21/01/2022 p. 2

E2) n = 1 mol di gas perfetto biatomico a $p_A = 1\,atm$, $T_A = 56°C$ raddoppia p ($p_B = 2p_A = 2\,atm$) in processo A→B politropico $T_A V_A^{-2} = cost$ compresso a T = cost (reversibile) fino a $V_C = \frac{1}{2} V_B$. C→D ad irr. fino a $p_D = p_A$, D→A isobara con singola sorgente di Q (irrev.)

Sapendo $W_{TOT} = 0$

1) pVT

2) ΔS_a, ΔS_{gas}

3) ΔS_Ω

4) pV, TS

1) $p_A = 1 atm$, $T_A = 56°C$, $p_B = 2\,atm$, $V_C = \frac{1}{2} V_B$, $p_D = p_A$

A) $V_A = \dfrac{nRT_A}{p_A} = \dfrac{(1\,mol)(8.314\,J/mol\,K)(329.15\,K)}{(101325\,Pa)} = 0.0270\,m^3$

A→B) $\dfrac{T_A}{V_A^2} = \dfrac{T_B}{V_B^2}$, $\dfrac{p_A V_A}{nRV_A^2} = \dfrac{p_B V_B}{nRV_B^2} \rightarrow \dfrac{p_A}{V_A} = \dfrac{p_B}{V_B}$ $\alpha = -1$

B) $V_B = 2V_A = \dfrac{2nRT_A}{p_A}$, $p_B = 2\,atm$, $T_B = \dfrac{p_B V_B}{nR} = 1316.23\,K$

$V_B = 0.0540\,m^3$, $p_B = 202650\,Pa$, $T_B = 1316.23\,K$

$V_C = \frac{1}{2} V_B = 0.0270\,m^3$, $T_C = T_B = 1316.23\,K$ (C)

$p_C = \dfrac{nRT_C}{V_C} = \dfrac{2nRT_B}{V_B} = 405301.34\,Pa$

recap

	A	B	C	D
P	101325 Pa	202650 Pa	405301.34 Pa	101325 Pa
V	0.0270 m³	0.0540 m³	0.0270 m³	
T	329.15 K	1316.23 K	1316.23 K	

Ok!

D) $\oint dU = \oint đQ - \oint đW = 0 \Rightarrow \oint đQ = \oint đW = 0$

A→B $\displaystyle\int_A^B đQ_\alpha = nC_\alpha \int_A^B dT$

B→C) $dT = 0$, $đQ = dU + đW = p\,dV = \dfrac{nRT}{V} dV$

$\displaystyle\int_B^C đQ = nRT_B \int_B^C \frac{1}{V} dV$

C→D) $đQ = 0$

D→A) p = cost ⇒ $đQ = nc_v dT + nR dT = nc_p dT$

$\displaystyle\int_D^A đQ = nc_p \int_D^A dT$

$Q_{TOT} = 3nR(T_B - T_A) + nRT_B \log\left(\dfrac{V_C}{V_B}\right) + nc_p(T_A - T_D) = 0$

$$n c_p (T_D - T_A) = 3nR(T_B - T_A) + nR \log\left(\frac{V_C}{V_B}\right) T_B$$

$$\frac{7}{2}(T_D - T_A) = 3(T_B - T_A) + \log\left(\frac{1}{2}\right) T_B$$

$$T_D = \frac{6}{7}(T_B - T_A) + \frac{2}{7} T_B \log\left(\frac{1}{2}\right) + T_A = 914.55 \text{ K}$$

$$P_D = P_A \Rightarrow V_D = \frac{nRT_D}{P_A} = 0.0750 \text{ m}^3$$

	A	B	C	D
P[Pa]	101325	202650	405301.34	101325
V[m³]	0.0270	0.0540	0.0270	0.0750
T[K]	329.15	1316.23	1316.23	914.55

A	B	C	D	
1	2	4	1	P [atm]
27	54	27	75	V [ℓ]
56	1043.8	1043.8	641.4	T [°C]

2) ΔS_a, ΔS_g

$$dS = \frac{đQ}{T} \Rightarrow \Delta S_g = \oint \frac{đQ}{T} = \int_A^B \frac{n c_\alpha}{T} dT + \int_B^C \frac{nR}{V} dV + \int_C^D \frac{n c_V}{T} dT + \frac{nR}{V} dV + \int_D^A \frac{n c_p}{V} dV$$

$$\Delta S_g = 3nR \log\left(\frac{T_B}{T_A}\right) + nR \log\left(\frac{V_C}{V_B}\right) + n c_V \log\left(\frac{T_D}{T_C}\right) + nR \log\left(\frac{V_D}{V_C}\right) + n c_p \log\left(\frac{V_A}{V_D}\right)$$

$$c_V = \frac{5}{2}R, \quad c_p = \frac{7}{2}R$$

$$\Delta S_g = 3nR \log\left(\frac{T_B}{T_A}\right) + nR \log\left(\frac{1}{2}\right) + \frac{5}{2} nR \log\left(\frac{T_D}{T_C}\right) + nR \log\left(\frac{V_D}{V_C}\right) + \frac{7}{2} nR \log\left(\frac{V_A}{V_D}\right) \simeq 0 \quad ✓$$

$\Delta S_{AB} = 34.57$ J/K

$\Delta S_{BC} = -5.76$ J/K

ok!

$\Delta S_{CD} = 0.92$ J/K

$\Delta S_{DA} = -29.73$ J/K

$$\Delta S_a = -\frac{1}{T_A}\left(Q_{AB} + Q_{BC} + Q_{DA}\right)$$

A→B, B→C rev, C→D, D→A irr

$\Delta S^a_{AB} = -\Delta S^g_{AB} = -34.57$ J/K

$\Delta S^a_{BC} = -\Delta S^g_{BC} = 5.76$ J/K

$\Delta S^a_{CD} = -\frac{Q_{CD}}{T_a} = 0$

$\Delta S^a_{DA} = \frac{-n c_p (T_A - T_D)}{T_A} = 51.75$ J/K ✓

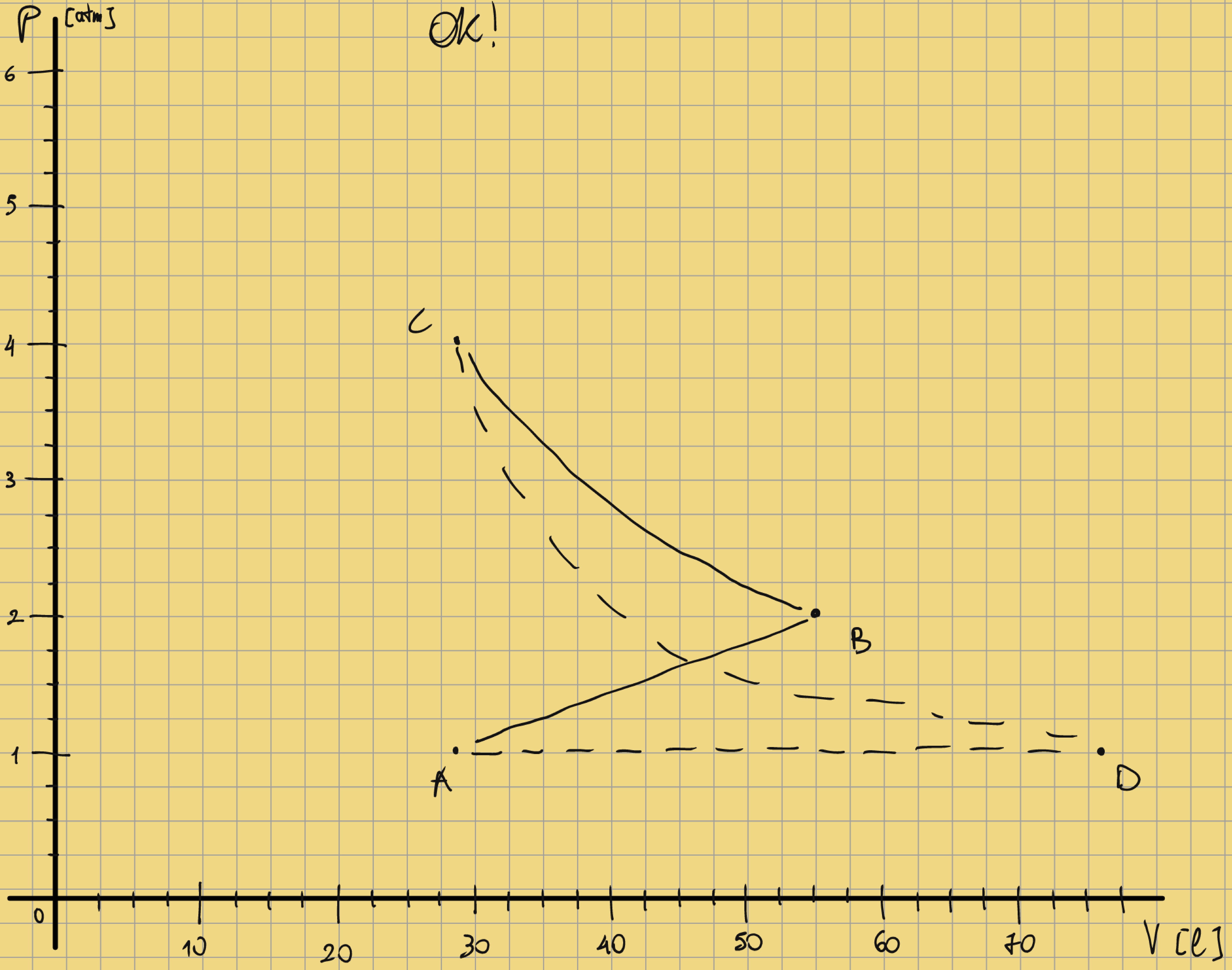
$\Delta S^a_T = 22.94$ J/K

3)

$\Delta S_\Omega = \Delta S^a_T + \Delta S^g_T = \Delta S^a_T = 22.94$ J/K

4) pV , TS

OK!



29/01/2021

E1) Ciclo Diesel ideale e reversibile (2 ad + isop (T_H) + isoV (T_L)). A stato di avvio della compressione isoentropica; B, C, D stati di equilibrio successivi.

$p_A = 1$ atm, $W_T = 1421$ J

1) natura del gas sapendo che:

$T_A = 304$ K, $T_B = 667$ K, $T_C = 904$ K, $T_D = 465.3$ K, $\eta = 51.4\%$

2) Tab. ABCD + pV

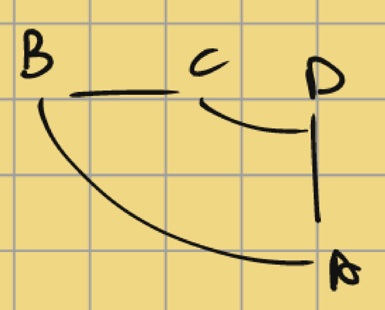
E1a) fissati A, B, ad irreversibili, $\eta' = 6\%$, $\Delta S_{isop} = 2.4$ J/K

3) B^*, D^*, posizione in pV

4) ΔS_Ω

1) $T_A = 304$ K, $T_B = 667$ K, $T_C = 904$ K, $T_D = 465.3$ K, $\eta = 51.4\%$, $p_A = 1$ atm $= 101325$ Pa, $W_T = 1421$ J

$$\oint dU = \oint (đQ - đW) = 0 \Rightarrow \oint đW = \oint đQ$$



$$W_T = \oint đQ = \int_{isoV} đQ + \int_{isop} đQ$$

$$\eta = \left| \frac{W}{Q_{abs}} \right| \to \eta |Q_{abs}| = |W_T| \Rightarrow \frac{|W_T|}{\eta} = |Q_{abs}| \Rightarrow |Q_{abs}| = \frac{1421}{51.4\%}\ \text{J} = 2764.59\ \text{J}$$

$$\Delta U = 0 \Rightarrow W_T = Q_{abs} + Q_{ced} \Rightarrow Q_{ced} = W_T - Q_{abs} = -1343.59\ \text{J}$$

$$\begin{cases} Q_{abs} = n c_p (T_C - T_B) \\ Q_{ced} = n c_V (T_A - T_D) \end{cases} \Rightarrow \frac{Q_{abs}}{Q_{ced}} = \gamma \frac{T_C - T_B}{T_A - T_D} \to \gamma = \frac{T_A - T_D}{T_C - T_B} \frac{Q_{abs}}{Q_{ced}} = \frac{304 - 465}{904 - 667} \frac{2764.59}{-1343.59} = 1.40 = \frac{7}{5}\ ! \to \text{biatomico}$$

$$c_V = \frac{5}{2}R,\ c_p = \frac{7}{2}R \Rightarrow n = \frac{Q_{abs}}{c_p (T_C - T_B)} = \frac{2 Q_{abs}}{7R(T_C - T_B)} = \frac{2 \cdot 2764.59\ \text{J}}{7 \cdot 8.314 (\text{J/Kmol})(904 - 667)\text{K}} = 0.401\ \text{mol}$$

$\gamma = \frac{7}{5}$, $n = 0.401$ mol

2) $T_A = 304$ K, $T_B = 667$ K, $T_C = 904$ K, $T_D = 465.3$ K, $\eta = 51.4\%$, $\gamma = \frac{7}{5}$, $n = 0.401$ mol

$p_A = 1$ atm $= 101325$ Pa

$$V_A = \frac{nRT_A}{p_A}, \quad p_A V_A^\gamma = p_B V_B^\gamma$$

$$pV = nRT \to V = \frac{nRT}{p} \to p_A \left(\frac{nRT_A}{p_A} \right)^\gamma = p_B \left(\frac{nRT_B}{p_B} \right)^\gamma \to p_A^{1-\gamma} T_A^\gamma = p_B^{1-\gamma} T_B^\gamma \to T_A p_A^{\frac{1-\gamma}{\gamma}} = T_B p_B^{\frac{1-\gamma}{\gamma}}$$

$$p_B^{\frac{1-\gamma}{\gamma}} = \frac{T_A}{T_B} p_A^{\frac{1-\gamma}{\gamma}} \to p_B = p_A \left(\frac{T_A}{T_B} \right)^{\frac{\gamma}{1-\gamma}};\quad \gamma = \frac{7}{5} \to 1 - \gamma = 1 - \frac{7}{5} = -\frac{2}{5} \to \frac{\gamma}{1-\gamma} = \frac{7/5}{-2/5} = -\frac{7}{2}$$

$$p_B = p_A \left(\frac{T_B}{T_A} \right)^{\frac{7}{2}}$$

A → B AD, B → C iso p, C → D AD, D → A isoV

$$V_B = \frac{nRT_B}{p_B} = \frac{nRT_B}{p_A} \left(\frac{T_B}{T_A} \right)^{\frac{7}{2}} = \frac{nRT_B^{9/2}}{p_A} \frac{1}{T_A^{7/2}}$$

p_A, V_A, T_A ✓, p_B, V_B, T_B ✓

$P_A = 1$ atm, $V_A = \frac{nRT_A}{P_A}$, $T_A = 304$ K

$P_B = P_A\left(\frac{T_B}{T_A}\right)^{7/2}$, $V_B = \frac{nRT_B}{P_B} = \frac{nRT_B}{P_A}\left(\frac{T_B}{T_A}\right)^{7/2}$, $T_B = 667$ K

$P_C = P_B = P_A\left(\frac{T_B}{T_A}\right)^{7/2}$; $V_C = \frac{nRT_C}{P_C} = \frac{nRT_C}{P_A}\left(\frac{T_B}{T_A}\right)^{7/2}$; $T_C = 904$ K

$P_D = P_C\left(\frac{T_D}{T_C}\right)^{7/2} = P_B\left(\frac{T_D}{T_C}\right)^{7/2} = P_A\left(\frac{T_BT_D}{T_AT_C}\right)^{7/2}$; $V_D = \frac{nRT_D}{P_A}\left(\frac{T_BT_D}{T_AT_C}\right)^{7/2}$, $T_D = 465.3$ K

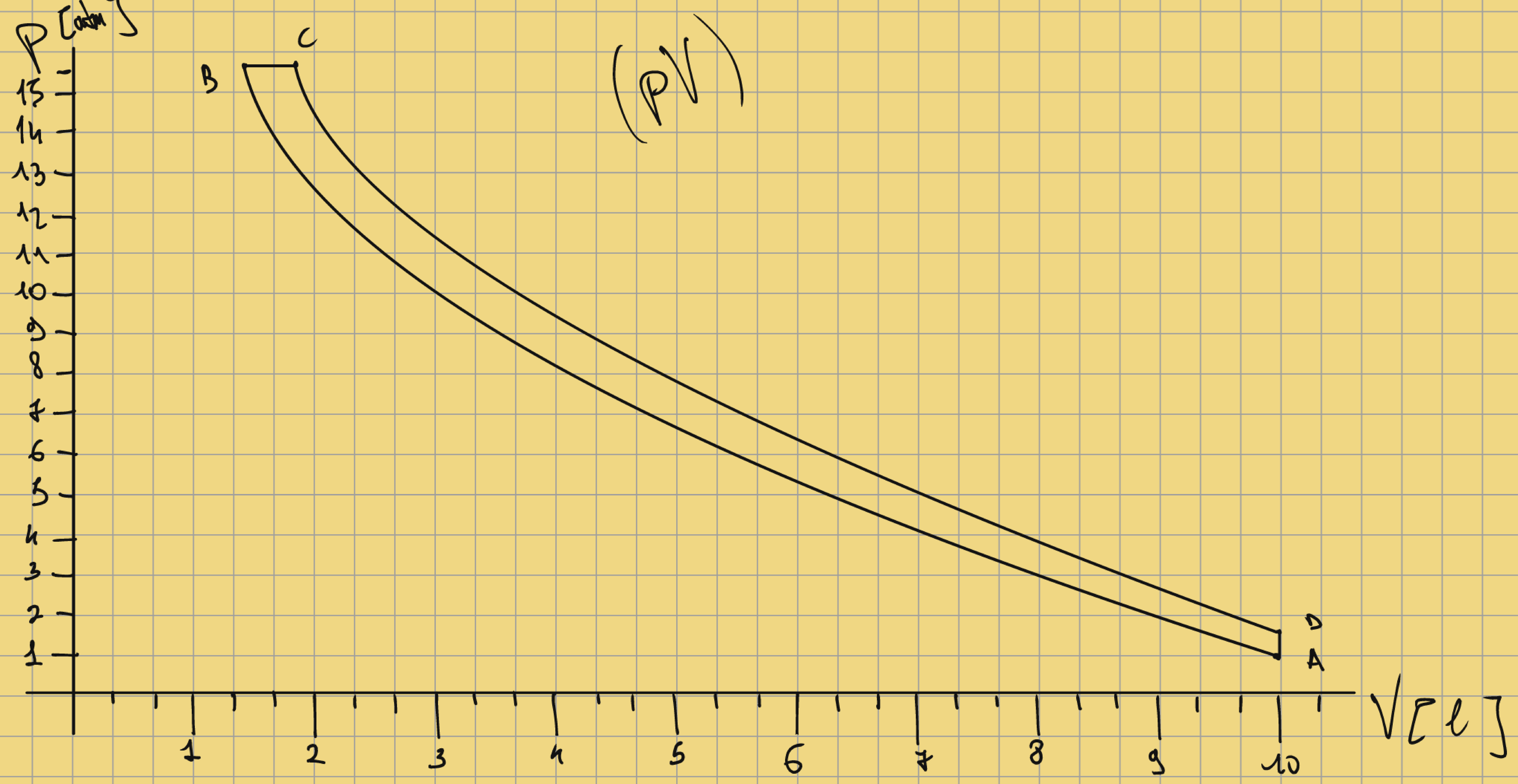
$n = 0.401$ mol!

$P_B = 1385256.30$ Pa, $V_B = 1.40\cdot 10^{-3}$ m^3, $T_B = 667$ K ✓ 1.4 l 13.64 atm

$P_C = 1385256.30$ Pa, $V_C = 1.90\cdot 10^{-3}$ m^3, $T_C = 904$ K ✓ 1.9 l ok 1 & 2

$P_D = 155086.80$ Pa, $V_D = 0.010$ m^3, $T_D = 465.3$ K ✓ 10 l

$P_A = 101325$ Pa, $V_A = 0.010$ m$^3 = V_D$, $T_A = 304$ K ✓ 10 l



P2) $\Delta S_{BC} = 2.4$ J/K, AB & CD IRR

$$\Delta S_{BC} = nc_p \log\left(\frac{T_C}{T_B}\right) = nc_p\log\left(\frac{V_C}{V_B}\right)$$

a) $P_B^* = P_C$, $V_D^* = V_A \to T_B^*, V_B^*$; T_D^*, P_D^*; $\eta^* = 6\%$

$$\Delta S_{B^*C} = nc_p\log\left(\frac{V_C}{V_B^*}\right) = 2.4\ \frac{J}{K} \Rightarrow \log\left(\frac{V_C}{V_B^*}\right) = \frac{2.4\ J/K}{nc_p} \to \frac{V_C}{V_B^*} = \exp\left(\frac{2.4\ J/K}{nc_p}\right) \to V_B^* = V_C\exp\left(-\frac{2.4\ J/K}{nc_p}\right);\ \begin{cases} n = 0.401\ \text{mol} \\ c_p = \frac{7}{2}R \\ V_C = 1.80\cdot 10^{-3}\ \text{m}^3 \end{cases}$$

$V_B^* = 1.55\cdot 10^{-3}$ m$^3 \gtrsim V_B$

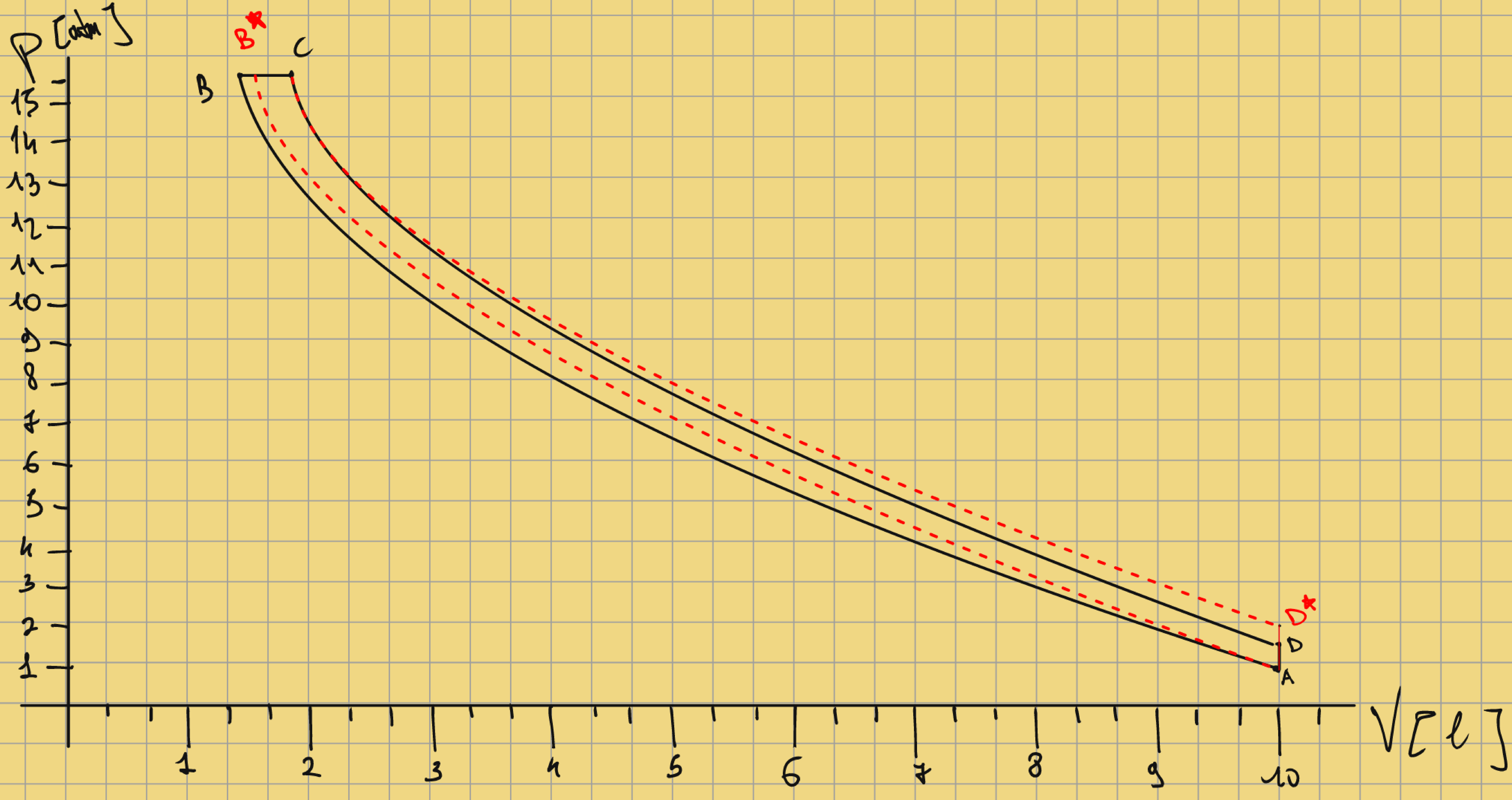
$$T_B^* = \frac{P_BV_B^*}{nR} = \frac{(1385256.30\ \text{Pa})(1.55\cdot 10^{-3}\ \text{m}^3)}{(0.401\ \text{mol})(8.314\ \text{J/mol K})} = 737.02\ \text{K}$$

$$\eta^* = \frac{|Q_T|}{|Q_{ass}^*|} = \frac{Q_{ass}^* + Q_{ced}^*}{Q_{ass}^*} = 1 + \frac{Q_{ass}^*}{Q_{ced}^*} \Rightarrow (\eta^*-1)Q_{ced}^* = Q_{ass}^*;\quad Q_{ass}^* = nc_p(T_C - T_B^*) = \frac{7}{2}nR(T_C - T_B^*);\quad Q_{ced}^* = \frac{nc_p(T_C - T_B^*)}{\eta^*-1} = nc_v(T_A - T_D^*)$$

$\frac{7}{2}/\frac{5}{2} = 7/5$

$$\therefore \frac{\gamma}{\eta^*-1}(T_C - T_B^*) = T_A - T_D^*;\quad T_D^* = \frac{\gamma}{\eta - 1^*}(T_B^* - T_C) + T_A = \frac{7/5}{-0.94}(732.02\ \text{K} - 904\ \text{K}) + 304\ \text{K} = 560.14\ \text{K}$$

$$P_D^* = \frac{nRT_D^*}{V_A} = \frac{0.401\cdot 8.314\cdot 560.14}{0.010}\ \text{Pa} = 186745.86\ \text{Pa}$$



4) ΔS_Ω.

$$\Delta S_g = \Delta S_{AB^*} + \Delta S_{B^*C} + \overset{+\Delta S_{CD^*}}{\Delta S_{D^*A}} = \underbrace{\left[n c_v \log\left(\frac{T_B^*}{T_A}\right) + nR \log\left(\frac{V_B^*}{V_A}\right)\right]}_{1.11\ J/K} + 2.4\ J/K + \underbrace{\left[n c_v \log\left(\frac{T_D^*}{T_C}\right) + nR \log\left(\frac{V_D^*}{V_C}\right)\right]}_{1.54\ J/K} + \underbrace{n c_v \log\left(\frac{T_A}{T_D^*}\right)}_{-5.09\ J/K} \simeq 0\ J/K \quad (!\,ok)$$

$$\Delta S_a = -\Delta S^g_{B^*C} + \frac{Q_{D^*A}}{T_A} = -2.4\ J/K - \int_{T_D^*}^{T_A} \frac{n c_v\, dT}{T} = -2.4\ J/K + n c_v \log\left(\frac{T_D^*}{T_A}\right) = 5.09\ \frac{J}{K} - 2.40\ \frac{J}{K} = 2.69\ J/K \quad \checkmark\ (ok,\ T_D^*)$$

$$\Delta S_\Omega = \Delta S_a + \Delta S_g = \Delta S_a = 2.69\ J/K$$

E2) Giacca con $n_S = 5$ strati, spessi $d_S = 0.1$ mm, $K_S = 0.13$ W/mK, tra i quali $n_A = 4$ strati d'aria $K_A = 0.025$ W/mK, $d_A = 2$ mm

Assunto $T_{int} = 28°C$, $S_{ext} = 1\ m^2$ e regime stazionario:

ammaccatura al segno esercizio di verifica ✓

1) Potenza dissipata quando $T_{ext} = -5°C$, $h_c = 25\ W/m^2K$

2) Potenza se $n_S = 1$

3) Spessore se $K_L = 0.035$ W/mK, $n_L = 1$, stesse proprietà del $n_S = 5$

1) $\dot{q} = h\Delta T$; $h^{-1} = \sum_{i=1}^{N} h_i^{-1}$.

per def, $h_S = \frac{K_S}{d_S} \Rightarrow h^{-1} = \sum_{S=1}^{n_S} h_S^{-1} + \sum_{A=1}^{n_A} h_A^{-1} + h_c^{-1}$

$n_S = 5$, $n_A = 4 \Rightarrow h^{-1} = 5h_S^{-1} + 4h_A^{-1} + h_c^{-1}$

$$\therefore \dot{q} = h\Delta T \longrightarrow T_{int} - T_{ext} = (5h_S^{-1} + 4h_A^{-1} + h_c^{-1})\frac{1}{S}\frac{dQ}{dt} = \left(5\frac{d_S}{K_S} + 4\frac{d_A}{K_A} + \frac{1}{h_c}\right)\frac{1}{S}\frac{dQ}{dt}$$

$$\therefore \frac{dQ}{dt} = S\left(\frac{K_A K_S h_c}{5 d_S K_A h_c + 4 d_A K_S h_c + K_A K_S}\right)\Delta T$$

$S = 1\ m^2$, $h_c = 25\ W/m^2K$, $K_A = 0.025$ W/mK, $K_S = 0.13$ W/mK, $d_S = 0.1$ mm, $d_A = 2$mm, $\Delta T = 23$ K

$$\frac{dQ}{dt} = \left[\frac{(0.025)(0.13)(25)}{5(0.0001)(0.025)(25) + 4(0.002)(0.13)(25) + (0.13)(0.025)}\right](33)\ W = 30.70\ W$$

2) $$\frac{dQ}{dt} = S\left[\frac{d_S}{K_S} + \frac{1}{h_c}\right]^{-1}\Delta T = S\left[\frac{h_c K_S}{d_S h_c + K_S}\right]\Delta T = \left[\frac{(25)(0.13)}{(1\cdot 10^{-4})(25) + 0.13}\right] 33\ W = 809.43\ K$$

3) Lana $K_L = 0.035$ W/mK, $n_L = 1$, $h_L = h_M$

$$\frac{1}{h_M} = \frac{4}{h_A} + \frac{5}{h_S} = \frac{d_L}{K_L} = h_L \Rightarrow d_L = K_L\left(\frac{4}{h_A} + \frac{5}{h_S}\right) = 0.035\frac{W}{mK}\left(\frac{4\cdot(0.002)\,m}{0.025\ W/mK} + \frac{5(0.0001)\,m}{0.13\ W/mK}\right) = 0.0113\ m = 1.13\ cm$$ ✓

20/06/2019

E1) Recipiente ad lungo $L = 1\,m$ diviso in 2 camere da pistone & molla ($K = 2\cdot10^4\,N/m$, $L_0 = L/2$). Presente valvola

Camera del pistone vuota, camera 2 con gas monoatomico, $n = 1$, $T_A = 300\,K$. Nuovo eq. quando viene aperta la valvola.

1) ℓ iniziale della molla

2) T_B gas

3) ΔS_Ω

1) $p_A V_A = nRT_A \to p_A = \frac{nRT_A}{V_A}$; $p = \frac{F}{S}$; $F = -K(\ell - L_0) = K(\ell - L/2)$; $\therefore p_A = -\frac{1}{S}K(\ell - L/2)$

$$\begin{cases} \frac{-K}{S}(\ell - L/2) = \frac{nRT_A}{V_A} \\ V_A = (L-\ell)S \end{cases} \quad \left\{ -\frac{K}{S}\left(\ell - \frac{L}{2}\right) = \frac{nRT_A}{S(L-\ell)} \Rightarrow -K\left(\ell - \frac{L}{2}\right) = \frac{nRT_A}{L-\ell} \to (L-\ell)\left(\frac{L}{2} - \ell\right) = \frac{nRT_A}{K} \right.$$

$$(L-\ell)\left(\frac{L}{2} - \ell\right) = \frac{nRT_A}{K} \to \frac{L^2}{2} - L\ell - \frac{\ell L}{2} + \ell^2 = \ell^2 - \frac{3}{2}L\ell + \frac{L^2}{2} - \frac{nRT_A}{K} = 0$$

$-b = \frac{3L}{2}$, $c = \frac{L^2}{2} - \frac{nRT_A}{K}$

$$\ell_{1,2} = \frac{-b}{2} \pm \sqrt{\frac{b^2}{4} - c} = \frac{3L}{4} \pm \sqrt{\frac{1}{4}\left(\frac{3L}{2}\right)^2 + \frac{nRT_A}{K} - \frac{L^2}{2}} = \frac{3}{4}(1\,m) \pm \sqrt{\frac{1}{4}\left(\frac{9}{4}\,m^2\right) + \underbrace{\frac{(1\,mol)(8.314\,J/Kmol)(300\,K)}{20000\,N/m}}_{=0.12471\,m^2} - \frac{1}{2}m^2} = \begin{cases} 1.183\,m \\ 0.32\,m \end{cases}$$

Per esclusione $\ell = 0.32\,m$ √

2) T_B?

(da $U(\ell) = \frac{1}{2}K(L/2 - \ell)^2$ a 0)

Espansione adiabatica nella camera, $V_A \to V_B = LS$ ad $\Rightarrow Q = 0$ $dU = -đW \Rightarrow \Delta U = -W_{mecc}$; $W_{mecc} = \frac{1}{2}K\left(\frac{L}{2} - \ell\right)^2$

$$\therefore nc_V\Delta T = \frac{1}{2}K\left(\frac{L}{2} - \ell\right)^2 \to T_B = \frac{1}{2}\frac{K}{nc_V}\left(\frac{L}{2} - \ell\right)^2 + T_A \to T_B = \frac{1}{2}\frac{2K}{3nR}\left(\frac{L}{2} - \ell\right)^2 + T_A = \frac{1}{3}\frac{K}{nR}\left(\frac{L}{2} - \ell\right)^2 + T_A = 325.98\,K$$

$T_B = 325.98\,K$

3) ΔS_Ω

$$dU = TdS - pdV \to \frac{1}{T}dU + \frac{1}{T}đW = dS \; ; \; dS = \frac{nc_V}{T}dT + \frac{p}{T}dV = \frac{nc_V}{T}dT + \frac{nR}{V}dV$$

$$\Delta S = nc_V \log\left(\frac{T_B}{T_A}\right) + nR\log\left(\frac{V_B}{V_A}\right) = nc_V\log\left(\frac{T_B}{T_A}\right) + nR\log\left(\frac{L}{L-\ell}\right) = 4.24\,\frac{J}{K} \quad √$$

E2) $n = 1\,mol$ di gas BIATOMICO compie ciclo Stirling IRR, $T_H = 540°C$, $T_L = 270°C$, con $r = 2$. $\eta = 10\%$ e $W_{exp} = 4.2\,kJ$

a) piano PV

b) W, Q (tutti)

c) ΔS_Ω, TS

d) η_S con stesso r e T_H, T_L

e) η_C tra T_H, T_L

a) pV. Stirling → 2 isocore + 2 isoterme

So:

$T_H = 540°C = 813.15\ K$

$T_L = 270°C = 543.15\ K$

$\frac{V_A}{V_B} = 2$

$\eta = \left|\frac{W}{Q_a}\right| = 0.10$

$W_{CD} = 4200\ J$

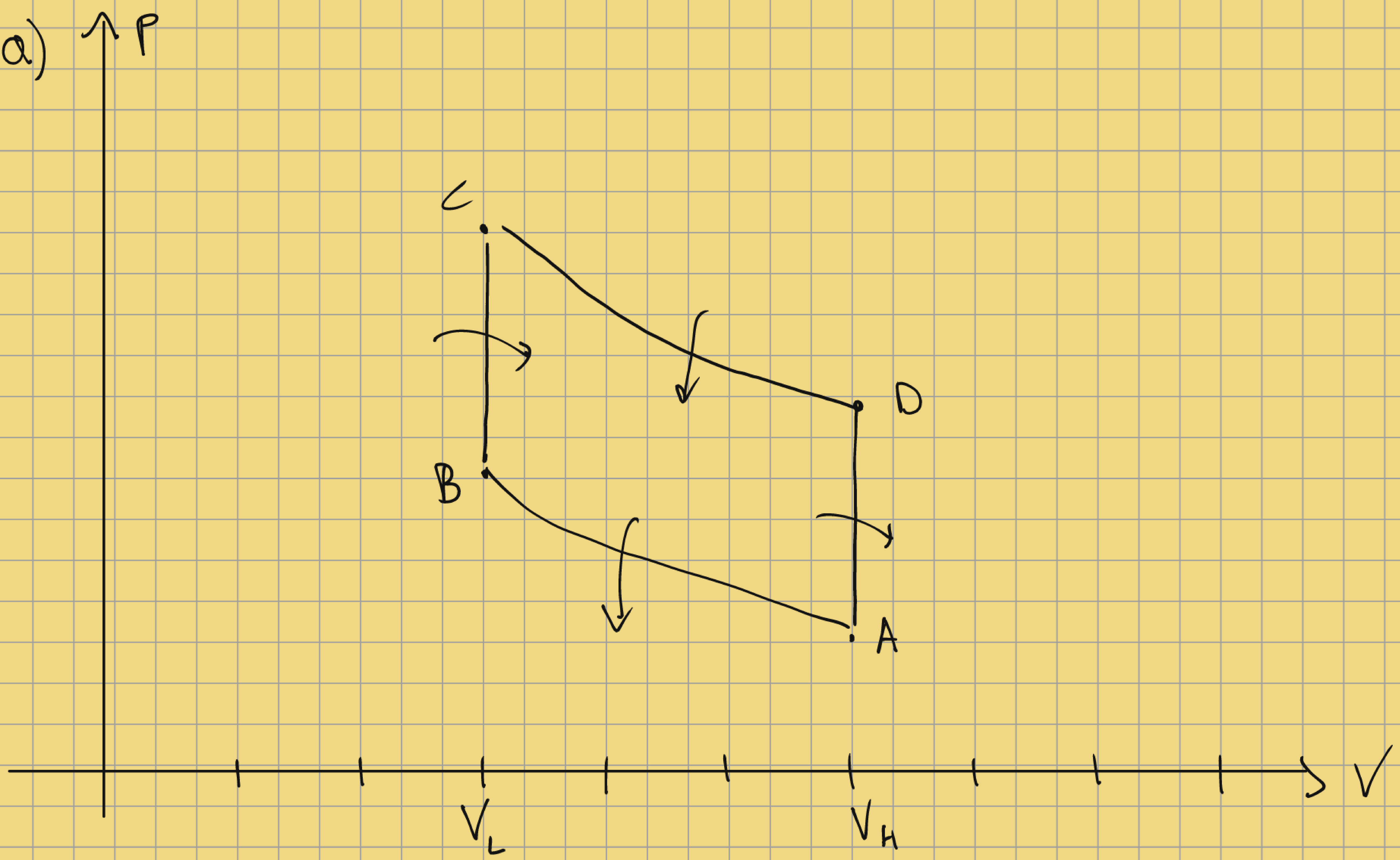
$V_A = V_D$

$V_B = V_C$

$n = 1\ mol$

[PORCODIO <u>NON</u> CHIEDE
LE COORDINATE pVT]

a)



b) W_{TOT} ?

$W_{AB} = nR\log\left(\frac{V_B}{V_A}\right) = nR\log\left(\frac{1}{2}\right)$ $Q_{AB} = W - W_{CD} = -3218.81\ J$

$W_{BC} = 0$ $Q_{BC} = nc_v(T_H - T_L) = 5611.95\ J$ ✓

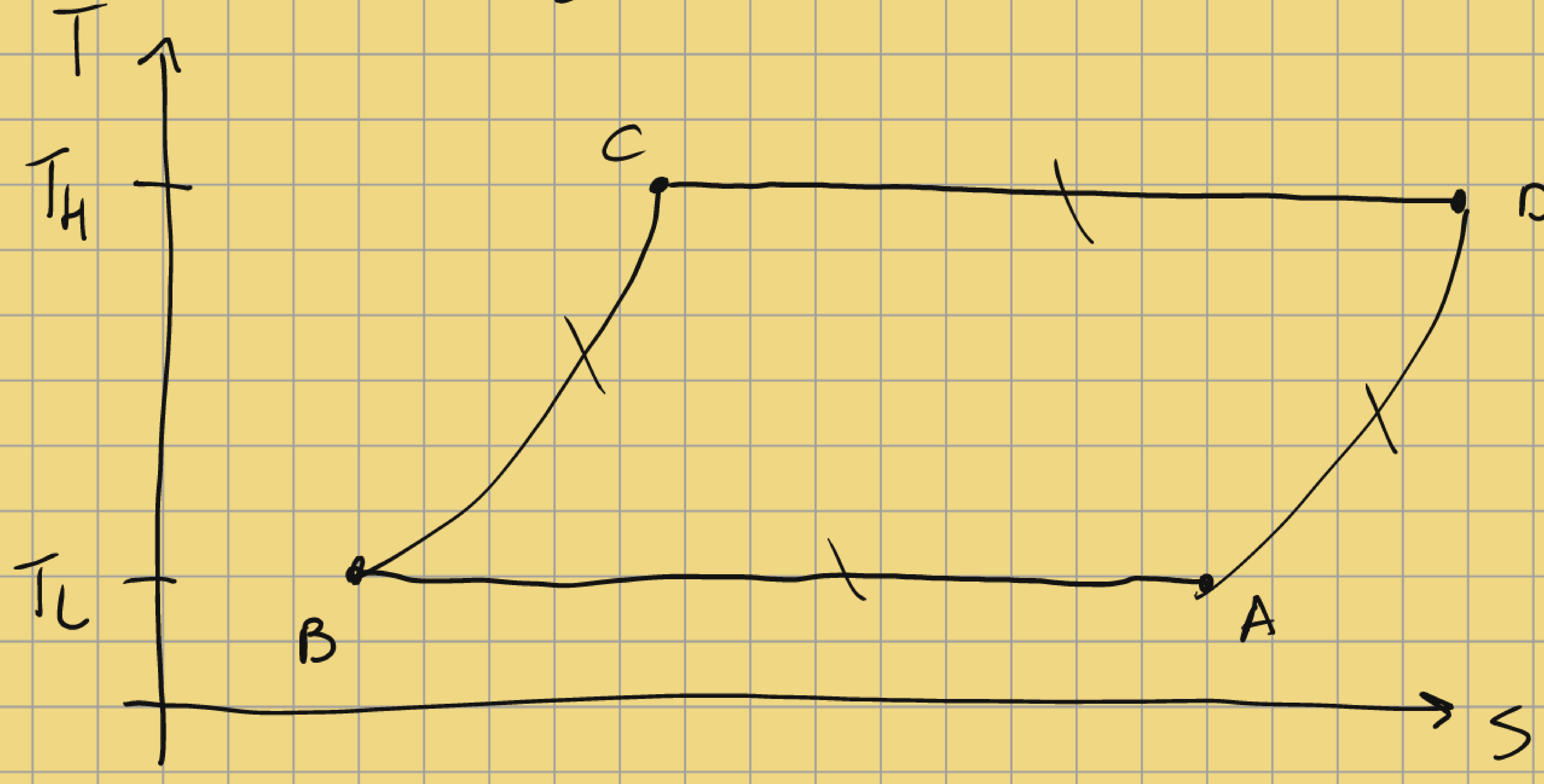
$W_{CD} = 4200\ J = Q_{CD}$ ✓

$W_{DA} = 0$ $Q_{DA} = nc_v(T_L - T_H) = -5611.95\ J$ ✓

$\eta = \frac{W}{Q_a} \rightarrow W = \eta Q_a = \eta(Q_{BC} + Q_{DC}) = \eta(nc_v(T_H - T_L) + W_{BC}) = 981.19\ J$ ✓

c) $\Delta S_{\Omega} = \Delta S_a$

$$\Delta S_a = -\left[\frac{Q_{AB}+Q_{DA}}{T_L} + \frac{Q_{BC}+Q_{CD}}{T_H}\right] = -\left[\frac{-3218.81-5611.95}{543.15} + \frac{3611.95+4200}{813.15}\right] = 4.19\ \frac{J}{K}\ \checkmark$$



d) η_S con T_H, T_L, $r=2$ reversibile

$$\eta_S = \frac{W}{Q_{ass}} = \frac{W_{AB}+W_{CD}}{Q_{BC}+Q_{CD}} = \frac{nR(T_H-T_L)\log(2)}{nc_v[T_H-T_L]+nRT_H\log(2)} = \frac{\cancel{n}R(T_H-T_L)\log(2)}{\cancel{n}T_H[c_v+R\log(2)]-\cancel{n}c_vT_L} = \frac{R(T_H-T_L)\log(2)}{T_H[c_v+R\log(2)]-c_vT_L} =$$

$$= \frac{R(T_H-T_L)\log(2)}{RT_H[5/2+\log(2)]-\frac{5}{2}RT_L} = \frac{(T_H-T_L)\log(2)}{T_H[\frac{5}{2}+\log(2)]-\frac{5}{2}T_L} = \frac{(813.15-543.15)\log(2)}{813.15[3/2+\log(2)]-\frac{3}{2}(543.15)} = 0.15\ \checkmark$$

e)

$$\eta_C = 1-\frac{T_L}{T_H} = 0.33\ \checkmark$$

11/02/2020 (pausa h. 22.04 05/07/23)

E1) Contenitore cilindrico ad diviso in 2 comparti da setto isolante rigido.

dx: fondo conduttore con tappo isolante, Sx: pistone ad senza attrito con n=1 mol monoatomico, $P_A = 1$ atm, $T_A = 93°C$

Compressione su pistone con $P_B = 4P_A$, $\Delta S_{AB} = 5$ J/K

rottura → espansione istantanea fino a V_C

tappo ad → parete dt con sorgente T_B fino a $P_D = P_A$

isobara irr con sorgente a T_A

a) Coordinate

b) Q, W, η

c) ΔS_g, ΔS_a

d) ΔS_Ω

e) TS

---

a) pVT (n=1 mol)

1) $P_A = 101325$ Pa, $T_A = 366.15$ K

2) $\Delta S_{AB} = 5$ J/K, $P_B = 4P_A = 405300$ Pa

compressione isoterma AB irreversibile

↯) exp rapida, V_C espansione libera BC

isoterma irr a $T_C = T_D$

3) $P_D = P_A$, $T_D = T_B$

4) P_A, T_A

isobara irr DA

A → B ad! irr $\Delta S_{AB} = 5$ J/K

B → C esp. libera irr V_C, $T_C = T_B$

C → D isoterma irr $V_D = V_C$, $T_D = T_B$, $P_D = P_A$

D → A isobara irr @ T_A

A) $P_A = 101325$ Pa, $T_A = 366.15$ K, $V_A = \frac{nRT_A}{P_A} = 0.0300\ m^3$

B) $P_B = 4P_A$, $\Delta S_{AB}(T,p) = nc_p\log\left(\frac{T_B}{T_A}\right) - nR\log(4) \rightarrow \frac{\Delta S_{AB}}{nc_p} + \frac{R}{c_p}\log(4) = \log\left(\frac{T_B}{T_A}\right) \rightarrow T_B = T_A e^{\frac{\Delta S_{AB}}{nc_p} + \frac{R}{c_p}\log(4)} = 4^{\frac{R}{c_p}} T_A \log(4)$

$P_B = 4P_A$, $T_B = 4^{\frac{R}{c_p}} T_A \exp\left(\frac{\Delta S_{AB}}{nc_p}\right)$, $V_B = \frac{nRT_B}{P_B} = \frac{nR}{P_A} 4^{\frac{R}{c_p}-1} \exp\left(\frac{\Delta S_{AB}}{nc_p}\right)$

A) $P_A = 101325$ Pa, $V_A = \frac{nRT_A}{P_A} = 0.0300\ m^3$, $T_A = 366.15$

B) $P_B = 4P_A = 405300$ Pa, $V_B = \frac{nR}{P_A} 4^{\frac{R}{c_p}-1} \exp\left(\frac{\Delta S_{AB}}{nc_p}\right) = 4.54 \overset{??}{\cdot} 10^{-5}\ m^3$, $T_B = 4^{\frac{R}{c_p}} T_A \exp\left(\frac{\Delta S_{AB}}{nc_p}\right) = 810.88$ K ✓

$V_B = \frac{nR}{4P_A} 4^{\frac{R}{c_p}} T_A \exp\left(\frac{\Delta S_{AB}}{nc_p}\right) = \frac{nR}{P_A} 4^{\frac{R}{c_p}-1} T_A \exp\left(\frac{\Delta S_{AB}}{nc_p}\right) = \frac{nR}{P_A} 4^{-\frac{3}{5}} T_A \exp\left(\frac{2\Delta S_{AB}}{5nR}\right) = 0.0166\ m^3$ ✓

$(nc_p = \frac{5}{2}nR)$

C) esp libera, $T_C = T_B$, $Q_{BC} = 0$, V_C, P_C

$\Delta S_{BC} = nR\log\left(\frac{V_C}{V_B}\right) = -nR\log\left(\frac{P_C}{4P_A}\right)$

$\Delta S_{CD} = nR\log\left(\frac{V_D}{V_C}\right) = -nR\log\left(\frac{P_A}{P_C}\right)$ (isoterma rev)

D) isoterma irr, $T_D = T_B$, $P_D = P_A$, V_D

DA) isobara irr. con sorgente a T_A

$\Delta S_{DA} = nc_p\log\left(\frac{T_A}{T_B}\right) = nc_p\log\left(\frac{V_A}{V_D}\right)$

$\oint dS = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CD} + \Delta S_{DA} = 0$, $\Delta S_{AB} + nR\log\left(\frac{V_C}{V_B}\right) + nR\log\left(\frac{V_D}{V_C}\right) + \frac{5}{2}nR\log\left(\frac{V_A}{V_D}\right) = 0$

$\therefore\ \Delta S_{AB} + nR\left[\log\left(\frac{V_D}{V_B}\right) + \log\left(\frac{V_A}{V_D}\right)^{\frac{5}{2}}\right] = 0$

$\Delta S_{AB} + nR\left\{\log\left[\frac{V_D}{V_B}\left(\frac{V_A}{V_D}\right)^{\frac{5}{2}}\right]\right\} = 0 \Rightarrow \frac{V_D^{-\frac{3}{2}}}{V_B} V_A^{\frac{5}{2}} = e^{\frac{-\Delta S_{AB}}{nR}}$

$V_D^{-\frac{3}{2}} = V_B V_A^{-\frac{5}{2}} e^{-\frac{\Delta S_{AB}}{nR}} \to V_D = V_B^{-\frac{2}{3}} V_A^{\frac{5}{3}} e^{\frac{2\Delta S_{AB}}{3nR}} = 0.0667\ m^3$ ✓!!! sborra pompino

A ✓
B ✓
D, p_A, V_D, T_B ✓
C → $T_C = T_B$, p_C ? V_C ? $V_C = V_B + V_A = 0.0466\ m^3$, $p_C = \frac{nRT_B}{V_C} = 144670.74\ Pa$

$\therefore$

$p_A = 101325\ Pa$ ✓	$p_B = 4p_A = 405300\ Pa$ ✓	$p_C = \frac{nRT_B}{V_A+V_B} = 144670.74\ Pa$ ✓	$p_D = p_A = 101325\ Pa$ ✓
$V_A = \frac{nRT_A}{p_A} = 0.0300\ m^3$ ✓	$V_B = \frac{nR}{p_A} 4^{-\frac{3}{5}} T_A \exp\left(\frac{2\Delta S_{AB}}{5nR}\right) = 0.0166\ m^3$ ✓	$V_C = V_A + V_B = 0.0466\ m^3$ ✓	$V_D = V_B^{-\frac{2}{3}} V_A^{\frac{5}{3}} \exp\left(\frac{2\Delta S_{AB}}{3nR}\right) = 0.0667\ m^3$ ✓
$T_A = 366.15\ K$ ✓	$T_B = 4^{\frac{5}{2}} T_A \exp\left(\frac{2\Delta S_{AB}}{5nR}\right) = 810.88\ K$ ✓	$T_C = T_B = 810.88\ K$ ✓	$T_D = T_B = 810.88\ K$ ✓

b) Q ? W ? η ?

AB adiabatica, BC esp. libera, CD isoterma, DA isobara

$Q_{AB} = Q_{BC} = 0$

CD) $Q_{CD} = W_{CD} \to Q_{CD} = nRT_C\int_C^D \frac{1}{V}\,dV = nRT_C \log\left(\frac{V_D}{V_C}\right) = 2417.59\ J$

DA) $Q_{DA} = nc_p\int_D^A dT = \frac{5}{2}nR(T_A - T_D) = -9243.71\ J$

$W_{AB} = -\Delta U_{AB} = -nc_v\int_A^B dT = \frac{3}{2}nR(T_A - T_B) = -5546.23\ J$ ✓

$dU = đQ = 0 \Rightarrow W_{BC} = 0$!

$Q_{AB} = Q_{BC} = 0$	$W_{AB} = -5546.23\ J$	$W_{tot} = -6847.71\ J$ ✓
$Q_{CD} = W_{CD} = 2417.19\ J$	$W_{BC} = 0$	$Q_{tot} = -6830.52\ J \simeq W_{tot}$ ✓
$Q_{DA} = -9243.71\ J$	$W_{CD} = Q_{CD} = 2417.19\ J$	
	$W_{DA} = -3718.64\ J$	

(W<0 frigorifero)

$W_{DA} = p_A(V_A - V_D) = -3718.67\ J$

$W_{CD} = Q_{CD} = 2417.19\ J$

$W_{tot} < 0 \Rightarrow COP_f = \frac{Q_a}{|W_{tot}|} = \frac{2417.19}{6847.11} = 0.353$, $COP_p = COP_f + 1 = 1.353$ ✓

c) ΔS_{amb}, ΔS_{gas}

$\oint dS = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CD} + \Delta S_{DA} = 0$, $\Delta S_{AB} + nR\log\left(\frac{V_C}{V_B}\right) + nR\log\left(\frac{V_D}{V_C}\right) + \frac{5}{2}nR\log\left(\frac{V_A}{V_D}\right) = \Delta S_{gas} = 0$

$\Delta S_{BC} = nR\log\left(\frac{V_C}{V_B}\right) = 8.58\ J/K$, $\Delta S_{CD} = nR\log\left(\frac{V_D}{V_C}\right) = 2.98\ J/K$, $\Delta S_{DA} = \frac{5}{2}nR\log\left(\frac{V_A}{V_D}\right) = -16.61\ J/K$, $\Delta S_{AB} = 5\ J/K$

$\Delta S_a = \Delta S^a_{AB} + \Delta S^a_{BC} + \Delta S^a_{CD} + \Delta S^a_{DA}$; $\Delta S^a_{AB} = \Delta S^a_{BC} = 0\ J/K$

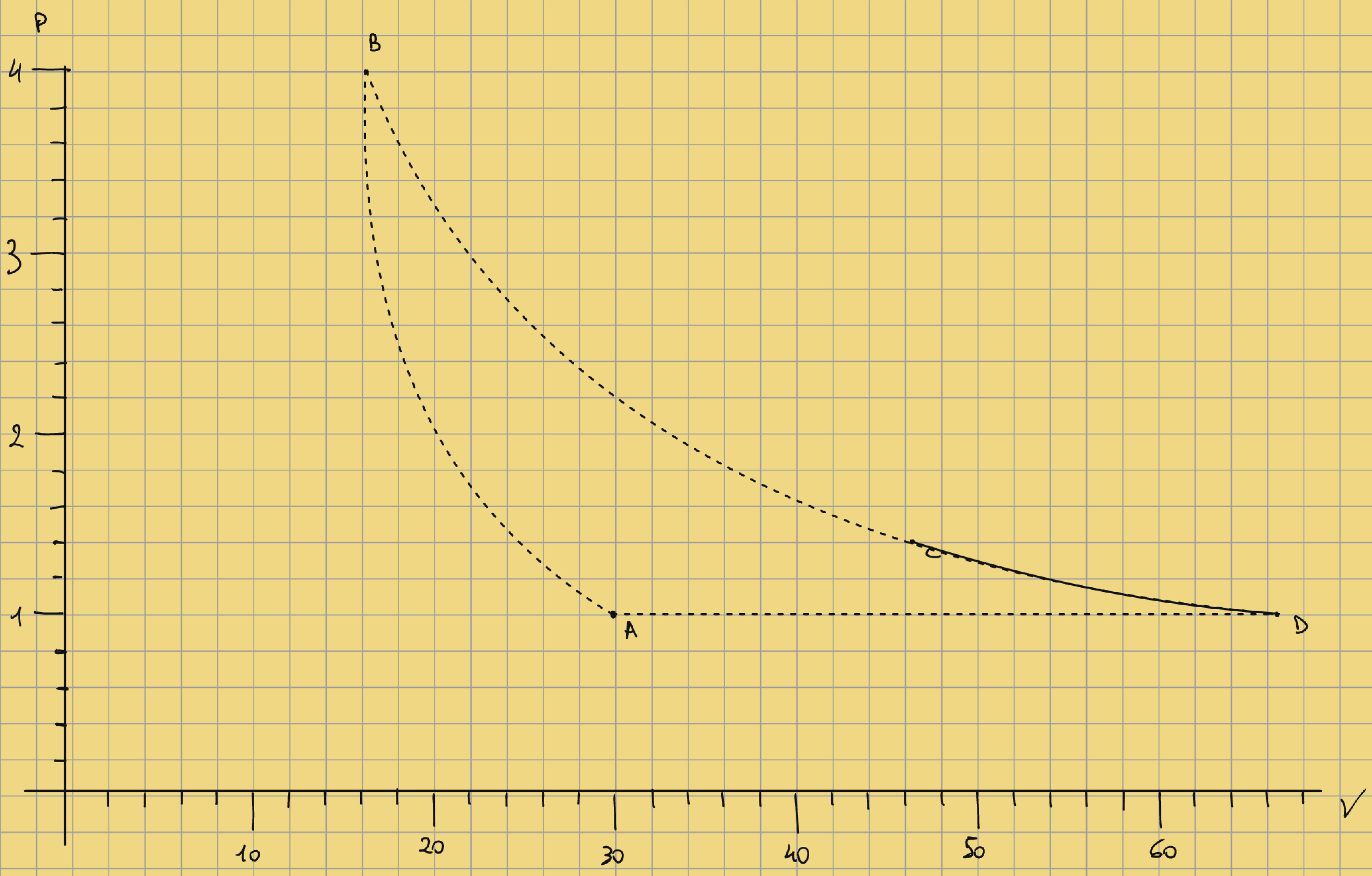
$\Delta S^a_{CD} = -\Delta S_{CD} = nR\log\left(\frac{V_C}{V_D}\right) = -2.98\ J/K$, $\Delta S^a_{DA} = -\frac{Q_{DA}}{T_A} = \frac{9243.71}{366.15}\ \frac{J}{K} = 25.25\ J/K$ ✓

$\Delta S_a = nR\log\left(\frac{V_C}{V_D}\right) - \frac{Q_{DA}}{T_A} = 22.27\ J/K = \Delta S_u$

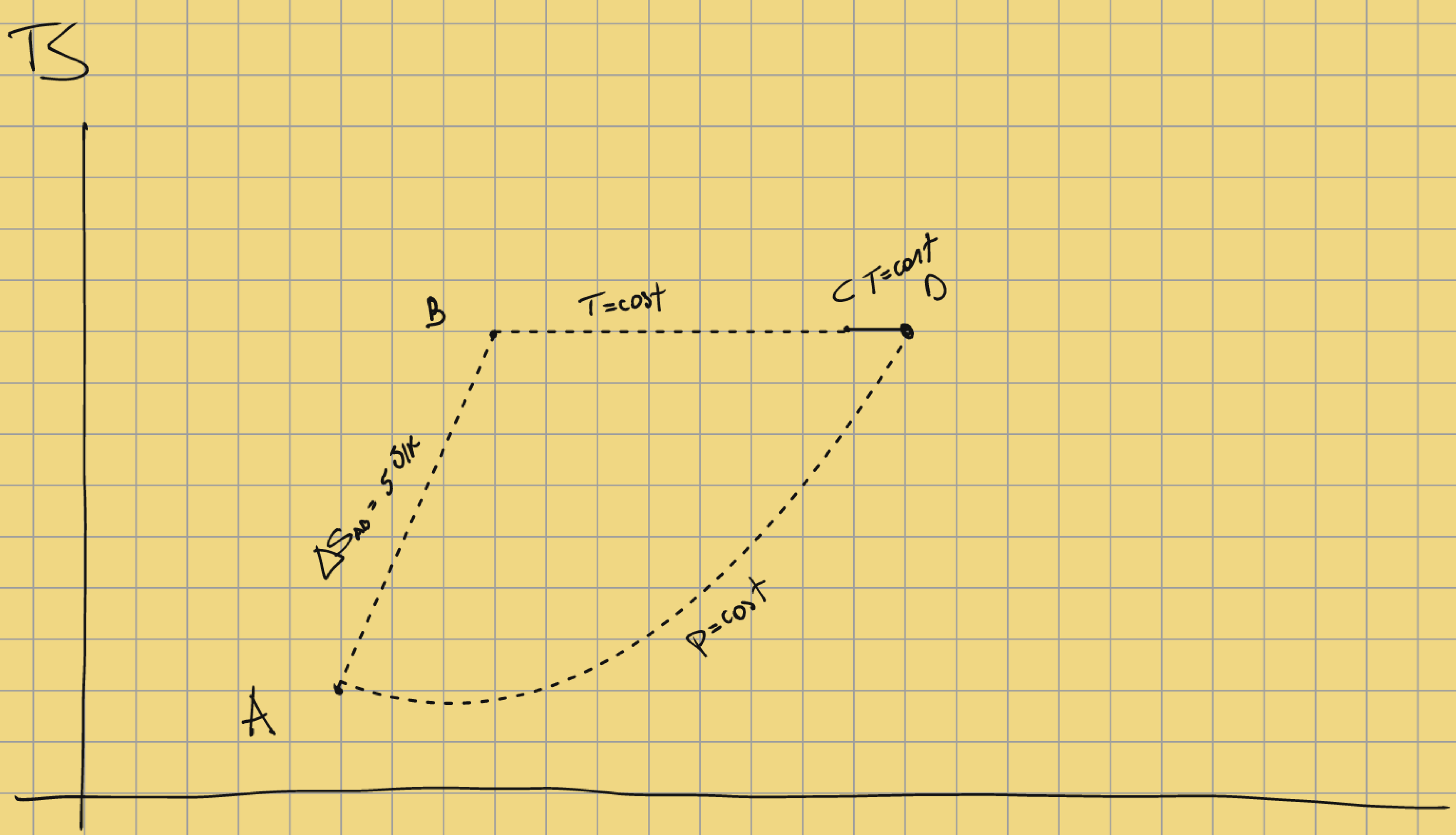
e) pV, TS

$V[l] = (30\ l, 16.6\ l, 46.6\ l, 66.7\ l)$

$p[atm] = (1, 4, 1.43, 1)$



P
B
4
3
2
1
A
C
D
V
10
20
30
40
50
60



TS
B
T=cost
C
T=cost
D
ΔS = 5.31K
p=cost
A

E2) Finestra composta con doppio vetro con cornice di legno.

Parametri geom: $W = 80\ cm$, $H = 160\ cm$, $L = 10\ cm$, $D = 2cm$, $d = 3mm$

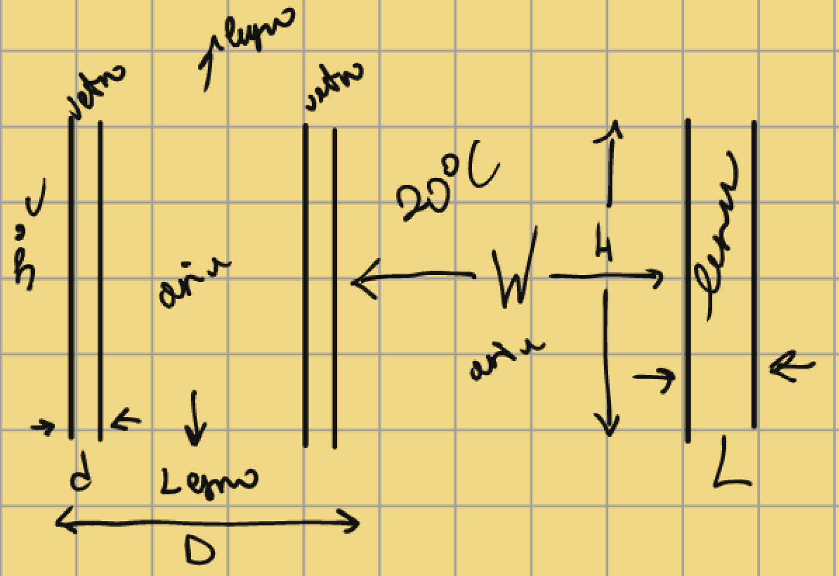
Parametri termodinamici: $k_v = 0.92\ \frac{W}{Km}$, $k_a = 0.025\ \frac{W}{Km}$, $k_\ell = 0.15\ \frac{W}{Km}$

1) Potenza termica $\{\dot{Q}\}$ che attraversa cornice quando $T_s = 20°C$, $T_{ext} = 5°C$ $\left(h_{int} = 8\ \frac{W}{Km^2},\ h_{ext} = 30\ \frac{W}{Km^2}\right)$

2) Potenza parte centrale stanza

3) W necessario per tenere $T = cost = 20°C$ con macchina $COP_p = 2$

(no effetti di bordo / irraggiamento)



1) Quanto esce da cornice?

$$\frac{1}{S_c}\frac{dQ}{dt} = h\,\Delta T, \quad \frac{1}{h} = \sum \frac{1}{h_i} = \sum \frac{d_i}{k_i}$$

$$h_v = \frac{k_v}{d},\quad h_a = \frac{k_a}{D-2d},\quad h_L = \frac{k_L}{D} \Rightarrow \frac{1}{S}\frac{dQ}{dt} = \left(\frac{1}{h_{int}} + \frac{1}{h_{out}} + \frac{1}{h_L}\right)^{-1}(T_{int} - T_{ext})$$

$$\Delta T = 15K,\quad h_T = \left(0.15 + \underbrace{\frac{0.1}{0.15}}_{}\right)^{-1}\frac{W}{m^2K} = 0.015\ \frac{W}{m^2K}$$

$$\frac{dQ}{dt} = h_T S \Delta T = S(0.015)(15) = (0.88)m^2\left(0.015\ W/m^2K\right)(15K) = 0.198\ W$$

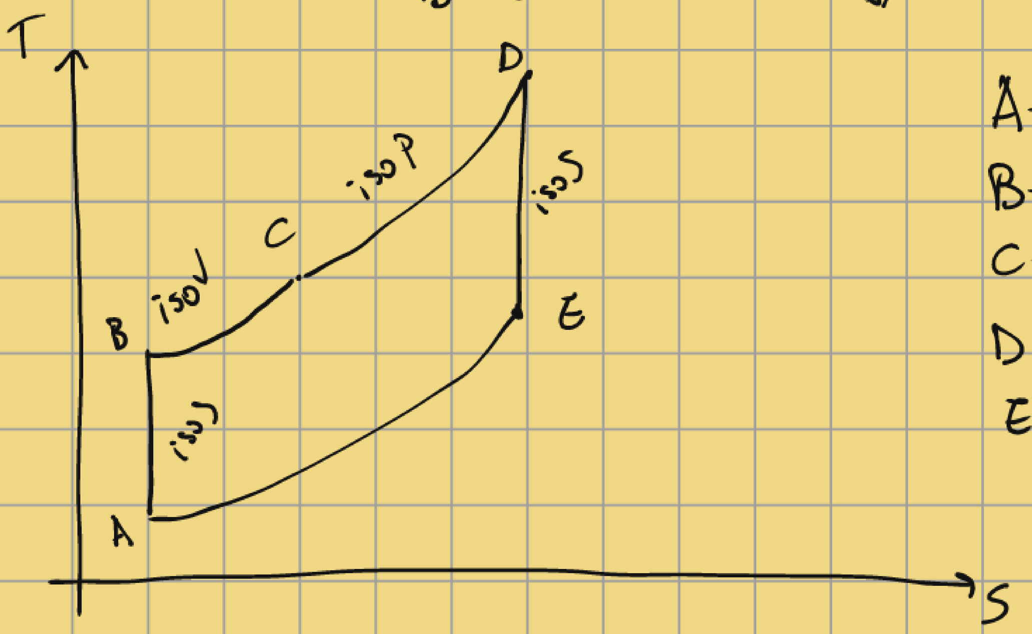
$$S = 2(H+W)L + 4L^2 = \left[2(0.8+1.6)(0.1) + 4(0.1)^2\right]cm^2 = 0.88\ m^2$$

RICCI UOMO DI MERDA

12/02/2019

E1) $n = 1\ mol$, $T_A = 40°C$, $p_A = 1atm$.

$V_c = V_B$, $V_E = V_A$, $p_D = p_C$, $r = 15$, $p_{max} = 60\ atm$, $Q_{abs} = 161\ kJ$



A → B isoS
B → C isoV
C → D isoP
D → E isoS
E → A isoP

$p_A = 101325\ Pa$, $T_A = 313.15\ K$, $V_A = \frac{nRT_A}{p_A} = 0.02569\ m^3 = V_E$

$nc_v = \frac{5}{2}nR$, $nc_p = \frac{7}{2}nR$, $\gamma = \frac{c_p}{c_v} = \frac{7}{5}$

$p_A V_A^\gamma = p_B V_B^\gamma$

$V_B = V_C$

$p_C = p_D$

$p_D V_D^\gamma = p_E V_E^\gamma$

$p_E = p_A$

$$\begin{cases} p_A V_A^\gamma = p_B V_c^\gamma \\ p_C V_D^\gamma = p_A V_E^\gamma \end{cases}$$

$p_A = p_B \left(\frac{V_C}{V_A}\right)^\gamma \Rightarrow p_B = p_A r^\gamma$

$p_C = p_A \left(\frac{V_E}{V_D}\right)^\gamma \quad p_C = p_A r^{-\gamma}$

$p_B = 15^{7/5} p_A = 4489979.67\ Pa$

$p_C = 15^{-7/5} p_A = p_D = 2286.59\ Pa$

Termodinamica e Laboratorio a.a. 2018-2019

prof. R. Di Leonardo, F. Ricci, N. Saini

Appello straordinario

Esame scritto del 17 Maggio 2019

Esercizio 1

Un gas perfetto biatomico di numero di moli n=1.5 è posto in un contenitore adiabatico di forma cilindrica con sezione $S = 0.01\ m^2$. Il contenitore è chiuso da un pistone adiabatico di massa trascurabile, libero di muoversi. La pressione esterna è pari a 1 atm e il gas è in equilibrio alla temperatura $T_A = 27\ °C$. Si poggia sul pistone un oggetto di massa 20 kg così che il gas raggiunge rapidamente il nuovo stato di equilibrio B. Mantenendo il pistone bloccato nella sua posizione, il gas viene poi scaldato lentamente fino alla temperatura $T_C = 400$ K. Successivamente, rimosso il blocco del pistone, il gas compie un'espansione isoterma reversibile fino allo stato D con volume $V_D = V_A$. Un raffreddamento isocoro reversibile riporta infine il gas nello stato A.

Determinare:

a) le coordinate termodinamiche degli stati di equilibrio A, B, C, D;
b) il calore e il lavoro totale scambiati nel ciclo e l'efficienza dello stesso;
c) confrontare l'efficienza trovata con quella di un ciclo di Carnot che opera tra le temperature estreme del ciclo;
d) la variazione totale di entropia del sistema, dell'ambiente e dell'universo nel ciclo.

a) Coordinate pVT

A) $n = 1.5\ mol$, $S = 0.01\ m^2$, $P_{ext} = 1\ atm = 101325\ Pa$ $T_A = 27°C = (27 + 273.15)K = 300.15\ K$

Gas in equilibrio $\Rightarrow P_A = P_{ext} \Rightarrow V_A = \frac{nRT_A}{P_A} = \frac{(1.5\ mol)(8.314\ J/Kmol)(300.15\ K)}{101325\ Pa} = 0.03694\ m^3$

STATO A

$P_A = 101325\ Pa$, $V_A = 0.03694\ m^3$, $T_A = 300.15\ K$

STATO B

$m = 1kg \Rightarrow P_B = \frac{F_g}{S} + P_A = \frac{mg}{S} + P_A = \frac{(20\ kg)(9.81\ m/s^2)}{0.01\ m^2} + 101325\ Pa = 120945\ Pa$

$P = \frac{F_{tot}}{S} = \frac{F_g + F_{atm}}{S} = \frac{mg}{S} + \frac{F_{atm}}{S} = \frac{mg}{S} + P_{atm} = \frac{mg}{S} + P_A$

$A \to B$ adiabatica $(\Delta P \neq 0, \Delta V \neq 0, \Delta T \neq 0)$

Gas biatomico $\Rightarrow c_p = \frac{7}{2}R$, $c_v = \frac{5}{2}R$; $\gamma = \frac{7}{5} = \frac{c_p}{c_v}$

$P_A V_A^\gamma = P_B V_B^\gamma \Rightarrow V_B = \left(\frac{P_A}{P_B}\right)^{\frac{1}{\gamma}} V_A = \left(\frac{P_A}{P_B}\right)^{\frac{5}{7}} V_A = \left(\frac{P_A}{P_A + \frac{mg}{S}}\right)^{\frac{5}{7}} V_A = 0.03669\ m^3$

$T_B = \frac{P_B V_B}{nR} = \frac{(120945\ Pa)(0.03669\ m^3)}{(1.5\ mol)(8.314\ J/mol K)} = 355.82\ K$

STATO B)

$P_B = P_A + \frac{mg}{S} = 120945\ Pa$, $V_B = \left(\frac{P_A}{P_B}\right)^{\frac{5}{7}} V_A = 0.03255\ m^3$, $T_B = 355.82\ K$

STATO C)

$T_C = 400$ K (sarà detto lentamente {quasi statico} $Q \neq 0$ $TdS \neq 0$, $dU \neq 0$, $dW \neq 0$)

CD) isoterma rev t.c. $V_D = V_A$ $(T_D = T_C = 400 \text{ K})$ ✓

DA) isocora rev. ✓

$$dU = TdS - pdV \; ; \; \oint dU = \oint dS = 0 \; \{TdS = dU + pdV\}$$

$$\oint dS = \underbrace{\Delta S_{AB}}_{\substack{= 0 \\ (\text{adiabatica})}} + \Delta S_{BC} + \Delta S_{CD} + \Delta S_{DA} = 0 \quad \{nc_V = \frac{5}{2} nR$$

$$\Delta S_{BC} = \int_B^C dS = \int_B^C \frac{dU}{T} + \int_B^C \frac{nR}{V} dV = \frac{5}{2} nR \log\left(\frac{T_C}{T_B}\right) + nR \log\left(\frac{V_C}{V_B}\right) \Big] BC$$

$$\Delta S_{CD} = \int_C^D \frac{nR}{V} dV = nR \log\left(\frac{V_D}{V_C}\right) = -nR \log\left(\frac{P_D}{P_C}\right)$$

$\{dU = 0\}$

$$\Delta S_{DA} = \int_D^A dS = nc_V \int_D^A \frac{dT}{T} = \frac{5}{2} nR \log\left(\frac{T_A}{T_D}\right) \Big) DA$$

$(pdV = 0)$

$$\oint dS = \underbrace{nR \left[\log\left(\frac{T_C}{T_B}\right)^{\frac{5}{2}} + \log\left(\frac{V_C}{V_B}\right) \right]}_{B \to C} + \underbrace{nR \log\left(\frac{V_D}{V_C}\right)}_{C \to D} + \underbrace{\frac{5}{2} nR \log\left(\frac{T_A}{T_D}\right)}_{D \to A} = 0$$

V_C? $\qquad V_C, V_D$?

$$\Delta S_{BC} = -\Delta S_{DA} - \Delta S_{CD}$$

$$nR \left[\log \left[\left(\frac{T_C}{T_B}\right)^{5/2} \left(\frac{V_C}{V_B}\right) \right] \right] = -nR \left[\log \left(\frac{V_D}{V_C}\right) \left(\frac{T_A}{T_D}\right)^{\frac{5}{2}} \right]$$

$T_C = 400$ K, $T_B = 300.99$ K, $T_A = 300.15$ K, $V_A = 0.03694$ m^2, $V_B = 0.03669$ m^3

STATO D) $P_D = 135040.61$ Pa, $V_D = V_A = 0.03694$ m^3, $T_D = T_C = 400$ K

$$V_D = V_A \, , \; T_D = T_C \rightarrow P_D = \frac{nRT_C}{V_A} = 135040.61 \text{ Pa}$$