INTRODUCTION TO ROBOTICS

03/06/24

Robot
- Mechanical Moving Parts.
- Electrical actuation.
- Some autonomy (usually sensing and some control actions based on the sensing - implemented through codes).

~~Common Ro~~ Common Robot Types -
1. Manipulators.
2. Mobile / Legged Robots.
3. Flying / Aerial Robots.

Types of Robots :

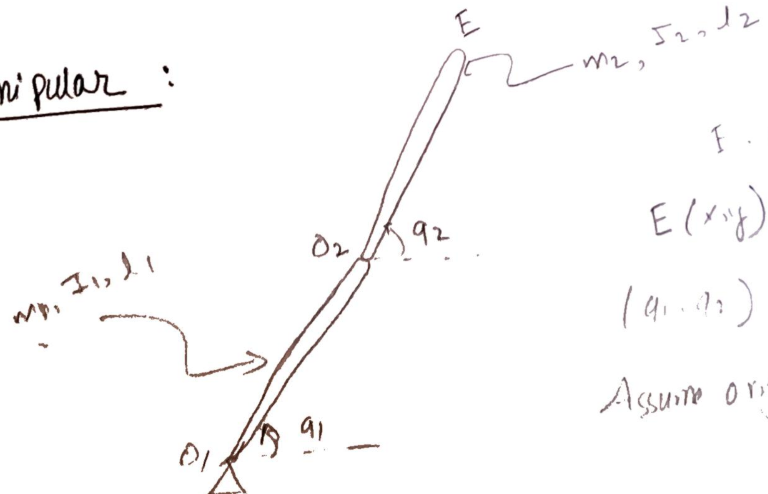
1. Manipulator.
   - Serial → limbs connected in series.
   - Parallel.
2. Mobile
3. Aerial
4. Higher-level robots.

Let us start with 2R manipulator (also called Planar Show Manipulator)

2R = 2 Revolute Joint.

Joints
- Revolute (R)
- Prismatic (P).

2R Manipular :



E : end effector

E(x, y) - end effector coordinates

(q_1, q_2) .. joint angles.

Assume origin is at O_1.

Let us assume that motors are connected to both joints O_1 and O_2 and we have the ability to control either the torques τ_1 and τ_2 applied at these joints or control the angles q_1 and q_2.

We will study later how (hardware, algorithm, software) we can control τ_1 and τ_2, or q_1 and q_2.

Let us consider 3 tasks.

<u>Task 1 (T_1)</u>: Given arbitrary trajectory of end effector. (given x, y function of time), make the robot follow the trajectory.

<u>Task 2 (T_2)</u>: Given a location of a wall → make the robot touch the wall and apply a constant predefined force on the wall.

<u>Task 3 (T_3)</u>: Make the robot behave like a virtual spring. (that has stiffness k and connects E to a specified point (x_0, y_0)) E(x, y).

Now,

$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2.$$

or using simplified notations.

$$\left.\begin{aligned} x &= l_1 cq_1 + l_2 cq_2 \\ y &= l_1 sq_1 + l_2 sq_2. \end{aligned}\right\} \text{— (1)}$$

FORWARD KINEMATICS.

Differentiating (1), we get -

$\cancel{\dot{x} = -l_1 s q_1 - l_2 s q_2}$

$$\dot{x} = -l_1 s q_1 \dot{q}_1 - l_2 s q_2 \dot{q}_2$$

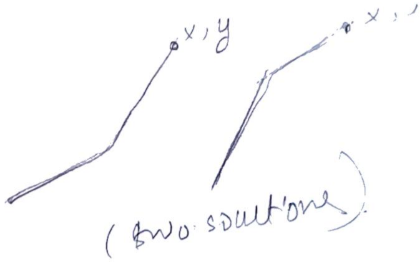
$$\dot{y} = l_1 c q_1 \dot{q}_1 + l_2 c q_2 \dot{q}_2$$

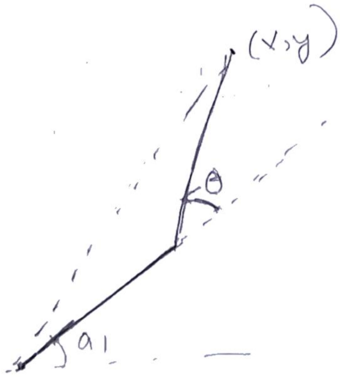
=> End-effector velocity:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & -l_2 s q_2 \\ l_1 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad - (2)$$

We will also need the reverse relationships. Given x and y, we need to be able to solve for q_1 and q_2.

Option 1 - solve numerically.
Option 2 - Derive a closed-form expression.
- Hard in general
- Multiple solutions. →





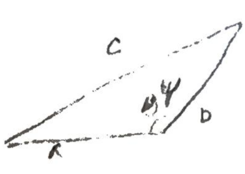
$\cancel{\theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)}$

$$\theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)$$

$$\cos\theta = \frac{x^2+y^2-l^2-l^2}{2l_1l_2}$$

$$2l_1l_2\cos\theta = x^2+y^2-l_1^2-l_2^2$$

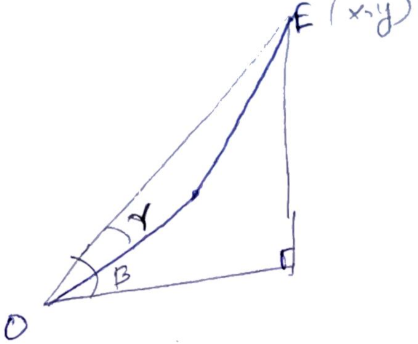
Pythagorous, Cosine Rule



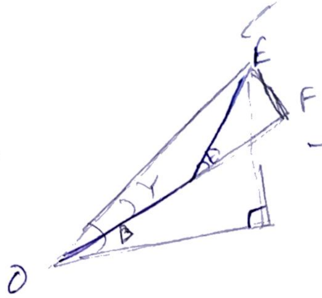
$$\boxed{c^2 = a^2+b^2-2ab\cos\psi}$$

← Cosine rule.

first $\theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right)$ [illegible] acute angle



$$q_1 = \beta - \gamma = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right)$$



$\rightarrow \tan^{-1}\left(\frac{EF}{OF}\right)$

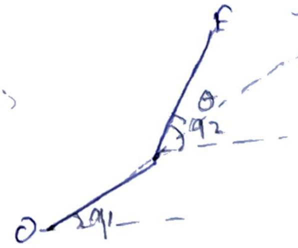
$$q_2 = q_1 + \theta.$$

$$\theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2 l_1 l_2}\right)$$

$$q_1 = \beta - \gamma = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right)$$

(3)

$$q_2 = q_1 + \theta$$

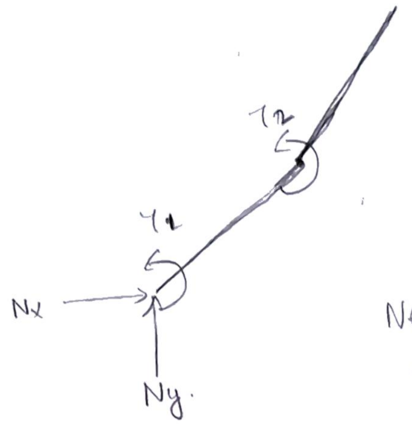


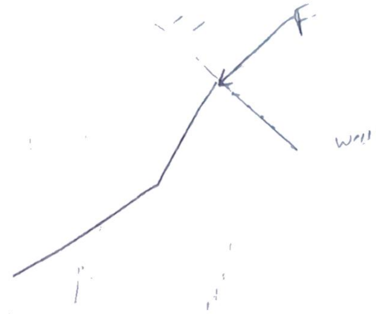
→ First-level answer to T1:
- control both motors in position control mode to achieve above q_1 and q_2 at each time step.

x_{d}, y_{d}
x_{1d}, q_{1d} — Later use

Later use these terminology for desired values.

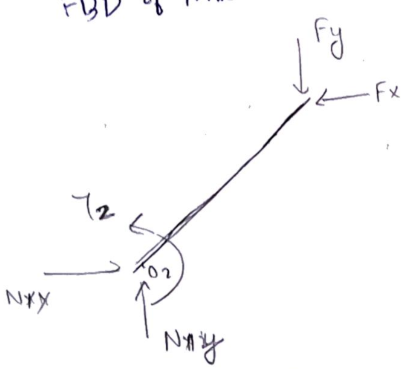
Task 2 : FBD of entire robot.

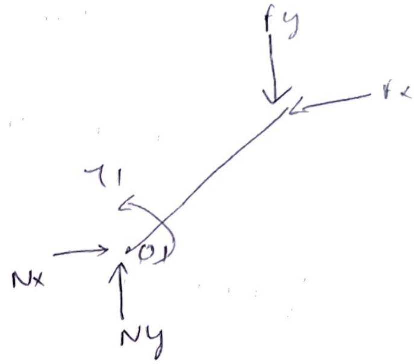




Neglect gravity for the moment.

FBD of link 2





Static equilibrium

$\Rightarrow \ \Sigma M_{o_2} = 0$ & $\Sigma M_{o_1} = 0$. (from FBD).

$$\boxed{\begin{aligned} F_y l_2 Cq_2 - F_x . l_2 Sq_2 &= \tau_2 \\ F_y l_1 Cq_1 - F_x l_1 Sq_1 &= \tau_1 \end{aligned}} \quad (4).$$

(3) along with (4) answers T2.

$$\begin{bmatrix} -l_2 Sq_2 & l_2 Cq_2 \\ -l_1 Sq_1 & l_1 Cq_1 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} \tau_2 \\ \tau_1 \end{bmatrix}$$

Dynamic effects:

Need to understand the dynamics

Lagrange's Equation:

K - KE
V - P.E

Lagrangian: $L = K - V$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i'}$$ (5).

q_i - independent degrees of freedom generalized coordinates.

Q_i' - generalized forces derived using principle of virtual work.

$i = 1, 2 \ldots$, no. of ~~DOF~~ DOF

KE

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{pure rotation of link } l_1} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2}_{\text{pure rotation of link } l_2} + \underbrace{\frac{1}{2} m_2 V_{c2}^2}_{\text{translation of } l_2}$$

$$V_{c2}^2 = (l_1\dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2}\dot{q}_2 C(q_2 - q_1).$$

$$V = m_1 g \frac{l_1}{2} S q_1 + m_2 g\left(l_1 S q_1 + \frac{l_2}{2} S q_2\right)$$

$L = K - V$

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 C(q_2 - q_1) - m_2 \frac{l_1 l_2}{2} \dot{q}_2(\dot{q}_2 - \dot{q}_1) S(q_2 - q_1)$$

$$+ m_1 g \frac{l_1}{2} C q_1 + m_2 g l_1 C q_1 = \tau_1$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + \frac{m_2 l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 C(q_2 - q_1)$$
$$- m_2 \frac{l_1 l_2}{2} \dot{q}_1(\dot{q}_2 - \dot{q}_1) S(q_2 - q_1) + m_2 g \frac{l_2}{2} S q_2 = \tau_2$$

(6)

(captures dynamic effects (equations of motion)

TASK. 3

(4) is valid for any end-effector forces f_x and f_y.

$F_x = Kx$

$F_y = Ky$.

[• more generally
$F_x = K(x - x_0)$
$F_y = K(y - y_0)$]

then, using (1) and (4):

$$\boxed{\begin{aligned} &K(l_1 sq_1 + l_2 sq_2)\, l_2 cq_2 - K(l_1 cq_1 + l_2 cq_2)\, l_2 sq_2 = \tau_{2s} \\ &K(l_1 sq_1 + l_2 sq_2)\, l_1 cq_1 - K(l_1 cq_1 + l_2 cq_2)\, l_1 sq_1 = \tau_{1s} \end{aligned}}$$

(7) Answer to T3

Q1: Write down equations (1) to equation (7) in a pdf. form.

Q2: code forward & reverse ~~bi~~ inverse kinematic in python & perform 3 simple tests. (animation end effector ...)