

## Homework Assignment #2

**Due date:** before class on Friday, 9/25

1. Suppose that a stock price is \$20 per share at current time. Consider a European call option which allows to buy 100 shares of the stock at \$21 per share at the end of three months and the premium (for 100 shares) of the option is \$100.00. The risk free rate is 12% per annum. Draw a diagram (at the time of purchase) illustrating how the profit from a short position in the option depends on the stock price at maturity of the option.
2. Consider a European call option and a put option on a stock each with a strike price of  $K = \$22$ . The price of call is  $C = \$4$  and the price of put is  $P = \$3$ . The 6-month risk free interest rate is  $r = 10\%$ . The current stock price is  $S_0 = \$20$ . Show how to create an arbitrage strategy and calculate the arbitrage traders profit (could be a function of  $S_T$ ).
3. A *butterfly spread* is a combination of three option positions with different strike prices on the same underlying asset. To create a butterfly spread, a trader purchases a European call option at \$10 with a low strike price \$20 and a second European call option at \$4 with a high strike price \$30, and sells two other European call options at \$6 with an intermediate strike price \$25.

- (a) Construct a table showing both the payoff and net profit at time of maturity as a function of  $S_T$  based on this strategy.

Price ranges	Payoff	Net Profit

- (b) Based on the table, construct a diagram showing the net profit as a function of  $S_T$ .
- (c) Which price  $S_T$  provides maximum profit? For what range of prices of the underlying asset would the butterfly spread lead to a loss?
4. Suppose that a stock price is \$10 per share at current time and at the end of three months the stock price may either move up to \$11 per share or down to \$9 per share. Consider a European call option which allows to buy the stocks at \$10.5 per share at the end of three months. The risk free rate is 12% per annum.
  - (a) Find the fair price for the option.
  - (b) If the option can be bought or sold at \$0.8 per share, devise an investment strategy to show the existence of arbitrage.

5. Suppose that a binomial tree has  $n$  steps, and the stock has initial price  $S_0$  and then at each step, its price can only move up by a factor  $u$  or down by a factor  $d$ . Let  $S_k, k = 0, 1, \dots, n$ , be the price of the stock at the end of the  $k$ -th step. Denote by  $\tau$  time length between consecutive steps, and  $r$  the risk-free interest rate. Consider a call option with strike price  $K$  with maturity  $n\tau$ .

- (a) Show that  $S_k = S_0 u^X d^{k-X}$ , where  $X$  is equal to the number of times during the past  $k$  steps where the price moves up along the tree.
- (b) In the risk-neutral world, what is the probability that the stock moves down at each step?
- (c) For  $n = 3$ , calculate the fair price for the option at current time corresponding to the initial node of the tree.

Assume  $n = 10$ ,  $\tau = 1$ ,  $r = 6\%$ ,  $S_0 = 100$ ,  $u = 1.1$ ,  $d = 0.9$ ,  $K = 110$ .

- (d) In the risk-neutral world, find the probability that the stock price moves up three times and down four times in the first seven steps. What is the corresponding price value?
- (e) Write a R code to evaluate the fair price for the option at current time.