



# Time Series

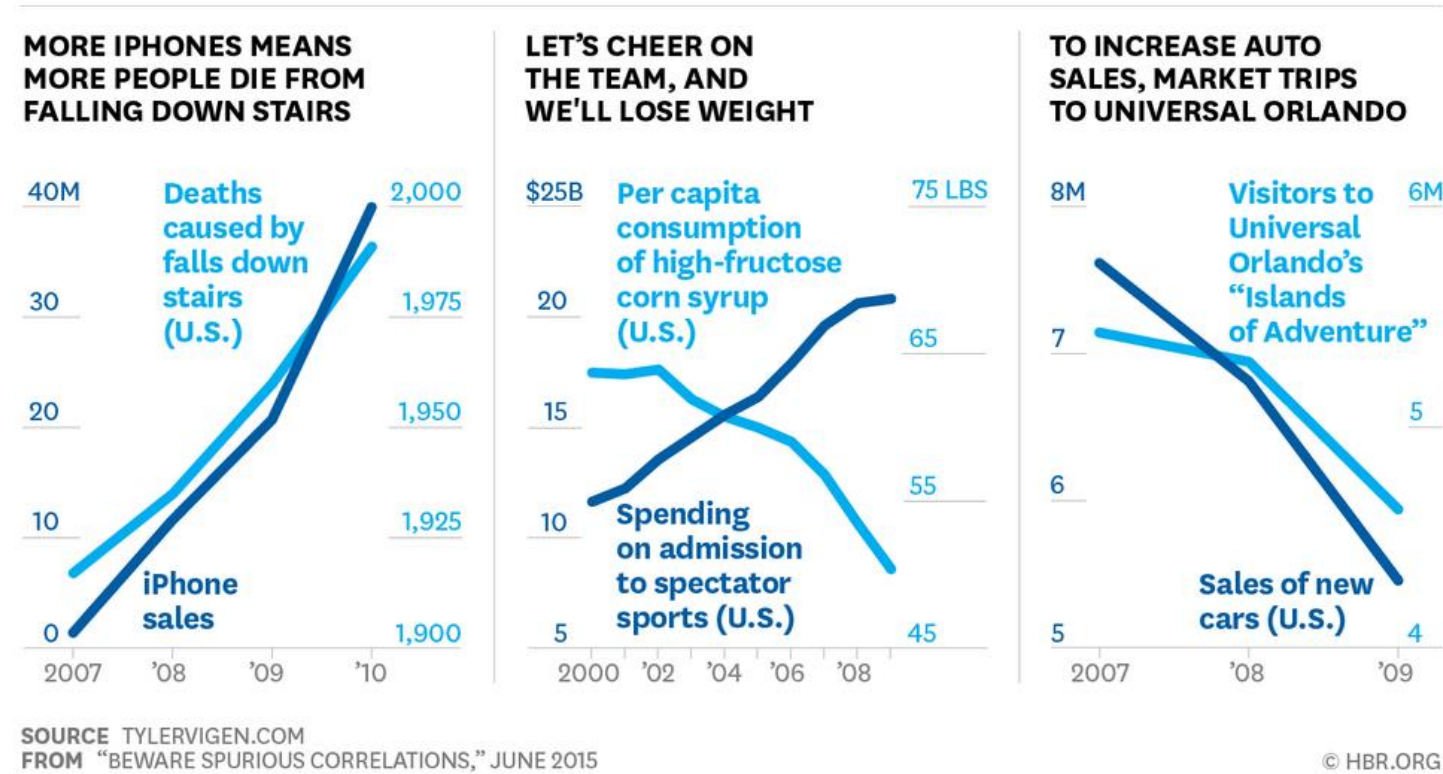
To infinity and beyond....

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# Objectives of this lecture

- Time Series data: Different data points represent different points in time
- This introduces some additional challenges
- We will discuss how to deal with those

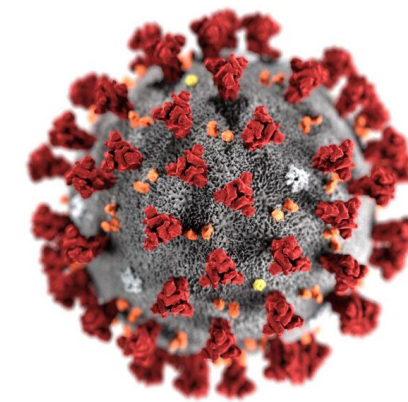
# What's the challenge of time series data?



Can you spot the problem?

Time becomes a confounding variable  
Non-stationary: characteristics of data vary with time

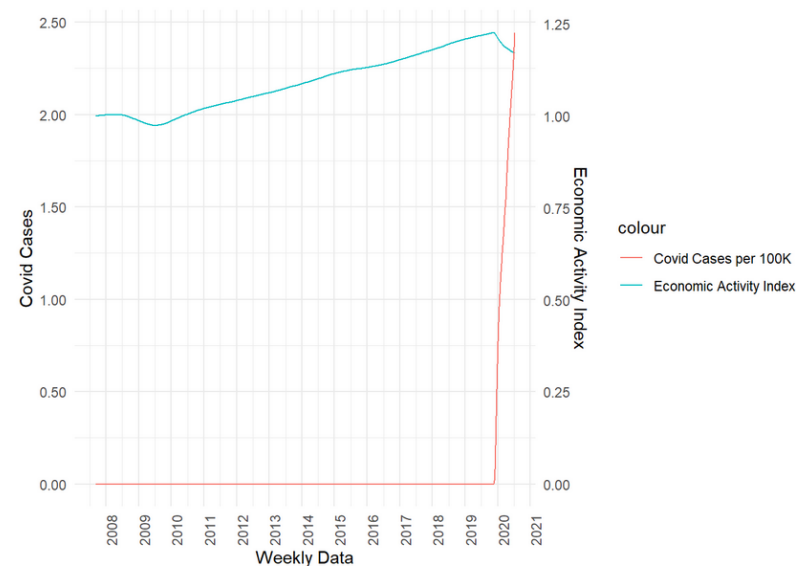
# COVID vs GDP



head(df)

```
##      week  WEI  Index cases deaths  lnindex lockshare
## 1 2008-01-05 1.42 1.00000    0      0 0.0000000000      0
## 2 2008-01-12 1.46 1.00028    0      0 0.0002799608      0
## 3 2008-01-19 1.40 1.00055    0      0 0.0005498488      0
## 4 2008-01-26 0.96 1.00073    0      0 0.0007297337      0
## 5 2008-02-02 0.73 1.00088    0      0 0.0008796130      0
## 6 2008-02-09 0.78 1.00103    0      0 0.0010294699      0
```

What you  
think is  
going to  
happen?



```
lm(lnindex~cases,df) %>% summary()
```

```
##
## Call:
## lm(formula = lnindex ~ cases, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.108375 -0.064942 -0.002043  0.055511  0.121388
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.082359   0.002731  30.156 < 2e-16 ***
## cases        0.050576   0.007800   6.484 1.74e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06928 on 669 degrees of freedom
## Multiple R-squared:  0.05913,    Adjusted R-squared:  0.05772
## F-statistic: 42.04 on 1 and 669 DF,  p-value: 1.736e-10
```

More COVID = more GDP?  
100K more = 5% more GDP?



# Taking control of time...with a timeline

```
df=df %>% mutate(t=1:n())  
lm(lnindex~cases+t,df) %>% summary()
```

```
##  
## Call:  
## lm(formula = lnindex ~ cases + t, data = df)  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max   
## -0.024859 -0.004965 -0.001175  0.003861  0.038124   
##  
## Coefficients:  
##              Estimate Std. Error t value Pr(>|t|)      
## (Intercept) -3.850e-02  9.170e-04  -41.98  <2e-16 ***  
## cases       -2.262e-02  1.393e-03  -16.23  <2e-16 ***  
## t           3.752e-04  2.466e-06   152.11  <2e-16 ***  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 0.01161 on 668 degrees of freedom  
## Multiple R-squared:  0.9736, Adjusted R-squared:  0.9735   
## F-statistic: 2 and 668 DF,  p-value: < 2.2e-16
```

100k more  
cases = 2.2%  
lower GDP



# What if time is not linear?

- Seasonal effects
- Recessions
- Natural disasters
- Political turmoil
- War
- Pandemic

## Panel data to the rescue

```
head(statsbyweek %>% arrange(state, week))
```

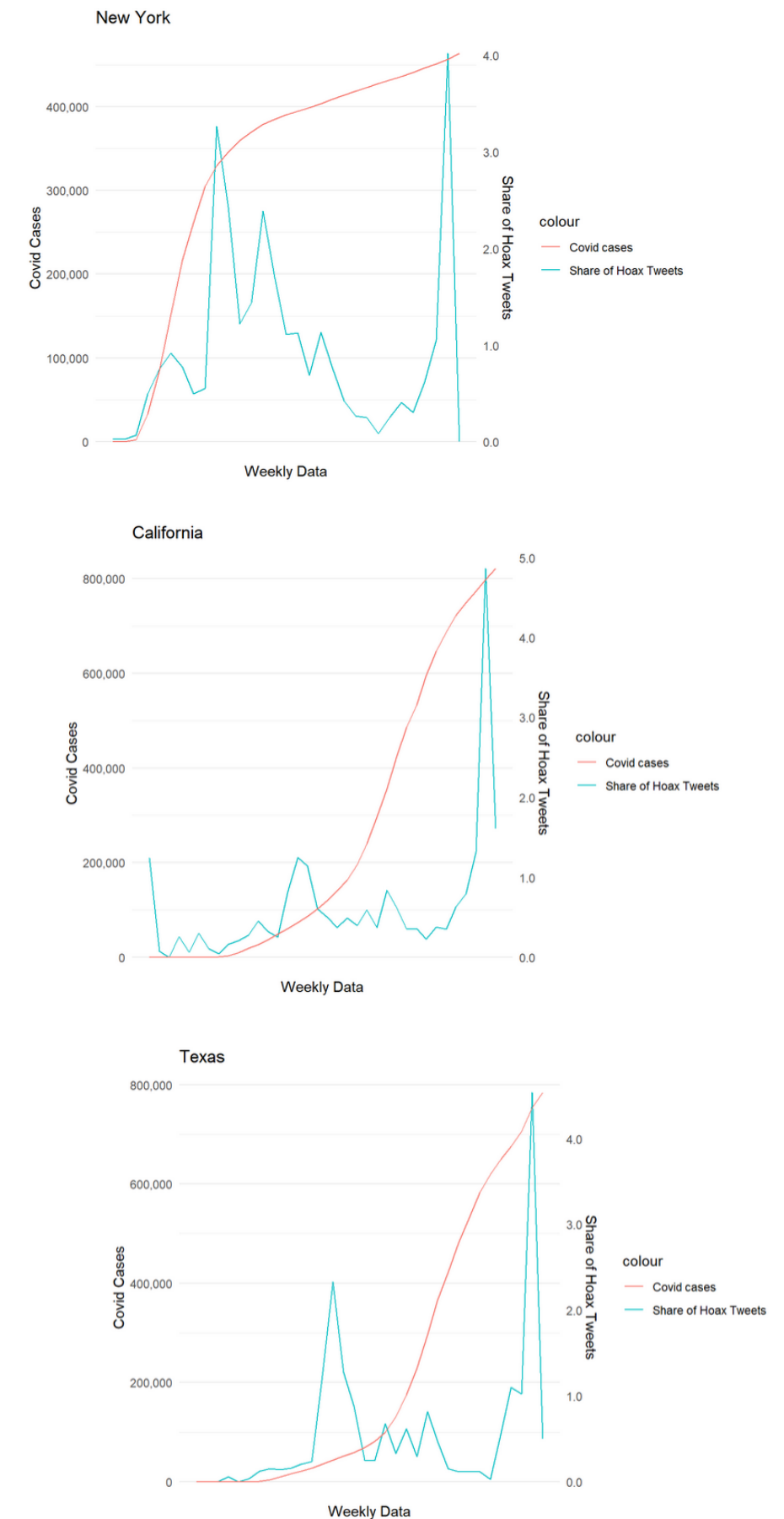
```
## # A tibble: 6 x 9
## # Groups:   state [1]
##   state   week      hoax tweets cases deaths hoaxsh Dcases Ddeaths
##   <chr> <date>    <int> <int> <int> <int> <dbl> <int> <int>
## 1 Alabama 2020-03-15      4  1503    51      0  0.266    NA     NA
## 2 Alabama 2020-03-22     62  4198   386      1  1.48    335      1
## 3 Alabama 2020-03-29     14  5218  1108     28  0.268    722     27
## 4 Alabama 2020-04-05     12  4793  2498     67  0.250   1390     39
## 5 Alabama 2020-04-12      9  4486  4241    123  0.201   1743     56
## 6 Alabama 2020-04-19      6  3570  5610    201  0.168   1369     78
```

```
statsbyweek %>% group_by(state) %>% summarise(n())
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
## # A tibble: 50 x 2
##   state   `n()`
##   <chr>   <int>
## 1 Alabama    29
## 2 Alaska     29
## 3 Arizona    36
## 4 Arkansas   30
## 5 California  36
## 6 Colorado   30
## 7 Connecticut 30
## 8 Delaware   30
## 9 Florida    31
## 10 Georgia   31
## # ... with 40 more rows
```

Multiple periods  
for the same cross  
section unit





# Panel data example

```
lm(cases~hoaxsh,statsbyweek) %>% summary()
```

```
##
## Call:
## lm(formula = cases ~ hoaxsh, data = statsbyweek)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -189328  -50914  -40048   7176   7176
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    50929      3108  16.388 < 2e-16 ***
## hoaxsh         11555      2380   4.855 1.33e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 108700 on 1544 degrees of freedom
## Multiple R-squared:  0.01504,    Adjusted R-squared:  0.0144
## F-statistic: 23.57 on 1 and 1544 DF,  p-value: 1.326e-06
```

Hoax share up by 1 percentage point means 11555 more cases

```
lm(cases~hoaxsh+factor(week),statsbyweek) %>% summary()
```

```
##
## Call:
## lm(formula = cases ~ hoaxsh + factor(week), data = statsbyweek)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -199861  -37318  -9820   1098  668461
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)         1.00    99956.85   0.000  0.99999
## hoaxsh              7865.20    2593.18   3.033  0.00246 **
## factor(week)2020-01-26 -2439.83    111758.05 -0.022  0.98259
## factor(week)2020-02-02  -95.32    107965.74 -0.001  0.99930
## factor(week)2020-02-09   1.00    106858.37  0.000  0.99999
## factor(week)2020-02-16 -254.24    106020.28 -0.002  0.99809
## factor(week)2020-02-23  -50.25    105363.77  0.000  0.99962
## factor(week)2020-03-01 -1014.28    102855.30 -0.010  0.99213
## factor(week)2020-03-08  -70.35    101086.35 -0.001  0.99944
## factor(week)2020-03-15 -133.49    100951.52 -0.001  0.99895
```

Smaller effect when controlling for time (week) effects

```
lm(cases~hoaxsh++factor(state)+factor(week),statsbyweek) %>% summary()
```

```
##
## Call:
## lm(formula = cases ~ hoaxsh + +factor(state) + factor(week),
##      data = statsbyweek)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -264367  -23041    593   22221  456332
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)         -1002      70567  -0.014  0.988669
## hoaxsh              3788      1863   2.033  0.042192 *
## factor(state)Alaska  -52626     17974  -2.928  0.003465 **
## factor(state)Arizona   41125     17193   2.392  0.016885 *
## factor(state)Arkansas -24748     17843  -1.387  0.165651
## factor(state)California 223775     17186  13.021 < 2e-16 ***
## factor(state)Califor 17833    -1.107  0.268403
```

Also controlling for state

```
library(plm)
```

```
plm(cases~hoaxsh+factor(week)+factor(state),statsbyweek,
    index=c("state","week"),
    model="within",
    effect="twoways") %>% summary()
```

```
## Twoways effects Within Model
##
## Call:
## plm(formula = cases ~ hoaxsh + factor(week) + factor(state),
##      data = statsbyweek, effect = "twoways", model = "within",
##      index = c("state", "week"))
##
## Unbalanced Panel: n = 50, T = 29-37, N = 1546
##
## Residuals:
##      Min.      1st Qu.      Median      3rd Qu.      Max.
## -264367.42 -23040.53    592.79   22221.48  456331.70
##
## Coefficients:
##              Estimate Std. Error t-value Pr(>|t|)
## hoaxsh       3788.3      1863.0   2.0334  0.04219 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Total Sum of Squares:    6.8535e+12
## Residual Sum of Squares: 6.8341e+12
## R-Squared:      0.0028259
## Adj. R-Squared: -0.055952
## F-statistic: 4.13474 on 1 and 1459 DF, p-value: 0.042192
```

Alternative command to include cross sectional and time effects in panel data  
Substantially more efficient with large datasets (many cross sectional units)

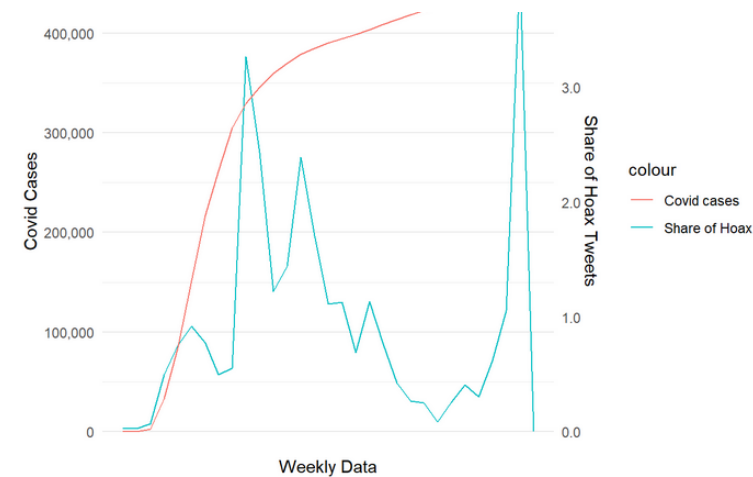
# Autoregression

- A particular concern in time series is the possibility that observations are correlated over time

- Simplest way to model this is via an Auto regression:

- $$Y_t = \beta_0 + \beta_1 Y_{t-1} + \epsilon_t$$

$Y_{t-1}$  becomes the X variable  
We can do normal OLS as long as  $-1 < \rho < 1$



- With  $\rho = 1$  we have non-stationarity because of path dependence
- The series can wander off into any direction and never come back
- If that happens OLS is no longer un-biased (different observations are too related to each other)
- Also: if you are interested in  $Y = \beta X$  and both Y and X have unit roots you will have a spurious correlation (the unit root becomes the confounder)
- Random Walk
- Of course we don't know if this is the case in our data before we start any analysis





# Dickey-Fuller test to the rescue



Rewrite original model by subtracting  $Y_{t-1}$  on both sides of the model equation:

$$\begin{aligned} Y_t &= \beta_0 + \beta Y_{t-1} + \epsilon_t \\ &\Downarrow \\ Y_t - Y_{t-1} &= \Delta Y_t = \beta_0 + \underbrace{(\beta - 1)}_{=\delta} Y_{t-1} + \epsilon_t \end{aligned}$$

Testing for a random walk (aka unit root) now boils down to

H0:  $\delta=0$

H1:  $\delta<0$  i.e. stationary process

- We cannot just compare the implied test statistic to a normal t-table
- Luckily R will help us

# R to the rrrrrrescue

```
library(urca)
```

```
## Warning: package 'urca' was built under R version 4.0.2
```

```
ur.df(df$cases,type="none",lags=1) %>% summary()
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression none
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.02151  0.00000  0.00000  0.00000  0.07106
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1      0.0004516  0.0006910   0.654   0.514
## z.diff.lag  0.9805725  0.0133827  73.272 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.004085 on 667 degrees of freedom
## Multiple R-squared:  0.9468, Adjusted R-squared:  0.9467
## F-statistic: 5938 on 2 and 667 DF, p-value: < 2.2e-16
##
##
## Value of test-statistic is: 0.6536
##
## Critical values for test statistics:
##      1pct   5pct  10pct
## tau1 -2.58 -1.95 -1.62
```

We cannot reject unit root  
because  $0.653 > -1.95$

```
ur.df(df$lnindex,type="none",lags=1) %>% summary()
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression none
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -8.343e-04 -2.643e-05  4.240e-06  4.131e-05  1.922e-04
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1      -2.155e-05  2.703e-05  -0.797   0.426
## z.diff.lag   9.924e-01  5.596e-03 177.334 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.414e-05 on 667 degrees of freedom
## Multiple R-squared:  0.9812, Adjusted R-squared:  0.9811
## F-statistic: 1.739e+04 on 2 and 667 DF, p-value: < 2.2e-16
##
##
## Value of test-statistic is: -0.7971
##
## Critical values for test statistics:
##      1pct   5pct  10pct
## tau1 -2.58 -1.95 -1.62
```

We cannot reject unit root  
because  $-0.7971 > -1.95$

# Getting rid of unit roots

```
ur.df(diff(df$cases,1),type="none",lags=1) %>% summary()
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.02330  0.00000  0.00000  0.00000  0.04392
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1      -0.036593   0.007388  -4.953 9.26e-07 ***
## z.diff.lag    0.604696   0.031607  19.132 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.003285 on 666 degrees of freedom
## Multiple R-squared:  0.3567, Adjusted R-squared:  0.3547
## F-statistic: 184.6 on 2 and 666 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -4.9534
##
## Critical values for test statistics:
##      1pct   5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

- Differencing:  $\Delta y_t = y_t - y_{t-1}$
- Checking that differenced series is not unit root

We cannot reject unit root  
because  $-4.9534 < -1.95$



# Getting rid of unit roots – Economic Activity index

```
ur.df(diff(df$lnindex,1),type="none",lags=1) %>% summary()
```

```
##
## #####
## # Augmented Dickey-Fuller Test Unit Root Test #
## #####
##
## Test regression none
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -8.236e-04 -3.079e-05  3.980e-06  4.133e-05  1.963e-04
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1      -0.010977   0.005233  -2.098   0.0363 *
## z.diff.lag    0.195464   0.038082   5.133 3.75e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.28e-05 on 666 degrees of freedom
## Multiple R-squared:  0.04215,    Adjusted R-squared:  0.03927
## F-statistic: 14.65 on 2 and 666 DF,  p-value: 5.92e-07
##
##
## Value of test-statistic is: -2.0976
##
## Critical values for test statistics:
##      1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

We can reject unit root (at least at 5%)

# Revisiting COVID vs GDP

```
df=df %>% arrange(week) %>% mutate(Dlnindex=lnindex-dplyr::lag(lnindex),  
                                   Dcases=cases-dplyr::lag(cases) ,  
                                   DDLnindex=Dlnindex-dplyr::lag(Dlnindex))  
  
lm(Dlnindex~Dcases+t,df) %>% summary()
```

```
##  
## Call:  
## lm(formula = Dlnindex ~ Dcases + t, data = df)  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max   
## -1.107e-03 -9.941e-05  4.439e-05  1.487e-04  1.041e-03   
##  
## Coefficients:  
##              Estimate Std. Error t value Pr(>|t|)      
## (Intercept)  1.429e-04  2.415e-05   5.918 5.20e-09 ***  
## Dcases      -2.316e-02  7.258e-04 -31.914 < 2e-16 ***  
## t           5.269e-07  6.490e-08   8.119 2.28e-15 ***  
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 0.000305 on 667 degrees of freedom  
## (1 observation deleted due to missingness)  
## Multiple R-squared:  0.6053  
## F-statistic: 1000.0 on 2 and 667 df, p-value: < 2.2e-16
```

100k more cases = 2.3% lower  
GDP...similar to what we had  
before....but of course we didn't  
know that would happen

# Summary

- Time series can be easy
- But you need to worry about how stationary your series is
- If the series clearly grows or shrinks continuously definitely include a time trend
- However, even if it doesn't grow (or shrink) the series might contain a unit root
- If that's the case a time trend is not enough
- Use the Dickey Fuller Test to make sure you are dealing with a stationary series





# Extra Slides

# Other considerations

```
lm(Dlnindex~Dcases+t+Dlockshare,df) %>% summary()
```

```
##
## Call:
## lm(formula = Dlnindex ~ Dcases + t + Dlockshare, data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.0011149 -0.0001001  0.0000414  0.0001472  0.0010273
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.401e-04  2.404e-05   5.831 8.61e-09 ***
## Dcases       -2.311e-02  7.221e-04 -32.010 < 2e-16 ***
## t             5.400e-07  6.471e-08   8.345 4.10e-16 ***
## Dlockshare  -1.253e-05  4.354e-06  -2.878  0.00412 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0003034 on 666 degrees of freedom
## (1 observation deleted due to missingness)
## Multiple R-squared:  0.6095
## Adjusted R-squared:  0.6095
## F-statistic: 34.00 on 3 and 666 df, p-value: < 2.2e-16
```

- If 100% of US population go into lockdown GDP goes down by -0.138% (seems low..more research needed)

## More lags AR(2)?

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + u_t.$$

Stationarity now requires

$$\beta_1 + \beta_2 < 1$$

while

$$\beta_1 + \beta_2 = 1$$

$$Y_t - Y_{t-1} = \beta_0 + (\beta_1 + \beta_2 - 1)Y_{t-1} - \beta_2(Y_{t-1} - Y_{t-2}) + \epsilon_t$$

We can test this again using the coefficient on  $Y_{t-1}$

## More lags and trend?

$$Y_t - Y_{t-1} = \beta_0 + (\beta_1 + \beta_2 - 1)Y_{t-1} - \beta_2(Y_{t-1} - Y_{t-2}) + \rho t + \epsilon_t$$



# Some remarks on Causality and unit roots

- $X$  causes  $Y$  then both need to be integrated in the same order
- i.e. if  $X$  has a unit root  $Y$  has a unit root as well
- If  $Y$  has a unit root but not  $X$  then  $X$  can (potentially) have a causal effect on  $\Delta Y$
- If  $X$  has a unit root but not  $Y$  we should be looking for a causal effect of  $\Delta X$  on  $Y$



# More lags

```
lm(Dlnindex~dplyr::lag(Dlnindex)+dplyr::lag(Dlnindex,2)+Dcases+dplyr::lag(Dcases)+dplyr::lag(Dcases,2)+t+Dlockshare+dplyr::lag(Dlockshare)+dplyr::lag(Dlockshare,2),df) %>% summary()
```

```
##
## Call:
## lm(formula = Dlnindex ~ dplyr::lag(Dlnindex) + dplyr::lag(Dlnindex,
##      2) + Dcases + dplyr::lag(Dcases) + dplyr::lag(Dcases, 2) +
##      t + Dlockshare + dplyr::lag(Dlockshare) + dplyr::lag(Dlockshare,
##      2), data = df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.264e-04 -3.238e-05 -2.604e-06  3.437e-05  1.893e-04
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    1.374e-06  4.565e-06   0.301   0.7634
## dplyr::lag(Dlnindex)  7.911e-01  3.793e-02  20.855 < 2e-16 ***
## dplyr::lag(Dlnindex, 2)  1.854e-01  3.743e-02   4.954 9.27e-07 ***
## Dcases        -7.495e-04  1.100e-03  -0.681   0.4960
## dplyr::lag(Dcases)   -3.192e-03  1.444e-03  -2.210   0.0275 *
## dplyr::lag(Dcases, 2)   3.703e-03  7.344e-04   5.043 5.94e-07 ***
## t              2.186e-08  1.270e-08   1.721   0.0857 .
## Dlockshare      -1.036e-05  8.940e-07 -11.586 < 2e-16 ***
## dplyr::lag(Dlockshare) -9.179e-06  1.167e-06  -7.867 1.49e-14 ***
## dplyr::lag(Dlockshare, 2) -3.171e-06  1.449e-06  -2.189   0.0290 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.585e-05 on 658 degrees of freedom
## (3 observations deleted due to missingness)
## Multiple R-squared:  0.987, Adjusted R-squared:  0.9868
## F-statistic: 5546 on 9 and 658 DF, p-value: < 2.2e-16
```