TMIRI

Writing Test
November 2019. Duration: 75'

1. (6 points) During a few minutes I will show you how to play a game called **Lights Off** (available in all ubuntu distributions) that you will have to describe. You must find the right level of abstraction and precision. As for abstraction, there are elements that are completely unrelated to what really matters in the game (e.g. clicking on a cell vs selecting a cell). As for precision, be as formal as possible without letting the formality obfuscate the text.

   Solution:

   Consider an $n \times n$ square matrix M whose elements are either 0 or 1. A cell (i, j) has four *neighbors* defined as the cell *above* (that is $(i-1, j)$, *below* (that is $(i+1, j)$), on its *left* (that is $(i, j-1)$) and on its *right* (that is $(i, j+1)$). When a cell is in the first or last row or column, the previous definition is naturally adapted to contain only those neighbors that exist.

   The operator *switch* $sw(M, i, j)$ takes a matrix M and the coordinates (i, j) of a cell and returns another matrix in which the value of cell (i, j) and its neighbors are switched (switching a $0/1$ value b means replacing it by $1 - b$).

   In the *Lights Off* game the player receives an arbitrary matrix M and the goal is to transform it into a matrix with all 0's by applying the switch operator. Ideally, the player should minimize the number of steps in the transformation.

   Formally, given the initial matrix M the goal is to find the shortest sequence of coordinates $(i_1, j_1), (i_2, j_2), \ldots, (i_k, j_k)$ such that,

   $$sw(...(sw(sw(M, i_1, j_1), i_2, j_2), ...), i_k, j_k) = M^0$$

   where M^0 denotes the matrix containing 0 in all its cells.

2. (4 points) Consider a $n \times n$ square matrix M whose elements are either 0 or 1. For simplicity, we will assume that n is an even number.

   Think of an imaginary horizontal line that separates the upper half of the matrix and the lower half. Similarly, think of an imaginary vertical line that separates the left half of the matrix and the right half. The two lines together separate the matrix into four quadrants (top-left, top-right, bottom-left and bottom-right). Each element in the top-left quadrant has a symmetric in every other quadrant. For instance, the symmetricals of $M_{1,1}$ are $M_{1,n}$, $M_{n,1}$ and $M_{n,n}$, respectively.

Give the arithmetic expression that defines a function $f(M)$ that returns the number of ones in the top-left quadrant such that all their symmetrical positions are one as well. For example, if M is,

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

then $f(M) = 2$ because there are two positions in the top-left quadrant (i.e, $M_{1,1}$ and $M_{1,2}$) that satisfy the condition.

Solution:

Note that the four symmetric elements of an element $M_{i,j}$ in the first quadrant are: $M_{i,n-j+1}$, $M_{n-i+1,j}$ and $M_{n-i+1,n-j+1}$. Thus, operator $f(M)$ is,

$$f(M) = \sum_{i=1}^{n/2} \sum_{j=1}^{n/2} (M_{i,j} \cdot M_{i,n-j+1} \cdot M_{n-i+1,j} \cdot M_{n-i+1,n-j+1})$$

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