EE7403 2023-2024 S2

3. (a) $\mu = \frac{1}{5}(x_1 + x_2 + x_3 + x_4 + x_5)$

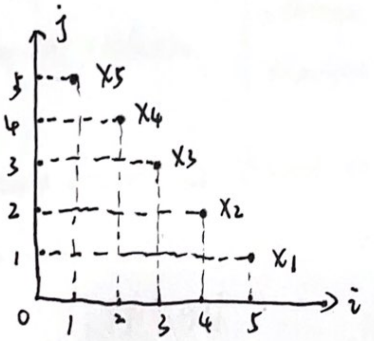
$$= \frac{1}{5}\left(\begin{bmatrix}5\\1\end{bmatrix}+\begin{bmatrix}4\\2\end{bmatrix}+\begin{bmatrix}3\\3\end{bmatrix}+\begin{bmatrix}2\\4\end{bmatrix}+\begin{bmatrix}1\\5\end{bmatrix}\right)$$

$$= \begin{bmatrix}3\\3\end{bmatrix}$$

Covariance matrix:

$$\Sigma = \frac{1}{5}\sum_{i=1}^{5}(x_i-\mu)(x_i-\mu)^T = \frac{1}{5}\begin{bmatrix}10 & -10\\-10 & 10\end{bmatrix} = \begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}$$

(b)




(c) Suppose the eigenvalue is $\lambda$, and the eigenvector is $v$

$\Sigma v = \lambda v$

$(\lambda I - \Sigma)v = 0 \Rightarrow \begin{vmatrix}\lambda-2 & 2\\2 & \lambda-2\end{vmatrix} = 0 \Rightarrow \lambda_1 = 0 \quad \lambda_2 = 4$

$v_1 = \begin{bmatrix}\frac{\sqrt{2}}{2}\\\frac{\sqrt{2}}{2}\end{bmatrix} \quad v_2 = \begin{bmatrix}\frac{\sqrt{2}}{2}\\-\frac{\sqrt{2}}{2}\end{bmatrix}$

(d) the eigenvalues tells how much variance is along each principal axis

the eigenvectors tells the directions of those variations, aligning with the direction of the line that connects all pixels.