**ENGG1203: Introduction to Electrical and Electronic Engineering**
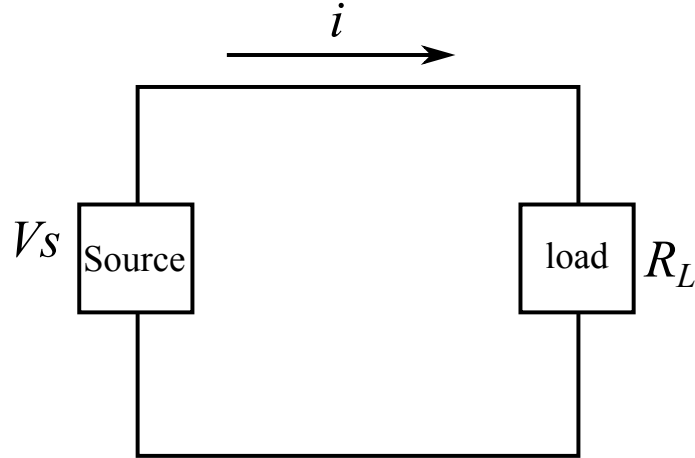
# Circuits (ENGG1203)

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# 1 Basics

We use a simple model as a starting point to discuss electronic circuits, which is shown on Figure 1.

FIGURE 1 – Fundamental circuit model



A basic circuit is made up of a <u>source</u> which provides voltage across its terminals, denoted by $V$.

## 1.1 Current

The current $i$ results from the stream of electric charge around the closed loop shown on Figure 1. The mathematic definition is equal to the amount of charge, $Q$, passing through a cross-section per second and it can be expressed as

$$i = \frac{dQ}{dt} \tag{1.1}$$

The unit of charge is the Coulomb. One Coulomb is equivalent to $6.24 \times 10^{18}$ electrons. The unit for current is the ampere, A. One ampere = 1 Coulomb/sec.

### 1.1.1 Ideal DC Current Source

The current source is a device that can provide a certain amount of current to a circuit. The symbol for a DC current source and the i / v characteristic curve of an ideal current source are shown on Figure 2.

FIGURE 2 – Ideal DC Current Source
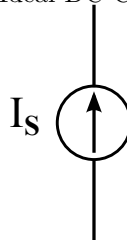
## 1.2 Voltage

Moving electrons along a conductor requires some amount of work that must be somehow supplied by an electromotive force provided by a battery or similar device. The electromotive force is the potential difference(voltage) between two points or across a component in circuit. The mathematical definition of voltage is given by

$$v = \frac{dW}{dQ} \tag{1.2}$$

where work(W) is measured in Joules and charge(Q) in Coulombs. Basically, the voltage is measured in volts(V) and $1 volt = 1 \frac{Joule}{Coulomb} = 1 \frac{Newton\ meter}{Ampere\ second}$

### 1.2.1 Ideal DC Voltage Sources

The voltage provided by a voltage source(usually battery) is constant in time and thus it is called DC voltage. In its ideal implementation the battery provides a specific voltage at all times and for all loads. The symbols of an ideal DC voltage source are shown as follows :

FIGURE 3 – Ideal DC voltage source
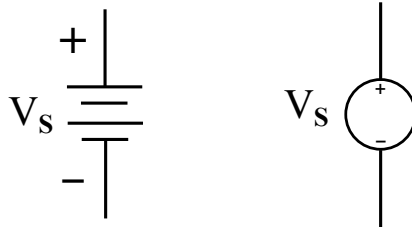
## 1.3 Ideal resistor

The ideal resistor is a passive, linear, two-terminal device whose resistance follows Ohm's law given by,
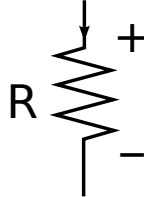
$$v = iR, \tag{1.3}$$

which states that the voltage across an element is directly proportional to the current flowing through the element. The constant of proportionality is the resistance R provided by the element. The resistance is measured in Ohms, $\Omega$, and

$$1\Omega = 1\frac{V}{A}, \tag{1.4}$$

the symbol for a resistor is,

FIGURE 4 – Ideal resistor

## 1.4 Resistor Colour Code

The coloured painted bands produce a system of identification generally known as a **Resistors Colour Code**. The resistor colour code markings are always read one band at a time starting from the left to the right, with the larger width tolerance band oriented to the right side indicating its tolerance. By matching the colour of the first band with its associated number in the digit column of the colour chart below the first digit is identified and this represents the first digit of the resistance value.

Again, by matching the colour of the second band with its associated number in the digit column of the colour chart we get the second digit of the resistance value and so on. Then the resistor colour code is read from left to right as illustrated below
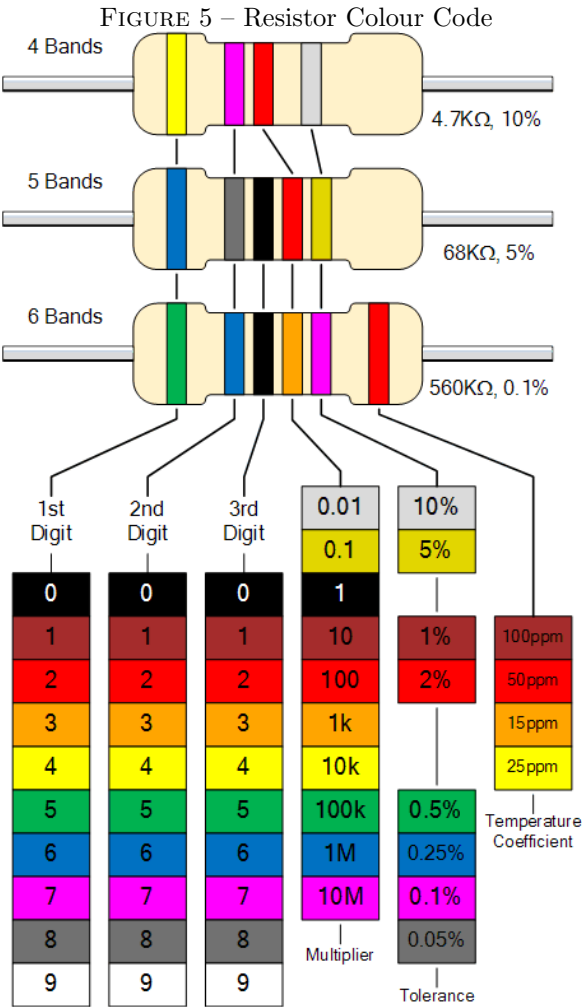
FIGURE 5 – Resistor Colour Code



FIGURE 6 – Colour Code Table

| Colour | Digit | Multiplier | Tolerance |
|---|---|---|---|
| Black | 0 | 1 | |
| Brown | 1 | 10 | ± 1% |
| Red | 2 | 100 | ± 2% |
| Orange | 3 | 1,000 | |
| Yellow | 4 | 10,000 | |
| Green | 5 | 100,000 | ± 0.5% |
| Blue | 6 | 1,000,000 | ± 0.25% |
| Violet | 7 | 10,000,000 | ± 0.1% |
| Grey | 8 | | ± 0.05% |
| White | 9 | | |
| Gold | | 0.1 | ± 5% |
| Silver | | 0.01 | ± 10% |
| None | | | ± 20% |

For example, a resistor has the following colored markings :

$$\text{Yellow Violet Red} = 4\ 7\ 2 = 4\ 7 \times 10^2 = 4700\Omega \text{ or } 4k7.$$

## 1.5   Power

When current flows through a resistor, energy is irreversibly lost(or we say dissipated) in overcoming the resistance. THe dissipated power shows up as heat most of the time. The mathematical definition of power is the rate as which energy is delivered that can be expressed as

$$P = \frac{dW}{dt}, \tag{1.5}$$

The units for power are *Joules/sec* or Watts, $W$. $(1 Joules/sec = 1W)$ Power can be related to voltage and current by rewriting Eq.(1.5) as,

$$P = \frac{dW}{dt} = \frac{dW}{dQ}\frac{dQ}{dt} = vi, \tag{1.6}$$

Then substitute Ohm's law in Eq.(1.6), we will get the power dissipated in a resistor of resistance R is a non-linear function of either $i$ or $v$.

$$P = i^2 R \quad or \quad P = \frac{v^2}{R} \tag{1.7}$$

Power rating is a fundamental constraint of resistors and electronic devices in general. The power rating is referred to the maximum power that the device can dissipate without adversely affecting its operation. When the power rating is exceeded the resistor overheats and it is destroyed by burning up.

# 2   Kirchhoffs Laws

Kirchhoffs laws also known as Kirchhoffs Current Law (KCL) and Kirchhoffs Voltage Law (KVL) are based respectively on the conservation of charge and the conservation of energy and are derived from Maxwells equations. They along with Ohm's law present the fundamental tools for circuit analysis.

## 2.1   Kirchhoffs Current Law

The current flowing out of any node in a circuit must be equal to the current flowing into the node. It is expressed mathematically as

$$\sum_{n=1}^{N} i_n = 0 \tag{2.1}$$

where N is the number of branches that are connected to the node.

## 2.2   Kirchhoffs Voltage Law

The algebraic sum of voltages around a closed loop is zero. It is expressed mathematically as

$$\sum_{n=1}^{N} v_n = 0 \tag{2.2}$$

where N is the number of voltages in the loop. The number of voltages is equal to the number of elements encountered as we go around the loop.

Next we will solve for the circuit shown on Figure 7

Figure 7 – Example resistive circuit
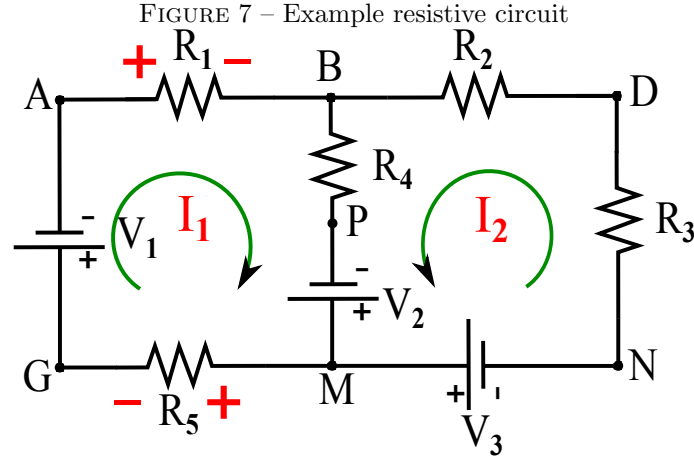


$$R_1 = 80\Omega, \ R_2 = 10\Omega, \ R_3 = 20\Omega, \ R_4 = 90\Omega, \ R_5 = 100\Omega, \ V_1 = 12V, \ V_2 = 24V, \ V_3 = 36V$$

Apply Kirchhoffs laws.
- Start with the nodes.
  1. The middle node P is already accounted for since we assigned the current above and below it the same value, $I_1$. This is just Kirchhoffs current law which says that the current going into a node is equal to that going out.
  2. The bottom and top nodes B and M are exactly the same and KCL for them is :
  $$I - I_1 + I_2 = 0$$
  since at node B current $I$ and $I_2$ flow in the node B and $I_1$ flow out the node B.
- Next let's apply KVL to the three loops
  1. **Loop 1($I_1$ loop) :** voltage source $V_1$, $V_2$ and resistors $R_1$, $R_4$, $R_5$

  $$V_1 + I_1 R_1 + (I_1 + I_2)R_4 - V_2 + I_1 R_5 = 0$$

  $$\Rightarrow 12 + 80I_1 + 90I_1 + 90I_2 - 24 + 100I_1 = 0$$

  $$\Rightarrow 270I_1 + 90I_2 = 12$$

  2. **Loop 2($I_2$ loop) :** voltage source $V_2$, $V_3$ and resistors $R_2$, $R_3$, $R_4$

  $$V_3 + I_2 R_3 + I_2 R_2 + (I_1 + I_2)R_4 - V_2 = 0$$

  $$\Rightarrow 36 + 20I_2 + 10I_2 + 90I_1 + 90I_2 - 24 = 0$$

  $$\Rightarrow 90I_1 + 120I_2 = -12$$

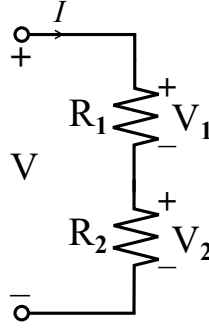- Calculate the required parameters using Ohm's law and relevant formulas.
  $$I_1 = \frac{14}{135}A$$

  $$I_2 = -\frac{8}{45}A$$

# 3 Voltage divider : Series Connection of Resistors

The voltage divider circuit is the most convenient means of passively stepping down the voltage from a fixed voltage source.
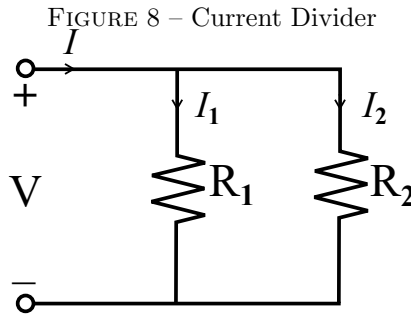$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = R_1 I = \frac{R_1}{R_1+R_2} V$$
$$V_2 = R_2 I = \frac{R_2}{R_1+R_2} V$$

By considering a circuit where we are using N resistors connected in series, we can show that the equivalent resistance is the sum of the resistances.

$$R_{eq} = \sum_{n=1}^{N} R_n = R_1 + R_2 + ... + R_N \tag{3.1}$$

# 4  Current Divider : Parallel connection of resistors

The schematic on Figure 8 shows a simple current divider circuit. Here the two resistors $R_1$ and $R_2$ are connected in parallel. Lets determine the current $I_1$ and $I_2$ flowing in the two resistors.



FIGURE 8 – Current Divider

$$V = (R_1 \parallel R_2)I$$
$$I_1 = \frac{V}{R_1} = \frac{R_1 \parallel R_2}{R_1} I = \frac{R_2}{R_1+R_2} I$$
$$I_2 = \frac{R_1}{R_1+R_2} I$$

For N resistors connected in parallel the equivalent resistance is

$$\frac{1}{R_{eq}} = \sum_{n=1}^{N} \frac{1}{R_n} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N} \tag{4.1}$$

Note that the equivalent resistance $R_{eq}$ is smaller than the smallest resistance in the parallel arrangement.

The equivalent conductance of N resistors connected in parallel is

$$G_{eq} = \sum_{n=1}^{N} G_n = G_1 + G_2 + ... + G_N \tag{4.2}$$

where $G_n = \frac{1}{R_n}$. The current flowing through resistor $R_n$ is

$$i_n = i \frac{G_n}{G_{eq}} \tag{4.3}$$

# 5 Appendix

## 5.1 Node Method & Mesh Method

With the help of Kirchhoff's laws and Ohm's law, we can analyze any circuit to determine the currents and voltages. For formal circuit analysis, the challenge is to derive the smallest set of simultaneous equations that completely define the operating characteristics of a circuit.

The following part, we will develop two very powerful methods for analyzing any circuit : The **node method** and the **mesh method**. We will explain the steps to solve the circuit problem shown on Figure 2.

### 5.1.1 The Node Method

Voltage is defined as the potential difference between two points. When we talk about the voltage at a certain point, we imply that the measurement is performed between that point and some other point in the circuit. Usually, this reference point is defined as ground.

Node method(or node voltage method) is an efficient circuit analysis method based on KCL, KVL, and Ohm's laws. Steps for using node method to analysis circuit are as follows :

1. Label all the circuit parameters and distinguish the unknown parameters from the known ones.
2. Identify all nodes of the circuit
3. Choose a reference node(Ground) and assign to it a potential of $0V$. All other voltages in the circuit are measured with respect to the reference node.
4. Label the voltage at all other nodes.
5. Assign and label polarities.
6. Apply KCL at each node and express the branch currents in terms of the node voltages
7. Solve the resulting simultaneous equations for the node voltages.
8. Obtain the branch currents by Ohm's law.

We will use the circuit of Figure 9 for a step by step demonstration of the node method.

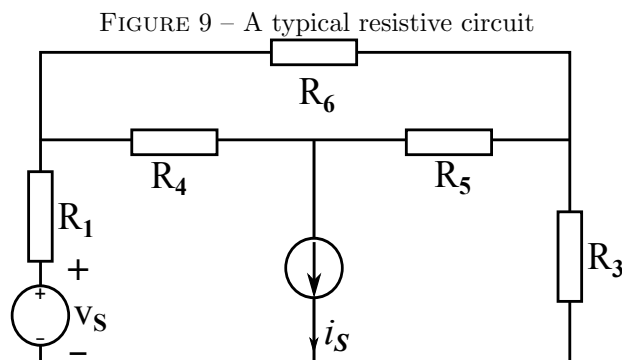FIGURE 9 – A typical resistive circuit



Figure 10. shows the implementation of steps 1 and 2. We have labeled all elements and identified all relevant nodes in the circuit.

The third step is to select one of the identified nodes as the reference node. We have four different choices for the assignment. In principle any of these nodes may be selected as the reference node. However, some nodes are more useful than others. Useful nodes are the ones which make the problem **easier to understand and solve**. There are a few general guidelines that we need to remember as we make the selection of the reference node.

* A useful reference node is one which should have the largest number of elements connected to it.
* A useful reference node is one which should be connected to the maximum number of voltage sources.

In this example the selection of node **0** as the reference node is the best choice. (you could have selected node **1** as our reference node.)

Next, we should label the voltages at the selected nodes. Figure 11 shows the circuit with the labeled nodal voltages. The reference node is assigned voltage 0 Volts indicated by the ground symbol. The remaining node voltages are labeled $V_{S1}$, $V_{S2}$, $V_{S3}$.

For the next step we assign current flow and polarities, see Figure 12. The circuit shown on Figure 4 is completely defined now. Once we determine the values for the node voltages $V_{S1}$, $V_{S2}$, $V_{S3}$. we will be ble to completely characterize this circuit. So lets go on to calculate the node voltages by applying KCL at the designated nodes.

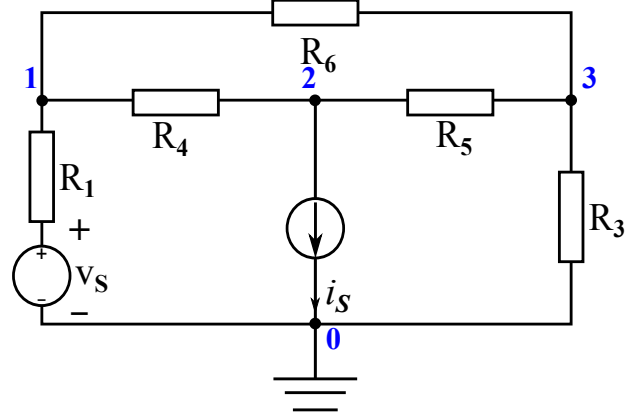FIGURE 10 – Circuit with labeled nodes



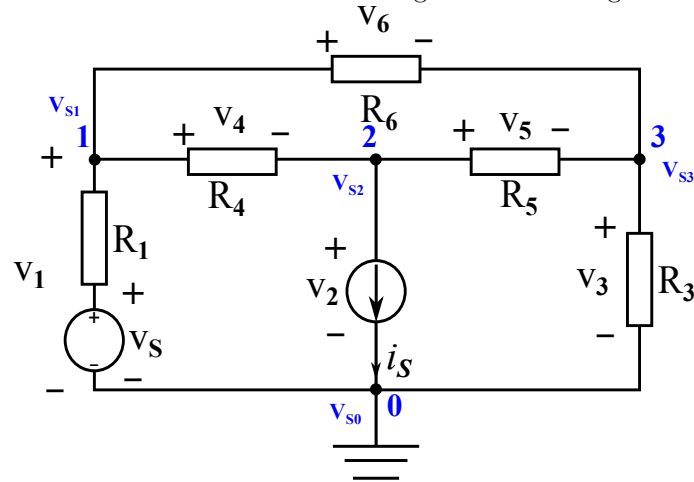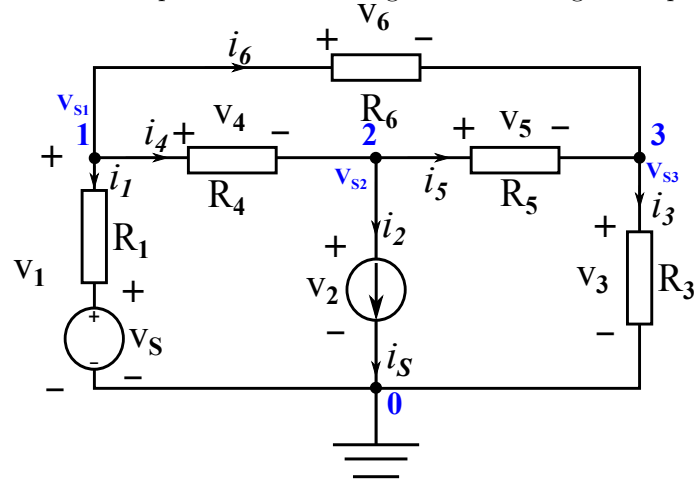FIGURE 11 – Circuit with assigned nodal voltages



FIGURE 12 – Example circuit with assigned node voltages and polarities



According to the ohm's law, the currents $i_1$ to $i_6$ are expressed in terms of the voltages $v_{s1}$, $v_{s2}$, $v_{s3}$ as follows.

$$i_1 = \frac{v_1 - v_s}{R_1} \tag{5.1}$$

$$i_2 = i_s \tag{5.2}$$

$$i_3 = \frac{v_3}{R_3} = \frac{v_{s3}}{R_3} \tag{5.3}$$

$$i_4 = \frac{v_4}{R_4} = \frac{v_{s1} - v_{s2}}{R_4} \tag{5.4}$$

$$i_5 = \frac{v_5}{R_5} = \frac{v_{s2} - v_{s3}}{R_5} \tag{5.5}$$

$$i_6 = \frac{v_6}{R_6} = \frac{v_{s1} - v_{s3}}{R_6} \tag{5.6}$$

KCL at node **1**,**2**,**3** we will get :

$$i_1 + i_4 + i_6 = 0 \tag{5.7}$$

$$i_2 - i_4 + i_5 = 0 \tag{5.8}$$

$$i_3 - i_5 - i_6 = 0 \tag{5.9}$$

By combining Eqs. $(5.1) - (5.9)$ we obtain :

$$\left(\frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_6}\right)v_{s1} - \frac{1}{R_4}v_{s2} - \frac{1}{R_6}v_{s3} = \frac{v_s}{R_1} \tag{5.10}$$

$$-\frac{1}{R_4}v_{s1} + \left(\frac{1}{R_4} + \frac{1}{R_5}\right)v_{s2} - \frac{1}{R_5}v_{s3} = -i_s \tag{5.11}$$

$$-\frac{1}{R_6}v_{s1} - \frac{1}{R_5}v_{s2} + \left(\frac{1}{R_3} + \frac{1}{R_5} + \frac{1}{R_6}\right)v_{s3} = 0 \tag{5.12}$$

If we assign $R_1 = R_5 = 2\Omega, \ R_3 = 6\Omega, \ R_4 = 4\Omega, \ R_6 = 1\Omega, \ v_s = 14V, \ i_s = 2A$
we can easily get the following equations by substituting the known parameters into the Eqs. $(5.10) - (5.12)$

$$1.75v_{s1} - 0.25v_{s2} - v_{s3} = 7 \tag{5.13}$$

$$-0.25v_{s1} + 0.75v_{s2} - 0.5v_{s3} = -2 \tag{5.14}$$

$$-v_{s1} - 0.5v_{s2} + \frac{5}{3}v_{s3} = 0 \tag{5.15}$$

The solutions of the above equations are as follows :
$$v_{s1} = 8V, v_{s2} = 4V, v_{s3} = 6V$$
The voltage of each branch($v_1$ to $v_6$) :
$$v_1 = v_{s1} = 8V$$
$$v_2 = v_{s2} = 4V$$
$$v_3 = v_{s3} = 6V$$
$$v_4 = v_{s1} - v_{s2} = 8 - 4 = 4V$$
$$v_5 = v_{s2} - v_{s3} = 4 - 6 = -2V$$
$$v_6 = v_{s1} - v_{s3} = 8 - 6 = 2V$$
The currents across each component are :
$$i_1 = \tfrac{1}{R_1}(v_1 - v_s) = 0.5 \times (8 - 14) = -3A$$
$$i_2 = i_s = 2A$$
$$i_3 = \tfrac{1}{R_3}v_3 = \tfrac{1}{6} \times 6 = 1A$$
$$i_4 = \tfrac{1}{R_3}v_3 = 0.25 \times 4 = 1A$$
$$i_5 = \tfrac{1}{R_5}v_5 = 0.5 \times (-2) = -1A$$

$$i_6 = \frac{1}{R_6}v_6 = 1 \times 2 = 2A$$

Using KCL rule to check the result

KCL at node **1** :

$$\sum i = i_1 + i_4 + i_6 = -3 + 1 + 2 = 0$$

KCL at node **2** :

$$\sum i = i_2 - i_4 + i_5 = 2 - 1 - 1 = 0$$

KCL at node **3** :

$$\sum i = i_3 - i_5 - i_6 = 1 + 1 - 2 = 0$$

Thus, the solution is right.

### 5.1.2 The Mesh Method

The mesh method uses the mesh currents as the circuit variables. The procedure for obtaining the solution is similar to that followed in the Node method and the various steps are given below.

1. Label all the circuit parameters and distinguish the unknown parameters from the known ones.
2. Identify all meshes of the circuit.
3. Assign mesh currents and label polarities.
4. Apply KVL at each mesh and express the voltages in terms of the mesh currents.
5. Solve the resulting simultaneous equations for the mesh currents.
6. Obtain the voltages by Ohm's law.

A **mesh** is defined as a loop which does not contain any other loops. The circuit example has seven loops but only three meshes as shown on Figure 13. Note that we have assigned a ground potential to a certain part of the circuit. Since the definition of ground potential is fundamental in understanding circuits this is a good practice and thus will continue to designate a reference (ground) potential as we continue to design and analyze circuits regardless of the method used in the analysis.



FIGURE 13 – Identification of the meshes
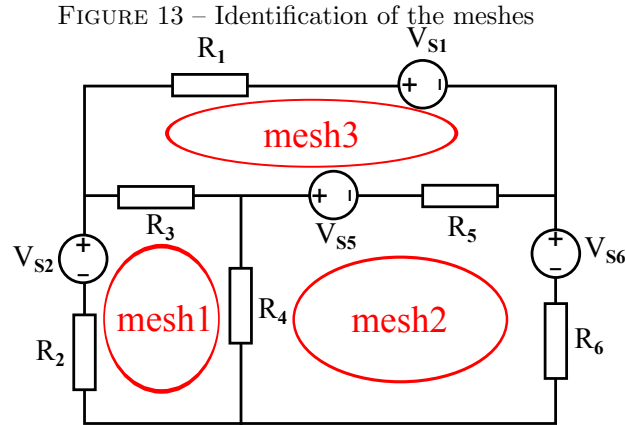
The meshes of interest are mesh1, mesh2 and mesh3. For the next step we will assign mesh currents, define current direction and voltage polarities.

The direction of the mesh currents $i_{m1}, i_{m2}, i_{m3}$ is defined in the clockwise direction as shown on Figure 14. The definition for the current direction is arbitrary but it helps if we maintain consistence in the way we define these current directions.
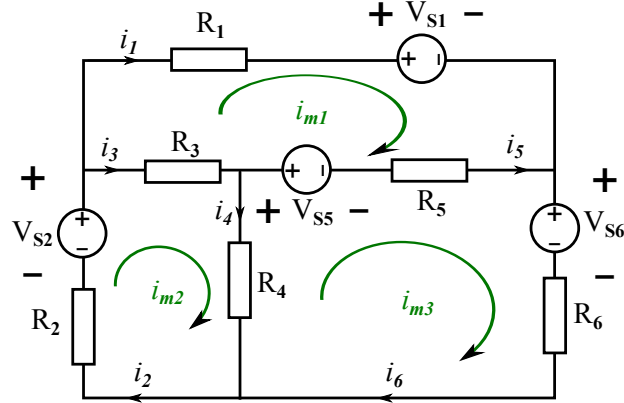
We first assume that the voltage drop of each branch is $u_1$ to $u_6$, and according to the Ohm's law :

$$v_1 = R_1 i_1 + V_{s1} = R_1 i_{m1} + V_{s1} \tag{5.16}$$

$$v_2 = R_2 i_2 - V_{s2} = R_2 \tag{5.17}$$

$$v_3 = R_3 i_3 = R_3(i_{m2} - i_{m1}) \tag{5.18}$$

FIGURE 14 – labeling mesh current direction



$$v_4 = R_4 i_4 = R_4(i_{m2} - i_{m1}) \tag{5.19}$$

$$v_5 = R_5 i_5 + V_{s5} = R_5(i_{m3} - i_{m1}) + V_{s5} \tag{5.20}$$

$$v_6 = R_6 i_6 + V_{s6} = R_6 i_{m3} + V_{s6} \tag{5.21}$$

Apply KVL to mesh1, mesh2 and mesh3 :

$$v_1 - v_3 - v_5 = 0 \tag{5.22}$$

$$v_2 + v_3 + v_4 = 0 \tag{5.23}$$

$$- v_4 + v_5 + v_6 = 0 \tag{5.24}$$

By combining Eqs. (5.16) – (5.24) we obtain :

$$(R_1 + R_3 + R_5)i_{m1} - R_3 i_{m2} - R_5 i_{m3} = V_{s5} - V_{s1} \tag{5.25}$$

$$- R_3 i_{m1} + (R_2 + R_3 + R_4)i_{m2} - R_4 i_{m3} = V_{s2} \tag{5.26}$$

$$- R_5 i_{m1} - R_4 i_{m2} + (R_4 + R_5 + R_6)i_{m3} = V_{s3} \tag{5.27}$$

If we assign $R_1 = R_2 = R_6 = 3\Omega$, $R_3 = 5\Omega$, $R_4 = 1\Omega$, $R_5 = 2\Omega$, $V_{s1} = 12V$, $V_{s2} = 10V$, $V_{s5} = 6V$, $V_{s6} = -20V$

$$10i_{m1} - 5i_{m2} - 2i_{m3} = -6 \tag{5.28}$$

$$- 5i_{m1} + 9i_{m2} - i_{m3} = 10 \tag{5.29}$$

$$- 2i_{m1} - i_{m2} + 6i_{m3} = 14 \tag{5.30}$$

Solve the equations we will get :

$$i_{m1} = 1A$$
$$i_{m2} = 2A$$
$$i_{m3} = 3A$$

The current of each branch($i_1$ to $i_6$) :

$$i_1 = i_{m1} = 1A$$
$$i_2 = i_{m2} = 2A$$
$$i_3 = i_{m2} - i_{m1} = 2 - 1 = 1A$$
$$i_4 = i_{m2} - i_{m3} = 2 - 3 = -1A$$

$$i_5 = i_{m3} - i_{m1} = 3 - 1 = 2A$$
$$i_6 = i_{m3} = 3A$$