atoms, the three atoms whose inner products were greatest were used in the estimations, i.e., $R = 3$ in Eq. 10. The windows with the lowest sidelobes only give the lowest error variance at very favourable SNRs, at real-world SNRs the original Hann window still performs best at estimating the parameters of a single component signal.

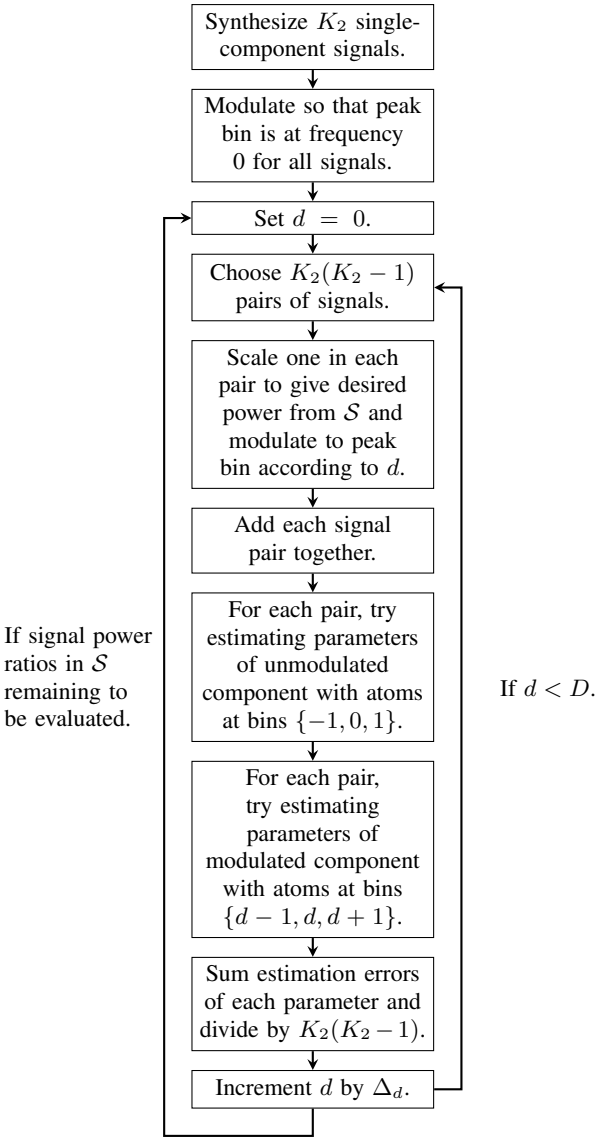

Figure 4: The evaluation procedure for 2-component signals.

7.2. Signals with 2 components

To evaluate the performance of the various windows when estimating the parameters of components in mixture we synthesized signals using Eq. 3 with $P = 2$ and $Q = 2$ and parameters chosen from the uniform distributions specified in [1]. We desired to see how the accuracy of estimation is influenced by the difference (in bins) between the locally maximized atoms and the difference in signal power between the two components. To obtain a set of components from which test signals exhibiting the desired differences could be constructed, we synthesized a set $\mathcal{C}$ of K_2 components for which the energy is maximized in bin 0. Test signals were obtained by choosing a pair of unique components from this set and modulating one to give the desired frequency and amplitude difference. This was carried out as follows: the atom r^* for which the inner product was maximized was determined for each unmixed chirp and the chirp was modulated by $\exp(-2\pi \frac{r^* n}{N} j)$ for $0 \leq n < N$ in order to move this maximum to $r = 0$. Then for each desired difference d, with $0 \leq d < D$ (for the evaluation $D = 40$), two unique chirps were selected from $\mathcal{C}$ and one chirp was modulated by $\exp(2\pi \frac{nd}{N} j)$ for $0 \leq n < N$ in order to give the desired difference between maxima. This component was also scaled by a constant to give a desired signal power ratio from set $\mathcal{S}$ with the other component (the power ratios $\mathcal{S}$ tested were 0 dB and -30 dB). As we assume perfect peak-atom selection for this evaluation no inner-product maximizing r^* is chosen, rather atoms with angular frequencies $\omega = 2\pi \frac{\hat{d}}{N}$ for $\hat{d} \in \{d-1, d, d+1\}$ in Eq. 20 (again, $R = 3$) were chosen to carry out the estimation. d was incremented by $\Delta_d = 0.25$ and so $\hat{d}$ was not generally integral valued in this case. The parameters of the unmodulated component were estimated using angular frequencies $\omega = 2\pi \frac{\hat{d}}{N}$ for $\hat{d} \in \{-1, 0, 1\}$ in Eq. 20. The squared estimation error for each parameter was summed and divided by $K_2(K_2-1)$ (the number of choices of two unique components) to give the averaged squared estimation error for each parameter at each difference d. The procedure is summarized in Fig. 4.

The behaviour of the windows when used to analyse mixtures of non-stationary signals is similar to the behaviour of windows used for harmonic analysis in the stationary case [16]; here we obtain further insight into how the estimation of each coefficient of the polynomial in Eq. 1 is influenced by main-lobe width and sidelobe height and slope. In Fig. 3 we see that there is generally less estimation error for components having similar signal power. This is to be expected as there will be less masking of the weaker signal in these scenarios. The estimation error is large when the atoms containing the most signal energy for each component are not greatly separated in frequency. This is due to the convolution of the Fourier transform of the window with the signal, and agrees with what was predicted by Eq. 17: indeed windows with a larger main lobe exhibit a larger "radius" (bandwidth) in which the error of the parameter estimation will be high. However, for signals where local inner-product maxima are from atoms sufficiently separated in frequency, windows with lower sidelobes are better at attenuating the other component and for these the estimation error is lowest.

8. CONCLUSIONS

Motivated by the need to analyse mixtures of frequency- and amplitude-modulated sinusoids (Eq. 3), we have shown that the DDM can be employed under a single-component assumption when components have roughly disjoint sets of atoms for which their inner products take on large values. This indicates the need for windows whose Fourier transform exhibits low sidelobes. We developed windows whose sidelobes are minimized while remaining everywhere once-differentiable: a requirement to generate valid atoms for the DDM. These windows were shown to only improve parameter estimation of $P = 1$ component with argument-polynomial of order $Q = 2$ in low amounts of noise. However, for $P = 2$ components of the same order in mixture without noise, granted the