EE 6222 2023-2024 - S1

2. (a) $g_i(x) = \ln p(x)$

$$= -\frac{1}{2}(x-\mu)^T\Sigma_i^{-1}(x-\mu) - \frac{n}{2}\ln 2\pi - \frac{1}{2}|\Sigma_i|$$

$$\Rightarrow g_i(x) = -(x-\mu_i)^T\Sigma_i^{-1}(x-\mu) - |\Sigma_i|$$

If Σ_i is diagonal, $\Sigma_i = \Lambda = \mathrm{diag}\{\sigma_{i1}^2, \sigma_{i2}^2, \cdots \sigma_{in}^2\}$

$$\Sigma_i^{-1} = \mathrm{diag}\{\frac{1}{\sigma_{i1}^2}, \frac{1}{\sigma_{i2}^2}, \cdots, \frac{1}{\sigma_{in}^2}\}$$

$$g_i(x) = -\sum_{j=1}^{n}(x_j-\mu_{ij})^2/\sigma_{ij}^2 \underbrace{- |\Sigma_i|}_{= +C_i}$$

$\therefore \Sigma_i$ should be diagonal

(b) in (a) we have:

$$g_i(x) = -x^T\Sigma_i^{-1}x + \mu_i^T\Sigma_i^{-1}x - \mu_i^T\Sigma_i\mu_i - |\Sigma_i|$$

when Σ_i are the same, $\Sigma_1 = \Sigma_2 = \cdots = \Sigma_n = \Sigma$

$g_i(x)$ can be: $g_i(x) = \mu_i^T\Sigma^{-1}x - \mu_i^T\Sigma^{-1}\mu_i - |\Sigma_i|$

$$= w_i^T x + b_i$$

$\therefore \Sigma$ are the same for every classes

(c) the first classifier is non-linear. require each μ_{ij} and σ_{ij}^2; and the second is linear, only dot product and bias are required, and easy to interpret.