

Tutorial on Circuit (Part 1)

Introduction to Electrical and Electronic Engineering

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Overview

- * **Learning Objectives:**

Analysis circuits through circuit laws (Ohm's Law, KCL and KVL)

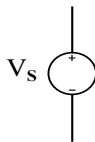
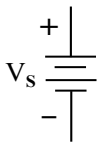
- * **Basics**

- Symbols & Rules
- Kirchhoffs Laws
 - Kirchhoffs Current Law(KCL)
 - Kirchhoffs Voltage Law(KVL)

- * **Questions & Summary**

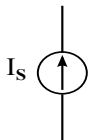
Symbols & Rules (Recap)

- **Voltage Source**



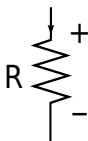
**Ideal direct
current(DC)
Voltage Source**

- **Current Source**



**Ideal DC
Current Source**

- **Resistor**



Resistor

Kirchhoffs Laws (Recap)

- **Kirchhoffs Current Law(KCL):** The current flowing out of any node in a circuit must be equal to the current flowing into the node.

$$\sum_{n=1}^N i_n = 0 \quad (1)$$

where N is the number of branches that are connected to the node.

- **Kirchhoffs Voltage Law(KVL):** The algebraic sum of voltages around a closed loop is zero.

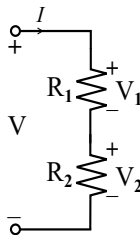
$$\sum_{n=1}^N v_n = 0 \quad (2)$$

where N is the number of voltages in the loop.

Voltage/Current Divider (Recap)

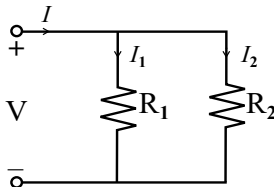
$$V_1 = \left(\frac{R_1}{R_1 + R_2} \right) V$$

$$V_2 = \left(\frac{R_2}{R_1 + R_2} \right) V$$



$$I_1 = \left(\frac{R_2}{R_1 + R_2} \right) I$$

$$I_2 = \left(\frac{R_1}{R_1 + R_2} \right) I$$

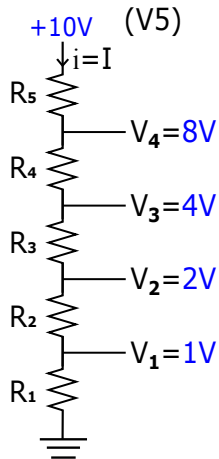


Solution(Q1)

Q: Determine the resistance value of R_1, R_2, \dots, R_5 in the circuits. (Assume the resistance of R_1 is R)

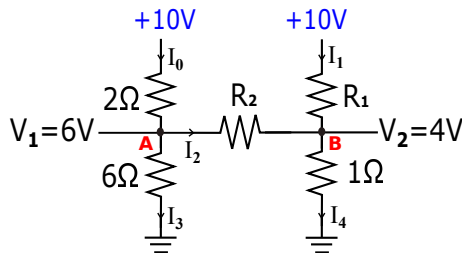
A: Because the resistors are in series, the resistance between successive nodes will be proportional to the voltage between the nodes.

$$\begin{aligned} * R_1 &\propto V_1 = 1R \\ R_2 &\propto V_2 - V_1 = 1R; \\ R_3 &\propto V_3 - V_2 = 2R; \\ R_4 &\propto V_4 - V_3 = 4R; \\ R_5 &\propto V_5 - V_4 = 2R; \end{aligned}$$



Solution(Q2)

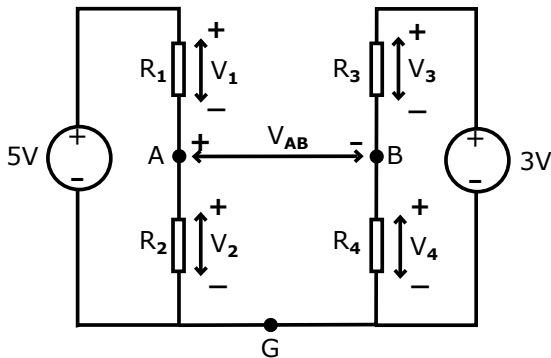
Q: Determine R_1 and R_2



- * KCL at the node **A** determines the current I_2 through R_2 , (i.e. $I_2 = 1A$)
- * To make $V_1 - V_2 = 2V = I_2 R_2$, it follows that $R_2 = 2\Omega$.
- * KCL at the node **B** then determines that $I_1 = 3A$, the current flows downward through R_1 .
To make $10V - V_2 = 6V$, it follows that $R_1 = 2\Omega$.

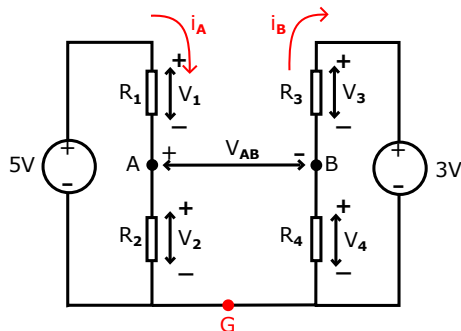
Question 3

- If $V_{AB} = 4V$, determine R_1, R_2, R_3 and R_4



Solution(Q3)

- * $V_{AB} = 4V$
- * If $V_B = -1.5V$,
then $V_A = 2.5V$
- * By potential divider,
 $R_1 : R_2 = 1 : 1$, $R_3 : R_4 = 1 : 1$

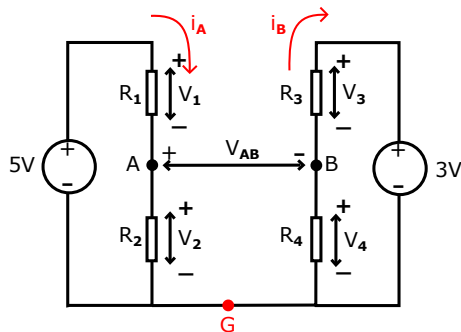


$$V_A = 5 \cdot \frac{R_2}{R_1 + R_2} = 2.5V, \quad V_B = -3 \cdot \frac{R_4}{R_3 + R_4} = -1.5V$$

- * You can pick any value for resistances.

Solution(Q3)

- * $V_{AB} = 4V$
- * If $V_B = -1V$,
then $V_A = 3V$
- * By potential divider,
 $R_1 : R_2 = 2 : 3$, $R_3 : R_4 = 2 : 1$



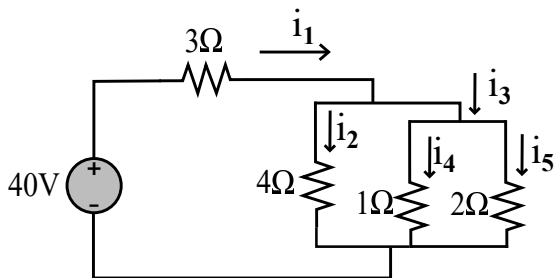
- * If $V_B = -2V \rightarrow V_A = 2V$
By potential divider,
 $R_1 : R_2 = 3 : 2$, $R_3 : R_4 = 1 : 2$

$$V_A = 5 \cdot \frac{R_2}{R_1 + R_2}, \quad V_B = -3 \cdot \frac{R_4}{R_3 + R_4}$$

$$V_{AB} = V_A - V_B = \frac{5R_2}{R_1 + R_2} - \frac{3R_4}{R_3 + R_4}$$

Question 4

- For the circuit in the figure, determine i_1 to i_5 .

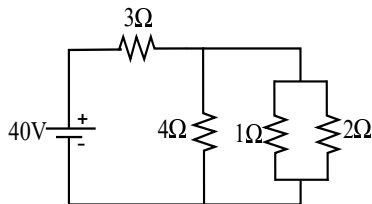


Solution(Q4)

* We apply:

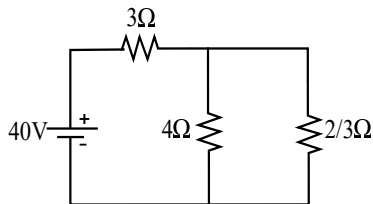
- $V = IR$
- Series / Parallel Combinations
- Current Divider

(i)



$$1\Omega \parallel 2\Omega = \frac{2}{3}\Omega$$

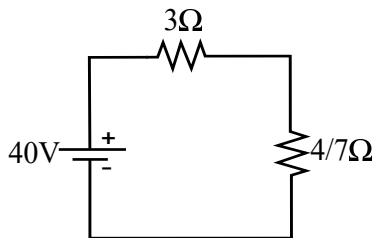
(ii)



$$4\Omega \parallel \frac{2}{3}\Omega = \frac{4}{7}\Omega$$

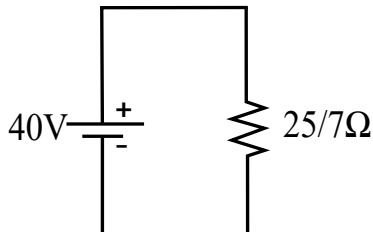
Solution(Q4)

(iii)



$$3\Omega + \frac{4}{7}\Omega = \frac{25}{7}\Omega$$

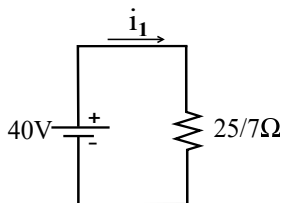
(iv)



Simplified Circuit

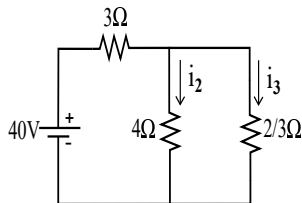
Solution(Q4)

(v)



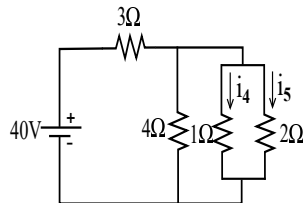
- $V = IR$
 $40 = i_1 \cdot \frac{25}{7}$
 $\Rightarrow i_1 = 11.2A$

(vi)



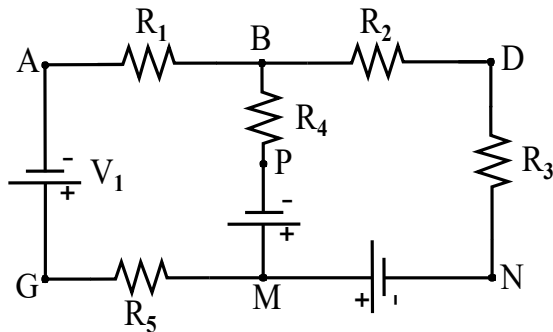
- $i_1 = i_2 + i_3$
 $i_2 = \frac{\frac{2}{3}}{4 + \frac{2}{3}} \cdot i_1$
 $= (\frac{1}{7})(11.2) = 1.6A$
 $i_3 = 11.2 - 1.6 = 9.6A$

(vii)



- $i_3 = i_4 + i_5$
 $i_4 = (\frac{2}{3})(9.6) = 6.4A$
 $i_5 = (\frac{1}{3})(9.6) = 3.2A$

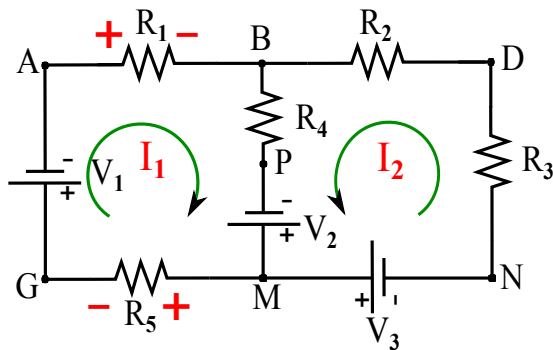
Question 5



- * $R_1 = 80\Omega, R_2 = 10\Omega, R_3 = 20\Omega, R_4 = 90\Omega, R_5 = 100\Omega$
- * Battery: $V_1 = 12V, V_2 = 24V, V_3 = 36V$
- * Resistor: $I_1(I_5), I_2(I_3), I_4 = ? \quad P_1, P_2, \dots, P_5 = ?$

Solution(Q5)

- * $R_1 = 80\Omega, R_2 = 10\Omega,$
 $R_3 = 20\Omega, R_4 = 90\Omega,$
 $R_5 = 100\Omega$
 $V_1 = 12V, V_2 = 24V,$
 $V_3 = 36V$



Loop 1:

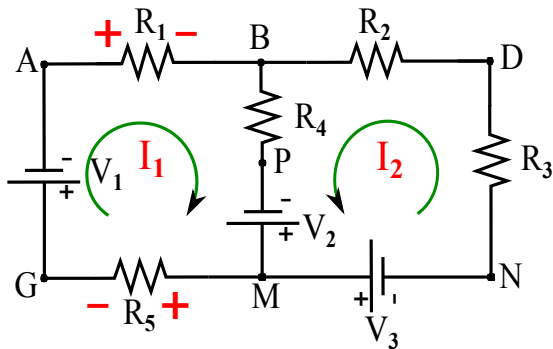
- * $V_1 + I_1 R_1 + (I_1 + I_2) R_4 - V_2 + I_1 R_5 = 0$

$$\Rightarrow 12 + 80I_1 + 90I_1 + 90I_2 - 24 + 100I_1 = 0$$

$$\Rightarrow 270I_1 + 90I_2 = 12 \quad (1)$$

Solution(Q5)

- * $R_1 = 80\Omega, R_2 = 10\Omega,$
 $R_3 = 20\Omega, R_4 = 90\Omega,$
 $R_5 = 100\Omega$
 $V_1 = 12V, V_2 = 24V,$
 $V_3 = 36V$

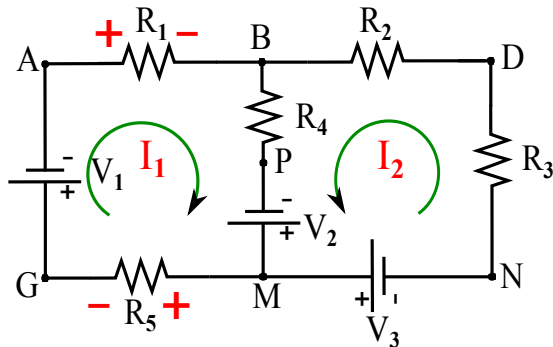


Loop 2:

- * $V_3 + I_2 R_3 + I_2 R_2 + (I_1 + I_2) R_4 - V_2 = 0$
 - * $36 + 20I_2 + 10I_2 + 90I_1 + 90I_2 - 24 = 0$
- $$\Rightarrow 90I_1 + 120I_2 = -12 \quad (2)$$

Solution(Q5)

- * $R_1 = 80\Omega, R_2 = 10\Omega,$
 $R_3 = 20\Omega, R_4 = 90\Omega,$
 $R_5 = 100\Omega$
 $V_1 = 12V, V_2 = 24V,$
 $V_3 = 36V$



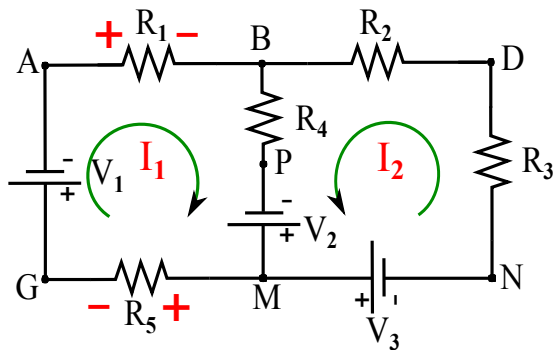
- * **Loop 1:** $270I_1 + 90I_2 = 12$ (1)

- * **Loop 2:** $90I_1 + 120I_2 = -12$ (2)

$$\Rightarrow I_2 = -\frac{8}{45}A, I_1 = \frac{14}{135}A$$

Solution(Q5)

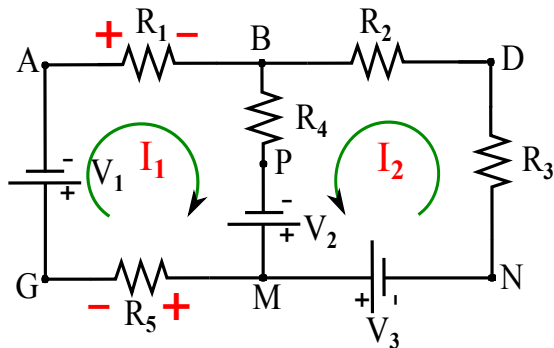
- * $R_1 = 80\Omega, R_2 = 10\Omega,$
 $R_3 = 20\Omega, R_4 = 90\Omega,$
 $R_5 = 100\Omega$
 $V_1 = 12V, V_2 = 24V,$
 $V_3 = 36V$



- * $I_1 = \frac{14}{135} A$
- * $I_2 = -\frac{8}{45} A$
- * **KCL of Node:** $I_{R_4} = I_1 + I_2$
- * $P = I^2 R$
 - $P_1 = I_1^2 R_1, P_4 = I_{R_4}^2 R_4, \dots$

Solution(Q5)

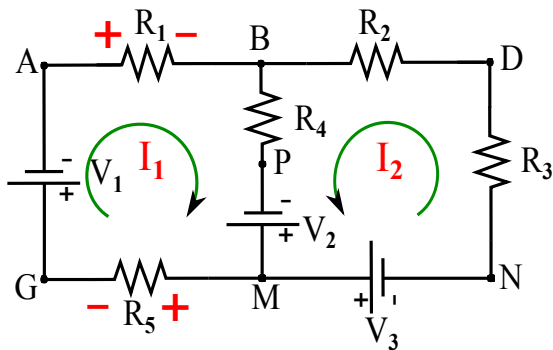
- * $R_1 = 80\Omega, R_2 = 10\Omega,$
 $R_3 = 20\Omega, R_4 = 90\Omega,$
 $R_5 = 100\Omega$
 $V_1 = 12V, V_2 = 24V,$
 $V_3 = 36V$



- * $I_1 = (24 - V_B)/180 = 14/135A = 0.104A$
- * $I_4 = (12 - V_B)/90 = 2/27A = 0.074A$
- * $I_2 = -V_B/30 = -8/45A = -0.178A$

Solution(Q5)

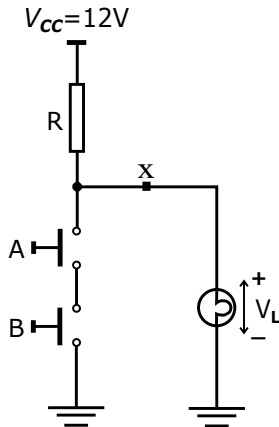
- * $R_1 = 80\Omega, R_2 = 10\Omega,$
 $R_3 = 20\Omega, R_4 = 90\Omega,$
 $R_5 = 100\Omega$
 $V_1 = 12V, V_2 = 24V,$
 $V_3 = 36V$



- * $P = I^2 R, P = V^2 / R \Rightarrow P_1 = (0.104)^2 \cdot 80 = 0.86528 W$
- * $P_2 = (-0.178)^2 \cdot 10 = 0.31684 W$
- * $P_4 = (0.074)^2 \cdot 90 = 0.49284 W = V_{R_4}^2 / R_4$
- * $P_3 = (-0.178)^2 \cdot 20 = 0.63368 W$
- * $P_5 = (0.104)^2 \cdot 100 = 1.0816 W$

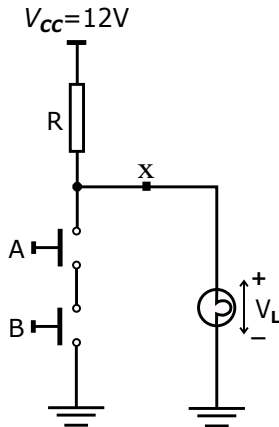
Question 6

- * You have connected the lamp, with $V_{CC} = 12V$. The datasheet of the lamp states that it only turns on when $V_L > 8V$. The lamp has an internal resistance of $1k\Omega$.
- * What is the range of R that would allow the circuit to function correctly with all input combinations.



Solution(Q6)

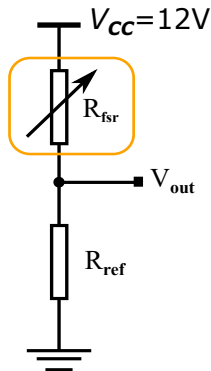
- * The range of R
- * When the lamp is turned on, $x = \text{"1"} , V_L > 8$
 $12 \times \frac{R_L}{R_L + R} > 8 \Rightarrow R < 500\Omega$
- * When the lamp is off, $x = \text{"0"} ,$ and $V_L < 8$. It will function regardless of the value of R .



Question 7a

- * A force sensitive resistor (FSR) is a resistor with its resistance changed according to the force applied to it.
- * For simplicity sake, your partner has wired up the FSR using a simple potential divider circuit

Force (N)	Resistance $R_{fsr}(\Omega)$
0	1M
0.5	10k
1	6k
10	1k

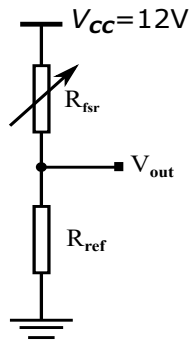


Solution(Q7a)

* Calculate the following quantities when $R_{ref} = 10k\Omega$:

- Voltage across the FSR;
- Voltage at V_{out} ;
- Current owing through the FSR.

R_{fsr}	I_{fsr}	V_{fsr}	V_{out}
1M	$11.9\mu A$	11.9V	0.1V
10k	0.6mA	6V	6V
6k	0.75mA	4.5V	7.5V
1k	1.09mA	1.09V	10.91V



Question 7b

- * The output V_{out} is used to detect the presence of a ball. Due to its light weight, the ball produces only 0.5N when it is located on top of the sensor. The rest of the system requires that $V_{IL} = 2V$ and $V_{IH} = 10V$
 - V_{IL} : Max. voltage that the system regards as logical LOW
 - V_{IH} : Min. voltage that the system regards as logical HIGH
- * Determine the range of value that R_{ref} may take for correct functioning of the circuit.
 - It should output a logical HIGH when a ball is presence and a logical LOW otherwise.

Solution(Q7b)

Voltage

12

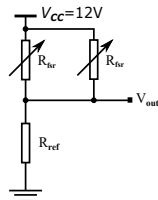
10

2

0

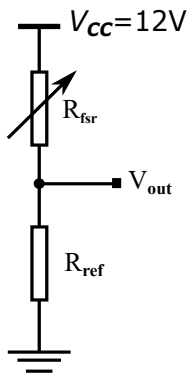


- ← $V_{DD} = 12V$
- ← (Presence of the ball / 0.5N)
- ← $V_{IH} = 10V$; Minimum HIGH
 $R_{fsr} = R_H = 10k\Omega$
- ← $10V - 2V$: (Unknown; Not in use)
- ← $V_{IL} = 2V$; Maximum LOW
- ← (Absence of the ball / 0N)
- ← $V_{GND} = 0V$



Solution(Q7b)

- * When the ball is on the sensor, $R_{fsr} = R_H = 10k\Omega$. The output voltage at that time should be higher than V_{IH} .
- * When the ball is not on the sensor, then $R_{fsr} = R_L = 1M\Omega$. The output voltage should be lower than V_{IL} .



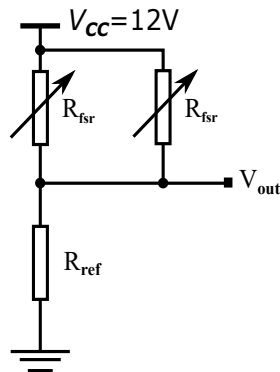
$$\begin{cases} V_{IH} < 12 \cdot \frac{R_{ref}}{R_H + R_{ref}} \\ 12 \cdot \frac{R_{ref}}{R_H + R_{ref}} < V_{IL} \end{cases} \Rightarrow \begin{cases} V_{IH} < 12 \cdot \frac{R_{ref}}{R_H + R_{ref}} \\ 12 \cdot \frac{R_{ref}}{R_H + R_{ref}} < V_{IL} \end{cases}$$

$$\Rightarrow 50k\Omega < R_{ref} < 200k\Omega$$

Question 7c

- * Your partner suggests that it may be possible to use 2 FSRs connected to perform a logical OR operation: When the ball rolls over either one of the 2 FSRs, the output V_{out} is HIGH, and is LOW otherwise.

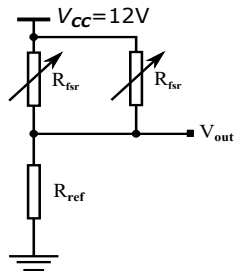
- * What is the output voltage V_{out} ?
 - one of the FSRs is under pressure of 0.5N;
 - both FSRs are under a pressure of 0.5N each;
 - none of the FSRs is under pressure;(assume R_{ref} is $100k\Omega$)



Solution(Q7c)

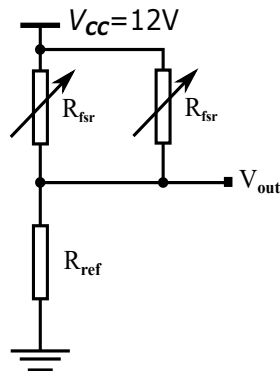
- * If $R_{ref} = 100k\Omega$,
- (i) one of the FSRs is under pressure of 0.5N;
 - (ii) both FSRs are under a pressure of 0.5N each;
 - (iii) none of the FSRs is under pressure;
 - R_{eqv} is the equivalent resistance of the parallel combination of the two FSRs.

Case	$R_{fsr1}(\Omega)$	$R_{fsr2}(\Omega)$	$R_{eqv}(\Omega)$	$V_{out}(V)$
(i)	10000	1000000	9901	10.9
(ii)	10000	10000	5000	11.4
(iii)	1000000	1000000	500000	2

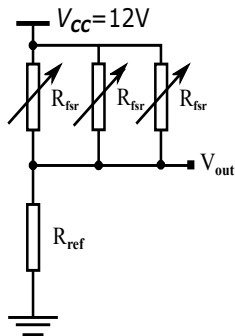


Question 7d

- * Recall that V_{IL} is 2V and V_{IH} is 10V, is the circuit functioning correctly as a 2-input OR?
- * If there are 3 FSRs connected in parallel, assuming R_{ref} remains at 100k, will the circuit behave as a 3-input OR?

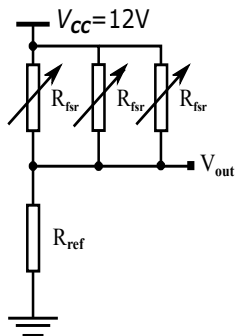


Solution(Q7d)



- * **Yes**, it works correctly as a 2-input OR gate because the output is HIGH when there is a ball on top of at least one of the input.
- * However, even if we connect 3 FSRs in parallel, the circuit cannot correctly function as a 3-input OR gate. In the case when there is **no ball** falling on the circuit, the **equivalent resistance** of the parallel combination of the 3 FSRs **drops too low** that $V_{out} > 2V$. As a result, the output fails to represent a logical LOW in this case.

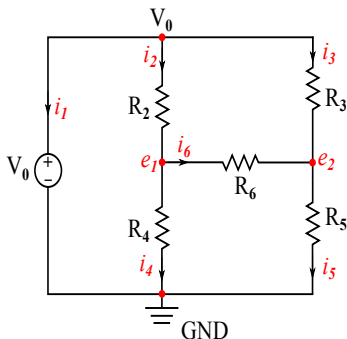
Solution(Q7d)



R_{fsr1}	R_{fsr2}	R_{fsr3}	R_{eqv}	V_{out}
1000000	1000000	1000000	333333.33	2.77
1000000	1000000	10000	9803.92	10.93
1000000	10000	1000000	9803.92	10.93
1000000	10000	10000	4975.12	11.43
10000	1000000	1000000	9803.92	10.93
10000	1000000	10000	4975.12	11.43
10000	10000	1000000	4975.12	11.43
10000	10000	10000	3333.33	11.61

Appendix(Question)

Wheatstone bridge



	NOT Always True	Always True
If $\frac{R_2}{R_4} = \frac{R_3}{R_5}$, then $i_6 = 0$		
$i_2 + i_3 = i_4 + i_5$		
$i_2 + i_6 = i_3$		
$e_1 = \frac{R_4}{R_2 + R_4} V_0$		
If $i_6 = 0$, then $\frac{R_2}{R_2 + R_4} = \frac{R_3}{R_3 + R_5}$		

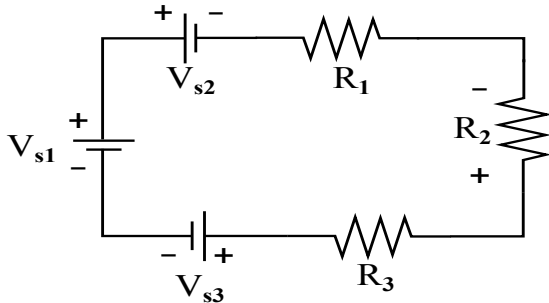
Appendix(Solution)

	NOT Always True	Always True
If $\frac{R_2}{R_4} = \frac{R_3}{R_5}$, then $i_6 = 0$		✓
$i_2 + i_3 = i_4 + i_5$		✓
$i_2 + i_6 = i_3$	✓	
$e_1 = \frac{R_4}{R_2 + R_4} V_0$	✓	
If $i_6 = 0$, then $\frac{R_2}{R_2 + R_4} = \frac{R_3}{R_3 + R_5}$		✓

Appendix(Question 8)

- * Find V_2 using single loop analysis
 - Without simplifying the circuit
 - Simplifying the circuit

$$V_{s1} = 2V, V_{s2} = 2V, V_{s3} = 2V, R_1 = 1\Omega, R_2 = 2\Omega, R_3 = 4\Omega$$

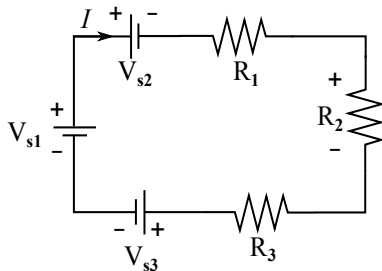


Appendix(Solution(Q8))

Without Simplifying the circuit

- * Choose loop current
 - * Apply KVL
 - Replace V_2 by $R_2 I$
 - $+V_{s2} + R_1 I + \textcolor{red}{R_2 I} + R_3 I + V_{s3} - V_{s1} = 0$
- $$\Rightarrow I = -\frac{2}{7} A$$
- * Find V_2
 - $V_2 = -R_2 I = \frac{4}{7} V$

**Notice: Ignore sign symbols(+/-)
on resistor when calculating
currents(R_2 in this example!!!)**



Appendix(Solution(Q8))

Simplifying the circuit

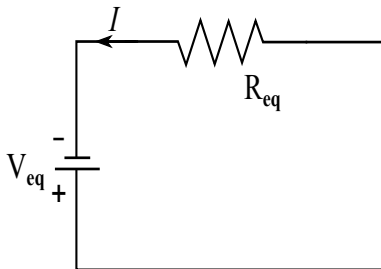
- * Simplifying the circuit with one voltage source and one resistor

- * $R_{eq} = R_1 + R_2 + R_3 = 7\Omega$

- * $V_{eq} = V_{s1} + V_{s2} + V_{s3} =$
 $-2 + 2 + 2 = 2V$

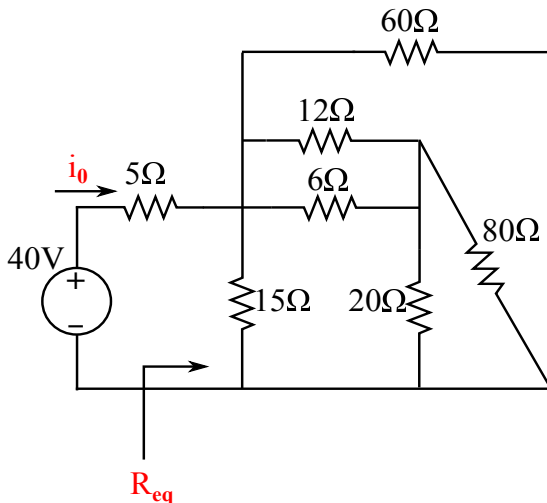
$\Rightarrow I = V_{eq} / R_{eq} = \frac{2}{7}A$

$\Rightarrow V_2 = \frac{4}{7}V$

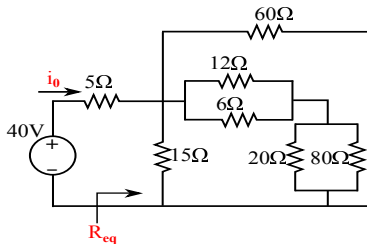


Appendix(Question 9)

- * Find R_{eq} and i_o in the circuit of the figure.

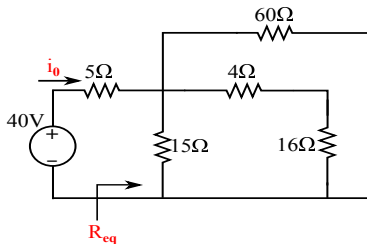


Appendix(Solution(Q9))



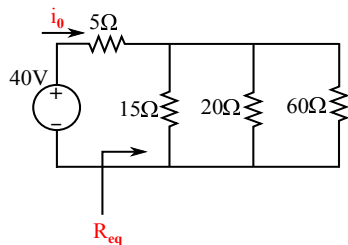
$$12\Omega \parallel 6\Omega = 4\Omega$$

$$20\Omega \parallel 80\Omega = 16\Omega$$



$$4\Omega + 16\Omega = 20\Omega$$

Appendix(Solution(Q9))



$$R_{eq} = 15 \parallel 20 \parallel 60\Omega = 7.5\Omega$$
$$V = IR \Rightarrow 40 = i_0 \cdot 7.5 + 5 \Rightarrow$$
$$i_0 = 3.2A$$

Appendix(Rules Governing Currents and Voltages)

Rule 1: Currents flow in loops

The same amount of current flows into the bulb (top path) and out of the bulb (bottom path)

Rule 2: Like the flow of water, the flow of electrical current (charged particles) is incompressible

Kirchoffs Current Law (KCL): the sum of the currents into a node is zero

Rule 3: Voltages accumulate in loops

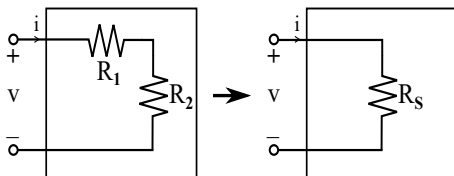
Kirchoffs Voltage Law (KVL): the sum of the voltages around a closed loop is zero

Appendix(Analyzing Circuits)

- Assign **node voltage variables** to every node except ground (whose voltage is arbitrarily taken as zero)
- Assign **component current variables** to every component in the circuit
- Write one **constructive relation** for each component in terms of the component current variable and the component voltage
- Express **KCL** at each node except ground in terms of the component currents
- **Solve** the resulting equations
- $Power = IV = I^2R = V^2/R$

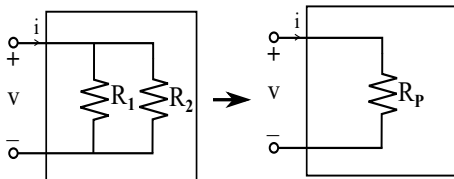
Appendix(Parallel/Series Combinations of Resistance)

- To simplify the circuit for analysis



$$\begin{aligned}v &= R_1 i + R_2 i \\v &= R_s i \\ \therefore R_s &= R_1 + R_2\end{aligned}$$

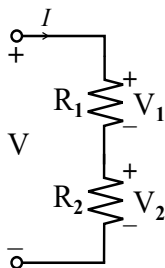
Series



$$\begin{aligned}R_p &= \frac{1}{1/R_1 + 1/R_2} = \frac{R_1 R_2}{R_1 + R_2} \\ &= R_1 \parallel R_2\end{aligned}$$

Parallel

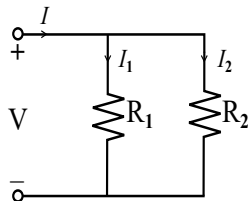
Appendix(Voltage/Current Divider)



← **Voltage Divider**

$$I = \frac{V}{R_1 + R_2}$$
$$V_1 = R_1 I = \frac{R_1}{R_1 + R_2} V$$
$$V_2 = R_2 I = \frac{R_2}{R_1 + R_2} V$$

Current Divider →



$$V = (R_1 \parallel R_2) I$$
$$I_1 = \frac{V}{R_1} = \frac{R_1 \parallel R_2}{R_1} I = \frac{R_2}{R_1 + R_2} I$$
$$I_2 = \frac{R_1}{R_1 + R_2} I$$

The End