

Tutorial 6 Questions

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Overview

* **Learning Objectives:**

- Signal Flow Graph
- Difference Equations

* **Basics**

- Building blocks of an LTI system
 - Three Building Blocks
 - Flow Graph Transformations
- Difference Equations
 - Conventions
 - Two Special Discrete-time signals
 - Flow Graphs

* **Questions & Summary**

Building blocks(Three Building Blocks)

The three building blocks of an LTI system: **multiplication**, **addition**, and **delay**

- **Multiplication(gain)**

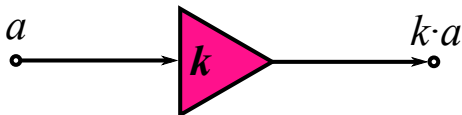


Figure : Output equals to the input with a gain k

- **Split/add(adder)**

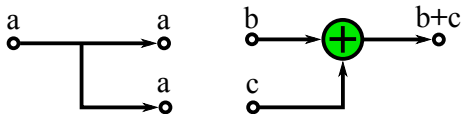


Figure : On the left, a signal is split into two paths. On the right, two signals are added together

Building blocks(Three Building Blocks)

The three building blocks of an LTI system: **multiplication**, **addition**, and **delay**

- **Delay**

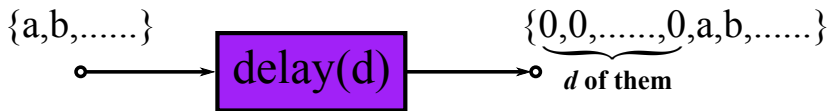


Figure : Output equals to input with a delay of d time units

Building blocks(Flow Graph Transformations)

Intuitively, some changes to the flow graphs are permitted:

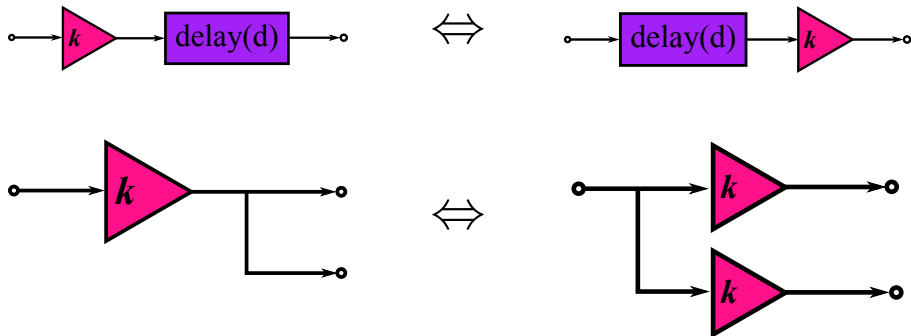


Figure : Some examples of flow graph transformations

Difference Equations(Expression & Conventions)

Expression:

- $y[n] = a_1y[n-1] + a_2y[n-2] + \dots + b_0x[n] + b_1x[n-1] + \dots$

Conventions:

- Signal: $x[n]$ (square bracket)
- Use $x[n]$ for an input signal, $y[n]$ for an output signal
- Often $n = 0, 1, \dots, N-1$ (integer) for a length- N signal. We may also have an “infinite” length signal where n can be any nonnegative integers.
- Assume $x[n] = 0$ outside this range.
 \Rightarrow No input, no output. System is “at rest”

Difference Equations(Two Special Discrete-time signals)

Impulse Signal(delta functions): $\delta[n]$

- $$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise}(n \neq 0) \end{cases}$$

This is called an impulse because it is active only at the first time instance, and then it returns to zero and stays there forever

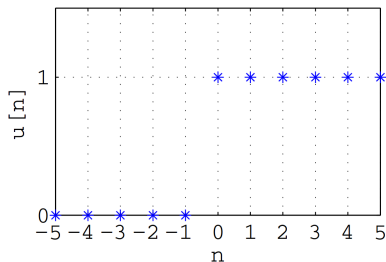
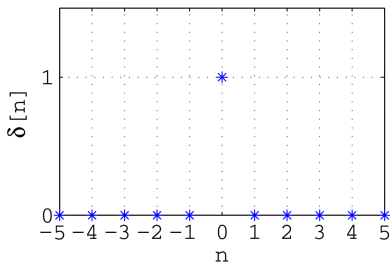
Unit Step Functions: $u[n]$

- $$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise}(n < 0) \end{cases}$$

Notice that because we have assumed that all signals with negative indices are zero, the unit step appears to be equal to 1 all the time

Difference Equations(Two Special Discrete-time signals)

Relation of these two signals: $\delta[n]$ and $u[n]$



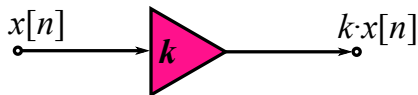
- $\delta[n] = u[n] - u[n - 1]$
- $u[n] = \sum_{m=-\infty}^n \delta[m] = \sum_{k=0}^{\infty} \delta[n - k]$

Difference Equations(Flow Graphs)

The three building blocks of an LTI system: **multiplication**, **addition**, and **delay**

- **Multiplication(gain)**

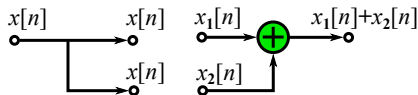
(k can be integer, fraction, negative number...)



- **Split/add(adder)**

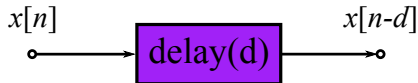
(A signal becomes two **identical** copies)

(Two signals added together)



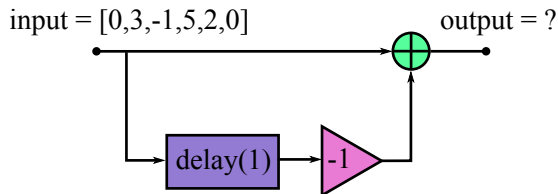
- **Delay**

(A signal is delayed by d **integer** units)



Question 1(a)

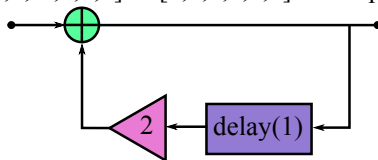
- * Find the output of the system?



Question 1(b)

- * Find the output of the system?

input = $[0, 3, -1, 5, 2, 0]$ or $[1, 0, 0, 0, 0, 0]$ output = ?

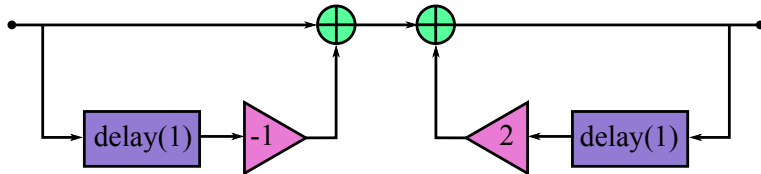


Question 1(c)

- * Find the output of the system?

input = $[1, 0, 0, 0, 0, 0]$

output = ?



Question 2(a)

(a) Sketch each of the following input signals

i. $x[n] = \delta[n] + \delta[n - 3]$

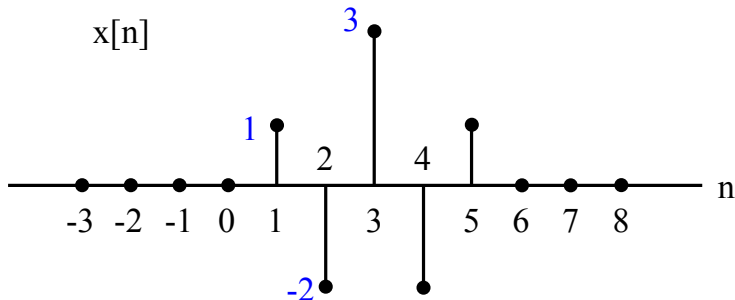
ii. $x[n] = u[n] - u[n - 5]$

iii. $x[n] = \delta[n] + \frac{1}{2}\delta[n - 1] + \frac{1}{2} \cdot 2\delta[n - 2] + \frac{1}{2} \cdot 3\delta[n - 3]$

where δ is unit impulse function and u is the unit step function.

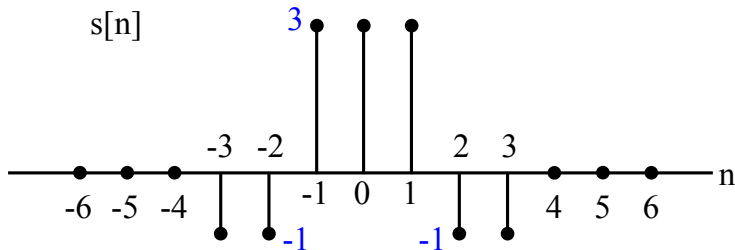
Question 2(b)

- (b) Express the following as sums of weighted delayed impulses, i.e. in the form $x[n] = \sum_{k=-\infty}^{\infty} a_k \delta[n - k]$



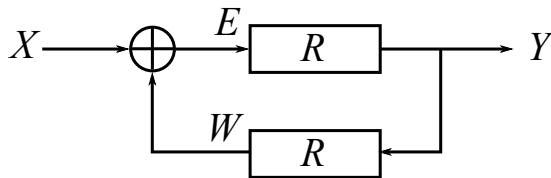
Question 2(c)

- (c) Express the following sequence as sum of unit step function, i.e. in the form $s[n] = \sum_{k=-\infty}^{\infty} a_k u[n - k]$



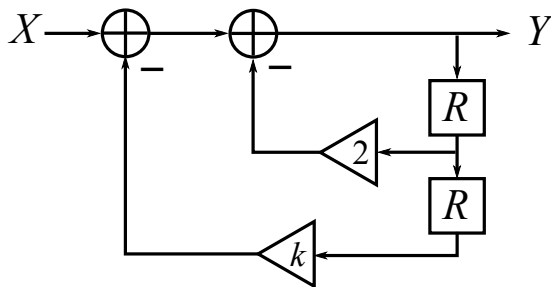
Question 3(a)

- * Determine the difference equation that relates X and Y ? **R: delay(1)**



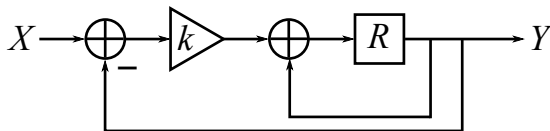
Question 3(b)

- * Determine the difference equation that relates X and Y ? **R: delay(1)**



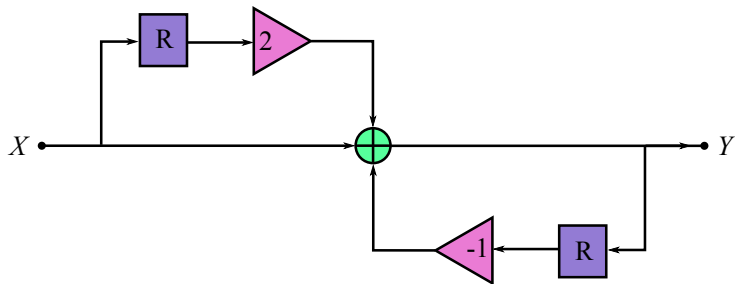
Question 3(c)

- * Determine the difference equation that relates X and Y ? **R: delay(1)**



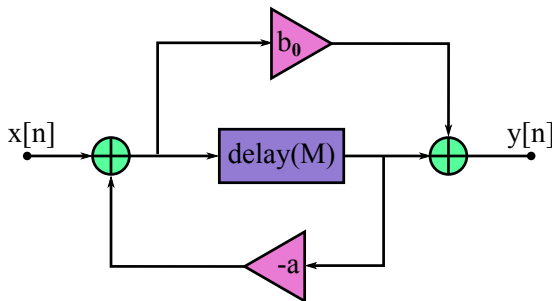
Question 3(d)

- * Determine the difference equation that relates X and Y ? **R: delay(1)**



Question 3(e)

- * Determine the difference equation that relates X and Y ? **R: delay(1)**



Question 4

[SP13 Final Exam] Consider the difference equation $y[n] = y[n-1] + k \cdot y[n-2] + x[n]$, where $x[n]$ is an impulse input. For what value(s) of k indicated below would the output converge to zero as n increases?

- i $k = 0$
- ii $k = -\frac{1}{2}$
- iii $k = -1$
- iv $k = -\frac{1}{2}$ and $k = 0$
- v $k = -1$, $k = -\frac{1}{2}$, and $k = 0$

Question 5(a)

[FA12 Final Exam] Consider the difference equation

$$y[n] = k \cdot y[n-1] + k \cdot y[n-2] + x[n].$$

Assume $x[n]$ is an impulse input, i.e. $x[0] = 1$ and $x[n] = 0$ for other values of n , and that $y[n] = 0$ for $n < 0$.

(a) Let $k = 1$. What is the value of $y[10]$?

- (i) 2
- (ii) 1
- (iii) 0
- (iv) -1
- (v) -2

Question 5(b)

[FA12 Final Exam] Consider the difference equation

$$y[n] = k \cdot y[n-1] + k \cdot y[n-2] + x[n].$$

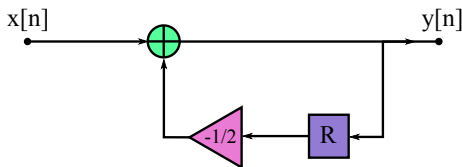
Assume $x[n]$ is an impulse input, i.e. $x[0] = 1$ and $x[n] = 0$ for other values of n , and that $y[n] = 0$ for $n < 0$.

(b) Let $k = -1$. What is the value of $y[10]$?

- (i) 34
- (ii) -34
- (iii) 55
- (iv) -55
- (v) 89

Question 6

- * Consider the block diagram relating the two signal $x[n]$ and $y[n]$ given in figure. **R: delay(1)**



- (a) Determine the difference equation relating $y[n]$ and $x[n]$.
- (b) Assume that a solution to the difference equation in part (a) is given by $y[n] = k\alpha^n u[n]$, where $u[n]$ is unit step function and $x[n] = \delta[n]$. Find the appropriate value of k and α , and verify that $y[n]$ satisfies the difference equation.
- (c) Verify your answer to part (b) by directly calculating $y[0]$, $y[1]$, and $y[2]$.