# Writing in Computer Science: Math

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# Writing Math

Consider the following three different ways to say the same thing,

1. Let $C$ be a set of cities. The population size of a city $c \in C$ is noted $size(c)$. The average size of cities in $C$ is,
$$m = \frac{\sum_{c \in C} size(c)}{|C|}$$
2. Suppose we have a set of $n$ cities each one identified with an index $1 \leq i \leq n$. The population of the $i$-th city is noted $s_i$. The average size of the cities is,
$$m = \frac{1}{n} \sum_{i=1}^{n} s_i$$
3. Let $C$ be a set of cities where each city is a pair $c = (id, s)$ with $id$ being its name and $s$ being its population. The average size of the cities in $C$ is,
$$m = \frac{\sum_{(n,s) \in C} s}{n}$$
where $n = |C|$.

# Symbols

Mathematical expressions are made out of symbols and operators. Symbols are used to name numbers, functions, sets,... A **symbol** is a letter in a given style. The usual symbol styles are:

- lower case italics: $x, y, p, q$
- lower case boldface: $\mathbf{x}, \mathbf{y}, \mathbf{p}, \mathbf{q}$
- upper case italics: $P, Q, R$
- upper case boldface: $\mathbf{P}, \mathbf{Q}, \mathbf{R}$
- Calligraphy: $\mathcal{P}, \mathcal{Q}, \mathcal{R}, \mathcal{X}$
- Greek alphabet (lower and upper case) $\alpha, \beta, \Delta, \Gamma$

# Plain text

## Warning

**Plain text is never a valid style** in mathematics because it is important to distinguish between the English and the Math. (e.g. It is wrong to write that n is the number of elements in set C; it is right to write that $n$ is the number of elements in set $C$).

# Styles

## Warning

In English, you can write a word in plain text, *italics* or **boldface**, lower or UPPER case and it is still the same word.
In math writing, if you define a symbol (e.g. $n$) the definition does not extend to other fonts or styles (e.g. $\mathrm{n}, \mathbf{n}, N, \mathcal{N} \ldots$). In other words, you cannot say that the size of a given set is $n$ and later on use **n** as a synonym because they are different symbols.

# Styles

## Hint

Do not to use the same letter in different, but similar, styles to denote different things, because it may be confusing.
For example, it is a bad idea to denote the size of a set of numbers $m$ and their mean $\mathrm{m}$.

The previous rule does not apply between lower and upper case. For instance, it is not unusual to name a set $M$ and an arbitrary element $m$.

I would avoid naming a set $C$ and an arbitrary element $c$ because they look much more similar.

# Notational Entity

## Notational Entity

Upper case has greater *notational entity* than lower case, and calligraphic upper case has greater notational entity than upper case. Additional **notational entity** can also be obtained using boldface.

A notation of greater entity is to be used to name more complex elements.

Example:

- Let $P$ be a set of integers
- Let $p$ be an integer
- Let $\mathcal{P}$ be a partition of a set of integers

# Modifiers

## Modifiers

Sometimes we need to name several mathematical objects that are very related. To emphasize their relation one can use the same letter with a **modifier**. Modifiers are sub-indexes, super-indexes, accents and priming. These elements can be very useful in some situations, but one should use with care.

- Accents and priming: useful to denote variations (e.g. derivative $f(x), f'(x)$) or special elements (e.g. optimum $x^*$)
- Indexes are useful for enumeration (e.g. $X = \{x_1, x_2, \ldots, x_i, \ldots, x_n\}$) or identification ($s_i$ and $w_i$ are the size and wealth of city $i$)

# Words as symbols

**Warning**

For complex definitions (specially functions) words are sometimes a good idea, because they are easy to memorize (e.g. ,...).

- E.g.sin for *sinus*, max for *maximum*
- E.g. $MC(G)$ set of maximal cliques of a graph $G$

Computer Scientists use words too often (probably because of our habit to name variables and algorithms). So it is good advise to restrict our notation to single letter symbols unless there is a good reason against it.

# Notational Conventions

Some usual naming conventions:

- **scalars** $a, b, x, y, p, q$
- **indexes** $i, j, k$
- **vectors** (i.e, fixed-size ordered sequences) $\vec{v} = (v_1, v_2, \ldots, v_n)$.
- In Computer Science we often talk about **arrays** and **lists**, which are finite (but not necessarily fixed-size) ordered sequences. Usual notation for arrays is $s = (s[1], s[2], \ldots, s[n])$, and for lists is $l = (l_1, l_2, \ldots, l_n)$
- **sets**, **multisets** $S = \{x_1, x_2, \ldots, x_n\}$
- **tuples** $P = (A, B)$
- **functions** $f, g, h$
  - $f : A \longrightarrow B$ (that means $x \in A$, then $f(x) \in B$)
  - $f : A \times A \longrightarrow B$ (that means $x, y \in A$, then $f(x, y) \in B$)

# Some hints from Donald Knuth

- Facilitate memorization (e.g. $\vec{v}$ for vectors, $p$ for points, $S$ for sets, ...)
- Avoid **unnecessary subscripts** (e.g. $x$ and $y$ is better than $x_1$ and $x_2$, or $x$ and $x'$)
- Avoid **piling subscripts** (e.g. $x_{i'}^{p_j}$)
- Avoid **unfamiliar symbols** (e.g. $\Xi$, $\Theta$) and **unfamiliar accents** (e.g. $\hat{a}$, $\check{a}$, $\dot{a}$)

# Some Advanced Operators

## Summation

$$\sum_{it} expr$$

**Example:**

- $\sum_{i=1}^{n} a_i$
- $\sum_{a \in S} a$
- $\sum_{i=1}^{n} \sum_{j=1}^{m} (a_i + b_j)$

**Similar syntax for:** $\prod$, max, min, $\forall$, $\exists$, etc

# Some Advanced Operators

## Set constructor

Allows to define a set based on a previous set and filtered with a condition

$$\{domain|\ filter\}$$

**Example:**

- $\{a \in \mathbb{N} |\ \forall_{0<b<a} \ \ b^2 \neq a\}$
- $\{(a, b) \in A \times B |\ a < b\}$

# On "efficient" definitions

**What is better?**

- $\{a \in \mathbb{N} | \; \forall_{0<b<a} \;\; b^2 \neq a\}$ because it is simpler
- $\{a \in \mathbb{N} | \; \forall_{0<b\leq a/2} \;\; b^2 \neq a\}$ because it is more efficient

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This is not an algorithm, you silly. When defining something you look for simplicity to make the definition as easy to grasp as possible. The very concept of efficiency does not even make sense in this context.

# Case-based definitions

## Case-based definitions

Allows to define based on parameter conditions

$$parameterizedname = \begin{cases} expr1, & cond1 \\ expr2, & cond2 \\ ... & \\ exprn, & condn \end{cases}$$

Conditions must be mutually exclusive and their union must cover all posible cases (this is why very often the last condition is "otherwise")

**Example:**

$$f(x) = \begin{cases} x^2, & -100 \leq x \leq 100 \\ |x|, & \text{otherwise} \end{cases}$$

# Examples

Consider a lists $l = (l_1, l_2, \ldots, l_n)$.

1. the **reverse** of $l$ is ,,,
2. the **one position shift** of $l$ is ,,,
3. **capping** the vector with $k$ is ,,,

# Examples

Consider a lists $l = (l_1, l_2, \ldots, l_n)$.

1. the **reverse** of list $l$ is a list $r = (r_1, r_2, \ldots, r_n)$ such that $r_i = l_{n-i+1}$
2. the **one position shift** of list $l$ is a list $r = (r_1, r_2, \ldots, r_n)$ such that
$$r_i = \begin{cases} l_{i-1}, & i > 1 \\ l_n, & i = 1 \end{cases}$$
3. **capping** list $l$ with $k$ is a list $r = (r_1, r_2, \ldots, r_n)$ such that
$r_i = \min\{l_i, k\}$

# Examples

Consider a lists $l = (l_1, l_2, \ldots, l_n)$.

1. the maximum among the elements of $l$ is ...
2. the position of the maximum is...
3. the number of elements in $l$ that are equal to their symmetric...
4. the number of elements that are repeated in $l$ is ...

# Examples

Consider a lists $l = (l_1, l_2, \ldots, l_n)$.

1. the maximum among the elements of $l$ is $\max_{1 \le i \le n}\{l_i\}$
2. the position of the maximum, $\mathrm{argmax}_{1 \le i \le n}\{l_i\}$
3. the number of elements in $l$ that are equal to their symmetric,

$$|\{i \in 1..(n+1)/2 |\ l_i = l_{n-i+1}\}|$$

4. the number of elements that are repeated in $l$ is

$$|\{l_i \in l \mid \exists_{j \neq i}\ l_i = l_j\ \}|$$

Note that sets do not have repetitions

# Examples with sets

Given two sets of integers $A$ and $B$,

1. the elements of $A$ whose square is in $B$ is...
2. the pairs whose first and second element is in $A$ and $B$, respectively that are no further apart than $k$ is...
3. the set of even numbers common to both sets is...
4. the set of subsets of $A$ whose sum is 0 is...
5. the set of subsets of $A$ that are also subsets of $B$ is...

# Examples with sets

Given two sets of integers $A$ and $B$,

1. the elements of $A$ whose square is in $B$ is, $\{a \in A | \ a^2 \in B\}$
2. the pairs whose first and second element is in $A$ and $B$, respectively that are no further apart than $k$ is, $\{(a, b) \in A \times B | \ |a - b| \leq k\}$
3. the set of even numbers common to both sets is, $\{a \in A \cap B | \ (a \bmod 2) = 0\}$
4. the set of subsets of $A$ whose sum is 0 is, $\{P \subseteq A | \ \sum_{p \in P} p = 0\}$
5. the set of subsets of $A$ that are also subsets of $B$ is, $\{P | \ P \subseteq A \cap B\}$

# Examples with matrices

Consider a matrix $A$ of dimension $n \times n$ where $a_{ij}$ denotes the element in row $i$ and column $j$

1. the vector containing the maximum of each row is ...
2. the minimum among the maximum of each row is ...
3. the vector with sum of each ascending diagonal is...
4. the elements in $A$ that are equal to their symmetric (the symmetry is defined with respect to the matrix diagonal) is ...
5. the elements that are larger than any of its neighbors is...

# Examples with matrices

Consider a matrix $A$ of dimension $n \times n$ where $a_{ij}$ denotes the element in row $i$ and column $j$

1. the vector containing the maximum of each row is $\vec{v} = (v_1, v_2, \dots, v_n)$ with $v_i = \max_{1 \leq j \leq n} a_{ij}$
2. the minimum among the maximum of each row is $\min_{1 \leq i \leq n} \{\max_{1 \leq j \leq n} a_{ij}\}$

# Examples with matrices

Consider a matrix $A$ of dimension $n \times n$ where $a_{ij}$ denotes the element in row $i$ and column $j$

1. the vector with sum of each ascending diagonal is $\vec{v} = (v_1, v_2, \ldots, v_{2n-1})$ where $v_k = \begin{cases} s_{k,1}, & 1 \leq k \leq n \\ s_{n,k-n+1}, & n < k \leq 2n-1 \end{cases}$
   with $s_{ij} = \sum_{r=0}^{\min\{i-1,n-j\}} a_{i-r,j+r}$

2. (alternatively) the vector with sum of each ascending diagonal is $\vec{v} = (v_1, v_2, \ldots, v_{2n-1})$ where
   $v_k = \begin{cases} \sum_{r=0}^{k-1} a_{k-r,1+r}, & 1 \leq k \leq n \\ \sum_{r=0}^{2n-k-1} a_{n-r,k-n+1+r}, & n < k \leq 2n-1 \end{cases}$

Because simplicity is also a desirable feature, may be the first one (probably with some words explaining the meaning of things) is the best option.

# Examples with matrices

Consider a matrix $A$ of dimension $n \times n$ where $a_{ij}$ denotes the element in row $i$ and column $j$

1. the elements in $A$ that are equal to their symmetric (the symmetry is defined with respect to the matrix diagonal) is $\{(i,j), 1 \leq j < i \leq n |\ a_{ij} = a_{ji}\}$
2. the elements that are larger than any of its neighbors are,

$$\{(i,j), 1 \leq i,j \leq n |\ a_{ij} > b \text{ for all } b \in N(a_{ij})\}$$

   where,

$$N(a_{ij}) = \{a_{i-1,j}, a_{i+1,j}, a_{i,j-1}, a_{i,j+1}\}$$

   where we assume for convenience $a_{0j} = a_{i0} = 0$

# Examples with graphs

Consider an undirected graph $G = (V, E)$

1. A path is ...
2. A loop is ...
3. The graph's shortest loop is ...
4. The graph embedded by $U \subset V$ is ...
5. Graph $G = (V, E)$ is a clique if ...
6. The MaxClique problem is...
7. Graph $H = (W, A)$ is an isomorphism of $G = (V, E)$ if ...

# Examples with graphs

Consider an undirected graph $G = (V, E)$

1. A path is a sequence of vertices $(u_1, u_2, \ldots, u_k)$ such that $(u_i, u_{i+1}) \in E$ for all $1 \leq i < k$
2. A loop is a path $(u_1, u_2, \ldots, u_k)$ such that $k > 1$ and $u_1 = u_k$
3. The graph's shortest loop is (just define the lenght)
4. The graph embedded by $U \subset V$ is $G' = (U, E')$ with $E' = \{(u, w) \in E |\ u \in U, w \in U\}$
5. Graph $G = (V, E)$ is a clique if $E = V \times V$
6. Given a graph $G = (V, E)$ the MaxClique problem consists on finding the largest (with respect to the number of vertices) embedded clique
7. Graph $H = (W, A)$ is an isomorphism of $G = (V, E)$ if there is a mapping (i.e, bijection) $f : V \rightarrow W$ such that $(u, w) \in E$ if and only if $(f(u), f(w)) \in A$