

1) (a) We define a new vector $\vec{u} = (u_1, u_2, \dots, u_{2n})$

$$\text{where } \begin{cases} u_i \in \vec{v} \text{ or } u_i \in \vec{w} & \forall 1 \leq i \leq 2n \\ u_i \leq u_{i+1} & \forall 1 \leq i \leq 2n-1 \end{cases}$$

$$\text{and } \vec{v} = (v_1, v_2, \dots, v_n)$$

$$\vec{w} = (w_1, w_2, \dots, w_n)$$

$$\text{and } \forall \begin{matrix} 1 \leq i \leq n-1 \\ i \leq j \leq n \end{matrix} \quad v_i \neq w_j$$

(b) Given a set of integers A ,

we define the set that contains the sum of all pairs as

$$\{c = a + b \mid a \in A, b \in A \text{ and } a \neq b\}$$

(c) Given the squared matrices A , B and C .

We define the max-min-mean as the matrix D of dimension $n \times n$

where d_{ij} denotes the element in row i and column j such that

$$D = \begin{pmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & & & \\ \vdots & & & \\ d_{n1} & & & d_{nn} \end{pmatrix} \quad \text{where } d_{ij} = \frac{\max(a_{ij}, b_{ij}, c_{ij}) + \min(a_{ij}, b_{ij}, c_{ij})}{2}$$

$$\text{and } d_{ij} \in \mathbb{R}^{n \times n}$$

Given 3 empty stacks of cards and

2) n cards, where each card is numbered from 1 to n

and each card is uniformly randomly placed in one of the three stacks of cards in descending order.

The goal is to place all the cards on a single stack.

The player can only make one move at a time

Each move consists of picking one of the stacks, taking the top card of the stack and moving it to the top of another stack.

The move is only valid if no larger card is placed on top of a smaller one.