# Time Series

To infinity and beyond….

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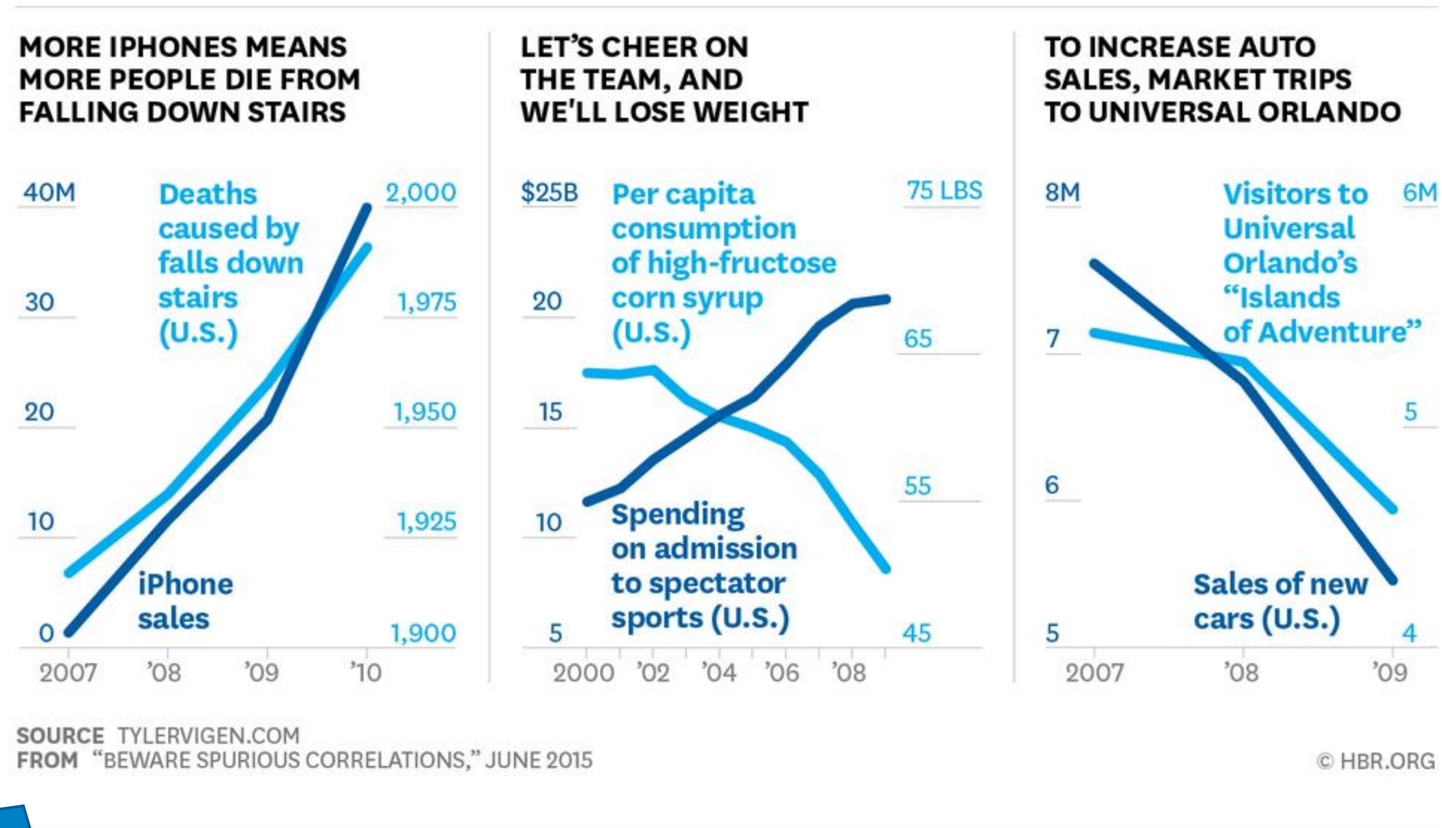
# Objectives of this lecture

- Time Series data: Different data points represent different points in time
- This introduces some additional challenges
- We will discuss how to deal with those

# What's the challenge of time series data?



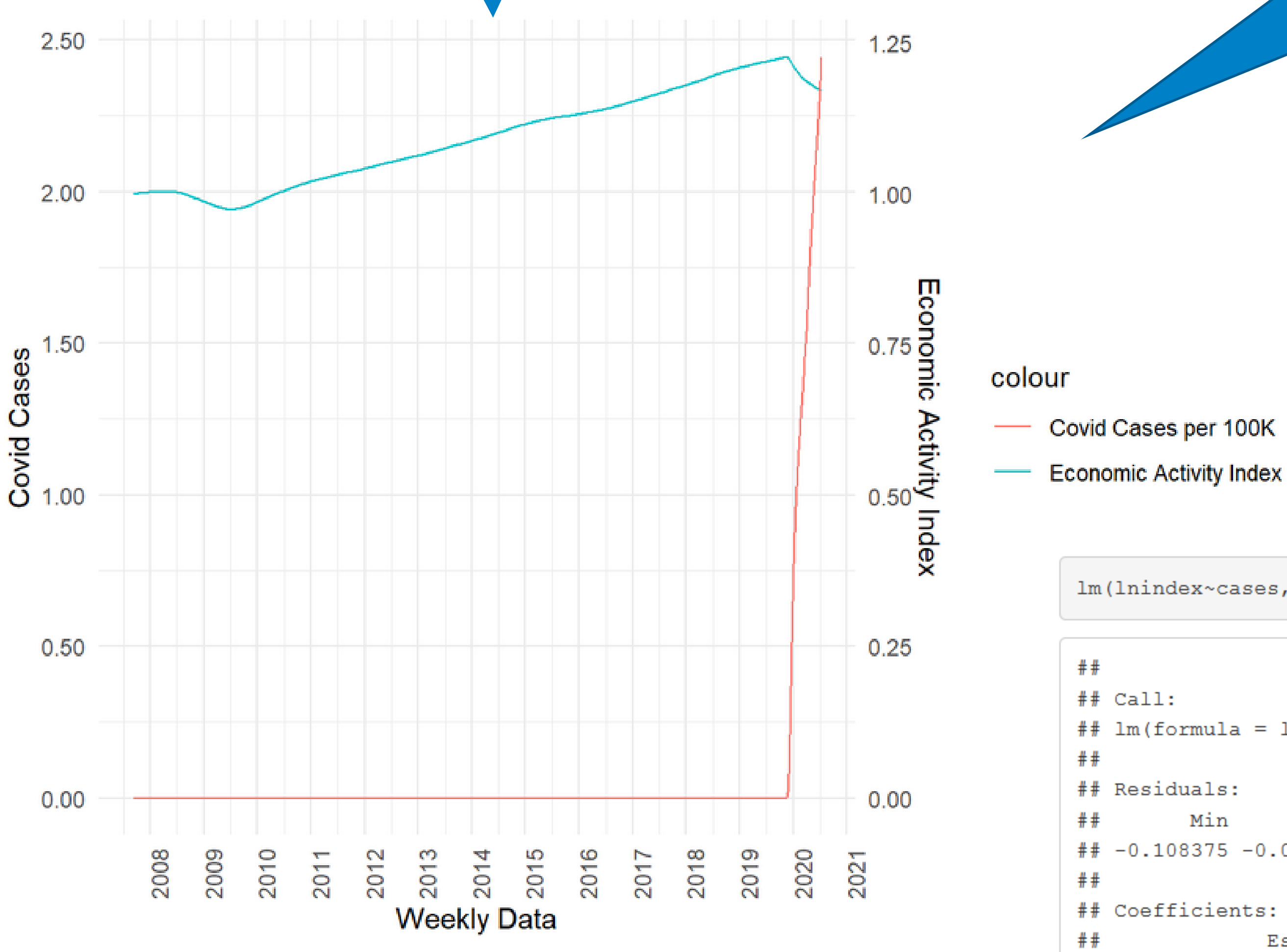

Can you spot the problem?

Time becomes a confounding variable
Non-stationary: characteristics of data vary with time

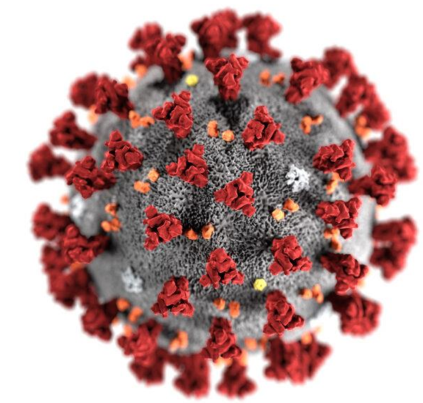
# COVID vs GDP

```
head(df)
```

```
##         week  WEI   Index cases deaths      lnindex lockshare
## 1 2008-01-05 1.42 1.00000     0      0 0.0000000000         0
## 2 2008-01-12 1.46 1.00028     0      0 0.0002799608         0
## 3 2008-01-19 1.40 1.00055     0      0 0.0005498488         0
## 4 2008-01-26 0.96 1.00073     0      0 0.0007297337         0
## 5 2008-02-02 0.73 1.00088     0      0 0.0008796130         0
## 6 2008-02-09 0.78 1.00103     0      0 0.0010294699         0
```




What you think is going to happen?



```
lm(lnindex~cases,df) %>% summary()
```

```
##
## Call:
## lm(formula = lnindex ~ cases, data = df)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.108375 -0.064942 -0.002043  0.055871  0.121388
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.082359   0.002731  30.156   < 2e     ***
## cases       0.050576   0.007800   6.484 1.74e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.06928 on 669 degrees of freedom
## Multiple R-squared:  0.05913,    Adjusted R-squared:  0.05772
## F-statistic: 42.04 on 1 and 669 DF,  p-value: 1.736e-10
```

More COVID = more GDP?
100K more = 5% more GDP?

# Taking control of time….with a timeline

```
df=df %>% mutate(t=1:n())
lm(lnindex~cases+t,df) %>% summary()
```

```
##
## Call:
## lm(formula = lnindex ~ cases + t, data = df)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.024859 -0.004965 -0.001175  0.003861  0.038124
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.850e-02  9.170e-04  -41.98   <2e-16 ***
## cases       -2.262e-02  1.393e-03  -16.23   <2e-16 ***
## t            3.752e-04  2.466e-06  152.11   <2e-16 ***
## ---
## Signif. c  es:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual         error: 0.01161 on 668 degrees of freedom
## Multiple           .9736, Adjusted R-squared:  0.9735
## F-statist           2 and 668 DF,  p-value: < 2.2e-16
```

100k more cases = 2.2% lower GDP

# What if time is not linear?

- Seasonal effects
- Recessions
- Natural disasters
- Political turmoil
- War
- Pandemic

## Panel data to the rescue

```
head(statsbyweek %>% arrange(state,week))
```

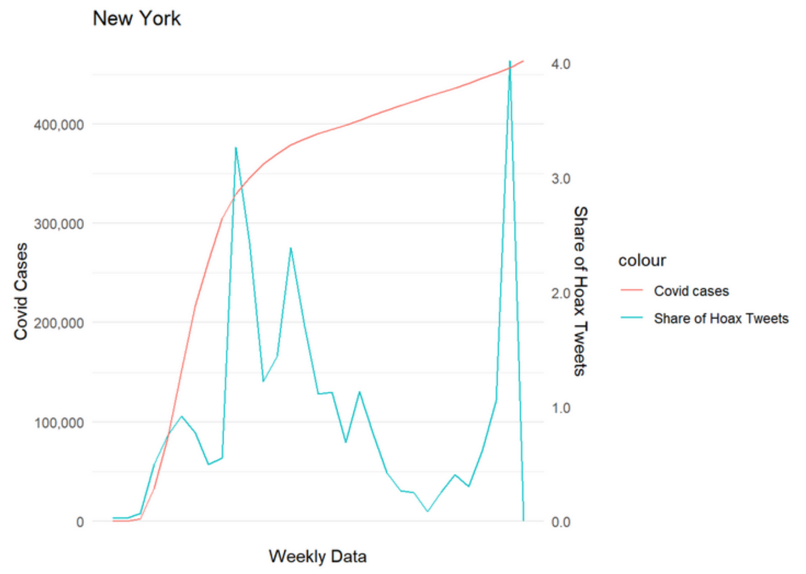
```
## # A tibble: 6 x 9
## # Groups:   state [1]
##   state   week        hoax tweets cases deaths hoaxsh Dcases Ddeaths
##   <chr>   <date>     <int>  <int> <int>  <int>  <dbl>  <int>   <int>
## 1 Alabama 2020-03-15     4   1503    51      0  0.266     NA      NA
## 2 Alabama 2020-03-22    62   4198   386      1  1.48     335       1
## 3 Alabama 2020-03-29    14   5218  1108     28  0.268    722      27
## 4 Alabama 2020-04-05    12   4793  2498     67  0.250   1390      39
## 5 Alabama 2020-04-12     9   4486  4241    123  0.201   1743      56
## 6 Alabama 2020-04-19     6   3570  5610    201  0.168   1369      78
```

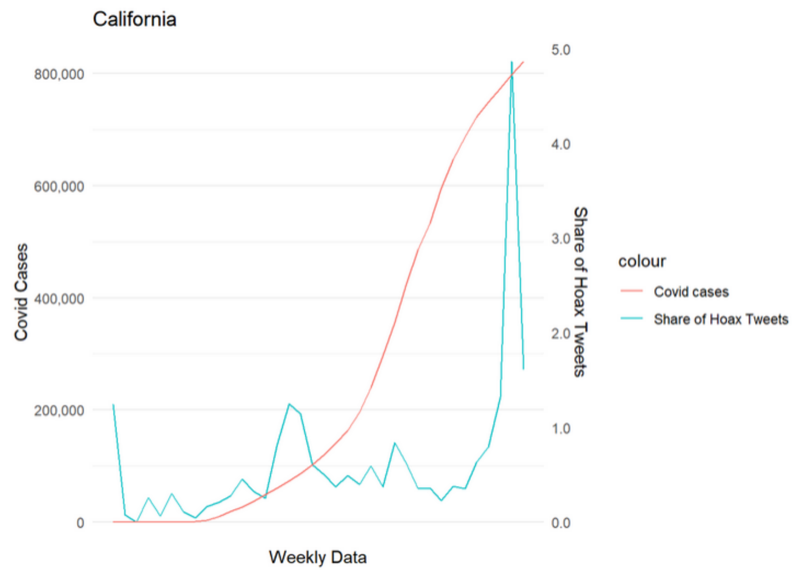
```
statsbyweek %>% group_by(state) %>% summarise(n())
```

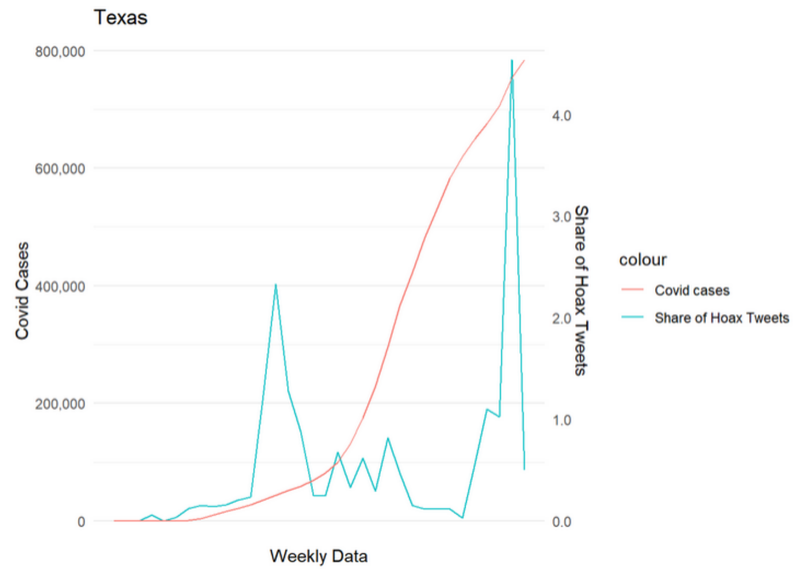
```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
## # A tibble: 50 x 2
##    state       `n()`
##    <chr>       <int>
##  1 Alabama        29
##  2 Alaska         29
##  3 Arizona        36
##  4 Arkansas       30
##  5 California     36
##  6 Colorado       30
##  7 Connecticut    30
##  8 Delaware       30
##  9 Florida        31
## 10 Georgia        31
## # ... with 40 more rows
```

Multiple periods for the same cross section unit



# Panel data example

```
lm(cases~hoaxsh,statsbyweek) %>% summary()
```

Hoax share up by 1 percentage point means 11555 more cases

```
##
## Call:
## lm(formula = cases ~ hoaxsh, data = statsbywe
##
## Residuals:
##     Min      1Q  Median      3Q     M
## -189328  -50914  -40048    7176  7
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)    50929       3108  16.388  < 2e-16 ***
## hoaxsh         11555       2380   4.855 1.33e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 108700 on 1544 degrees of freedom
## Multiple R-squared:  0.01504,    Adjusted R-squared:  0.0144
## F-statistic: 23.57 on 1 and 1544 DF,  p-value: 1.326e-06
```

```
lm(cases~hoaxsh+factor(week),statsbyweek) %>% summary()
```

Smaller effect when controlling for time (week) effects

```
##
## Call:
## lm(formula = cases ~ hoaxsh + factor(wee
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -199861  -37318   -9820    1098  668461
##
## Coefficients:
##                        Estimate Std.       t value Pr(>|t|)
## (Intercept)                1.00   99956.85   0.000  0.99999
## hoaxsh                  7865.20    2593.18   3.033  0.00246 **
## factor(week)2020-01-26 -2439.83  111758.05  -0.022  0.98259
## factor(week)2020-02-02   -95.32  107965.74  -0.001  0.99930
## factor(week)2020-02-09     1.00  106858.37   0.000  0.99999
## factor(week)2020-02-16  -254.24  106020.28  -0.002  0.99809
## factor(week)2020-02-23   -50.25  105363.77   0.000  0.99962
## factor(week)2020-03-01 -1014.28  102855.30  -0.010  0.99213
## factor(week)2020-03-08   -70.35  101086.35  -0.001  0.99944
## factor(week)2020-03-15  -133.49  100951.52  -0.001  0.99895
```

```
lm(cases~hoaxsh++factor(state)+factor(week),statsbyweek) %>% summary()
```

```
##
## Call:
## lm(formula = cases ~ hoaxsh + +factor(state) + factor(week),
##     data = statsbyweek)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -264367  -23041     593   22221  456332
##
## Coefficients:
##                              Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     -1002      70567  -0.014 0.988669
## hoaxsh                           3788       1863   2.033 0.042192 *
## factor(state)Alaska            -52626      17974  -2.928 0.003465 **
## factor(state)Arizona            41125      17193   2.392 0.016885 *
## factor(state)Arkansas          -24748      17843  -1.387 0.165651
## factor(s    )California        223775      17186  13.021  < 2e-16 ***
## factor(st      rado            -19744      17833  -1.107 0.268403
```

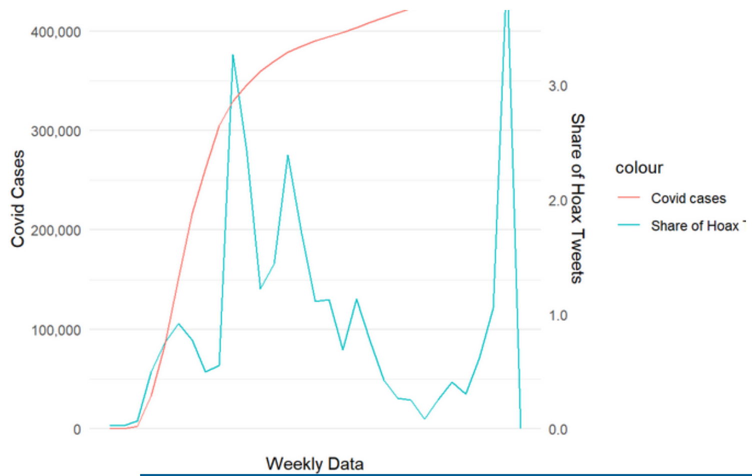
Also controlling for state

```
library(plm)
plm(cases~hoaxsh+factor(week)+factor(state),statsbyweek,
    index=c("state","week"),
    model="within",
    effect="twoways") %>% summary()
```

Alternative command to include cross sectional and time effects in panel data
Substantially more efficient with large datasets (many cross sectional units)

```
## Twoways effects Within Model
##
## Call:
## plm(formula = cases ~ hoaxsh + factor(we
##     data = statsbyweek, effect = "twoway
##     index = c("state", "week"))
##
## Unbalanced Panel: n = 50, T = 29-37, N = 1546
##
## Residuals:
##      Min.    1st Qu.     Median    3rd Qu.       Max.
## -264367.42  -23040.53     592.79   22221.48  456331.70
##
## Coefficients:
##        Estimate Std. Error t-value Pr(>|t|)
## hoaxsh   3788.3     1863.0  2.0334  0.04219 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Total Sum of Squares:    6.8535e+12
## Residual Sum of Squares: 6.8341e+12
## R-Squared:      0.0028259
## Adj. R-Squared: -0.055952
## F-statistic: 4.13474 on 1 and 1459 DF, p-value: 0.042192
```

# Autoregression




- A particular concern in time series is the possibility that observations

- Simplest way to model this is via an Auto regression:

- $Y_t = \beta_0 + \beta_1 Y_{t-1} + \epsilon_t$

$Y_{t-1}$ becomes the X variable
We can do normal OLS as long as $-1 < \rho < 1$

- With $\rho = 1$ we have non-stationarity because of path dependence
- The series can wander off into any direction and neve come back

- If that happens OLS is no longer un-biased (different observations are too related to each other)
- Also: if you are interested in $Y = \beta X$ and both Y and X have unit roots you will have a spurious correlation (the unit root becomes the confounder)
- Random Walk

- Of course we don't know if this is the case in our data before we start any analysis


# Dickey-Fuller test to the rescue

Rewrite original model by subtracting $Y_{t-1}$ on both sides of the model equation:

$$Y_t = \beta_0 + \beta Y_{t-1} + \epsilon_t$$
$$\Downarrow$$
$$Y_t - Y_{t-1} = \Delta Y_t = \beta_0 + \underbrace{(\beta - 1)}_{=\delta} Y_{t-1} + \epsilon_t$$

Testing for a random walk (aka unit root) now boils down to
H0: $\delta$=0
H1: $\delta$<0  i.e. stationary process

- We cannot just compare the implied test statistic to a normal t-table
- Luckily R will help us

# Dickey-Fuller with more lags

Rewrite original model by subtracting $Y_{t-1}$ on both sides of the model equation:

$$Y_t = \beta_0 + \beta Y_{t-1} + \epsilon_t$$
$$\Downarrow$$
$$Y_t - Y_{t-1} = \Delta Y_t = \beta_0 + \underbrace{(\beta - 1)}_{=\delta} Y_{t-1} + \epsilon_t$$

Testing for a random walk (aka unit root) now boils down to
H0: $\delta$=0
H1: $\delta$<0 i.e. stationary process

- We cannot just compare the implied test statistic to a normal t-table
- Luckily R will help us

# R to the rrrrrescue

```
library(urca)
```

```
## Warning: package 'urca' was built under R version 4.0.2
```

```
ur.df(df$cases,type="trend",lags=2) %>% summary()
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.025064 -0.000346 -0.000133  0.000080  0.044320
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.731e-04  2.556e-04  -1.068   0.2857
## z.lag.1      2.942e-03  5.660e-04   5.199 2.67e-07 ***
## tt           1.275e-06  6.914e-07   1.844   0.0656 .
## z.diff.lag1  1.559e+00  3.058e-02  50.980  < 2e-16 ***
## z.diff.lag2 -6.456e-01  3.175e-02 -20.332  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.003206 on 663 degrees of freedom
## Multiple R-squared:  0.966,  Adjusted R-squared:  0.9658
## F-statistic:  4708 on 4 and 663 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: 5.1991 12.0772 16.9937
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

We cannot reject unit root becase 5.1991>-3.41

```
ur.df(df$lnindex,type="trend",lags=4) %>% summary()
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -8.210e-04 -3.159e-05  2.000e-07  3.877e-05  1.935e-04
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.642e-05  1.015e-05  -1.617  0.10641
## z.lag.1     -6.131e-04  2.191e-04  -2.799  0.00528 **
## tt           2.136e-07  8.103e-08   2.636  0.00858 **
## z.diff.lag1  1.138e+00  3.862e-02  29.452  < 2e-16 ***
## z.diff.lag2 -9.104e-02  5.871e-02  -1.551  0.12145
## z.diff.lag3  6.759e-02  5.869e-02   1.152  0.24991
## z.diff.lag4 -1.278e-01  3.887e-02  -3.289  0.00106 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.154e-05 on 659 degrees of freedom
## Multiple R-squared:  0.9786, Adjusted R-squared:  0.9784
## F-statistic:  5026 on 6 and 659 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -2.7988 3.1876 4.1331
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

We cannot reject unit root becase -2.79>-3.41

# More lags AR(2)?

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + u_t.$$

Stationarity now requires

$$\beta_1 + \beta_2 < 1$$

while

$$\beta_1 + \beta_2 = 1$$

$$Y_t - Y_{t-1} = \beta_0 + (\beta_1 + \beta_2 - 1)Y_{t-1} - \beta_2(Y_{t-1} - \mathrm{Y}_{\mathrm{t}-2}) + \epsilon_{\mathrm{t}}$$

We can test this again using the coefficient on $Y_{t-1}$

# More lags and trend?

$$Y_t - Y_{t-1} = \beta_0 + (\beta_1 + \beta_2 - 1)Y_{t-1} - \beta_2(Y_{t-1} - \mathrm{Y}_{\mathrm{t}-2}) + \rho t + \epsilon_{\mathrm{t}}$$

# Getting rid of unit roots

```
ur.df(diff(df$cases,1),type="trend",lags=4) %>% summary()
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.024165 -0.000400 -0.000149  0.000102  0.042341
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.315e-04  2.604e-04  -1.273 0.203337
## z.lag.1     -3.335e-02  8.604e-03  -3.876 0.000117 ***
## tt           1.521e-06  7.023e-07   2.166 0.030695 *
## z.diff.lag1  6.401e-01  3.831e-02  16.708  < 2e-16 ***
## z.diff.lag2 -6.874e-02  4.585e-02  -1.499 0.134309
## z.diff.lag3 -1.117e-02  4.590e-02  -0.243 0.807849
## z.diff.lag4 -9.044e-02  3.952e-02  -2.289 0.022422 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.003248 on 658 degrees of freedom
## Multiple R-squared:  0.3781, Adjusted R-squared:  0.3725
## F-statistic: 66.69 on 6 and 658 DF,  p-value: < 2.2e-16
##
##
## Value of test-statistic is: -3.8762 5.3464 7.8731
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

We cannot reject unit root becase -3.42<-3.87

- Differencing: $\Delta y_t = y_t - y_{t-1}$
- Checking that differenced series is not unit rood

# Getting rid of unit roots – Economic Activity index

```
ur.df(diff(df$lnindex,1),type="trend",lags=4) %>% summary()
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression trend
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -8.250e-04 -3.113e-05  8.000e-07  3.858e-05  1.986e-04
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  6.860e-06  5.899e-06   1.163 0.245241
## z.lag.1     -1.772e-02  5.904e-03  -3.001 0.002789 **
## tt          -9.213e-09  1.459e-08  -0.631 0.527986
## z.diff.lag1  1.634e-01  3.879e-02   4.214 2.87e-05 ***
## z.diff.lag2  7.379e-02  3.905e-02   1.889 0.059266 .
## z.diff.lag3  1.386e-01  3.913e-02   3.541 0.000426 ***
## z.diff.lag4  3.180e-02  3.912e-02   0.813 0.416596
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.198e-05 on 658 degrees of freedom
## Multiple R-squared:  0.07249,    Adjusted R-squared:  0.06403
## F-statistic: 8.571 on 6 and 658 DF,  p-value: 5.487e-09
##
##
## Value of test-statistic is: -3.0014 3.0809 4.6052
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau3 -3.96 -3.41 -3.12
## phi2  6.09  4.68  4.03
## phi3  8.27  6.25  5.34
```

Still a unit root?

We cannot reject zero trend either

```
ur.df(diff(df$lnindex,1),type="none",lags=4) %>% summary()
```

```
##
## ###############################################
## # Augmented Dickey-Fuller Test Unit Root Test #
## ###############################################
##
## Test regression none
##
##
## Call:
## lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -8.248e-04 -2.843e-05  3.170e-06  4.148e-05  1.996e-04
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## z.lag.1     -0.014340   0.005255  -2.729 0.006523 **
## z.diff.lag1  0.162660   0.038764   4.196 3.09e-05 ***
## z.diff.lag2  0.072129   0.039000   1.849 0.064837 .
## z.diff.lag3  0.136314   0.039050   3.491 0.000514 ***
## z.diff.lag4  0.028763   0.039006   0.737 0.461138
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.197e-05 on 660 degrees of freedom
## Multiple R-squared:  0.07012,    Adjusted R-squared:  0.06308
## F-statistic: 9.954 on 5 and 660 DF,  p-value: 3.401e-09
##
##
## Value of test-statistic is: -2.729
##
## Critical values for test statistics:
##       1pct  5pct 10pct
## tau1 -2.58 -1.95 -1.62
```

Now we can reject unit root

# Revisiting COVID vs GDP

```
df=df %>% arrange(week) %>% mutate(Dlnindex=lnindex-dplyr::lag(lnindex),
                                   Dcases=cases-dplyr::lag(cases) ,
                                   DDlnindex=Dlnindex-dplyr::lag(Dlnindex))


lm(Dlnindex~Dcases+t,df) %>% summary()
```

```
##
## Call:
## lm(formula = Dlnindex ~ Dcases + t, data = df)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -1.107e-03 -9.941e-05  4.439e-05  1.487e-04  1.041e-03
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.429e-04  2.415e-05   5.918 5.20e-09 ***
## Dcases      -2.316e-02  7.258e-04 -31.914  < 2e-16 ***
## t            5.269e-07  6.490e-08   8.119 2.28e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual stan          r: 0.000305 on 667 degrees of freedom
##   (1 observat             to missingness)
## Multiple R-sq                  d R-squared:  0.6053
## F-statistic:                       e: < 2.2e-16
```

100k more cases = 2.3% lower GDP…similar to what we had before….but of course we didn't know that would happen

# Other considerations

```
lm(Dlnindex~Dcases+t+Dlockshare,df) %>% summary()
```

```
##
## Call:
## lm(formula = Dlnindex ~ Dcases + t + Dlockshare, data = df)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0011149 -0.0001001  0.0000414  0.0001472  0.0010273
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  1.401e-04  2.404e-05   5.831 8.61e-09 ***
## Dcases      -2.311e-02  7.221e-04 -32.010  < 2e-16 ***
## t            5.400e-07  6.471e-08   8.345 4.10e-16 ***
## Dlockshare  -1.253e-05  4.354e-06  -2.878  0.00412 **
## ---
## Signif. codes:   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standa          0.0003034 on 666 degrees of freedom
##   (1 observati             o missingness)
## Multiple R-squa               ted R-squared:  0.6095
## F-statistic: 3                   value: < 2.2e-16
```

- If 100% of US population go into lockdown GDP goes down by -0.138% (seems low..more research needed)

# Summary

- Time series can be easy
- But you need to worry about how stationary your series is
- If the series clearly grows or shrinks continuously definitely include a time trend
- However, even if it doesn't grow (or shrink) the series might contain a unit root
- If that's the case a time trend is not enough
- Use the Dickey Fuller Test to make sure you are dealing with a stationary series


# Extra Slides

# Some remarks on Causality and unit roots

- X causes Y then both need to be integrated in the same order
- i.e. if X has a unit root Y has a unit root as well
- If Y has a unit root but not X then X can (potentially) have a causal effect on $\Delta Y$
- If X has a unit root but not Y we should be looking for a causal effect of $\Delta X$ on $Y$

# More lags

```r
lm(Dlnindex~dplyr::lag(Dlnindex)+dplyr::lag(Dlnindex,2)+Dcases+dplyr::lag(Dcases)+dplyr::lag(Dcases,2)+t+Dlockshar
e+dplyr::lag(Dlockshare)+dplyr::lag(Dlockshare,2),df) %>% summary()
```

```
##
## Call:
## lm(formula = Dlnindex ~ dplyr::lag(Dlnindex) + dplyr::lag(Dlnindex,
##     2) + Dcases + dplyr::lag(Dcases) + dplyr::lag(Dcases, 2) +
##     t + Dlockshare + dplyr::lag(Dlockshare) + dplyr::lag(Dlockshare,
##     2), data = df)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -2.264e-04 -3.238e-05 -2.604e-06  3.437e-05  1.893e-04
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)                1.374e-06  4.565e-06   0.301   0.7634
## dplyr::lag(Dlnindex)       7.911e-01  3.793e-02  20.855  < 2e-16 ***
## dplyr::lag(Dlnindex, 2)    1.854e-01  3.743e-02   4.954 9.27e-07 ***
## Dcases                    -7.495e-04  1.100e-03  -0.681   0.4960
## dplyr::lag(Dcases)        -3.192e-03  1.444e-03  -2.210   0.0275 *
## dplyr::lag(Dcases, 2)      3.703e-03  7.344e-04   5.043 5.94e-07 ***
## t                          2.186e-08  1.270e-08   1.721   0.0857 .
## Dlockshare                -1.036e-05  8.940e-07 -11.586  < 2e-16 ***
## dplyr::lag(Dlockshare)    -9.179e-06  1.167e-06  -7.867 1.49e-14 ***
## dplyr::lag(Dlockshare, 2) -3.171e-06  1.449e-06  -2.189   0.0290 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.585e-05 on 658 degrees of freedom
##   (3 observations deleted due to missingness)
## Multiple R-squared:  0.987,  Adjusted R-squared:  0.9868
## F-statistic:  5546 on 9 and 658 DF,  p-value: < 2.2e-16
```