Question 1

$P(\text{cancer}) = 0.008$

$P(\neg\text{cancer}) = 0.992$

$P(\oplus|\text{cancer}) = 0.98$

$P(\oplus|\neg\text{cancer}) = 0.03$

$\oplus\oplus$ represents the fact that the patient gets positive results in both the first and second test.

$P(\oplus\oplus|\text{cancer}) = P(\oplus|\text{cancer}) \times P(\oplus|\text{cancer})$

$= 0.98 \times 0.98 = 0.9604$

$P(\oplus\oplus|\neg\text{cancer}) = P(\oplus|\neg\text{cancer}) \cdot P(\oplus|\neg\text{cancer})$

$= 0.03 \cdot 0.03 = 0.0009$

Applying the law of total probability gives:

$P(\oplus\oplus) = P(\oplus\oplus|\text{cancer}) \cdot (P(\text{cancer}) +$
$P(\oplus\oplus|\neg\text{cancer}) \cdot P(\neg\text{cancer})$

$= 0.9604 \times 0.008 + 0.0009 \cdot 0.992$

$= 0.008576$

probability of cancer =

$$P(cancer \mid \oplus\oplus) = \frac{P(\oplus\oplus \mid cancer)\,P(cancer)}{P(\oplus\oplus)}$$

$$= \frac{0.9604 \times 0.008}{0.008576} = 0.8959$$

$$P(\neg cancer \mid \oplus\oplus) = \frac{P(\oplus\oplus \mid \neg cancer)\,P(\neg cancer)}{P(\oplus\oplus)}$$

$$= \frac{0.0009 \cdot 0.992}{0.008576} = 0.1041$$

Question 2

For target value yes

$$p(yes) \cdot p(sunny | yes) \cdot p(cool | yes) \cdot p(high | yes) \cdot p(strong | yes) = \frac{8}{12} \cdot \frac{2}{8} \cdot \frac{3}{8} \cdot \frac{3}{8} \cdot \frac{3}{8}$$

$= 0.0088$

For target value no

$$p(no) \cdot p(sunny | no) \cdot p(cool | no) \cdot p(high | no) \cdot p(strong | no) = \frac{4}{12} \cdot \frac{3}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{2}{4}$$

$= 0.0234$

The prediction for playtennis for the same new instance is no because it has a higher score, thus a higher probability

Question 3

Forward pass 1st Iteration

$output_c = \sigma(w_{ca} \cdot a + w_{cb} \cdot b + w_{c0} \cdot 1)$

$= \sigma(0.1 \cdot 1 + 0.1 \cdot 0 + 0.1 \cdot 1) = 0.2$

$\sigma(0.2) = \frac{1}{1+e^{-0.2}} = 0.5498$

$output_d = \sigma(w_{dc} \cdot output_c + d \cdot w_{d0})$

$= \sigma(0.1 \cdot 0.5498 + 0.1 \cdot 1) = \sigma(0.15498)$

$= \frac{1}{1+e^{0.15498}} = 0.5387$

$\delta_d = O_d(1 - O_d)(t_d - O_d)$

$= 0.5387(1-0.5387)(1-0.5387)$

$= 0.1146$

$\delta_c = O_c(1-O_c) \sum_{k \in outputs} w_{kh} \delta_k$

$= 0.5387(1-0.5387) \cdot w_{dc} \cdot \delta_d$

$= 0.5387(1-0.5387) \cdot 0.1 \cdot 0.1146 = 0.0028$

Correction terms where $a = 1$ $b = 0$ $n = 0.3$

$\Delta w_{ji} = n \delta_j x_{ji}$

$\Delta w_{d0} = 0.3 \cdot 0.1146 \cdot 1 = 0.0342$

$\Delta w_{dc} = 0.3 \cdot 0.1146 \cdot 0.5498 = 0.0189$

$\Delta w_{c0} = 0.3 \cdot 0.0028 \cdot 1 = 0.00084$

$\Delta w_{ca} = 0.3 \cdot 0.0028 \cdot 1 = 0.00084$

$\Delta w_{cb} = 0.3 \cdot 0.0028 \cdot 0 = 0$

Updating weights

$w_{ji} = w_{ji} + \Delta w_{ji}$

$w_{d0} = 0.1 + 0.0342 = 0.1342$

$w_{dc} = 0.1 + 0.0189 = 0.1189$

$w_{c0} = 0.1 + 0.00084 = 0.10084$

$w_{ca} = 0.1 + 0.00084 = 0.10084$

$w_{cb} = 0.1$

Forward pass - second iteration

$$output_c = \sigma(w_{ca} \cdot a + w_{cb} \cdot b + w_{c0} \cdot 1)$$

$$= \sigma(0.10084 \cdot 0 + 0.1 \cdot 1 + 0.10084 \cdot 1)$$

$$= \sigma(0.20084) =$$

$$\frac{1}{1+e^{-0.20084}} = 0.55004$$

$$output_d = \sigma(w_{dc} \cdot output_c + 1 \cdot w_{d0})$$ note 1 is bias term

$$= \sigma(0.1189 \cdot 0.55004 + 1 \cdot 0.1342)$$

$$= \sigma(0.1996) = \frac{1}{1+e^{-0.1996}}$$

$$= 0.54973$$

this is the sample value

Error terms

$$\delta_d = o_d(1-o_d)(t_d - o_d)$$

$$= 0.54973(1-0.54973)(0-0.54973)$$

$$= -0.13607$$

$$\delta_c = o_c(1-o_c) \sum w_{kh} \cdot \delta_k$$

$$= 0.55004(1-0.55004) \cdot 0.1189 \cdot -0.13607$$

$= -0.004$

Correction terms where $a = 1$ $b = 0$ $n = 0.3$ $\alpha = 0.9$

$\Delta w_{ji} = n \, \delta_j \, x_{ji} \cdot a \cdot \Delta w_{ji} \, prev.$

$\Delta w_{d0} = (0.3 \cdot -0.13607 \cdot 1) + (0.9 \cdot 0.0342)$

$= -0.01004$

$\Delta w_{dc} = (0.3 \cdot -0.13607 \cdot 0.55004) + (0.9 \cdot 0.0189) = -0.00544$

$\Delta w_{c0} = (0.3 \cdot -0.004 \cdot 1) + (0.9 \cdot 0.00084)$

$= -0.00044$

$\Delta w_{ca} = (0.3 \cdot -0.004 \cdot 0) + (0.9 \cdot 0.00084)$

$= 0.00076$

$\Delta w_{cb} = (0.3 \cdot -0.004 \cdot 1) + (0.9 \cdot 0)$

$= -0.0012$

Updating weights

$w_{do} = 0.1342 + (-0.01004) = 0.12416$

$w_{dc} = 0.1189 + (-0.00544) = 0.11346$

$w_{co} = 0.10084 + (-0.00044) = 0.1004$

$w_{ca} = 0.10084 + 0.00076 = 0.1016$

$w_{cb} = 0.1 + (-0.0012) = 0.0988$