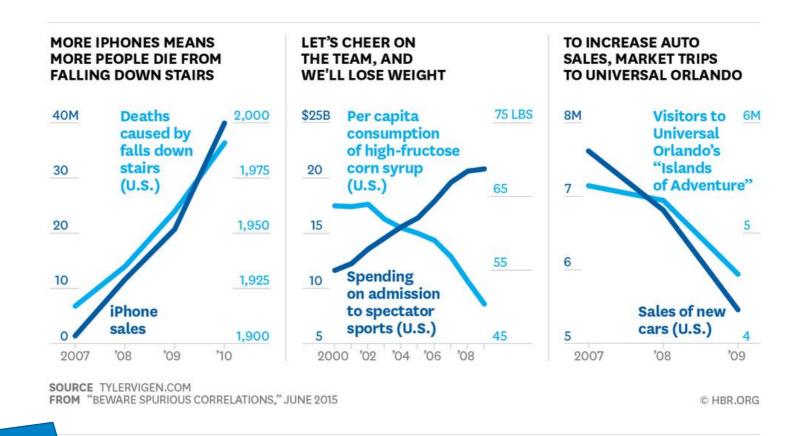


Objectives of this lecture

- Time Series data: Different data points represent different points in time
- This introduces some additional challenges
- We will discuss how to deal with those

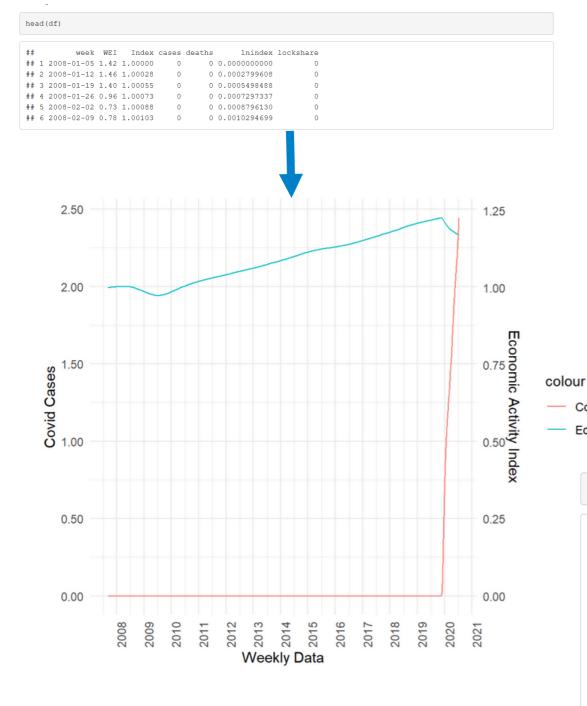
What's the challenge of time series data?



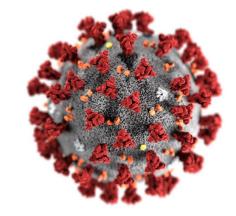
Can you spot the problem?

Time becomes a confounding variable Non-stationary: characteristics of data vary with time

COVID vs GDP



What you think is going to happen?



```
Economic Activity Index
```

lm(lnindex~cases,df) %>% summary()

Covid Cases per 100K

```
##
## Call:
## lm(formula = lnindex ~ cases, data = df)
##
## Residuals:
                  1Q Median
                                     3Q
                                                                      More COVID = more GDP?
## -0.108375 -0.064942 -0.002043 0.055871 0.121388
                                                                    100K more = 5% more GDP?
##
## Coefficients:
             Estimate Std. Error t value Pr(>|t|
## (Intercept) 0.082359 0.002731 30.156
             0.050576 0.007800
                                 6.484 1.74e-10 ***
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.06928 on 669 degrees of freedom
## Multiple R-squared: 0.05913, Adjusted R-squared: 0.05772
## F-statistic: 42.04 on 1 and 669 DF, p-value: 1.736e-10
```

Taking control of time....with a timeline

```
df=df %>% mutate(t=1:n())
lm(lnindex~cases+t,df) %>% summary()
```

```
##
## Call:
## lm(formula = lnindex ~ cases + t, data = df)
## Residuals:
        Min
                 1Q Median
                                               Max
## -0.024859 -0.004965 -0.001175 0.003861 0.038124
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.850e-02 9.170e-04 -41.98 <2e-16 ***
          -2.262e-02 1.393e-03 -16.23 <2e-16 ***
             3.752e-04 2.466e-06 152.11 <2e-16 ***
## t
## Signif. c s: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
                  error: 0.01161 on 668 degrees of freedom
## Residual
## Multiple
                       2.9736, Adjusted R-squared: 0.9735
## F-statist:
                           2 and 668 DF, p-value: < 2.2e-16
```

100k more cases = 2.2% lower GDP



What if time is not linear?

- Seasonal effects
- Recessions
- Natural disasters

- Political turmoil
- War
- Pandemic

Panel data to the rescue

```
head(statsbyweek %>% arrange(state,week))
```

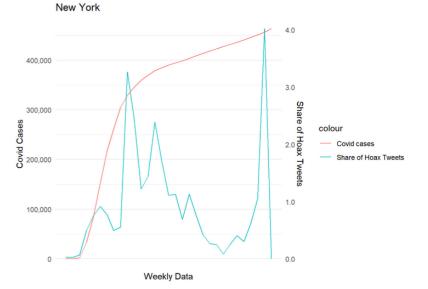
```
A tibble: 6 x 9
    Groups:
              state [1]
     state
            week
                        hoax tweets cases deaths hoaxsh Dcases Ddeaths
     <chr>
             <date>
                               <int> <int>
                                                  <dbl>
    Alabama 2020-03-15
                               1503
                                                0 0.266
                                       51
                                                             NA
                                                                     NA
    Alabama 2020-03-22
                          62
                                4198
                                      386
                                                1 1.48
                                                            335
                                                                      1
    Alabama 2020-03-29
                          14
                               5218
                                     1108
                                                  0.268
                                                            722
                                                                     27
    Alabama 2020-04-05
                                4793 2498
                                               67 0.250
                                                           1390
                                                                     39
                          12
## 5 Alabama 2020-04-12
                               4486
                                     4241
                                              123
                                                  0.201
                                                           1743
                                                                     56
## 6 Alabama 2020-04-19
                                                                     78
                               3570 5610
                                              201 0.168
                                                           1369
```

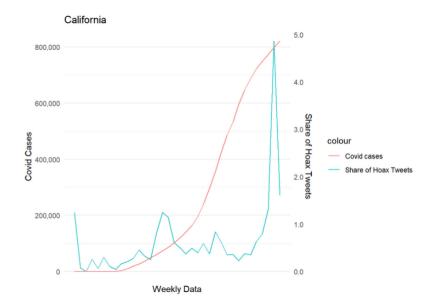
```
statsbyweek %>% group_by(state) %>% summarise(n())
```

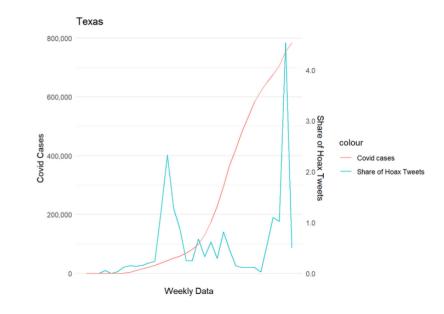
```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
## # A tibble: 50 x 2
      state
                   `n() `
      <chr>
                   <int>
   1 Alabama
                      29
    2 Alaska
                      29
    3 Arizona
                      36
    4 Arkansas
                      30
    5 California
                      36
    6 Colorado
                      30
    7 Connecticut
                      30
    8 Delaware
                      30
                      31
   9 Florida
## 10 Georgia
## # ... with 40 more rows
```

Multiple periods for the same cross section unit







Panel data example

lm(cases~hoaxsh,statsbyweek) %>% summary()

Hoax share up by 1 percentage point means 11555 more cases

```
## Call:
## lm(formula = cases ~ hoaxsh, data = statsbywe
## Residuals:
               10 Median
      Min
  -189328 -50914 -40048
                              7176
## Coefficients:
              Estimate Std.
                              rror t value Pr(>|t|)
                             3108 16.388 < 2e-16 ***
                  50929
## (Intercept)
## hoaxsh
                  11555
                                    4.855 1.33e-06 ***
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 108700 on 1544 degrees of freedom
## Multiple R-squared: 0.01504,
                                   Adjusted R-squared: 0.0144
## F-statistic: 23.57 on 1 and 1544 DF, p-value: 1.326e-06
```

```
lm(cases~hoaxsh+factor(week),statsbyweek) %>% summary()
```

```
##
## Call:
## lm(formula = cases ~ hoaxsh + factor(wee
                                              Smaller effect when
## Residuals:
                                              controlling for time
      Min
               10 Median
                               30
                                                  (week) effects
## -199861 -37318 -9820
                            1098 668461
## Coefficients:
                          Estimate Std.
                                          or t value Pr(>|t|)
                             1.00
                                     9956.85
                                               0.000 0.99999
## (Intercept)
## hoaxsh
                           7865.20
                                     2593.18
                                               3.033 0.00246 **
## factor(week) 2020-01-26 -2439.83 111758.05
                                             -0.022 0.98259
## factor(week) 2020-02-02
                           -95.32
                                  107965.74
                                              -0.001
                                                     0.99930
## factor(week) 2020-02-09
                           1.00 106858.37
                                               0.000 0.99999
## factor(week)2020-02-16
                           -254.24 106020.28
                                             -0.002 0.99809
## factor(week) 2020-02-23
                           -50.25 105363.77
                                               0.000 0.99962
## factor(week)2020-03-01 -1014.28 102855.30 -0.010 0.99213
## factor(week) 2020-03-08
                           -70.35 101086.35 -0.001 0.99944
## factor(week)2020-03-15 -133.49 100951.52 -0.001 0.99895
```

```
lm(cases~hoaxsh++factor(state)+factor(week), statsbyweek) %>% summary()
```

```
##
## Call:
## lm(formula = cases ~ hoaxsh + +factor(state) + factor(week),
      data = statsbyweek)
##
## Residuals:
     Min
             10 Median
                             30
## -264367 -23041 593
                          22221 456332
## Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                            -1002 70567 -0.014 0.988669
## hoaxsh
                             3788
                                         1863 2.033 0.042192 *
## factor(state)Alaska
                                        17974 -2.928 0.003465 **
                             -52626
## factor(state)Arizona
                             41125
                                        17193 2.392 0.016885 *
## factor(state)Arkansas
                              -24748
                                        17843 -1.387 0.165651
## factor(s
                             223775
                                        17186 13.021 < 2e-16 ***
           California
## factor(st
                             -19744
                                        17833 -1.107 0.268403
```

Also controlling for state

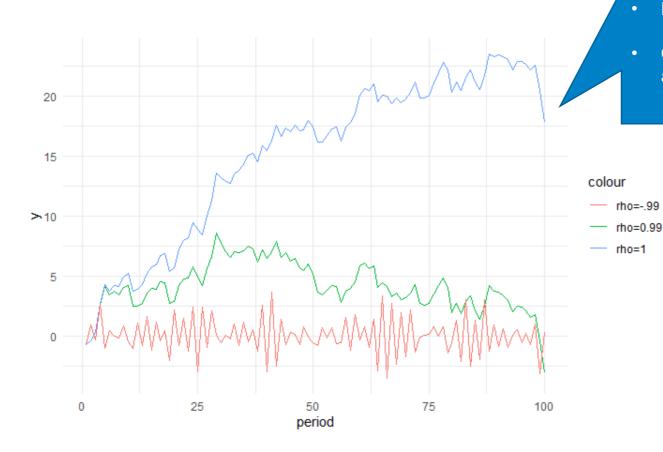
```
library (plm)
plm(cases~hoaxsh+factor(week)+factor(state), statsbyweek,
   index=c("state", "week"),
   model="within",
   effect="twoways") %>% summary()
                                          Alternative command to include cross
                                         sectional and time effects in panel data
## Twoways effects Within Model
                                          Substantially more efficient with large
## Call:
## plm(formula = cases ~ hoaxsh + factor(w
                                           datasets (many cross sectional units)
     data = statsbyweek, effect = "twowa"
     index = c("state", "week"))
##
## Unbalanced Panel: n = 50, T = 29-37, N = 1546
##
## Residuals:
      Min. 1st Ou.
                         Median 3rd Ou.
## -264367.42 -23040.53
                         592.79 22221.48 456331.70
## Coefficients:
    Estimate Std. Error t-value Pr(>|t|)
## hoaxsh 3788.3 1863.0 2.0334 0.04219 *
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Total Sum of Squares: 6.8535e+12
## Residual Sum of Squares: 6.8341e+12
## R-Squared:
                0.0028259
## Adj. R-Squared: -0.055952
## F-statistic: 4.13474 on 1 and 1459 DF, p-value: 0.042192
```

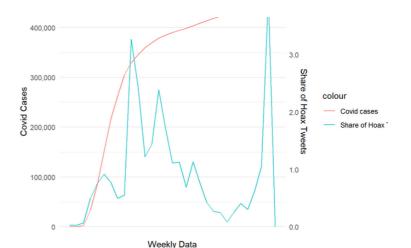
Autoregression

- A particular concern in time series is the possibility that observations
- Simplest way to model this is via an Auto regression:

•
$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \epsilon_t$$

 Y_{t-1} becomes the X variable We can do normal OLS as long as $-1 < \rho < 1$





- With $\rho = 1$ we have non-stationarity because of path dependence
- The series can wander off into any direction and neve come back
- If that happens OLS is no longer un-biased (different observations are too related to each other)
- Also: if you are interested in $Y = \beta X$ and both Y and X have unit roots you will have a spurious correlation (the unit root becomes the confounder)
- Random Walk
- Of course we don't know if this is the case in our data before we start any analysis



Dickey-Fuller test to the rescue



Rewrite original model by subtracting Y_{t-1} on both sides of the model equation:

$$Y_{t} = \beta_{0} + \beta Y_{t-1} + \epsilon_{t}$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$Y_{t} - Y_{t-1} = \Delta Y_{t} = \beta_{0} + \underbrace{(\beta - 1)}_{=\delta} Y_{t-1} + \epsilon_{t}$$

Testing for a random walk (aka unit root) now boils down to

H0: δ =0

H1: δ <0 i.e. stationary process

- We cannot just compare the implied test statistic to a normal t-table
- Luckily R will help us

Dickey-Fuller with more lags

Rewrite original model by subtracting Y_{t-1} on both sides of the model equation:

$$Y_{t} = \beta_{0} + \beta Y_{t-1} + \epsilon_{t}$$

$$\downarrow \downarrow$$

$$Y_{t} - Y_{t-1} = \Delta Y_{t} = \beta_{0} + \underbrace{(\beta - 1)}_{=\delta} Y_{t-1} + \epsilon_{t}$$

Testing for a random walk (aka unit root) now boils down to

H0: δ =0

H1: δ <0 i.e. stationary process

- We cannot just compare the implied test statistic to a normal t-table
- Luckily R will help us

R to the rrrrrescue

```
library (urca)
## Warning: package 'urca' was built under R version 4.0.2
 ur.df(df$cases,type="trend",lags=2) %>% summary()
##
## # Augmented Dickey-Fuller Test Unit Root Test #
  ## Test regression trend
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
## Residuals:
               10 Median
       Min
## -0.025064 -0.000346 -0.000133 0.000080 0.044320
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.731e-04 2.556e-04 -1.068 0.2857
## z.lag.1 2.942e-03 5.660e-04 5.199 2.67e-07 ***
        1.275e-06 6.914e-07 1.844 0.0656 .
## tt
## z.diff.lag1 1.559e+00 3.058e-02 50.980 < 2e-16 ***
## z.diff.lag2 -6.456e-01 3.175e-02 -20.332 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.003206 on 663 degrees of freedom
## Multiple R-squared: 0.966, Adjusted R-squared: 0.9658
## F-statistic: 4708 on 4 and 663 DF, p-value: < 2.2e-16
##
## Value of test-statistic is: 5.1991 12.0772 16.9937
## Critical values for test statistics:
       1pct 5pct 10pct
                                      We cannot reject unit root
## tau3 -3.96 -3.41 -3.12
## phi2 6.09 4.68 4.03
                                        becase 5.1991>-3.41
## phi3 8.27 6.25 5.34
```

```
ur.df(df$lnindex,type="trend",lags=4) %>% summary()
```

```
** **********************************
## # Augmented Dickey-Fuller Test Unit Root Test #
## Test regression trend
##
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
## Residuals:
## Min 1Q Median
                                       3Q
                                                Max
## -8.210e-04 -3.159e-05 2.000e-07 3.877e-05 1.935e-04
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.642e-05 1.015e-05 -1.617 0.10641
## z.lag.1 -6.131e-04 2.191e-04 -2.799 0.00528 **
           2.136e-07 8.103e-08 2.636 0.00858 **
## z.diff.lag1 1.138e+00 3.862e-02 29.452 < 2e-16 ***
## z.diff.lag2 -9.104e-02 5.871e-02 -1.551 0.12145
## z.diff.lag3 6.759e-02 5.869e-02 1.152 0.24991
## z.diff.lag4 -1.278e-01 3.887e-02 -3.289 0.00106 **
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 7.154e-05 on 659 degrees of freedom
## Multiple R-squared: 0.9786, Adjusted R-squared: 0.9784
## F-statistic: 5026 on 6 and 659 DF, p-value: < 2.2e-16
## Value of test-statistic is: -2.7988 3.1876 4.1331
## Critical values for test statistics:
        1pct 5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2 6.09 4.68 4.03
## phi3 8.27 6.25 5.34
```

We cannot reject unit root becase -2.79>-3.41

More lags AR(2)?

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + u_t.$$

Stationarity now requires

$$\beta_1 + \beta_2 < 1$$

while

$$\beta_1 + \beta_2 = 1$$

$$Y_t - Y_{t-1} = \beta_0 + (\beta_1 + \beta_2 - 1)Y_{t-1} - \beta_2(Y_{t-1} - Y_{t-2}) + \epsilon_t$$

We can test this again using the coefficient on Y_{t-1}

More lags and trend?

$$Y_t - Y_{t-1} = \beta_0 + (\beta_1 + \beta_2 - 1)Y_{t-1} - \beta_2(Y_{t-1} - Y_{t-2}) + \rho t + \epsilon_t$$

Getting rid of unit roots

```
ur.df(diff(df$cases,1), type="trend", lags=4) %>% summary()
```

```
## # Augmented Dickey-Fuller Test Unit Root Test #
## Test regression trend
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
## Residuals:
            1Q Median 3Q
## -0.024165 -0.000400 -0.000149 0.000102 0.042341
## Coefficients:
           Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.315e-04 2.604e-04 -1.273 0.203337
## z.laq.1 -3.335e-02 8.604e-03 -3.876 0.000117 ***
      1.521e-06 7.023e-07 2.166 0.030695
## z.diff.lag1 6.401e-01 3.831e-02 16.708 < 2e-16 ***
## z.diff.lag2 -6.874e-02 4.585e-02 -1.499 0.134309
## z.diff.lag3 -1.117e-02 4.590e-02 -0.243 0.807849
## z.diff.lag4 -9.044e-02 3.952e-02 -2.289 0.022422 *
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.003248 on 658 degrees of freedom
## Multiple R-squared: 0.3781, Adjusted R-squared: 0.3725
## F-statistic: 66.69 on 6 and 658 DF, p-value: < 2.2e-16
## Value of test-statistic is: -3.8762 5.3464 7.8731
## Critical values for test statistics:
       1pct 5pct 10pct
                                           We cannot reject unit root
## tau3 -3.96 -3.41 -3.12
## phi2 6.09 4.68 4.03
                                              becase -3.42<-3.87
## phi3 8.27 6.25 5.34
```

- Differencing: $\Delta y_t = y_t y_{t-1}$
- Checking that differenced series is not unit rood

Getting rid of unit roots – Economic Activity index

```
ur.df(diff(df$lnindex,1),type="trend",lags=4) %>% summary()
## # Augmented Dickey-Fuller Test Unit Root Test #
## Test regression trend
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
## Residuals:
      Min
                10 Median
## -8.250e-04 -3.113e-05 8.000e-07 3.858e-05 1.986e-04
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 6.860e-06 5.899e-06 1.163 0.245241
## z.lag.1 -1.772e-02 5.904e-03 -3.001 0.002789 **
            -9.213e-09 1.459e-08 -0.631 0.527986
## z.diff.lag1 1.634e-01 3.879e-02 4.214 2.87e-05 ***
## z.diff.lag2 7.379e-02 3.905e-02 1.889 0.059266 .
## z.diff.lag3 1.386e-01 3.913e-02 3.541 0.000426 ***
## z.diff.lag4 3.180e-02 3.912e-02 0.813 0.416596
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 7.198e-05 on 658 degrees of freedom
## Multiple R-squared: 0.07249, Adjusted R-squared: 0.06403
## F-statistic: 8.571 on 6 and 658 DF, p-value: 5.487e-09
## Value of test-statistic is: -3.0014 3.0809 4.6052
## Critical values for test statistics:
       1pct 5pct 10pct
## tau3 -3.96 -3.41 -3.12
                                               Still a unit root?
## phi2 6.09 4.68 4.03
## phi3 8.27 6.25 5.34
                      We cannot reject
```

zero trend either

```
ur.df(diff(df$lnindex,1),type="none",lags=4) %>% summary()
## # Augmented Dickey-Fuller Test Unit Root Test #
## Test regression none
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
## Residuals:
                        Median
        Min
                10
                                      30
## -8.248e-04 -2.843e-05 3.170e-06 4.148e-05 1.996e-04
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
## z.lag.1 -0.014340 0.005255 -2.729 0.006523 **
## z.diff.lag1 0.162660 0.038764 4.196 3.09e-05 ***
## z.diff.lag2 0.072129 0.039000 1.849 0.064837 .
## z.diff.lag3 0.136314 0.039050 3.491 0.000514 ***
## z.diff.lag4 0.028763 0.039006 0.737 0.461138
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 7.197e-05 on 660 degrees of freedom
## Multiple R-squared: 0.07012, Adjusted R-squared: 0.06308
## F-statistic: 9.954 on 5 and 660 DF, p-value: 3.401e-09
## Value of test-statistic is: -2.729
                                        Now we can reject
## Critical values for test statistics:
                                             unit root
       1pct 5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

Revisiting COVID vs GDP

```
## lm(formula = Dlnindex ~ Dcases + t, data = df)
## Residuals:
                  10 Median
## -1.107e-03 -9.941e-05 4.439e-05 1.487e-04 1.041e-03
## Coefficients:
            Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.429e-04 2.415e-05 5.918 5.20e-09 ***
## Dcases -2.316e-02 7.258e-04 -31.914 < 2e-16 ***
          5.269e-07 6.490e-08 8.119 2.28e-15 ***
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual stand
                        🙄 0.000305 on 667 degrees of freedom
    (1 observat:
                               o missingness)
## Multiple R-sq
                                    R-squared: 0.6053
## F-statistic:
                                           e: < 2.2e-16
```

100k more cases = 2.3% lower GDP...similar to what we had before....but of course we didn't know that would happen

Other considerations

```
lm(Dlnindex~Dcases+t+Dlockshare,df) %>% summary()
## Call:
## lm(formula = Dlnindex ~ Dcases + t + Dlockshare, data = df)
## Residuals:
## Min 1Q Median 3Q
## -0.0011149 -0.0001001 0.0000414 0.0001472 0.0010273
## Coefficients:
## Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.401e-04 2.404e-05 5.831 8.61e-09 ***
## Dcases -2.311e-02 7.221e-04 -32.010 < 2e-16 ***
## t 5.400e-07 6.471e-08 8.345 4.10e-16 ***
## Dlockshare -1.253e-05 4.354e-06 -2.878 0.00412 **
## Signif. codes: \Q '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standa
                         Q.0003034 on 666 degrees of freedom
## (1 observation
                              missingness)
## Multiple R-squa
                                 ted R-squared: 0.6095
## F-statistic: 3
                                     value: < 2.2e-16
```

 If 100% of US population go into lockdown GDP goes down by -0.138% (seems low..more research needed)

Summary

- Time series can be easy
- But you need to worry about how stationary your series is
- If the series clearly grows or shrinks continuously definitely include a time trend
- However, even if it doesn't grow (or shrink) the series might contain a unit root
- If that's the case a time trend is not enough
- Use the Dickey Fuller Test to make sure you are dealing with a stationary series



Extra Slides



Some remarks on Causality and unit roots

- X causes Y then both need to be integrated in the same order
- i.e. if X has a unit root Y has a unit root as well
- If Y has a unit root but not X then X can (potentially) have a causal effect on ΔY
- If X has a unit root but not Y we should be looking for a causal effect of ΔX on Y



More lags

```
##
## Call:
## lm(formula = Dlnindex ~ dplyr::lag(Dlnindex) + dplyr::lag(Dlnindex,
      2) + Dcases + dplyr::lag(Dcases) + dplyr::lag(Dcases, 2) +
      t + Dlockshare + dplyr::lag(Dlockshare) + dplyr::lag(Dlockshare,
      2), data = df)
##
## Residuals:
        Min
                 1Q Median 3Q
## -2.264e-04 -3.238e-05 -2.604e-06 3.437e-05 1.893e-04
## Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.374e-06 4.565e-06 0.301 0.7634
## dplyr::lag(Dlnindex) 7.911e-01 3.793e-02 20.855 < 2e-16 ***
## dplyr::lag(Dlnindex, 2) 1.854e-01 3.743e-02 4.954 9.27e-07 ***
                        -7.495e-04 1.100e-03 -0.681 0.4960
## Dcases
## dplyr::lag(Dcases) -3.192e-03 1.444e-03 -2.210 0.0275 *
## dplyr::lag(Dcases, 2) 3.703e-03 7.344e-04 5.043 5.94e-07 ***
## t
                         2.186e-08 1.270e-08 1.721 0.0857.
              -1.036e-05 8.940e-07 -11.586 < 2e-16 ***
## Dlockshare
## dplyr::lag(Dlockshare) -9.179e-06 1.167e-06 -7.867 1.49e-14 ***
## dplyr::lag(Dlockshare, 2) -3.171e-06 1.449e-06 -2.189 0.0290 *
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 5.585e-05 on 658 degrees of freedom
## (3 observations deleted due to missingness)
## Multiple R-squared: 0.987, Adjusted R-squared: 0.9868
## F-statistic: 5546 on 9 and 658 DF, p-value: < 2.2e-16
```