

Math 170A HW3

Neo Lee

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Problem 4. $B = X^\top AX = X^\top R^\top RX = (RX)^\top (RX)$. Let $M = RX$, then $B = (RX)^\top (RX) \Leftrightarrow B = M^\top M$. Since R, X are both invertible with determinant $\neq 0$, M is also invertible with determinant $\neq 0$.
 $B^\top = (M^\top M)^\top = M^\top M = B$. So B is symmetric.

Then let $\vec{x} \neq \vec{0}$. $\vec{x}^\top B \vec{x} = \vec{x}^\top M^\top M \vec{x} = (M\vec{x})^\top M\vec{x} = M\vec{x} \cdot M\vec{x}$. Let $y = M\vec{x}$. Since M is invertible and $\vec{x} \neq \vec{0}$, $y \neq \vec{0}$. Hence, $M\vec{x} \cdot M\vec{x} = y \cdot y > 0$.

Therefore, B is positive definite.