

Note: A square-root sign is to be understood over *every* coefficient, e.g., for  $-8/15$  read  $-\sqrt{8/15}$ .

Notation:

$J$	$J$	...
$M$	$M$	...

  

$m_1$	$m_2$	
$m_1$	$m_2$	Coefficients
$\vdots$	$\vdots$	
$\vdots$	$\vdots$	

$$1/2 \times 1/2$$

1		
+1	1	0
+1/2 +1/2	1	0
+1/2 -1/2	1/2	1/2
-1/2 +1/2	1/2	-1/2
-1/2 -1/2	1	

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}} \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi}$$

$$Y_2^2 = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{2i\phi}$$

$$2 \times 1/2$$

5/2		
+5/2	1	3/2
+2 +1/2	1	3/2 +3/2
+2 -1/2	1/5	4/5
+1 +1/2	4/5 -1/5	5/2 3/2

2/5	3/5	5/2	3/2
+1 -1/2	2/5	3/5	5/2 3/2
0 +1/2	3/5 -2/5	-1/2 -1/2	
0 -1/2	3/5	2/5	5/2 3/2
-1 +1/2	2/5 -3/5	-3/2 -3/2	
-1 -1/2	4/5	1/5	5/2
-2 +1/2	1/5 -4/5	-5/2	
-2 -1/2	1		

$$3/2 \times 1/2$$

2		
+2	1	
+3/2 +1/2	1	1
+3/2 -1/2	1/4	3/4
+1/2 +1/2	3/4 -1/4	2 1
+1/2 -1/2	1/2	1/2
-1/2 +1/2	1/2 -1/2	-1 -1
-1/2 -1/2	3/4	1/4
-3/2 +1/2	1/4 -3/4	2
-3/2 -1/2	1	

$$2 \times 1$$

3		
+3	3	2
+2 +1	1	2
+2 0	1/3	2/3
+1 +1	2/3 -1/3	3 2 1
+2 -1	1/15	1/3
+1 0	8/15	1/6 -3/10
0 +1	2/5	-1/2 1/10
3	2	1
0	0	0

$$3/2 \times 1$$

5/2		
+5/2	5/2	3/2
+3/2 +1	1	3/2 +3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5 -2/5	5/2 3/2 1/2
+3/2 -1	1/10	2/5
+1/2 0	3/5	1/15 -1/3
-1/2 +1	3/10	-8/15 1/6
5/2	3/2	1/2
-1/2 -1/2	-1/2	-1/2

$$1 \times 1$$

2		
+2	2	1
+1 +1	1	1
+1 0	1/2	1/2
0 +1	1/2 -1/2	2 1 0
+1 -1	1/6	1/2
0 0	2/3	0 -1/3
-1 +1	1/6 -1/2	1/3
2	1	
-1	-1	

$$3 \times 2$$

3	2	1
0	0	0
+1 -1	1/5	1/2
0 0	3/5	0 -2/5
-1 +1	1/5 -1/2	3/10
3	2	1
-1	-1	-1

$$5/2$$

5/2	3/2	1/2
-1/2 -1	3/10	8/15
-1/2 0	3/5	-1/15 -1/3
-3/2 +1	1/10	-2/5 1/2
-1/2 -1	3/5	2/5
-3/2 0	2/5 -3/5	-5/2
-3/2 -1	1	

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

0	-1	1/2	1/2	2
-1	0	1/2 -1/2	-2	
-1	-1	1		

$$d_{\ell,0}^\ell = \sqrt{\frac{4\pi}{2\ell+1}} Y_\ell^m e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M \rangle = (-1)^{J-j_1-j_2} \langle j_2 j_1 m_2 m_1 | j_2 j_1 J M \rangle$$

$$d_{m',m}^j = (-1)^{m-m'} d_{m,m'}^j = d_{-m,-m'}^j$$

$$2 \times 3/2$$

7/2		
+7/2	7/2	5/2
+2 +3/2	1	5/2 +5/2
+2 +1/2	3/7	4/7
+1 +3/2	4/7 -3/7	7/2 5/2 3/2
+2 -1/2	1/7	16/35
+1 +1/2	4/7	1/35 -2/5
0 +3/2	2/7 -18/35	1/5
7/2	5/2	3/2
+1/2	+1/2	+1/2

$$3/2 \times 3/2$$

3		
+3	3	2
+3/2 +3/2	1	2
+3/2 +1/2	1/2	1/2
+1/2 +3/2	1/2 -1/2	3 2 1
+3/2 -1/2	1/5	1/2
+1/2 +1/2	3/5	0 -2/5
-1/2 +3/2	1/5 -1/2	3/10
3	2	1
+1	+1	+1

$$d_{1,0}^1 = \cos \theta$$

$$d_{1/2,1/2}^{1/2} = \cos \frac{\theta}{2}$$

$$d_{1,1}^1 = \frac{1 + \cos \theta}{2}$$

$$d_{1/2,-1/2}^{1/2} = -\sin \frac{\theta}{2}$$

$$d_{1,0}^1 = -\frac{\sin \theta}{\sqrt{2}}$$

$$d_{1,-1}^1 = \frac{1 - \cos \theta}{2}$$

$$2 \times 2$$

4		
+4	4	3
+2 +2	1	3
+2 +1	1/2	1/2
+1 +2	1/2 -1/2	4 3 2
+2 0	3/14	1/2
+1 +1	4/7	0 -3/7
0 +2	3/14 -1/2	2/7
4	3	2
+1	+1	2

$$7/2$$

+1/2	+1/2	+1/2	+1/2	+1/2	-1/2	9/2
1/35	6/35	2/5	2/5	-1/2	+1/2	9/2
12/35	5/14	0	-3/10	-3/2	+3/2	1/2
18/35	-3/35	-1/5	1/5	7/2	5/2	3/2
4/35	-27/70	2/5	-1/10	-1/2	-1/2	-1/2

$$3/2$$

3/2	1/2	3/10
+1/2 +1/2	3/5	0 -2/5
-1/2 +3/2	1/5 -1/2	3/10
+3/2 -3/2	1/20	1/4
+1/2 -1/2	9/20	1/4 -1/20 -1/4
-1/2 +1/2	9/20 -1/4	-1/20 1/4
-3/2 +3/2	1/20 -1/4	9/20 -1/4
3	2	1
0	0	0

$$1/2$$

1/2	1/2	3/10
+1/2 -3/2	1/5	1/2
-1/2 -1/2	3/5	0 -2/5
-3/2 +1/2	1/5 -1/2	3/10
-1/2 -3/2	1/2	1/2
-3/2 -1/2	1/2 -1/2	-3
-3/2 -3/2	1	

$$d_{3/2,3/2}^{3/2} = \frac{1 + \cos \theta}{2} \cos \frac{\theta}{2}$$

$$d_{3/2,1/2}^{3/2} = -\sqrt{3} \frac{1 + \cos \theta}{2} \sin \frac{\theta}{2}$$

$$d_{3/2,-1/2}^{3/2} = \sqrt{3} \frac{1 - \cos \theta}{2} \cos \frac{\theta}{2}$$

$$d_{3/2,-3/2}^{3/2} = -\frac{1 - \cos \theta}{2} \sin \frac{\theta}{2}$$

$$d_{1/2,1/2}^{3/2} = \frac{3 \cos \theta - 1}{2} \cos \frac{\theta}{2}$$

$$d_{1/2,-1/2}^{3/2} = -\frac{3 \cos \theta + 1}{2} \sin \frac{\theta}{2}$$

$$d_{2,2}^2 = \left( \frac{1 + \cos \theta}{2} \right)^2$$

$$d_{2,1}^2 = -\frac{1 + \cos \theta}{2} \sin \theta$$

$$d_{2,0}^2 = \frac{\sqrt{6}}{4} \sin^2 \theta$$

$$d_{2,-1}^2 = -\frac{1 - \cos \theta}{2} \sin \theta$$

$$d_{2,-2}^2 = \left( \frac{1 - \cos \theta}{2} \right)^2$$

$$d_{1,1}^2 = \frac{1 + \cos \theta}{2} (2 \cos \theta - 1)$$

$$d_{1,0}^2 = -\sqrt{\frac{3}{2}} \sin \theta \cos \theta$$

$$d_{1,-1}^2 = \frac{1 - \cos \theta}{2} (2 \cos \theta + 1)$$

$$d_{0,0}^2 = \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right)$$

$$1/4$$

1/4	3/10	3/7	1/5
0 -1	3/7	1/5 -1/14 -3/10	
-1 0	3/7	-1/5 -1/14	3/10
-2 +1	1/14 -3/10	3/7 -1/5	
0	-2	3/14	1/2
-1 -1	4/7	0 -3/7	
-2 0	3/14 -1/2	2/7	
-1	-2	1/2	1/2
-2 -1	1/2 -1/2	-4	
-2	-2	1	

$$3/4$$

3/4	1/2	2/7
-1 -1	4/7	0 -3/7
-2 0	3/14 -1/2	2/7
-1	-2	1/2
-2 -1	1/2 -1/2	-4
-2	-2	1