EE6222 2024-2025-S1

the same as EE7403 2021-2022 S2

2. (a) $\mu_{ki} = \frac{1}{n}\sum_{j=1}^{n} x_{kij}$

$$\sigma_{kpq} = \frac{1}{n}\sum_{j=1}^{n}(x_{kpj} - \mu_{kp})(x_{kqj} - \mu_{kq})$$

where $\mu_{kp} = \frac{1}{n}\sum_{j=1}^{n} x_{kpj}$, $\mu_{kq} = \frac{1}{n}\sum_{j=1}^{n} x_{kqj}$

(b) $x_{kj} = \begin{bmatrix} x_{k1j} \\ x_{2j} \\ \vdots \\ x_{mj} \end{bmatrix}$ $\mu_k = \frac{1}{n}\sum_{j=1}^{n} x_{kj}$

$$\Sigma_k = \frac{1}{n}\sum_{j=1}^{n}(x_{kj} - \mu_k)(x_{kj} - \mu_k)^T$$

(c) $X_k = [(x_{k1} - \mu_k)\ \ (x_{k2} - \mu_k)\ \ \cdots\ \ (x_{kn} - \mu_k)]$

$$\text{mean}(X_k) = \frac{1}{n}\sum_{j=1}^{n}(x_{kj} - \mu_k) = \frac{1}{n}\sum_{j=1}^{n} x_{kj} - \frac{1}{n}\cdot n\cdot \mu_k = \mu_k - \mu_k = 0$$

$$\Sigma_k = \frac{1}{n}\sum_{j=1}^{n}(x_{kj} - \mu_k)(x_{kj} - \mu_k)^T$$

$$= \frac{1}{n} X_k X_k^T$$

(d) $y = a^T X_k$. $y = [y_1, y_2, \ldots, y_n]$

$$\bar{y} = \sum_{j=1}^{n} y_j = \frac{1}{n} a^T \sum_{j=1}^{n}(x_{kj} - \mu_k) = 0$$

$$V_k = \frac{1}{n}\sum_{j=1}^{n}(y_j - \bar{y})^2$$

$$= \frac{1}{n}\sum_{j=1}^{n} y_i^2$$

$$= \frac{1}{n}\sum_{j=1}^{n} a^T(x_{kij} - \mu_k)(x_{kij} - \mu_k)^T a$$

$$= a^T\left(\frac{1}{n}\sum_{j=1}^{n}(x_{kij} - \mu_k)(x_{kij} - \mu_k)^T\right) a$$

$$= a^T \Sigma_k a$$