§ 0.1 Multi-Index Notation

In order to ease various calculations one can utilize more abstract index constructions. One of these is the *multi-index* notation, where instead of having an index $i \in \mathbb{N}$ or $j \in \mathbb{Z}$, one constructs a "vector" of indices, like $\alpha = (a_1, \cdots, a_n) \in \mathbb{N}^n$ or $\beta = (b_1, \cdots, b_n) \in \mathbb{Z}^n$.
This notation includes a set of operations on such multi-indexes, defined as follows

Theorem 0.1 (Operations on Multi-indexes)**.** *Given a multi-index $\alpha \in \mathbb{N}^n$, one can define the following operations on them*

$$\begin{aligned} |\alpha| &= \sum_{i=1}^{n} a_i \\ \alpha! &= \prod_{i=1}^{n} a_i! \end{aligned} \tag{1}$$

Given $x \in \mathbb{R}^n$ and the del operator ∂ one can also write

$$\begin{aligned} x^\alpha &= \prod_{i=1}^{n} x_i^{a_i} \\ \partial^\alpha &= \prod_{i=1}^{n} \partial_i^{a_i} = \frac{\partial^{|\alpha|}}{\partial x_1^{a_1} \cdots \partial x_n^{a_n}} = \frac{\partial^{|\alpha|}}{\partial x^\alpha} \end{aligned} \tag{2}$$