EE7403 2022-2023 S2

2. (a) $g_i(x) = \ln\left[\frac{p(w_i)\cdot p(x|w_i)}{p(x)}\right]$

$= \ln[p(w_i)] + \ln[p(x|w_i)] - \ln[p(x)]$

for all i. $p(x)$ ~~is~~ the same

simplify $g_i = \ln[p(w_i)] + \ln[p(x|w_i)]$

$= -\frac{d}{2}\ln 2\pi - \frac{1}{2}\ln|\Sigma_i| - \frac{1}{2}(x-\mu_i)^T\Sigma_i^{-1}(x-\mu_i) + \ln[p(w_i)]$

$-\frac{d}{2}\ln 2\pi$ is a constant

$g_i(x) = -\frac{1}{2}(x-\mu_i)^T\Sigma_i^{-1}(x-\mu_i) + \ln[p(w_i)] - \frac{1}{2}\ln|\Sigma_i|$

decide $w_i = \underset{w_i}{\operatorname{argmax}}\ g_i(x)$.

(b) for w_1: $\mu_1 = \frac{1}{3}\left(\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}2\\1\end{bmatrix}\right) = \begin{bmatrix}\frac{4}{3}\\\frac{4}{3}\end{bmatrix}$ $p(w_1) = \frac{3}{6} = \frac{1}{2}$

$\Sigma_1 = \frac{1}{3}\sum_{i=1}^{3}(x_i-\mu_1)(x_i-\mu_1)^T = \frac{1}{9}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$

w_2: $\mu_2 = \frac{1}{3}\left(\begin{bmatrix}2\\2\end{bmatrix} + \begin{bmatrix}2\\3\end{bmatrix} + \begin{bmatrix}3\\2\end{bmatrix}\right) = \frac{7}{3}\begin{bmatrix}1\\1\end{bmatrix}$

$\Sigma_2 = \frac{1}{3}\sum_{i=4}^{6}(x_i-\mu_2)(x_i-\mu_2)^T = \frac{1}{9}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$ $p(w_2) = \frac{3}{6} = \frac{1}{2}$

obviously, $\Sigma_1 = \Sigma_2 = \frac{1}{9}\begin{bmatrix}2 & -1\\-1 & 2\end{bmatrix}$ $p(w_1) = p(w_2)$

simplify $g_i(x) = -\frac{1}{2}(x-\mu_i)^T\Sigma^{-1}(x-\mu_i)$

$= -\frac{1}{2}x^T\Sigma^{-1}x + \mu_i^T\Sigma^{-1}x - \frac{1}{2}\mu_i^T\Sigma^{-1}\mu_i$

$-\frac{1}{2}x^T\Sigma^{-1}x$ is irrelative to i

$\therefore g_i(x) = \mu_i^T\Sigma x - \frac{1}{2}\mu_i^T\Sigma^{-1}\mu_i$

decide $w_i = \underset{w_i}{\operatorname{argmax}}\ g_i(x)$