

## Homework Assignment #5

**Due date:** before class on Wednesday, 11/18.

1. Consider the GARCH(1,1) model,

$$a_t = \sigma_t \epsilon_t \quad \text{and} \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2,$$

where  $\epsilon_t \sim_{iid} N(0, 1)$ ,  $\epsilon_t$  and  $\sigma_t$  are independent for all  $t = 1, \dots, T$ . Assume model stationary.

- (a) Calculate  $E(a_t)$ .
- (b) Calculate  $\text{Var}(a_t^2)$ .
- (c) Show that the kurtosis of  $a_t$  is given by

$$\frac{Ea_t^4}{(Ea_t^2)^2} = \frac{3[1 - (\alpha_1 + \beta_1)^2]}{1 - (\alpha_1 + \beta_1)^2 - 2\alpha_1^2}.$$

2. Consider the daily log returns of Apple (AAPL) stock based on adjusted prices from October 31, 2005 to October 31, 2015.

- (a) Is the expected value of  $r_t$  significant different from 0?
- (b) Are there any serial correlations in  $r_t$ ?
- (c) Build an appropriate Gaussian ARMA model to the  $r_t$  series. Test ARCH effect on the residuals by applying both Box.test statistics and F test statistics.
- (d) Build a Gaussian ARMA-GARCH model to the  $r_t$  series. Obtain the normal QQ-plot of the standardized residuals, and write down the fitted model. Is the model adequate? Why?
- (e) Build an ARMA-GARCH model with Student- $t$  innovations for the log return series. Perform model checking and write down the fitted model.
- (f) Plot estimated volatilities using the fitted ARMA-GARCH model with Student- $t$  innovations.
- (g) Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted ARMA-GARCH model with Student- $t$  innovations.
- (h) Fit an IGARCH(1,1) model for the log returns. Write down the fitted model. Is the IGARCH model adequate? Why?
- (i) Fit a TGARCH(1,1) model to the log returns. Write down the fitted model. Is the leverage effect statistically significant? Is the TGARCH model adequate? Why?
- (j) Fit a GARCH-M(1,1) model to the log returns. Write down the fitted model. Is the risk premium statistically significant? Is the GARCH-M model adequate? Why?