Gateway Quiz MATH 267 Term Exam 2 W2024,v1

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PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 1. (1 point)

Which of the following is a geometric series?

- **A.** $\sum_{n=1}^{\infty}(-1)^n \frac{3^{2n}}{5^n}$
- **B.** $\sum_{n=1}^{\infty} \frac{1}{n^2}$
- **C.** $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$
- **D.** $\sum_{n=1}^{\infty}(-1)^n \frac{2^n}{n!}$
- **E.** $\sum_{n=1}^{\infty}(-1)^n \frac{1}{2n}$

preview answers

Entered	Answer Preview
A	A

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 2. (1 point)

Assume we are trying to determine the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{5n^2+6n^4}{n^8-7n^2}.$$

Which of the following statements accurately describes the series?

- (selected) **A.** The series converges by the Limit Comparison Test with the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$.
- **B.** The series diverges by the Divergence Test.
- **C.** The series converges by the Limit Comparison Test with the series $\sum_{n=1}^{\infty} \frac{5}{n^6}$.
- **D.** The series converges conditionally.
- **E.** It is impossible to tell if the series converges or diverges.

preview answers

Entered	Answer Preview
A	A

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 3. (1 point)

Let $\{a_n\}_{n=1}^{\infty}$ and $\{c_n\}_{n=1}^{\infty}$ be sequences of non-negative real numbers.

For each of the following statements, determine if the statement is TRUE or FALSE.

Hint: A statement is true if and only if it is true for all choices of the sequences $\{a_n\}_{n=1}^{\infty}$ and/or $\{c_n\}_{n=1}^{\infty}$.

FALSE **1.** If $c_n \leq a_n$ for all n and $\sum_{n=1}^{\infty} c_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

FALSE **2.** If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty}(-1)^n a_n$ converges to 0.

TRUE **3.** If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{N\to\infty} \sum_{n=1}^{N} a_n$ exists.

Note: ***You can earn partial credit on this problem.***

preview answers

Entered	Answer Preview
FALSE	FALSE

FALSE	FALSE
TRUE	TRUE

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 4. (1 point)

Which of the following series are **conditionally convergent** ?

Select ALL correct answers.

- [x] **A.** $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{5}{\sqrt{n}}$
- [x] **B.** $\sum_{n=1}^{\infty}(-1)^{n}\frac{n^2-5}{5^n}$
- [x] **C.** $\sum_{n=4}^{\infty}(-1)^{n+1}\frac{4}{\sqrt{n^5}}$
- [x] **D.** $\sum_{n=3}^{\infty}(-1)^{n}\frac{1}{4\sqrt[5]{n^7}}$
- [x] **E.** $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{7+\sqrt[5]{n^4}}$

preview answers

Entered	Answer Preview
ABCDE	ABCDE

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 5. (1 point)

Consider the series $\sum_{n=1}^{\infty}\frac{9n^3}{n^6+7}$.

Which of the following is the best test to use in order to determine the convergence/divergence of the series?

- (●) **A.** The Limit Comparison Test with the series $\sum_{n=1}^{\infty}\frac{1}{n^3}$.
- ( ) **B.** The Root Test.
- ( ) **C.** The Integral Test.

- **D.** The Limit Comparison Test with the sequence $\left\{\dfrac{9}{n^3}\right\}_{n=1}^{\infty}$.
- **E.** The Comparison Test with the series $\displaystyle\sum_{n=1}^{\infty}\frac{9}{7n^6}$.
- **F.** The Ratio Test.

preview answers

Entered	Answer Preview
A	A

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 6. (1 point)

Consider the series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{4n^5-7}{7n+16n^5}.$$

Which of the following statements accurately describes the series?

- **A.** The series converges conditionally.
- **B.** The series converges to $\dfrac{1}{4}$.
- **C.** The series diverges by the Divergence Test.
- **D.** The series diverges by the Alternating Series Test.
- **E.** The series converges absolutely by the Ratio Test.

preview answers

Entered	Answer Preview
D	D

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 7. (1 point)

Choose the correct conclusion that you can make after applying the **Ratio Test** to each of the given series.

[The series diverges] **1.** $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{6^{n+1}}{5^{n+2}\sqrt{n}}$

The series converges absolutely

2. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{7^n}{n^2\,8^{n+2}}$

Note: ***You can earn partial credit on this problem.***

preview answers

Entered	Answer Preview
DIV	DIV
ABS	ABS

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 8. (1 point)

Suppose that $\{a_n\}_{n=1}^{\infty} = \{a_1, a_2, a_3, \ldots\}$ is a sequence of positive real numbers.

Suppose that $a_{n+1} < a_n$ for all $n \geq 1$ and suppose that $\lim\limits_{n\to\infty} a_n = 0$.

Which of the following statements is always TRUE?

- ○ **A.** The series $\sum_{n=1}^{\infty} a_n$ converges.
- ○ **B.** The series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ diverges.
- ○ **C.** The series $\sum_{n=1}^{\infty} a_n$ converges to 0.
- ○ **D.** The series $\sum_{n=1}^{\infty}(-1)^{n}a_n$ converges absolutely.
- ● **E.** The series $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$ converges.

preview answers

Entered	Answer Preview
E	E

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 9. **(1 point)**

Consider the series

$$\sum_{n=1}^{\infty} \frac{3n^2 + (-1)^n 2n + \cos(n)}{4n^3 - 2n + \sin(n)}.$$

Exactly which of the following statements is true?

- ○ **A.** The series diverges by the Comparison Test with the series $\sum_{n=1}^{\infty} \frac{1}{5n}$
- ○ **B.** The series diverges by the Divergence Test.
- ○ **C.** The series converges by the Alternating Series Test.
- ○ **D.** The series diverges by the Ratio Test.
- ◉ **E.** The series converges by the Limit Comparison Test with the series $\sum_{n=1}^{\infty} \frac{3n^2}{4n^3}$

preview answers

Entered	Answer Preview
E	E

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 10. **(1 point)**

Consider the power series

$$\sum_{n=1}^{\infty} \frac{n^8 + 3n + 1}{n^2 + 2n + 5}(4x - 5)^n$$

Which of the following is equal to the radius of convergence?

- ○ **A.** $R = 4$
- ○ **B.** $R = \frac{1}{5}$
- ○ **C.** $R = 0$
- ○ **D.** $R = 5$
- ◉ **E.** $R = \frac{1}{4}$
- ○ **F.** $R = \infty$

preview answers

Entered	Answer Preview

E	$\mathbf{E}$

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 11. (1 point)

Consider the power series given by

$$\sum_{n=1}^{\infty} \frac{5^n(6x-2)^n}{n^2+3n}.$$

Find the centre of the series and the radius of convergence.

the centre is at $c = \boxed{\frac{1}{3}}$,

and the radius of convergence is $R = \boxed{\frac{1}{30}}$.

Note: If your answer is a fraction, just enter the fraction.

Note: ***You can earn partial credit on this problem.***

preview answers

Entered	Answer Preview
1/3	$\frac{1}{3}$
1/30	$\frac{1}{30}$

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 12. (1 point)

Given that a function $f(x)$ has a tangent line at $x = 2$ given by $y = 5(x-2)+11$. Which of the following could be the Taylor Series representation for $f(x)$?

- **A.** $\sum_{n=0}^{\infty} \frac{5}{n!}(x-2)^n$
- **B.** $\sum_{n=0}^{\infty} \frac{11}{n!}(x-2)^n$

- (selected) C. $\sum_{n=0}^{\infty} \frac{10+(-5)^n}{n!}(x-2)^n$
- D. $\sum_{n=0}^{\infty} \frac{11+(5)^n}{n!}(x-2)^n$

preview answers

Entered	Answer Preview
C	C

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 13. (1 point)

Consider the function

$$f(x) = \int_0^x \frac{\cos(t)-1}{t^2}\, dt.$$

Which of the following is the Taylor Series for $f(x)$ centred at $x = 0$?

- A. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n-1)(2n)!} x^{2n-1} + C.$
- B. $\sum_{n=1}^{\infty} \frac{(-1)^n(2n-2)}{(2n)!} x^{2n-3}.$
- C. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n-2}.$
- (selected) D. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)(2n)!} x^{2n-1}.$

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preview answers

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D	D

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 14. (1 point)

Consider the power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{\sqrt{n}\,3^n}(x-1)^n.$$

Determine the center and the interval of convergence of the given series.

- ○ **A.** The center is $\frac{2}{3}$ and the interval of convergence is $\left(-\frac{1}{2}, \frac{5}{2}\right)$.
- ◉ **B.** The center is 1 and the interval of convergence is $\left[-\frac{1}{2}, \frac{5}{2}\right]$.
- ○ **C.** The center is $\frac{2}{3}$ and the interval of convergence is $\left[-\frac{1}{2}, \frac{5}{2}\right]$.
- ○ **D.** The center is 1 and the interval of convergence is $\left[-\frac{1}{2}, \frac{5}{2}\right)$.
- ○ **E.** The center is 1 and the interval of convergence is $\left(-\frac{1}{2}, \frac{5}{2}\right)$.
- ○ **F.** The center is 1 and the interval of convergence is $\left(-\frac{1}{2}, \frac{5}{2}\right]$.

preview answers

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B	**B**

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 15. (1 point)

Suppose that the Taylor series of the function f centred at 3 is given by

$$\sum_{n=1}^{\infty}(-1)^n \frac{(3n)!}{5^n}(x-3)^{2n-1}.$$

Find the value of $f^{(17)}(3)$, that is, the order 17 derivative of f evaluated at $x=3$.

Answer: $\frac{(-1)^9(3(9))!}{5^9}\cdot 17!$

preview answers

Entered	Answer Preview
-1.98299E+36	$\frac{(-1)^9((3\cdot 9)!)}{5^9}(17!)$

PREVIEW ONLY -- ANSWERS NOT RECORDED

Problem 16. (1 point)

Suppose that the sequence $\{c_n\}_{n=1}^{\infty}$ has the property that $\lim_{n\to\infty}\sqrt[n]{|c_n|}=4$.

What is the radius of convergence of the power series $\sum_{n=1}^{\infty}(-6)^n c_n (x+6)^n$?

Answer: $\frac{1}{24}$

preview answers

Entered	Answer Preview
0.0416667	$\frac{1}{24}$

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Preview Test Grade Test