Math 109 HW1

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(1)

$$((P \wedge (Q \to P)) \vee (Q \wedge (P \to Q))) \to (P \wedge Q)$$

P	Q	$P \wedge (Q \to P)$	$Q \wedge (P \to Q)$	$((P \wedge (Q \to P)) \vee (Q \wedge (P \to Q))) \to (P \wedge Q)$
T	T	T	T	T
T	F	T	F	F
F	T	F	T	F
F	F	F	F	T

(2)

- **Proposition 1.** $(P \to Q) \vee R \equiv P \to (Q \vee R)$

  Proof.

  $$\begin{aligned} (P \to Q) \vee R &\equiv (\neg P \vee Q) \vee R && (1)\\ &\equiv \neg P \vee (Q \vee R) && (2)\\ &\equiv P \to (Q \vee R) && (3) \end{aligned}$$

  □

- **Proposition 2.** $(P \vee Q) \to R \not\equiv P \vee (Q \to R)$

  Proof.

P	Q	R	$(P \vee Q) \to R$	$P \vee (Q \to R)$
T	F	F	F	T

  □

- **Proposition 3.** $(P \to Q) \wedge R \not\equiv P \to (Q \wedge R)$

  Proof.

P	Q	R	$(P \to Q) \wedge R$	$P \to (Q \wedge R)$
F	T	F	F	T

  □

- **Proposition 4.** $(P \wedge Q) \to R \not\equiv P \wedge (Q \to R)$

  Proof.

P	Q	R	$(P \wedge Q) \to R$	$P \wedge (Q \to R)$
F	T	T	T	F

  □

(3)

$$((P \to Q) \land (Q \to R)) \to (R \to P) \equiv ((\neg P \lor Q) \land (\neg Q \lor R)) \to (\neg R \lor P) \tag{4}$$
$$\equiv \neg((\neg P \lor Q) \land (\neg Q \lor R)) \lor (\neg R \lor P) \tag{5}$$
$$\equiv \neg(\neg P \lor Q) \lor \neg(\neg Q \lor R) \lor (\neg R \lor P) \tag{6}$$