ON THE DESIGN AND USE OF ONCE-DIFFERENTIABLE HIGH DYNAMIC RESOLUTION ATOMS FOR THE DISTRIBUTION DERIVATIVE METHOD

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ABSTRACT

The accuracy of the Distribution Derivative Method (DDM) [1] is evaluated on mixtures of chirp signals. It is shown that accurate estimation can be obtained when the sets of atoms for which the inner product is large are disjoint. This amounts to designing atoms with windows whose Fourier transform exhibits low sidelobes but which are once-differentiable in the time-domain. A technique for designing once-differentiable approximations to windows is presented and the accuracy of these windows in estimating the parameters of sinusoidal chirps in mixture is evaluated.

1. INTRODUCTION

Additive synthesis using a sum of sinusoids plus noise is a powerful model for representing audio [2], allowing for the easy implementation of many manipulations such as time-stretching [3] and timbre-morphing [4]. In these papers, [2–4] the phase evolution of the sinusoid is assumed linear over the analysis frame, only the phase and frequency of the sinusoids at these analysis points are used to fit a plausible phase function after some the analysis points are connected to form a partial [5]. Recently, there has been interest in using higher-order phase functions [6] as the estimation of their parameters has been made possible by a new set of techniques of only moderate computational complexity using signal derivatives [7]. The use of higher-order phase models allows for accurate description of highly modulated signals, for example in the analysis of birdsong [8]. The frequency modulation information has also been used in the regularization of mathematical programs for audio source separation [9].

The sinusoidal model approximating signal s typically considered is

$$\tilde{s}(t) = \exp(a_0 + \sum_{q=1}^{Q} a_q t^q) + \eta(t) \tag{1}$$

where $\tilde{s}$ is the approximating signal, t the variable of time, the $a_q \in \mathbb{C}$ coefficients of the argument's polynomial, and $\eta(t)$ white Gaussian noise. Although this technique can be extended to describe a single sinusoid of arbitrary complexity simply by increasing Q, it remains essential to consider signals featuring a sum of P such components, whether they represent the harmonic structure of a musical sound or the union of partials resulting from a mixture of multiple signal sources (e.g., recordings of multiple speakers or performers), i.e.,

$$x(t) = \sum_{p=1}^{P} x_p(t) + \eta(t) \tag{2}$$

with

$$x_p(t) = \exp(a_{p,0} + \sum_{q=1}^{Q} a_{p,q} t^q) \tag{3}$$

As regards the design and evaluation of signal-derivatives analysis techniques, previous work has generally assumed signals containing a single component, i.e., P = 1 or assumed the influence of other components to be negligible. Later we will refine when this assumption can be made. In [10] the authors provide a comprehensive evaluation of various signal-derivatives analysis methods applied to a single-component signal. In [11] the extent to which two components in mixture can corrupt estimations of the frequency slope ($\Im\{a_{0,2}\}$ and $\Im\{a_{1,2}\}$) is investigated in the context of the reassignment method, one of the signal-derivatives techniques, but the corruption of the other parameters is not considered.

In this paper, we revisit the quality of signal-derivatives estimation of *all* the a_q when analyzing a *mixture* of components. We focus on the DDM [1] analysis method for its convenience as it can simply be considered as an atomic decomposition (see Sec. 2), and does not require computing derivatives of the signal to be analysed.

The DDM does, however, require a once-differentiable analysis window. As we are interested in windows with lower sidelobes in order to better estimate parameters of sinusoidal chirp signals in mixture, we seek windows that combine these two properties. For this, a technique to design once-differentiable approximations to arbitrary symmetrical windows is proposed and presented along with a design example for a high-performance window. Finally we evaluate the performance of various once-differentiable windows in estimating the parameters a_q.

2. ESTIMATING THE PARAMETERS a_q

We will now show briefly how the DDM can be used to estimate the a_q. Based on the theory of distributions [12], the DDM makes use of "test functions" or atoms ψ. These atoms must be once differentiable with respect to time variable t and be non-zero only on a finite interval $[-\frac{L_t}{2}, \frac{L_t}{2}]$. First, we define the inner product

$$\langle x, \psi \rangle = \int_{-\infty}^{\infty} x(t)\overline{\psi}(t)dt \tag{4}$$

and the operator

$$\mathcal{T}^{\alpha} : (\mathcal{T}^{\alpha} x)(t) = t^{\alpha} x(t) \tag{5}$$

Consider the weighted signal

$$f(t) = x(t)\overline{\psi}(t) \tag{6}$$

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