EE7403 2021-2022-S2

4\. (a) $\mu_{ki} = \frac{1}{n}\sum_{j=1}^{n} x_{kij}$

σ_{kpq} is the element of covariance matrix between feature p and feature q.

$$\sigma_{kqp} = \frac{1}{n}\sum_{j=1}^{n}(x_{kpj} - \mu_{kp})(x_{kqj} - \mu_{kq})$$

where $\mu_{kp} = \frac{1}{n}\sum_{j=1}^{n} x_{pj}$, $\mu_{kq} = \frac{1}{n}\sum_{j=1}^{n} x_{qj}$

(b) $x_{kj} = [x_{k1j} \quad x_{k2j} \dots x_{kmj}]^T$

$$\mu_k = \frac{1}{n}\sum_{j=1}^{n} x_{kj}$$

$$\Sigma_k = \frac{1}{n}\sum_{j=1}^{n}(x_{kj} - \mu_k)(x_{kj} - \mu_k)^T$$

(c) $X_k = [(x_{k1} - \mu_k) \quad (x_{k2} - \mu_k) \quad \dots \quad (x_{km} - \mu_k)]$

$$\Sigma_k = \frac{1}{n} X_k X_k^T$$

(d) $y = a^T X_k = [a^T(x_{k1} - \mu_k) \quad a^T(x_{k2} - \mu_k) \quad \dots \quad a^T(x_{km} - \mu_k)]$

$= (y_{k1} \quad y_{k2} \dots y_{kn})$

where y_{kj} is scalar

$$\bar{y} = a^T\left(\sum_{j=1}^{n} x_{kj} - n\mu_k\right) = 0$$

$$\therefore V_k = \frac{1}{n}\sum_{j=1}^{n}(y_{kj} - \bar{y})^2$$

$$= \frac{1}{n}\sum_{j=1}^{n} y_{kj}^2$$

$$= \frac{1}{n} y y^T$$

$$= \frac{1}{n} a^T X_k X_k^T a$$

$$= a^T\left(\frac{1}{n} X_k X_k^T\right) a$$

$$= a^T \Sigma_k a$$