# LINEAR REGRESSION EXAMPLE

#### THE RUNNER

A runner wants to know what will be its time in a 10 Kms race. The time done on previous races (with other distances) are shown on the next table.

KM'S	TIME
5	18
5	19
21	80
21	79
40	170
40	169
30	120
30	119

$$s_{xy} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{n - 1}$$

$$s_x^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}$$

$$b_1 = \frac{s_{xy}}{s_x^2}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} \qquad \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

WHAT IS THE EXPRESSION OF THIS REGRESSION MODEL?

$$y = \beta_0 + \beta_1 x + \varepsilon$$

ESTIMATE THE PREDICTION OF THE INTERVAL. CALCULATE THE VALUE FOR 10.

$$SSE = (n-1) \left( s_y^2 - \frac{s_{xy}^2}{s_x^2} \right) \quad s_\varepsilon = \sqrt{\frac{SSE}{n-2}} \quad \hat{y} \pm t_{\alpha/2, n-2} s_\varepsilon \sqrt{1 + \frac{1}{n} + \frac{(x_g - \overline{x})^2}{(n-1)s_x^2}}$$

2,446912 =T.INV.2T(0,95;8-2)

Interpret this value.

WE CAN ALSO TEST THE SLOPE, FIND ITS CONFIDENCE INTERVAL.

$$b_1 \pm t_{\alpha/2} s_{b_1} = \frac{s_{\varepsilon}}{\sqrt{(n-1)s_x^2}}$$

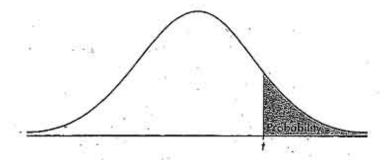


TABLE B: 1-DISTRIBUTION CRITICAL VALUES

1 1,000 1,3 2 ,816 1,0 3 ,765 9 4 ,741 9,5 5 ,727 9,6 6 ,718 9,7 7 ,711 8,8 8 ,706 8,8 9 ,703 8,8 10 ,700 8,8 11 ,697 8,8 12 ,695 8,8 13 ,694 8,8 14 ,692 8,6 15 ,691 8,6 16 ,690 8,6 17 ,689 8,8 18 ,688 8,8 19 ,688 8,8 20 ,687 8,8 21 ,686 8,3 22 ,686 8,3 24 ,685 8,3 25 ,684 8,3 26 ,684 8,3 27 ,684 8,3 28 ,683 8,3 29 ,683 8,3 30 ,681 8,3 30 ,681 8,3 30 ,681 8,3 30 ,681 8,3 30 ,687 8,4	_					Tai	l probabi	hty p				7.	
2	ďf	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.000
3 .765 .99 4 .741 .94 5 .727 .99 6 .718 .99 7 .711 .89 8 .706 .83 9 .703 .83 10 .700 .83 11 .697 .83 12 .695 .83 13 .694 .83 14 .692 .86 15 .691 .86 16 .690 .86 17 .689 .86 18 .688 .86 20 .687 .86 21 .686 .83 22 .686 .83 23 .685 .83 24 .685 .83 25 .684 .83 26 .684 .83 27 .684 .83 28 .683 .83 29 .683 .83 30 .683 .83 30 .683 .83 30 .683 .83 30 .683 .83 30 .681 .83 30 .681 .83 30 .687 .86 30 .679 .84 30 .677 .84	1	1.000	1,376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
4 .741 .94 5 .727 .99 6 .718 .99 7 .711 .81 8 .706 .83 9 .703 .83 10 .700 .83 11 .697 .83 12 .695 .83 14 .692 .84 15 .691 .86 16 .690 .86 17 .689 .86 18 .688 .86 20 .687 .86 21 .686 .83 22 .686 .83 23 .685 .83 24 .685 .83 25 .684 .83 26 .684 .83 27 .684 .83 28 .683 .83 29 .683 .83 30 .683 .83 40 .681 .83 50 .679 .84 80 .678 .84 100 .677 .84 100 .677 .84 100 .677 .84	2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
5 .727 .99 6 .718 .99 7 .711 .81 8 .706 .83 9 .703 .83 10 .700 .83 11 .697 .83 12 .695 .83 13 .694 .83 14 .692 .86 15 .691 .86 16 .690 .86 17 .689 .86 18 .688 .86 20 .687 .86 21 .686 .83 22 .686 .83 23 .685 .83 24 .685 .83 25 .684 .83 27 .684 .83 28 .683 .83 29 .683 .83 30 .683 .83 40 .681 .83 50 .679 .84 100 .677 .84 100 .677 .84		.765	.978	1.250	1.638	2.353	3,182	3.482	4.541	5.841	7.453	10.21	12.92
6 .718 .99 7 .711 .81 8 .706 .83 9 .703 .83 10 .700 .8 11 .697 .8 12 .695 .8 13 .694 .8 14 .692 .8 15 .691 .8 16 .690 .8 17 .689 .8 18 .688 .8 19 .688 .8 20 .687 .8 21 .686 .8 22 .686 .8 23 .685 .8 24 .685 .8 25 .684 .8 26 .684 .8 27 .684 .8 28 .683 .8 30 .683 .8 30 .683 .8 30 .683 .8 30 .681 .8 30 .679 .8 40 .677 .8 40 .677 .8 41000 .677 .8 44	4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
7 .711 .81 8 .706 .83 9 .703 .83 10 .700 .8 11 .697 .8 12 .695 .8 13 .694 .8 14 .692 .8 15 .691 .8 16 .690 .8 17 .689 .8 18 .688 .8 19 .688 .8 20 .687 .8 21 .686 .8 22 .686 .8 23 .685 .8 24 .685 .8 25 .684 .8 27 .684 .8 28 .683 .8 29 .683 .8 30 .683 .8 30 .683 .8 30 .681 .8 30 .679 .8 40 .677 .8 40 .677 .8 4100 .677 .8 4100 .677 .8 44	5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
8 .706 .83 9 .703 .83 10 .700 .87 11 .697 .87 12 .695 .87 13 .694 .87 14 .692 .86 15 .691 .86 16 .690 .86 17 .689 .86 18 .688 .86 20 .687 .86 21 .686 .83 22 .686 .83 24 .685 .83 25 .684 .83 26 .684 .83 27 .684 .83 29 .683 .83 30 .683 .83 40 .681 .83 50 .679 .84 60 .679 .84 100 .677 .84	6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317		5.959
9 .703 .8i 10 .700 .8' 11 .697 .8' 12 .695 .8' 13 .694 .8' 14 .692 .8i 15 .691 .8i 16 .690 .8i 17 .689 .8i 19 .688 .8i 20 .687 .8i 21 .686 .8i 22 .686 .8i 23 .685 .8i 24 .685 .8i 25 .684 .8i 27 .684 .8i 28 .683 .8i 29 .683 .8i 30 .683 .8i 30 .683 .8i 30 .681 .8i 50 .679 .8i 80 .678 .8i	7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5,40
10 .700 .8° 11 .697 .8° 12 .695 .8° 13 .694 .8° 14 .692 .8° 15 .691 .8° 16 .690 .8° 17 .689 .8° 18 .688 .8° 20 .687 .8° 21 .686 .8° 22 .686 .8° 23 .685 .8° 24 .685 .8° 25 .684 .8° 27 .684 .8° 28 .683 .8° 29 .683 .8° 30 .683 .8° 30 .683 .8° 30 .681 .8° 50 .679 .8° 80 .678 .8° 100 .677 .8° 1000 .677 .8° 1000 .675 .8°	8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5:04
11	9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.78
12	10	.700	.879	1.093	1.372	1.812	2.228	2,359	2.764	3.169	3.581	4.144	4.58
13	11	.697	.876	1.088	1.363	1.796	2,201	2.328	2.718	3.106	3.497	4.025	4.43
14	12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.31
15	13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.22
16	14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
17	15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
18	16	.690	.865	1.071	1.337	1.746	2,120	2.235	2.583	2.921	3.252-	3.686	4.01
19	17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
20 .687 .86 21 .686 .83 22 .686 .83 23 .685 .83 24 .685 .83 25 .684 .85 26 .684 .85 27 .684 .85 28 .683 .85 29 .683 .85 30 .683 .85 40 .681 .85 50 .679 .84 60 .679 .84 100 .677 .84	18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197		3.92
21 .686 .83 22 .686 .83 23 .685 .83 24 .685 .83 25 .684 .85 26 .684 .85 27 .684 .85 28 .683 .85 29 .683 .85 30 .683 .85 40 .681 .85 50 .679 .84 60 .679 .84 100 .677 .84	19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
22 .686 .83 23 .685 .83 24 .685 .83 25 .684 .85 26 .684 .85 27 .684 .85 28 .683 .85 29 .683 .85 30 .683 .85 40 .681 .85 50 .679 .84 60 .679 .84 100 .677 .84	20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3,153	3.552	3.850
23 .685 .83 24 .685 .83 25 .684 .85 26 .684 .85 27 .684 .85 28 .683 .85 29 .683 .85 30 .683 .85 40 .681 .85 50 .679 .84 60 .679 .84 100 .677 .84	21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
24	22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
25	23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
26	24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467.	3.745
27	25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3,450	3.725
28	26	.684	.856	1.058	1.315	1.706	2.056	2,162	2.479	2.779	3.067	3.435	3.707
29 .683 .83 30 .683 .85 40 .681 .85 50 .679 .84 60 .679 .84 80 .678 .84 100 .677 .84	27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
30 .683 .85 40 .681 .85 50 .679 .84 60 .679 .84 80 .678 .84 100 .677 .84	28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
40 .681 .85 50 .679 .84 60 .679 .84 80 .678 .84 100 .677 .84	29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
50 .679 .84 60 .679 .84 80 .678 .84 100 .677 .84 000 .675 .84	30	.683	.854	1.055	1.310	1.697	2.042	2.147	2:457	2.750	3.030	3.385	3.646
60 .679 .84 80 .678 .84 100 .677 .84 000 .675 .84	40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
80 .678 .84 100 .677 .84 000 .675 .84	50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.49€
100 .677 .84 000 .675 .84	60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3,460
000 .675 .84	80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
.000 .675 .84	00	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
	000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3,300
107.1		.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
50% 60		50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%

## WITH R, CARS

We use "cars" data.

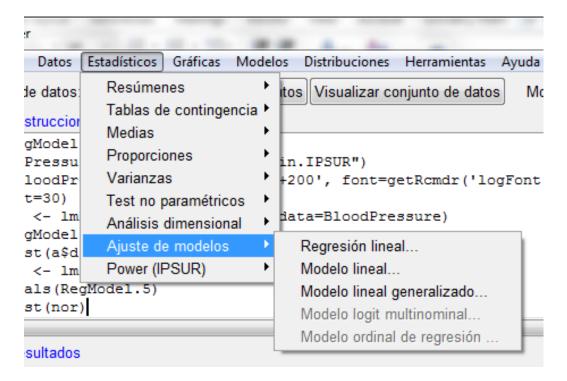
> head(cars)

>a<-cars

Perform the linear model with R

> cars.lm <- Im(dist ~ speed, data = cars)

Or with RCmndr



#### ANALYZE THE RESULTS.

There is a relation between speed and distance?

Draw a Scatterplots to represent the relation.

What is the confidence interval for the two parameters?

> confint(cars.lm)

#### TEST THE HYPOTHESES

Load package zoo and Imtest

## NORMALITY OF THE RESIDUALS

> shapiro.test(residuals(<the model>))

Draw a plot, we must reject it?

# CONSTANT VARIANCE ASSUMPTION

Breusch-Pagan test

> bptest(<the model>)

## INDEPENDENCE ASSUMPTION

**Durbin-Watson test** 

> dwtest(<the model>, alternative = "two.sided")