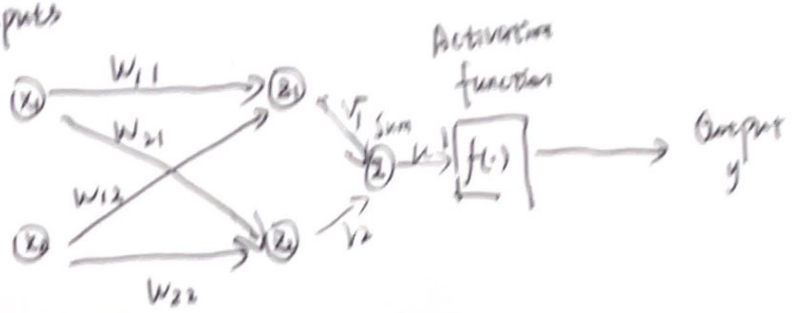
EE6222 2024-2025-S1

3. (a) inputs



(b) $z_j = f(w_{j1}x_1 + w_{j2}x_2)$

$y = g(v_1 z_1 + v_2 z_2)$

$$z_1 = f(w_{11}x_1 + w_{12}x_2) = \frac{1}{1+\exp(-w_{11}x_1 - w_{12}x_2)}$$

$$z_1 = f(w_{21}x_1 + w_{22}x_2) = \frac{1}{1+e(-w_{21}x_1 - w_{22}x_2)}$$

$$y = v_1 z_1 + v_2 z_2 = \frac{v_1}{1+e(-w_{11}x_1 - w_{12}x_2)} + \frac{v_2}{1+\exp(-w_{21}x_1 - w_{22}x_2)}$$

(c) $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $W = \begin{bmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \end{bmatrix}$ $V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$J(w,v) =$

(d) $E\{\|e\|^2\} = E\{\|t-y\|^2\} = E\{\|t - g(V^T f(W^T x))\|^2\}$

$$= \frac{1}{2}(t-y)^2 = \frac{1}{2}(t-g(z))^2 = \frac{1}{2}\left[t - g\left(\sum_{j=1}^{2} v_j z_j\right)\right]^2$$

$$\frac{\partial J(w,v)}{\partial v_j} = \frac{\partial J(w,v)}{\partial z} \cdot \frac{\partial z}{\partial v_j} = -(t-y)\cdot v_j$$

$$J(w,v) = \frac{1}{2}\left[t - g\left(\sum_{j=1}^{2} v_j f\left(\sum_{i=1}^{2} w_{ji} x_i\right)\right)\right]^2 = \frac{1}{2}\left(t - g\left(\sum_{j=1}^{2} v_j f(q_j)\right)\right)$$

$$\frac{\partial J(w,v)}{\partial w_{ji}} = \frac{\partial J(w,v)}{\partial z} \cdot \frac{\partial z}{\partial z_j} \cdot \frac{\partial z_j}{\partial w_{ji}}$$

$$= -\sum_{j=1}^{2}(t-y)\cdot v_j \cdot f(q_j)(1-f(q_j))\, x_i$$

$$= -\sum_{j=1}^{2}(t-y)\cdot v_j \frac{+\exp\left(\sum_{i=1}^{2} w_{ji} x_i\right)}{\left(1+\exp\left(\sum_{i=1}^{2} w_{ji} x_i\right)\right)^2} \cdot x_i$$

$v_j \Leftarrow v_j + \eta(t-y)v_j$

$w_{ji} \Leftarrow w_{ji} + \eta\left[\sum_{j=1}^{2}(t-y)\cdot v_j \frac{\exp\left(-\sum_{i=1}^{2} w_{ji} x_i\right)}{1+\exp\left(-\sum_{i=1}^{2} w_{ji} x_i\right)}\right] x_i$