07/02/2023

1) Due macchine di Carnot A, B in serie tra sorgenti con $T_1 = 891°C$ & $T_2 = 11°C$

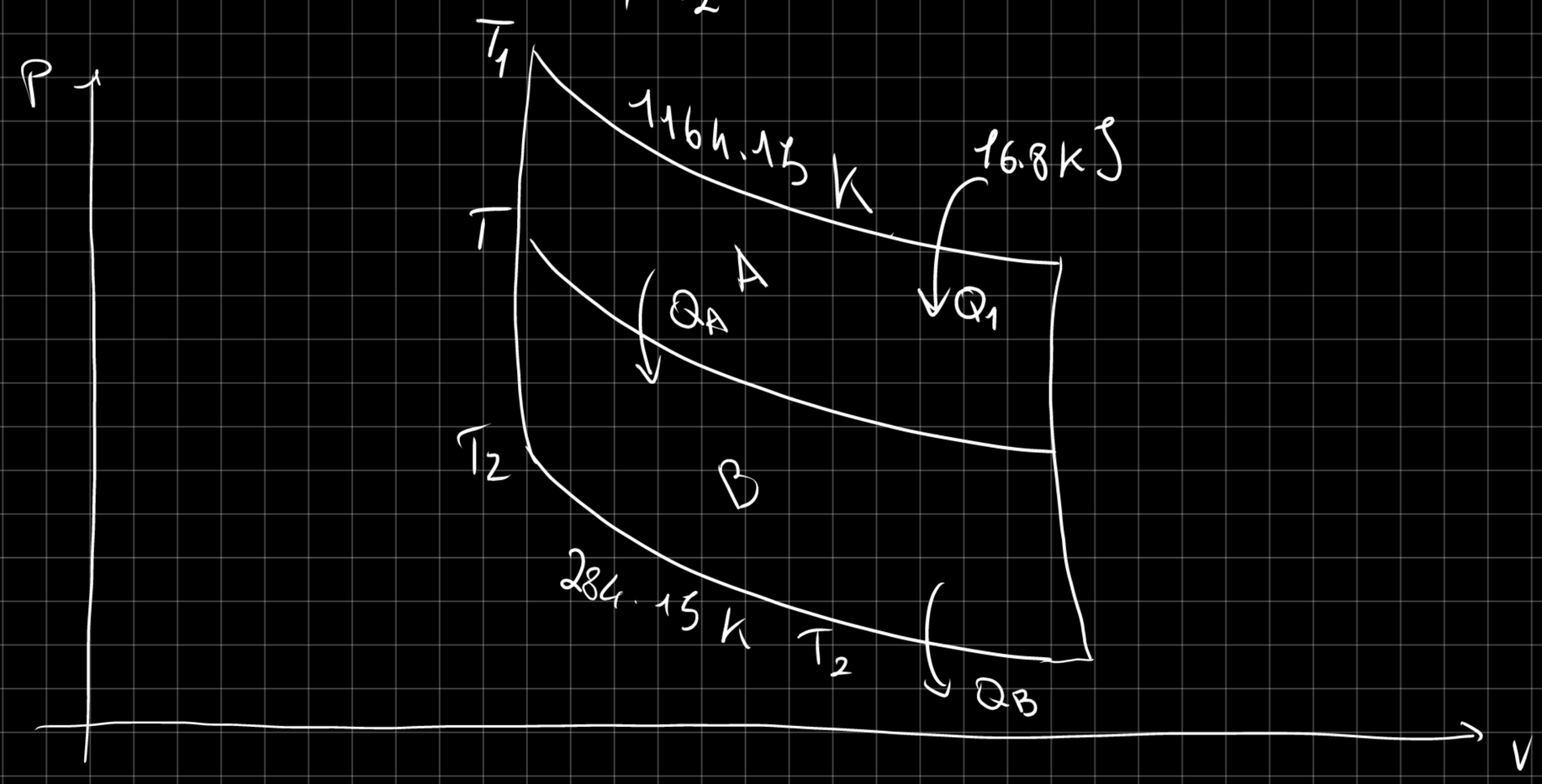
•) A assorbe $Q_1 = 16.8\,kJ$ da T_1 e cede calore a sorgente int. @ T

•) Q_1 assorbito da B che cede Q_2 a T_2

1a) $\eta_A = \eta_B \rightarrow$ trova T; Q_A, Q_B ceduti; L_A, L_B delle macchine

1b) $L_A = L_B \Rightarrow$ trova T'; Q'_A, Q'_B ceduti; η_A, η_B

$T_1 = 891°C = 1164.15\,K$, $T_2 = 11°C = 284.15\,K$



1a) $\eta_A = \eta_B = \eta$

i) trova $T \Rightarrow \eta = 1 + \frac{Q_A}{Q_1} = 1 - \frac{T}{T_1} = 1 + \frac{Q_B}{Q_A} = 1 - \frac{T_2}{T}$

$\cancel{1} + \frac{T}{T_1} = \cancel{1} + \frac{T_2}{T} \Rightarrow T^2 = T_1 T_2 \Rightarrow T = \sqrt{T_1 T_2} = (1164.15 \cdot 284.15)^{1/2} K$

$T = \sqrt{T_1 T_2} = 575.15\,K$

ii) trova Q_A, Q_B

$\eta = \cancel{1} + \frac{Q_A}{Q_1} = \cancel{1} - \frac{T_2}{T} \Rightarrow Q_A = -\frac{T_2}{T} Q_1 = \frac{284.15}{575.15}(16.8)\,kJ = -8.3\,kJ$

$\eta = 1 + \frac{Q_B}{Q_A} = 1 + \frac{T_2}{T}\frac{Q_B}{Q_1} = 1 - \frac{T}{T_1} \Rightarrow -\frac{T_2}{T}\frac{Q_B}{Q_1} = \frac{T}{T_1} \Rightarrow Q_B = -\frac{T_2}{T_1} Q_1 = 4.1\,kJ$

$$\begin{cases} Q_A = -\frac{T_2}{T} Q_1 = -8.3\,kJ \\ Q_B = \frac{T_2}{T_1} Q_1 = -4.1\,kJ \end{cases}$$

iii) trovare L_A, L_B

$$\Delta U_A = \Delta U_B \Rightarrow Q_T - L = 0 \Rightarrow \begin{cases} Q_1 + Q_A = L_A \\ Q_A + Q_B = L_B \end{cases}$$

$$L_A = 16.8\,kJ - 8.3\,kJ = 8.5\,kJ$$

$$L_B = 8.3\,kJ - 4.1\,kJ = 4.2\,kJ$$

1b) $L_A = L_B$

i) T'; $\eta = \left|\frac{L}{Q}\right| \Rightarrow \eta |Q| = L \Rightarrow \eta_A |Q_1| = |L_A| = \eta_B |Q'_A| = |L_B|$

$$\begin{cases} \eta_A = 1 + \left|\frac{Q'_A}{Q_1}\right| = 1 - \frac{T'}{T_1} \Rightarrow |Q'_A| = -\frac{T'}{T_1}|Q_1| \\ \eta_B = 1 + \left|\frac{Q'_B}{Q'_A}\right| = 1 - \frac{T_2}{T'} \Rightarrow |Q'_B| = -\frac{T_2}{T'}|Q'_A| = \frac{T'}{T_1}\frac{T_2}{T'}|Q_1| = \frac{T_2}{T_1}|Q_1| \end{cases}$$

$$\eta_A |Q_1| = \eta_B |Q'_A| \Rightarrow \left(1 - \frac{T'}{T_1}\right)|Q_1| = \left(1 - \frac{T_2}{T'}\right)|Q'_A| = -\left(1 - \frac{T_2}{T'}\right)Q'_A$$

$$\left(1 - \frac{T'}{T_1}\right)Q_1 = -\left(-\frac{T'}{T_1}\right)\left(1 - \frac{T_2}{T'}\right)Q_1 \Rightarrow 1 - \frac{T'}{T_1} = \frac{T'}{T_1}\left(1 - \frac{T_2}{T'}\right)$$

$$1 - \frac{T'}{T_1} = \frac{T'}{T_1} - \frac{T_2}{T_1} \Rightarrow 1 + \frac{T_2}{T_1} = 2\frac{T'}{T_1} \Rightarrow T' = \frac{T_1 + T_2}{2}$$

$$T' = \frac{1}{2}(T_1 + T_2) = 724.15$$

ii) Q'_A, Q'_B

$$|Q'_B| = \frac{T_2}{T_1}|Q_1| = 4.1\,kJ$$

$$|Q'_A| = -\frac{T'}{T_1}|Q_1| = 10.45\,kJ$$

iii) η_A, η_B

$$\Delta U_i = 0 \Rightarrow L_T = Q_T \Rightarrow \begin{cases} |L_A| = Q_1 - Q'_A = (16.8 - 10.45)\,kJ = 6.35\,kJ \\ |L_B| = Q'_A - Q'_B = (10.45 - 4.1)\,kJ = 6.35\,kJ \end{cases}$$

✓

$\therefore \eta_A = \left|\frac{L}{Q_1}\right| = \frac{6.33}{16.8} = 0.378$, $\eta_B = \left|\frac{L}{Q_A}\right| = 0.61$

$\eta_A = 38\%$, $\eta_B = 61\%$ ✓

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2) 1 mol di gas BIATOMICO ($M = 28\,g/mol$) è in stato iniziale A con $P = P_{atm}$ compie ciclo reversibile ABCDA

- A) $V_q = 529\,m/s = \sqrt{\langle v^2 \rangle}$

- A→B) $V_q = a_1$
- B→C) $V_q V^{-1} = a_2$
- C→D) $V_q^2 V^{-1} = a_3$
- D→A) $V_q^2 P^{-1} = a_4$

Sapendo che $r = \frac{V_{max}}{V_{min}} = 3$, & $Q_{BC} = 10\,kJ$

a) rapp. PV

b) tabella coord ABCD

d) Q_{TOT} , η

e) rapp. TS

Punto a $\Rightarrow$ rappresentazione sul piano pV

Dalla TCG abbiamo che

$$V_q = \sqrt{\frac{3K_BT}{m}} = \sqrt{\frac{3RT}{\mu}}$$

Pertanto:

Trasformazione $A \to B$

$$V_q = a_1 \Rightarrow \frac{3RT}{\mu} = a_1^2 \Rightarrow T = \frac{1}{3}\frac{\mu a_1^2}{R} = \text{const.}$$

isoterma

Trasformazione $B \to C$

$$V_q = a_2 V \Rightarrow \frac{3RT}{\mu} = a_2^2 V^2 \Rightarrow TV^{-2} = \frac{1}{3}\frac{\mu a_2^2}{R} = \text{const}$$

Poiché $pV = nRT$, $T = \frac{pV}{nR}$

$$\therefore pV^{-1} = \text{const}$$

politropica $\alpha = -1$

Trasformazione $C \to D$

$$V_q^2 = a_3 V \Rightarrow \frac{3RT}{\mu} = a_3 V \Rightarrow TV^{-1} = \text{const}$$

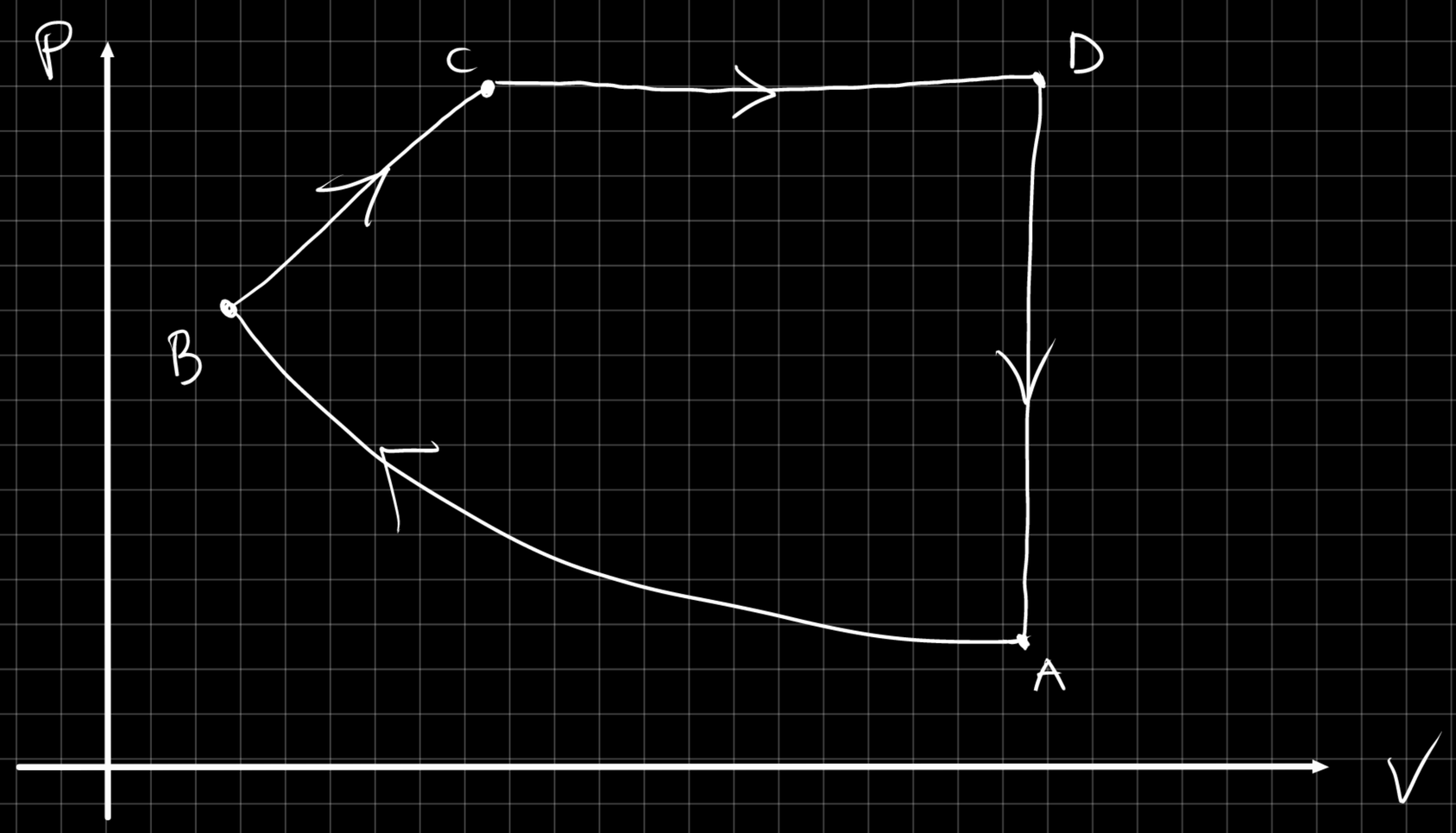
$$\therefore p = \text{const}$$

Trasf. $D \to A$

$$V_q^2 = a_4 p \Rightarrow \frac{3RT}{\mu} = a_3 p \Rightarrow p^{-1}T = \text{const}$$

$$\therefore V = \text{const}$$

Piano pV:



A→B $\quad p(V) = \frac{k}{V}$

B→C $\quad p(V) = kV$

C→D $\quad p(V) = k$

D→A $\quad V(p) = k$

Punto b

al punto A è noto $v_{q_A} = 529 \frac{m}{s}$, $P_A = P_{atm} = 101325$ Pa

$$\therefore \quad v_{q_A} = \sqrt{\frac{3RT_A}{M}} \Rightarrow T_A = \frac{M}{3R} v_{q_A}^2$$

$$P_A V_A = nRT_A \Rightarrow V_A = \frac{nRT_A}{P_A} = \frac{nMR}{3RP_A} v_{q_A}^2 = \frac{1}{3} \frac{nM}{P_A} v_{q_A}^2$$

$\therefore$ A

$P_A = 101325$ Pa

$V_A = 0.02578\ m^3 = 25.78\ l$

$T_A = 314.15$ K

A→B isoterma $\Rightarrow T_B = T_A = 314.15$ K

$$P_A V_A = P_B V_B \Rightarrow \frac{V_A}{V_B} = \frac{P_B}{P_A} = r \ ; \quad \therefore \quad P_B = r P_A, \quad \frac{V_A}{V_B} = r \Rightarrow V_B = \frac{V_A}{r}$$

$r = 3$ (def)

B: $P_B = 303975$ Pa $\quad V_B = 0.00859\ m^3 \quad T_B = 314.15$ K

$B \to C$ politropica $\alpha = -1$, dove $Q_{BC} = 10\ kJ$

Def:

$pV^{\alpha} = cost.$

$đQ = nc_{\alpha}dT = nc_V dT + \frac{nR}{1-\alpha}dT \Rightarrow \int_B^C đQ = nc_{\alpha}(T_C - T_B)$

$c_{\alpha} = c_V + \frac{R}{1-\alpha}$

Biatomico $\Rightarrow c_V = \frac{5}{2}R$, $c_{\alpha} = \frac{5}{2}R + \frac{R}{2} = \frac{6}{2}R = 3R$

$\therefore\ Q_{BC} = 3nR(T_C - T_B) \Rightarrow T_C = \frac{Q_{BC}}{3nR} + T_B$

$\therefore\ T_C = \frac{10\ kJ}{3 \cdot 8.314\ \frac{J}{K}} + 314.15\ K = 715.08\ K$

$T_C V_C^{-2} = T_B V_B^{-2} \Rightarrow V_C^2 = \frac{T_C}{T_B} V_B^2 \Rightarrow V_C = V_B\sqrt{\frac{T_C}{T_B}}$

$V_C = 0.01296\ m^3$

$p_C V_C = nRT_C \Rightarrow p_C = nR\frac{T_C}{V_C} = 3.814\ \frac{J}{K} \cdot \frac{715.08\ K}{0.01296\ m^3} = 458732.65\ Pa$

C:

$p_C = 458733\ Pa \qquad V_C = 0.01296\ m^3 \qquad T_C = 715.08\ K$

$C \to D$

isobara $\Rightarrow p_C = \overset{p_{C0}}{p_D}$. Poiché $D \to A$ isocora $V_D = V_A$ pertanto

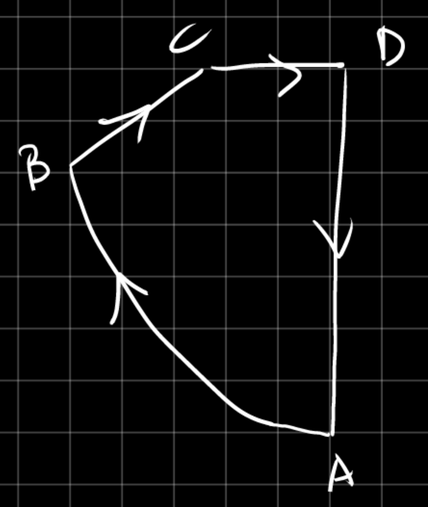
$p_D V_D = nRT_D$, $p_C V_A = nRT_D \Rightarrow T_D = \frac{p_C V_A}{nR} = \frac{(458732.65\ Pa)(0.02578\ m^3)}{8.314\ J/K} = 1422.44\ K$

D:

$p_D = p_C = 458733\ Pa \qquad V_C = V_A = 0.02578\ m^3 \qquad T_C = 1422.44\ K$ ✓

A	B	C	D
$p_A = 101325\ Pa$	$p_B = 303976$	$p_C = 458733\ Pa$	$p_D = 458733\ Pa$
$V_A = 0.02578\ m^3$	$V_B = 8.59 \cdot 10^{-3}\ m^3$	$V_C = 0.01296\ m^3$	$V_D = 0.02578\ m^3$
$T_A = 314.15\ K$	$T_B = 314.15\ K$	$T_C = 715.06$	$T_D = 1422.44\ K$

Punto c) Trova Q_{TOT} scambiato & $\eta = \frac{W}{Q}$



1) $A \to B$; isoterma $\Rightarrow \Delta U = 0$; $đQ = đW \Rightarrow đQ = pdV$

$pV = nRT \Rightarrow p = \frac{nRT}{V} \Rightarrow đQ = nRT\, d\log V$

$$\therefore \int_A^B đQ = nRT_A \log\left(\frac{V_B}{V_A}\right) = -8.314\ \frac{J}{K} \cdot 314.15\ K \cdot \log\left(\frac{8.59 \cdot 10^{-3}}{0.02578}\right) = -2870.42\ J$$

2) $Q_{BC} = 10\ kJ = 10\,000\ J$

3) $đQ_{CD} = nc_p dT \Rightarrow Q_{CD} = \frac{7}{2} nR(T_D - T_C) = 20584.05\ J$

4) $đQ_{DA} = nc_V dT \Rightarrow Q_{DA} = \frac{5}{2} nR(T_A - T_D) = -23035.81\ J$

$Q_T = 4677.82\ J$

$\Delta U_T = Q_T - W_T = 0 \Rightarrow Q_T = W_T$;

$Q_{CED} = Q_{DA} + Q_{AB} = 25905.23 \Rightarrow \eta = \frac{W_T}{Q_{CED}} = \frac{4677.82}{25905.23} = 0.181 = 18.1\ \%$

punto d

ΔS in tutte e ΔS_Ω

$dS = \frac{1}{T} đQ$

$$\therefore A \to B \quad \Delta S_{AB} = \int_A^B \frac{1}{T_A} đQ = nR \log\left(\frac{V_B}{V_A}\right) = -9.14\ J/K$$

$B \to C$)

$$\Delta S_{BC} = nc_Q \int_B^C \frac{1}{T} dT = nc_Q \log\left(\frac{T_C}{T_B}\right) = 20.51\ \frac{J}{K}$$

$$\Delta S_{CD} = nc_p \int_C^D \frac{1}{T} dT = nc_p \log\left(\frac{T_D}{T_C}\right) = 20.01\ J/K$$

$$\Delta S_{DA} = n c_V \log\left(\frac{T_A}{T_D}\right) = -31.39 \ \frac{J}{K}$$

$$\therefore \ \Delta S_{tot} = (-9.14 + 20.51 + 20.91 - 31.39)\frac{J}{K} = 0 \ \frac{J}{K} \Rightarrow \text{ciclo reversibile}$$

$$\Rightarrow \Delta S_{\Omega} = 0$$

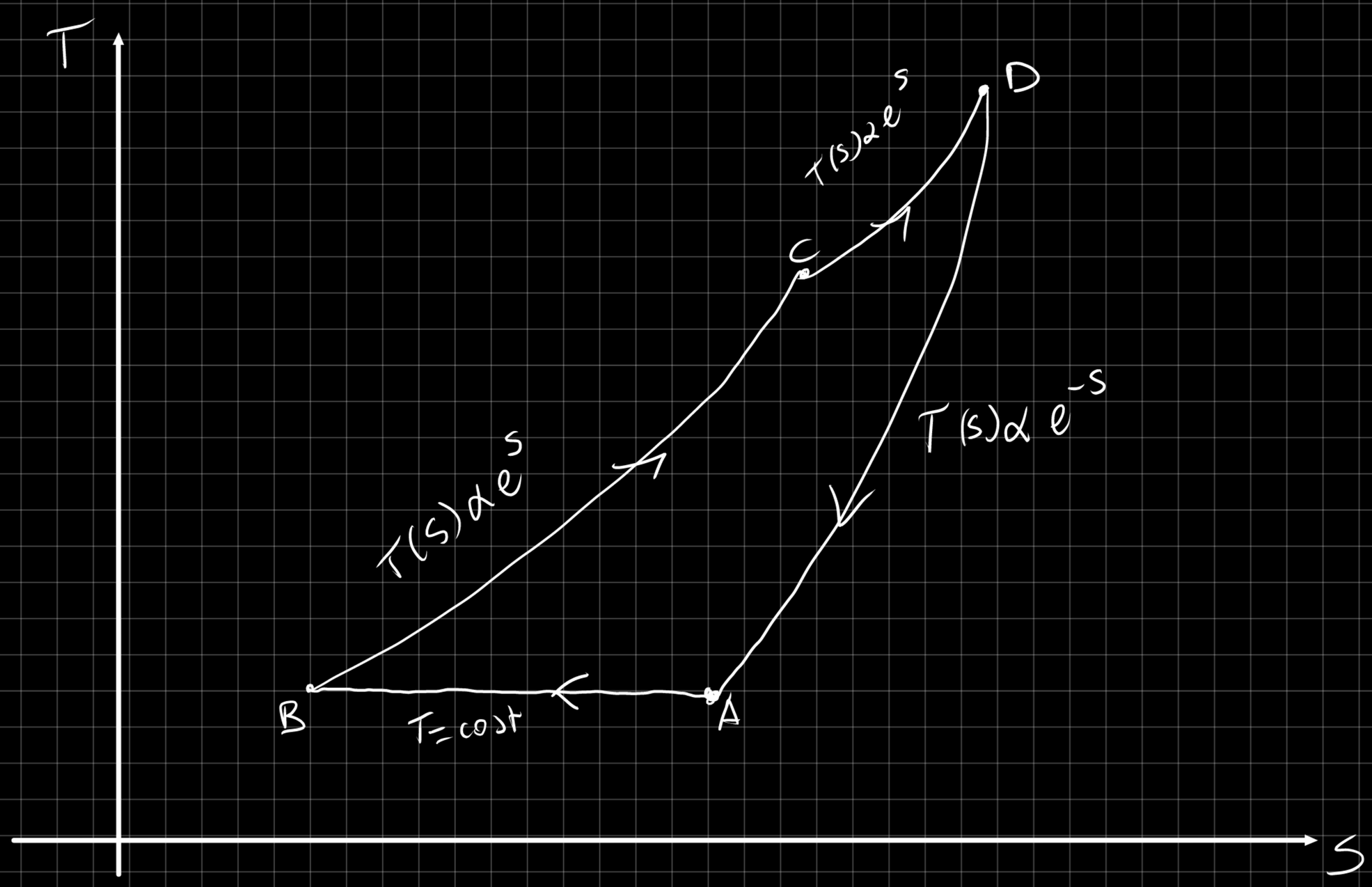
punto e, rapp. TS $(T(S))$

$A \to B$ $T(S) = \text{cost.}$ $\Delta S \simeq -10 \ J/K$

$B \to C$ $S(T) \propto \log(T)$ $\Delta S \simeq 20 \ J/K \Rightarrow T(S) \propto e^{S}$

$C \to D$ $S(T) \propto \log(T)$ $\Delta S \simeq 20 \ J/K$ $T(S) \propto e^{S}$

$D \to A$ $S(T) \propto \log(T)$ $\Delta S \simeq -30 \ J/K$ $T(S) \propto e^{S}$



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E1) Macchina reversibile usa come fonti $T_1 = T_{amb} = 20°C$ e serbatoio con $m_{H_2O} = 10\ kg$ e ghiaccio fondente $m_g = 5\ kg$ @ $T = 0°C$ (Isoluto)

1) W_1 fino a scioglimento

2) W_2 dopo scioglimento

3) Trova η con $W_{TOT} = W_1 + W_2$

4) ΔS_{serb}

5) ΔS_Ω quando serbatoio in contatto con amb.

$\lambda_f = 333\ J/g \quad c_{H_2O} = 1\ \frac{cal}{gK}$

1) Fino a scioglimento $Q = m\lambda_f = 5 \cdot 10^3\ g \cdot 333\ J/g = 1665000\ J$

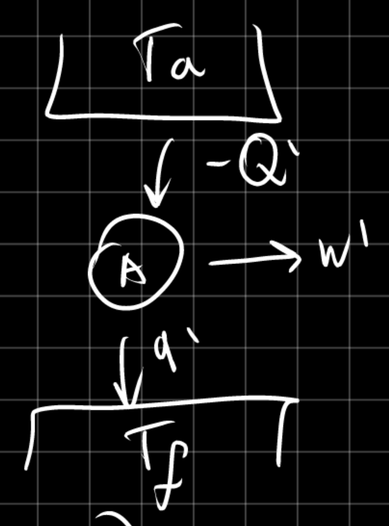
Poiché rev $\Rightarrow \eta = \eta_{car} = 1 - \frac{T_2}{T_1} = 1 - \frac{T_f}{T_{amb}} = 1 - \frac{273.15}{293.15} = 0.068 \simeq 6.8\ \%$

$\eta = \frac{W}{Q} \Rightarrow W = \eta Q = 0.068 \cdot 1665000\ J = 113220\ J$

$\frac{Q_f}{Q_{ass}} = \frac{T_f}{T_a} \Rightarrow Q_{ass} = \frac{T_a}{T_f} Q_f = \frac{T_a}{T_f} m_g \lambda_f = 178611.04\ J$

$\Delta W = \Delta Q = Q_{ass} - Q_f = 121911.04\ J$

2) Dopo scioglimento $m'_{H_2O} = m_{H_2O} + m_g$ (sciolto) $\rightarrow$ macchina scalda H_2O da T_f a T_a



$1\ cal = 4.186\ J$

Processo isoentropico ($\Delta S_\Omega = 0$)

$\Delta S_\Omega = \Delta S_a + \Delta S_{H_2O} = m'_{H_2O} c_{H_2O} \log\left(\frac{T_a}{T_f}\right) - \frac{Q'}{T_a} = 0$

$Q' = m'_{H_2O} c_{H_2O} T_a \log\left(\frac{T_a}{T_f}\right) = 4.186\ J/cal \cdot 1\ \frac{cal}{gK} \cdot 15 \cdot 10^3\ g \cdot 293.15\ K \cdot \log\left(\frac{293.15}{273.15}\right) = 1300692.01\ J$

$q' = m'_{H_2O} c_{H_2O} (T_a - T_f) = 1255800\ J$

$\therefore\ W' = Q' - q' = 44892\ J$

3) 1) $Q_{ass} = Q' + Q$, $W = W + W'$

$\therefore\ \eta = \frac{W+W'}{Q+Q'} = 0.054 = 5.4\%$ ✓

4) ΔS_a durante processo $1 \to 3$

$$\Delta S_a = \Delta S_{SC} + \Delta S_{TE} = \frac{Q_f}{T_f} + \int_{T_f}^{T_a} \frac{đQ}{T} = \frac{m_g \lambda_f}{T_f} + m'_{H_2O}\, c_{H_2O} \log\left(\frac{T_a}{T_f}\right) = 10532.50\ J/K$$

5) ΔS_Ω

a) S potenziale $\Rightarrow \Delta S_{contatto} = \Delta S_{SC} + \Delta S_{TE}$

$$\therefore\ \Delta S_{H_2O} = \frac{-q_T}{T_a} = -\frac{(Q_f + q')}{T_a} = -9963.30\ J/K$$

b) $\Delta S_\Omega = \Delta S_{H_2O} + \Delta S_a = 10532.50\ J/K - 9963.30\ J/K = 569\ J/K$

E2) Gas perfetto monoatomico, inizialmente a $p_A = 1\ atm$ & $T_A = 23°C$ viene compresso fino a B con processo $T p^{-1/2} = cost.$ con $W_{BC} = -3.2\ kJ$. ($T_B = 2T_A$)
Processo fino a C con $p T^{-1/2} = const.$ fino a $V_C = V_A$. Da C espansione adiabatica con $\Delta S_{CD} = 5.53\ J/K$ fino a $P_C = P_A$. Riportato ad A con isobara con singola sorgente

6) Tabella (p, V, T) di ABCD

7) Q_{TOT}, W_{TOT}

8) ΔS_{gas}, ΔS_{amb}

9) ΔS_Ω

10) rapp. pV & TS

6) Tabella pVT

$A \to B$ $\frac{T}{\sqrt{p}} = const$, $B \to C$ $\frac{p}{\sqrt{T}} = const$ $(V_C = V_A)$, $C \to D$ $đQ = 0$, $\Delta S = 5.53\ J/K$, $p_D = p_A$

$D \to A$ $p = const$

$$\frac{T_A}{\sqrt{p_A}} = \frac{T_B}{\sqrt{p_B}} \qquad \frac{p_B}{\sqrt{T_C}} = \frac{p_C}{\sqrt{T_C}} \qquad 1)\ T = \frac{pV}{nR}$$

$$\frac{p_A V_A}{\cancel{nR}} \frac{1}{\sqrt{p_A}} = \frac{p_B V_B}{\cancel{nR}} \frac{1}{\sqrt{p_B}}, \quad V_A\sqrt{p_A} = V_B\sqrt{p_B}, \quad p_A V_A^2 = p_B V_B^2 \quad (AB)$$

$$\frac{p_B}{\sqrt{p_B V_B}} = \frac{p_C}{\sqrt{p_C V_C}}, \quad \sqrt{\frac{p_B}{V_B}} = \sqrt{\frac{p_C}{V_C}}, \quad p_B V_B^{-1} = p_C V_C^{-1} \quad (BC)$$

$dU = TdS - đW$, $TdS = dU + đW$, $TdS = nc_v dT + pdV$ $\qquad$ $C \checkmark \rightarrow D$ (P_D)

$pdV = d(pV) - Vdp$, $pdV = nRdT - Vdp$

$$TdS = nc_v dT + nRdT - pdV = nc_p dT - Vdp = nc_p dT - \frac{nRT}{p} dp$$

$$dS = \frac{nc_p}{T} dT - \frac{nR}{p} dp \Rightarrow \Delta S_{CD} = nc_p \log\left(\frac{T_D}{T_C}\right) - nR \log\left(\frac{P_D}{P_C}\right) \quad (CD)$$

(DA) $\quad P_D = P_A \qquad \frac{nRT_D}{V_D} = \frac{nRT_A}{V_A} \Rightarrow \frac{T_D}{V_D} = \frac{T_A}{V_A} \quad (DA)$

$$\begin{cases} P_A V_A^2 = P_B V_B^2 \\ P_B V_B^{-1} = P_C V_C^{-1} \\ V_C = V_A \\ P_D = P_A \quad T_B = 2T_A \\ \frac{T_D}{V_D} = \frac{T_A}{V_A} \end{cases} \qquad \begin{cases} P_A V_A^2 = P_B V_B^2 \\ P_B V_B^{-1} = P_C V_A^{-1} \\ \frac{T_D}{V_D} = \frac{T_A}{V_A} \end{cases}$$

$P_A = 1$ atm

$T_B = 2T_A = 2 \cdot (293.15\ K)$

$W_{AB} = -3200\ J$

$$W_{AB} = n\frac{R}{1-\alpha}\Delta T = \frac{nR}{-1}(2T_A - T_A) = -nRT_A = W_{AB}$$

$$n = \frac{-W_{AB}}{RT_A} \qquad P_A V_A = nRT_A \Rightarrow P_A V_A = -\frac{W_{AB}}{\cancel{RT_A}}\cancel{RT_A} \Rightarrow V_A = -\frac{W_{AB}}{P_A} = 0.0316\ m^3$$

$$n = \frac{3200}{8.314 \cdot 293.15}\ mol = 1.313\ mol$$

$$V_A = -\frac{W_{AB}}{P_A} \Rightarrow P_A V_A^2 = P_A\left(\frac{W_{AB}}{P_A}\right)^2 = \frac{W_{AB}^2}{P_A}$$

$$\frac{W_{AB}^2}{P_A} = P_B V_B^2$$

$$P_B = \frac{nRT_B}{V_B} \Rightarrow P_B V_B^2 = nRV_B T_B = 2nRT_A V_B$$

$$\frac{W_{AB}^2}{P_A} = 2nRT_A V_B = -2\frac{W_{AB}}{\cancel{RT_A}}\cancel{RT_A} V_B \rightarrow \frac{W_{AB}^{\cancel{2}}}{P_A} = -2\cancel{W_{AB}} V_B \Rightarrow V_B = -\frac{W_{AB}}{2P_A} = 0.0158\ m^3$$

$$\begin{cases} P_A V_A^2 = P_B V_B^2 \\ \frac{nR}{1-\alpha}(T_B - T_A) \overset{*}{=} W_{AB} \\ T_B = 2T_A \end{cases} \qquad \begin{cases} P_A V_A^2 = P_B V_B^2 \\ -nRT_A = W_{AB} \end{cases} \qquad \begin{cases} P_A V_A^2 = P_B V_B^2 \\ W_{AB} = -P_A V_A \Rightarrow V_A = -\frac{W_{AB}}{P_A} \end{cases}$$

$$P_A V_A = nRT_A$$

$$\begin{cases} \frac{W_{AB}^2}{P_A} = P_B V_B^2 \\ V_A = -\frac{W_{AB}}{P_A} \\ P_B V_B = 2nRT_A \end{cases} \qquad \begin{cases} P_B V_B^2 = -\frac{W_{AB}^2}{P_A} \\ V_A = -\frac{W_{AB}}{P_A} \\ P_B = \frac{2nRT_A}{V_B} \end{cases} \qquad \begin{cases} \overbrace{2nRT_A}^{+\cancel{W_{AB}}\ -W_{AB}} V_B = \frac{W_{AB}^2}{P_A} \\ n = -\frac{W_{AB}}{RT_A} \\ P_B = \frac{2nRT_A}{V_B} \end{cases} \qquad \begin{cases} -2\cancel{W_{AB}} V_B = \frac{W_{AB}^{\cancel{2}}}{P_A} \\ \\ \end{cases}$$

$n = -\frac{W_{AB}}{RT_A}$, $V_A = -\frac{W_{AB}}{P_A}$, $V_B = -\frac{W_{AB}}{2P_A}$, $T_B = 2T_A$, $P_B = \frac{2nRT_A}{V_B} = -\frac{2W_{AB}}{V_B}$

T_A ✓ P_B ?

P_A ✓ V_B ?

V_A ? T_B ✓

n ?

$V_A = -\frac{W_{AB}}{P_A}$, $V_B = -\frac{W_{AB}}{2P_A}$ $P_B = \frac{nRT_B}{V_B} = nRT_B\left(-\frac{2P_A}{W_{AB}}\right)$

$P_B = -\frac{\cancel{W_{AB}}}{\cancel{RT_A}}\cancel{R}\cancel{T_B}^{2}\left(-\frac{2P_A}{\cancel{W_{AB}}}\right) = 4P_A$ $n = \frac{-W_{AB}}{RT_A}$

$\begin{cases} n = -\frac{W_{AB}}{RT_A} \\ V_A = -\frac{W_{AB}}{P_A} \end{cases}$ $\begin{cases} P_B = 4P_A = 4\,atm = 405300\ Pa \\ V_B = -\frac{W_{AB}}{2P_A} = \frac{V_A}{2} = 0.0158\ m^3 \end{cases}$ $\begin{cases} n = -\frac{W_{AB}}{RT_A} = 1.313\ mol \\ V_A = 0.0316\ m^3 \end{cases}$

(AB)

$\begin{cases} P_B V_B^{-1} = P_C V_C^{-1} \\ V_C = V_A \end{cases}$ $\begin{cases} P_B V_B^{-1} = P_C V_A^{-1} \\ V_A = -\frac{W_{AB}}{P_A} \\ V_B = \frac{V_A}{2} \end{cases}$ $\begin{cases} P_B\left(\frac{V_A}{2}\right)^{-1} = P_C V_A^{-1} \\ P_C V_A = nRT_C \end{cases}$ $\begin{cases} \frac{2P_B}{V_A} = \frac{P_C}{V_A} \rightarrow P_C = 2P_B \ ✓ \\ P_C V_A = nRT_C \rightarrow T_C = \frac{2P_B V_A}{nR} \ ✓ \end{cases}$

$C \rightarrow D$

$\Delta S_{CD} = nc_p \log\left(\frac{T_D}{T_C}\right) - nR\log\left(\frac{P_A}{P_C}\right) \rightarrow \frac{\Delta S_{CD}}{nc_p} + \frac{R}{c_p}\log\left(\frac{P_A}{P_C}\right) = \log\left(\frac{T_D}{T_C}\right)$

$P_D = P_A$

$\left(\frac{P_A}{P_C}\right)^{\frac{R}{c_p}} + e^{\frac{\Delta S_{CD}}{nc_p}} = \frac{T_D}{T_C} \Rightarrow T_D = T_C\left(\frac{P_A}{P_C}\right)^{\frac{2}{5}} + T_C e^{\frac{\Delta S_{CD}}{nc_p}} = T_C\left(\frac{P_A}{P_C}\right)^{\frac{2}{5}} + T_C e^{-\frac{2T_A\Delta S_{CD}}{5W_{AB}}}$

$c_p = \frac{5}{2}R$ $n = -\frac{W_{AB}}{RT_A}$ $P_C = 2P_B = 8P_A$ $\frac{P_A}{P_C} = \frac{1}{8}$ $T_C = \frac{2P_B V_A}{nR} = 8T_A$

$T_D = 8T_A\left(\frac{1}{8}\right)^{\frac{2}{5}} + 8T_A\exp\left(-\frac{2T_A\Delta S_{CD}}{W_{AB}}\right) = 8^{\frac{3}{5}}T_A + 8T_A\exp\left(-2T_A\frac{\Delta S_{CD}}{W_{AB}}\right)$ Gesù!

$1 - \frac{2}{5} = \frac{5-2}{5} = \frac{3}{5}$

BRILLIANT CALCULATION

$T_D = T_A\sqrt[5]{8^3} + 8T_A e^{-\frac{2T_A\Delta S_{CD}}{W_{AB}}}$ X

$T_C = \frac{2P_B V_A}{nR} = \frac{8P_A V_A}{nR}$, $V_A = -\frac{W_{AB}}{P_A}$, $T_C = \frac{-8W_{AB}}{nR} = -\frac{8\cancel{W_{AB}}}{\cancel{R}}\frac{\cancel{R}T_A}{\cancel{W_{AB}}} = 8T_A$

$T_B = 2T_A$

$V_A = -\frac{W_{AB}}{P_A}$ $P_A V_D = |W_{AB}|\frac{T_D}{T_A} \Rightarrow V_D = |W_{AB}|\frac{T_D}{P_A T_A}$ $P_B = 4P_A$

$V_B = -\frac{W_{AB}}{2P_A}$ $V_D = |W_{AB}|\frac{1}{P_A}\left(\sqrt[5]{8^3} + \frac{1}{T_A}e^{-\frac{2T_A\Delta S_{CD}}{W_{AB}}}\right)$ $P_C = 8P_A$

$V_C = -\frac{W_{AB}}{P_A}$ $P_D = P_A$

A

$P_A = 101325\ Pa$ ✓

$V_A = \frac{|W_{AB}|}{P_A} = 0.0316\ m^3$ ✓

$T_A = 296.15\ K$ ✓

B

$P_B = 4P_A = 405300\ Pa$ ✓

$V_B = \frac{|W_{AB}|}{2P_A} = 0.0158\ m^3$ ✓

$T_B = 2T_A = 592.3\ K$ ✓

C

$P_C = 8P_A = 810600\ Pa$ ✓

$V_C = V_A = 0.0316\ Pa$ ✓

$T_C = 8T_A = 2369.2\ K$ ✓

D

$P_D = P_A = 101325\ Pa$ ✓

$$dU = TdS + pdV \Rightarrow dS = \frac{1}{T}dU + \frac{1}{T}\left(d(pV) - Vdp\right)$$

$$V = \frac{nR}{p}T, \quad dU = nc_v dT$$

$$dS = \frac{nc_v}{T}dT + \frac{1}{T}\left(nRdT - \frac{nRT}{p}dp\right)$$

$$dS = \frac{nc_v}{T}dT + \frac{nR}{T}dT - \frac{nR}{p}dp = \frac{nc_p}{T}dT - \frac{nR}{p}dp$$

$$S_{CD} = \log\left[\left(\frac{T_D}{T_C}\right)^{nc_p}\left(\frac{P_D}{P_C}\right)^{-nR}\right]$$

$$e^{S_{CD}} = \left(\frac{T_D}{T_C}\right)^{nc_p}\left(\frac{P_D}{P_C}\right)^{-nR} \rightarrow e^{S_{CD}} = \left(\frac{T_D}{8T_A}\right)^{nc_p}\left(\frac{P_A}{8P_A}\right)^{-nR}$$

$$n = \frac{W_{AB}}{RT_A}, \quad c_p = \frac{5}{2}R \Rightarrow nR = \frac{W_{AB}}{RT_A}R = \frac{W_{AB}}{T_A}; \quad nc_p = \frac{W_{AB}}{RT_A}\frac{5}{2}R = \frac{5W_{AB}}{2T_A}$$

$$8^{-nR}e^{S_{CD}} = \left(\frac{T_D}{8T_A}\right)^{nc_p} \rightarrow \frac{8^{nc_p}}{8^{nR}}T_A^{nc_p}e^{S_{CD}} = T_D^{nc_p} \rightarrow T_D = \frac{8T_A}{8^{\frac{nR}{nc_p}}}e^{\frac{S_{CD}}{nc_p}}$$

$$8^{1-\frac{R}{c_p}}T_A e^{\frac{S_{CD}}{nc_p}} = 8^{3/5}T_A e^{\frac{2T_A S_{CD}}{5W_{AB}}} = 1265.62\ K$$ ✓

$$V_D = \frac{nRT_D}{P_A} = \frac{W_{AB}}{\cancel{T_A}P_A}\left(8^{3/5}\cancel{T_A}e^{\frac{2T_A S_{CD}}{5W_{AB}}}\right) = \frac{8^{3/5}W_{AB}}{P_A}e^{\frac{2T_A S_{CD}}{5W_{AB}}} = 0.135\ m^3$$ ✓

A

$P_A = 101325\ Pa$ ✓

$V_A = \frac{|W_{AB}|}{P_A} = 0.0316\ m^3$ ✓

$T_A = 296.15\ K$ ✓

B

$P_B = 4P_A = 405300\ Pa$ ✓

$V_B = \frac{|W_{AB}|}{2P_A} = 0.0158\ m^3$ ✓

$T_B = 2T_A = 592.3\ K$ ✓

C

$P_C = 8P_A = 810600\ Pa$ ✓

$V_C = V_A = 0.0316\ Pa$ ✓

$T_C = 8T_A = 2369.2\ K$ ✓

D

$P_D = P_A = 101325\ Pa$ ✓

$V_D = 8^{3/5}\frac{W_{AB}}{P_A}\exp\left(\frac{2T_A S_{CD}}{5W_{AB}}\right) = 0.135\ m^3$ ✓

$T_D = 8^{3/5}T_A\exp\left(\frac{2T_A S_{CD}}{5W_{AB}}\right) = 1265.62\ K$ ✓

7) Q_{TOT} ? W_{TOT} ?

$A \to B$ politropica $\alpha = 2$, $B \to C$ politropica $\alpha = -1$ $C \to D$ ad. irr $D \to A$ isobara

$A \to B$) $đQ = nc_\alpha dT = \frac{nc_v}{1-\alpha} dT$

$B \to C$) $\quad \parallel$

$C \to D$) $đQ = 0$

$D \to A$) $đQ = nc_p dT$

$$đQ_\alpha = nc_\alpha dT = n\left(c_v + \frac{R}{1-\alpha}\right) dT = n\left(\frac{3}{2} + \frac{1}{1-\alpha}\right) R\, dT = \frac{W_{AB}}{T_A}\left(\frac{3}{2} + \frac{1}{1-\alpha}\right) dT$$

$$đQ_{AB} = đQ_2 = \frac{W_{AB}}{T_A}\left(\frac{3}{2} - 1\right) dT = \frac{W_{AB}}{2T_A} dT$$

$$đQ_{BC} = đQ_{-1} = \frac{W_{AB}}{T_A}\left(\frac{3}{2} + \frac{1}{2}\right) dT = \frac{2W_{AB}}{T_A} dT$$

$$Q_{AB} = \frac{W_{AB}}{2} = 1600\ J$$

$$Q_{BC} = 12 W_{AB} = 38400\ J \quad \checkmark$$

$$đQ_{DA} = nc_p dT = \frac{|W_{AB}|}{RT_A} \frac{5}{2} R\, dT = \frac{5}{2} \frac{W_{AB}}{T_A} dT$$

$$Q_{DA} = \frac{5W_{AB}}{T_A}(T_A - T_D) = 5W_{AB}\left(1 - 8^{3/5} \exp\left(\frac{2T_A S_{CD}}{5W_{AB}}\right)\right) = -52342.27\ J \quad ?$$ ✓ fatto 2

$$\frac{5}{2} nR\, dT = \frac{5}{2} \frac{W}{T_A} dT$$

$$Q_{DA} = \frac{5}{2} W\left(1 - \frac{T_D}{T_A}\right) = \frac{5}{2} W\left(1 - 8^{\frac{3}{5}} \exp\left(\frac{2T_A S_{CD}}{5W_{AB}}\right)\right) = -26186.135\ J$$

$$\oint dU = \oint đQ - \oint đW = 0 \Rightarrow \oint đQ = \oint đW \Rightarrow Q_{TOT} = W_{TOT}$$

$$Q_{TOT} = Q_{AB} + Q_{BC} + Q_{DA} = 13813.86\ J$$

$$\eta = \frac{W_{TOT}}{Q_{ASS}} = \frac{Q_{TOT}}{Q_{AB} + Q_{BC}} = 0.3453 \simeq 34.53\% \quad \checkmark$$

8) ΔS_{gas}, ΔS_{amb}

$$dS = \frac{đQ}{T} \Rightarrow dS_{AB,BC} = dS_\alpha = \frac{đQ_\alpha}{T} = \frac{W_{AB}}{T_A}\left(\frac{3}{2} + \frac{1}{1-\alpha}\right) \frac{1}{T} dT$$

$$dS_{DA} = \frac{đQ_p}{T} = \frac{nc_p}{T} dT = \frac{5}{2} \frac{W_{AB}}{T_A} \frac{1}{T} dT$$

$$S_\alpha = \frac{W_{AB}}{T_A}\left(\frac{3}{2} + \frac{1}{1-\alpha}\right)\log\left(\frac{T_f}{T_i}\right)$$

$$S_{AB} = S_2 = \frac{W_{AB}}{2T_A}\log(2) = 3.745\ J/K \checkmark$$

$$S_{BC} = S_{-1} = \frac{2W_{AB}}{T_A}\log(4) = 29.959\ J/K \checkmark$$

$$S_{CD} = 5.53\ J/K \checkmark$$

$$S_{DA} = \frac{5}{2}\frac{W_{AB}}{T_A}\log\left(\frac{T_A}{T_D}\right) = \frac{5}{2}\frac{W_{AB}}{T_A}\log\left(8^{\frac{3}{5}} e^{\frac{2T_A S_{CD}}{5W_{AB}}}\right) = \frac{5W_{AB}}{2T_A}\frac{3}{5}\log 8 + \frac{\cancel{5}\cancel{W_{AB}}}{\cancel{2}\cancel{T_A}}\frac{\cancel{2}}{\cancel{5}}\frac{\cancel{T_A}S_{CD}}{\cancel{W_{AB}}}$$

$$S_{DA} = -\frac{3W_{AB}}{2T_A}\log 8 - S_{CD} = -39.234\ J/K \checkmark$$

$$dS_\Omega = dS^a + dS_{gas} \Rightarrow dS_{AMB,\alpha} = -dS_\alpha \Rightarrow \begin{array}{l} S^a_{AB} = -3.745\ J/K \\ S^a_{BC} = -29.959\ J/K \end{array}$$

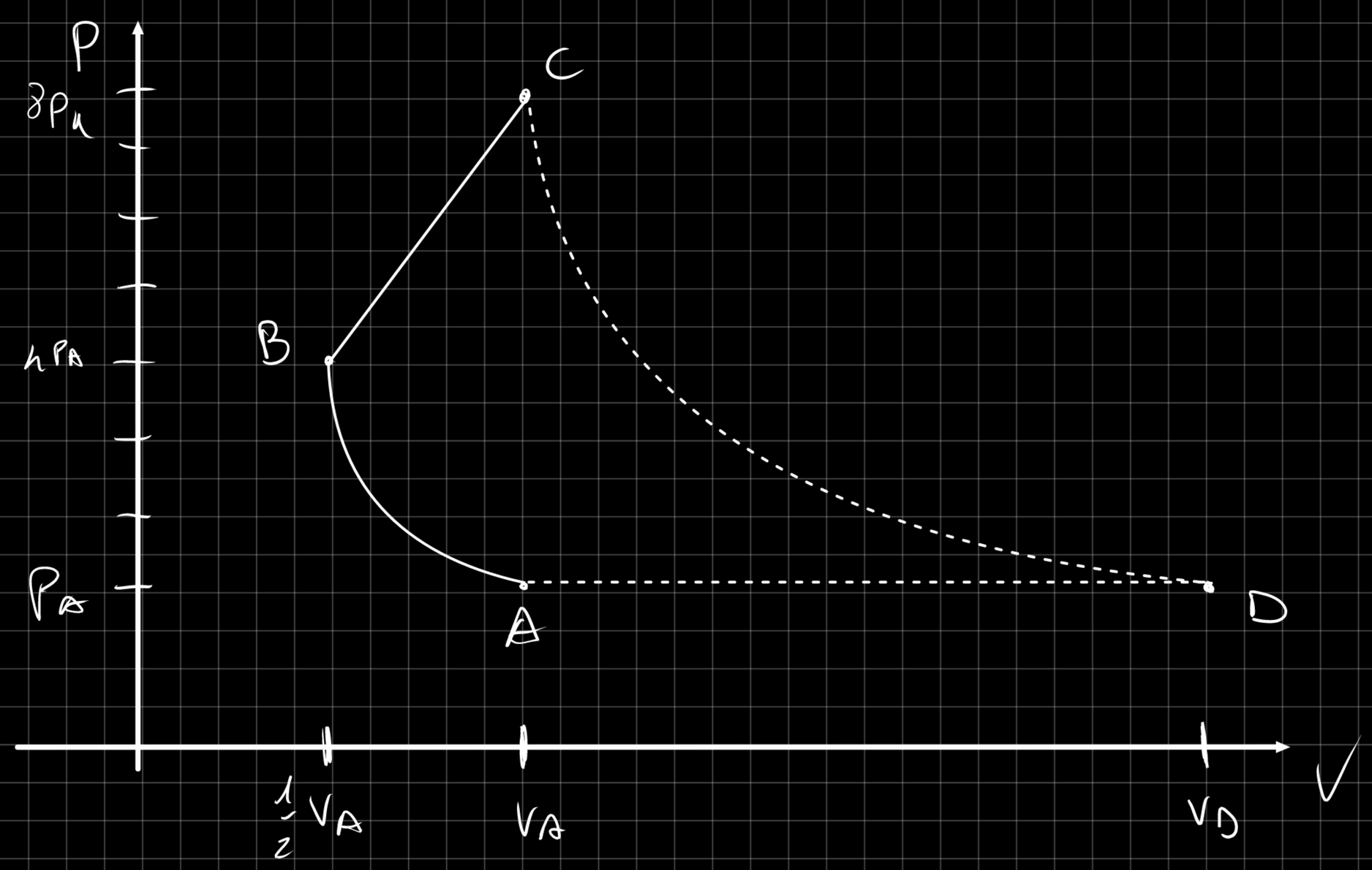
$$S^a_{CD} = 0$$

$$S^a_{DA} = -\frac{Q_{DA}}{T_A} = -\frac{5}{2}\frac{W_{AB}}{T_A}\left(1 - 8^{3/5}\exp\left(\frac{2T_A S_{DA}}{5W_{AB}}\right)\right) = 88.422\ J/K \checkmark$$

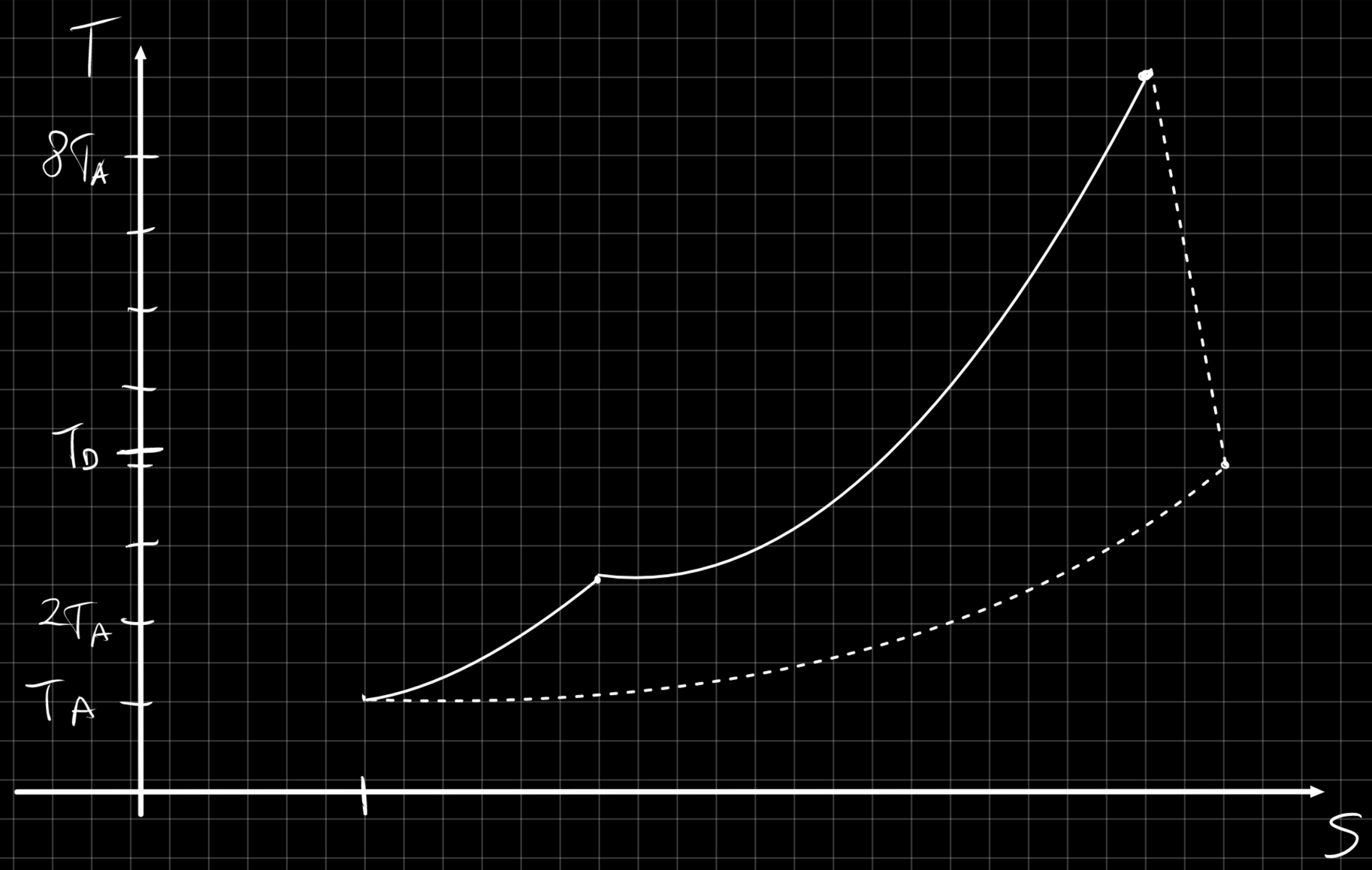
$$S^a_{TOT} = 54.72\ J/K, \quad S_\Omega = S^a_{TOT} + S_{gas} = 54.72\ J/K$$

10) (P,V), (T,S)

$A \to B$ $p(V) = kV^{-2}$, $B \to C$ $p = kV$, $C \to D$ $p = kV^{-\gamma}$, $D \to A$ $p = \text{cost}$



(T, S)



08/09/2021

E1) $M_{Cu} = 0.25\ kg$ a $T = T_x$ immersu in recipiente con $C_r \simeq 0$ con $m_{H_2O} = 0.1\ kg$ a $T = 320\ K$. Ad equilibrio $m_{H_2O} = 0.09\ kg$. Trasurando perdite verso amb:

a) T_x ? , b) $\Delta S_{H_2O} + \Delta S_{Cu}$, c) ΔS_Ω

Assumendo il vapore disperso in amb con $T_a = 300\ K$

d) Q tra vap. amb.

e) ΔS_Ω

$C_{Cu} = 387\ \frac{J}{kgK}$, $C_{H_2O} = 4187\ \frac{J}{kgK}$, $C_{vap} = 1.970\ \frac{kJ}{kgK}$, $\lambda_{ev} = 22.6 \cdot 10^5\ J/K$

a) processo evap. , $T_x \to T_{amb}$

$dQ_{Cu} = m_{Cu} C_{Cu}\, dT$, $Q_{H_2O} = (m_{H_2O} - m_{fH_2O})\lambda_{ev} + m_{H_2O} C_{H_2O} (T_{ev} - T_{H_2O})$

$-Q_{Cu} = Q_{H_2O} \Rightarrow M_{Cu}(T_x - T_{ev}) = m_{H_2O} C_{H_2O} (T_{ev} - T_{H_2O}) + (m_{H_2O} - m_{fH_2O})\lambda_{ev}$

$$T_x = \frac{m_{H_2O}}{M} \frac{C_{H_2O}}{C_{Cu}} (T_{ev} - T_{H_2O}) + \frac{m_{H_2O} - m_{fH_2O}}{M_{Cu}} \lambda_{ev} + T_{ev} = 836.76\ K$$

$$Q = M C_{Cu} (T_x - T_{ev}) = 44853.91\ J$$

b)

$$\Delta S_{H_2O} = \int_{T_{H_2O}}^{T_{ev}} \frac{m_{H_2O} C_{H_2O}}{T} dT + \frac{(m_{H_2O} - m'_{H_2O})\lambda_{ev}}{T_{ev}} = m_{H_2O} C_{H_2O} \log\left(\frac{T_{ev}}{T_{H_2O}}\right) + \frac{m_{H_2O} - m'_{H_2O}}{T_{ev}} \lambda_{ev}$$

$= 124.90\ J/K$

$$\Delta S_{Cu} = \int_{T_x}^{T_{ev}} \frac{M C_{Cu}}{T} dT = M_{Cu} \log\left(\frac{T_{ev}}{T_x}\right) = -78.13\ \frac{J}{K}$$

$$\Delta S_{TOT} = \Delta S_{Cu} + \Delta S_{H_2O} = 46.77\ J/K \quad \checkmark$$

c) $\Delta S_\Omega = \Delta S_{TOT} = 46.77\ \frac{J}{K}$ ✓

d) Vapore disperso in amb. a $T_a = 300\ K$

Vap -> amb $dQ = m_{vap} C_{vap}\, dT \Rightarrow Q_{v \to a} = m_{vap} C_{vap} (T_{ev} - T_a) = -1441.05\ J$

e) $$\Delta S_{vap} = \frac{Q_{vap}}{T_a} + \Delta S_{vap \to amb} = \frac{Q_{vap}}{T_a} + m_{vap} C_{vap} \int_{T_{ev}}^{T_a} \frac{1}{T} dT = -\frac{Q_{vap}}{T_a} + m_{vap} C_{vap} \log\left(\frac{T_a}{T_{ev}}\right)$$

$\Delta S_\Omega = \Delta S_{tot} + \Delta S_{vap} = 47.28 \frac{J}{K}$, $\Delta S_{vap} = 0.505 \frac{J}{K}$ V

E2)

$n=1$ di g.p. monoatomico a $p_A = 1 atm$ fa ciclo ABCDA. $A \to B$ $pT^{-1/2} = cost$, con $p_B = 2p_A$.

$B \to C$ compressione lenta, $W_{BC} = 31.9\ kJ$ con gas a contatto su serb. a $1200\ K$.

$C \to D$) esp. ad. irr. con $S_{CD} = 0.41 \frac{J}{K}$.

$D \to A$) esp. isot. irr. con $Q_{DA} = 0.8\ kJ$

1) tabella (p, V, T) di ABCD
2) W_{tot}, coeff. di produzione $(\eta, r\ ?)$
3) ΔS_{gas}, ΔS_{amb} in tutto
4) ΔS_Ω
5) (p, V), (T, S)

1) $A \to B$; $p_A = 1 atm$, $p_B = 2p_A$ $\quad p_A T_A^{-1/2} = p_B T_B^{-1/2}$ $\quad T_A = \frac{p_A V_A}{nR}$

$$p_A \left(\frac{p_A V_A}{nR}\right)^{-1/2} = \frac{p_A^{1/2} V_A^{-1/2}}{nR} = \frac{p_B^{1/2} V_B^{-1/2}}{nR} \Rightarrow p_A V_A^{-1} = p_B V_B^{-1} \quad (\text{politropica } \alpha = -1)$$

$B \to C$

$W_{BC} = 31.9\ kJ = 31900\ J$, $T_C = T_B = T_S$

$$p_A T_A^{-1/2} = p_B T_S^{-1/2} \Rightarrow T_A = \left(\frac{p_B}{p_A} T_S^{-1/2}\right)^{-2} \Rightarrow T_A = \left(\frac{p_A}{p_B}\right)^2 T_S \qquad p_B = 2p_A$$

$$T_A = \frac{1}{4} T_S \qquad V_B = \frac{nRT_B}{p_B} = \frac{nRT_S}{2p_A}$$

$$V_A = \frac{nRT_A}{p_A} = \frac{nR}{4p_A} T_S \qquad T_B = T_S$$

$$dW = p\,dV = \frac{nRT_S}{V} dV \Rightarrow W_{BC} = nRT_S \log\left(\frac{V_C}{V_B}\right) = nRT_S \log\left(\frac{2p_A V_C}{nRT_S}\right)$$

$$e^{\frac{W_{BC}}{nRT_S}} = \frac{2p_A V_C}{nRT_S} \Rightarrow V_C = \frac{nRT_S}{2p_A} e^{\frac{W_{BC}}{nRT_S}}, \quad T_C = T_S$$

$$p_C = \frac{nRT_S}{V_C} = \frac{nRT_S}{\frac{nRT_S}{2p_A}} e^{-\frac{W_{BC}}{nRT_S}} = 2p_A e^{\frac{W_{BC}}{nRT_S}}$$

C→D) exp. ad. irr $\quad Q=0,\ S_{CD}=0.46\ J/K \quad T_D=T_A$

p_D, V_D? $(T_C=T_S, p_C, V_C\ \checkmark)$

$$TdS = dU + \delta W = nC_v dT + pdV \rightarrow dS = \frac{nC_v}{T}dT + \frac{p}{T}dV$$

$$dS = \frac{nC_v}{T}dT + \frac{nR}{V}dV$$

$$T_A = T_D = \frac{1}{4}T_S$$

$$S_{CD} = nC_v \log\left(\frac{T_D}{T_S}\right) + nR\log\left(\frac{V_D}{V_C}\right) = nC_v\log\left(\frac{1}{4}\right) + nR\log\left(\frac{V_D}{V_C}\right)$$

$$\frac{S_{CD}}{nR} + \frac{C_v}{R}\log(4) = \log\left(\frac{V_D}{V_C}\right) \Big/ p_D = \frac{nRT_D}{V_D} = \frac{\cancel{nR}T_A}{\frac{4\cancel{nR}T_S}{p_A}} e^{-\frac{1}{nR}\left(S_{CD}+\frac{W_{BC}}{T_S}\right)} = \frac{p_A T_A}{T_S} e^{-\frac{1}{nR}\left(S_{CD}+\frac{W_{BC}}{T_S}\right)}$$

$$4^{\frac{C_v}{R}} e^{\frac{S_{CD}}{nR}} = \frac{V_D}{V_C} \rightarrow V_D = 4^{\frac{C_v}{R}} V_C e^{\frac{S_{CD}}{nR}} \qquad C_v = \frac{3}{2}R \rightarrow \frac{C_v}{R} = \frac{3}{2}$$

$W_{BC} < 0$

$$4^{\frac{3}{2}} e^{\frac{S_{CD}}{nR}} V_C = 4^{3/2} e^{\frac{S_{CD}}{nR}} \left(\frac{nRT_S}{2p_A}\right) e^{\frac{W_{BC}}{nRT_S}} = \frac{4nRT_S}{p_A} e^{\frac{1}{nR}\left(S_{CD}+\frac{W_{BC}}{T_S}\right)} = 0.0170\ m^3 \ \checkmark$$

A

$p_A = 101325\ Pa \ \checkmark$

$V_A = \frac{nR}{4p_A}T_S = 0.0246\ m^3 \ \checkmark$

$T_A = \frac{1}{4}T_S = 300\ K \ \checkmark$

B

$p_B = 2p_A = 202650\ Pa \ \checkmark$

$V_B = \frac{nRT_S}{2p_A} = 0.0492\ m^3 \ \checkmark$

$T_B = T_S = 1200\ K \ \checkmark$

C

$p_C = 2p_A e^{\frac{-W_{BC}}{nRT_S}} = 4958697.4\ Pa \ \checkmark$

$V_C = \frac{nRT_S}{2p_A} e^{\frac{W_{BC}}{nRT_S}} = 2.01\cdot 10^{-3}\ m^3 \ \checkmark$

$T_C = T_S = 1200\ K \ \checkmark$

D

$$p_D = \frac{nRT_D}{V_D} = \frac{nRT_S}{4V_D} = \frac{\cancel{nRT_S}}{4}\left(\frac{p_A}{4\cancel{nRT_S}}\right) e^{-\frac{1}{nR}\left(S_{CD}+\frac{W_{BC}}{T_S}\right)} = \frac{p_A}{16} e^{-\frac{1}{nR}\left(S_{CD}+\frac{W_{BC}}{T_S}\right)} = 146618.52\ Pa \ \checkmark$$

$$V_D = \frac{4nRT_S}{p_A} e^{\frac{1}{nR}\left(S_{CD}+\frac{W_{BC}}{T_S}\right)} = 0.0170\ m^3 \ \checkmark$$

$$T_D = T_A = \frac{T_S}{4} = 300\ K \ \checkmark$$

non ho considerato $Q_{DA} = 0.8\ kJ$ (è corretto più?)

2) W_{TOT}? η? r?

$$dU = \delta Q - \delta W \rightarrow \oint dU = \oint \delta Q - \oint \delta W = 0 \Rightarrow \oint \delta Q = \oint \delta W$$

$$\oint \delta Q = n\left(C_v + \frac{R}{1-\alpha}\right)\int_A^B dT + W_{BC} + Q_{DA} = nR\left(\frac{3}{2} + \frac{1}{1-\alpha}\right)\int_A^B dT + W_{BC} + Q_{DA}$$

$T_B = T_S,\ T_A = \frac{1}{4}T_S,\ W_{BC} = -31900\ J,\ Q_{DA} = 800\ J,\ T_S = 1200\ K$

$$\oint \delta Q = 2nR\left(T_S - \frac{1}{4}T_S\right) + W_{BC} + Q_{DA} = \frac{3}{2}nRT_S + W_{BC} + Q_{DA} = -16134.8\ J = \oint \delta W < 0 \text{ (frigorifero)}$$

eff. $\rightarrow COP_f = \left|\frac{Q_{ASS}}{W_T}\right| = \frac{\frac{3}{2}nRT_S + Q_{DA}}{W_T} = 0.977$

(1)

$COP_{PL} = COP_f + 1 = 1.98$

$$COP_{PC} = COP_f + 1 = \frac{1}{\eta}$$

3) ΔS_{gas}, ΔS_{amb} in AB, BC, CD, DE

$$\Delta S_{gas} = \oint dS = 0 \,, \quad \oint dS = \int_A^B dS + \int_B^C dS + \int_C^D dS + \int_D^A dS = 0$$

$$\int_A^B dS = nc_v \int_A^B \frac{dT}{T} + nR \int_A^B \frac{dV}{V} = nc_v \log\left(\frac{T_B}{T_A}\right) + nR \log\left(\frac{V_B}{V_A}\right) = nc_v \log(4) + nR\log(2) =$$

$$= n\left(2c_v \log(2) + R\log(2)\right) = n\left(2c_v + R\right)\log(2) = 4nR\log(2) = nR\log(16) = 23.05 \frac{J}{K} = S_{AB}$$

$$TdS = nc_v dT + \frac{nR}{V} dV \Rightarrow dS_{BC} = \frac{nR}{V} dV \rightarrow S_{BC} = nR\log\left(\frac{V_C}{V_B}\right) = -26.59 \ J/K$$

$$S_{CD} = 0.46 \ J/K \qquad dS = nc_v dT + \frac{nR}{V} dV \qquad S = nc_v \log\left(\frac{T_A}{}\right) + nR\log\left(\frac{V_B}{V_A}\right)$$

$$dS_{DA} = \frac{nR}{V} dV = nR\log\left(\frac{V_A}{V_D}\right) = -3.07 \ J/K$$

$$S_{AB} = 23.05 \ J/K \,, \quad S_{BC} = -26.59 \ J/K \,, \quad S_{CD} = 0.46 \ J/K \,, \quad S_{DA} = -3.07 \ J/K \ \checkmark$$

$$\Delta S_{gas} = \oint dS = S_{AB} + S_{BC} + S_{CD} + S_{DA} = 0 \ J/K \ \checkmark$$

AMBIENTE:

$$S^a_{AB} = -S_{AB} \,, \quad S^a_{BC} = -S^a_{BC} \,, \quad S^a_{CD} = 0 \,, \quad S^a_{DA} = \frac{-Q_{DA}}{T_A} = -2.67 \ J/K$$

$$\Delta S_{amb} = S^a_{DA} - S_{AB} - S_{BC} = 26.59 \ J/K - 23.05 \ J/K - 2.67 \ J/K = 0.87 \ J/K \ \checkmark$$

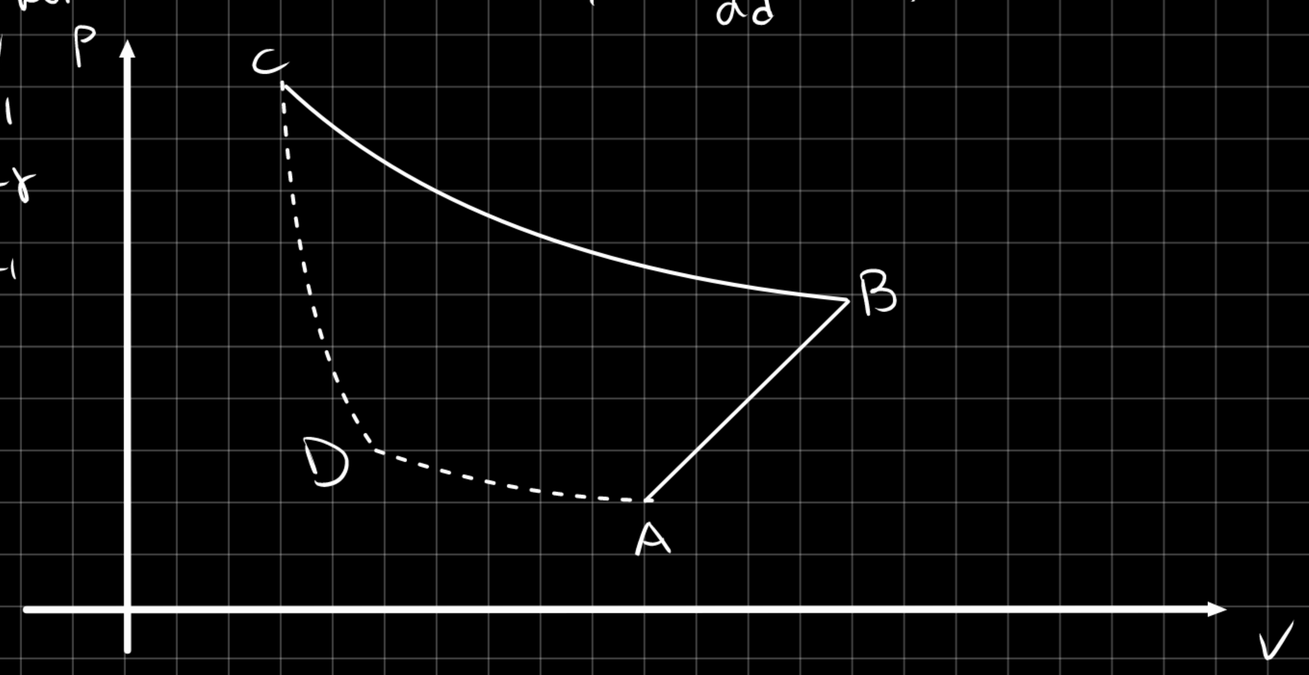
4) ΔS_Ω ?

$$\Delta S_\Omega = \Delta S_{amb} + \Delta S_{gas} = \Delta S_{amb} = 0.87 \ J/K \ \checkmark$$

5) pV, Ts

a) $pV \rightarrow A \xrightarrow[pol]{\alpha = -1} B \,, \quad B \xrightarrow{T} C \,, \quad C \underset{ad}{\rightsquigarrow} D \,, \quad D \overset{T}{\rightsquigarrow} A$

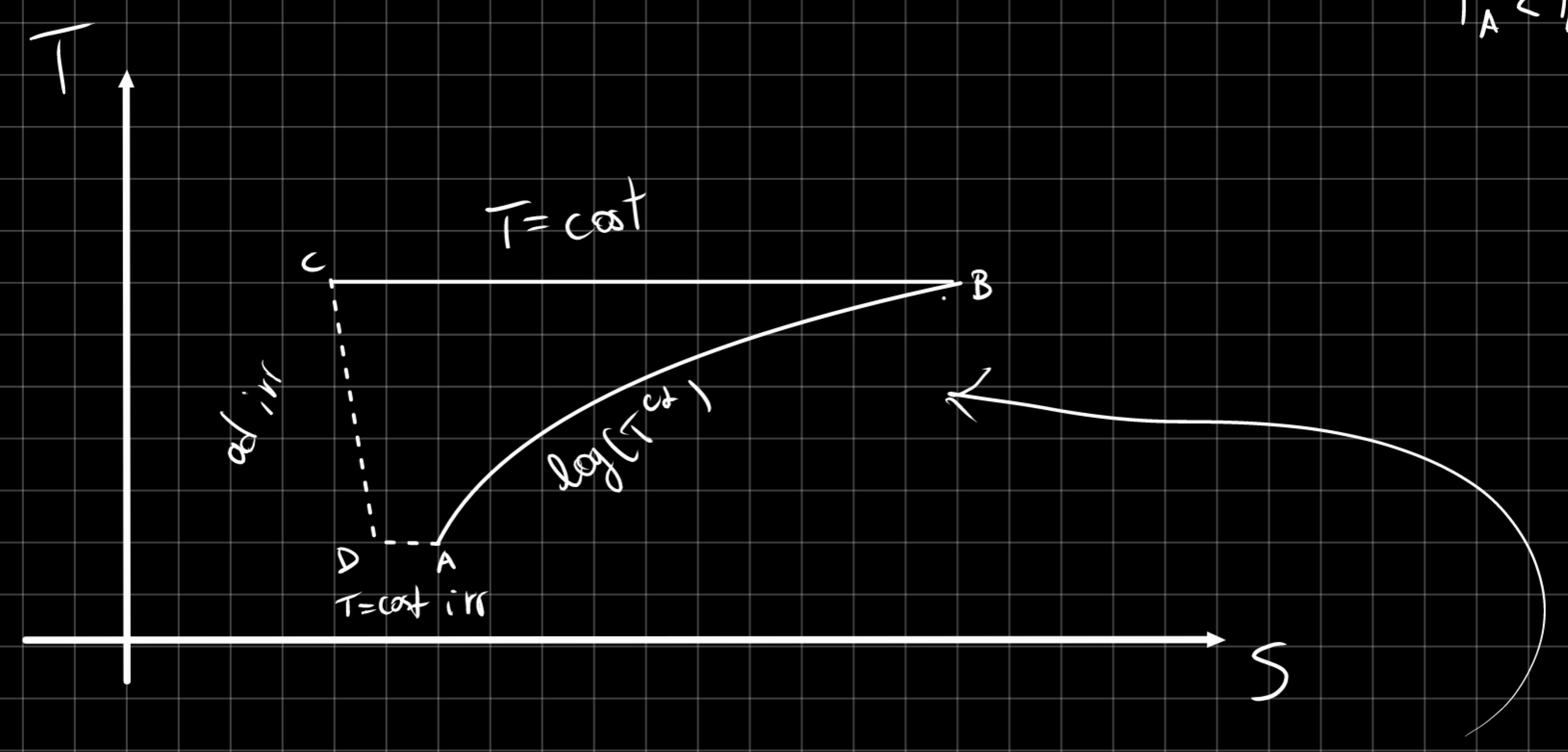
$$p(V) = \begin{cases} p = kV \\ p = V^{-1} \\ p = V^{-\gamma} \\ p = V^{-1} \end{cases}$$



b) TS

$T_A < T_B = T_C > T_D = T_A$

$$S(T) \propto \begin{cases} \log(T) \\ \log(T) \\ K \\ \log(T) \end{cases}$$



A B espansione politropica $\alpha = -1$ $\quad dQ = nc_\alpha dT \rightarrow S = nc_\alpha \log(T) = \log(T^{nc_\alpha})$

16/02/2021

E1) Cilindro in contatto termico con ambiente + pistone bloccato

$\Rightarrow n = 1^{mol}$ gas di Van der Waals $a = 4.54\ atm\,l^2/mol^2$, $b = 0.04\ l/mol$, $V_A = 1.0\ l$.

$A \to B$ espansione rapida contro P_{ATM} fino a $V_B = 4.0\ l$

Assunta $T_A = 300\ K$

1) P_{gas}

2) Q_{AB}

3) ΔS_Ω

4) Q_{REV}

5) $W_{DEG} = |W_{REV} - W_{IRR}|$

$$f(p,V,T) = \left[p + a\left(\frac{n}{V}\right)^2\right](V - nb) - nRT = 0 \qquad \frac{V}{n} = \nu$$

$$\left(p + \frac{a}{\nu^2}\right)(\nu - b) = RT$$

$$P_A = \frac{RT_A}{\nu_A - b} - \frac{a}{\nu_A^2} \quad ; \quad a = 4.54\ l^2 atm/mol^2 = 4.54\,(10^{-3}\ m^3)^2\,(101325\ Pa)/mol^2$$

$$a = 460.015.5 \cdot 10^{-6}\ m^6 Pa/mol = 0.46\ m^6 Pa/mol^2$$

$$b = 0.04\ l/mol = 0.04 \cdot 10^{-3}\ m^3/mol = 4 \cdot 10^{-5}\ m^3/mol, \quad \nu_A = \frac{V_A}{n} = 1 \cdot 10^{-3}\ \frac{m^3}{mol}$$

$$P_A = \frac{(8.314\ J/K\,mol)(300\ K)}{1 \cdot 10^{-3}\ m^3 - 4 \cdot 10^{-5}\ m^3} - \frac{0.46\ m^6 Pa/mol^2}{1 \cdot 10^{-6}\ m^6/mol^2} = 2138125\ Pa$$

2) Q_{AB}

$$T\frac{\partial S}{\partial T} = \frac{đQ}{dT}$$

a) $dU = đQ - đW \to dU = TdS - pdV$

$$\begin{cases} TdS(T,V) = T\left(\frac{\partial S}{\partial T}\right)_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV \\ \left(\frac{\partial S}{\partial T}\right)_V = \frac{1}{T}\left(\frac{đQ}{dT}\right)_V = \frac{C_V}{T} \end{cases} \to TdS = T\frac{C_V}{T}dT + T\left(\frac{\partial S}{\partial V}\right)_T dV$$

$$-dF = SdT + pdV \to \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$\therefore TdS = \frac{TC_V}{T}dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$$

$$\therefore dU = \frac{TC_V}{T}dT + T\left(\frac{\partial P}{\partial T}\right)_V dV - pdV = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV - pdV$$

$$p(V,T) = \frac{RT}{V-b} - \frac{a}{V^2} \rightarrow \left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V-b}$$

$$\therefore dU = C_V dT + \cancel{\frac{RT}{V-b}}dV - \left(\cancel{\frac{RT}{V-b}} - \frac{a}{V^2}\right)dV = C_V dT + \frac{a}{V^2}dV \quad \checkmark$$

$$U(T,V) = C_V T - \frac{a}{V}$$

Tra A, B ho

$V_A = 1\,l$, $V_B = 4\,l$, $P_{ext} = 1$ atm

W_{ext} su sist (<0)

$$đQ = dU - đW \rightarrow Q_{AB} = (U_B - U_A) + P_{ext}(V_B - V_A)$$

$$U_B - U_A = C_V(T_B - T_A) + a\left(\frac{1}{V_A} - \frac{1}{V_B}\right)$$

$$\therefore Q_{AB} = C_V(T_B - T_A) + P_{ext}(V_B - V_A) + a\left(\frac{1}{V_A} - \frac{1}{V_B}\right) = P_{ext}\Delta V_{AB} + a\frac{\Delta V_{AB}}{V_A V_B}$$

$a = 0.46\ m^6 Pa\, mol^{-2}$, $T_A = T_B = 300\ K$, $\Delta V_{AB} = 3\,l$, $P_{ext} = 101325\ Pa$

$V_A V_B = 4\,l^2 = 4 \cdot 10^{-6}\ m^6$, $\Delta V_{AB} = 3 \cdot 10^{-3}\ m^3$, $\Delta T_{AB} = 0$

$$Q_{AB} = P_{ext}(V_B - V_A) + a\left(\frac{1}{V_A} - \frac{1}{V_B}\right) = 648.975\ J \quad \checkmark$$

3) ΔS_Ω ?

$$\Delta S_\Omega = \Delta S_{gas} + \Delta S_{atm}$$

$$\Delta S_{atm} = \frac{Q_{AB}}{T_A} \quad \checkmark$$

$$TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV \rightarrow dS = \frac{C_V}{T}dT + \left(\frac{\partial P}{\partial T}\right)_V dV$$

$dT = 0$

$$\therefore \Delta S_{gas} = \int_A^B \left(\frac{\partial P}{\partial T}\right)_V dV = R\int_A^B \frac{1}{V-b}dV = R\log\left(\frac{V_B - b}{V_A - b}\right)$$

$$\Delta S_\Omega = -\frac{Q_{AB}}{T_A} + R\log\left(\frac{V_B - b}{V_A - b}\right) = \frac{-648.975}{300}\frac{J}{K} + 8.314 \cdot \log\left(\frac{4 \cdot 10^{-3}\ m^3 - 4 \cdot 10^{-5}\ m^3}{1 \cdot 10^{-3}\ m^3 - 4 \cdot 10^{-5}\ m^3}\right)\frac{J}{K} = 9.62\ \frac{J}{K}$$

OK

4) Q_{rev} ?

Se rev $\rightarrow$ $đQ = TdS \Rightarrow Q_{AB} = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV = C_V dT + \frac{RT}{V-b} dV$

$\therefore Q_{AB} = RT_A \log\left(\frac{V_B - b}{V_A - b}\right) = 8.314 \cdot 300 \log\left(\frac{4 - 4\cdot 10^{-2}}{1 - 4\cdot 10^{-2}}\right) J = 3534.45\ J$ ✓

5) E_{dg} ?

$E_{dg} = Q_{rev} - Q_{irr} = 3534.45\ J - 648.975\ J = 2885.475\ J$ ✓

NOTA: per E1 con gas reale:

1- equazioni $TdS \oplus dU$

2- con dU trovi irr usando $dU = TdS - pdV$, $TdS = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV$

3- con TdS trovi rev (ricordu)

$$Q_{AB}^{rev} = \int_A^B T dS$$

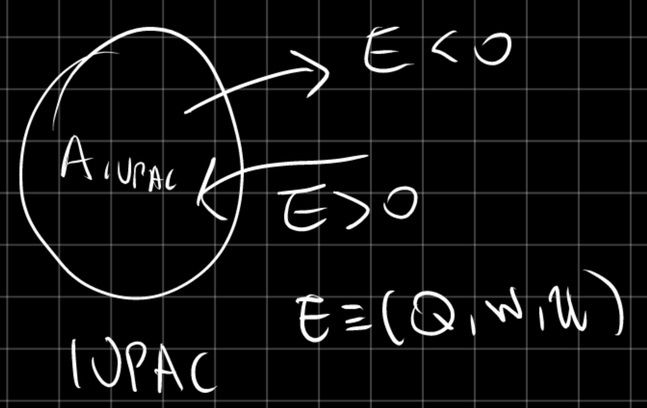
4- per irr usu I° PT

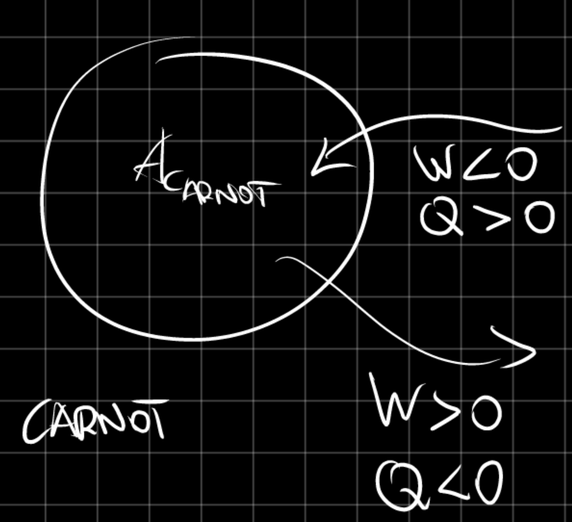
$$\begin{cases} đQ = dU + đW & A \rightsquigarrow B \ (irr) \\ TdS = dU + pdV & A \longrightarrow B \ (rev) \end{cases}$$

5 - STANDARD SEGNI

IUPAC (esam) $\begin{cases} đQ = dU + đW \\ đQ = dU - đW \end{cases}$ $\begin{cases} đW_{rev} = pdV & \text{(lavoro fatto sul sistema)} \\ đW_{rev} = -pdV & \text{(lavoro fatto dal sistema)} \end{cases}$

Carnot (libro)





NOTA PER E2:

PROCESSI IN CICLI (OTTO, DIESEL, STIRLING, ...)

O: 2 ad + 2 isoV, D: 2 ad + isoV + iso p, Brayton: 2 ad + 2 isop, Stirling: 2 isot. + 2 isoV

E_2) $n = 1$ mol gas perfetto biatomico con $T_A = 31.6°C$, $p_A = 1$ atm compie ciclo ABCD irr. con 2 sorgenti $T_L = 31.6°C$ e $T_H = 869°C$.

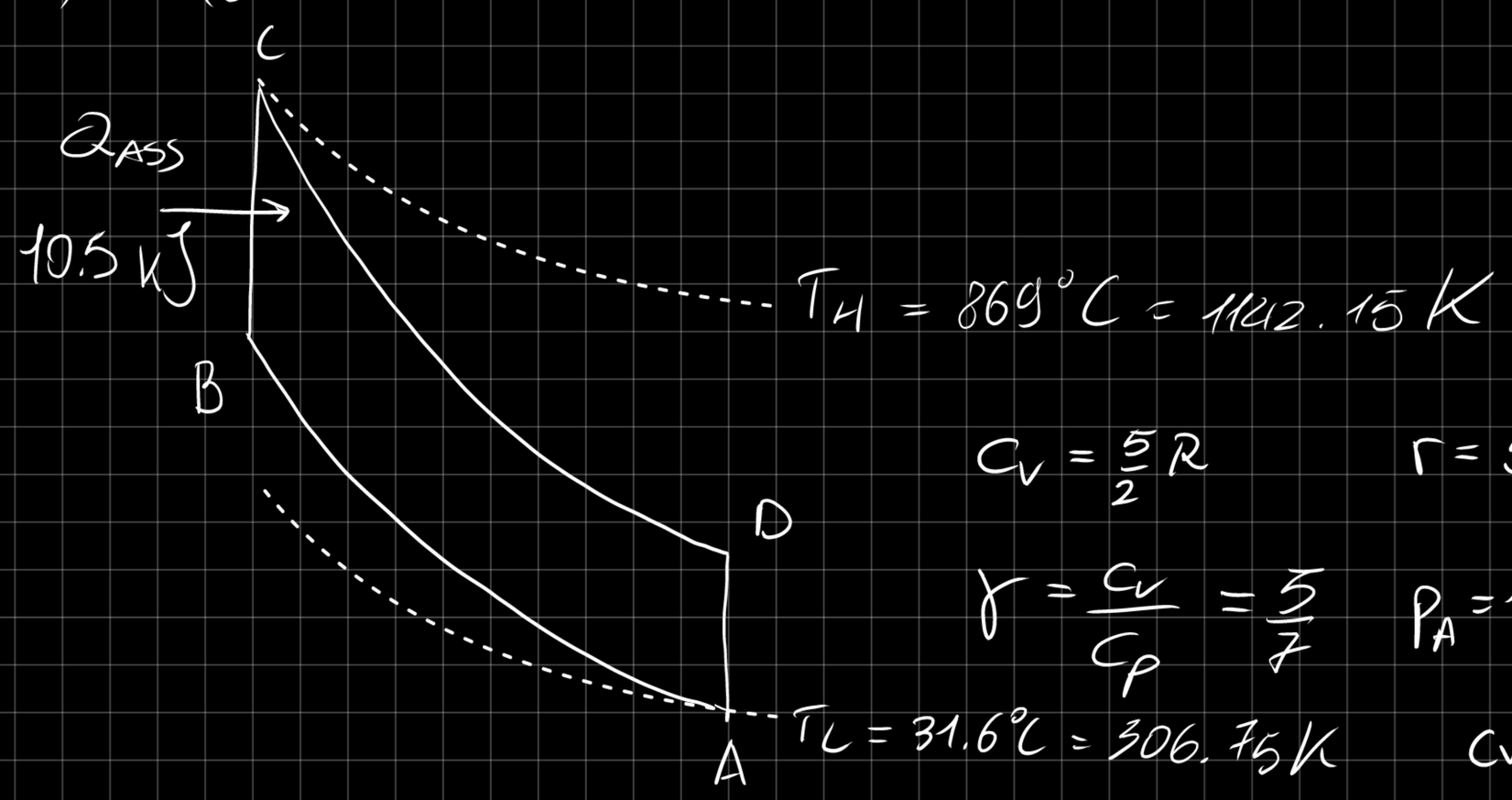
Dato $\eta = 0.41\,\eta_{CAR}$, $r = 5$, $Q_{ass} = 10.5$ kJ trova

1) tabella pVT
2) Q_{TOT}
3) ΔS_{gas}, ΔS_{amb}
4) ΔS_{Ω}
5) pV, TS

---

1) pVT

a) OTTO



$c_V = \frac{5}{2}R$ $\quad r = 5$

$\gamma = \frac{c_V}{c_p} = \frac{5}{7}$ $\quad p_A = 1$ atm $= 101325$ Pa

$c_V = \frac{5}{2}R \rightarrow \frac{1}{c_V} = \frac{2}{5R}$

$Q_{ASS} = nc_V(T_C - T_B) \Rightarrow T_B = T_H - \frac{Q_{ASS}}{nc_V}$

$V_A = 0.0252\ m^3$
$V_B = 0.0050\ m^3$
$p_A = 101325$ Pa
$p_B = 1059167.02$ Pa ✓
$T_B = 636.98$ K

$r = \frac{V_A}{V_B} \rightarrow V_B = \frac{1}{5}V_A$

$p_A V_A = nRT_L \rightarrow V_A = \frac{nRT_L}{p_A} \Rightarrow V_B = \frac{nRT_L}{5p_A} = \frac{V_A}{5}$

$p_B V_B = nRT_B = nR\left(T_H - \frac{Q_{ASS}}{nc_V}\right) \Rightarrow p_B = \frac{nR}{V_B}\left(T_H - \frac{Q_{ASS}}{nc_V}\right)$ $\quad T_C = T_H$

$p_B = \frac{5nRp_A}{nRT_L} = \frac{5p_A}{T_L}\left(T_H - \frac{Q_{ASS}}{nc_V}\right)$

A	B	C
$p_A = 101325$ Pa	$p_B = 1059167.02$ Pa	$p_C = \frac{nRT_H}{V_B} = 1899167.02$ Pa
$V_A = 0.0252\ m^3$	$V_B = 0.0050\ m^3$	$V_C = V_B = 0.0050\ m^3$
$T_A = 306.75$ K	$T_B = 636.98$ K	$T_C = T_H = 1142.15\ m^3$

D) $V_D = V_A = 0.0232\ m^3$

$$\eta = 0.41\,\eta_C = 0.41\left(1 - \frac{T_L}{T_H}\right) = 1 + \frac{Q_{ced}}{Q_{ass}} = 1 + \frac{nc_v(T_L - T_D)}{nc_v(T_H - T_B)}$$

$$\eta = 0.30 = 1 + \frac{T_L - T_D}{T_H - T_B} \rightarrow \quad T_D = 0.7(T_H - T_B) + T_L = 660.37\,K$$

$$P_D = \frac{nRT_D}{V_D} = \frac{nRT_D}{V_A} = 217869.69\ Pa$$

A	B	C	
$P_A = 101325\ Pa$	$P_B = 1059167.02\ Pa$	$P_C = 1899167.02\ Pa$	$P_D = 217869.69\ Pa$ ✓
$V_A = 0.0252\ m^3$	$V_B = 0.0050\ m^3$	$V_C = 0.0050\ m^3$	$V_D = 0.0252\ m^3$
$T_A = 306.75\ K$	$T_B = 636.98\ K$	$T_C = 1142.15\ K$	$T_D = 660.37\ K$

2)

$$\eta = \left|\frac{W}{Q_A}\right| \rightarrow \quad W = \eta Q_A = 0.30 \cdot 10.5\,kJ = 3.15\ kJ = 3150\ J$$

3)

ΔS_{gas}, ΔS_{amb}

$A \rightarrow B$ (ad) $\rightarrow \Delta S^a_{AB} = 0 \quad \Delta S^g_{AB} = nc_v \log\left(\frac{T_B}{T_L}\right) + nR\log\left(\frac{V_B}{V_A}\right) = 1.74\ J/K$

$B \rightarrow C$ (isoV) $\rightarrow \Delta S^g_{BC} = 12.14\ \frac{J}{K}$.

$$dU = TdS - pdV \rightarrow TdS = dU + pdV = nc_v dT + pdV \qquad S_{BC}(T,V) = nc_v \log\left(\frac{T_C}{T_B}\right) = \frac{5}{2}R\log\left(\frac{T_C}{T_B}\right)\frac{J}{K}$$

$$pdV = nRdT - \frac{nRT}{p}dp$$

$$\frac{nc_p}{T}dT - \frac{nR}{p}dp = dS \rightarrow \Delta S^g_{BC} = nc_p \log\left(\frac{T_H}{T_B}\right) - nR\log\left(\frac{P_C}{P_B}\right) \quad \text{ok!}$$

$$\Delta S^{amb}_{BC} = \frac{-Q_{ass}}{T_H} = -9.19\ \frac{J}{K}$$

$C \rightarrow D \quad \Delta S^g_{CD} = nc_v \log\left(\frac{T_D}{T_C}\right) + nR\log\left(\frac{V_D}{V_C}\right) = 2.06\ J/K$

$D \rightarrow A \quad \Delta S^a_{DA} = \frac{Q_{ced}}{T_L}, \quad \Delta S^g_{DA} = nc_v \log\left(\frac{T_L}{T_D}\right) = -15.94\ \frac{J}{K}; \quad \Delta S^a_{DA} = 23.96\ J/K$

$$\eta = \left|\frac{Q_T}{Q_{ass}}\right| = \left|\frac{Q_{ced} + Q_{ass}}{Q_{ass}}\right| = 1 + \left|\frac{Q_{ced}}{Q_{ass}}\right| \rightarrow |Q_{ced}| = |(\eta - 1)\,Q_{ass}| = |(0.30 - 1)Q_{ass}| = 7350\ J$$

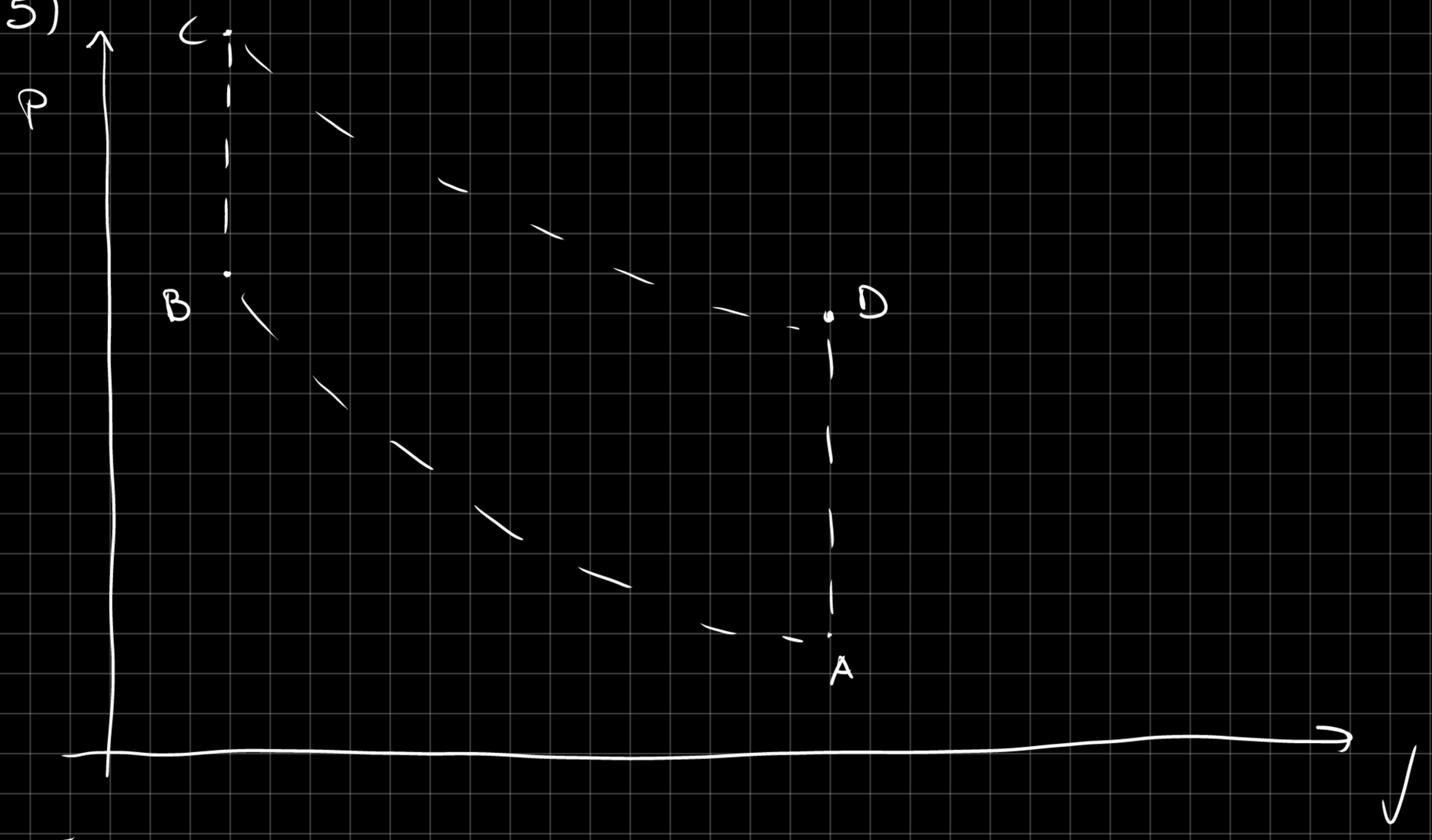
$$\Delta S^g_{TOT} = (1.74 + 12.14 + 2.06 - 15.94)\ \frac{J}{K} = 0\ J/K \quad \text{ok}$$

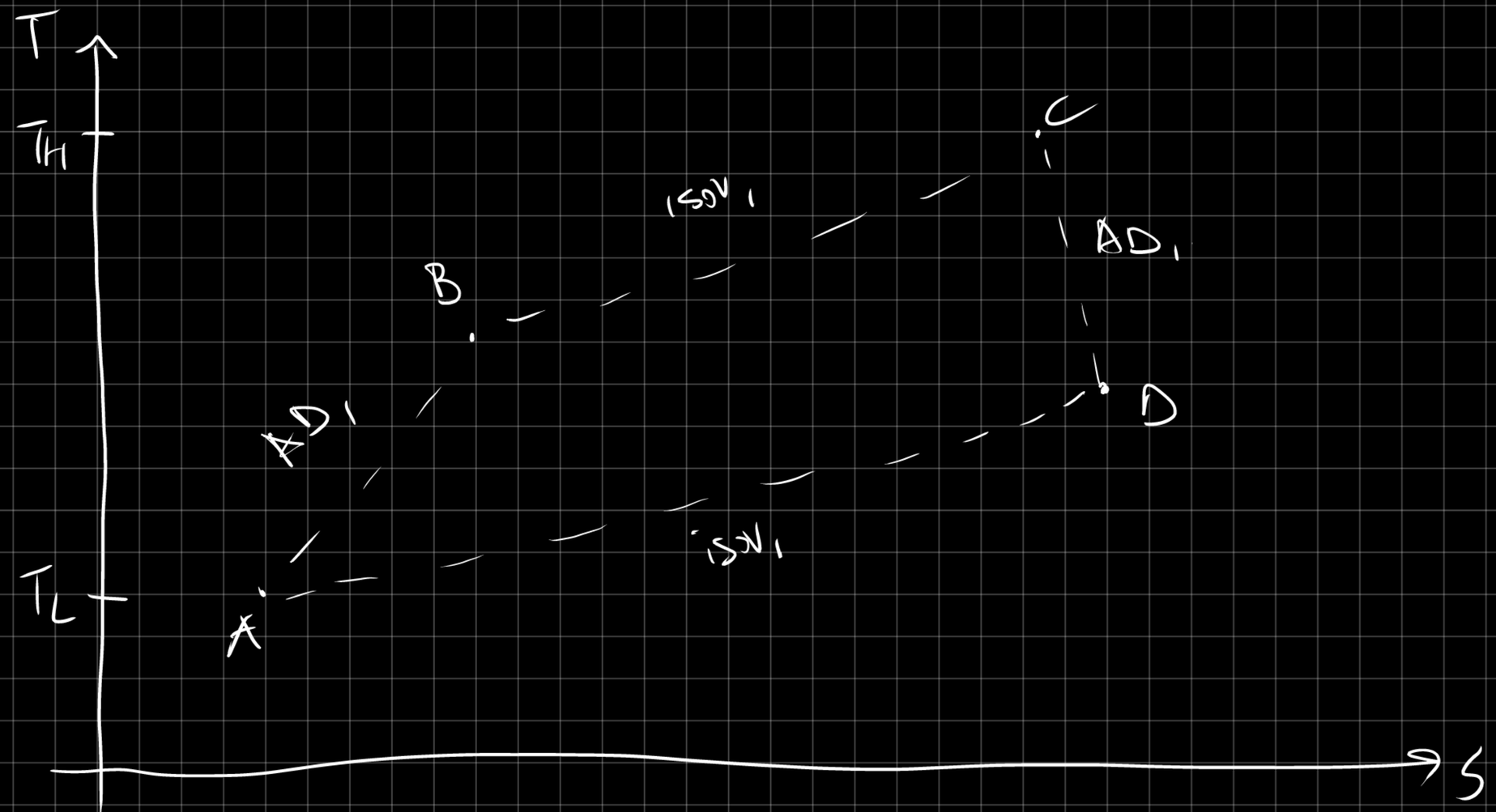
$$\Delta S^a_{TOT} = (-9.19 + 23.96)\ J/K = 14.77\ J/K$$

4)

$$\Delta S_\Omega = \Delta S_{gas} + \Delta S_{amb} = \Delta S_{amb} = 14.77 \ J/K$$

5)





20/01/2023 ← Recente!

E1) 10 sassi ($m_S = 100g = 0.1kg$), 1 mela ($m_m = 300g = 0.3kg$), 2 p. ghiaccio ($m_g = 150g = 0.15kg$)

1) 5 sassi in lago

2) 1 gh. in lago

3) 5 sassi in calorimetro ad. con $m_{H_2O} = 450\,g$ a $T_{H_2O} = 50°C$; $m^* = 50g$

4) ghiaccio in calorimetro

5) mela cadutta da $h = 100\,m$ (attrito trascurabile)

→ $T_{amb} = 30°C$, $T_{lago} = 5°C$, trova ΔS_Ω

N.B.:

$c_{H_2O} = 1\ cal/g°C$, $c_S = 0.2\ cal/g°C$, $c_m = 0.8\ cal/g°C$, $\lambda_g = 80\ cal/g$

$1J = 0.2389\ cal$

$4.186\ J/cal$

E1)

a) $c_{H_2O} = 4.186\ \frac{J}{gK}$, $c_S = 0.8372\ \frac{J}{gK}$, $c_m = 3.3488\ \frac{J}{gK}$, $\lambda_g = 334.88\ J/g$

b) $T_a = 303.15\,K$, $T_e = 278.15\,K$

1) 5 sassi nel lago

$$\Delta S_\Omega = \Delta S_a + \Delta S_S$$

$$\Delta S_S = \sum_{i=1}^{5} m_S c_S \log\left(\frac{T_e}{T_a}\right) = 5 m_S c_S \log\left(\frac{T_e}{T_a}\right) = 5 \cdot 100 \cdot 0.2 \cdot \log\left(\frac{278.15}{303.15}\right) \frac{J}{K} = -36.03\ J/K$$

$$\Delta S_e = \frac{Q}{T_e} = \frac{1}{T_e}\left(-m_S c_S (T_e - T_S)\right) = \frac{1}{278.15}\left(-5 \cdot 100 \cdot 0.8372 \cdot (278.15 - 303.15)\right) J/K$$

$$\Delta S_e = 37.62\ J/K$$

$$\Delta S_\Omega = \Delta S_e + \Delta S_S = (37.62 - 36.03)\ J/K = 1.59\ J/K$$

2) 1 palla di $H_2O_{(s)}$ nel lago

$m_g = 150\,g$, $c_{H_2O} = 4.186\ J/gK$, $\lambda_g = 334.88\ J/g$

$$Q_{fus} + Q_{H_2O} = Q$$

$$\Delta S_g = \frac{m_g \lambda_g}{273.15} + m_{H_2O} c_{H_2O} \log\left(\frac{T_e}{273.15}\right) = \frac{150 \cdot 334.88}{273.15} \frac{J}{K} + 150 \cdot 4.186 \cdot \log\left(\frac{278.15}{273.15}\right) \frac{J}{K}$$

$$\Delta S_g = 195.29 \ J/K$$

$$\Delta S_a = -\frac{m_g \lambda_g}{T_e} - \frac{m_{H_2O} c_{H_2O} (T_e - 273.15)}{T_e} = -191.88 \ J/K$$

$$\Delta S_\Omega = 3.41 \ J/K \ \checkmark$$

3) 5 Sassi $m_s = 100g$, $c_s = 0.8372 \ J/gK$, $m_{H_2O} = 450 \ g$, $T_{H_2O} = 323.15 \ K$, $m^* = 50g$

$c_{H_2O} = 4.186 \ J/gK$, $T_a = 303.15 \ K$

$$\Delta S_\Omega = \Delta S_{H_2O} + \Delta S_s + \Delta S_a$$

$$\Delta S_{H_2O} = (m_{H_2O} + m^*) c_{H_2O} \log\left(\frac{T_{eq}}{T_{H_2O}}\right)$$

$$\Delta S_s = 5 m_s c_s \log\left(\frac{T_{eq}}{T_a}\right)$$

$$-Q_{H_2O} = (m_{H_2O} + m^*) c_{H_2O} (T_{H_2O} - T_{eq}) = 5 m_s c_s (T_a - T_{eq}) = Q_s$$

$$\left(5 m_s c_s - (m_{H_2O} + m^*) c_{H_2O}\right) T_{eq} = 5 m_s c_s T_a - (m_{H_2O} + m^*) T_{H_2O}$$

$$T_{eq} = \frac{5 m_s c_s T_a + (m_{H_2O} + m^*) T_{H_2O}}{5 m_s c_s + (m_{H_2O} + m^*) c_{H_2O}} = 319.82 \ K \ \checkmark$$

$$\Delta S_{H_2O} = -21.68 \ J/K \ \checkmark, \quad \Delta S_s = 22.41 \ J/K \ \checkmark$$

$$\Delta S_\Omega = 0.73 \ J/K \ \checkmark$$

4) palla di ghiaccio $m_g = 150 \ g$, $m_{H_2O} = 450 g$, $m^* = 50 \ g$, $T_{H_2O} = 323.15 \ K$

$T_f = 273.15 \ K$, $\lambda_f = 334.88 \ J/g$, $c_{H_2O} = 4.186 \ J/mol \cdot K$

fassan @ $T = T_f$ → eq. con calorimetro

$$Q_{fus} = m_g \lambda_f, \quad Q_{H_2O} = m_g c_{H_2O} (T_{eq} - T_f)$$

$$Q_{cal} = -Q_{H_2O} - Q_f \rightarrow T_{eq} = \frac{(m_{H_2O} + m^*) c_{H_2O} T_{H_2O} + m_g c_{H_2O} T_f}{(m_g + m_{H_2O} + m^*) c_{H_2O}} - \frac{m_g \lambda_f}{(m_g + m_{H_2O} + m^*) c_{H_2O}} = 293.22 \ \checkmark$$

$$\Delta S_\Omega = \Delta S_c + \Delta S_{H_2O} + \Delta S_f = (m_{H_2O} + m^*) c_{H_2O} \log\left(\frac{T_{eq}}{T_{H_2O}}\right) + m_g c_{H_2O} \log\left(\frac{T_{eq}}{T_f}\right) + \frac{m_g \lambda_f}{T_f} = 24.82 \ J/K \ \checkmark$$

5) mela $m_m = 300\ g$, cade su lago $T_e = 278.15$ da $h = 100\ m$ (no attrito)

$T_a = 303.15\ K$, $c_m = 0.8\ cal/g°C$

lago a $h = 0\ m$

1) $W = mg\Delta h \rightarrow W = m_m g h = (0.3\ Kg)\cdot(9.81\ m/s^2)\cdot(100\ m) = 294.3\ J$ ✓

$Q_m = \Delta U_m + W_m \Rightarrow Q = m_m c_m (T_e - T_a) + m_m g h = -25122\ J$

$Q_e = -Q_m$

$\Delta S_m = m_m c_m \log\left(\frac{T_e}{T_a}\right) = -86.49\ J/K$

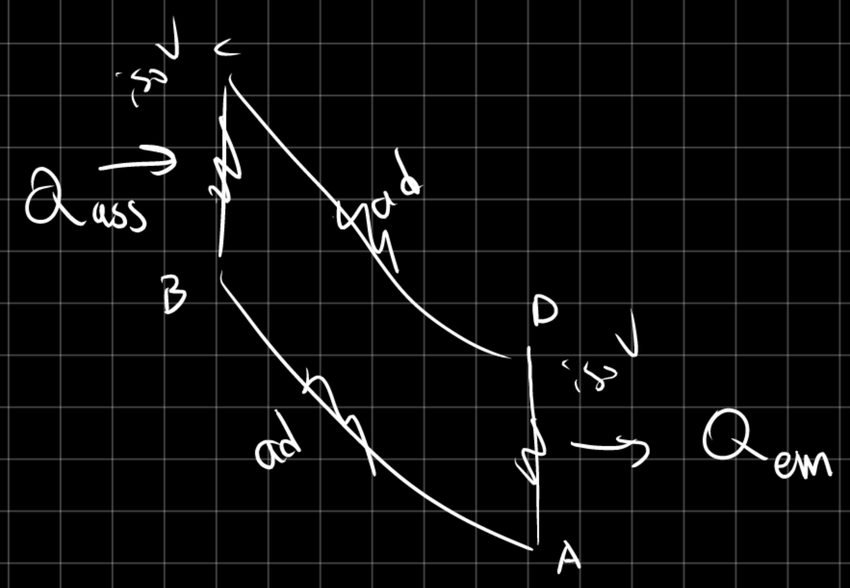
$\Delta S_e = \frac{Q_e}{T_e} = \frac{-Q_m}{T_e} = 90.31\ J/K$

$\Delta S_\Omega = \Delta S_m + \Delta S_e = 3.82\ J/K$ ~ ok?

E2)

Gas perfetto biatomico ($c_v = \frac{5}{2}R$), $n = \frac{1}{2}$ mol, è in $T_A = 27°C$, $p_A = 1\ atm$ compie ciclo Otto (2 ad, 2 isoV) ABCD irr il ciclo è realizzato con 2 sorgenti $T_H = 928°C$, $T_L = 27°C$. $\eta = \frac{1}{5}\eta_c$. $r = \frac{V_{max}}{V_{min}} = 3$ e $\Delta S_{BC} = 5\Delta S_{AB}$

1) pVT
2) W_{TOT}
3) ΔS_g, ΔS_a
4) ΔS_Ω
5) pV, TS



1) pVT

$p_A = 101325\ m^3$, $T_A = T_L = 300.15\ K$ (A) ✓

$V_A = \frac{nRT_A}{p_A} = \frac{1}{2}\frac{RT_A}{p_A} = 0.0123\ m^3$

$\frac{V_A}{V_B} = r = 3 \Rightarrow V_B = \frac{1}{3}V_A = 0.0041\ m^3 = V_C$ ✓ ; $T_H = T_C = 1201.15\ K$ ✓

1217848.91 Pa

$P_C = \frac{nRT_C}{V_C} = 1248295.14\ Pa \simeq 12.32\ atm$ ✓

$$\Delta S_{BC} = 5\Delta S_{AB} \qquad (P_A, V_A, T_A)\ \checkmark;\ (P_C, V_C, T_C)\ \checkmark,\ (?, V_B, ?)$$

$$\Delta S_{BC}(T,V) = nc_v \log\left(\frac{T_C}{T_B}\right) + nR\log\left(\frac{V_C}{V_B}\right) = 5\left(nc_v\log\left(\frac{T_B}{T_A}\right) + nR\log\left(\frac{V_B}{V_A}\right)\right) = 5\,\Delta S_{AB}(T,V)$$

$$T_C = T_H,\quad V_C = V_B,\quad \frac{V_B}{V_A} = \frac{1}{3},\quad V_B = V_C,\quad T_A = T_L$$

$$nc_v\log\left(\frac{T_H}{T_B}\right) + \cancel{nR\log(1)} = 5nc_v\log\left(\frac{T_B}{T_L}\right) + 5nR\log\left(\frac{1}{3}\right)$$

$$nc_v\log T_H - nc_v\log(T_B) = 5nc_v\log T_B + 5\left(nR\log\left(\frac{1}{3}\right) - nc_v\log(T_L)\right)$$

$$6nc_v\log T_B = nc_v\log T_H - 5n\left(R\log\left(\frac{1}{3}\right) - c_v\log(T_L)\right)$$

$$n = \frac{1}{2}\text{ mol},\quad c_v = \frac{5}{2}R$$

$$\frac{15}{2}R\log T_B = \frac{5}{4}R\log T_H - \frac{5}{2}R\left(\log\left(\frac{1}{3}\right) - \frac{5}{2}\log(T_L)\right)$$

$$\log(T_B) = \frac{\cancel{2}}{\cancel{15}_{3}}\,\frac{\cancel{5}}{\cancel{4}_{2}}\log T_H - \frac{\cancel{2}}{\cancel{15}_{3}}\,\frac{\cancel{5}}{\cancel{2}}\left(\log\left(\frac{1}{3}\right) - \frac{5}{2}\log(T_L)\right)$$

$$\log T_B = \frac{1}{6}\log T_H - \frac{1}{3}\log\left(\frac{1}{3}\right) + \frac{5}{6}\log(T_L)$$

$$\log T_B = \log\left(\sqrt[6]{T_H}\right) + \log\left(\sqrt[3]{3}\right) + \log\left(T_L^{\frac{5}{6}}\right)$$

$$T_B = \sqrt[3]{3}\,\sqrt[6]{T_H}\,T_L^{\frac{5}{6}} = 545.45\text{ K}\ \checkmark\ !!$$

$$\underline{P_B} = \frac{nRT_B}{V_B} = \frac{1}{2}\frac{RT_B}{V_B} = \underline{553033.08}$$

$P_A = 101325$ Pa	$P_B = 553033.08$ Pa	$P_C = 1217848.91$ Pa	$P_D = 289803.73$
$V_A = 0.0123\ m^3$	$V_B = 0.0041\ m^3$	$V_C = 0.0041\ m^3$	$V_D = 0.0123\ m^3$
$T_A = 300.15$ K	$T_B = 545.45$ K	$T_C = 1201.15$ K	$T_D = 857.49$ K

$$\eta_0 = \frac{1}{5}\left(1 - \frac{T_L}{T_H}\right) = \frac{|Q_{rev}|}{|Q_{abs}|} = 1 + \frac{Q_{ced}}{Q_{abs}};\quad \eta_c = 0.75 \Rightarrow \eta_0 = 0.15 \Rightarrow Q_{ced} = -0.85\,Q_{abs}$$

$$Q_{abs} = nc_v(T_C - T_B),\quad Q_{ced} = -0.85\,Q_{abs} = nc_v(T_A - T_D)$$

$$0.85\,\cancel{nc_v}(T_C - T_B) = \cancel{nc_v}(T_D - T_A) \rightarrow T_D = 0.85(T_C - T_B) + T_A = \underline{857.49\text{ K}}$$

$$P_D = \frac{nRT_D}{V_A} = 289803.73\text{ Pa}$$

2) W_{TOT}

per def: $\eta = \left|\frac{W_{TOT}}{Q_{abs}}\right| = 0.15 \rightarrow W_{TOT} = 0.15 Q_{abs} = 0.15 n c_v (T_C - T_B) = 1022.15\ J$ ✓

(Soluzione del prof. $W_{TOT} = Q_{TOT} = Q_{AB} + Q_{DA} = 1022.1\ J$)

3) ΔS_{gas}, ΔS_{amb} in tutte le trasf.

a) GAS

$$\left.\begin{aligned}
\Delta S_{AB} &= n c_v \log\left(\frac{T_B}{T_A}\right) + nR\left(\frac{1}{r}\right) = 1.64\ J/K \checkmark \\
\Delta S_{BC} &= n c_v \log\left(\frac{T_C}{T_B}\right) = 8.20\ J/K \checkmark \\
\Delta S_{CD} &= n c_v \log\left(\frac{T_D}{T_C}\right) + nR \log(r) = 1.06\ J/K \checkmark \\
\Delta S_{DA} &= n c_v \log\left(\frac{T_A}{T_D}\right) = -10.90\ J/K \checkmark
\end{aligned}\right\} \Delta S_{gas} = 0\ J/K$$

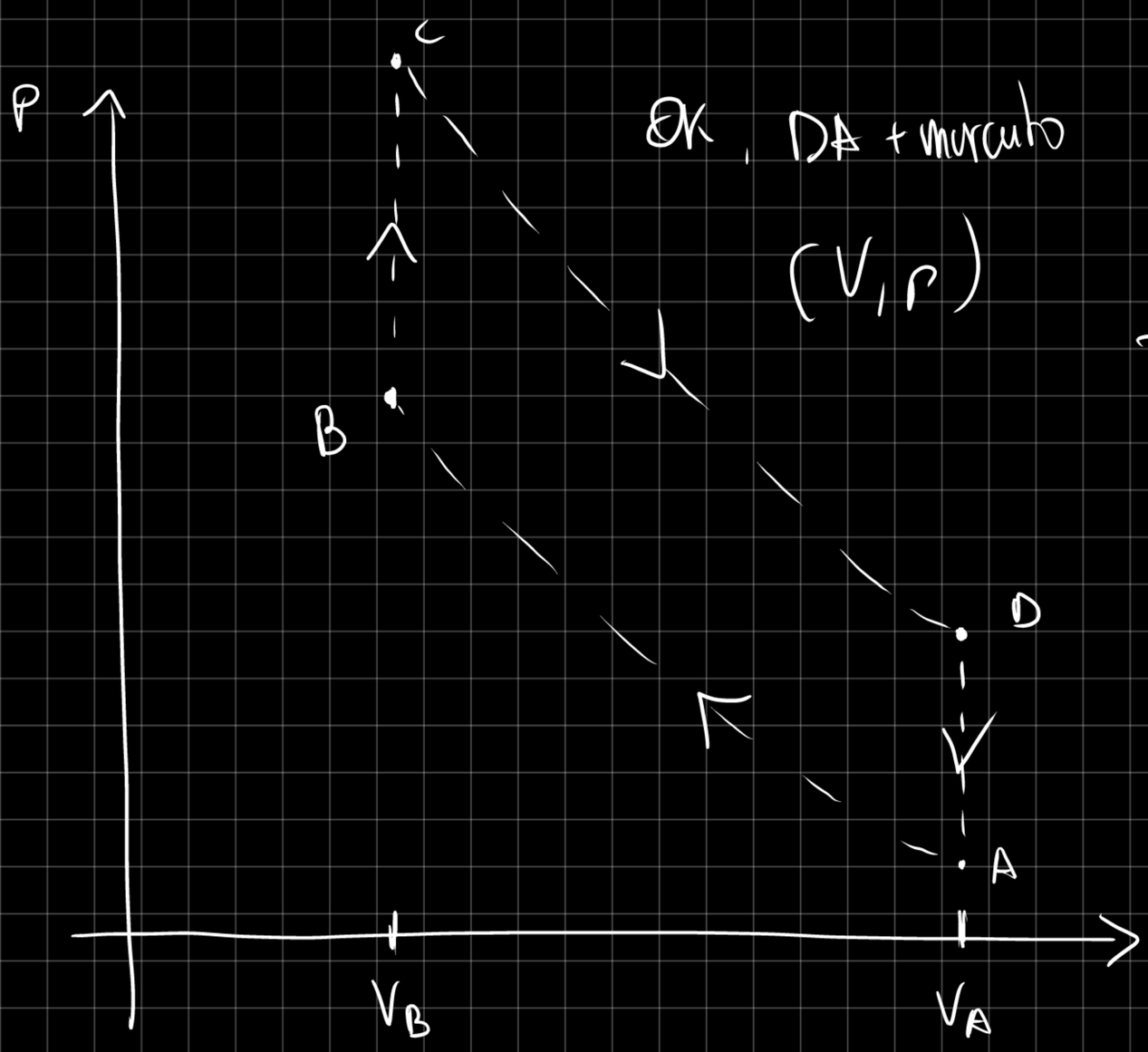
b) AMB

$$\left.\begin{aligned}
\Delta S_{BC} &= -\frac{Q_{abs}}{T_H} = \frac{n c_v (T_B - T_H)}{T_H} = -5.67\ J/K \checkmark \\
\Delta S_{DA} &= \frac{Q_{ceb}}{T_L} = \frac{n c_v (T_D - T_A)}{T_L} = 19.30\ J/K \checkmark
\end{aligned}\right\} \Delta S_{amb} = 13.63\ J/K$$

4) ΔS_Ω

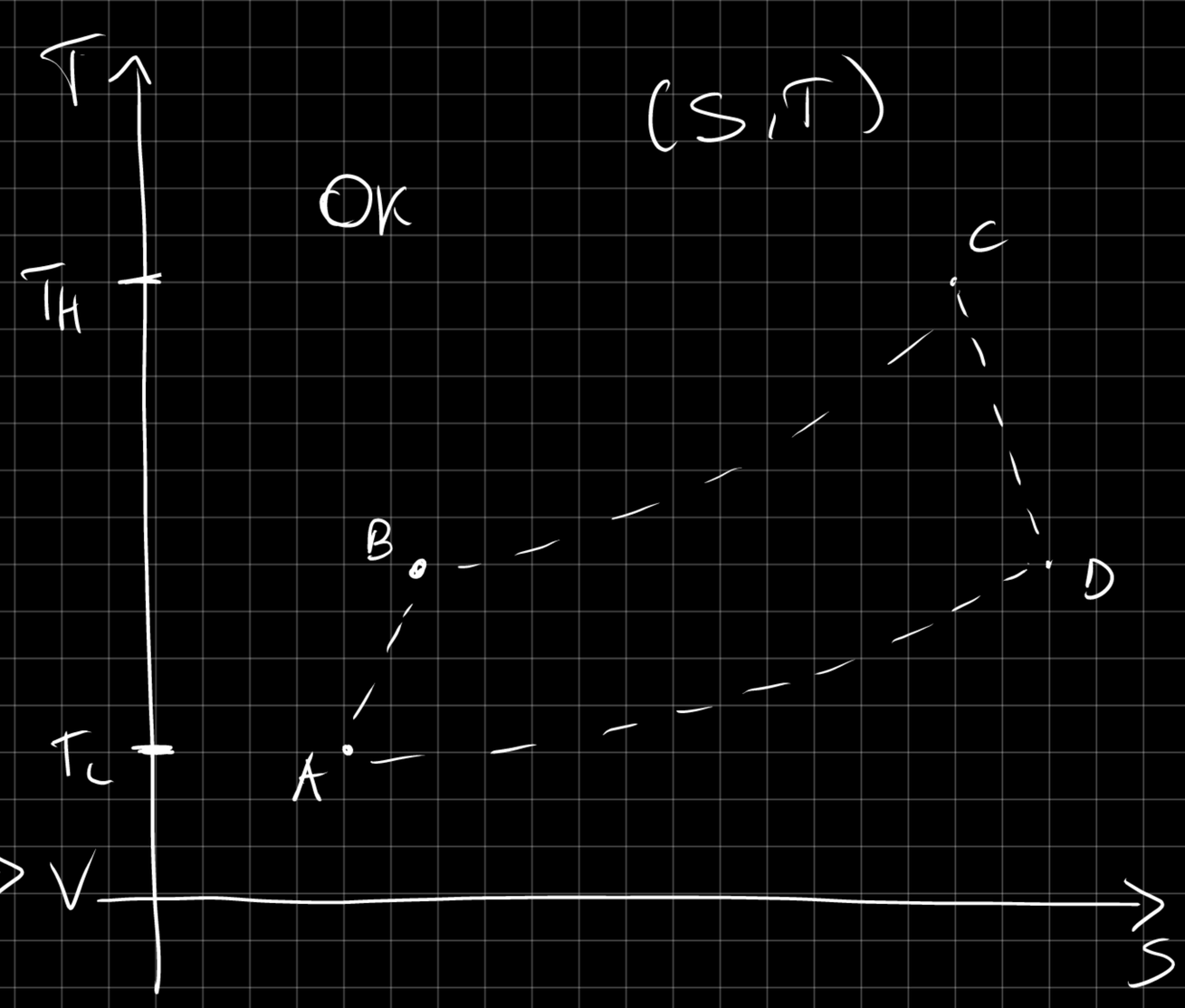
$$\Delta S_\Omega = \Delta S_{amb} + \Delta S_{gas} = \Delta S_{amb} = 13.63\ J/K$$

5) pV, TS



OK, DA + mercurio

(V, p)



OK

(S, T)

21/01/2022

E1) Cilindro rigido ad diviso da una parete da pareti ad e dall'altra da pareti dt in contatto con amb a $T_0 = 300K$

Diviso in 3 ambiti A B C, $V_0 = 1\ l$. A è riempita con gas perfetto monoatomico a $p_0 = 1atm$ e $T = T_0$. Gas in B a T_0. In C vuoto.

A | B da pistone ad senza attrito, tra B C membrana ad con $p_{max} = 2atm$

Una valvola con $V_v \simeq 0$, $C_{ve} \simeq 0$ ruota in A dissipando $P = 1W$

Supponendo il riscaldamento da valvola lento:

1) stati iniziali in A, B

2) stati A, B prima di rottura membrana

3) ΔU prima

4) Dopo quanto tempo cede

5) Stato dopo rottura con valvola ferma, sistema cede $Q = 50\ J$

1) stati iniziali

$$(p_0, V_0, T_0) = (p_A, V_A, T_A) = (p_B, V_B, T_B)$$

$$pV = nRT \rightarrow n = \frac{p_0 V_0}{RT_0} = \frac{101325 \cdot 10^{-3}}{8.314 \cdot 300} mol = 0.041\ mol$$

2) B ad, pistone

$$\frac{F}{S} = \frac{N}{m^2} = \frac{kg\ m}{s^2 m^2} = \frac{kg}{m s^2} = Pa$$

$$p_0 \rightarrow p_B = 2atm = 202650\ \overset{Pa}{\overset{\shortparallel}{kg/ms^2}}$$

$$p_0 V_0^\gamma = p_B V_B^\gamma \rightarrow V_B = V_0 \left(\frac{p_0}{p_B}\right)^{1/\gamma} = 1 \cdot 10^{-3} m^3 \left(\frac{101325}{202650}\right)^{\frac{3}{5}} = 6.60 \cdot 10^{-4}\ m^3 \checkmark$$

$$c_v = \frac{3}{2}R, \quad c_p = \frac{5}{2}R \rightarrow \gamma = \frac{5}{3}$$

$$\text{pistone} \rightarrow V_A = 2V_0 - V_B = 1.34 \cdot 10^{-3}\ m^3$$

$$T_A = \frac{p_B V_A}{nR} = \frac{202650 \cdot 1.34 \cdot 10^{-3}}{0.041 \cdot 8.314} K = 796.63\ K, \quad T_B = \frac{p_B V_B}{nR} = 392.37\ K$$

$B \rightarrow (2\,atm,\ 0.66\,l,\ 119.22\,°C) = (202650\,Pa,\ 6.6\cdot10^{-4}\,m^3,\ 392.37\,K)$

$A \rightarrow (2\,atm,\ 1.34\,l,\ 523.48\,°C) = (202650\,Pa,\ 1.34\cdot10^{-3}\,m^3,\ 796.63\,K)$ pre

3) $\Delta U_{TOT} = nc_V \int_0^A dT + nc_V \int_0^B dT = nc_V (T_A - T_0 + T_B - T_0) = nc_V (T_A + T_B - 2T_0)$

$\Delta U_T = 0.041 \cdot \frac{3}{2} R \cdot (796.63 + 392.37 - 2\cdot 300)\,J = 301.16\,J$

4) $P = \frac{\Delta E}{\Delta t} \rightarrow \Delta t = \frac{\Delta E}{P} = \frac{\Delta U_{TOT}}{P} = \frac{301.16}{1}\,\frac{J}{J/s} = 301.16\,s = 5\,m\,0.6\,s$

5)

1) $Q_g = -50\,J$

$P_B' V_B' = nRT_0$ $\quad P_B' = P_A'$, senza movimento $\rightarrow V = 3V_0$

$P_A' V_A' = nRT_A'$ $\quad \therefore 3p'V_0 = p'(V_B' + V_A') = nR(T_0 + T_A')$ ✓

Variazione spontanea $\Rightarrow \Delta U = -50\,J = Q$

$(T_A' - T_A + T_0 - T_B) = \frac{Q}{nc_V} \rightarrow T_0 + T_A' = \frac{Q}{nc_V} + T_A + T_B$

$3p'V_0 = nR\left(\frac{Q}{nc_V} + T_A + T_B\right) \rightarrow p' = \frac{nR}{3V_0}\left(\frac{Q}{nc_V} + T_A + T_B\right) = 128433.06\,Pa$ ✓ $\simeq 1.26\,atm$

$p'V_B' = nRT_0 \rightarrow V_B' = 7.96\cdot10^{-4}\,m^3$ ✓

$V_A' = 3V_0 - V_B' = 2.2\cdot10^{-3}\,m^3$ ✓

$T_A' = \frac{p'V_A'}{nR} = 828.90\,K$ ✓