Math 128A HW1

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Section 1.1

Problem 2c

Proposition 1. *$f(x) = -3 \cdot tan(2x) + x = 0$ has at least one solution for $x \in [0, 1]$.*

Proof. Note that the interval is end point inclusive. We have $f(0) = 0$, which is immediately one solution to the equation. □

Problem 2d

Proposition 2. *$f(x) = ln(x) - x^2 + \frac{5}{2}x - 1 = 0$ has at least one solution for $x \in [\frac{1}{2}, 1]$.*

Proof. $f(\frac{1}{2}) \approx -0.693, f(1) = 0.5$. Hence, by the intermediate value theorem, there exists a solution in the interval. □

Problem 4d

Find interval containing solutions to $x^3 + 4.001x^2 + 4.002x + 1.101 = 0$.

Problem 6a

Find $max_{a \le x \le b}|f(x)|$ for $f(x) = \frac{2x}{x^2+1}$ on $[0, 2]$.

Solution. We proceed by finding the critical points of $f(x)$ on $[0, 2]$.

$$\begin{aligned} f'(x) &= \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2} \\ &= \frac{2 - 2x^2}{(x^2+1)^2} \\ &= 0 \text{ when } x = \pm 1. \end{aligned}$$

Then, we have $f(0) = 0, f(1) = 1, f(2) = \frac{4}{5}$. Hence, the maximum value of $f(x)$ on $[0, 2]$ is 1 when $x = 1$. □

Problem 14

Let $f(x) = 2x \cdot cos(2x) - (x-2)^2$ and $x_0 = 0$.

(a) Find the third Taylor polynomial $P_3(x)$ and use it to approximate $f(0.4)$.

Solution.

$$\begin{aligned} f'(x) &= 2cos(2x) - 4xsin(2x) - 2(x-2), \\ f''(x) &= -8sin(2x) - 8xcos(2x) - 2 \\ f'''(x) &= 16xsin(2x) - 24cos(2x). \end{aligned}$$

Now, we have $f(0) = -4, f'(0) = 6, f''(0) = -2, f'''(0) = -24$. Hence, the third Taylor polynomial $P_3(x) = -4 + 6x - x^2 - 4x^3$, and $f(0.4) \approx -2.016$. □

(b) Use the error formula in Taylor's Theorem to find an upper bound for the error $|f(0.4) - P_3(0.4)|$

Solution.

$$\begin{aligned} f^4(x) &= 64sin(2x) + 32xcos(2x) \\ R_3(x) &= \frac{f^4(\xi(x))}{4!}(0.4)^4 \qquad \textit{for } 0 \le \xi(x) \le 0.4. \end{aligned}$$

Hence,

$$\begin{aligned} |f(0.4) - P_3(0.4)| = |R_3(0.4)| &= \frac{f^4(\xi(x))}{4!}(0.4)^4 \\ &= \frac{64sin(2\xi(x)) - 32 \cdot \xi(x)cos(2\xi(x))}{24} \times 0.0256 \\ &\le \frac{64 - 32 \cdot \xi(x)}{24} \times 0.0256 \qquad (\textit{notice } 0 \le sin(2\xi(x)), cos(2\xi(x)) \le 1) \\ &\le \frac{64}{24} \times 0.0256 \\ &\le 0.06827. \end{aligned}$$

□

Problem 26

Note that the variable $n-1$ needs to be replace by n.

Section 1.2

Problem 2c

Problem 4b

Problem 12

Problem 22

Section 1.3

Problem 8

Problem 15

Discussion Question 2 (p. 38)

Section 2.1

0.1 Problem 6d

0.2 Problem 8

0.3 Problem 20