EE7403 2022-2023-S2

4. (a) w_1 for male students. w_2 for female students

$$p(w_1)=\frac{300}{300+700}=0.3 \qquad p(w_2)=\frac{700}{300+700}=0.7$$

$$p(e_1)=1-p(w_1)=0.7 \qquad p(e_2)=1-p(w_2)=0.3$$

My decision is female, and $p(e)=p(e_2)=0.3$

(b) assume the distributions of w_1 and w_2 have the same σ and satisfy $X\sim N(\mu_1,\sigma^2)$, $X\sim N(\mu_2,\sigma^2)$ respectively

$$p(x|w_1)=\frac{1}{\sqrt{2\pi}\,\sigma}\exp\left(-\frac{(x-\mu_1)^2}{2\sigma^2}\right)$$

$$p(x|w_2)=\frac{1}{\sqrt{2\pi}\,\sigma}\exp\left(-\frac{(x-\mu_2)^2}{2\sigma^2}\right)$$

as σ^2 is the same. the prior probability is decide on $|x-\mu_i|$. $|x-\mu_i|\uparrow$ $p(x|w_i)\downarrow$

$|x-\mu_1|=|1.67-1.7|=0.03$ $\quad |x-\mu_2|=|1.67-1.62|=0.05$ $\quad \therefore p(x|w_1)>p(x|w_2)$

$$p(w_1|x)=\frac{p(x|w_1)p(w_1)}{p(x)} \qquad p(w_2|x)=\frac{p(x|w_2)p(w_2)}{p(x)}$$

since $p(w_1)=p(w_2)=0.5$ $\quad p(x)$ is a constant.

$p(x|w_1)>p(x|w_2)$ $\quad \therefore p(w_1|x)>p(w_2|x)$ decide w_1.

the student's male.

(c) $$p(x|w_1)=\frac{1}{\sqrt{2\pi}\times 0.2}\exp\left[-\frac{(x-1.7)^2}{0.08}\right]$$

$$p(x|w_2)=\frac{1}{\sqrt{2\pi}\times 0.3}\exp\left[-\frac{(x-1.62)^2}{0.18}\right]$$

$x=1.67$ $\quad p(1.67|w_1)\simeq 1.97$ $\quad p(1.67/w_2)\simeq 1.31$

$p(w_1)=0.3$ $\quad p(w_2)=0.7$

since $p(x)$ is constant

decide $w_i=\underset{w_i}{\operatorname{argmax}}\ p(x|w_i)p(w_i)$

$p(w_1)p(x|w_1)=0.3\times 1.97=0.59$

$p(w_2)p(x|w_2)=0.7\times 1.31=0.92$

$p(w_2)p(x|w_2)>p(w_1)p(x|w_1)$

$\therefore$ decide w_2 the student's female.