EE7403 2022-2023 S2

(a) for $wx - \sqrt{3}wy = b$ (b is a constant)

$$y = \frac{1}{\sqrt{3}}x - \frac{b}{\sqrt{3}}$$

$f(x,y) = h(b)$ is a constant.

$$\phi = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

(b) suppose $z = wx - \sqrt{3}wy$

$$\frac{\partial f(x,y)}{\partial x} = \frac{\partial f(x,y)}{\partial z}\frac{\partial z}{\partial x} = h'(wx - \sqrt{3}wy)\cdot w$$

$$\frac{\partial f(x,y)}{\partial y} = \frac{\partial f(x,y)}{\partial z}\frac{\partial z}{\partial y} = h'(wx - \sqrt{3}y)\cdot(-\sqrt{3}w)$$

the direction of gradient : $\arctan^{-1}\left(\frac{\frac{\partial f(x,y)}{\partial y}}{\frac{\partial f(x,y)}{\partial x}}\right) = \arctan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

the orientation ϕ is perpendicular to the direction of gradient

$$\phi = -\frac{\pi}{3} + \frac{\pi}{2} = \frac{\pi}{6}$$

(c) $f(x,y) = h(wx - \sqrt{3}wy) = \sin(wx - \sqrt{3}wy)$

$$\nabla F(m,n) = \begin{bmatrix} \sin(w(m+1)\Delta - \sqrt{3}wn\Delta) - \sin(w(m-1)\Delta - \sqrt{3}wn\Delta) \\ \sin(wm\Delta - \sqrt{3}w(n+1)\Delta) - \sin(wm\Delta - \sqrt{3}w(n-1)\Delta) \end{bmatrix}$$

$$= \begin{bmatrix} 2\cos\frac{w(m+1)\Delta - \sqrt{3}wn\Delta + w(m-1)\Delta - \sqrt{3}wn\Delta}{2}\cdot\sin\frac{w[(m+1)-(m-1)\Delta]}{2} \\ 2\cos\frac{2wm\Delta - \sqrt{3}w(n+1+n-1)\Delta}{2}\cdot\sin\frac{-\sqrt{3}w(n+1-n+1)\Delta}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 2\cos(wm\Delta - \sqrt{3}wn\Delta)\cdot\sin w\Delta \\ 2\cos(wm\Delta - \sqrt{3}wn\Delta)\cdot\sin(-\sqrt{3}w\Delta) \end{bmatrix}$$

the gradient's orientation $\varphi = \arctan^{-1}\left(\frac{\cancel{2\cos(wm\Delta - \sqrt{3}wn\Delta)}\sin w\Delta}{\cancel{2\cos(wm\Delta - \sqrt{3}wn\Delta)}\cdot\sin(-\sqrt{3}w\Delta)}\right)$

$$= -\arctan^{-1}\left(\frac{\sin\sqrt{3}w\Delta}{\sin w\Delta}\right)$$

for $\Delta = \frac{\pi}{2w}$ $\varphi = -\arctan^{-1}\left(\frac{\sin\frac{\sqrt{3}}{2}\pi}{\sin\frac{\pi}{2}}\right) \approx -0.38\,\text{rad}$

$\phi = \varphi \pm \frac{\pi}{2}$ and $\phi \in (-\frac{\pi}{2}, \frac{\pi}{2}]$ $\Rightarrow \phi = \varphi + \frac{\pi}{2} \approx 1.19\,\text{rad}$

for $\Delta = \frac{\pi}{8w}$ $\varphi = -\arctan^{-1}\left(\frac{\sin\frac{\sqrt{3}}{8}\pi}{\sin\frac{\pi}{8}}\right) \approx -1.02\,\text{rad}$

$\phi = \varphi + \frac{\pi}{2} \approx 0.55\,\text{rad}$

(d). according to (a) and (b), we can see the orientation of the image is $\frac{\pi}{6}$ rad, which is equal to 0.52 rad. in part (c), $\Delta = \frac{\pi}{8w}$ whose Δ is smaller is much more closer to the real angle.

Besides, according to (c), the $\varphi = -\tan^{-1}\left(\frac{\sin(\sqrt{3}w\Delta)}{\sin w\Delta}\right)$.

for $g(\Delta) = \frac{\sin(\sqrt{3}w\Delta)}{\sin w\Delta}$

$$\lim_{\Delta \to 0} g(\Delta) = \lim_{\Delta \to 0} \frac{\sqrt{3}w\cos(\sqrt{3}w\Delta)}{w\cos(w\Delta)} = \sqrt{3}$$

We can know that, when $\Delta \to 0$, Δ is smaller, the $g(\Delta) \to \sqrt{3}$

$\varphi \to -\arctan^{-1}\sqrt{3}$ which is closer to (b)