Math 180A HW4

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Problem 1.

(a) $E[3X + 2] = 5$

(b) $E[X^2] = 2$

(c) $E[(2X + 1)^2] = E[4X^2 + 4X + 1] = 8 + 4 + 1 = 13$

(d) $Var(-2X+7) = E[(-2X+7)^2] - (E[-2X+7])^2 = E[4X^2 - 28X + 49] - (-2+7)^2 = 8 - 28 + 49 - 25 = 4$

Problem 2.

(a)

$$p_x(k) = \begin{cases} \frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6} = \frac{5}{24}, & 1 \le k \le 4 \\ \frac{1}{12}, & 5 \le k \le 6 \quad ; \forall k \in \mathbb{Z}. \\ 0, & otherwise \end{cases} \tag{1}$$

(b) $E[X] = E[X \le 4] + E[5 \le X \le 6] = \frac{5}{24} \times (1 + 2 + 3 + 4) + \frac{1}{12} \times (5 + 6) = 3.$

(c) $Var(X) = E[X^2] - (E[X])^2 = \frac{5}{24} \times (1 + 4 + 9 + 16) + \frac{1}{12} \times (25 + 36) = \frac{34}{3} - 3^2 = \frac{7}{3}.$

Problem 3.

(a) Let X be random variable of number of correct questions.

$$P(X \ge 3) = P(X = 3) + P(x = 4) \tag{2}$$

$$= \binom{4}{3} \times 0.8^3 \times 0.2 + 0.8^4 \tag{3}$$

$$= 0.8192. \tag{4}$$

(b) $P = \binom{3}{2} \times 0.8^2 \times 0.2 + 0.8^3 = 0.896.$

Problem 4.

(a) Let X be the number of winning games. So the experiment is a $Binomial(20, p)$.

$$P(X \ge 12) = \sum_{k=12}^{20} \binom{20}{k} p^k (1 - p)^{20-k} \tag{5}$$

(b) Let A be at leat one wins.

$$P(A) = 1 - P(A^c) \tag{6}$$

$$= 1 - \left(\sum_{k=0}^{11} \binom{20}{k} p^k (1 - p)^{20-k} \right)^{20} \tag{7}$$

Problem 5.

(a) It is $Binomial(9, \frac{3}{7})$.

$$P(X \geq 1) = 1 - P(X = 0) \tag{8}$$

$$= 1 - \left(\frac{4}{7}\right)^9 \tag{9}$$

$$\approx 0.9935 \tag{10}$$

$$P(X \leq 5) = 1 - \sum_{k=6}^{9} \binom{9}{k} \left(\frac{3}{7}\right)^k \left(\frac{4}{7}\right)^{9-k} \tag{11}$$

$$\approx 0.8653 \tag{12}$$

(b) It is $Geometric(\frac{4}{7})$.

$$P(X \leq 9) = \sum_{k=1}^{9} \left(\frac{3}{7}\right) \left(\frac{4}{7}\right)^{k-1} \tag{13}$$

$$\approx 0.9935 \tag{14}$$

(c) Yes, because $P(X \leq 9)$ can also be written as $1 - P(X > 9) = 1 - \left(\frac{4}{7}\right)^9$.

Problem 6.

(a) X is $Geometric(1/5)$. $p_x(k) = (1/5)(4/5)^{k-1} \ \forall k \in \mathbb{Z}^+$

(b) $E(X) = \frac{1}{\frac{1}{5}} = 5$.

$Var(X) = \frac{1 - \frac{1}{5}}{\left(\frac{1}{5}\right)^2} = 20$.

$\sigma = \sqrt{Var(X)} = \sqrt{20}$.

Problem 7. $Binomial(n, p)$ can be represented by a collection of Indicator Random Variables for which $I_i = I_{i\text{th trial succeds}}$. Thus, $X = I_1 + \cdots + I_p$. Note $P(I_1 = 1) = \cdots = P(I_p = 1) = p$. Therefore,

$$E[X] = np, \tag{15}$$

$$Var(X) = \sum_{k=1}^{n} Var(I_k) \tag{16}$$

$$= \sum_{k=1}^{n} \left(E[I_k^2] - (E[I_k])^2\right) \tag{17}$$

$$= n(p - p^2) \tag{18}$$

$$= np(1 - p) \tag{19}$$