Math 109 HW4

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Problem 8.1

Proposition 1. $g(x,y) = \begin{cases} x & if x \geq y \\ y & if x \leq y \end{cases}$ *is well defined for* $g : \mathbb{R}^2 \to \mathbb{R}$.

Proof. For all $(x,y) \in \mathbb{R}^2$, it is exclusively that $x > y$, $x < y$, or $x = y$. If $x > y$, $g(x,y)$ is uniquely defined as $x \in \mathbb{R}$. If $x < y$, $g(x,y)$ is uniquely defined as $y \in \mathbb{R}$. If $x = y$, $g(x,y)$ is uniquely defined as $x = y \in \mathbb{R}$. $\square$

Proposition 2. *Let* $f(x,y) = \frac{x+y}{2} + \frac{|x-y|}{2}$ *for* $f : \mathbb{R}^2 \to \mathbb{R}$, $f = g$.

Proof. If $x > y$, $f(x,y) = \frac{x+y}{2} + \frac{x-y}{2} = x$. If $x < y$, $f(x,y) = \frac{x+y}{2} + \frac{y-x}{2} = y$. If $x = y$, $f(x,y) = \frac{x+x}{2} + \frac{x-x}{2} = x = y$. Hence, $f(x,y) = g(x,y)$ for all $(x,y) \in \mathbb{R}^2$. $\square$

Problem 8.2

(i) $f \circ f = f(f(x)) = f(x^3) = x^{3^3} = x^9$ for $\mathbb{R} \to \mathbb{R}$.

(ii) $f \circ g = f(g(x)) = f(1-x) = (1-x)^3$ for $\mathbb{R} \to \mathbb{R}$.

(iii) $g \circ f = g(f(x)) = g(x^3) = 1 - x^3$ for $\mathbb{R} \to \mathbb{R}$.

(iv) $g \circ g = g(g(x)) = g(1-x) = 1 - (1-x) = x$ for $\mathbb{R} \to \mathbb{R}$.

$fg(x) = gf(x) \Leftrightarrow (1-x^3) = 1 - x^3 \Leftrightarrow 1 - 3x + 3x^2 - x^3 = 1 - x^3 \Leftrightarrow x(x-1) = 0 \Leftrightarrow x = 0$ or $x = 1$. Hence, $\{x \in \mathbb{R} | fg(x) = gf(x)\} = \{0, 1\}$.

Problem 8.3

(i) $f_1(x) = x$ for $\mathbb{R} \to \mathbb{R}$.

(ii) $f_2(x) = |x|$ for $\mathbb{R} \to \mathbb{R}$.

(iii) $f_3(x) = \begin{cases} x & if x \notin \mathbb{Z} \\ 0.1 & if x \in \mathbb{Z} \end{cases}$ for $\mathbb{R} \to \mathbb{R}$.

(iv) $f_4(x) = \lfloor x \rfloor$ for $\mathbb{R} \to \mathbb{R}$.

Problem 8.5 (i) and (iv) are graphs of a function $f : X \to Y$.

x	$f_i(x)$	$f_{iv}(x)$
a	z	y
b	y	z
c	z	w
d	x	x

For (ii), $\{c\} \times Y$ contains no elements, which means not every element in X is mapped to Y. For (iii), $\{b\} \times Y$ contains more than one element, which mean $f(x)$ is not uniquely defined in Y for $x = b$.

Problem 9.1

Problem 9.2

Problem 9.3

Problem 9.4

Problem 9.6

Problem 14

Problem 15

Problem 16