LEGGE DI OHM

In metallo $\rightarrow$ mare di elettroni liberi:

collisioni!

Cammino libero medio $\propto v \rightarrow L = \tau v$

$F = -eE$ (Coulomb), $F = ma \Rightarrow a = -\frac{e}{m}E$

$\langle v \rangle = a\tau = -\frac{e\tau}{m}E$

Per def:

$\underline{J} = Nq\underline{v} \Rightarrow \underline{J} = -Ne\underline{v}$

$\underline{v} = -\frac{e\tau}{m}E \Rightarrow \underline{J} = -Ne\left(\frac{e\tau}{m}\right)E = \frac{Ne^2}{m}\tau E$

<u>Def</u> (Ohm statica)

$\underline{J} = \sigma \underline{E} \Rightarrow \underline{J} = \frac{Ne^2}{m}\tau E \Rightarrow \sigma = \frac{Ne^2}{m}\tau$ (conducibilità)

Per filo metallico omogeneo lineare sez. S

$\frac{\partial \sigma}{\partial E} = 0 \Rightarrow \sigma$ costante

<u>DEF</u>

Resistività (ρ)

$\rho = \frac{1}{\sigma} \Rightarrow \underline{E} = \rho \underline{J}$

Sia filo lungo l e ΔV ai capi

<u>PER DEF</u>:

$\Delta V = \int_l \underline{E}\, d\underline{l} = El$; $E = \frac{\Delta V}{l}$

$I = \int_S \underline{J}\, \underline{n}\, d^2S = JS = \sigma S E = \frac{\sigma S}{l}\Delta V$

DEF (Resistenza)

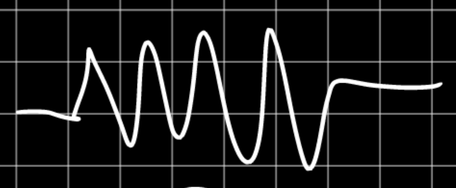
$$R = \frac{l}{\sigma S} = \frac{\rho}{S} l \quad (\text{resistività} / \text{sup})$$

Allora:

$$I = \frac{\sigma S}{l} \Delta V \Rightarrow \Delta V = \frac{l}{\sigma S} I \Rightarrow \Delta V = R I$$

OHM: $\Delta V = R I$

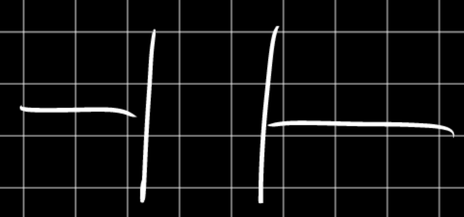
COMPONENTI



R

Resistore

$$\begin{cases} V = RI \\ I = \frac{V}{R} \end{cases}$$



C

Capacitore

$$\begin{cases} V = \frac{1}{C}\int_0^t I \, dt \\ I = C \frac{dV}{dt} \end{cases}$$

L

Induttore

$$V = L \frac{dI}{dt}$$

$$I = \frac{1}{L}\int_0^t V dt$$

Resistori:

- filo metallico
- impasto
- film metallico

$\Rightarrow$ Resistori Variabili (trimmer, potenziometri)

Condensatori:

- mica
- ceramica
- poliestere
- elettrolitici

$\Rightarrow$ Condensatori variabili

Induttori:

-) con nucleo di ferrite

-) senza ferrite

RESISTORI

R possibile fra $0.01\,\Omega$ a $10^{12}\,\Omega$

Potenze dissipabili da $\frac{1}{8}$ W a 250 W

Precisione da 0.005 % a 20 %

→ Impasto carbone più economico, garantito $R = R_{Nom} \pm 5\%$

ATTENZIONE:

-) saldatura → cambio permanente del 2 %

-) vibrazioni (2G) & shock (100G) → 2 % permanente

-) umidità (95 % a 40°C) → (6 ÷ 10) % (non permanente)

-) 10V cambiano da −0.15 % a −0.3 %

-) $T_1 = 25°C \rightarrow T_2 = -15°C$ da 2.5 % a 4.5 %

-) $T_1 = 25°C \rightarrow T_2 = 85°C$ da 3.3 % a 5.9 %

DIFETTI ULTERIORI (IMPASTO)

-) L ad alte freq ω

-) Rumore a basse ω

RESISTORI A FILM METALLICO

→ tolleranza dell' 1 %

→ stabilità migliori di 0.1 %

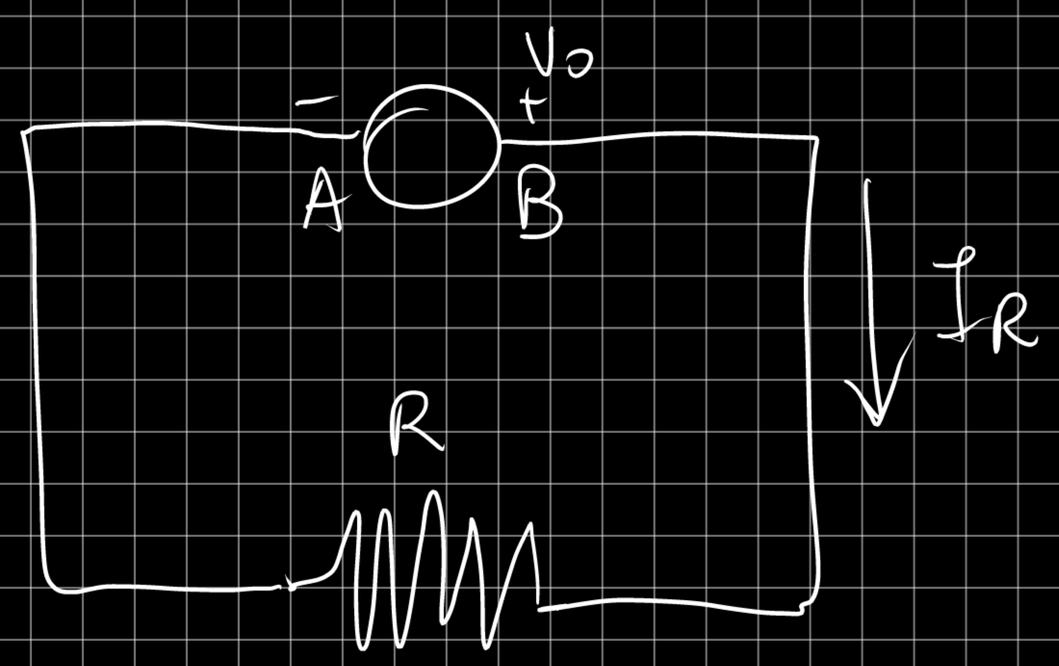
CONDENSATORI

(. . .)

GENERATORI IDEALI

TENSIONE:

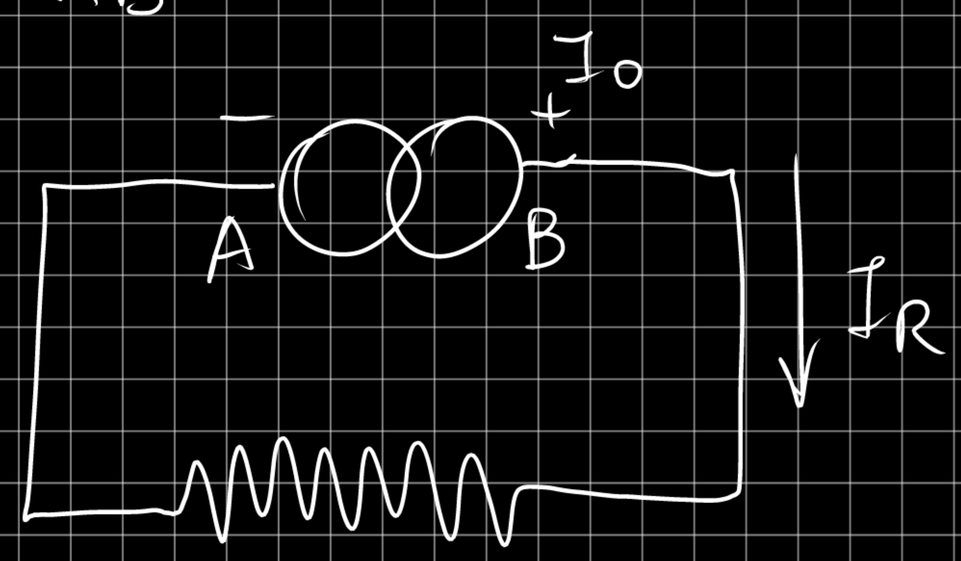
Sorgente di energia che tiene ΔV costante (V_0) tra punti A, B indipendentemente da connessioni



CORRENTE

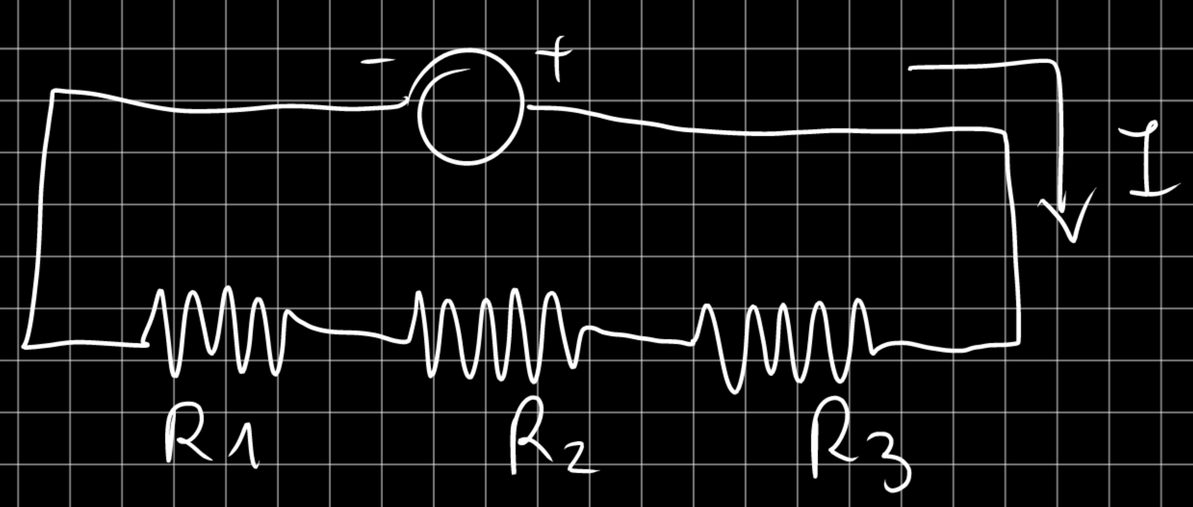
Corrente costante a punti A, B

→ Varia ΔV_{AB} in modo che I costante



CIRCUITI SEMPLICI

Resistenze in serie



$$\begin{cases} V_1 = R_1 I \\ V_2 = R_2 I \\ V_3 = R_3 I \end{cases}$$

Ma $V = V_1 + V_2 + V_3 \Rightarrow V = (R_1 + R_2 + R_3) I$

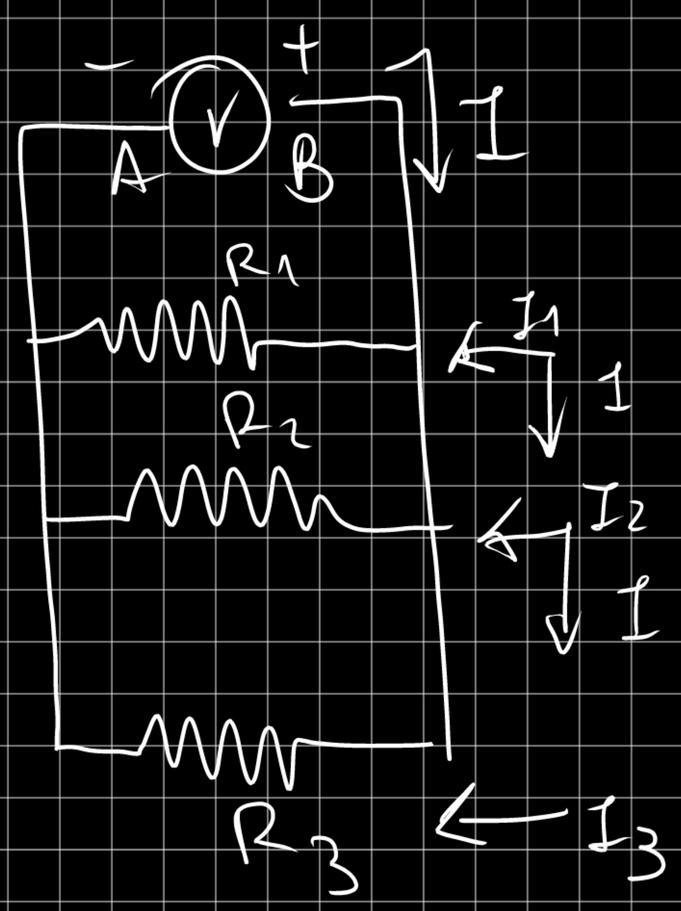
<u>Def</u>:

$$R = R_1 + R_2 + R_3 \Rightarrow V = R I$$

$$\boxed{\therefore R_S = \sum_i R_i} \longrightarrow \text{SERIE}$$

Resistenza in parallelo (N.B. $\Rightarrow$ serie I cost)

(N.B $I \rightarrow$ parallelo V cost)



Da $V = R I \Rightarrow I = \frac{V}{R} \Rightarrow \begin{cases} I_1 = \frac{V_1}{R_1} \\ I_2 = V_2/R_2 \\ I_3 = V_3/R_3 \end{cases}$

Ma $V_1 = V_2 = V_3 = V$ (stessa V, parallelo)

$$I_1 = V/R_1 \,,\; I_2 = V/R_2 \,,\; I_3 = V/R_3$$

$$I_1 + I_2 + I_3 = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V = R V$$

<u>Def</u>

PARALLELO $\boxed{\frac{1}{R_P} = \sum \frac{1}{R_i}}$

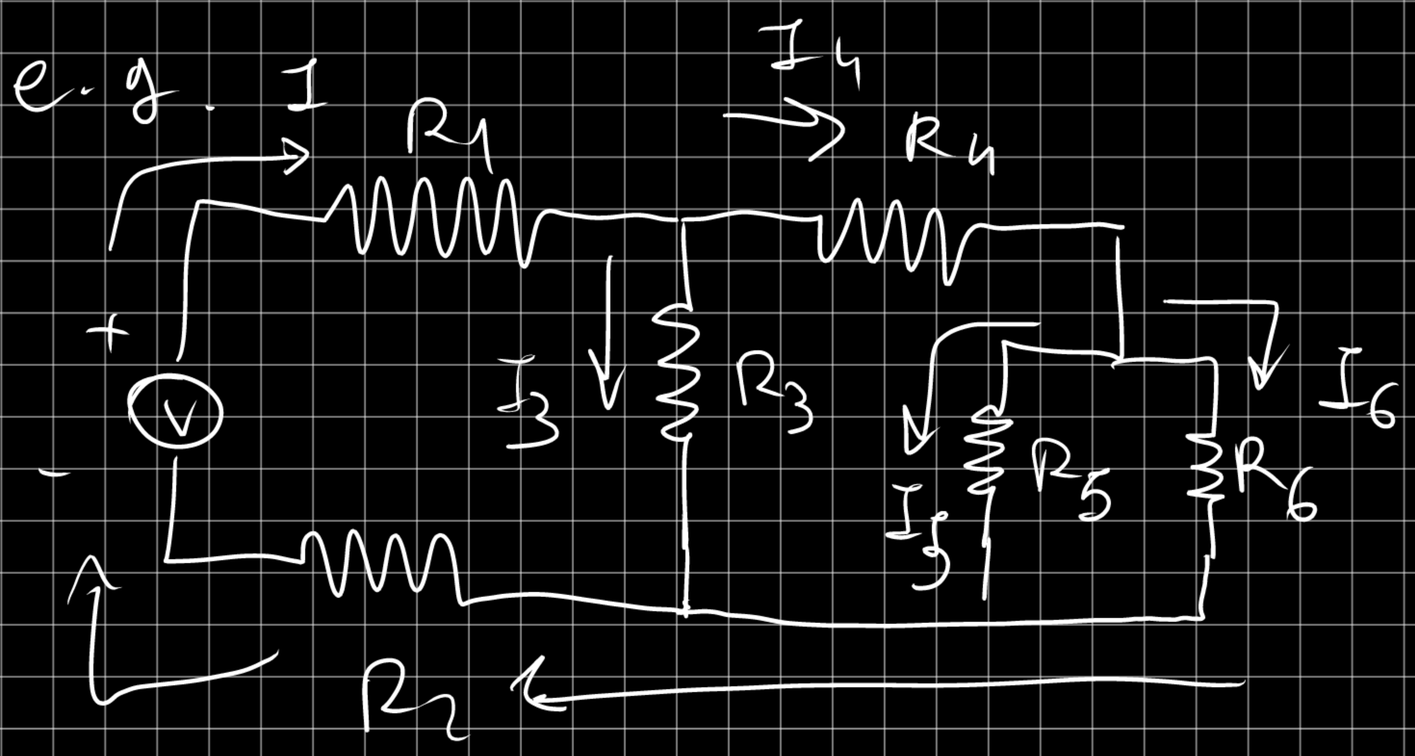
RETE

CONNESSIONE DI CIRCUITI SERIE - PARALLELO

APPUNTI

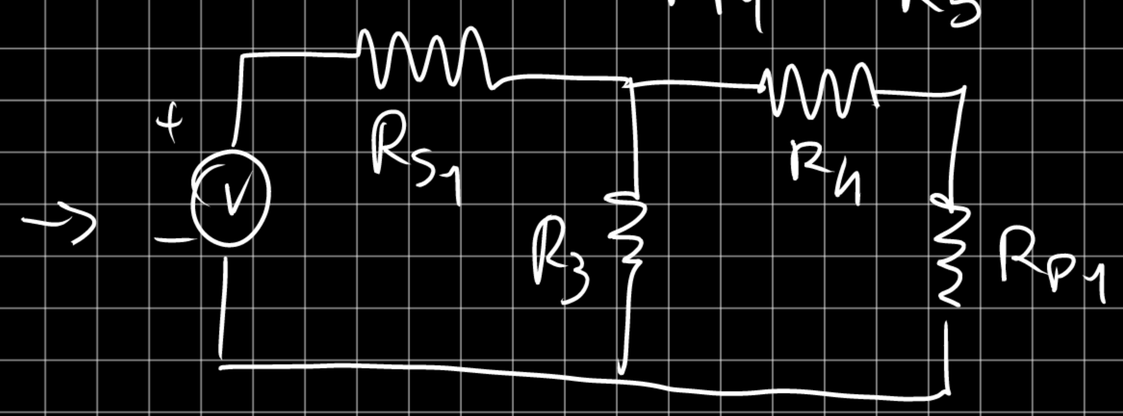
$$\begin{cases} \dfrac{1}{R_P} = \sum \dfrac{1}{R_{i_P}} \\ R_S = \sum R_S \end{cases} \rightarrow \text{ottieni } R_{eq} \text{ circuito}$$

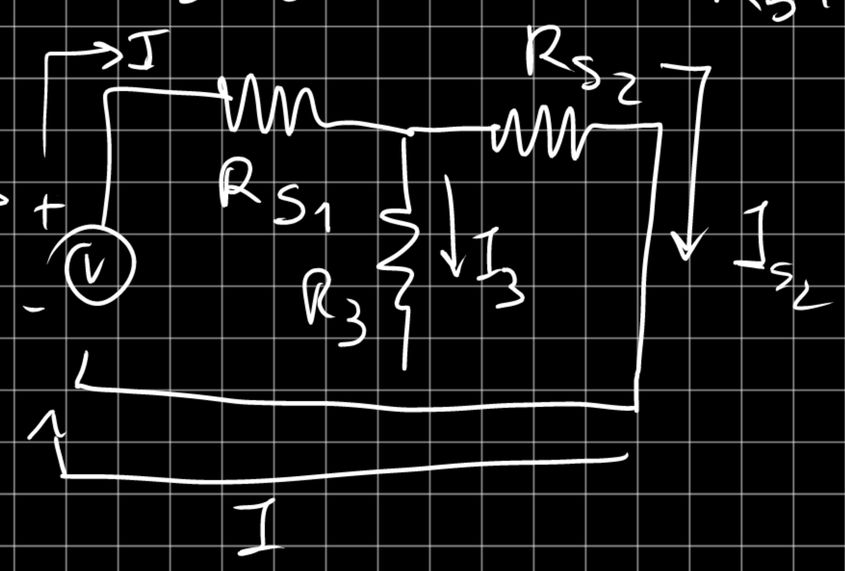
e.g.



1) $I_1 = I_2 \Rightarrow R_{s_1} = R_1 + R_2$

2) $V_5 = V_6 \rightarrow \dfrac{1}{R_{P_1}} = \dfrac{1}{R_5} + \dfrac{1}{R_6} = \dfrac{R_6 + R_5}{R_5 R_6} \rightarrow R_{P_1} = \dfrac{R_5 R_6}{R_5 + R_6}$

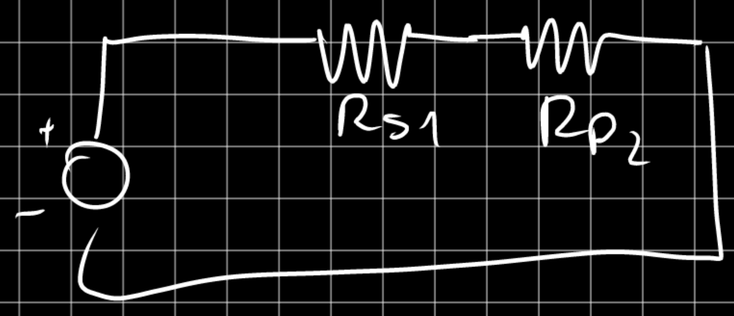




$R_{S_2} = R_4 + R_{P_1} =$

$= R_4 + \dfrac{R_5 R_6}{R_5 + R_6}$

$\dfrac{1}{R_{P_2}} = \dfrac{1}{R_{S_2}} + \dfrac{1}{R_3} \Rightarrow R_{P_2} = \dfrac{R_{S_2} R_3}{R_{S_2} + R_3} = R_3 \left(R_4 + \dfrac{R_5 R_6}{R_5 + R_6} \right) \Big/ \left(R_3 + R_4 + \dfrac{R_5 R_6}{R_5 R_6} \right)$



$R_T = R_{S_1} + R_{P_2}$

$$\therefore R_T = R_1 + R_2 + R_3\left(\frac{R_4 + \frac{R_5R_6}{R_5+R_6}}{R_3 + R_4 + \frac{R_5R_6}{R_5+R_6}}\right)$$ QUESTO √!

PRINCIPI DI KIRCHHOFF

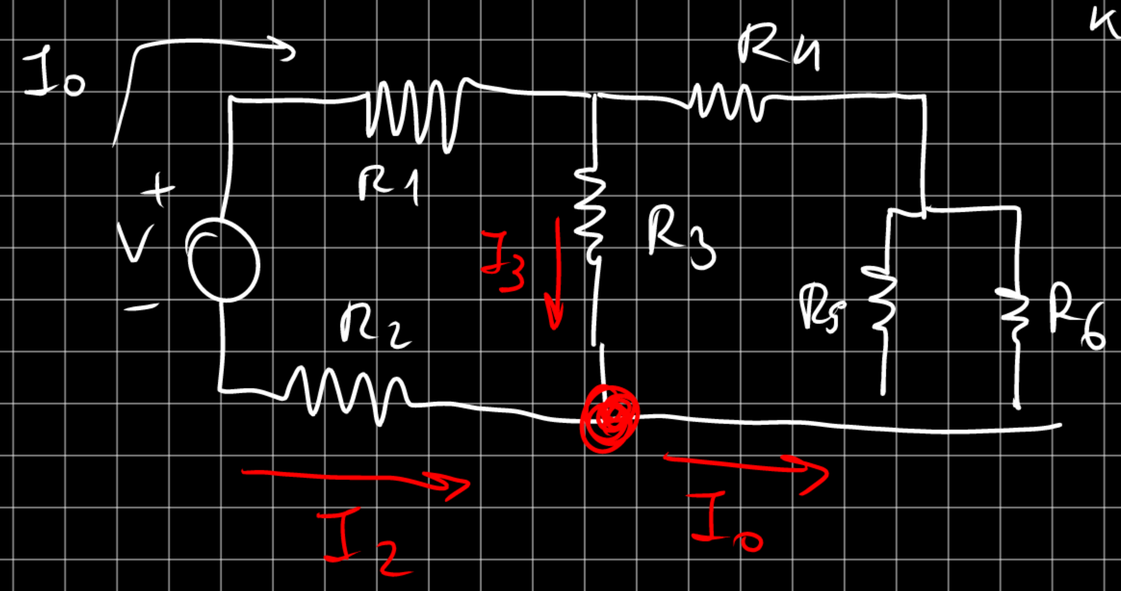
Rete di conduttori: nodi + maglie

NODO: PUNTO DI CONVERGENZA DI CONDUTTORI

MAGLIA: PERCORSO SINGOLO (CHIUSO) DA 1 NODO

PRIMO PRINCIPIO DI KIRCHHOFF

In un nodo $\sum_k I_k = 0$ $\begin{cases} I_k > 0 & \text{ENTRANTE} \\ I_k < 0 & \text{USCENTE} \end{cases}$



$$I_3 + I_2 - I_0 = 0$$

SECONDO PRINCIPIO

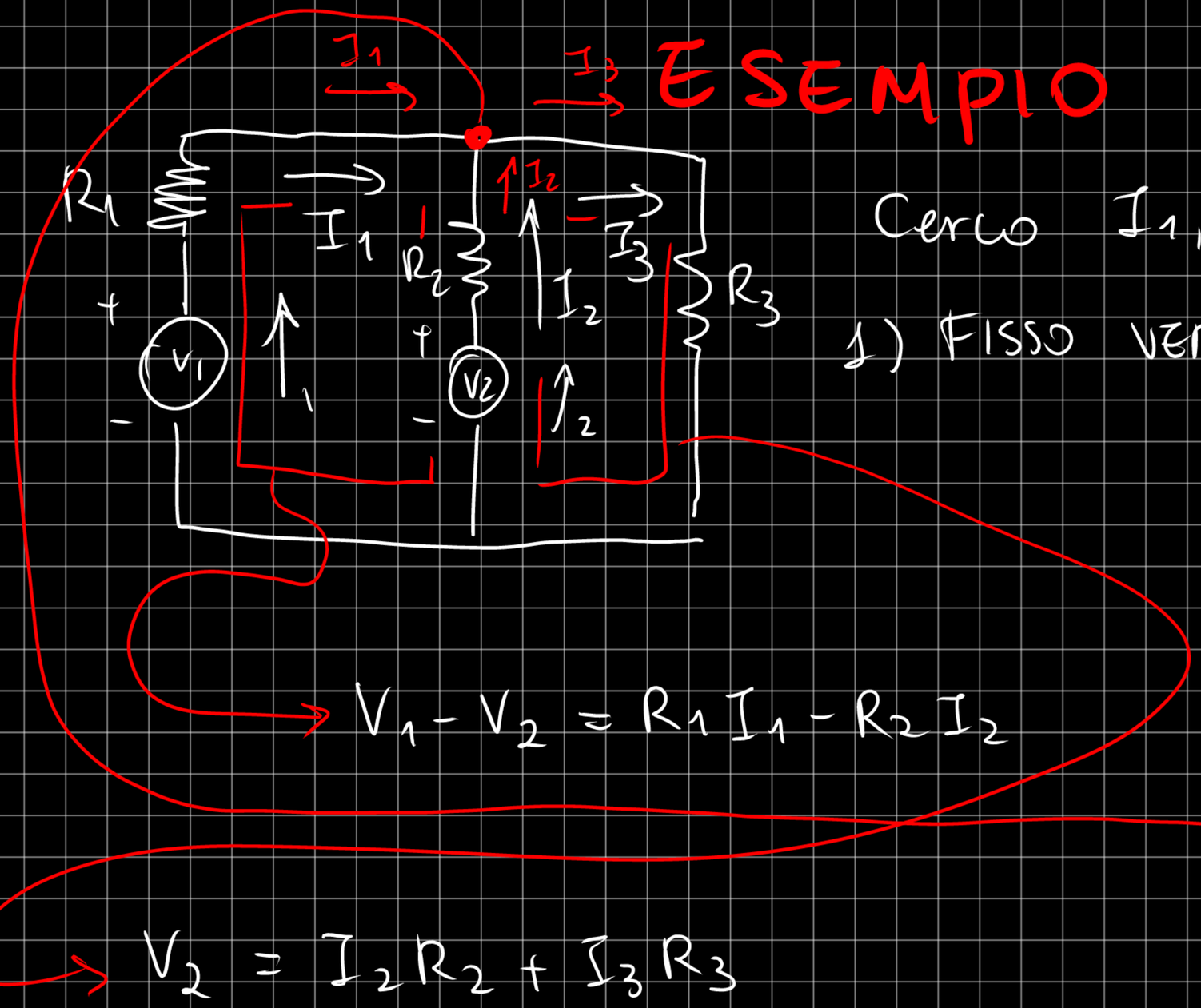
In una maglia $\sum_k V_k = 0$

N.B. Per Ohm se $R_k I_k$ caduta di tens. su res. allora:

$$\sum V_k = \sum R_k I_k = 0$$

$$(V_k = R_k I_k \rightarrow \text{OHM})$$

ESEMPIO



Cerco I_1, I_2, I_3

1) FISSO VERSO DI PERCORRENZA (>0)

$$V_1 - V_2 = R_1 I_1 - R_2 I_2$$

$$I_1 + I_2 - I_3 = 0$$

$$V_2 = I_2 R_2 + I_3 R_3$$

$$\begin{cases} V_1 - V_2 = R_1 I_1 - R_2 I_2 \\ I_1 + I_2 - I_3 = 0 \quad \Rightarrow I_3 = I_1 + I_2 \\ V_2 = I_2 R_2 + I_3 R_3 \end{cases}$$

$$\begin{cases} V_2 = I_2 R_2 + (I_1 + I_2) R_3 = (R_3 + R_2) I_2 + I_1 R_3 \\ V_1 - V_2 = R_1 I_1 - R_2 I_2 \end{cases}$$

$$\frac{V_2 - I_1 R_3}{R_3 + R_2} = I_2$$

$$V_1 - V_2 = R_1 I_1 - \frac{R_2}{R_3 + R_2}(V_2 - I_1 R_3)$$

I_2

$$V_1 - V_2 + \frac{R_2}{R_3 + R_2} V_2 = \left(R_1 + \frac{R_2 R_3}{R_3 + R_2}\right) I_1$$

$$V_1 + \left(\frac{R_2}{R_3 + R_2} - 1\right) V_2 = \left(R_1 + \frac{R_2 R_3}{R_3 + R_2}\right) I_1$$

$$V_1 + \left(\frac{R_2 - R_3}{R_3 + R_2}\right) V_2 = \frac{R_1(R_3 + R_2) + R_1 R_2 R_3}{R_3 + R_2} I_1$$

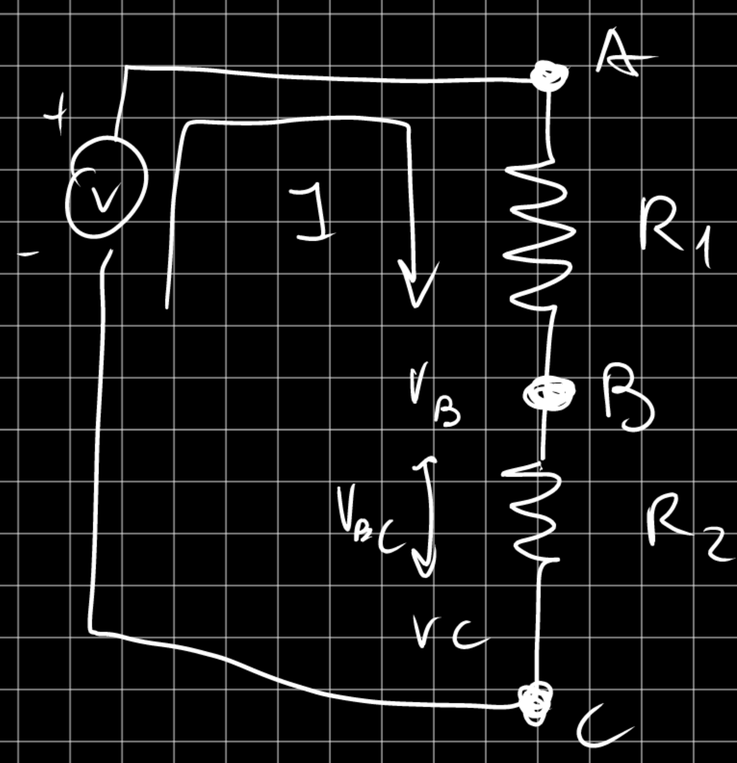
$$\frac{R_2 + R_3}{R_1(R_3 + R_2) + R_1 R_2 R_3} V_1 + \frac{R_2 - R_3}{R_1(R_3 + R_2) + R_1 R_2 R_3} V_2 = I_1$$

$$\begin{cases} I_1 + I_2 - I_3 = 0 \\ I_1 = \dfrac{1}{R_1(R_3+R_2)+R_1R_2R_3}\left[(R_2+R_3)V_1 + (R_2-R_3)V_2\right] \\ I_2 = \dfrac{1}{R_3+R_2}\left(V_2 - R_3 I_1\right) \end{cases}$$

$$I_2 = \frac{V_2}{R_3+R_2} - \frac{R_3}{R_1(R_3+R_2)+R_1R_2R_3}\left[V_1 + \frac{(R_2-R_3)V_2}{R_3+R_2}\right]$$

$$I_3 = I_1 + I_2$$

PARTITORE DI TENSIONE



Trova ddp (ΔV) ai capi di R_2

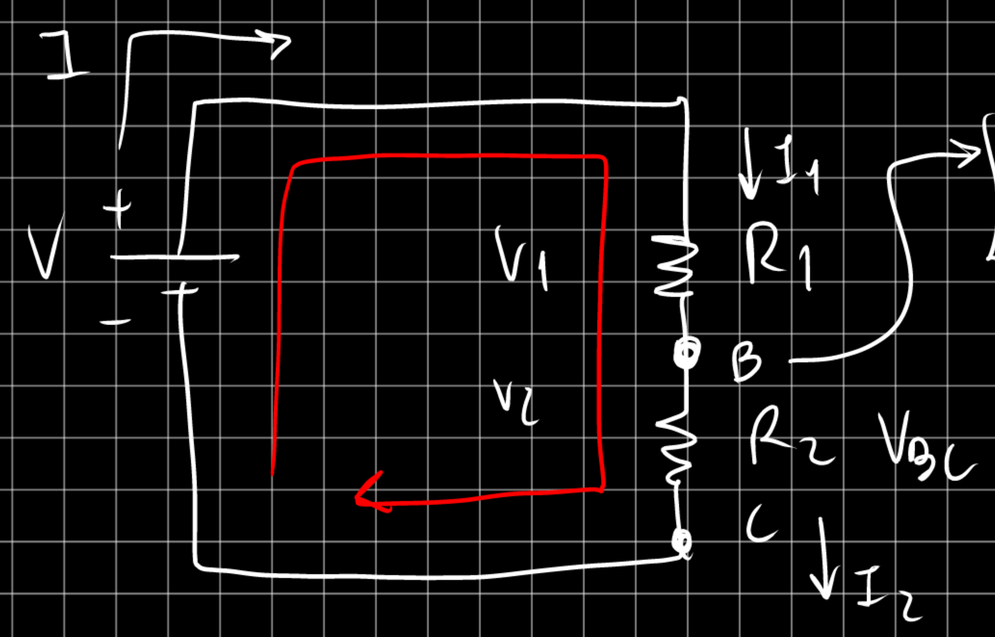
$$V = R_1 I + R_2 I$$

$$I = \frac{V}{R_1+R_2} = I_1 = I_2$$

$$R_2 I_2 = V_{BC} = R_2 I = \frac{R_2 V}{R_1+R_2}$$

$$\Longrightarrow V_{BC} = \frac{R_2}{R_1+R_2} V$$ $\longrightarrow$ posso scegliere R_2 tale che cambio V_{BC}

PROVO CON KIRCHHOFF

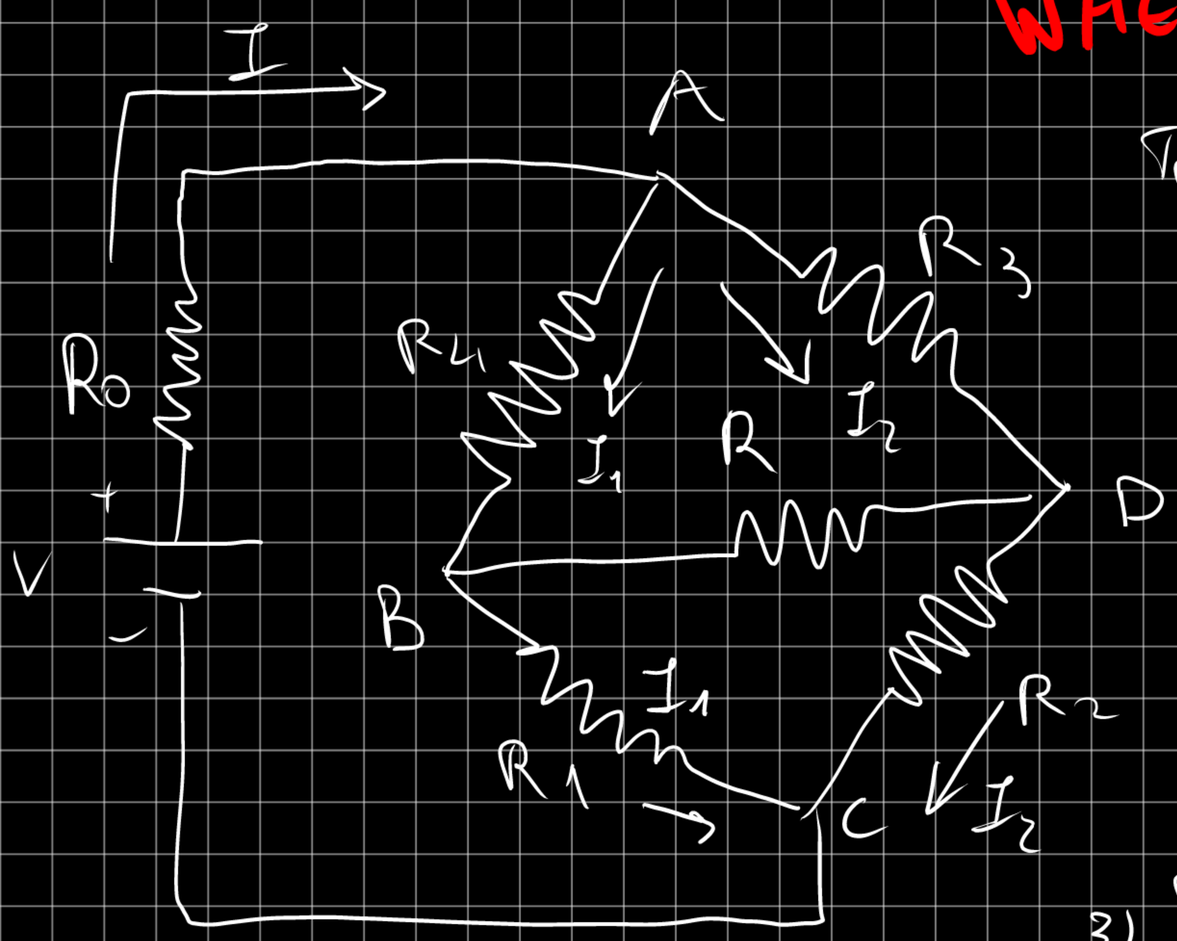


$$\begin{cases} I_1 - I_2 = 0 \rightarrow I_1 = I = I_2 \\ V = R_1 I_1 + R_2 I_2 = (R_1+R_2) I \end{cases}$$

$$\therefore I = \frac{V}{R_1+R_2}$$

$$V_2 = R_2 I = \frac{R_2}{R_1+R_2} V = V_{BC}$$ ok

PONTE DI WHEATSTONE



Trova R_i t.c. corrente tra B & D

$I_{BD} = 0$ (No corrente su R)

1) $I_{BD} = \dfrac{V_B - V_D}{R}$

2) $\begin{cases} I_1 = \dfrac{V}{R_4+R_1} \\ I_2 = \dfrac{V}{R_3+R_2} \end{cases}$

3) $\begin{cases} V_B - V_C = V_D - V_C \\ V_B - V_C = R_1 I_1 = R_2 I_2 \end{cases}$

$$R_1 I_1 = \frac{R_1}{R_4+R_1} V \quad , \quad R_2 I_2 = \frac{R_2}{R_3+R_2} V$$

$$I_{BD} = \frac{V_B - V_D}{R} = \frac{(V_B - V_C) - (V_D - V_C)}{R} = \frac{V}{R}\left(\frac{R_1}{R_4+R_1} - \frac{R_2}{R_3+R_2}\right) =$$

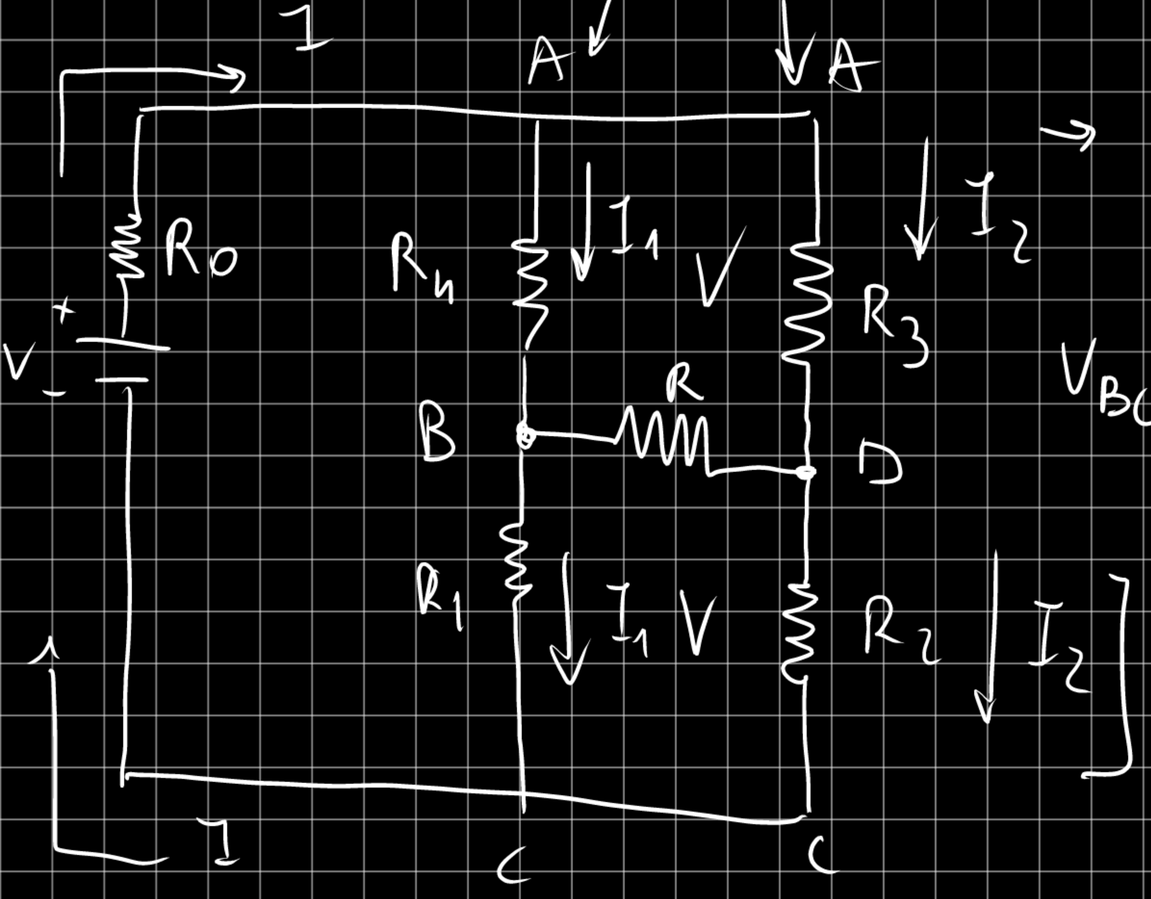
$$= \frac{V}{R}\left(\frac{R_1(R_3+R_2) - R_2(R_4+R_1)}{(R_4+R_1)(R_3+R_2)}\right) = I_{BD}$$

$$I_{BD} = 0 \Rightarrow R_1R_3 + \cancel{R_1R_2} - R_2R_4 - \cancel{R_2R_1} = 0$$

$$R_1R_3 - R_2R_4 = 0 \Rightarrow R_1R_3 = R_2R_4$$

N.B.

PARTITORI DI V



$$V_{BC} = \frac{R_1}{R_4+R_1} V$$

$$V_{DC} = \frac{R_2}{R_3+R_2} V$$

$$V_{BC} - V_{DC} = V\left(\frac{R_1}{R_4+R_1} - \frac{R_2}{R_2+R_3}\right)$$

$$\shortparallel$$

$$V_B - V_D$$

$$\shortparallel$$

$$RI \Rightarrow I = \frac{V}{R}\left(\frac{R_1}{R_4+R_1} - \frac{R_2}{R_2+R_3}\right) = 0$$

$$\therefore \frac{R_1}{R_1+R_4} = \frac{R_2}{R_2+R_3}$$

$$R_1(R_2+R_3) = R_2(R_1+R_4)$$

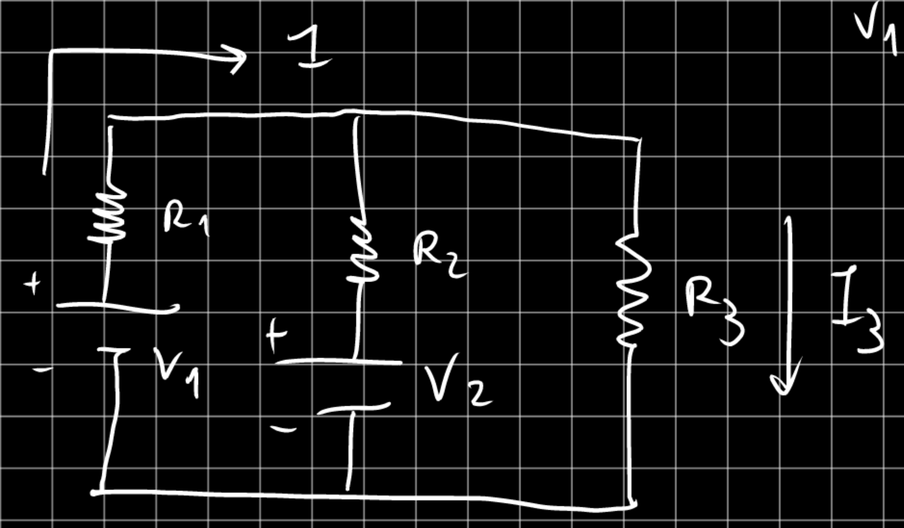
$$R_1 R_3 = R_3 R_4 \quad \checkmark$$

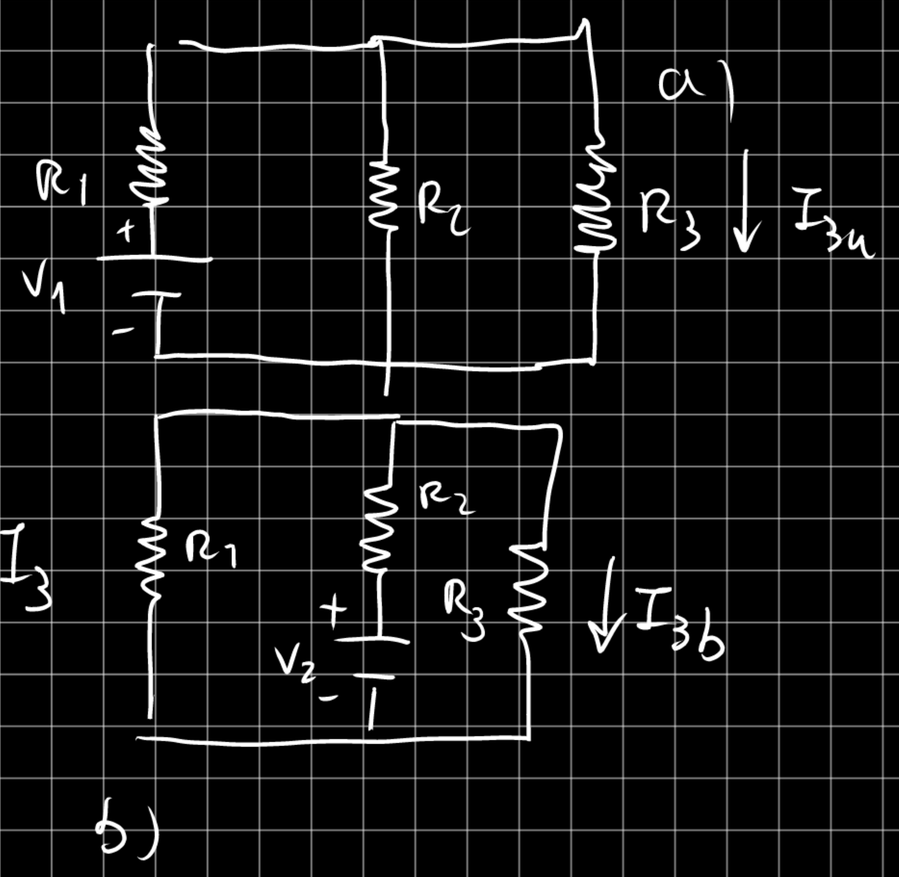
TEOREMA DI SOVRAPPOSIZIONE

→ Più generatori in circuito allora I_{TOT} in un ramo è somma correnti da singoli generatori isolati

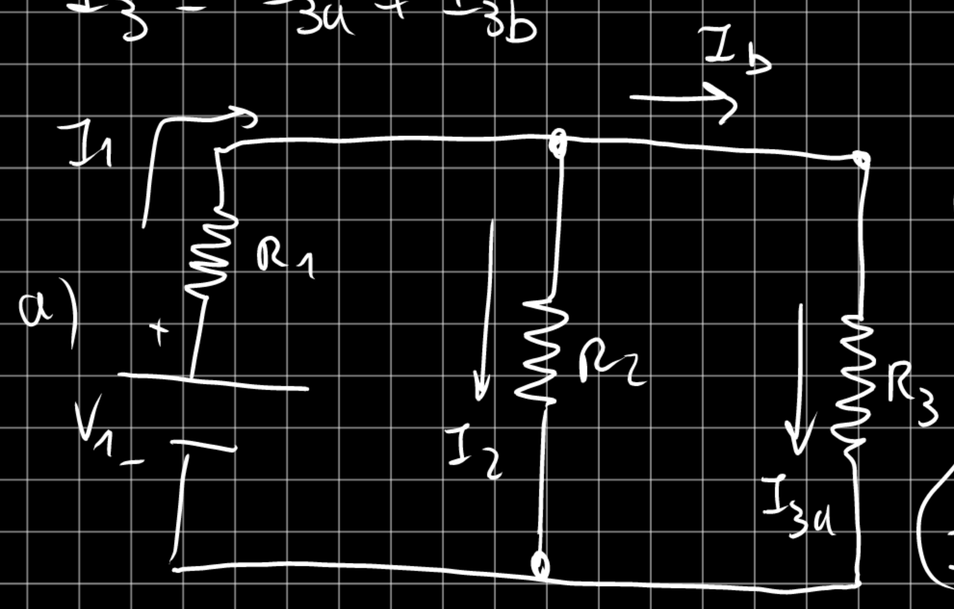
→ Deriva da Kirchhoff

E.G.





$$I_3 = I_{3a} + I_{3b}$$



$$\begin{cases} I_2 R_2 = I_{3a} R_3 \\ I_1 = \dfrac{V_1}{R_{eq}} \\ I_1 - I_2 - I_{3a} = 0 \end{cases}$$

$$\begin{cases} I_{3a} = \dfrac{R_2}{R_3} I_2 \\ I_1 = V_1/R_{eq} \\ \dfrac{V_1}{R_{eq}} - \left(1 + \dfrac{R_2}{R_3}\right) I_2 = 0 \end{cases}$$

$$\frac{1}{R_2} + \frac{1}{R_3} = \frac{R_2+R_3}{R_2 R_3} = \frac{1}{R_a} \qquad I_{3a} =$$

$$I_2 = \left(1 + \frac{R_2}{R_3}\right)^{-1} \frac{V_1}{R_{eq}} = \frac{R_3}{R_2+R_3} \frac{V_1}{R_{eq}}$$

$$R_{eq} = R_1 + R_a$$

$$I_{3a} = \frac{R_2 V_1}{R_2+R_3}\left(R_1 + \frac{R_2 R_3}{R_2+R_3}\right)^{-1}$$

b) come a) ma $R_2 \to R_1$ $\therefore I_{3b} = \frac{R_1 V_2}{R_1+R_3}\left(R_2 + \frac{R_1 R_3}{R_2+R_3}\right)^{-1}$

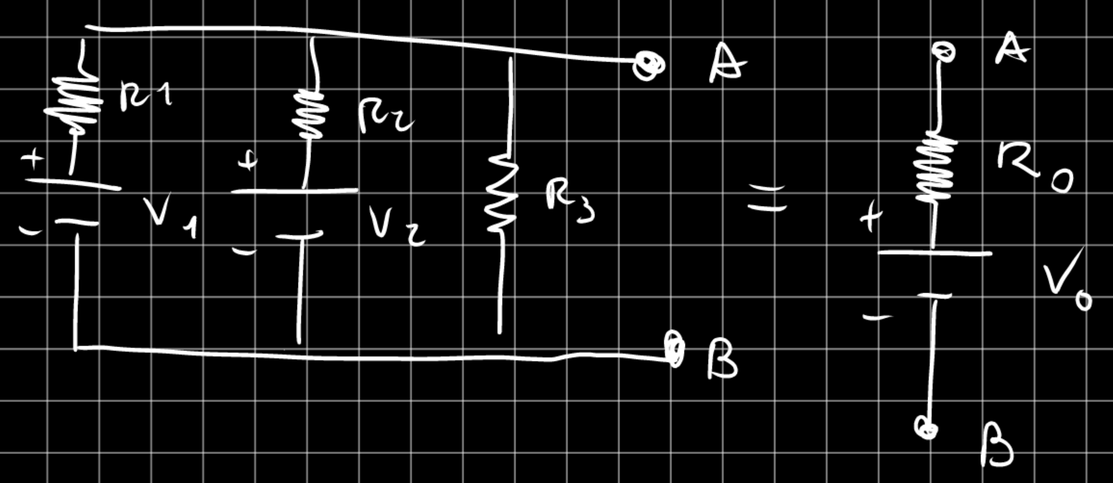
$$I_3 = I_{3a} + I_{3b} = \frac{R_2 V_1}{R_2+R_3}\left(\frac{R_1(R_2+R_3)+R_3R_2}{R_2+R_3}\right)^{-1} + \frac{R_1 V_2}{R_1+R_3}\left(\frac{R_2(R_1+R_3)+R_1R_3}{R_1+R_3}\right)^{-1}$$

THEVENIN

→ Necessario per semplificare circuiti

→ Sostituisce con $\overset{+}{\underset{-}{=}}$ ideale con R in serie

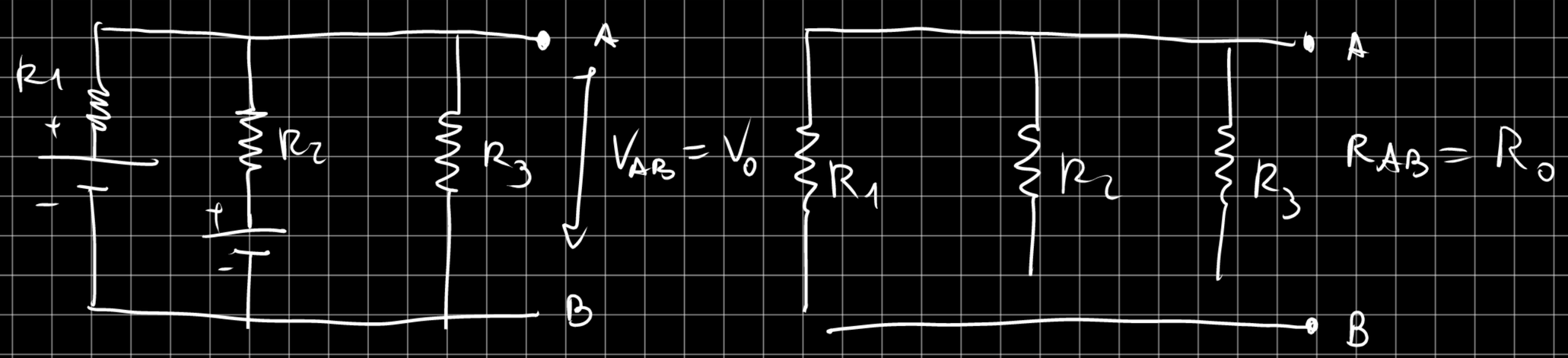
Qualsiasi circuito è schematizzabile in gen + res in serie



Generatori tensione in corto

Generatori di corrente aperti

Fissati due punti A, B, V_0 è $V_A - V_B$ e R_0 è R_{AB} quando tutti i gen sono in corto e quelli di corrente sono aperti. Quindi

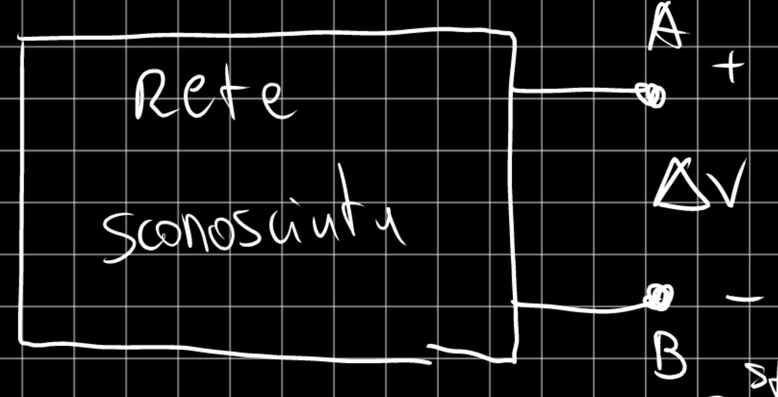


Se connetto una resistenza R in serie A-B ⇒ $R_{AB} = R + R_0$ ⇒ $I_{AB} = \frac{V_0}{R_0+R}$ ✓

⇒ Misuro V_0, notando che $V_{AB} = V_0$ (aperto),

⇒ Misuro R_0, notando che $I_{AB} = \frac{V_0}{R_0}$ (corto)

In generale

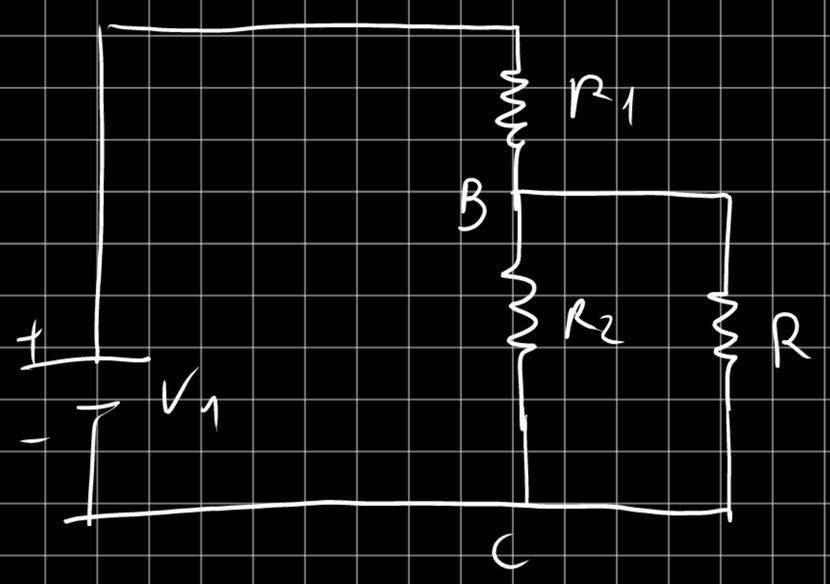


$$\begin{cases} I = \overbrace{\sum_i G_i V_i}^{\text{maglia}} - \overbrace{G \Delta V}^{AB} \\ [G] = [\Omega]^{-1} = \mho \text{ o Siemens} \\ \Delta V = \sum_i \frac{G_i}{G} V_i - \frac{I}{G} \end{cases}$$

DEF: $V_0 = \sum_i \overbrace{\frac{G_i}{G} V_i}^{\text{stacco } I}$, $R_0 = \frac{1}{G}$ } stacco ⇒ $\Delta V = V_0 - R_0 I$

Generatore in serie con R_0

PARTITORE CON THEVENIN

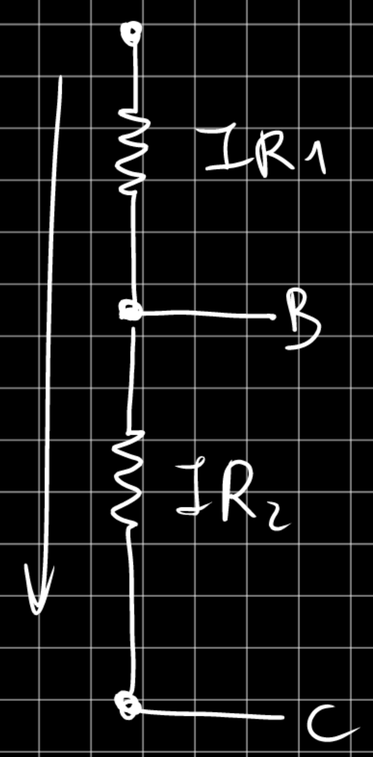


Cerco I_R.

1) Cortocircuito V (metto filo

2) Stacco a B e C

5)



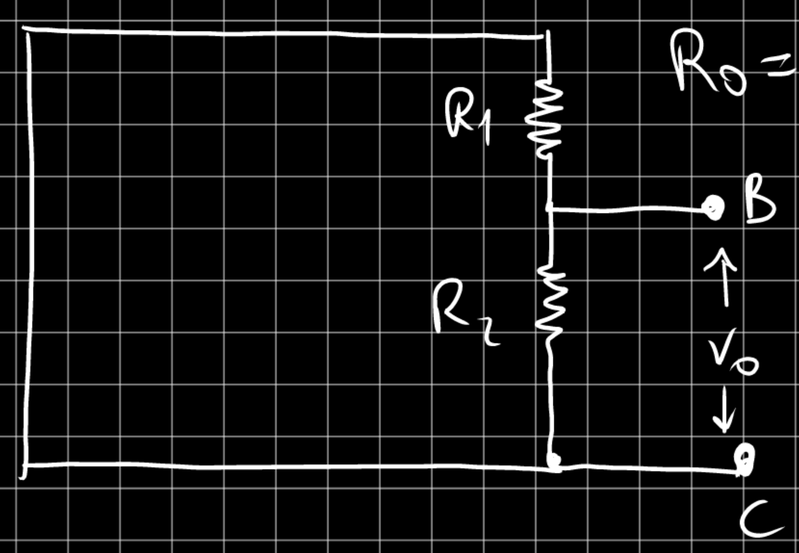
$$V_1 = IR_1 + IR_2$$

$$V_1 = (R_1 + R_2)\, I$$

$$I = \frac{V_1}{R_1 + R_2}$$

$$IR_2 = V_{BC} = \frac{R_2}{R_1 + R_2} V_1 = V_0$$

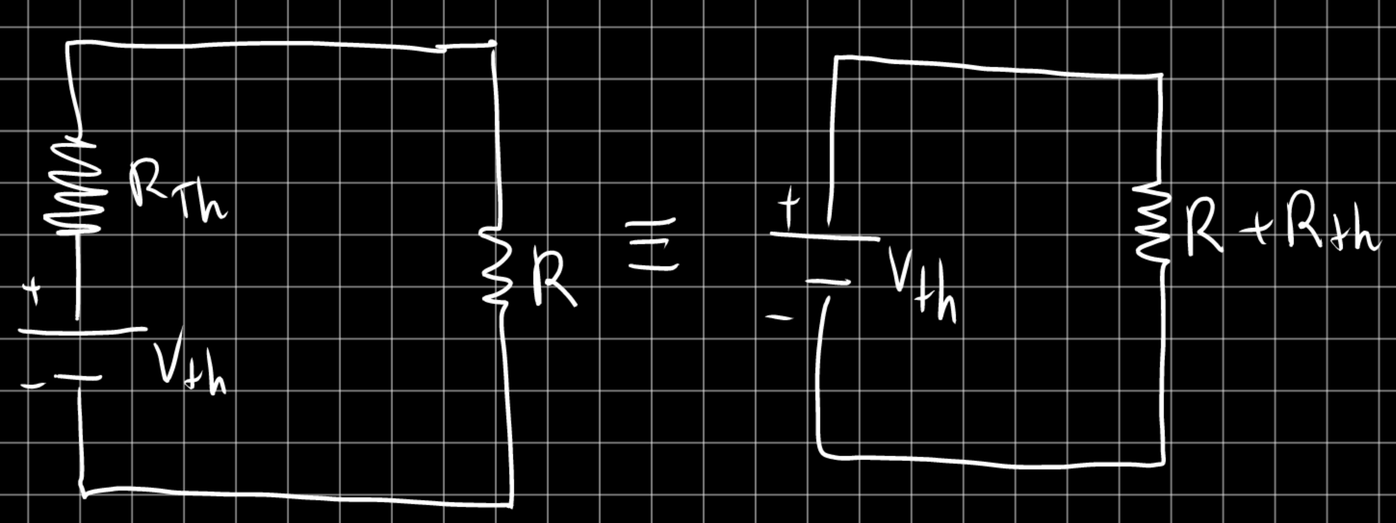
4)



$$R_0 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_0 = \frac{R_2}{R_1 + R_2} V_1 \quad , \quad R_0 = \frac{R_1 R_2}{R_1 + R_2}$$

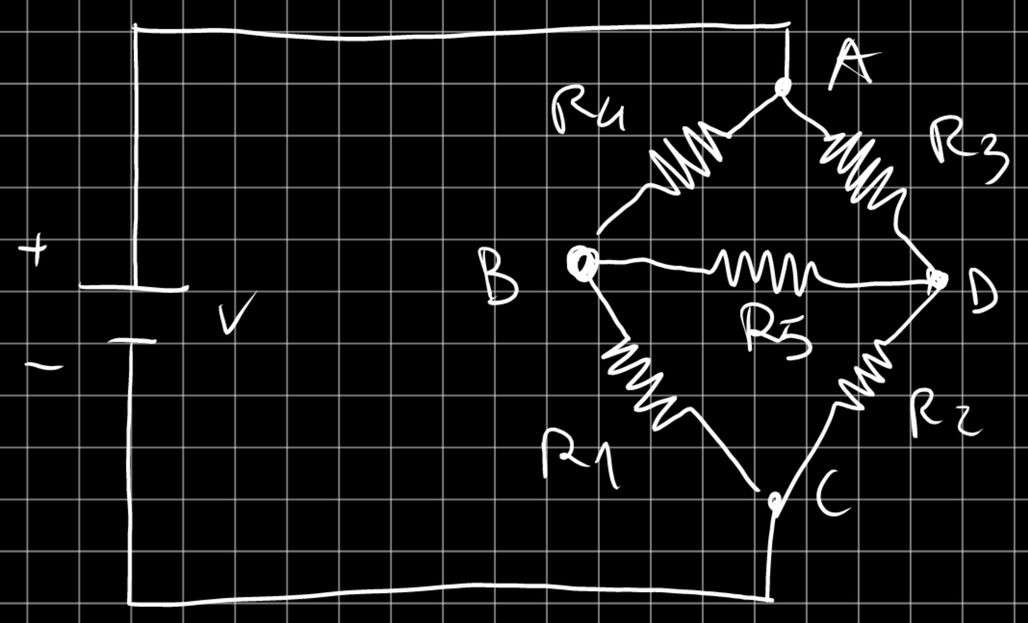
5)



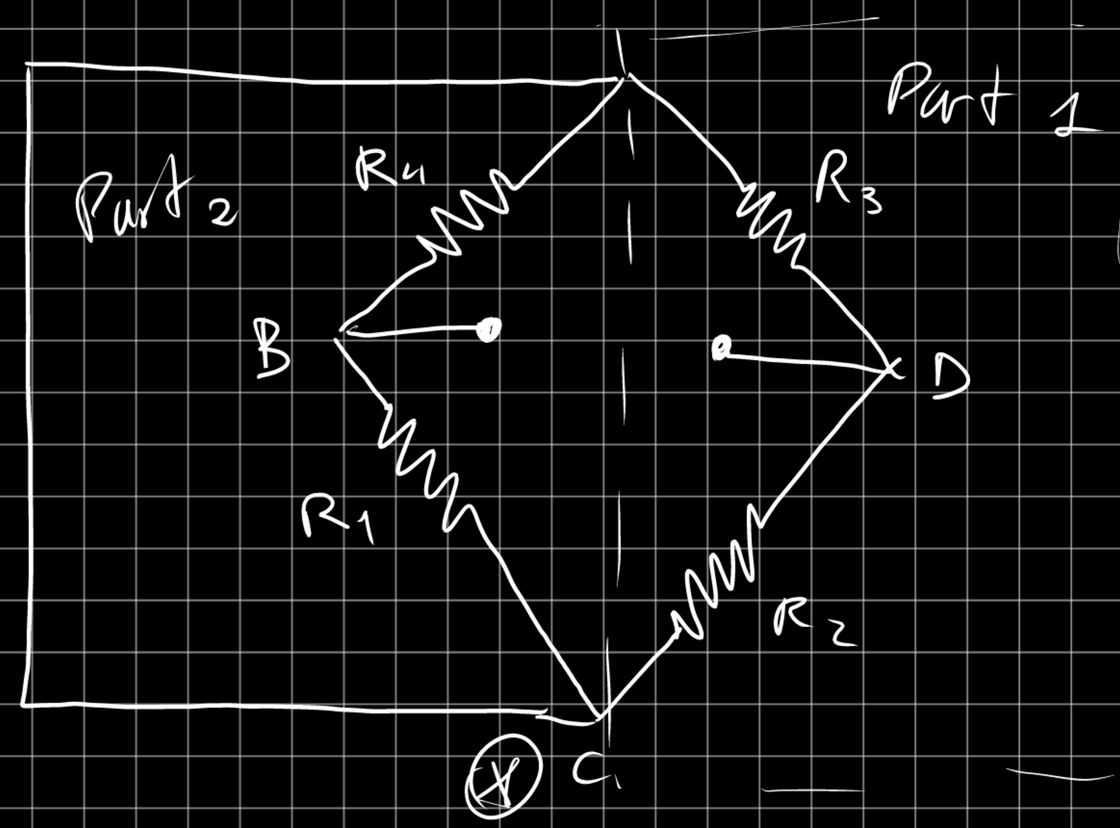
$$I_R = \frac{V_{Th}}{R + R_{Th}} = \frac{\frac{R_2}{R_1 + R_2} V_1}{R + \frac{R_1 R_2}{R_1 + R_2}} = \frac{R_2 V_1}{R R_1 + R R_2 + R_1 R_2}$$

ok! ✓

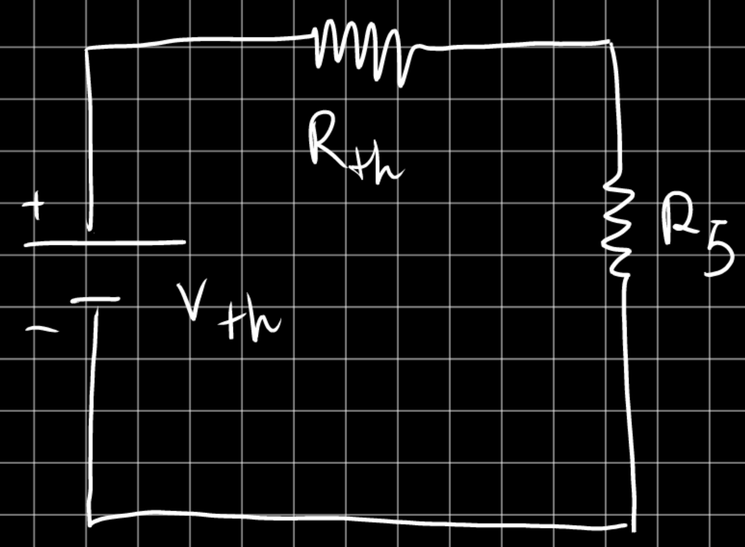
PONTE DI WHEATSTONE (THEVENIN)



$\xrightarrow{Th_1}$



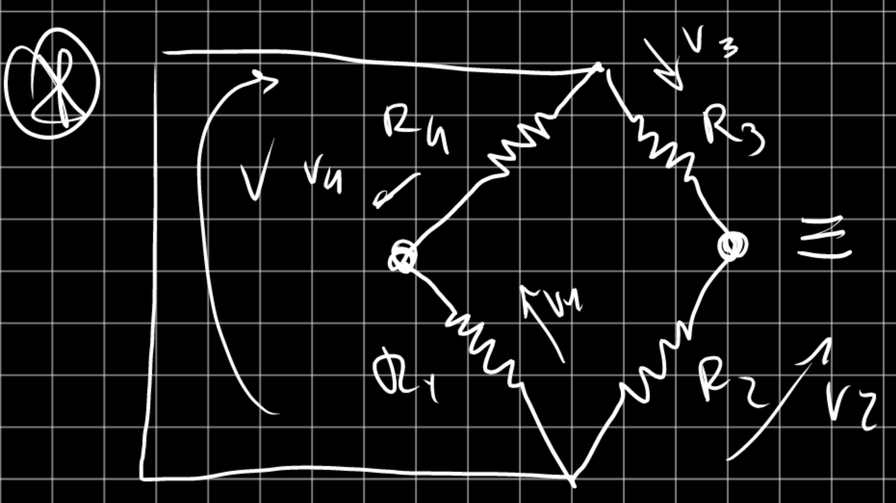
$Th_2)$

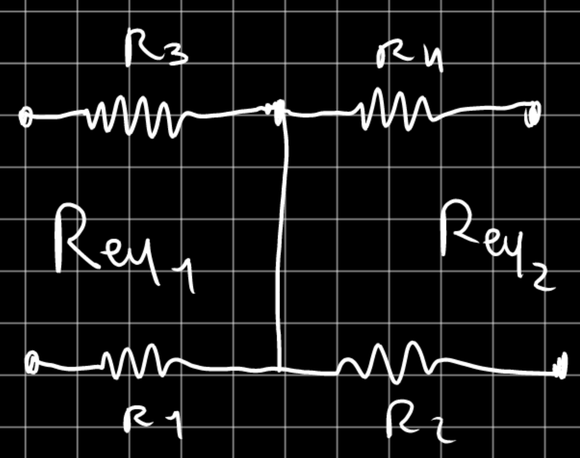


$\Rightarrow I_5 = \dfrac{V_{th}}{R_{th}+R_5}$ ✓

$$V_{part_2} = \frac{R_1 V}{R_1+R_4} \qquad V_{part_1} = \frac{R_2 V}{R_2+R_3}$$

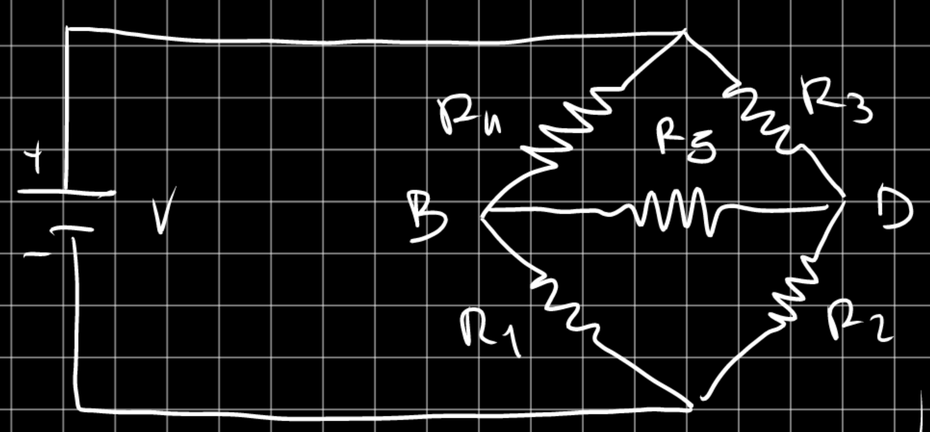
$$V_{th} = V_{part_2} - V_{part_1} = V\left(\frac{R_1}{R_1+R_4} - \frac{R_2}{R_2+R_3}\right) \checkmark$$





$$\begin{cases} Req_1 = \dfrac{R_3 R_1}{R_1+R_3} \\[2ex] Req_2 = \dfrac{R_4 R_2}{R_2+R_4} \end{cases}$$

$$R_{th} = \frac{R_3 R_1}{R_1+R_3} + \frac{R_4 R_2}{R_2+R_4}, \quad V_{th} = V\left(\frac{R_1}{R_1+R_4} - \frac{R_2}{R_2+R_3}\right), \quad I_5 = \frac{V_{th}}{R+R_{th}}$$

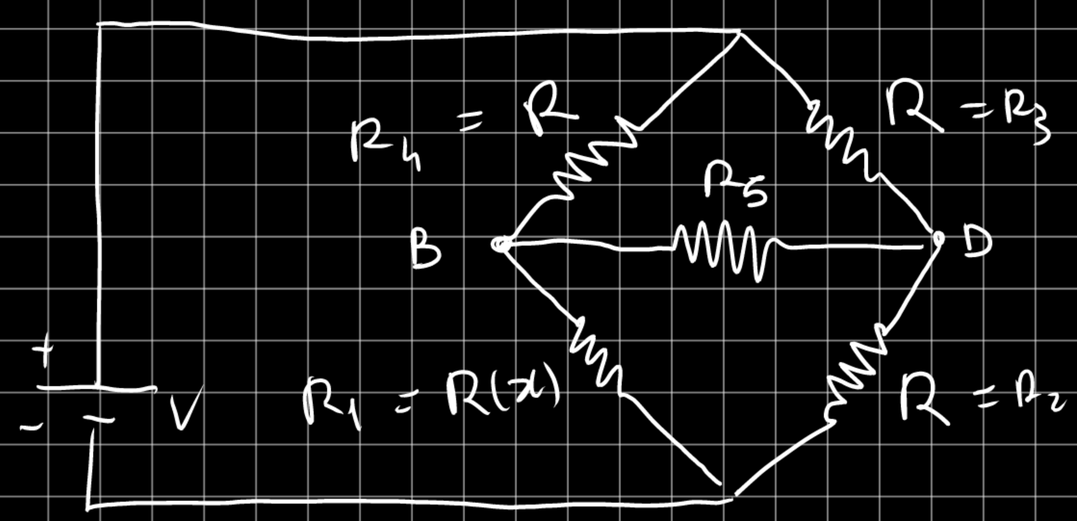


R_5 alta → come se fosse aperto! $V_5 = V_{th}$

$V_{th} \neq 0$ iff $\dfrac{R_1(R_2+R_3) - R_2(R_1+R_4)}{(R_1+R_4)(R_2+R_3)} \neq 0$

$$\boxed{R_1 R_3 \neq R_2 R_4} \checkmark$$

MISURE SENSIBILI CON WHEATSTONE



1) $R_5 \gg R$

2) $R_1 \longrightarrow R(x)$

So che

$$V_{BD} = V\left(\underbrace{\frac{R_1}{R_1+R_4}}_{\frac{R(x)}{R(x)+R}} - \underbrace{\frac{R_2}{R_2+R_3}}_{1/2}\right)$$

$$\therefore V_{BD}(x) = \left(\frac{R(x)}{R(x)+R} - \frac{1}{2}\right)V \simeq \frac{V}{4R}\left(R(x) - R\right)$$

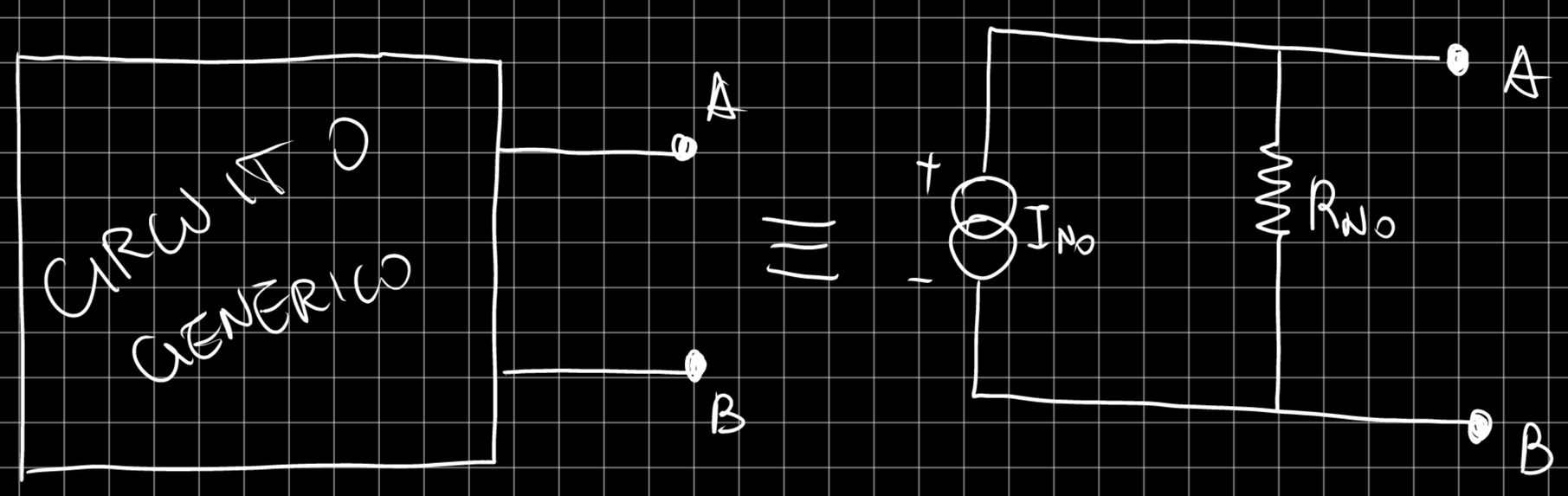
e.g. Cerco se $R = 10^4\,\Omega$ oppure $(10^4+1)\,\Omega$, misura diretta richiede strumento ipersensibile.

$\Rightarrow$ Costruisco ponte di Wheatstone e faccio variare $R(x)$ fino a bilanciamento! ($V_{BD} = 0$)

$R(x)$ è un trasduttore (potenziometro?)

TEOREMA DI NORTON

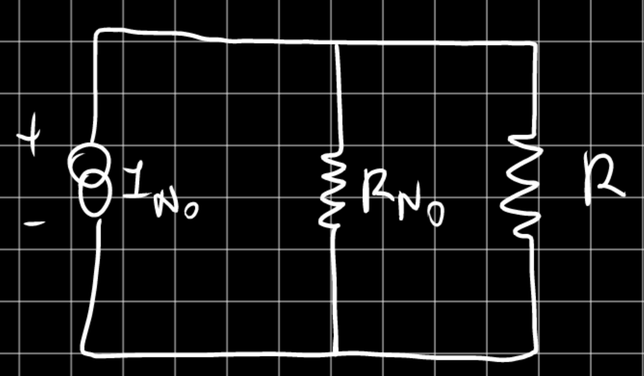
Qualsiasi circuito lineare attivo può essere schematizzato con generatore di corrente più R_{N_0} in parallelo.



1) I_{N_0} è corrente di corto tra A e B (attraverso filo)

2) R_{N_0} è resistenza dove: $\begin{cases} \text{in corto} \\ \text{aperto} \end{cases}$

3) Duale a Thevenin



$$I_{R_{N_0}} R_{N_0} = R I_R \Rightarrow I_{R_{N_0}} = \frac{R}{R_{N_0}} I_R$$

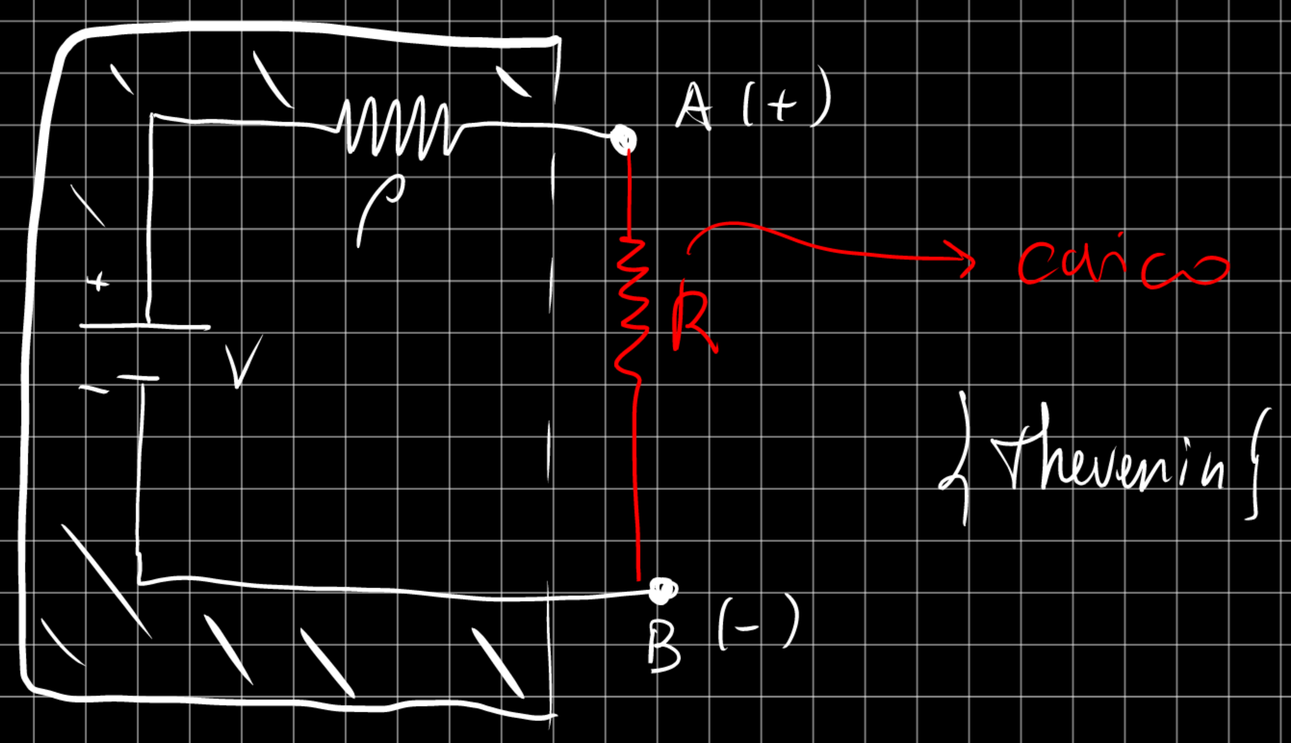
$$I_{N_0} = I_{R_{N_0}} + I_R = \frac{R}{R_{N_0}} I_R + I_R = \left(\frac{R}{R_{N_0}} + 1\right) I_R$$

$$\therefore V_{Th} = I_{N_0} R_{N_0} = \left(\frac{R}{\cancel{R_{N_0}}} + R_{N_0} \right) I_R \cancel{R_{N_0}} = (R + R_{N_0}) I_R$$

$$\Rightarrow I_R = \frac{V_{Th}}{R + R_{N_0}} \quad \begin{cases} 1)\ R_{Th} = R_{N_0} \\ 2)\ V_{Th} = I_{N_0} R_{N_0} \\ 3)\ I_{N_0} = \dfrac{V_{Th}}{R_{Th}} = \dfrac{V_{Th}}{R_{N_0}} \end{cases}$$

GENERATORI REALI

Un generatore REALE può essere schematizzato come serie tra gen (V) & resistore ρ (resistenza interna

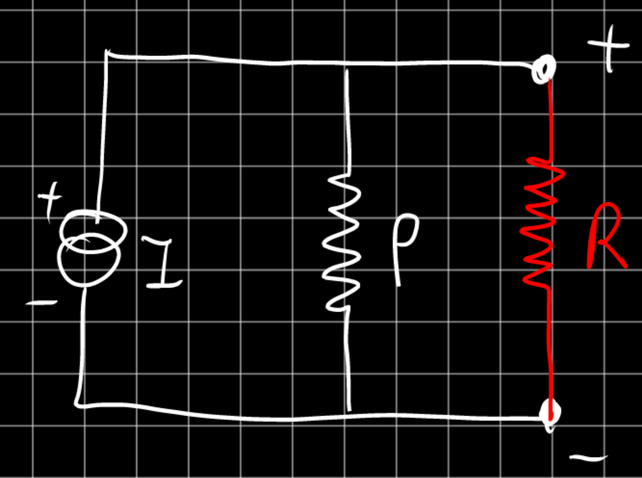


{Thevenin}

Con carico R $\frac{\partial V}{\partial I_R} \neq 0$;

$$I_R = \frac{V}{R+\rho} \; ; \; V_R = R I_R = \frac{R}{R+\rho} V \approx V\left(1 - \frac{\rho}{R}\right) \quad (\rho << R)$$

Per un generatore di corrente invece ho



(Norton)

Generatore di corrente $\simeq$ ideale se $R << \rho$

$$I = I_\rho + I_R \; ; \; \rho I_\rho = R I_R \rightarrow I_\rho = \frac{R}{\rho} I_R$$

$$I = I_R \left(1 + \frac{R}{\rho}\right) = I_R \left(\frac{\rho + R}{\rho}\right)$$

$$\therefore I_R \simeq I \left(1 - \frac{R}{\rho}\right)$$

TRASFERIMENTO DI POTENZA DA GEN A CARICO

-) Batteria converte $E_{CHIM} \rightarrow E_{EL}$

Se carico è $R \Rightarrow E_{EL} \rightarrow E_{THERM}$ (Joule)

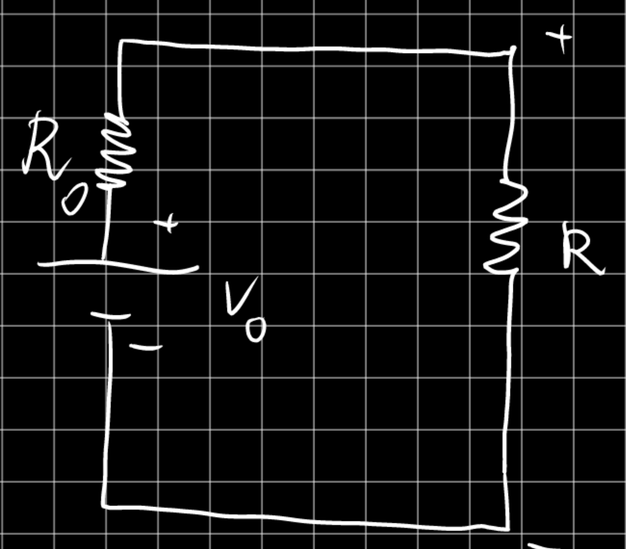
EFFETTO JOULE

Circuito con resistenza R e ddp V. Corrente $I = \frac{dQ}{dt}$

$$dW = VdQ \Rightarrow dW = VIdt$$

$$P = \frac{dW}{dt} = VI \; ; \; [P] = W = J/s \quad (\text{watt})$$

$$[P] = [V][I] = VA = J/s$$



$$\begin{cases} P_R = V_R I_R \; ; \quad V_R = \frac{R}{R+R_0} V_0 \quad (\text{partitore}) \\ P_R = \frac{R_0}{R+R_0} I_R \qquad I_R = \frac{V_R}{R} = \frac{1}{R+R_0} V_0 \; ✓ \end{cases}$$

$$P_R = \left(\frac{R}{R+R_0} V_0\right)\left(\frac{1}{R+R_0} V_0\right) = \frac{RV_0^2}{(R+R_0)^2}$$

$$\frac{\partial P_R}{\partial R} = \left(\frac{V_0}{R+R_0}\right)^2 - \frac{2(R+R_0)RV_0^2}{(R+R_0)^4} = V_0^2\left(\frac{1}{(R+R_0)^2} - \frac{2(R+R_0)R}{(R+R_0)^4}\right)$$

$$\frac{\partial P_R}{\partial R} = V_0^2\left(\frac{(R+R_0)^{\cancel{2}} - 2\cancel{(R+R_0)}R}{(R+R_0)^3}\right) = V_0^2 \frac{R_0 - R}{(R+R_0)^2}$$

$\frac{\partial P_R}{\partial R} = 0$ iff $R = R_0 \Rightarrow \max_R \{P_R\} = P_R(R_0) = \frac{1}{4R_0} V_0^2$] CARICO ADATTATO

$\max_R \{P_R\} \not\Rightarrow$ max efficienza. $\boxed{\text{DEF}}$: Efficienza trasferimento

$$\eta = \frac{P_R}{P_{TOT}} = \frac{P_R}{P_{Gen} + P_R}$$

$P_R = V_R I_R = R I_R^2$, $P_{Gen} = V_0 I_0 = R_0 I_0^2$

$$\eta = \frac{R I_R^2}{R_0 I_0^2 + R I_R^2} = \frac{R}{R_0 + R} \quad \{I_R = I_0\}$$

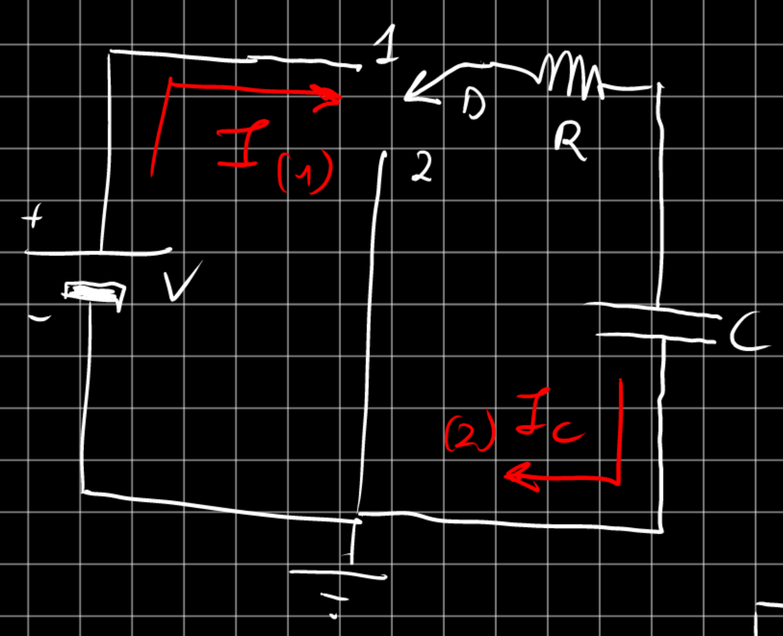
$\eta = 1$ iff $R \gg R_0$! $\eta = 50\%$ per $R = R_0$

CORRENTI VARIABILI

CIRCUITO RC

N.B. $V = \frac{1}{C}\int_0^t I\,dt$; $I = C\frac{dV}{dt}$

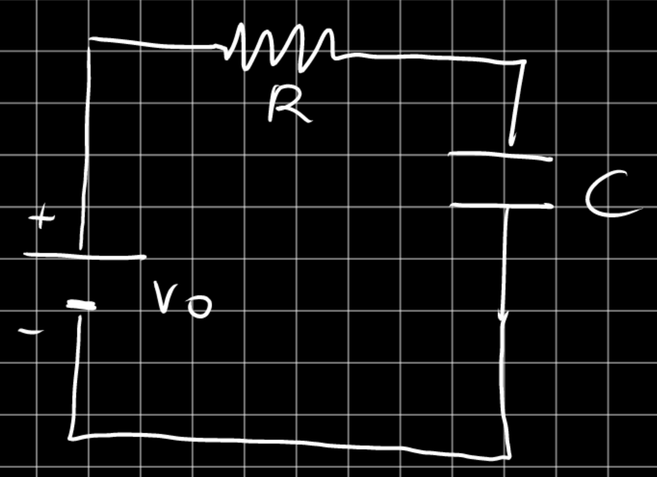
Carica & Scarica di C



1 → carica $I \to C$ $t = t_0$

2 → scarica $C \to \overline{\overline{\overline{\ }}}$ $t = t_1$

Switch su 1 →



Quando C è carico ⇒ $V_C = \frac{Q}{C}$ (def.)

$$V_0 = V_R + V_C = RI + \frac{Q}{C} = R\frac{dQ}{dt} + \frac{Q}{C} = V_0$$

$$CV_0 - Q = RC\frac{dQ}{dt} \to \frac{dQ}{dt} = \frac{CV_0 - Q}{RC}$$ (Risolvi ODE separabile 1° ord)

$$\left[\frac{dQ}{CV_0 - Q} = \frac{1}{RC}\,dt\right]$$

$$\therefore \frac{1}{CV_0 - Q}\,dQ = -\int d\log(CV_0 - Q) = \int \frac{1}{RC}\,dt$$

$$\Rightarrow -\log\left(\frac{CV_0 - Q}{CV_0 - Q_0}\right) = \frac{t}{RC} \quad ; \quad Q_0 = Q(0) = 0$$

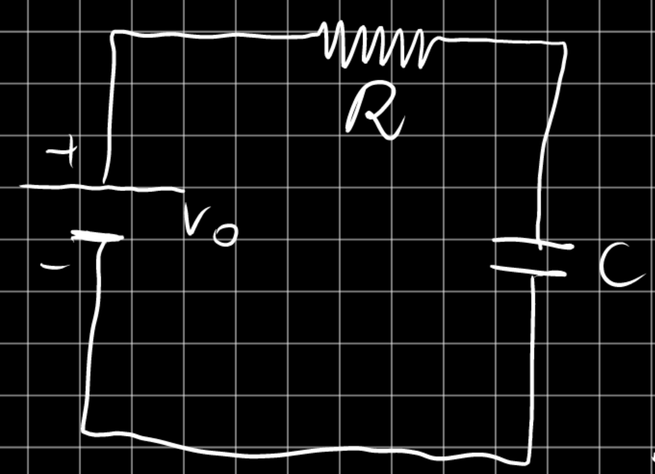
$$\therefore CV_0 - Q = CV_0\,e^{-\frac{t}{RC}}$$

$$Q = CV_0\left(1 - e^{-t/RC}\right)$$

DEF $RC = \tau$ (tempo di rilassamento)

$$\Rightarrow V_C(t) = \frac{1}{C} Q(t) = V_0\left(1 - e^{-t/RC}\right) = V_0\left(1 - e^{-t/\tau}\right)$$

Per $t \gtrsim 4\tau \Rightarrow$ a regime

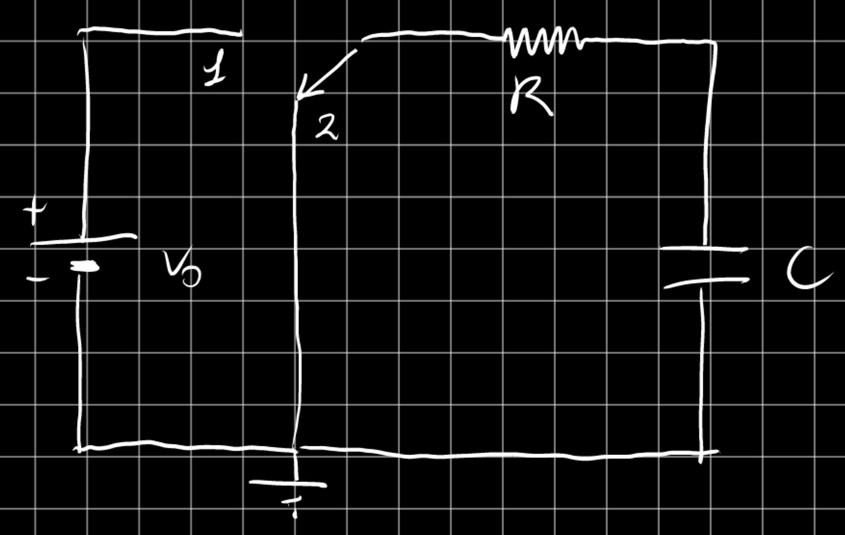


$$V_R = RI = R\frac{dQ}{dt}$$

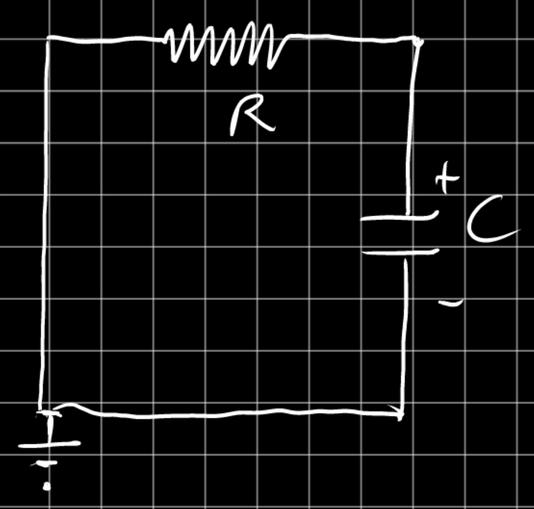
$$\frac{dQ}{dt} = \frac{CV_0}{\tau} e^{-t/\tau}$$

$$\therefore V_R = \frac{RC}{\tau} V_0 e^{-t/\tau} = V_0 e^{-t/\tau}$$

Scarico



$\equiv$



$$IR + V_C = 0$$

$$R\frac{dQ}{dt} = -V_C$$

$$V_C(t) = \frac{1}{C} Q(t) \Rightarrow R\frac{dQ}{dt} = -\frac{1}{C} Q(t) \Rightarrow \frac{d\log(Q)}{dt} = \frac{-1}{RC}$$

$$\log(Q(t)) + k = -\frac{t}{RC} \Rightarrow Q(t) = k e^{-\frac{t}{RC}}$$

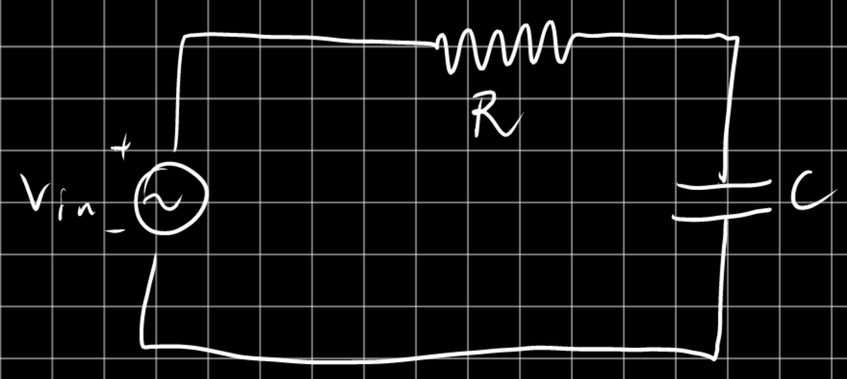
$$V_C = \frac{Q(t)}{C} = \frac{k}{C} e^{-t/RC} \; ; \; V_C(0) = V_0 \Rightarrow k = CV_0$$

$$\therefore \begin{cases} Q(t) = CV_0 e^{-t/RC} \\ V_C(t) = V_0 e^{-t/RC} \end{cases}$$

AC

Consideriamo adesso un circuito RC con $V_{in} = V_{in}(t)$ armonico

$$V_{in}(t) = V_0 \cos(\omega t)$$



$$V_C = \frac{Q}{C} \; , \; V_R = RI = R\frac{dQ}{dt}$$

$$\boxed{V_{in} = R\frac{dQ}{dt} + \frac{Q}{C}}$$ OK

$\therefore \; V(t) - R\dot{Q} - \frac{Q}{C} = 0 \; ; \; V(t) = V_0 \cos(\omega t)$

$\frac{Q}{C} + R\dot{Q} = V_0 \cos(\omega t) \; ; \; \frac{Q}{C} = V_c(t) \; , R\dot{Q} = RC\dot{V}_c(t)$ ok

$V_c(t) + RC\dot{V}_c(t) = V_0 \cos(\omega t)$

$\rightarrow$ similitudine $\therefore \; V_c(t) \propto \cos(\omega t) \; ; \; V_c(t) = V_{0c} \cos(\omega t + \varphi)$

$\dot{V}_c(t) = -\omega V_{0c} \sin(\omega t + \varphi)$

$V_{0c} \cos(\omega t + \varphi) - \omega RC \sin(\omega t + \varphi) = V_0 \cos(\omega t)$ {merda}

$\rightarrow$ Salgo $V_c^{(M)}(t) = Re\{V_{0c} e^{j\omega t}\} \Leftarrow$ Misuro $V_c^{(M)}$ <u>NON</u> $V_c(t)$!

$V_c(t) = V_{0c} e^{j\omega t + j\varphi} \; , \; \dot{V}_c(t) = j\omega V_{0c} e^{j\omega t + j\varphi}$ {$j^2 = -1$, notazione del corso}

$V_c(t) + RC\dot{V}_c(t) = V_{0c} \cancel{e^{j\omega t}} e^{+j\varphi} + RCj\omega V_{0c} \cancel{e^{j\omega t}} e^{+j\varphi} = V_0 \cancel{e^{j\omega t}}$

$V_{0c} e^{j\varphi} + RCj\omega V_{0c} e^{j\varphi} = V_0$

$(1 + RCj\omega) V_{0c} e^{j\varphi} = V_0 \Rightarrow V_{0c} e^{j\varphi} = \frac{1}{1 + RCj\omega} V_0 = \frac{1 - RCj\omega}{1 + R^2C^2\omega^2} V_0$

$V_{0c} e^{j\varphi} = \frac{1}{1 + (RC\omega)^2} V_0 - \frac{jRC\omega}{1 + (RC\omega)^2} V_0$

$\therefore \; V_{0c}^{(M)} = \frac{1}{1 + (RC\omega)^2} V_0$ (1)

(2) $\varphi = \arg\{V_{0c} e^{j\varphi}\} = \arctan\left(\frac{Im\{V_c(t)\}}{Re\{V_c(t)\}}\right)$

$Im\{V_c(t)\} = -\frac{RC\omega}{1 + (RC\omega)^2} V_0 \; , \; Re\{V_c(t)\} = \frac{1}{1 + (RC\omega)^2} V_0 \Rightarrow \frac{Im\{V_c(t)\}}{Re\{V_c(t)\}} = -RC\omega$

$\therefore \; V_{0c}^{(M)} = \frac{1}{1 + (RC\omega)^2} V_0 \cos\left(\omega t + \arctan(-RC\omega)\right)$ $\quad RC = \tau$

$\therefore \begin{cases} V_{0c}^{(M)} = [1 + (\tau\omega)^2]^{-1} V_0 \\ \varphi = \arctan(-\omega\tau) \end{cases} \Rightarrow \begin{cases} \lim_{\omega \to 0} V_c^{(M)} = V_0 \\ \lim_{\omega \to \infty} V_c^{(M)} = 0 \end{cases}$

$\rightarrow 0 < \omega < \infty \quad V_c^{(M)}(t) < V_0(t)$ con dephasing

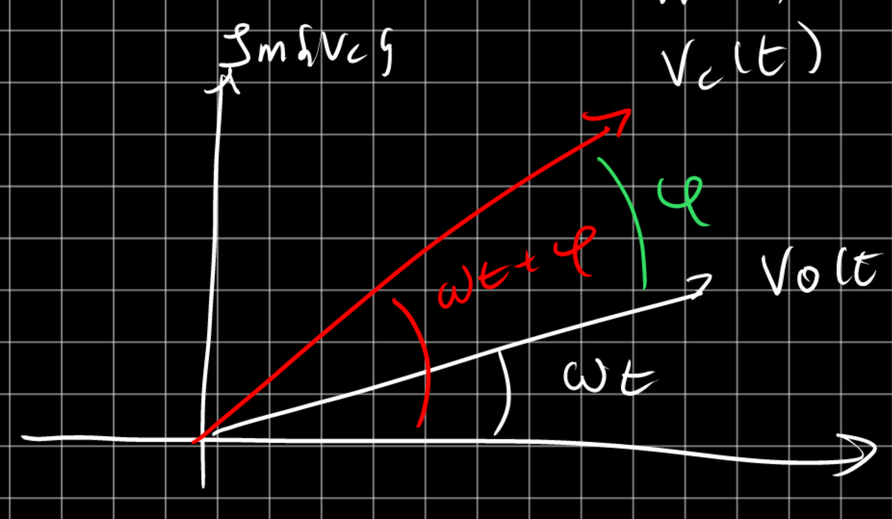
① $\omega \ll (RC)^{-1} \rightarrow V_C \neq 0$

② $\omega \gg (RC)^{-1} \rightarrow V_C = 0$

} FILTRO IN FREQUENZA PASSA BASSO

$\omega_T = \frac{1}{RC}$ → freq. taglio

Yuddu yuddu $V_C(t) \propto e^{j\omega t} \in \mathbb{C}$ → rapp. fasori



Rotazione CCW con vel ang $\omega = 2\pi\nu = 2\pi \underbrace{f}$ → frequenza (not. uzzo)

CORRENTE?

1) $V_C(t) = \frac{1}{C} Q(t) \Rightarrow i(t) = \dot{Q} \Rightarrow i(t) = C\dot{V}_C(t)$! ✓

$$i(t) = C\frac{d}{dt}\left(V_{0c}\, e^{j\omega t+\varphi}\right) = j\omega V_{0c} e^{j\omega t+\varphi} = j\omega C V_c$$

$$i_c(t) = i(t) = j\omega C V_c(t) \leftarrow !!$$

$V = RI \Rightarrow \begin{cases} I = Z_e V \\ Z_e = \frac{1}{R} \end{cases} \rightarrow \begin{cases} i(t) = Z_{ec} V_c(t) \\ Z_{ec}(t) = j\omega C \end{cases}$ } LEGGE DI OHM GENERALIZZATA

$$\begin{cases} V_c(t) e^{j\varphi} = \frac{V_0}{1+j\omega RC} e^{j\omega t} \\ i(t) e^{j\varphi} = \frac{Z_{ec} V_0}{1+j\omega RC} e^{j\omega t} = \frac{j\omega C V_0}{1+j\omega RC} e^{j\omega t} \end{cases}$$

$V_0 e^{j\omega t} = V_{in}(t)$

$$\therefore\ i(t) = \frac{j\omega C}{1+j\omega RC} V_{in}(t)$$

$i(t) = I_0 e^{j\omega t + j\varphi_I}$

$$j\omega C\left(\frac{1-j\omega C}{1+(\omega RC)^2}\right) = \frac{j\omega C + (\omega C)^2}{1+(\omega RC)^2} \quad (*)$$

(*) $I_0^2 = \mathrm{Re}\left\{ \frac{j\omega C + (\omega C)^2}{1+(\omega RC)^2} \right\} = \frac{(\omega C)^2}{1+(\omega RC)^2} V_0^2$ (Now $\sqrt{\ }$?)

$$\therefore I_0 = \frac{\omega C}{(1+(\omega RC)^2)^{1/2}} V_0 = \frac{\omega C}{(1+(\omega\tau)^2)^{1/2}} V_0$$

$$\varphi_I = \arctan\left(\frac{\mathrm{Im}\{i\}}{\mathrm{Re}\{i\}}\right) = \arctan\left(\frac{1}{\omega \underset{\tau}{C}}\right) = \arctan\left(\frac{\overset{1}{\cancel{R}}}{\omega\tau}\right) \ ?$$

$$\frac{j\omega C}{1+(\omega RC)^2} + \frac{\omega^2 C^2}{1+(\omega RC)^2} \qquad \frac{\dfrac{\cancel{\omega C}}{\cancel{1+(\omega RC)^2}}}{\dfrac{\omega^2 C^2}{\cancel{1+(\omega RC)^2}}} = \frac{1}{\omega C}$$

è sfunta!

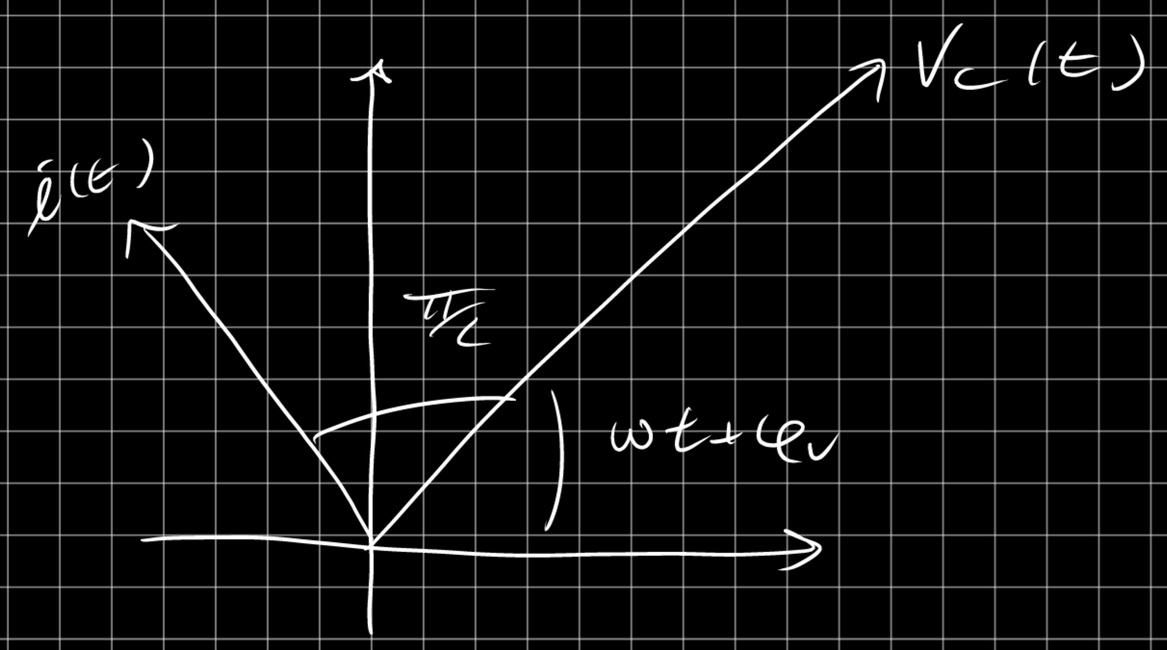
$$\begin{cases} I_0 = \dfrac{\omega C}{(1+(\omega RC)^2)^{1/2}} V_0 \\ \varphi_I = \arctan\left(\dfrac{\cancel{R}\,1}{\omega\tau}\right) \end{cases} \qquad \begin{cases} V_{C0} = \dfrac{1}{(1+\omega^2\tau^2)^{1/2}} V_0 \\ \varphi_V = \arctan(-\omega\tau) \end{cases}$$

1) $\tan\varphi_I = \dfrac{R}{\omega\tau}$, $\tan\varphi_V = -\omega\tau$

CRISTO MALATTE

$$\tan(\varphi_I - \varphi_V) = \frac{\tan(\varphi_I) - \tan(\varphi_V)}{1+\tan(\varphi_V)\tan(\varphi_I)} = \frac{(\omega\tau)^{-1} + \omega\tau}{1 - \dfrac{\omega\tau}{\omega\tau}} = \frac{(\omega\tau)^{-1} + \omega\tau}{0} = \infty$$

$$\therefore \varphi_I - \varphi_V = \frac{\pi}{2} = \Delta\varphi_{VI}$$



LEGGI DI OHM GENERALIZZATE

CONDENSATORI

-) AC $\Rightarrow V_C(t) = V_{0C} e^{j\omega t + j\varphi} \Rightarrow i(t) = \dot{q} = C\dot{V}_C = C j\omega V_{0C} e^{j\omega t + j\varphi}$

$$i(t) = j\omega C\, V_C(t)$$

$$\therefore V_C(t) = \frac{1}{j\omega C} i(t)$$

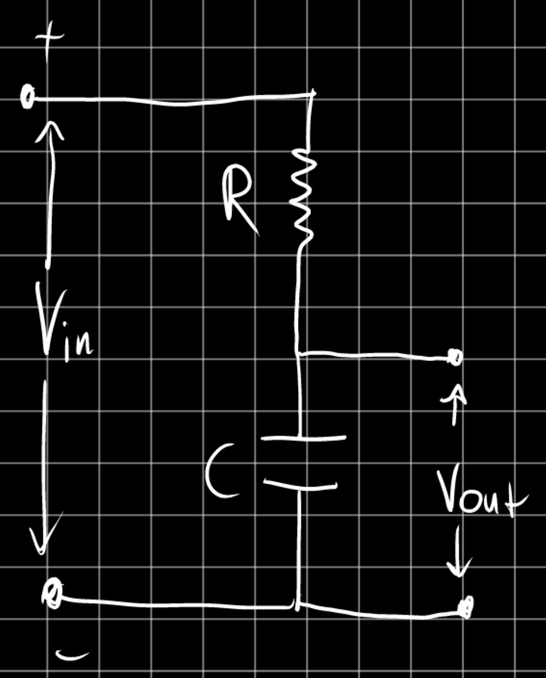
$\Rightarrow V = RI \longrightarrow V(t) = Z_C I$; $Z_C = (j\omega C)^{-1}$

INDUTTORE

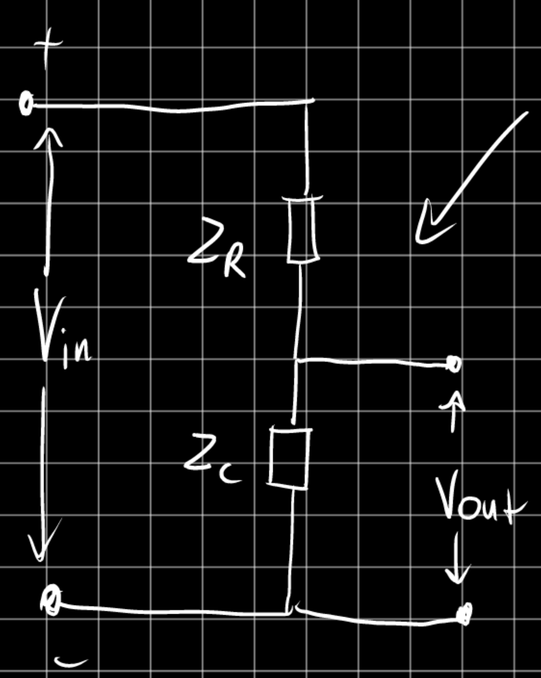
LENZ: $V_L = L\dot{I}_L$, $I_L(t) = i_{0_L} e^{j\omega t + j\varphi}$, $\dot{I}_L = j\omega I_L$

$$\therefore V_L(t) = j\omega L I_L = Z_L I_L \; ; \; Z_L = j\omega L$$

Di conseguenza per AC con <u>solo</u> R, L, C vale $V = Z_e I$. E.g.



$\equiv$



Partitore di Tensione!

$$V_{out} = \frac{Z_C}{Z_C + Z_R} V_{in}$$

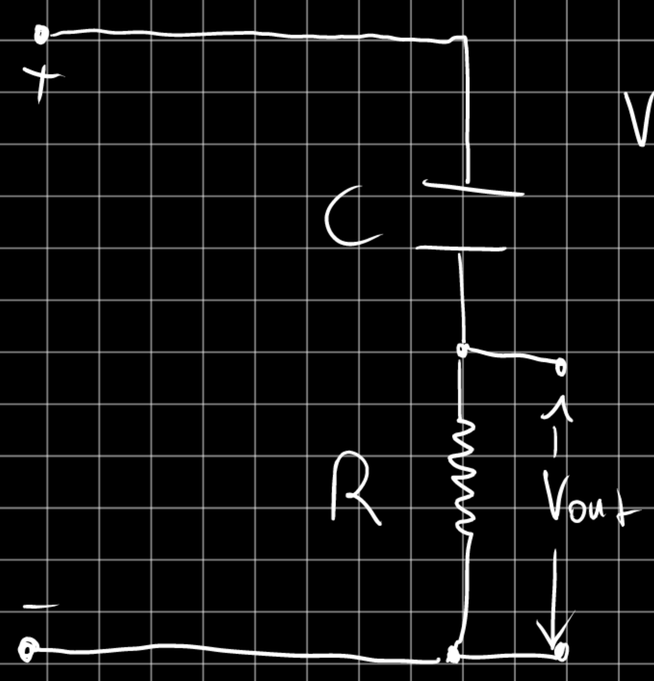
$$\begin{cases} Z_R = R \\ Z_C = (j\omega C)^{-1} \end{cases}$$

$$V_{out} = \frac{1}{j\omega C} \frac{V_{in}}{\frac{1}{j\omega C} + R} = \frac{V_{in}}{1 + j\omega RC} \checkmark \; , \; V_{0C} = \frac{1}{\sqrt{1 + (\omega RC)^2}} V_{in}$$

Ricordo che $V_{out} \propto e^{j\omega t + j\varphi}$, $V_{in} \propto e^{j\omega t}$ → fasori

$$\varphi = \arctan(-\omega RC)$$

Risolvo CR



$$V_{out}(t) = \frac{R}{R + Z_C} V_{in}(t) = \frac{R}{R + \frac{1}{j\omega C}} V_{in}(t) = R \frac{V_{in}(t)}{\frac{j\omega RC + 1}{j\omega C}}$$

$$V_{out}(t) = \frac{j\omega RC}{j\omega RC + 1} V_{in}(t)$$

$$\frac{j\omega RC}{j\omega RC + 1} = \left(\frac{1 - j\omega RC}{1 + (\omega RC)^2}\right) j\omega RC = \frac{j\omega RC + (\omega RC)^2}{1 + (\omega RC)^2}$$

$$\mathrm{Im}\left\{\frac{V_{out}}{V_{in}}\right\} = \frac{\omega RC}{1 + (\omega RC)^2} \qquad \mathrm{Re}\left\{\frac{V_{out}}{V_{in}}\right\} = \frac{(\omega RC)^2}{1 + (\omega RC)^2}$$

$$\varphi = \arctan\left(\frac{\frac{\cancel{\omega RC}}{\cancel{1 + (\omega RC)^2}}}{\frac{(\omega RC)^2}{\cancel{1 + (\omega RC)^2}}}\right) = \arctan\left(\frac{1}{\omega RC}\right)$$

$$V_{0R} = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}} V_{in}$$

RC

$$\therefore \frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{1+(\omega RC)^2}} \propto \frac{1}{\sqrt{1+\omega^2}}$$

RC → passa basso!

$$\omega \gg \frac{1}{RC} \Rightarrow \frac{V_{out}}{V_{in}} \simeq 0$$

CR

$$\frac{V_{out}}{V_{in}} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} \propto \frac{\omega}{\sqrt{1+\omega^2}}$$

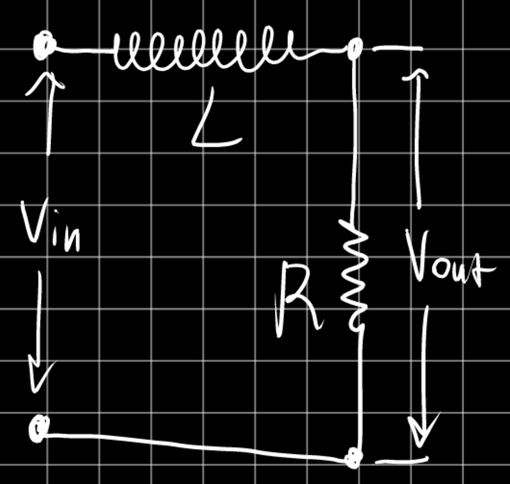
CR → passa alto!

$$\omega \ll \frac{1}{RC} \Rightarrow \frac{V_{out}}{V_{in}} \simeq 0$$

FILTRI

FILTRI CON INDUTTORI

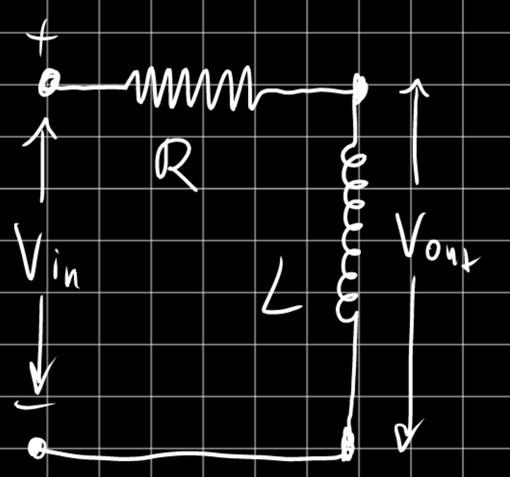
LR



$$V_{out} = \frac{Z_R}{Z_L+Z_R} V_{in} = \frac{R}{j\omega L + R} V_{in} = \frac{1}{j\omega\left(\frac{R}{L}\right)+1} V_{in}$$

DEF $\tau = \frac{R}{L}$

$$\frac{V_{out}}{V_{in}} = \frac{1}{j\omega\tau+1} \Rightarrow V_{OR} = \frac{1}{\sqrt{1+(\omega\tau)^2}} V_0$$



$$\frac{V_{out}}{V_{in}} = \frac{Z_L}{Z_L+Z_R} = \frac{j\omega L}{j\omega L + R} = \frac{j\omega\tau}{j\omega\tau+1}$$

$$V_{0L} = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}} V_0$$

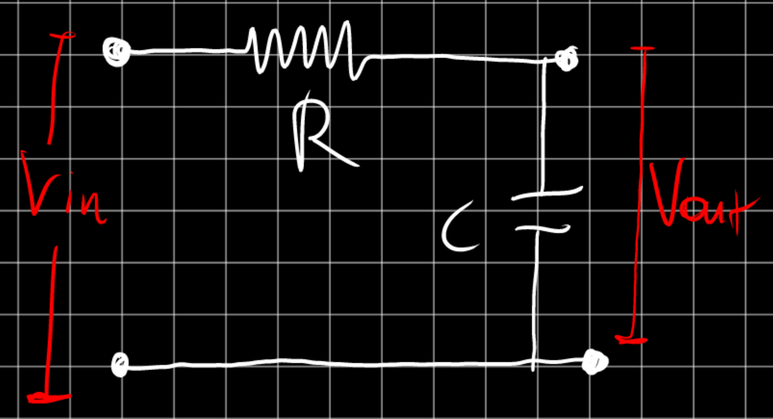
INTEGRATORI & DERIVATORI

Circuiti t.c. $V_{out} \propto \int V_{in}\,dt$ o $V_{out} \propto \frac{dV_{in}}{dt}$

N.B.

$$\left[\underbrace{\begin{aligned} V &= \frac{1}{C}\int_0^t I\,dt \\ I &= C\frac{dV}{dt} \end{aligned}}_{RC} \qquad \underbrace{\begin{aligned} V &= L\frac{dI}{dt} \\ I &= \frac{1}{L}\int V\,dt \end{aligned}}_{RL}\right]$$

1) Integratori (RC) +1



$$V_{in} = V_C + V_R$$

1a) $|V_C(t)| << |V_R(t)|$

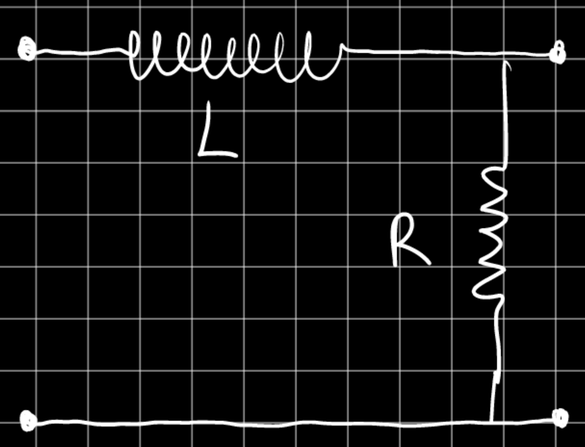
$$V_{in}(t) \approx V_R(t) = R\,I(t)$$

$$I(t) \simeq \frac{1}{R} V_{in}(t)$$

$$\therefore V_{out} = V_C(t) = \frac{1}{C}\int_0^t I(t)\,dt = \frac{1}{RC}\int_0^t V_{in}(t)\,dt$$

$$V_{out} = \frac{1}{\tau}\int_0^t V_{in}(t)\,dt$$

RL



$$V_{in}(t) = V_R(t) + V_L(t)$$

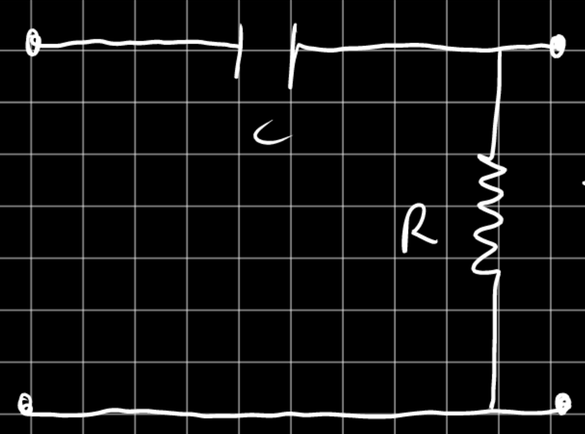
1b) $|V_R(t)| << |V_L(t)|$

$$V_{in}(t) \approx V_L(t) \approx L\frac{dI}{dt}$$

$$I(t) = \frac{1}{L}\int_0^t V_{in}(t)\,dt + I_0$$

$$V_{out}(t) = V_R(t) = R\,I(t) = \frac{R}{L}\int_0^t V_{in}(t)\,dt + R I_0 = \frac{1}{\tau}\int_0^t V_{in}(t)\,dt + R I_0$$

DERIVATORI (CR)



$$V_{in}(t) = V_R(t) + V_C(t)$$

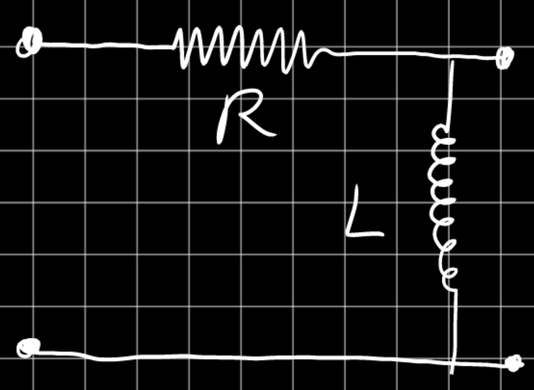
2a) $|V_R(t)| << |V_C(t)|$

$$V_{in}(t) \approx V_C(t) = \frac{1}{C}\int_0^t I(t)\,dt$$

$$\frac{dV_{in}}{dt} = \frac{1}{C} I(t)$$

$$V_{out}(t) = R\,I(t) = RC\frac{dV_{in}}{dt} = \tau\frac{dV_{in}}{dt}$$

RL



$$V_{in}(t) = V_R(t) + V_C(t)$$

2b) $|V_L(t)| << |V_R(t)|$

$$V_{in}(t) \approx V_R(t) = R\,I(t), \quad V_{out} = L\frac{dI}{dt}$$

$$\rightarrow V_{out} = \frac{L}{R}\frac{dV_{in}}{dt} = \tau\frac{dV_{in}}{dt}$$

→ PASSA BASSO RC, LR → INTEGRATORI

→ PASSA ALTO RL, CR → DERIVATORI

$\therefore V_{in} = V e^{j\omega t}$, $V_{out} = V_c e^{j\omega t + j\varphi}$; $V_c = \frac{1}{\sqrt{1+(\omega\tau)^2}} V$, $\varphi = \arctan(-\omega\tau)$

Più $V_{out} < V$ più $V_{out} \propto \frac{dV}{dt} / \propto \int V dt$

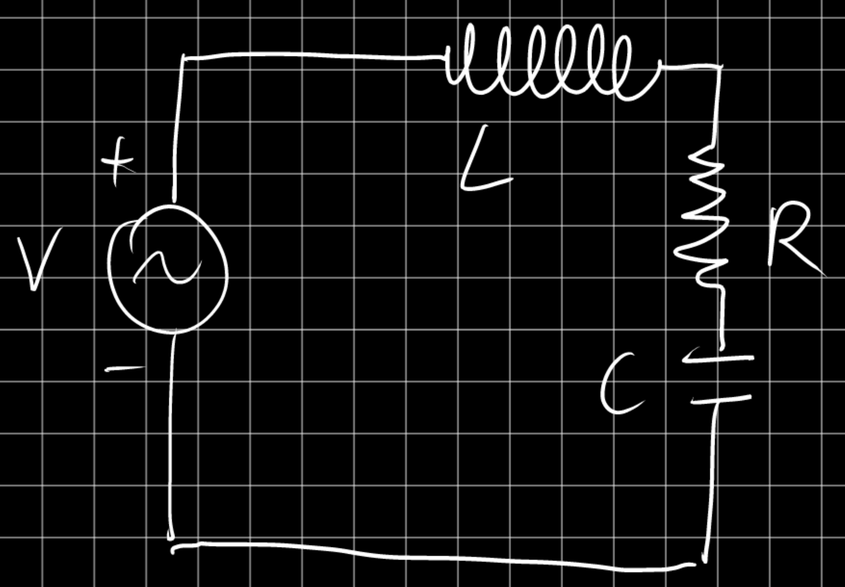
DERIVATORI | più rispettato quando $\omega < \omega_0 = \frac{1}{\tau}$

INTEGRATORI quando $\omega > \omega_0 = \frac{1}{\tau}$

LJ

RLC

RC + L



$V(t) = V_0 \cos(\omega t + \varphi_V)$

per Kirchhoff II $V = V_L + V_R + V_C$

$$\begin{cases} V_R = RI \\ V_L = L\partial_t I \\ V_C = \frac{1}{C}\int I dt \end{cases} \quad \left\{ \int I dt = Q \right\}$$

$V(t) = L\frac{\partial I}{\partial t} + RI + \frac{1}{C}\int I dt \Rightarrow \frac{\partial V}{\partial t} = L\frac{\partial^2 I}{\partial t^2} + R\frac{\partial I}{\partial t} + \frac{I}{C}$ risolvendo:

$V(t) = V_0(t) + V_p(t)$; $L\frac{\partial^2 I}{\partial t^2} + R\frac{\partial I}{\partial t} + \frac{I}{C} = 0$

$L\lambda^2 + R\lambda + \frac{I}{C} = 0 \Rightarrow \lambda = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{I}{LC}} \Rightarrow I(t) = I_\pm e^{\left(-\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{I}{LC}}\right)t}$

posto $a = \frac{R}{2L}$, $b = \left(\left(\frac{R}{2L}\right)^2 - \frac{I}{LC}\right)^{1/2} \Rightarrow I = I_+ e^{-(a-b)t} + I_- e^{-(a+b)t}$

CASISTICHE:

$$\begin{cases} \left(\frac{R}{2L}\right)^2 > \frac{1}{LC}, \ b \in \mathbb{R} \quad (1) \\ \left(\frac{R}{2L}\right)^2 = \frac{1}{LC}, \ b = 0 \quad (2) \\ \left(\frac{R}{2L}\right)^2 < \frac{1}{LC}, \ b \in \mathbb{C} \quad (3) \end{cases}$$

(1) → SOVRASMORZATO

Impongo CI Cauchy ; $I(0) = 0$

$I(0) = I_+ + I_- = 0 \Rightarrow I_+ = -I_-$, $|I_+| = |I_-| = I$

$\therefore RI(0) + L\frac{\partial I}{\partial t}\Big|_0 + \frac{q(0)}{C} = 0 \quad \{ q(0) = q_0$

$$\frac{\partial I}{\partial t} = -I_+(a-b)e^{-(a-b)t} - I_-(a+b)e^{-(a+b)t}, \quad \frac{\partial I}{\partial t}\Big|_{t=0} = -I_+(\cancel{a}-b) - I_-(\cancel{a}+b) = 2bI$$

$$2LbI = -\frac{q_0}{C} \Rightarrow I = -\frac{q_0}{2bLC} = -\frac{q_0}{2b}\omega_0^2 \; ; \quad \omega_0^2 = \frac{1}{LC}$$

$$\therefore I(t) = -\frac{q_0}{2b}\omega_0^2 e^{-at}\left(e^{bt} - e^{-bt}\right) = -\frac{q_0}{b}\omega_0^2 e^{-at}\sinh(bt)$$

$b=0$ SMORZAMENTO CRITICO

$$V(t) = V_R(t) + V_L(t) + V_C(t) = RI(t) + L\frac{\partial I}{\partial t} + \frac{1}{C}\int_0^t I(t)\,dt$$

$$\begin{cases} \frac{\partial V}{\partial t} = R\frac{\partial I}{\partial t} + L\frac{\partial^2 I}{\partial t^2} + \frac{1}{C}I(t) \\ I(0) = 0 \end{cases} \Rightarrow R\frac{\partial I}{\partial t} + L\frac{\partial^2 I}{\partial t} + \frac{1}{C}I(t) = 0$$

$$R\lambda + L\lambda^2 + \frac{1}{C} = 0 \quad \lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC} = 0 \Rightarrow -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$a = \frac{R}{2L}, \quad b = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad b = 0 \Rightarrow \left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \; ; \quad \lambda_{1,2} = \lambda = -\frac{R}{2L}$$

$$\therefore I = I_1 e^{-\frac{R}{2L}t} + tI_2 e^{-\frac{R}{2L}t} = (I_1 + tI_2)e^{-at}$$

but $I(0) = I_1 = 0 \Rightarrow I(t) = tI_2 e^{-at}$

$$\frac{\partial I}{\partial t} = I_2 e^{-at} - atI_2 e^{-at} = (1-at)I_2 e^{-at}, \quad \boxed{\frac{\partial I}{\partial t}\Big|_0 = I_2} \rightarrow \text{STABILE} \quad \left(V_C(t) = \frac{q_0}{C}\right.$$

$$LI_2 = -\frac{q_0}{C} \Rightarrow I_2 = -\frac{q_0}{LC} = -q_0\omega_0^2$$

$$I(t) = tI_2 e^{-at} = -q_0 t\omega_0^2 e^{-at} = -q_0 t\omega_0^2 \exp\left(-\frac{Rt}{2L}\right)$$

III bed, OSCILLATORIO SMORZATO

$$\left(\frac{R}{2L}\right)^2 < \omega_0^2 \Rightarrow \lambda_{1,2} = -a \pm jb$$

$$I(t) = I_+ e^{(a+jb)t} + I_- e^{(a-jb)t} = \left(I_+ e^{jbt} + I_- e^{-jbt}\right)e^{-at}$$

CAUCHY $\rightarrow I(0) = 0 \Rightarrow I_+ = -I_-$

$$\therefore I(t) = I_0 e^{-at}\left(e^{jbt} - e^{-jbt}\right) = 2jI_0 e^{-at}\sin(bt) \; ; \quad I_0 = -\frac{q_0}{2bLC} = -\frac{q_0\omega_0^2}{2b}$$

$$I(t) = -j\frac{\omega_0^2 q_0}{b}e^{-at}\sin(bt)$$ N.B. $\rightarrow$ dissipazione Joule su R

RLC SERIE

No Dissipazione $\rightarrow \frac{\partial E}{\partial t} = 0$

Per Cauchy:

$$E(0) = E_{max} = \frac{1}{2}\frac{q_0^2}{C} \; ; \; E_C(t) = \frac{1}{C}\int_0^t q\,dq = \frac{q^2(t)}{2C} = \frac{1}{2}CV^2$$

$$E_L = \int W dt = \int I L dI = \frac{1}{2}LI^2$$

$$E(t) = E_C(t) + E_L(t) = \frac{1}{2}\left[CV^2 + LI^2\right] \overset{?}{=} E(0)$$

1) $I(t) = \frac{1}{\beta} q_0 \omega_0^2 \sin(\beta t) = I_0 \sin(\omega_0 t)$

2) $V(t) = V_R(t) + V_C(t) + V_L(t) = \underbrace{\cancel{R I(t)}}_{\text{(no Joule)}} + L\frac{\partial I}{\partial t} + V_C(t) \Rightarrow -L\frac{\partial I}{\partial t} = V_C(t)$

3) $E_C(t) = \frac{1}{2}CV^2 = \frac{1}{2}CL^2 I_0^2 \omega_0^2 \cos^2(\omega_0 t) = \frac{1}{2}LI_0^2\cos^2(\omega_0 t)$

$E_L(t) = \frac{1}{2}LI^2 = \frac{1}{2}LI_0^2\sin^2(\omega_0 t)$

$E(t) = E_C + E_L = \frac{1}{2}\left[CI_0^2 + LI_0^2\right] = E(0)$ ✓

RLC AC

$\rightarrow V(t) \propto e^{j\omega t}$

$$\frac{\partial V}{\partial t} = L\frac{\partial^2 I}{\partial t^2} + R\frac{\partial I}{\partial t} + \frac{I}{C}, \quad \text{con} \quad \begin{cases} V = V_0 e^{j\omega t + j\varphi_V} \\ I = I_0 e^{j\omega t + j\varphi_I} \end{cases}$$

$$\therefore \frac{\partial V}{\partial t} = j\omega V_0 e^{j\omega t + j\varphi_V} \; ; \; \frac{\partial I}{\partial t} = j\omega I_0 e^{j\omega t + j\varphi_I}, \; \frac{\partial^2 I}{\partial t^2} = -\omega^2 I_0 e^{j\omega t + j\varphi_I}$$

$$\cancel{j\omega} V_0 e^{j\varphi_V} = -\omega^{\cancel{2}} L I_0 e^{j\varphi_I} + \cancel{j\omega} R e^{j\varphi_I} + \frac{I_0}{\omega C} e^{j\varphi_I}$$

$$V_0 e^{j\varphi_V} = \left[R - \left(\frac{\omega L}{j} - \frac{1}{j\omega C}\right)\right] I_0 e^{j\varphi_I}$$

$$V_0 e^{j\varphi_V - j\varphi_I} = \left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right] I_0 \quad \left\{ \text{Ohm gen} \; \left(R_L = j\omega L, \; R_C = \frac{1}{j\omega C} = -\frac{j}{\omega C} \right) \right.$$

Posto $\varphi = \varphi_V - \varphi_I$, $Z_{RLC} = R + j\omega L + \frac{1}{j\omega C}$

$$V_0 e^{j\varphi} = Z I_0 \Rightarrow \begin{cases} I_0 = \frac{V}{|Z_0|} = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{-1/2} V_0 \\ \varphi = \arctan\left[\frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right)\right] \end{cases}$$

$Z \in \mathbb{R}$ iff $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$ & $Z = R$ $\therefore \varphi = 0$ (Risonanza)

Def FATTORE DI QUALITÀ

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \; ; \; \therefore \; I_0 = \frac{V}{\sqrt{1 + Q_0^2\left(\frac{\omega^2-\omega_0^2}{\omega\omega_0}\right)^2}}$$

FILTRO PASSA-BANDA $Q_0 \to \infty \Rightarrow$ filtro più selettivo.

Def LARGHEZZA DI BANDA $\Delta\omega$; distanza tra punti di ris. in freq dove

$$I_{\Delta\omega} = \frac{1}{\sqrt{2}} I_{max}$$

N.B. $\Delta\omega = \Delta\omega(Q_0)$

$$\therefore \; I = \frac{I_0}{\sqrt{g/R}} = \frac{1}{\sqrt{1 + Q_0^2\left(\frac{\omega^2-\omega_0^2}{\omega\omega_0}\right)^2}} = \frac{1}{\sqrt{2}} \Rightarrow 1 + Q_0^2\left(\frac{\omega^2-\omega_0^2}{\omega\omega_0}\right)^2 = 1$$

$$\therefore \; \frac{\omega^2-\omega_0^2}{\omega\omega_0} = \pm\frac{1}{Q_0} \Rightarrow \omega^2 - \omega_0^2 \mp \frac{\omega\omega_0}{Q_0} = 0$$

Risolvo per $\omega \Rightarrow \pm\frac{\omega_0}{2Q_0} \pm \sqrt{\left(\frac{\omega_0}{2Q_0}\right)^2 + \omega_0^2} > 0 \Rightarrow \begin{cases} \omega_1 = -\frac{\omega_0}{2Q_0} + \sqrt{\left(\frac{\omega_0}{2Q_0}\right)^2 - \omega_0^2} \\ \omega_2 = \frac{\omega_0}{2Q_0} + \sqrt{\left(\frac{\omega_0}{2Q_0}\right)^2 - \omega_0^2} \end{cases}$

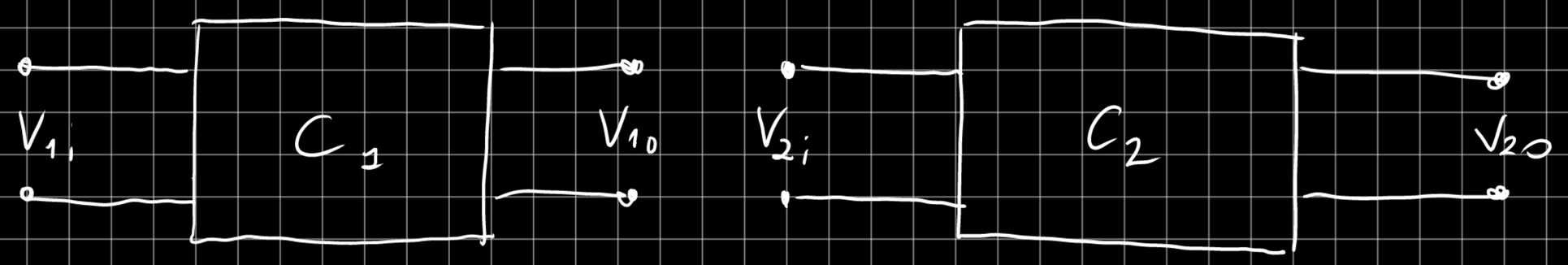
$$\Delta\omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q_0} = \frac{R}{L} \quad ✓$$

$R_{min} = R_L \Rightarrow$ commerciali $Q_0 \propto 10^2$; $Q_0 \gg 10^2$ iff L composta da superconduttori.

$L \to 0$ ✓

Funzione di Trasferimento

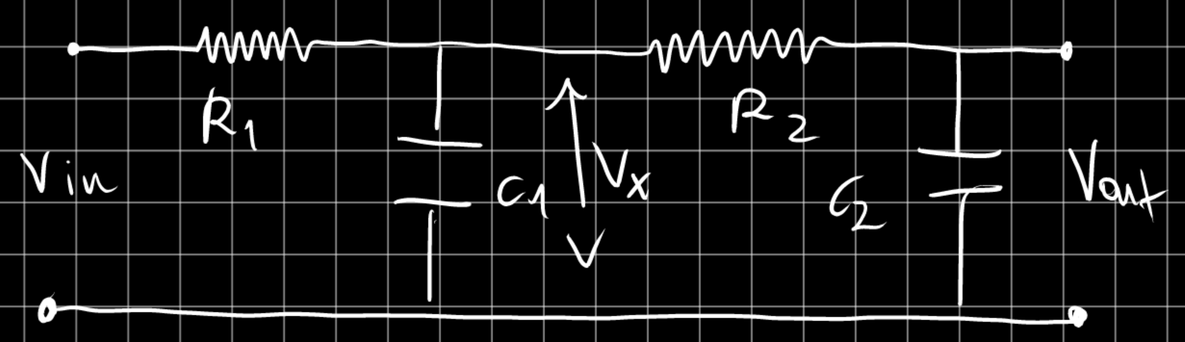
Circuiti lineari $\Rightarrow$ Sovrapposizione $\Rightarrow$ fdt di trasferimento



Def: $T(\omega) = \frac{V_{in}(\omega)}{V_{out}(\omega)}$, $\varphi(\omega) = \Delta\varphi_{io}$

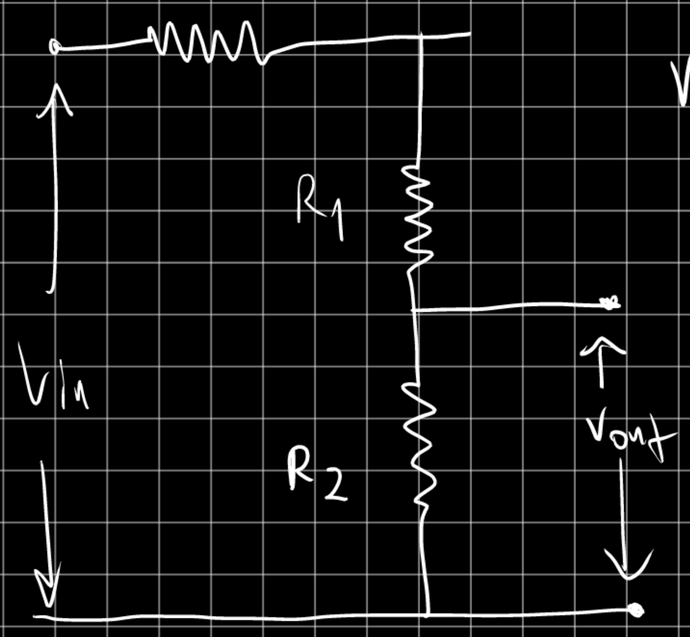
(int. pb)

$T_{C_1+C_2} \neq T_1 T_2$! C_2 modifica risposta C_1. E.g., C_i circuito RC



$$T = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x}\frac{V_x}{V_{in}}$$

Uso partitore di tensione



$$V_{out} = \frac{R_2}{R_1+R_2} V_{in}$$

$$\frac{V_x}{V_{in}} = \frac{Z_{e11}}{R_1+Z_{e11}} \qquad \frac{V_{out}}{V_x} = \frac{Z_{ec1}}{R_2+Z_{c1}}$$

$$Z_{ec_1} = \frac{1}{j\omega C_1} \Rightarrow \frac{V_{out}}{V_x} = \frac{1}{j\omega C_2}\left(\frac{1}{R_2+\frac{1}{j\omega C_2}}\right)$$

$$\frac{V_{out}}{V_x} = \frac{1}{1+j\omega C_2 R_2}$$

$Z_{e_{11}} \rightarrow$ Serie C_2 R_2 in parallelo con $C_1 \Rightarrow Z_{e_2} = Z_{c_2}+R_2 = \frac{1}{j\omega C_2} + R_2 = \frac{1+j\omega C_2 R_2}{j\omega C_2}$

$$Z_{c_1} = \frac{1}{j\omega C_1} \Rightarrow Z_{11} = \left(\frac{1}{Z_{c_1}}+\frac{1}{Z_2}\right)^{-1} = \frac{Z_{c1}Z_2}{Z_{c1}+Z_2} \checkmark$$

$$= \frac{\frac{1}{j\omega C_1}\left(R_2+\frac{1}{j\omega C_2}\right)}{R_2+\frac{1}{j\omega C_1}+\frac{1}{j\omega C_2}} = \frac{\frac{1}{j\omega C_1}\left(\frac{1+j\omega C_2 R_2}{j\omega C_2}\right)}{\frac{1}{j\omega C_1}+\frac{1+j\omega C_2 R_2}{j\omega C_2}} = \frac{\frac{1+j\omega C_2 R_2}{j\omega C_1 j\omega C_2}}{\frac{j\omega C_2 + j\omega C_2 R_2 j\omega C_1 + j\omega C_1}{j\omega C_1 j\omega C_2}}$$

$$= \frac{1+j\omega C_2 R_2}{j\omega(C_1+C_2)-\omega^2 R_2 C_1 C_2} = Z_1$$

$$\frac{V_x}{V_{in}} = \left(\frac{1+j\omega C_2 R_2}{j\omega(C_1+C_2)-\omega^2 R_2 C_1 C_2}\right)\left(R_1+\frac{1+j\omega C_2 R_2}{j\omega(C_1+C_2)-\omega^2 R_2 C_1 C_2}\right)^{-1}$$

$$\frac{V_x}{V_{in}} = \frac{1+j\omega C_2 R_2}{j\omega(C_2+C_1)-\omega^2 R_2 C_1 C_2} \quad \frac{j\omega(C_1+C_2)-\omega^2 R_2 C_1 C_2}{j\omega R_1(C_1+C_2)-\omega^2 R_1 R_2 C_1 C_2 + j\omega R_2 C_2 + 1} = \frac{1+j\omega C_2 R_2}{j\omega[R_2 C_1 + (C_1+C_2)R_1] - \omega^2 R_1 R_2 C_1 C_2}$$

$$T = \frac{V_x}{V_{in}}\frac{V_{out}}{V_x} = \frac{1+j\omega C_2 R_2}{j\omega[R_2 C_1 + R_1(C_1+C_2)] - \omega^2 R_1 R_2 C_1 C_2} \quad \frac{1}{1+j\omega C_2 R_2} = \{j\omega[R_2 C_1 + R_1(C_1+C_2)] - \omega^2 R_1 R_2 C_1 C_2\}^{-1}$$

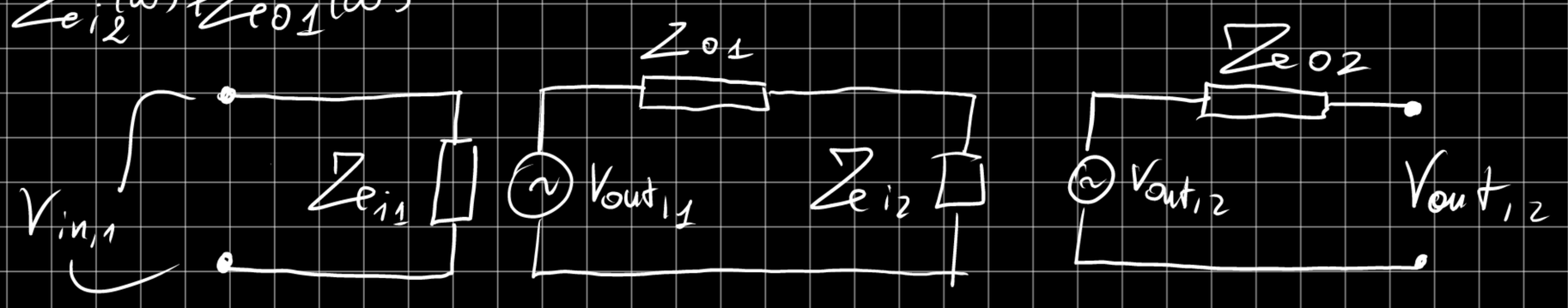
i.e. $T(\omega) = \frac{1}{A(\omega)+j\omega B(\omega)} = \frac{A(\omega)-j\omega B(\omega)}{A^2(\omega)+\omega^2 B^2(\omega)}$

$$\begin{cases} \|T(\omega)\| = \left(A^2(\omega)+B^2(\omega)\right)^{-1/2} \\ \arg(T(\omega)) = \arctan\left(-\frac{\omega B(\omega)}{A(\omega)}\right) \end{cases}$$

In generale:

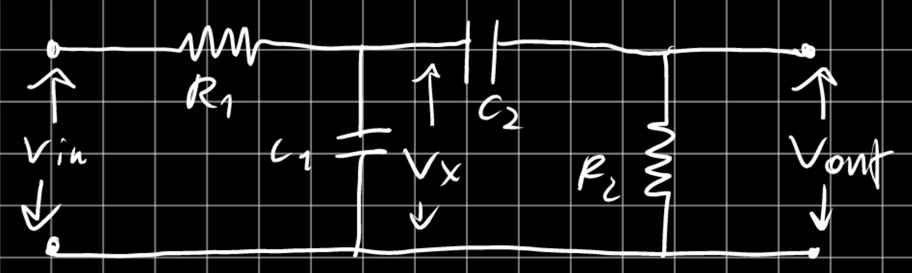
Dati due circuiti C_1, C_2 con $T_1(\omega)$, $T_2(\omega)$ fni trasferimento

$$T(\omega) = \frac{Z_{ei2}(\omega)}{Z_{ei2}(\omega)+Z_{eo1}(\omega)} T_1(\omega)T_1(\omega) \quad \text{DA SCHEMA}$$



Se $Z_{in,2}(\omega) \gg Z_{out,1} \Rightarrow \frac{Z_{i2}}{Z_{i2}+Z_{o1}} \approx 1 \Rightarrow T(\omega) \approx T_1(\omega)\, T_2(\omega)$

RC + CR



$$T = \frac{V_{out}}{V_x}\,\frac{V_x}{V_{in}}$$

$$\frac{V_{out}}{V_x} = \frac{R_2}{R_2+\frac{1}{j\omega C_2}} = \frac{R_2}{j\omega C_2}\,\frac{1}{1+j\omega C_2 R_2} \qquad \frac{V_{in}}{V_x} = \frac{Z_{11}}{R_1+Z_{11}}$$

$$Z_2 = \frac{1}{j\omega C_2} + R_2 = \frac{1+j\omega C_2 R_2}{j\omega C_2}, \quad Z_{11} = \left(j\omega C_1 + \frac{j\omega C_2}{1+j\omega C_2 R_2}\right)^{-1} = \frac{1+j\omega C_2 R_2}{j\omega C_1 - \omega^2 C_1 C_2 R_2}$$

$$\frac{V_{in}}{V_x} = \frac{1+j\omega C_2 R_2}{j\omega C_1 - \omega^2 C_1 C_2 R_2} \Big/ \frac{j\omega R_1 C_1 - \omega^2 C_1 C_2 R_1 R_2 + j\omega C_2 R_2 + 1}{j\omega C_1 - \omega^2 C_1 C_2 R_2} = \text{VAFFANCULO}$$

CIRCUITI RISONANTI

LC Non smorzato
RLC Smorzato

} oscillazione energetica E-H. E cost iff $R=0$ (No Joule)

Se $R>0 \Rightarrow$ Aggiungo forzante (Generatore) con $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$ freq. risonanza

Meccanicamente:

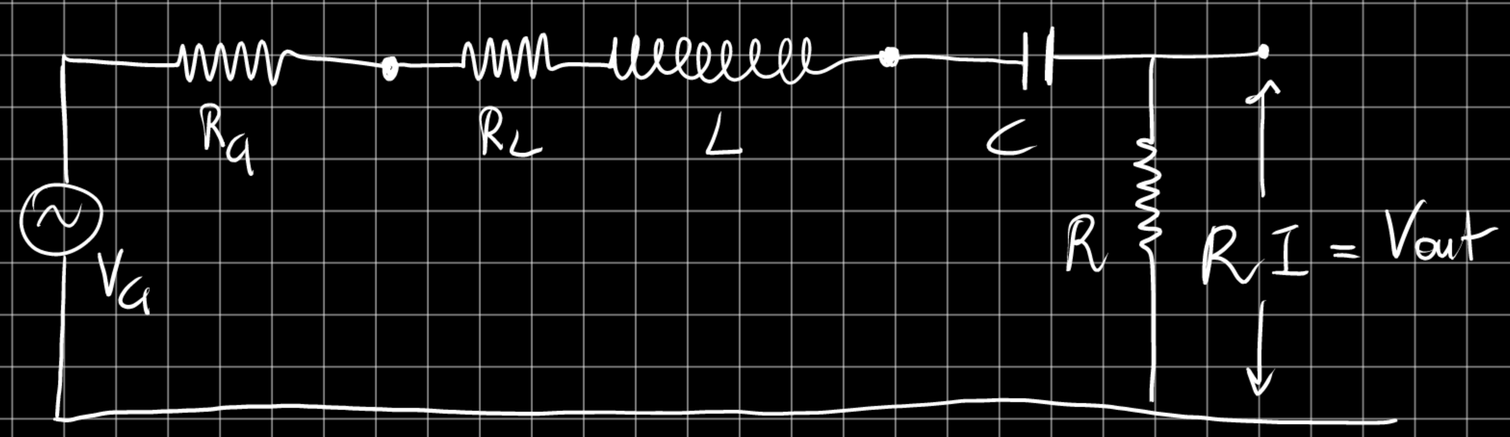
$$\frac{\partial^2 x}{\partial t^2} + \omega_S \frac{\partial x}{\partial t} + \omega_N^2 x = a_{0F}\cos(\omega_F t)$$

Dove

$$x(t) = Ae^{-\frac{\omega_S t}{2}}\cos(\omega_S t+\varphi) + B\cos(\omega_F t-\delta); \quad B = \frac{a_0}{\sqrt{(\omega_N-\omega_F)^2+\gamma^2\omega_F^2}}$$

RLC SERIE & RLC PARALLELO

RLC SERIE



$$R = R_G + R_L + j\omega L + \frac{1}{j\omega C} = R_G + R_L + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$I(\omega) = \frac{V_{out}}{R} = \left(R_G + R_L + j\left(\omega L - \frac{1}{\omega C}\right)\right)^{-1} V_G \leftarrow \text{trovo } Q$$

EXTRATENSIONI

$$V_{out}(t) = V_0 e^{j\omega t} = R I_0 e^{j\omega t + j\varphi_I} = \left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right] I_0 e^{j\omega t + j\varphi_I}$$

Extratensioni V_C, V_L (attivi)

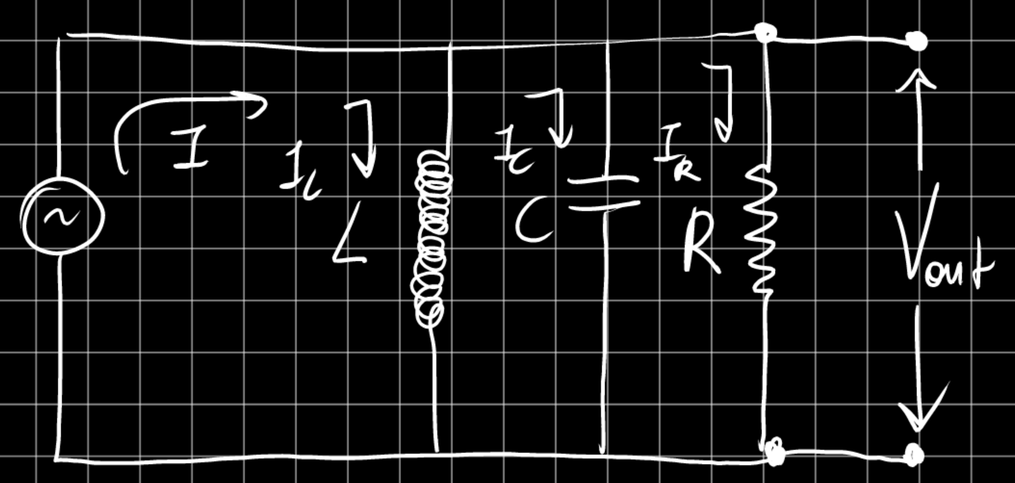
$$\begin{cases} V_C = Z_C I = \dfrac{1}{j\omega C} \dfrac{V_0(t)}{R_G + R_L + j\left(\omega L + \frac{1}{\omega C}\right)} \\ V_L = Z_L I = j\omega L \dfrac{V_0(t)}{R_G + R_L + j\left(\omega L + \frac{1}{\omega C}\right)} \end{cases} \qquad \begin{cases} \dfrac{V_C}{V_0} = \dfrac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \dfrac{1}{\omega C} \\ \dfrac{V_L}{V_0} = \dfrac{\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \end{cases}$$

A risonanza, $\omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$ $\quad \dfrac{V_C}{V_0} = \dfrac{V_L}{V_0} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = Q \Longrightarrow V_C = V_L = Q V_0$

$\therefore V_C = V_L > V_0$! N.B. $\varphi_C = -\varphi_L \rightarrow$ Non fanno scorrere corrente su R & ~

$$\max_{V_L, V_C}(\omega) = \begin{cases} \dfrac{1}{\sqrt{LC - \frac{1}{2}(RC)^2}} > \omega_0 \\ \sqrt{\dfrac{1}{LC} - \dfrac{1}{2}\left(\dfrac{R}{L}\right)^2} < \omega_0 \end{cases}$$

RLC PARALLELO



$$\frac{1}{Z_{eq}} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

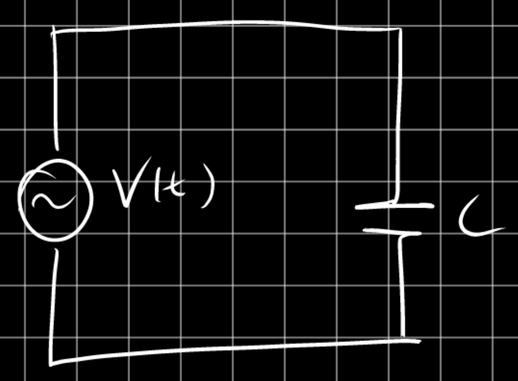
$$I = \frac{V(t)}{Z_{eq}} = \left[\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)\right] V(t) \; ; \; I_0 = V_0 \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$\varphi = \arctan\left[R\left(\omega C - \frac{1}{\omega L}\right)\right] = \arctan\left[\frac{1}{\omega L}\left(\frac{\omega^2}{\omega_0^2} - 1\right)\right]$$

24/03 fo8

POTENZA AC

Potenza dissipata AC : $W(t) = V(t)i(t)$. E.g.



$$V = Z_I I = \frac{I}{j\omega C} \Rightarrow I = j\omega C V(t) \,,\quad \varphi_I = \frac{\pi}{2}$$

$$\text{se } V(t) = \sin(\omega t) \Rightarrow I = \omega C V_0 \sin\Big(\omega t + \underbrace{\frac{\pi}{2}}_{\varphi_I}\Big)$$

Carica $\rightarrow$ $W > 0$ (fornisco a C)

Scarica $\rightarrow$ $W < 0$ (C dà)

Per def

$$T = \frac{2\pi}{\omega}$$

$$\langle W \rangle = \frac{1}{T}\int_0^T I(t)V(t)\,dt = \frac{\omega}{2\pi}\int_0^{\omega/2\pi} V_0 I_0 \sin(\omega t)\cos(\omega t)\,dt = 0$$

In generale:

$$\langle W \rangle = \frac{V_0 I_0}{T}\int_0^T \cos(\omega t + \varphi_I)\cos(\omega t)\,dt$$

$$\cos(\omega t + \varphi_I) = \cos(\omega t)\cos(\varphi_I) + \sin(\omega t)\sin(\varphi_I)$$

$$\langle W \rangle = \frac{V_0 I_0}{T}\cos\varphi_I \int_0^T \cos^2(\omega t)\,dt + \frac{V_0 I_0}{T}\sin(\varphi_I)\underbrace{\int_0^T \sin(\omega t)\cos(\omega t)\,dt}_{0}$$

$$\langle W \rangle = \frac{V_0 I_0}{T}\cos(\varphi_I)\int_0^T \cos^2(\omega t)\,dt = \frac{V_0 I_0}{2}\cos(\varphi_I)$$

<u>Def</u>

$$V_{eff} = \frac{V_0}{\sqrt{2}} \,,\quad I_{eff} = \frac{I_0}{\sqrt{2}} \Rightarrow \langle W \rangle = V_{eff} I_{eff} \underbrace{\cos(\varphi_I)} \,,\quad \varphi = \arctan\left\{\frac{Im\{Z_e\}}{Re\{Z_e\}}\right\}$$

$\cos(\varphi_I) \rightarrow$ FATTORE DI POTENZA

Circuiti puramente attivi $\Rightarrow Re\{Z_e\} = 0 \Rightarrow \cos(\varphi_I) = 0$

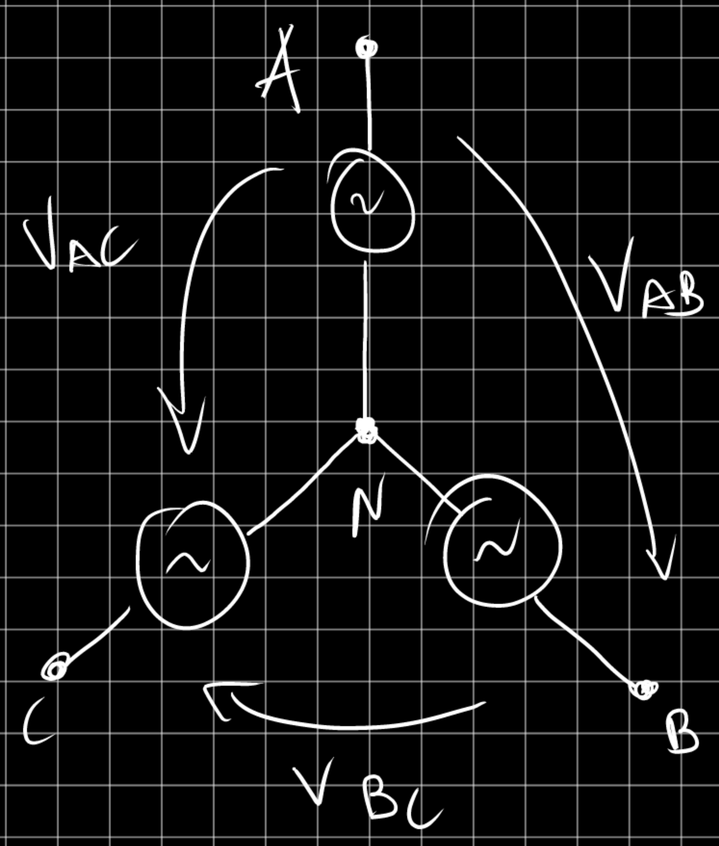
N.B. $I \neq 0 \Rightarrow$ in R_G $W \neq 0$! $\{\langle W \rangle = 0\}$

Q_0 (E)

Per def $$Q_0 = \omega_0 \frac{L}{R} = \frac{2\pi}{T}\frac{L}{R}\frac{I_0^2/2}{I_0^2/2} = \frac{2\pi}{T}\frac{L I_0^2}{2}\frac{2}{R I_0^2} = \frac{2\pi}{T}\frac{E_{ST}}{W_{Joule}} = 2\pi\frac{E_{ST}}{E_J}$$

$Q_0 \Rightarrow 2\pi$ Rapporto E risuonatore vs E Joule (dissipata)

GENERATORI TRIFASE



$$\begin{cases} V_{AN}(t) = V_A - V_N = V_0 e^{j\omega t} \\ V_{BN}(t) = V_B - V_N = V_0 e^{j\omega t + 2\pi j/3} \\ V_{CN}(t) = V_C - V_N = V_0 e^{j\omega t + 4\pi j/3} \end{cases}$$

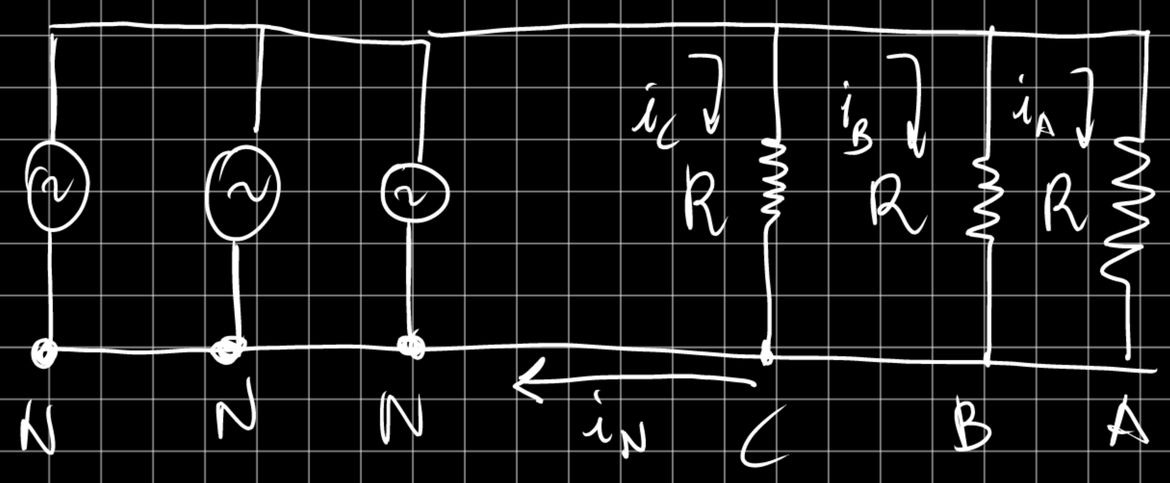
$$V_{AB} = V_A - V_N - V_B + V_N = V_0\left[1 - e^{-\frac{2\pi j}{4}}\right] e^{j\omega t} = V_0 e^{j\omega t}\left[1 + \frac{1}{\sqrt{2}} - j\frac{\sqrt{3}}{2}\right] = \sqrt{3}\, V_0 e^{j\omega t}\left[\frac{\sqrt{3}}{2} - \frac{j}{2}\right]$$

$$V_{AB} = \sqrt{3}\, V_0 e^{j\omega t - \pi j/6} \rightarrow \|V_{AB}\| = \sqrt{3}\,\|V_0\|$$

In IT: $V_{AN} = V_{BN} = V_{CN} = 230\sqrt{2}$ V, $V_{AB} = V_{BC} = V_{CA} = 400\sqrt{2}$ V

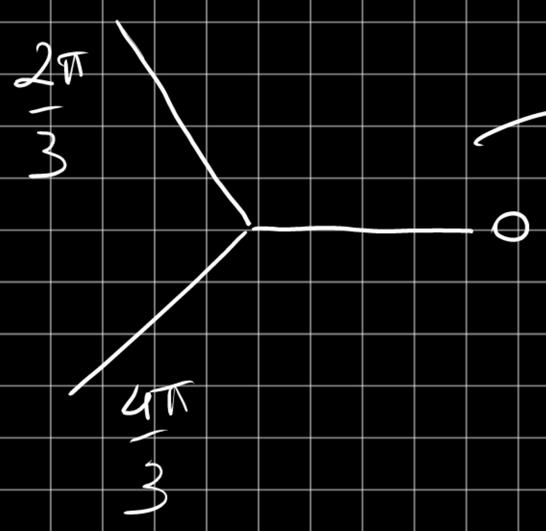
Prima $\quad 220\sqrt{2}$ V $\quad 380\sqrt{2}$ V

e.g. con 3 carichi uguali



$$i_N = i_A + i_B + i_C$$

$$i_N = \frac{V_0}{R} e^{j\omega t}\left[1 + e^{\frac{2\pi j}{3}} + e^{\frac{4\pi j}{3}}\right]$$



$$\rightarrow \mathrm{Im}\left\{e^{\frac{4\pi j}{3}}\right\} = -\mathrm{Im}\left\{e^{\frac{2\pi j}{3}}\right\},\ \mathrm{Re}\left\{e^{\frac{4\pi j}{3}}\right\} = \mathrm{Re}\left\{e^{\frac{2\pi j}{3}}\right\}$$

$$i_N = \frac{V_0}{R} e^{j\omega t}\left[1 + 2\,\mathrm{Re}\left\{e^{\frac{2\pi j}{3}}\right\}\right]$$

$$i_N = \frac{V_0}{R}\left[1 + 2\cos\left(\frac{2\pi}{3}\right)\right] = \frac{V_0}{R}\left[1 - 1\right] = 0\ !$$

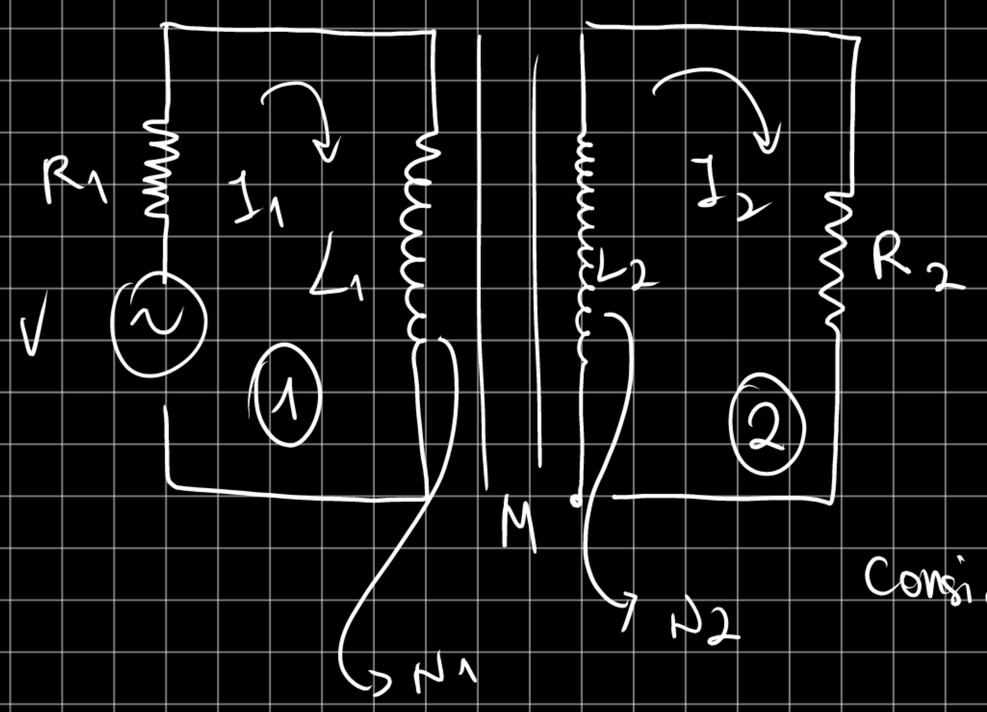
→ MONO FASE → $i_N = 3i$

→ TRIFASE → $i_N = 0$

TRASFORMATORI

Avvolgimenti di conduttore attorno a ferromagnete con perm. μ

AC impone H indotto in fase con I che genera I_2 per FNL



$$\begin{cases} V - L_1 \frac{\partial I_1}{\partial t} + M_{12} \frac{\partial I_2}{\partial t} = R I_1 \\ M_{21} \frac{\partial I_1}{\partial t} - L_2 \frac{\partial I_2}{\partial t} = R I_2 \end{cases}$$

Considerati L_i con N_i avvolgimenti, per MWG

$\nabla \times H = I \Rightarrow H_1 \ell = N_1 I_1$, $B_1 = \mu \frac{N_1}{\ell} I_1$, $\iint_S \underline{B} \cdot \hat{\underline{n}} \, ds = N_1 B S = \frac{\mu N_1^2}{\ell} S I_1$

Quindi $L_1 = \frac{\mu N_1^2 S}{\ell}$, $L_2 = \frac{\mu}{\ell} N_2^2 S$

Ma $M_{ij} = \Phi_i(B_j)/I_j \Rightarrow M = \frac{\mu S}{\ell} N_1 N_2$. Nota che $\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{N_1}{N_2} \Rightarrow$ ottengo AC con $V \gtrless V_0$

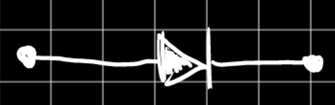
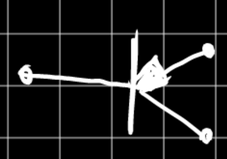
Lg V 46m 47.85s → L10 (28/03/23)

30/03/23

COMPONENTI NON LINEARI

Componenti per cui $I \not\propto V$

E.G.:

- LAMPADINA AD INCANDESCENZA
- VALVOLE TERMOIONICHE
- DIODI 
- TRANSISTOR 

} SEMICONDUTTORI

CONDUZIONE IN SEMICONDUTTORI

ISOLANTI	>	SEMICONDUTTORI	>	CONDUTTORI
$\sim 10^{15}\ \Omega\,cm$		$\sim 50\ \Omega\,cm$ = R_{Ge}		$\sim 1.7\ \mu\Omega\,cm$ = R_{Cu}

Derivato da struttura interna. Si, Ge i più usati (tetravalenti)

Si → struttura cristallina lega tutti i 4 e^- → conduzione per agitazione termica + $\underline{E}$ ho corrente intrinseca

Varia conducibilità di un semiconduttore con impurezze <<drogaggio>>

e.g. Si → P ⇒ +1 e^- debolmente legato

e.g. II Si → Ga ⇒ −1 e^- (lacuna)

In SC drogaggio a sude 1 ppm aumenta di 10^2 σ

TIPOLOGIE

- N (+ e^-, drogante pentavalente P, As, Sb)
- P (− e^-, drogante trivalente B, Ga, In

DIODI SEMICONDUTTORI

Giunzione di cristalli P & N

In gen: Genera $\underline{E}$ da N a P (dipolo elettrico) → stabilità

⇒ ∃ POTENZIALE ES φ su giunzione

∴ e^- in N passano in P iff $E_{e^-} > \varphi$

Elettroni termici ⇒ Statistica di Boltzmann

⇒ # e^- in stato dE $\left(\beta = \frac{1}{kT}\right)$

$$dN = e^{-\beta E} dE$$

$$\therefore N(E>\varphi) = C\int_{\varphi}^{\infty} e^{-\beta E} dE = \frac{C}{\beta} e^{-\beta\varphi}$$

Dove $N_T = C\int_{\mathbb{R}_+} e^{-\beta E} dE = \frac{C}{\beta}$

∴ Prob e^- con $E>\varphi$ e passaggio ⇒ $P(E>\varphi \vee N\to P) = \frac{N(E>\varphi)}{N_T} = e^{-\beta\varphi}$

e $I_{NP} \propto e^{-\beta\varphi}$. A regime $I_{NP} = I_{PN} = A e^{-\beta\varphi}$

POLARIZZAZIONE DIRETTA DIODI

Aggiungo $\underline{E}'$ con batteria, $\therefore E'_{e^-} = \varphi - e\|V\|$

$\therefore I_{Np} = Ae^{-\beta(\varphi - e|V|)}$, $I_{pN} = Ae^{-\beta\varphi}$

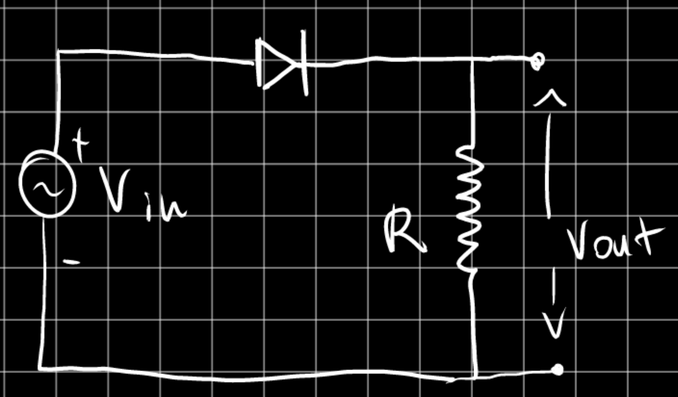
$\therefore I_{e^-} = I_{Np} - I_{pN} = Ae^{-\beta\varphi}\left(e^{\beta e\|V\|} - 1\right)$

si può invertire generatore e si ottiene $I = I_0\left(e^{-\beta eV} - 1\right)$

N.B. Diodo come Resistenza polarizzata $R_{eq} = \frac{\partial I}{\partial V} = \left(\frac{\partial V}{\partial I}\right)^{-1}$

UTILIZZO → RADDRIZZATORI

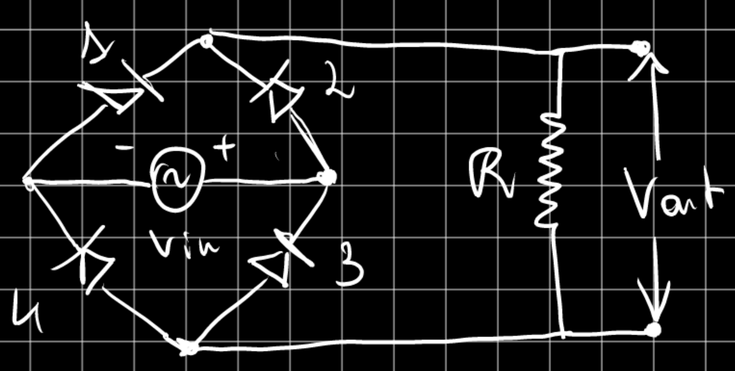
A SEMIONDA



$V_{in} \propto \sin(\omega t)$

$V_{out} \propto \begin{cases} \sin(\omega t) & V_{in} > 0 \\ 0 & V_{in} < 0 \end{cases}$

A PONTE



$V_{out} \propto |V_{in}|$

1, 3 pol. inversa $P \to N$

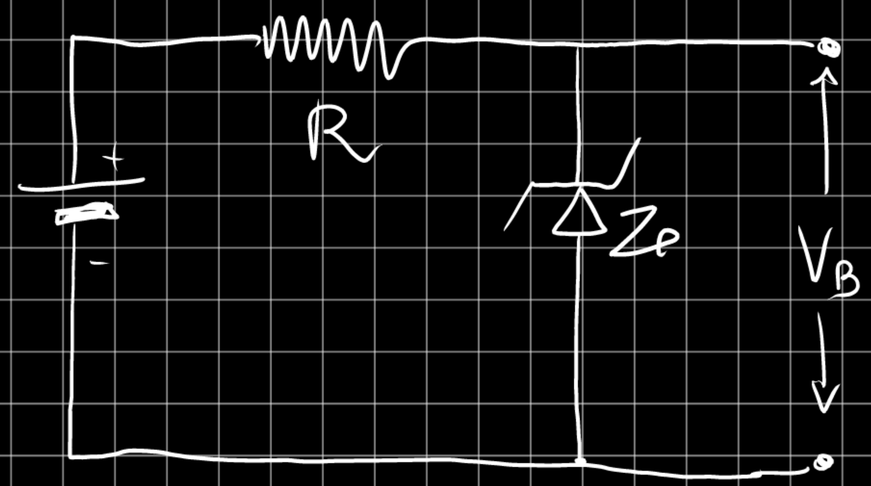
2, 4 pol diretta $N \to P$

DIODI ZENER

$\exists V_B$ (breakdown) T.C. → Ionizzazione di PN Zener costruiti per sopporta

$V > V_B$

Utili per stabilizzare V_{DC}



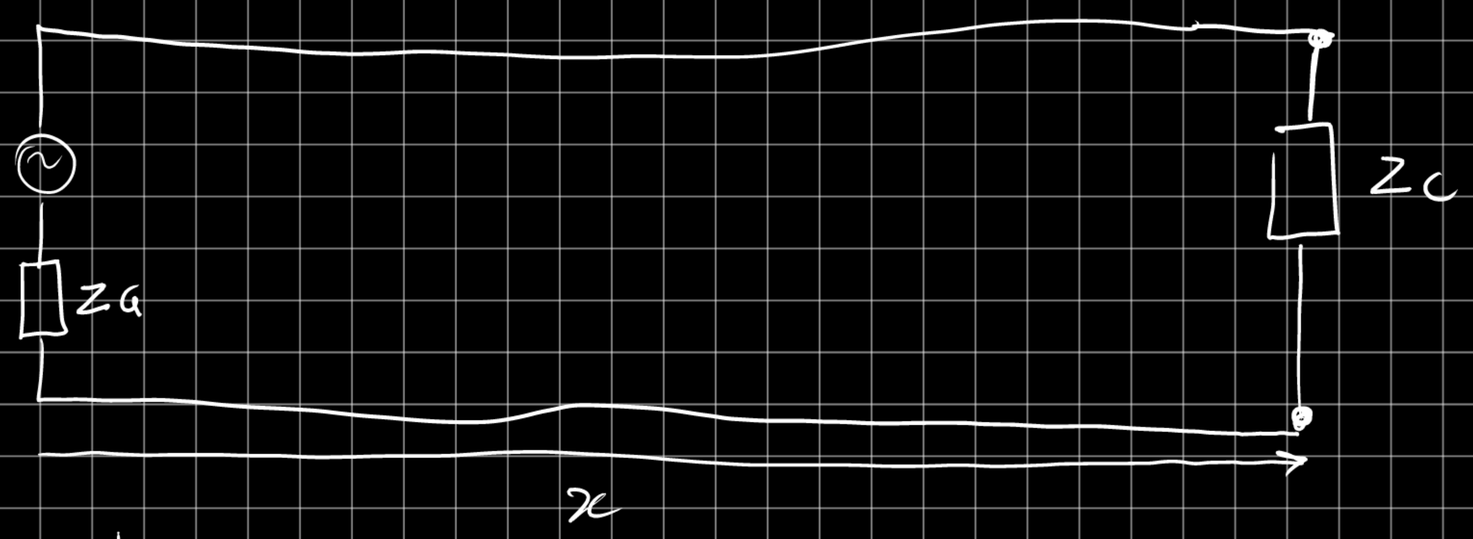
Zener anche per limitare V con clipping a $-0.6V$ & V_B

LINEE DI TRASMISSIONE

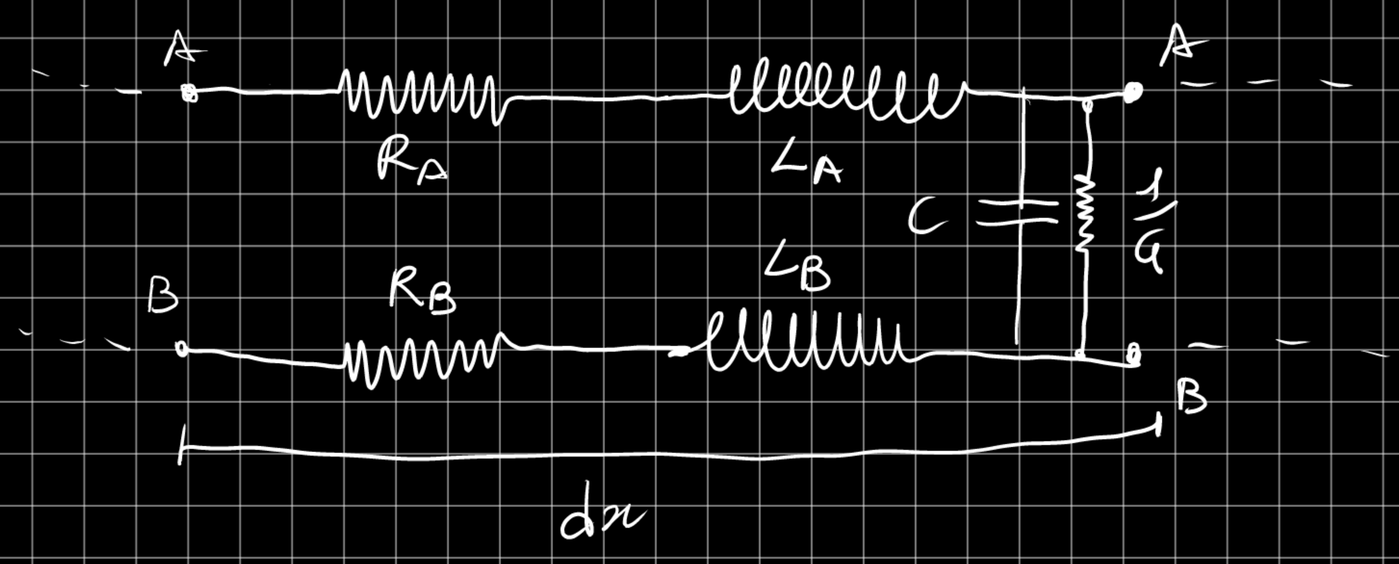
KIRCHHOFF EM

Relatività $\Rightarrow$ segnale a $V = c$

$V \approx \infty$ iff $\nu \ll 1\,GHz$



Schematizzabile in



$R, L \rightarrow$ cadute di V

$C, G \rightarrow$ perdite di I

$$\begin{cases} dV_A = -R_A I(x,t)\,dx - L_A \dfrac{\partial I}{\partial t}\,dx \\ dV_B = R_B I(x,t) + L_B \dfrac{\partial I}{\partial t} \end{cases} \Rightarrow \frac{\partial V}{\partial x} = -(R_B + R_A)\,I(x,t) - (L_A + L_B)\frac{\partial I}{\partial t}$$

def:

$R_u\,dx = R_A + R_B$, $L_u\,dx = L_A + L_B$

$$\therefore \frac{\partial V}{\partial x} = -\left[R_u + L_u \frac{\partial}{\partial t}\right] I(x,t) \quad , \text{def } \hat{Z}_{eu} = R_u + L_u \partial_t \Rightarrow \frac{\partial V}{\partial x} = -\hat{Z}_{eu}\, I(x,t)$$

$$\& \quad \frac{\partial I}{\partial x} = -\left[G_u + C_u \frac{\partial}{\partial t}\right] V(x,t) \quad , \text{def } \hat{Y}_u = G_u + C_u \frac{\partial}{\partial t} \Rightarrow \frac{\partial I}{\partial x} = -\hat{Y}_u\, V(x,t)$$

$\hat{Y}_u \rightarrow$ ammettenza per unità di lunghezza

Se $V \propto e^{j\omega t}$

$$\begin{cases} Z_e = R + j\omega L \\ Y = G + j\omega C \end{cases} ; \quad \begin{cases} \dfrac{\partial V}{\partial x} = -\hat{Z}_{eu}\, I \\ \dfrac{\partial I}{\partial x} = -\hat{Y}_u\, V \end{cases} \quad \begin{cases} \dfrac{\partial^2 V}{\partial x^2} = \hat{Z}_{eu}\hat{Y}_u V = \hat{\gamma}^2 V(x,t) \\ \dfrac{\partial^2 I}{\partial x^2} = \hat{Z}_u \hat{Y}_u I = \hat{\gamma}^2 I(x,t) \end{cases}$$

EQN TELEGRAFISTI

Soluzione:

$V(x) = A_1 e^{-\gamma x} + A_2 e^{\gamma x}$, $A_i = \tilde{A}_i e^{j\varphi_i}$, $Z_{e0} = Z_{e0} e^{j\psi}$

Def: $\gamma = \alpha + j\beta = \left(\hat{Z}_{eu}\hat{Y}_u\right)^{1/2} = \left(R_u + j\omega L\right)^{1/2}\left(G_u + j\omega C\right)^{1/2}$

$$\therefore V_t(x) = A_1 e^{-\gamma x} + A_2 e^{\gamma x} = A_1 e^{-(\alpha + j\beta)x} + A_2 e^{(\alpha + j\beta)x} =$$

$$V_t(x) = \tilde{A}_1 e^{-j(\beta x - \varphi_1)} e^{-\alpha x} + \tilde{A}_2 e^{j(\beta x + \varphi_2)} e^{\alpha x}$$

$$V(x,t) = V_t(x) e^{j\omega t} \Rightarrow V_M(x,t) = Re\{V(x,t)\} = \tilde{A}_1 e^{-\alpha x} Re\{e^{-j(\beta x - \varphi_1 - \omega t)}\} + \tilde{A}_2 e^{\alpha x} Re\{e^{j(\beta x + \varphi_1 + \omega}$$

$$V_M(x,t) = \tilde{A}_1 e^{-\alpha x} \cos(\omega t + \varphi_1 - \beta x) + \tilde{A}_2 e^{\alpha x} \cos(\omega t + \varphi_2 + \beta x)$$

TBP: $V_+(x,t) = V_t(x) e^{j\omega t}$, $V_-(x,t) = V_t(x) e^{-j\omega t}$, $V(x,t) = V_+(x,t) + V_-(x,t)$

$$\therefore V_M(x,t) = \tilde{A}_\pm e^{\pm \alpha x} \cos(\omega t + \varphi_i \mp \beta x)$$

$V_M \propto e^{-\alpha x}$; $V_-(x_1,t) > V_-(x_2,t)$, $V_+(x_2,t) > V(x_1,t)$ $\quad x_1 < x_2$

A FASE COSTANTE (ICONALE)

V. FASE

$$K = \omega t + \varphi_1 - \beta x \Rightarrow \frac{\partial}{\partial t}(\omega t + \varphi_1 - \beta x) = \omega - \beta \frac{\partial x}{\partial t} = 0 \Rightarrow \frac{\partial x}{\partial t} = \frac{\omega}{\beta} = u \;!$$

N.B.

$$u = Im\{\gamma\} = Im\{\sqrt{\hat{Z}_u \hat{Y}_u}\}$$

NON DISSIPATIVO

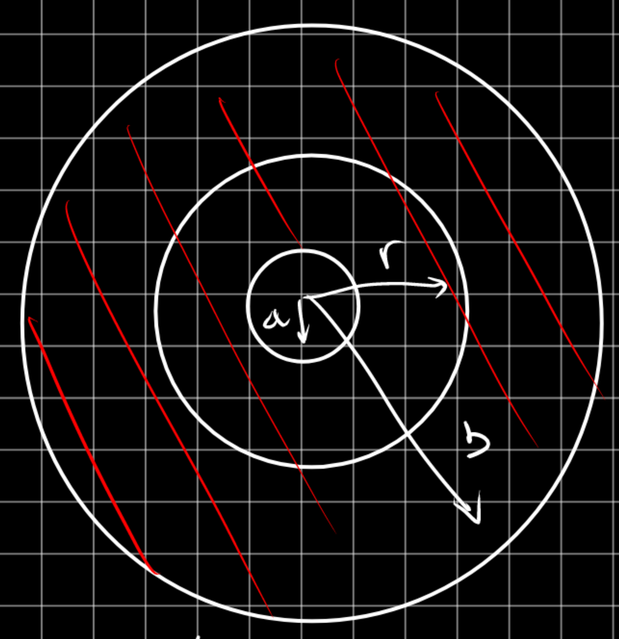
$G_u = 0$, $R_u = 0$

$$\therefore Z_u = j\omega L, \; Y_u = j\omega C \Rightarrow \gamma = \sqrt{j^2\omega^2 LC} = j\omega\sqrt{LC} = j\omega\tau \;!!$$

$$!\therefore \beta = \omega\tau \Rightarrow u = \frac{1}{\tau} \;!!$$

CAVI COASSIALI

1- Autoinduzione



Cavo coassiale RG-58

(Sez. semplificata toroidale)

conduttore $0 < r \leq a$

isolante $a < r \leq b$

1) $\begin{cases} \nabla \cdot \underline{B} = 0 & a) \\ \nabla \times \underline{B} = \mu \underline{J} & b) \end{cases}$ $\quad b) \Rightarrow \oint_{\partial cc} \underline{B} \cdot \underline{\ell} \, d\ell = 2\pi r B = \mu I \rightarrow B(r) = \frac{\mu}{2\pi r} I$

$$\therefore \iint_{cc} \underline{B} \cdot \underline{n} \, d^2s = \int_a^b B(r) \, l \, dr = \frac{\mu l I}{2\pi} \int_a^b \frac{1}{r} dr = \frac{\mu l I}{2\pi} \log\left(\frac{b}{a}\right)$$

Per def $L_u = \frac{L}{l} = \frac{\Phi}{I} = \frac{\mu}{2\pi} \log\left(\frac{b}{a}\right)$

- Capacità (formula nota per condensatore cilindrico)

$$C = \frac{Q}{V} = \frac{2\pi\epsilon l}{\log(b/a)} \Rightarrow C_u = \frac{C}{l} = 2\pi\epsilon \log^{-1}\left(\frac{b}{a}\right)$$

$$\therefore \gamma = \sqrt{L_u C_u} = \left[\left(\frac{\mu}{2\pi} \cancel{\log}\left(\frac{b}{a}\right)\right)\left(2\pi\epsilon \cancel{\log^{-1}}\left(\frac{b}{a}\right)\right)\right]^{1/2} = \sqrt{\mu\epsilon} = \underbrace{\sqrt{\mu_0\epsilon_0}}_{1/c} \underbrace{\sqrt{\mu_r\epsilon_r}}_{n}$$

$$\gamma = \frac{n}{c} \; ! \; ; \quad R_0 = \sqrt{\frac{L_u}{C_u}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}} \log\left(\frac{b}{a}\right)$$