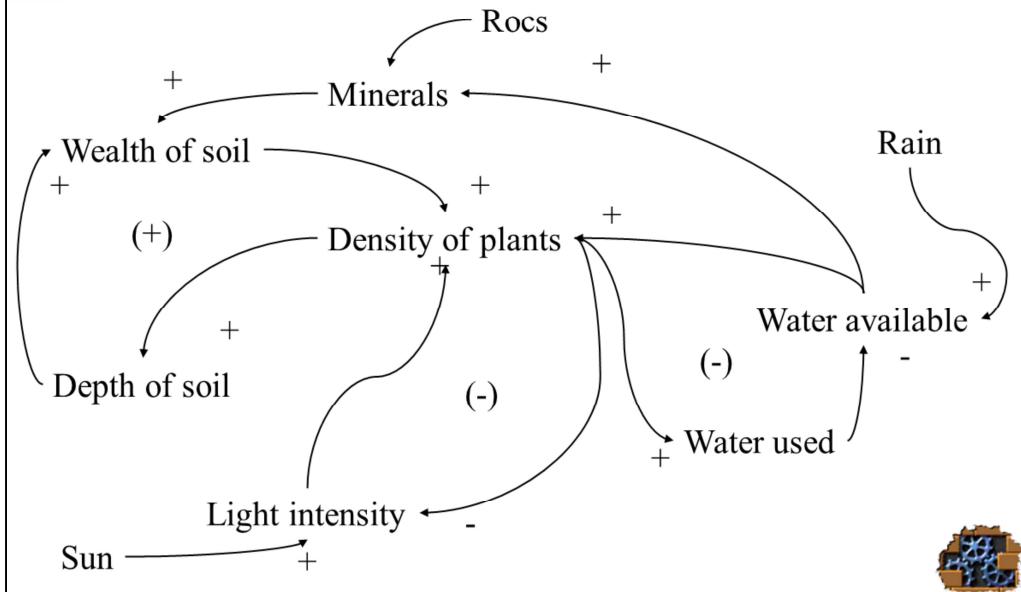
We know that the minerals depends directly in the amount of rocs.

We also know now that the amount of vegetables depends on the richness of the ground, the sun and the water.

Also we know that the vegetables increases the deeper of the ground, and this increases the richness of the ground, that will also affect the vegetables.

Finally we know that water depends on this model, exclusively from the rain.

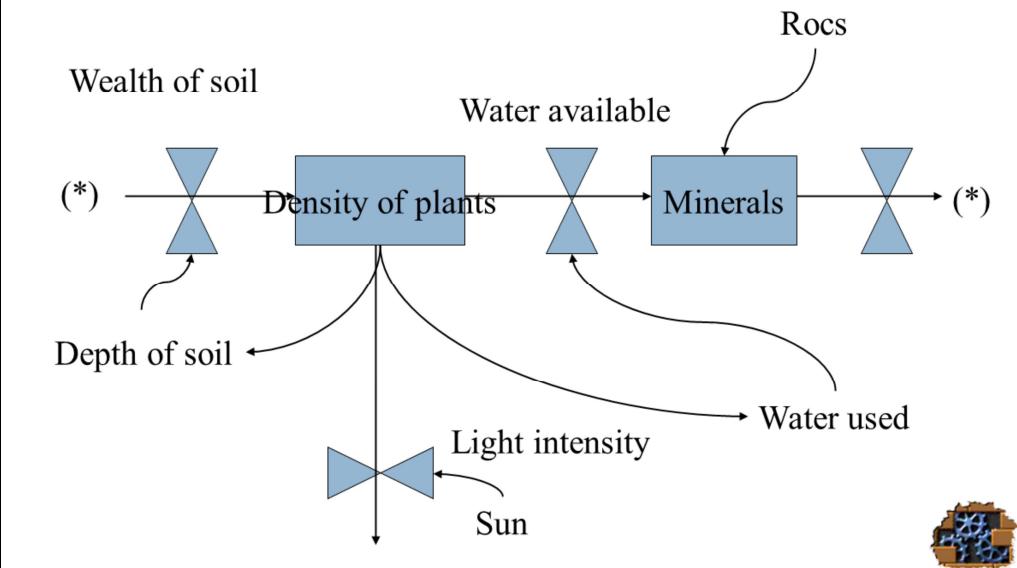
Example Minerals: Causal diagram



This is the Causal diagram that we will use on this model. Please, review it and define the Forrester diagram.

Stop the video and do it by yourself.

Example minerals: Forrester diagram



On this slide we show the Forrester diagram that is generated from the previous Causal diagram. Notice that the different elements we find are currently defined on the previous slide. Also, notice that the star we find represents a loop that makes a model with two levels nested.

Systems pathology's

- Resistance to change: All system prefer to remain in its current state.
- Objective erosion: The objectives are relaxed.
- Addiction: the need of an external corrective factor.
- Secondary effects.



At this point we will analyze the systems pathology, that are systems that behaves in a well-known pattern. This systems are:

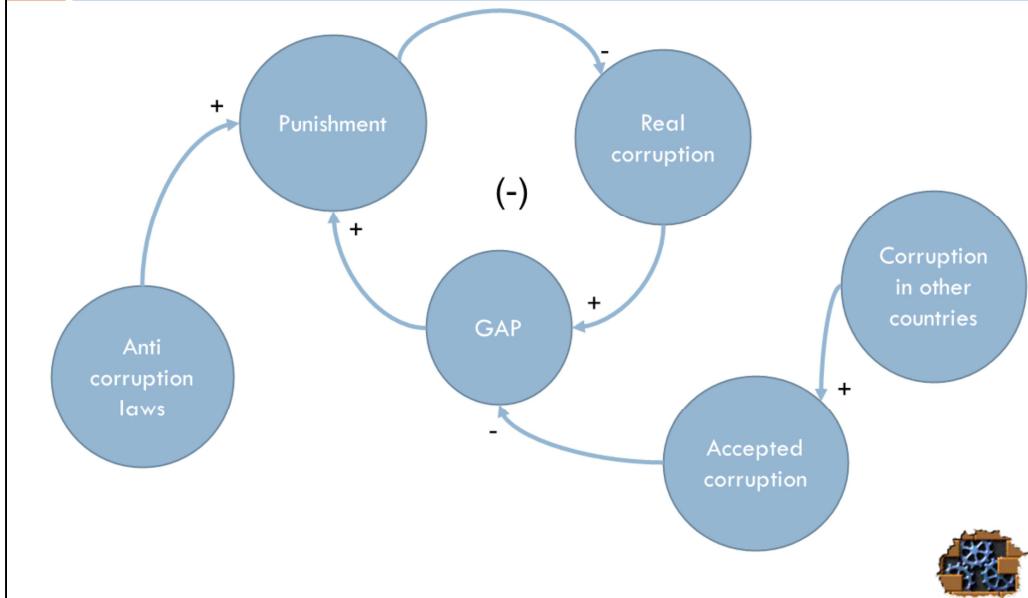
Resistance to change, where all the systems, because are systems, prefers to remain in its current state.

The second pathology that we are going to analyze is the Objective erosion, is similar to the resistance top the change, with the difference that the goal, the objectives are relaxed.

The third system typology to analyze is the addiction, that represents the need of an external corrective factor.

Finally we will see the secondary effects, where we will see how trying to do something, we achieve other thing.

Resistance to change

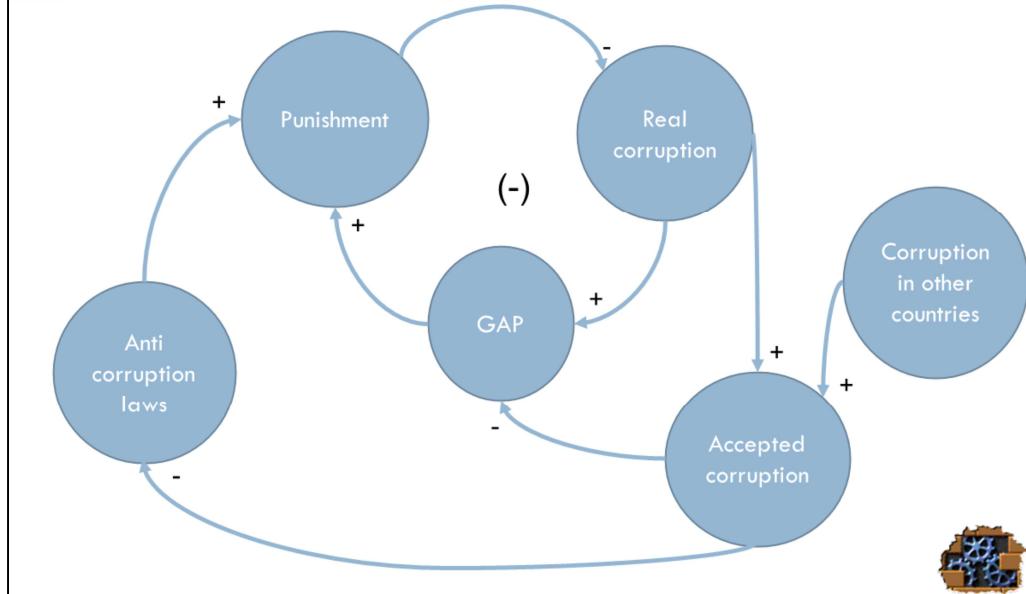


We will start with Resistance to change.

On this example we are focused on analyze the corruption in a country. As one can see in the Causal diagram, the Real corruption depends on the punishment, if we increase the punishment, we decrease the real corruption. However, this punishment depends on a GAP that tends to increase the punishment, notice also that if the gap decreases, the punishment also decreases.

Punishment depends on Anti corruption Laws, that are an element that is outside of our analysis, the same for the Accepted corruption, that depends on the corruption on other countries. This implies that the system tends to a stabilization in a point that depends on the Accepted corruption of our country.

Objective erosion



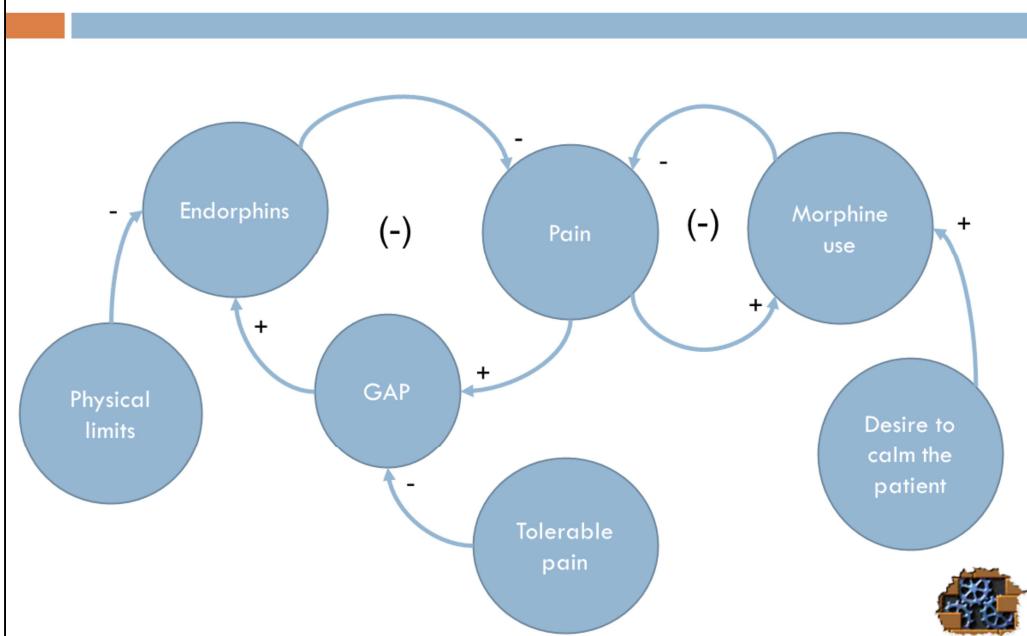
However, what happens if we include Anti Corruption Laws and Accepted corruption in our analysis?.

If we include Accepted corruption, this is going to be affected by the real corruption on our country. If we increase the real corruption, the accepter corruption is going to be increased, that implies that the GAP is going to go to a lower value, decreasing the Punishment.

But also, if the accepted corruption also affects the Anti Corruption Laws, we notice that with more accepted corruption, we will decrease the Anti Corruption Laws, implying that the punishment will be also decreased.

Including this relations imply that the final state we will find will be different from the original final state we desire, since Accepted corruption increases, this is an objective erosion.

Addiction



Addiction is other well-known schema or system pathology that is worth to mention.

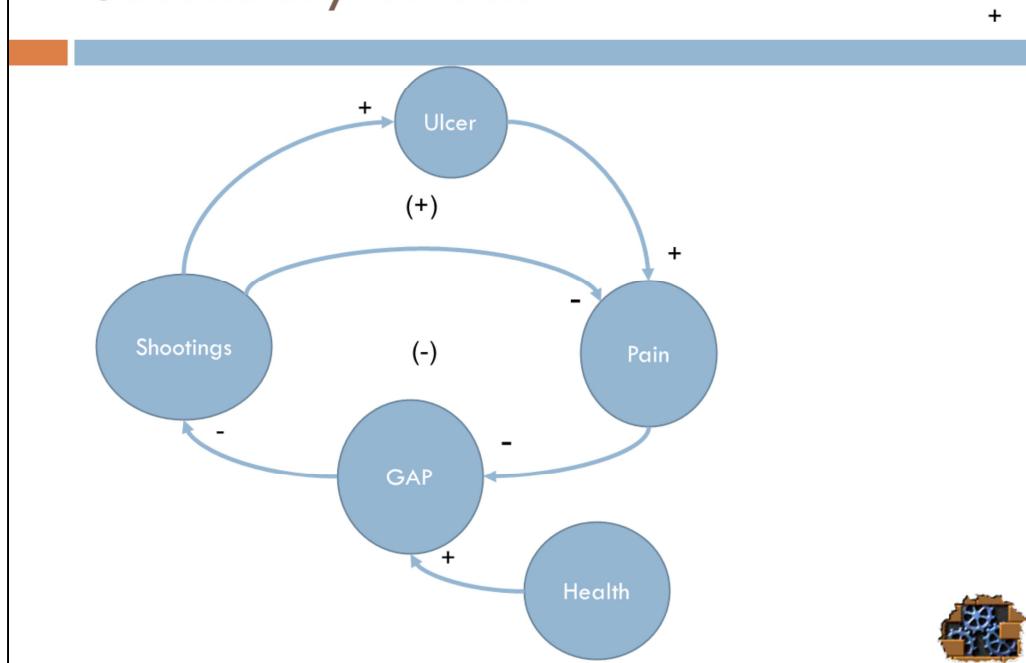
In this case the element that we want to analyze is the Pain, think that we want to minimize the patient pain. In this case the body generates naturally Endorphins that relieve this pain. However the body have a physical limitations to generate this endorphins, hence, when the negative loop on the endorphins owns a limit, represented by the Physical limit element.

On the other side exists other negative loop that also will affect the pain. In that case the Morphine use depends only on the desire to relief the patient, hence no physical limit exists. This imply that this second loop will increase during the time, implying that the GAP in the fist loop decreases, implying that the generation of endomorphisms by the body also decreases.

At the end, the body will lose its capability to generate endorphins and to relieve the pain of the patient we need the morphine that is delivered by an external element.

Is clear that this generates a dependency, an addiction.

Secondary effects



Finally we will review secondary effects schema. On this case, we see that the negative loop, that decreases the pain of the patient generates a secondary loop, that increases the pain in the long term. The answer that we want to solve with this schema is when the secondary loop will be bigger enough to generate more pain. Think in a medicine that cures something, but since you need to take it for moths, is needed to take also to protect the stomach, in order to avoid the ulcer. This is a classic example of secondary effects.

Other population dynamics models

□ Turmites

- Turing machine over a bidimensional space.
 - Similar to a Cellular automaton.
- Evolves from an easy rules.
- Shown repetitive patterns.
 - Emergent behavior.

<http://mathworld.wolfram.com/Turmite.html>

<https://mathworld.wolfram.com/CellularAutomaton.html>



There are other Methods to analyze population Dynamics models, the cellular automaton and the Turmites can be excellent alternatives to this type of models. You can access this two websites to review and understand the logics behind this other approaches.

Examples

Easter Island

L'altiplà de Kaibab

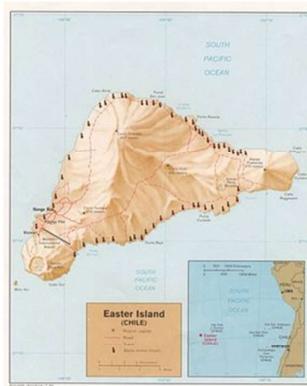
Easter Island

- Easter Island (Rapa Nui: Rapa Nui, Spanish: Isla de Pascua) is a Polynesian island in the southeastern Pacific Ocean, at the southeastern most point of the Polynesian triangle. The island is a special territory of Chile.



Easter Island

- The island was populated by Polynesians who navigated in canoes or double-hulled canoes from the Marquesas Islands (320 km away) or Tumotou Islands (Mangareva, 2000 km away) or Pitcairn (2000 km away). When Captain Cook visited the island, one of his crew member, who was a Polynesian from Bora Bora, was able to communicate with the Rapa Nui. In 1999, a reconstitution with Polynesian boats was carried out, rallying the Easter Island from Mangareva in 10 days.^{citation needed} According to legend, recorded by the missionaries in the 1860s, the island originally had a theocratic system with an ariki, king, wielding absolute god-like power even since Hotu Matu'a had arrived on the island. The most visible element in the culture was production of massive moai that were part of the ancestral worship. With a strictly unified appearance, moai were erected along most of the coastline, indicating a homogeneous society and centralized governance. For unknown reasons, a cult by military leaders, the cult matatua had brought a new cult based around a previously unexceptional god Makemake. The cult of the birdman (Rapapū; tangata manu) was largely to blame for the island's misery of the late 18th and 19th centuries. With the island's ecosystem fading, destruction of crops quickly resulted in famine, sickness and death.
- European accounts from 1722 and 1770 still saw only standing statues, but by Cook's visit in 1774 many were reported toppled. The hui mo ai - the statue-topping ceremony - into the 1820s as part of fierce intertribe wars. By 1838 they only remained Moai were on the slopes of Rano Raraku and Hoa Hakananai'a at Orongo.



Probablement va ser trobada pel pirata anglès Davis, el 1686. Com que no la va situar correctament, durant molt de temps els exploradors van estar buscant l'anomenada Terra de Davis.

Va ser el neerlandès Jakob Roggeveen que la va descobrir el dia de Pasqua de 1722.

El 1770 Felipe González de Haedo en pren possessió en nom del rei Carles III, i l'anomena illa de San Carlos. Però l'expedició no va tenir continuïtat.

L'explorador anglès James Cook, el 1774, i el francès La Perouse, el 1786, van descriure i estudiar els habitants i la geografia de l'illa.

Després, el seu aïllament i la falta de recursos van fer que caigués en l'oblit. L'illa va caure en mans de caçadors d'esclaus peruvians que van deportar els nadius a Perú i Xile, deixant només 111 habitants l'any 1877.

Va ser annexionada per Xile l'any 1888 per Policarpo Toro.

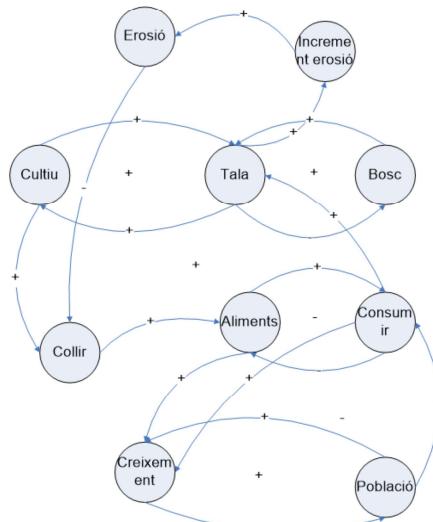
El nom polinesi de Rapa Nui és modern, originat per immigrants polinesis de l'illa de Rapa, o Rapa Iti (petita Rapa).

Easter Island

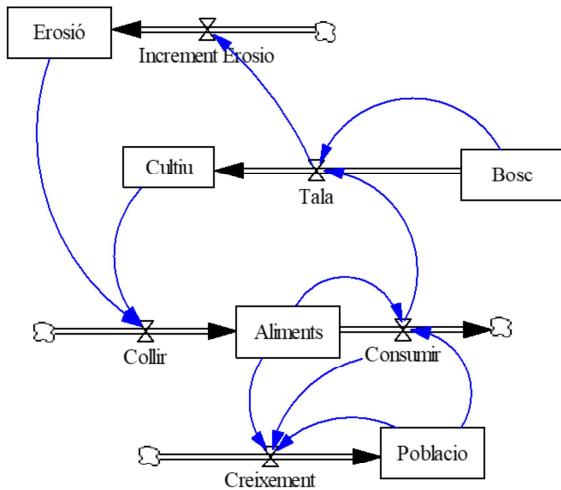
Initial population	5000 inhabitants
Wooden surface	1000 Km ²
Agricultural surface	10 km ²
Vegetative increment of the population	0.17%
Needed food	400 kg/year
Food production	200000 kg/km ²
Erosion increase	0.1 de la superficie talada
Erosion effects	1% per Km ²

1. Population only grows.
2. The amount of food is determined by the productivity of the land and the consumption of the population.
3. The land is a finite resource, which can be divided into two types agricultural land and wooded land.

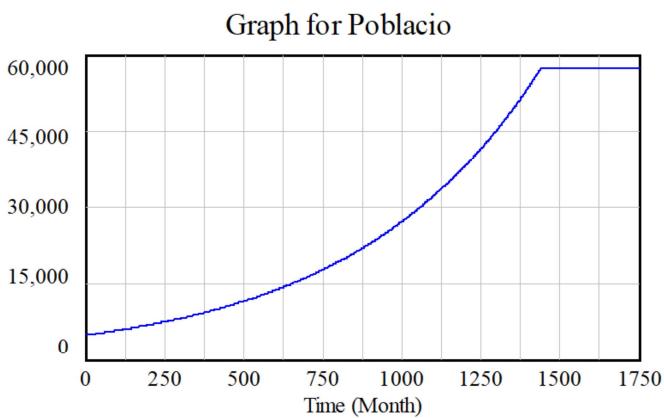
Causal diagram



Forrester diagram



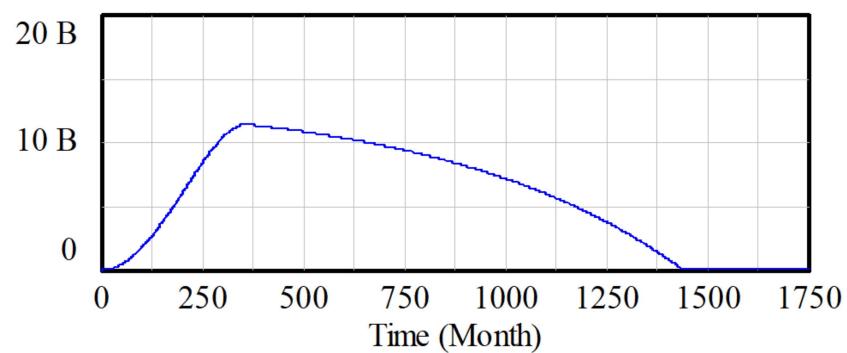
Results



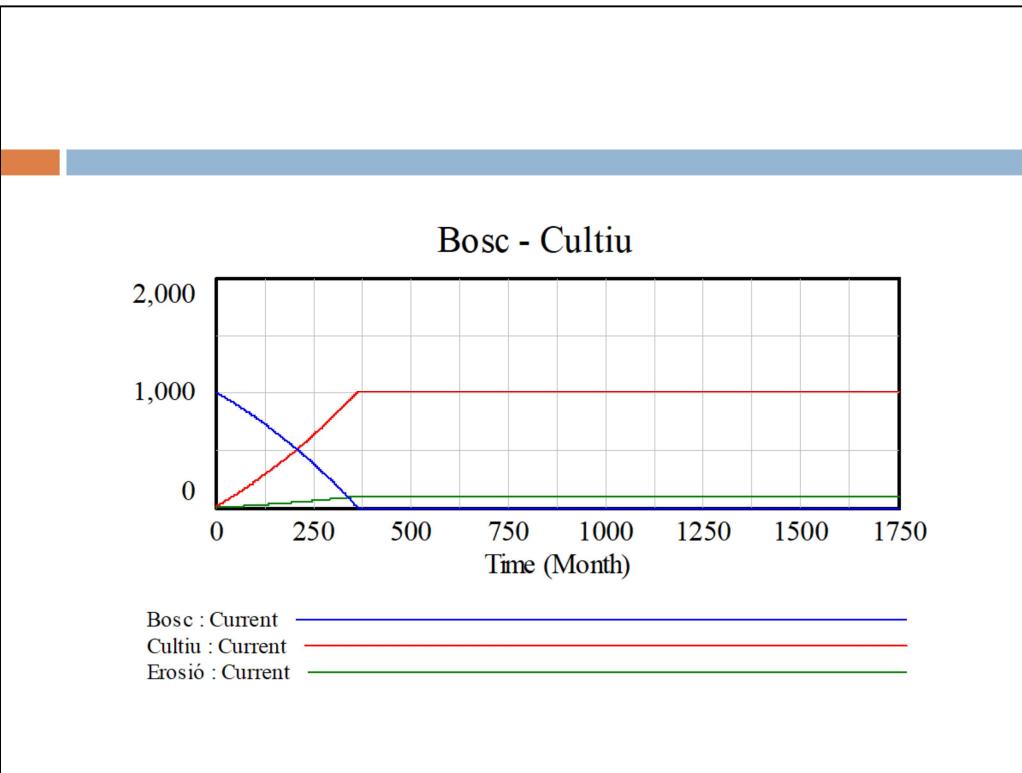
Poblacio : Current _____

Results

Graph for Aliments



Aliments : Current

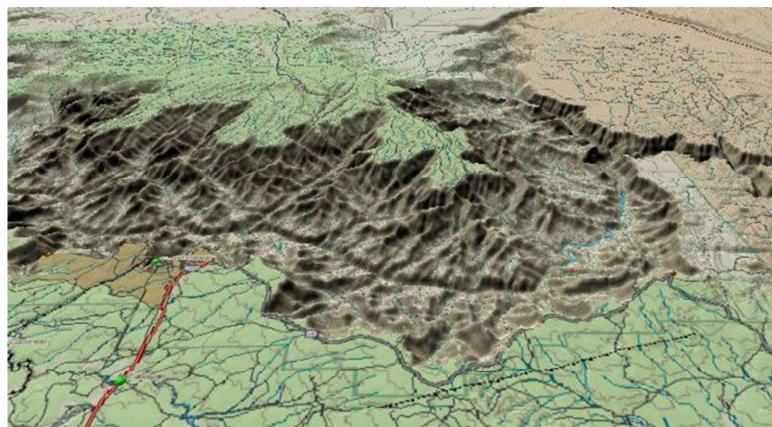


Kaibab National Forest



Lone Pinyon Pine, Yaki Point

Kaibab National Forest



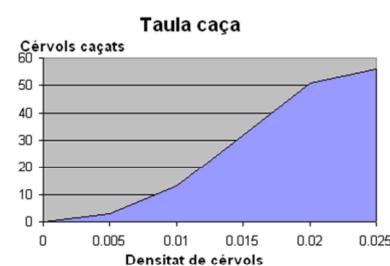
Data, populations

- Deer population: In the beginning **2.000 members**.
- Cougar population: Fixed population of **200 members**.

- <http://www.fs.fed.us/r3/kai/>

Data, hunting

Deer density	Deers killed by cougar
0	0
0.005	3
0.01	13
0.015	32
0.02	51
0.025	56

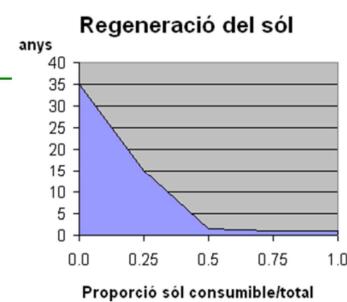


Data, surfaces and consume

- Surface: This data refers to the total number of hectares that compose the natural reservation under study. A constant with the value of **800.000 hectares**.
- Total graze: Takes as value the number total of hectares. A constant with the value of **800.000 hectares**.
- Deer consume: Represents the amount on graze eaten by each deer. A constant of **13,3333 hectares**.

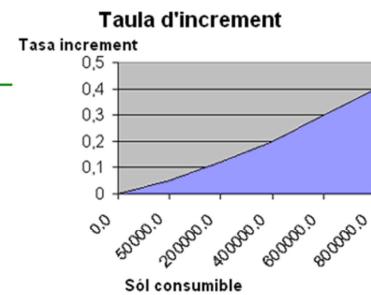
Data, eatable graze

Proportion (eatable graze/ total graze)	Regeneration time (in years)
0	35
0.25	15
0.5	1.5
0.75	1.1
1	1

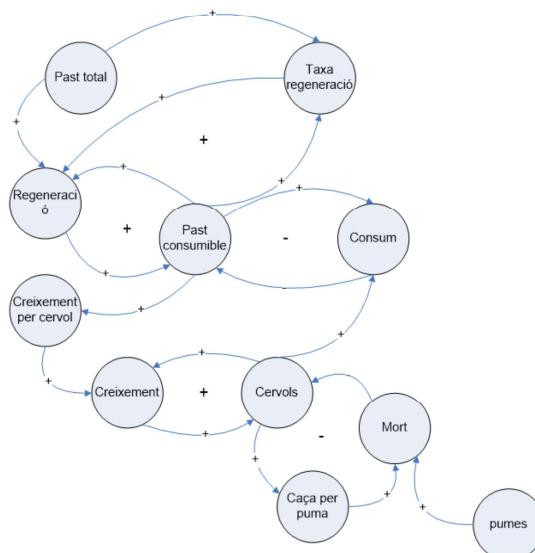


Data, vegetative increment

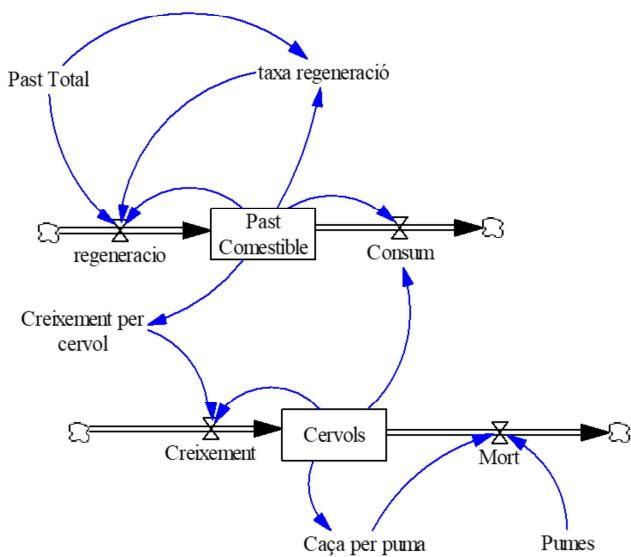
Eatable graze	Vegetative increment
0	0
50000	0.05
200000	0.12
400000	0.2
600000	0.3
800000	0.4



Causal diagram

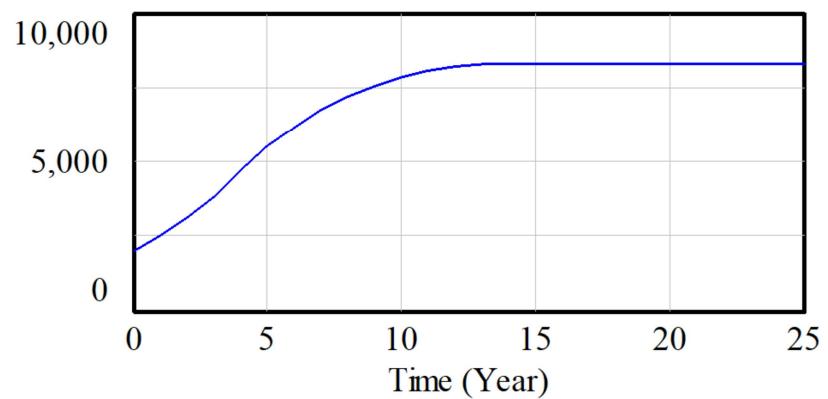


Forrester diagram



Results

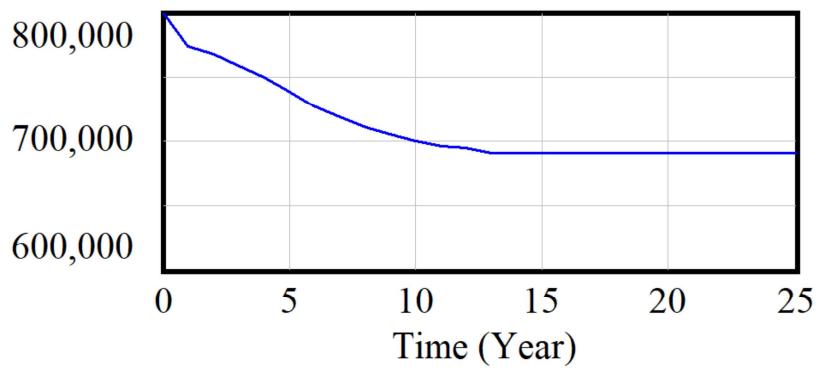
Graph for Cervols



Cervols : Current

Results

Graph for Past Comestible



Past Comestible : Current _____

Tools

□ <https://insightmaker.com/insight/2068>