Tutorial 6 Questions

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Overview

* **Learning Objectives:**
  * Signal Flow Graph
  * Difference Equations
* **Basics**
  * Building blocks of an LTI system
    * Three Building Blocks
    * Flow Graph Transformations
  * Difference Equations
    * Conventions
    * Two Special Discrete-time signals
    * Flow Graphs
* **Questions & Summary**

Building blocks(Three Building Blocks)

The three building blocks of an LTI system: **multiplication**, **addition**, and **delay**

- **Multiplication(gain)**

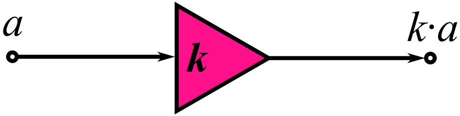


Figure : **Output equals to the input with a gain k**

- **Split/add(adder)**

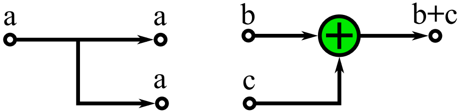


Figure : **On the left, a signal is split into two paths. On the right, two signals are added together**

Building blocks(Three Building Blocks)

The three building blocks of an LTI system: **multiplication**, **addition**, and **delay**

- **Delay**

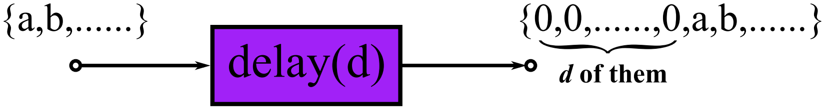


Figure : **Output equals to input with a delay of d time units**

Building blocks(Flow Graph Transformations)

Intuitively, some changes to the flow graphs are permitted:

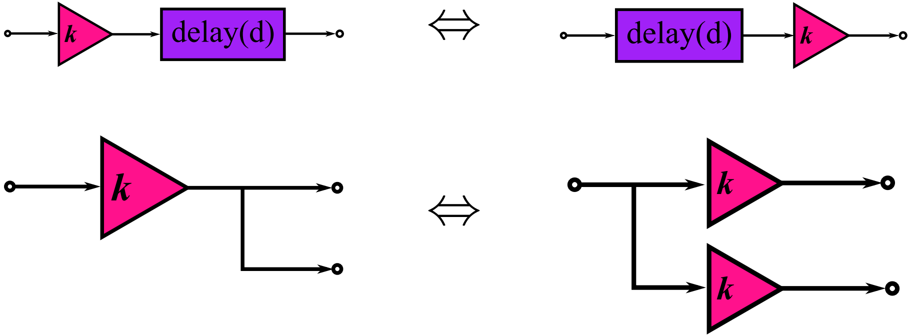


Figure : **Some examples of flow graph transformations**

Difference Equations(Expression & Conventions)

Expression:

- $y[n] = a_1 y[n-1] + a_2 y[n-2] + ... + b_0 x[n] + b_1 x[n-1] + ...$

Conventions:

- Signal: $x[n]$(square bracket)
- Use $x[n]$ for an input signal, $y[n]$ for an output signal
- Often $n = 0, 1, ..., N-1$(integer) for a length-N signal. We may also have an "infinite" length signal where n can be any nonnegative integers.
- Assume $x[n] = 0$ outside this range.

  $\Rightarrow$ No input, no output. System is "at rest"

Difference Equations(Two Special Discrete-time signals)

Impulse Signal(delta functions): $\delta[n]$

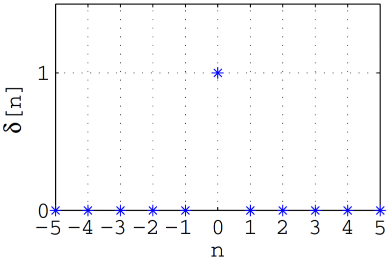
- $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & otherwise (n \neq 0) \end{cases}$
  This is called an impulse because it is active only at the first time instance, and then it returns to zero and stays there forever

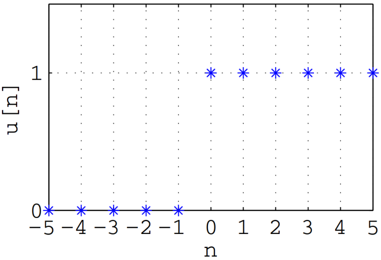
Unit Step Functions: $u[n]$

- $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & otherwise (n < 0) \end{cases}$
  Notice that because we have assumed that all signals with negative indices are zero, the unit step appears to be equal to 1 all the time

Difference Equations(Two Special Discrete-time signals)

Relation of these two signals: $\delta[n]$ and $u[n]$



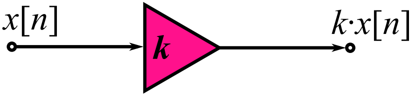


- $\delta[n] = u[n] - u[n-1]$
- $u[n] = \sum_{m=-\infty}^{n} \delta[m] = \sum_{k=0}^{\infty} \delta[n-k]$

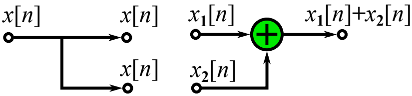
Difference Equations(Flow Graphs)

The three building blocks of an LTI system: **multiplication**, **addition**, and **delay**

- **Multiplication(gain)**
  (k can be integer, fraction, negative number...)



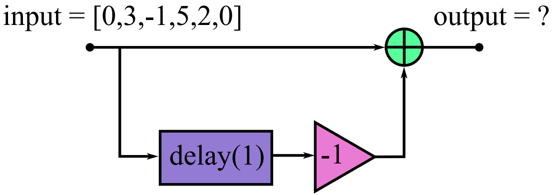
- **Split/add(adder)**
  (A signal becomes two **identical** copies)
  (Two signals added together)



- **Delay**
  (A signal is delayed by d **integer** units)

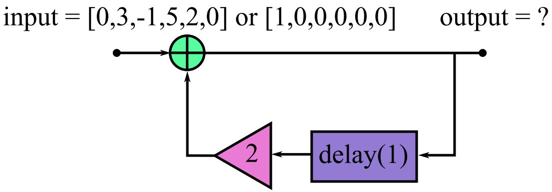
Question 1(a)

* Find the output of the system?



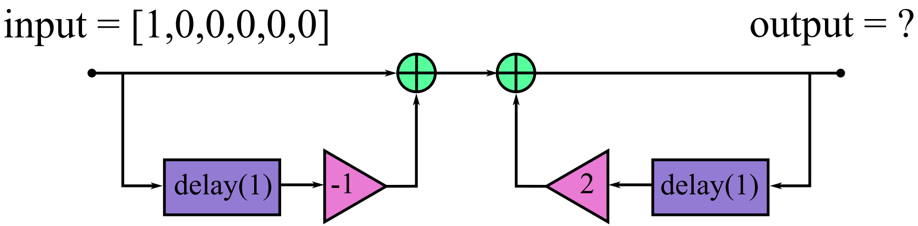
Question 1(b)

- Find the output of the system?



Question 1(c)

* Find the output of the system?



Question 2(a)

(a) Sketch each of the following input signals

i. $x[n] = \delta[n] + \delta[n-3]$

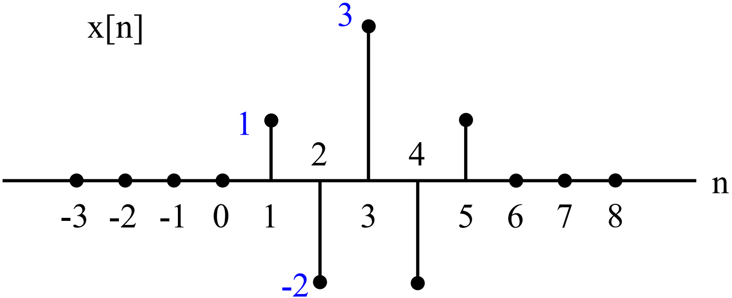
ii. $x[n] = u[n] - u[n-5]$

iii. $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + \frac{1}{2} \cdot 2\delta[n-2] + \frac{1}{2} \cdot 3\delta[n-3]$

where δ is unit impulse function and u is the unit step function.

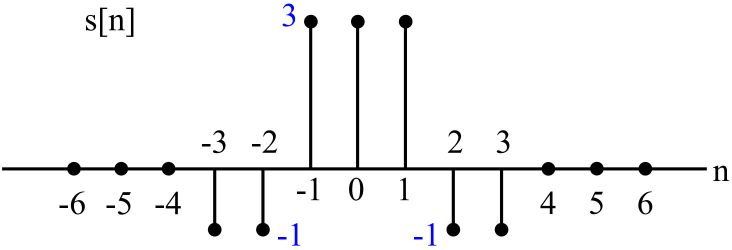
Question 2(b)

(b) Express the following as sums of weighted delayed impulses, i.e. in the form $x[n] = \sum_{k=-\infty}^{\infty} a_k \delta[n-k]$



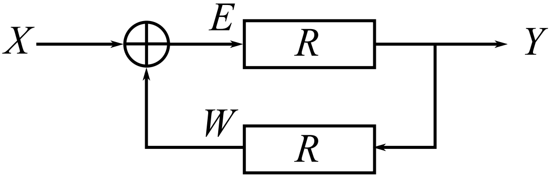
Question 2(c)

(c) Express the following sequence as sum of unit step function, i.e. in the form $s[n] = \sum_{k=-\infty}^{\infty} a_k u[n-k]$



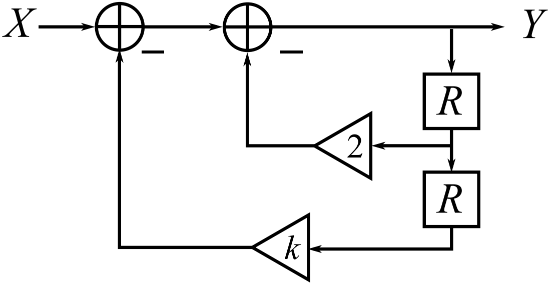
Question 3(a)

* Determine the difference equation that relates X and Y? **R: delay(1)**



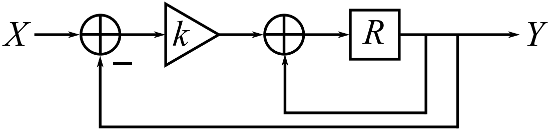
Question 3(b)

* Determine the difference equation that relates X and Y? **R: delay(1)**



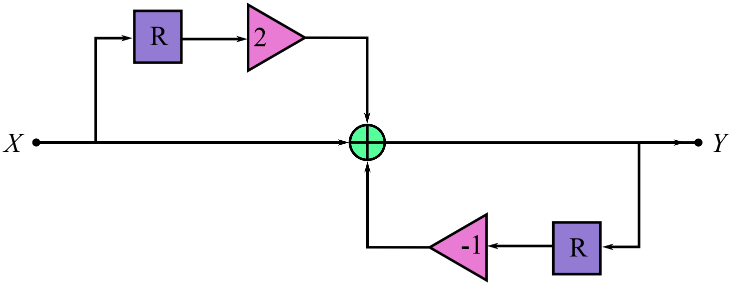
Question 3(c)

* Determine the difference equation that relates X and Y? **R: delay(1)**



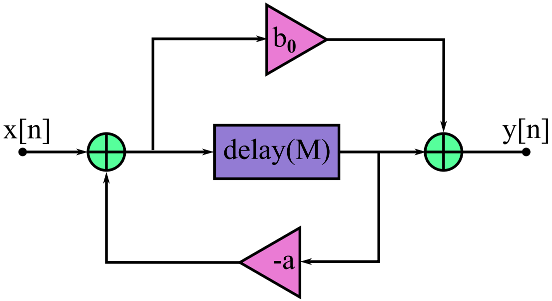
Question 3(d)

* Determine the difference equation that relates X and Y? **R: delay(1)**



Question 3(e)

* Determine the difference equation that relates X and Y? **R: delay(1)**



Question 4

[SP13 Final Exam] Consider the difference equation $y[n] = y[n-1] + k \cdot y[n-2] + x[n]$, where $x[n]$ is an impulse input. For what value(s) of k indicated below would the output converge to zero as n increases?

i $k = 0$

ii $k = -\frac{1}{2}$

iii $k = -1$

iv $k = -\frac{1}{2}$ and $k = 0$

v $k = -1$, $k = -\frac{1}{2}$, and $k = 0$

Question 5(a)

[FA12 Final Exam] Consider the difference equation
$y[n] = k \cdot y[n-1] + k \cdot y[n-2] + x[n]$.
Assume $x[n]$ is an impulse input, i.e. $x[0] = 1$ and $x[n] = 0$ for other values of n, and that $y[n] = 0$ for $n < 0$.

(a) Let $k = 1$. What is the value of $y[10]$?

(i) 2
(ii) 1
(iii) 0
(iv) -1
(v) -2

Question 5(b)

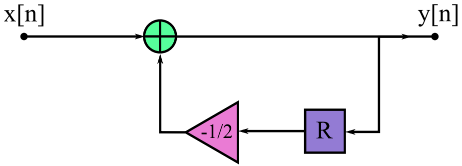
[FA12 Final Exam] Consider the difference equation
$y[n] = k \cdot y[n-1] + k \cdot y[n-2] + x[n]$.
Assume $x[n]$ is an impulse input, i.e. $x[0] = 1$ and $x[n] = 0$ for other values of n, and that $y[n] = 0$ for $n < 0$.

(b) Let $k = -1$. What is the value of $y[10]$?

(i) 34
(ii) -34
(iii) 55
(iv) -55
(v) 89

Question 6

* Consider the block diagram relating the two signal $x[n]$ and $y[n]$ given in figure. **R: delay(1)**



(a) Determine the difference equation relating $y[n]$ and $x[n]$.

(b) Assume that a solution to the difference equation in part (a) is given by $y[n] = k\alpha^n u[n]$, where $u[n]$ is unit step function and $x[n] = \delta[n]$. Find the appropriate value of k and α, and verify that $y[n]$ satisfies the difference equation.

(c) Verify your answer to part (b) by directly calculating $y[0]$, $y[1]$, and $y[2]$.