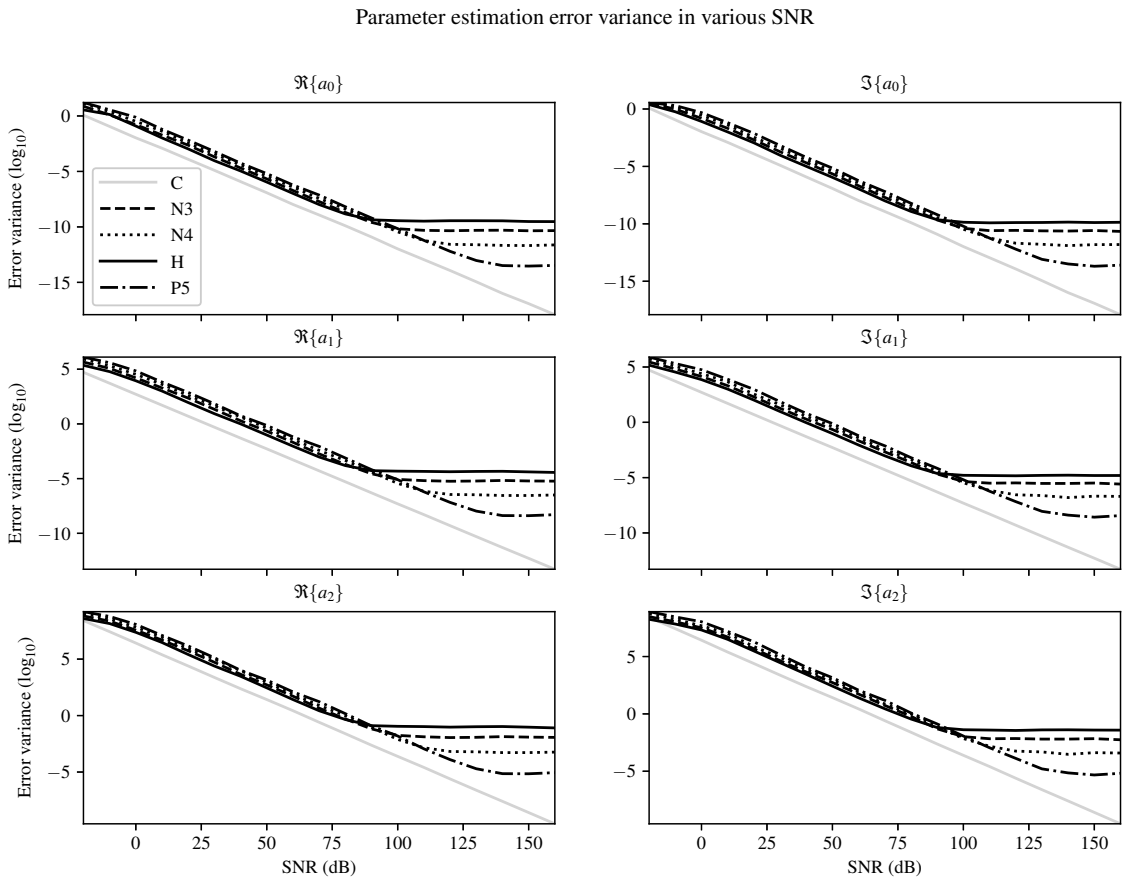

Figure 2: The estimation variance of random polynomial phase sinusoids averaged over $K_1 = 1000$ trials using atoms generated from various windows. *C* is the Cramér-Rao lower bound, *N3* and *N4* are the 3- and 4-cosine-term continuous Nuttall windows, *H* is the Hann window, and *P5* is the continuous 5-cosine-term approximation to a digital prolate window as described in Sec. 6.

Table 1: The coefficients of the once-differentiable approximation to a digital prolate window designed in Sec. 6.

$$\begin{aligned} b_0 &= 3.128 \times 10^{-1} \\ b_1 &= 4.655 \times 10^{-1} \\ b_2 &= 1.851 \times 10^{-1} \\ b_3 &= 3.446 \times 10^{-2} \\ b_4 &= 2.071 \times 10^{-3} \end{aligned}$$

resolution of 0.001. The window with the closest 3 dB bandwidth to the 4-term Nuttall window was obtained with $W = 0.008$. Its magnitude response is shown in Fig. 1. We see that this window's asymptotic falloff is 6 dB per octave and therefore has a discontinuity somewhere in its domain [15].

We designed an approximate window using Eq. 24 for M varying between 2 and $N/8$ to find the best approximation to the digital prolate window's main lobe using a small number of cosines. The M giving the best approximation was 5. The magnitude response of the approximation is shown in Fig. 1 and its coefficients are listed in Tab. 1; the temporal shape is very close to a digital prolate spheroidal window with $W = 0.008$ and is therefore omitted for brevity.

It is seen that a lower highest sidelobe level than the Nuttall and Prolate windows is obtained by slightly sacrificing the narrowness of the main lobe. More importantly, in Fig. 1 we observe that the falloff of the window is 18 dB per octave because it is once-differentiable at all points in its domain.

7. THE PERFORMANCE OF IMPROVED WINDOWS

7.1. Signals with single component

To compare the average estimation error variance with the theoretical minimum given by the Cramér-Rao bound we synthesized K_1 random chirps using Eq. 1 with $Q = 2$ and parameters chosen from uniform distributions justified in [1]. The original Hann window, the windows proposed by Nuttall and the new digital prolate based window were used to synthesize the atoms as described in Sec. 4 and their estimation error variance was compared (see Fig. 2). After performing the DFT to obtain inner products with the