

Tutorial on Signal Introduction to Electrical and Electronic Engineering

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Overview

* **Learning Objectives:**

- Signal Flow Graph
- Difference Equations

* **Basics**

- Building blocks of an LTI system
 - Three Building Blocks
 - Flow Graph Transformations
- Difference Equations
 - Conventions
 - Two Special Discrete-time signals
 - Flow Graphs

* **Questions & Summary**

Building blocks(Three Building Blocks)

The three building blocks of an LTI system: **multiplication**, **addition**, and **delay**

- **Multiplication(gain)**

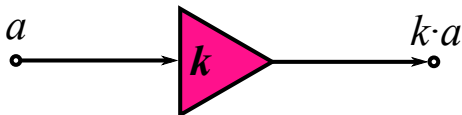


Figure : Output equals to the input with a gain k

- **Split/add(adder)**

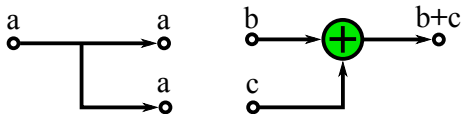


Figure : On the left, a signal is split into two paths. On the right, two signals are added together

Building blocks(Three Building Blocks)

The three building blocks of an LTI system: **multiplication**, **addition**, and **delay**

- **Delay**

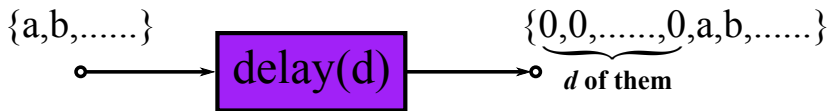


Figure : Output equals to input with a delay of d time units

Building blocks(Flow Graph Transformations)

Intuitively, some changes to the flow graphs are permitted:

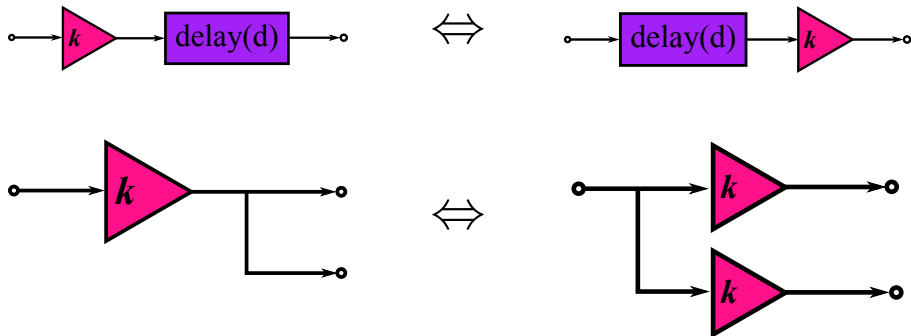


Figure : Some examples of flow graph transformations

Difference Equations(Expression & Conventions)

Expression:

- $y[n] = a_1y[n-1] + a_2y[n-2] + \dots + b_0x[n] + b_1x[n-1] + \dots$

Conventions:

- Signal: $x[n]$ (square bracket)
- Use $x[n]$ for an input signal, $y[n]$ for an output signal
- Often $n = 0, 1, \dots, N-1$ (integer) for a length- N signal. We may also have an “infinite” length signal where n can be any nonnegative integers.
- Assume $x[n] = 0$ outside this range.
 \Rightarrow No input, no output. System is “at rest”

Difference Equations(Two Special Discrete-time signals)

Impulse Signal(delta functions): $\delta[n]$

- $$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise}(n \neq 0) \end{cases}$$

This is called an impulse because it is active only at the first time instance, and then it returns to zero and stays there forever

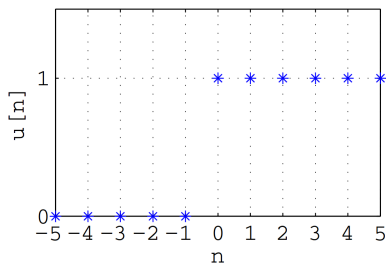
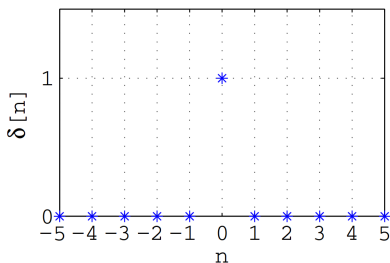
Unit Step Functions: $u[n]$

- $$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise}(n < 0) \end{cases}$$

Notice that because we have assumed that all signals with negative indices are zero, the unit step appears to be equal to 1 all the time

Difference Equations(Two Special Discrete-time signals)

Relation of these two signals: $\delta[n]$ and $u[n]$



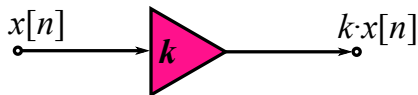
- $\delta[n] = u[n] - u[n - 1]$
- $u[n] = \sum_{m=-\infty}^n \delta[m] = \sum_{k=0}^{\infty} \delta[n - k]$

Difference Equations(Flow Graphs)

The three building blocks of an LTI system: **multiplication**, **addition**, and **delay**

- **Multiplication(gain)**

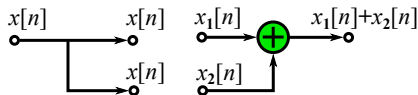
(k can be integer, fraction, negative number...)



- **Split/add(adder)**

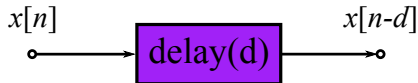
(A signal becomes two **identical** copies)

(Two signals added together)



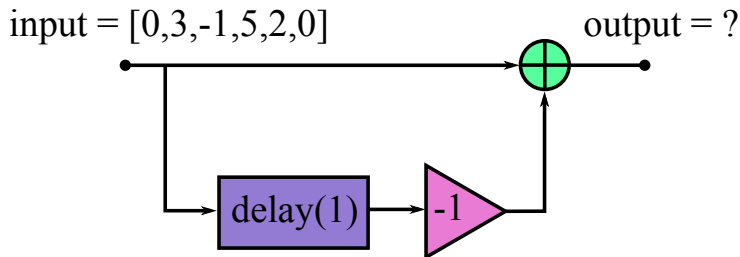
- **Delay**

(A signal is delayed by d **integer** units)



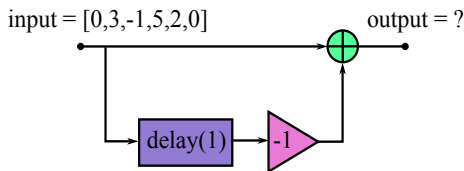
Question 1(a)

- * Find the output of the system?



Solution(Q1(a))

- * Find the output of the system?



Assume: The system has no signal before the input.

$$0 \rightarrow (0) = 0$$

$$3 \rightarrow (3) - 1(0) = 3$$

$$-1 \rightarrow (-1) - 1(3) = -4$$

$$5 \rightarrow (5) - 1(-1) = 6$$

$$2 \rightarrow (2) - 1(5) = -3$$

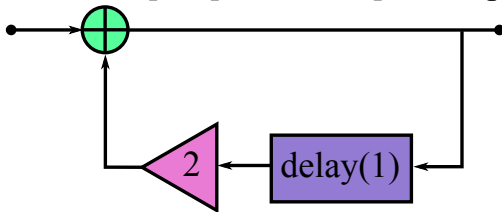
$$0 \rightarrow (0) - 1(2) = -2$$

Hence, output is [0, 3, -4, 4, -3, -2]

Question 1(b)

- * Find the output of the system?

input = $[0, 3, -1, 5, 2, 0]$ or $[1, 0, 0, 0, 0, 0]$ output = ?



Solution(Q1(b))

* Find the output of the system?

Assume: The system has no signal before the input.

$$0 \rightarrow (0) = 0$$

$$3 \rightarrow (3) + 2(0) = 3$$

$$-1 \rightarrow (-1) + 2(3) = 5$$

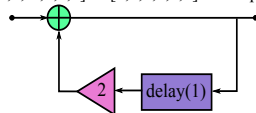
$$5 \rightarrow (5) + 2(5) = 15$$

$$2 \rightarrow (2) + 2(15) = 32$$

$$0 \rightarrow (0) + 2(32) = 64$$

Hence, output is $[0, 3, -4, 4, -3, -2]$

input = $[0, 3, -1, 5, 2, 0]$ or $[1, 0, 0, 0, 0, 0]$ output = ?



$$1 \rightarrow (1) = 1$$

$$0 \rightarrow (0) + 2(1) = 2$$

$$0 \rightarrow (0) + 2(2) = 4$$

$$0 \rightarrow (0) + 2(4) = 8$$

$$0 \rightarrow (0) + 2(8) = 16$$

$$0 \rightarrow (0) + 2(16) = 32$$

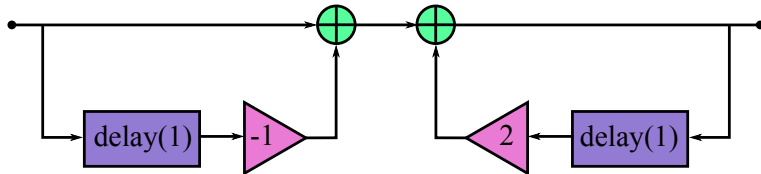
Output is $[1, 2, 4, 8, 16, 32]$

Question 1(c)

- * Find the output of the system?

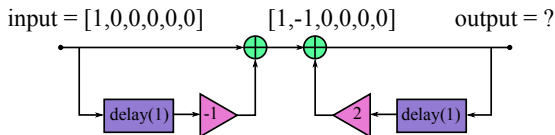
input = $[1, 0, 0, 0, 0, 0]$

output = ?



Solution(Q1(c))

- * Find the output of the system?



Assume: The system has no signal before the input.

$$\begin{aligned}
 1 &\rightarrow (1) &= 1 \\
 0 &\rightarrow (0) - 1(1) &= -1 \\
 0 &\rightarrow (0) - 1(0) &= 0 \\
 0 &\rightarrow (0) - 1(0) &= 0 \\
 0 &\rightarrow (0) - 1(0) &= 0 \\
 0 &\rightarrow (0) - 1(0) &= 0
 \end{aligned}$$

$$\begin{aligned}
 1 &\rightarrow (1) &= 1 \\
 -1 &\rightarrow (-1) + 2(1) &= 1 \\
 0 &\rightarrow (0) + 2(1) &= 2 \\
 0 &\rightarrow (0) + 2(2) &= 4 \\
 0 &\rightarrow (0) + 2(4) &= 8 \\
 0 &\rightarrow (0) + 2(8) &= 16
 \end{aligned}$$

Hence, output is [1, 1, 2, 4, 8, 16]

Question 2(a)

(a) Sketch each of the following input signals

i. $x[n] = \delta[n] + \delta[n - 3]$

ii. $x[n] = u[n] - u[n - 5]$

iii. $x[n] = \delta[n] + \frac{1}{2}\delta[n - 1] + \frac{1}{2} \cdot 2\delta[n - 2] + \frac{1}{2} \cdot 3\delta[n - 3]$

where δ is unit impulse function and u is the unit step function.

Solution(Q2(a))

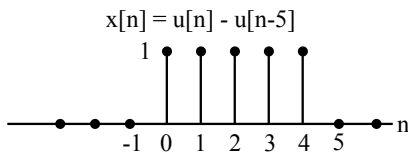
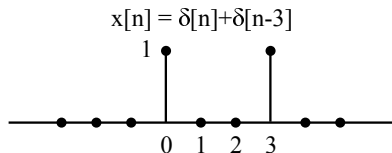
(a) Sketch each of the following input signals

i. $x[n] = \delta[n] + \delta[n-3]$

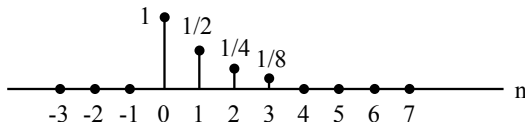
ii. $x[n] = u[n] - u[n-5]$

iii. $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + \frac{1}{2} \cdot 2\delta[n-2] + \frac{1}{2} \cdot 3\delta[n-3]$

where δ is unit impulse function and u is the unit step function.

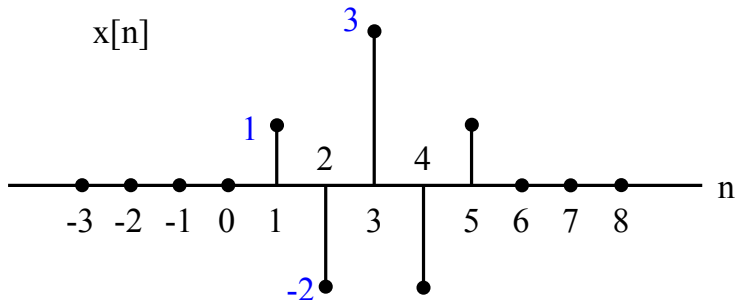


$x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + (\frac{1}{2})^2\delta[n-2] + (\frac{1}{2})^3\delta[n-3]$



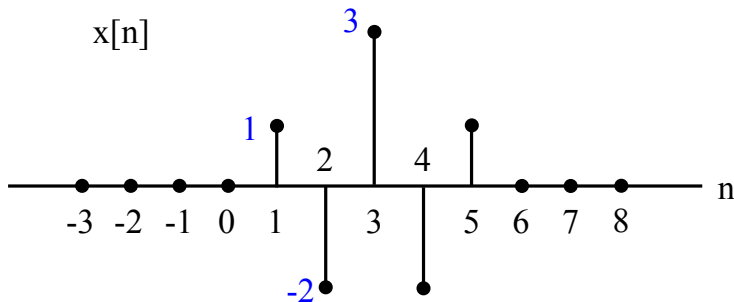
Question 2(b)

- (b) Express the following as sums of weighted delayed impulses, i.e. in the form $x[n] = \sum_{k=-\infty}^{\infty} a_k \delta[n - k]$



Solution(Q2(b))

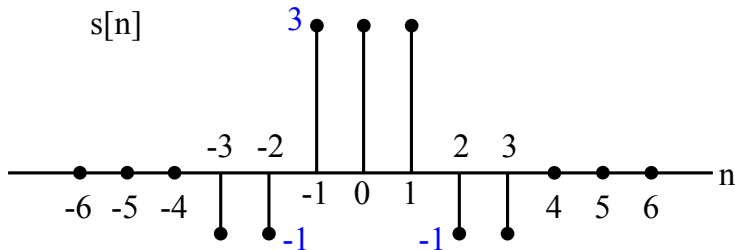
- (b) Express the following as sums of weighted delayed impulses, i.e. in the form $x[n] = \sum_{k=-\infty}^{\infty} a_k \delta[n - k]$



Ans: $x[n] = \delta[n - 1] - 2\delta[n - 2] + 3\delta[n - 3] - 2\delta[n - 4] + \delta[n - 5]$

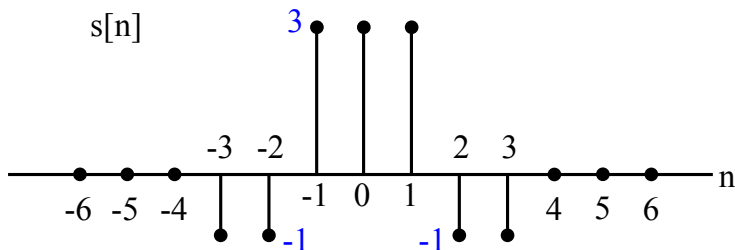
Question 2(c)

- (c) Express the following sequence as sum of unit step function, i.e. in the form $s[n] = \sum_{k=-\infty}^{\infty} a_k u[n - k]$



Solution(Q2(c))

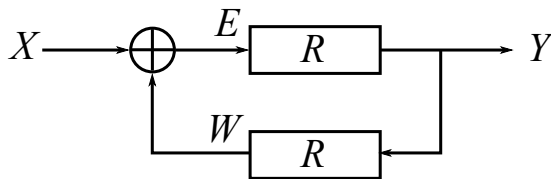
- (c) Express the following sequence as sum of unit step function, i.e. in the form $s[n] = \sum_{k=-\infty}^{\infty} a_k u[n - k]$



Ans: $s[n] = -u[n + 3] + 4u[n + 1] - 4u[n - 2] + u[n - 4]$

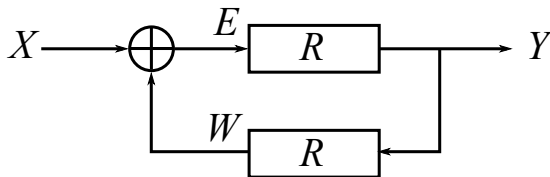
Question 3(a)

- * Determine the difference equation that relates X and Y ? **R: delay(1)**



Solution(Q3(a))

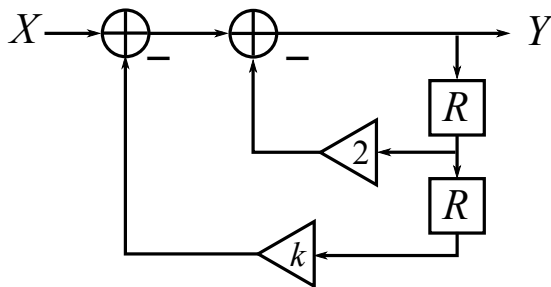
- * Determine the difference equation that relates X and Y ? **R: delay(1)**



- * Difference equation: $y[n] = x[n - 1] + y[n - 2]$

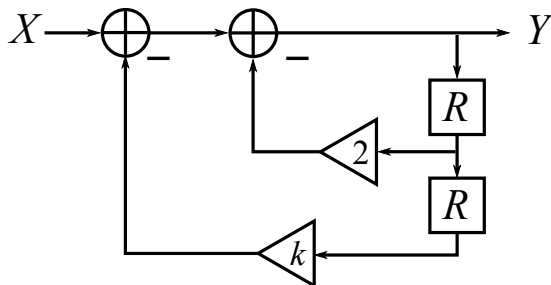
Question 3(b)

- * Determine the difference equation that relates X and Y ? **R: delay(1)**



Solution(Q3(b))

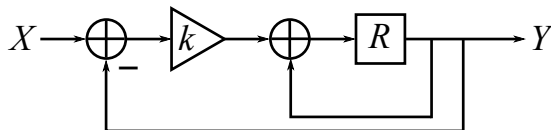
- * Determine the difference equation that relates X and Y ? **R: delay(1)**



- * Difference equation: $y[n] = x[n] - 2y[n-1] - ky[n-2]$

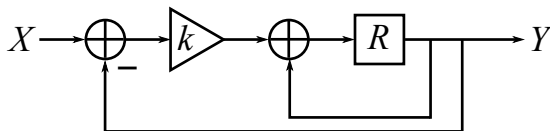
Question 3(c)

- * Determine the difference equation that relates X and Y ? **R: delay(1)**



Solution(Q3(c))

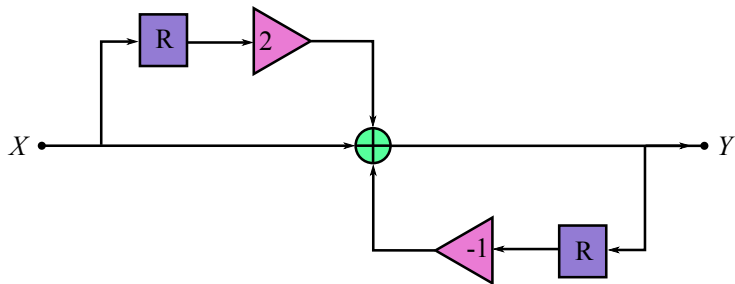
- * Determine the difference equation that relates X and Y ? **R: delay(1)**



- * Difference equation: $y[n] = y[n - 1] - kx[n - 1] - ky[n - 1]$

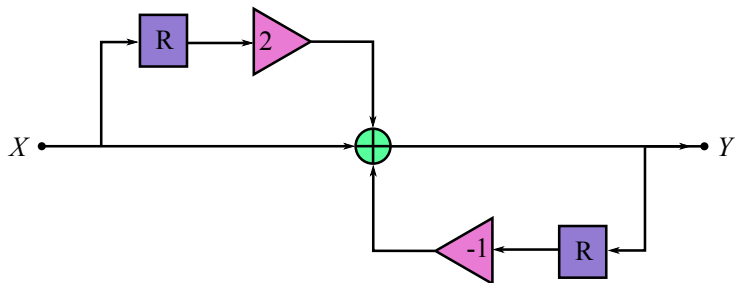
Question 3(d)

- * Determine the difference equation that relates X and Y ? **R: delay(1)**



Solution(Q3(d))

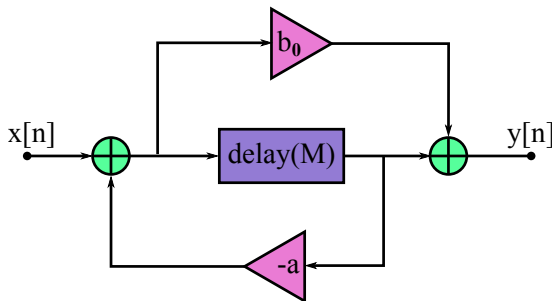
- * Determine the difference equation that relates X and Y? **R: delay(1)**



- * Difference equation: $y[n] = x[n] + 2x[n-1] - y[n-1]$

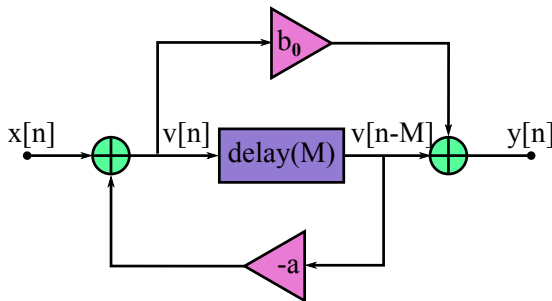
Question 3(e)

- * Determine the difference equation that relates X and Y ? **R: delay(1)**



Solution(Q3(e))

- * Determine the difference equation that relates X and Y? **R: delay(1)**



- * **Difference equation:**

$$\begin{aligned} v[n] &= x[n] - av[n - M]; \quad y[n] = b_0 v[n] + v[n - M] \\ \therefore y[n] &= b_0 \{x[n] - av[n - M]\} + x[n - M] - av[n - 2M] \\ &= b_0 x[n] + x[n - M] - a \{b_0 v[n - M] + v[n - 2M]\} \\ &= b_0 x[n] + x[n - M] - ay[n - M] \end{aligned}$$

Question 4

[SP13 Final Exam] Consider the difference equation $y[n] = y[n-1] + k \cdot y[n-2] + x[n]$, where $x[n]$ is an impulse input. For what value(s) of k indicated below would the output converge to zero as n increases?

- i $k = 0$
- ii $k = -\frac{1}{2}$
- iii $k = -1$
- iv $k = -\frac{1}{2}$ and $k = 0$
- v $k = -1$, $k = -\frac{1}{2}$, and $k = 0$

Solution(Q4)

Difference equation: $y[n] = y[n-1] + k \cdot y[n-2] + x[n]$

Impulse input $x[n] : x[n] = \delta[n] = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{otherwise} \end{cases}$

For $k = 0$ ($y[n] = y[n-1] + x[n]$)

n	$x[n]$	$y[n-1]$	+	$x[n]$	=	$y[n]$
0	1	0	+	1	=	1
1	0	1	+	0	=	1
2	0	1	+	0	=	1
3	0	1	+	0	=	1
4	0	1	+	0	=	1
5	0	1	+	0	=	1

Solution(Q4)

For $k = -\frac{1}{2}$ ($y[n] = y[n-1] - \frac{1}{2}y[n-2] + x[n]$)

n	$x[n]$	$y[n-1]$	$-$	$\frac{1}{2}y[n-2]$	$+$	$x[n]$	$=$	$y[n]$
0	1	0	$-$	$\frac{1}{2}(0)$	$+$	1	$=$	1
1	0	1	$-$	$\frac{1}{2}(0)$	$+$	0	$=$	1
2	0	1	$-$	$\frac{1}{2}(1)$	$+$	0	$=$	$\frac{1}{2}$
3	0	$\frac{1}{2}$	$-$	$\frac{1}{2}(1)$	$+$	0	$=$	0
4	0	0	$-$	$\frac{1}{2}(\frac{1}{2})$	$+$	0	$=$	$-\frac{1}{4}$
5	0	$-\frac{1}{4}$	$-$	$\frac{1}{2}(0)$	$+$	0	$=$	$-\frac{1}{4}$
6	0	$-\frac{1}{4}$	$-$	$\frac{1}{2}(-\frac{1}{4})$	$+$	0	$=$	$-\frac{1}{8}$

Solution(Q4)

For $k = -\frac{1}{2}$ ($y[n] = y[n-1] - \frac{1}{2}y[n-2] + x[n]$)

n	$x[n]$	$y[n-1]$	$-$	$\frac{1}{2}y[n-2]$	$+$	$x[n]$	$=$	$y[n]$
7	0	$\frac{-1}{8}$	$-$	$\frac{1}{2}(\frac{-1}{4})$	$+$	0	$=$	0
8	0	0	$-$	$\frac{1}{2}(\frac{-1}{8})$	$+$	0	$=$	$\frac{1}{16}$
9	0	$\frac{1}{16}$	$-$	$\frac{1}{2}(0)$	$+$	0	$=$	$\frac{1}{16}$
10	0	$\frac{1}{16}$	$-$	$\frac{1}{2}(\frac{1}{16})$	$+$	0	$=$	$\frac{1}{32}$
11	0	$\frac{1}{32}$	$-$	$\frac{1}{2}(\frac{1}{16})$	$+$	0	$=$	0
12	0	0	$-$	$\frac{1}{2}(\frac{1}{36})$	$+$	0	$=$	$\frac{-1}{72}$
13	0	$\frac{-1}{72}$	$-$	$\frac{1}{2}(0)$	$+$	0	$=$	$\frac{-1}{72}$

Solution(Q4)

For $k = -1$ ($y[n] = y[n-1] - y[n-2] + x[n]$)

n	$x[n]$	$y[n-1]$	$-$	$\frac{1}{2}y[n-2]$	$+$	$x[n]$	$=$	$y[n]$
0	1	0	$-$	(0)	$+$	1	$=$	1
1	0	1	$-$	(0)	$+$	0	$=$	1
2	0	1	$-$	(1)	$+$	0	$=$	0
3	0	0	$-$	(1)	$+$	0	$=$	-1
4	0	-1	$-$	(0)	$+$	0	$=$	-1
5	0	-1	$-$	(-1)	$+$	0	$=$	0
6	0	0	$-$	(-1)	$+$	0	$=$	1

Solution(Q4)

For $k = -1$ ($y[n] = y[n-1] - y[n-2] + x[n]$)

n	$x[n]$	$y[n-1]$	$-$	$\frac{1}{2}y[n-2]$	$+$	$x[n]$	$=$	$y[n]$
7	0	1	$-$	(0)	$+$	0	$=$	1
8	0	1	$-$	(1)	$+$	0	$=$	0
9	0	0	$-$	(1)	$+$	0	$=$	-1
10	0	-1	$-$	(0)	$+$	0	$=$	-1
11	0	-1	$-$	(-1)	$+$	0	$=$	0
12	0	0	$-$	(-1)	$+$	0	$=$	1
13	0	1	$-$	(0)	$+$	0	$=$	1

Question 5(a)

[FA12 Final Exam] Consider the difference equation

$$y[n] = k \cdot y[n-1] + k \cdot y[n-2] + x[n].$$

Assume $x[n]$ is an impulse input, i.e. $x[0] = 1$ and $x[n] = 0$ for other values of n , and that $y[n] = 0$ for $n < 0$.

(a) Let $k = 1$. What is the value of $y[10]$?

- (i) 2
- (ii) 1
- (iii) 0
- (iv) -1
- (v) -2

Solution(Q5(a))

$$k = 1, y[10] = ?$$

n	$x[n]$	$y[n]$	$=$	$y[n-1]$	$-$	$y[n-2]$	$+$	$x[n]$
0	1	1	$=$	0	$-$	0	$+$	1
1	0	1	$=$	1	$-$	0	$+$	0
2	0	0	$=$	1	$-$	1	$+$	0
3	0	-1	$=$	0	$-$	1	$+$	0
4	0	-1	$=$	-1	$-$	0	$+$	0
5	0	0	$=$	-1	$-$	-1	$+$	0

Solution(Q5(a))

$$k = 1, y[10] = ?$$

n	$x[n]$	$y[n]$	$=$	$y[n-1]$	$-$	$y[n-2]$	$+$	$x[n]$
6	0	1	$=$	0	$-$	-1	$+$	0
7	0	1	$=$	1	$-$	0	$+$	0
8	0	0	$=$	1	$-$	1	$+$	0
9	0	-1	$=$	0	$-$	1	$+$	0
10	0	-1	$=$	-1	$-$	0	$+$	0

Question 5(b)

[FA12 Final Exam] Consider the difference equation

$$y[n] = k \cdot y[n-1] + k \cdot y[n-2] + x[n].$$

Assume $x[n]$ is an impulse input, i.e. $x[0] = 1$ and $x[n] = 0$ for other values of n , and that $y[n] = 0$ for $n < 0$.

(b) Let $k = -1$. What is the value of $y[10]$?

- (i) 34
- (ii) -34
- (iii) 55
- (iv) -55
- (v) 89

Solution(Q5(b))

$$k = -1, y[10] = ?$$

n	$x[n]$	$y[n]$	$=$	$-y[n-1]$	$+$	$y[n-2]$	$+$	$x[n]$
0	1	1	$=$	$-(0)$	$+$	0	$+$	1
1	0	-1	$=$	$-(1)$	$+$	0	$+$	0
2	0	2	$=$	$-(-1)$	$+$	1	$+$	0
3	0	-3	$=$	$-(2)$	$+$	-1	$+$	0
4	0	5	$=$	$-(-3)$	$+$	2	$+$	0
5	0	-8	$=$	$-(5)$	$+$	-3	$+$	0

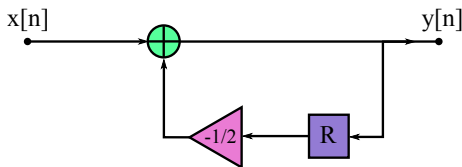
Solution(Q5(b))

$$k = -1, y[10] = ?$$

n	$x[n]$	$y[n]$	$=$	$-y[n-1]$	$+$	$y[n-2]$	$+$	$x[n]$
6	0	13	$=$	$-(-8)$	$+$	5	$+$	0
7	0	-21	$=$	$-(13)$	$+$	-8	$+$	0
8	0	34	$=$	$-(-21)$	$+$	13	$+$	0
9	0	-55	$=$	$-(34)$	$+$	-21	$+$	0
10	0	89	$=$	$-(-55)$	$+$	34	$+$	0

Question 6

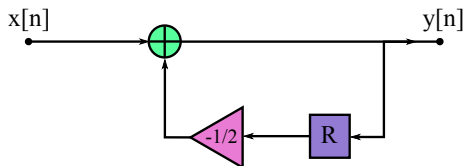
- * Consider the block diagram relating the two signal $x[n]$ and $y[n]$ given in figure. **R: delay(1)**



- (a) Determine the difference equation relating $y[n]$ and $x[n]$.
- (b) Assume that a solution to the difference equation in part (a) is given by $y[n] = k\alpha^n u[n]$, where $u[n]$ is unit step function and $x[n] = \delta[n]$. Find the appropriate value of k and α , and verify that $y[n]$ satisfies the difference equation.
- (c) Verify your answer to part (b) by directly calculating $y[0]$, $y[1]$, and $y[2]$.

Solution(Q6(a))

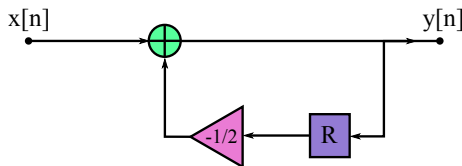
(a) Determine the difference equation relating $y[n]$ and $x[n]$.



- * Thus, $y[n] = x[n] - \frac{1}{2}y[n - 1]$
- * or $y[n] + \frac{1}{2}y[n - 1] = x[n]$

Solution(Q6(b))

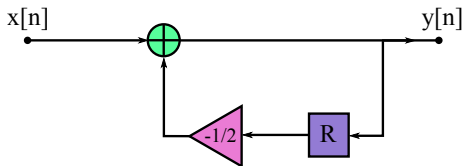
- (b) Assume that a solution to the difference equation in part (a) is given by $y[n] = k\alpha^n u[n]$, where $u[n]$ is unit step function and $x[n] = \delta[n]$. Find the appropriate value of k and α , and verify that $y[n]$ satisfies the difference equation.



- * For $n < 0$, $x[n] = \delta[n] = 0 \therefore y[n] = 0$
- * For $n = 0$, $y[0] + \frac{1}{2}y[-1] = x[0]$
- * Substituting $y[n] = k\alpha^n u[n]$
- * $k\alpha^0 u[0] + \frac{1}{2}k\alpha^{-1} u[-1] = 1$
- * $k(1)(1) + \frac{1}{2}k\alpha^{-1}(0) = 1 \Rightarrow k = 1$

Solution(Q6(b))

- (b) Assume that a solution to the difference equation in part (a) is given by $y[n] = k\alpha^n u[n]$, where $u[n]$ is unit step function and $x[n] = \delta[n]$. Find the appropriate value of k and α , and verify that $y[n]$ satisfies the difference equation.



- * For $n > 0$, $y[n] + \frac{1}{2}y[n-1] = x[n]$
- * $k\alpha^n u[n] + \frac{1}{2}k\alpha^{n-1}u[n-1] = 0$
- * $(1)(\alpha^n)(1) + \frac{1}{2}(1)\alpha^{n-1}(1) = 0$
- * $\alpha^n + \frac{1}{2}\alpha^{n-1} = 0 \Rightarrow \alpha = -\frac{1}{2}$

Solution(Q6(b))

(b) The difference equation: $y[n] + \frac{1}{2}y[n-1] = x[n]$

For $k = 1$, $\alpha = -\frac{1}{2}$, $y[n] = (-\frac{1}{2})^n u[n]$

Substituting into the left side of the difference equation,

We have

$$y[n] + \frac{1}{2}y[n-1] = (-\frac{1}{2})^n u[n] + \frac{1}{2}(-\frac{1}{2})^{n-1} u[n-1]$$

$$= (-\frac{1}{2})^n u[n] - (-\frac{1}{2})^n u[n-1]$$

$$= \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases} = \delta[n] = x[n] \quad \text{Verified.}$$

Solution(Q6)

- (c) We can successively calculate $y[n]$ by noting that $y[-1] = 0$ and that
- $$y[n] = -\frac{1}{2}y[n-1] + x[n]$$
- so

$$n = 0, y[0] = -\frac{1}{2} \cdot 0 + 1 = 1$$

$$n = 1, y[1] = -\frac{1}{2} \cdot 1 + 0 = -\frac{1}{2}$$

$$n = 2, y[2] = -\frac{1}{2} \cdot \left(-\frac{1}{2}\right) + 0 = \frac{1}{4}$$

Summary

- Building blocks
 - Three Building Blocks(**analyze signals one by one**)
 - Flow Graph Transformations
- Difference Equations
 - Conventions(**signals out of range is NULL or 0**)
 - Two Special Discrete-time signals($\delta[n]$ and $u[n]$)
 - Flow Graphs

The End