ENGG1203: Introduction to Electrical and Electronic Engineering

Circuits (ENGG1203)

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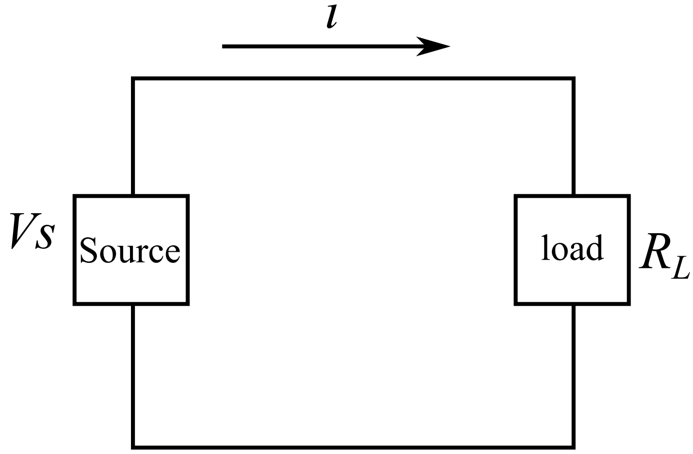
1 Basics

We use a simple model as a starting point to discuss electronic circuits, which is shown on Figure 1.

FIGURE 1 – Fundamental circuit model



A basic circuit is made up of a source which provides voltage across its terminals, denoted by V.

1.1 Current

The current i results from the stream of electric charge around the closed loop shown on Figure 1. The mathematic definition is equal to the amount of charge, Q, passing through a cross-section per second and it can be expressed as

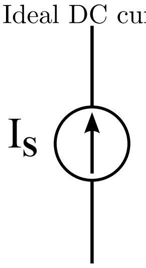
$$i = \frac{dQ}{dt} \tag{1.1}$$

The unit of charge is the Coulomb. One Coulomb is equivalent to 6.24×10^{18} electrons. The unit for current is the ampere, A. One ampere = 1 Coulomb/sec.

1.1.1 Ideal DC current source

The current source is a device that can provide a certain amount of current to a circuit. The symbol for a DC current source and the i / v characteristic curve of an ideal current source are shown on Figure 2.

FIGURE 2 – Ideal DC current source



1.2 Voltage

Moving electrons along a conductor requires some amount of work that must be somehow supplied by an electromotive force provided by a battery or similar device. The electromotive force is the potential difference(voltage) between two points or across a component in circuit. The mathematical definition of voltage is given by

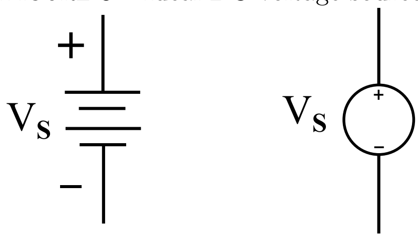
$$v = \frac{dW}{dQ} \tag{1.2}$$

where work(W) is measured in Joules and charge(Q) in Coulombs. Basically, the voltage is measured in volts(V) and $1volt = 1\frac{Joule}{Coulomb} = 1\frac{Newton\ meter}{Ampere\ second}$

1.3 Ideal DC voltage sources

The voltage provided by a voltage source(usually battery) is constant in time and thus it is called DC voltage. In its ideal implementation the battery provides a specific voltage at all times and for all loads. The symbols of an ideal DC voltage source are shown as follows :

Figure 3 – Ideal DC voltage source



1.4 Ideal resistor

The ideal resistor is a passive, linear, two-terminal device whose resistance follows Ohm's law given by,

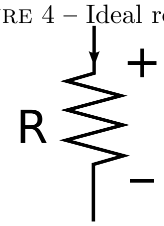
$$v = iR, \tag{1.3}$$

which states that the voltage across an element is directly proportional to the current flowing through the element. The constant of proportionality is the resistance R provided by the element. The resistance is measured in Ohms, Ω, and

$$1\Omega = 1\frac{V}{A}, \tag{1.4}$$

the symbol for a resistor is,

Figure 4 – Ideal resistor



1.5 Resistor Colour Code

The coloured painted bands produce a system of identification generally known as a **Resistors Colour Code**. The resistor colour code markings are always read one band at a time starting from the left to the right, with the larger width tolerance band oriented to the right side indicating its tolerance. By matching the colour of the first band with its associated number in the digit column of the colour chart below the first digit is identified and this represents the first digit of the resistance value.

Again, by matching the colour of the second band with its associated number in the digit column of the colour chart we get the second digit of the resistance value and so on. Then the resistor colour code is read from left to right as illustrated below

FIGURE 5 – Resistor Colour Code

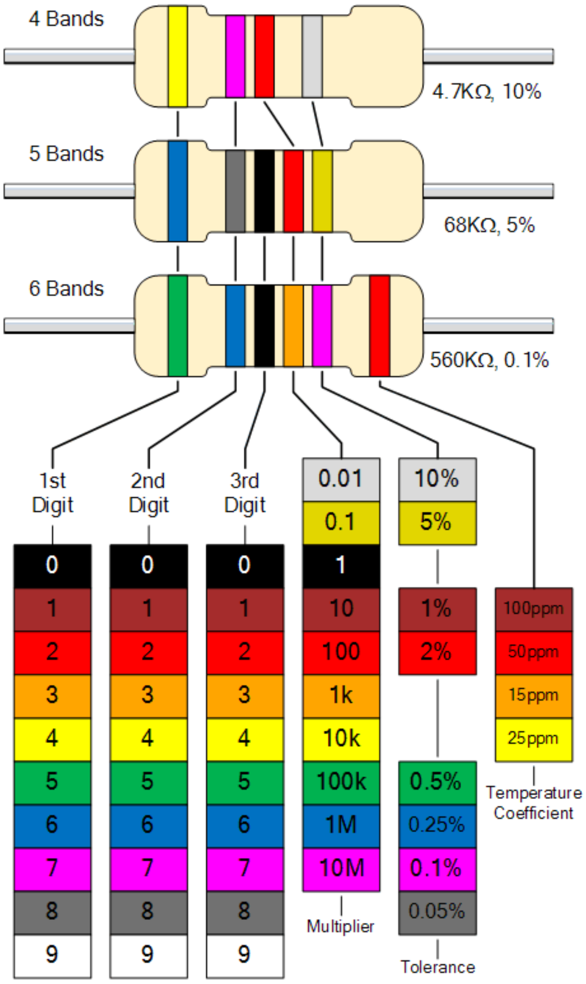


FIGURE 6 – Colour Code Table

Colour	Digit	Multiplier	Tolerance
Black	0	1	
Brown	1	10	± 1%
Red	2	100	± 2%
Orange	3	1,000	
Yellow	4	10,000	
Green	5	100,000	± 0.5%
Blue	6	1,000,000	± 0.25%
Violet	7	10,000,000	± 0.1%
Grey	8		± 0.05%
White	9		
Gold		0.1	± 5%
Silver		0.01	± 10%
None			± 20%

For example, a resistor has the following colored markings :

$$\text{Yellow Violet Red} = 4\ 7\ 2 = 4\ 7\times\ 10_2 = 4700\Omega \text{ or } 4k7.$$

1.6 Power

When current flows through a resistor, energy is irreversibly lost(or we say dissipated) in overcoming the resistance. THe dissipated power shows up as heat most of the time. The mathematical definition of power is the rate as which energy is delivered that can be expressed as

$$P = \frac{dW}{dt}, \tag{1.5}$$

The units for power are $Joules/sec$ or Watts, W. ($1Joules/sec = 1W$) Power can be related to voltage and current by rewriting Eq.(1.5) as,

$$P = \frac{dW}{dt} = \frac{dW}{dQ}\frac{dQ}{dt} = vi, \tag{1.6}$$

Then substitute Ohm‘s law in Eq.(1.6), we will get the power dissipated in a resistor of resistance R is a non-linear function of either i or v.

$$P = i^2 R \quad or \quad P = \frac{v^2}{R} \tag{1.7}$$

Power rating is a fundamental constraint of resistors and electronic devices in general. The power rating is referred to the maximum power that the device can dissipate without adversely affecting its operation. When the power rating is exceeded the resistor overheats and it is destroyed by burning up.

2 Kirchhoffs Laws

Kirchhoffs laws also known as Kirchhoffs Current Law (KCL) and Kirchhoffs Voltage Law (KVL) are based respectively on the conservation of charge and the conservation of energy and are derived from Maxwells equations. They along with Ohms law present the fundamental tools for circuit analysis.

2.1 Kirchhoffs Current Law

The current flowing out of any node in a circuit must be equal to the current flowing into the node. It is expressed mathematically as

$$\sum_{n=1}^{N} i_n = 0 \tag{2.1}$$

where N is the number of branches that are connected to the node. Consider the node shown on Figure 9. By adopting the sign convention that current flowing ito a node is positive (+) and current flowing out of the node is negative (-), application of KCL gives

2.2 Kirchhoffs Voltage Law

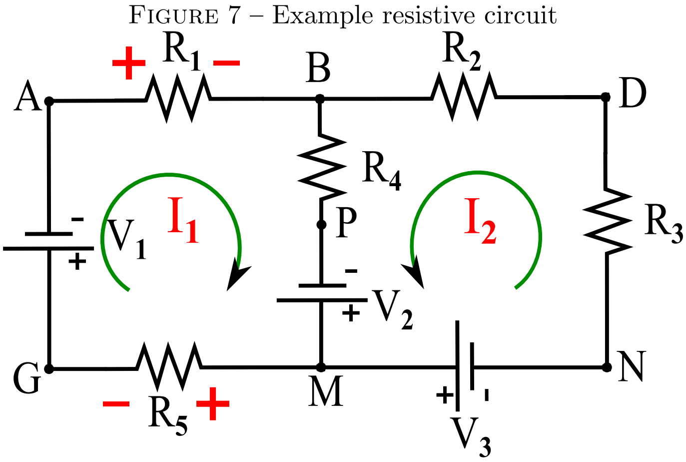
The algebraic sum of voltages around a closed loop is zero. It is expressed mathematically as

$$\sum_{n=1}^{N} v_n = 0 \tag{2.2}$$

where N is the number of voltages in the loop. The number of voltages is equal to the number of elements encountered as we go around the loop.

Next we will solve for the circuit shown on Figure 7

FIGURE 7 – Example resistive circuit



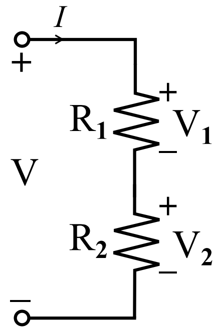
$R_1 = 80\Omega,\ R_2 = 10\Omega,\ R_3 = 20\Omega,\ R_4 = 90\Omega,\ R_5 = 100\Omega,\ V_1 = 12V,\ V_2 = 24V,\ V_3 = 36V$

Apply Kirchhoffs laws.

- Start with the nodes.
  1. The middle node P is already accounted for since we assigned the current above and below it the same value, I_1. This is just Kirchhoffs current law which says that the current going into a node is equal to that going out.
  2. The bottom and top nodes B and M are exactly the same and KCL for them is
  $$I - I_1 + I_2 = 0$$
  since at node B current I and I_2 flow in the node B and I_1 flow out the node B.
- Next let's apply KVL to the three loops
  1. **Loop 1(I_1 loop) :** voltage source V_1, V_2 and resistors R_1, R_4, R_5
  $$V_1 + I_1R_1 + (I_1 + I_2)R_4 - V_2 + I_1R_5 = 0$$
  $$\Rightarrow 12 + 80I_1 + 90I_1 + 90I_2 - 24 + 100I_1 = 0$$
  $$\Rightarrow 270I_1 + 90I_2 = 12$$
  2. **Loop 2(I_2 loop) :** voltage source V_2, V_3 and resistors R_2, R_3, R_4
  $$V_3 + I_2R_3 + I_2R_2 + (I_1 + I_2)R_4 - V_2 = 0$$
  $$\Rightarrow 36 + 20I_2 + 10I_2 + 90I_1 + 90I_2 - 24 = 0$$
  $$\Rightarrow 90I_1 + 120I_2 = -12$$
- Calculate the required parameters using Ohm‘s law and relevant formulas.
$$I_1 = \frac{14}{135}A$$
$$I_2 = -\frac{8}{45}A$$

3 Voltage divider : Series Connection of Resistors

The voltage divider circuit is the most convenient means of passively stepping down the voltage from a fixed voltage source.



$I = \frac{V}{R_1+R_2}$
$V_1 = R_1 I = \frac{R_1}{R_1+R_2} V$
$V_2 = R_2 I = \frac{R_2}{R_1+R_2} V$

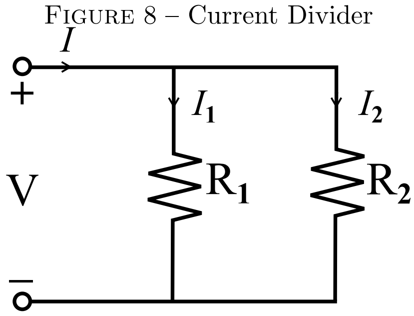
By considering a circuit where we are using N resistors connected in series, we can show that the equivalent resistance is the sum of the resistances.

$$R_{eq} = \sum_{n=1}^{N} R_n = R_1 + R_2 + ... + R_N \tag{3.1}$$

4 Current Divider : Parallel connection of resistors

The schematic on Figure 8 shows a simple current divider circuit. Here the two resistors R_1 and R_2 are connected in parallel. Lets determine the current I_1 and I_2 flowing in the two resistors.

FIGURE 8 – Current Divider



$V = (R_1 \parallel R_2) I$
$I_1 = \frac{V}{R_1} = \frac{R_1 \| R_2}{R_1} I = \frac{R_2}{R_1+R_2} I$
$I_2 = \frac{R_1}{R_1+R_2} I$

For N resistors connected in parallel the equivalent resistance is

$$\frac{1}{R_{eq}} = \sum_{n=1}^{N} \frac{1}{R_n} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N} \tag{4.1}$$

Note that the equivalent resistance R_{eq} is smaller than the smallest resistance in the parallel arrangement.

The equivalent conductance of N resistors connected in parallel is

$$G_{eq} = \sum_{n=1}^{N} G_n = G_1 + G_2 + ... + G_N \tag{4.2}$$

where $G_n = \frac{1}{R_n}$. The current flowing through resistor R_n is

$$i_n = i \frac{G_n}{G_{eq}} \tag{4.3}$$

5 Appendix

5.1 Node Method & Mesh Method

With the help of Kirchhoff's laws and Ohm's law, we can analyze any circuit to determine the currents and voltages. For formal circuit analysis, the challenge is to derive the smallest set of simultaneous equations that completely define the operating characteristics of a circuit.

Therefore, another two methods have been developed, namely node method and mesh method. We will explain the steps to solve the circuit problem shown on Figure 2.

The Node Method.

Note : Voltage is defined as the potential difference between two points. When we talk about the voltage at a certain point, we imply that the measurement is performed between that point and some other point in the circuit.

The node method or node voltage method is based on the application of KCL, KVL and Ohm's law. The following part list steps used to analysis a circuit with the node method.

1. Label all the circuit parameters and distinguish the unknown parameters from the known ones.
2. Identify all nodes of the circuit
3. Choose a reference node(Ground) and assign to it a potential of $0V$. All other voltages in the circuit are measured with respect to the reference node.
4. Label the voltage at all other nodes.
5. Assign and label polarities.
6. Apply KCL at each node and express the branch currents in terms of the node voltages
7. Solve the resulting simultaneous equations for the node voltages.
8. Obtain the branch currents by Ohm's law.