Math 181A HW3

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Problem 5.2.6

$$
\begin{aligned}
L(\theta) &= \prod_{i\in[1,4]} \frac{\theta}{2\sqrt{y_i}} e^{-\theta\sqrt{y_i}} \\
l(\theta) &= \sum_{i\in[1,4]} \ln\theta - \ln(2\sqrt{y_i}) - \theta\sqrt{y_i} \\
l'(\theta) &= \sum_{i\in[1,4]} \frac{1}{\theta} - \sqrt{y_i}.
\end{aligned}
$$

Then, set $l'(\theta) = 0$,

$$
\begin{aligned}
l'(\theta) = 0 &= \sum_{i\in[1,4]} \frac{1}{\theta} - \sqrt{y_i} \\
0 &= \frac{4}{\theta} - \sqrt{6.2} - \sqrt{7.0} - \sqrt{2.5} - \sqrt{4.2} \\
\hat{\theta}_{MLE} &= \frac{4}{\sqrt{6.2} + \sqrt{7.0} + \sqrt{2.5} + \sqrt{4.2}} \\
&\approx 0.456.
\end{aligned}
$$

Problem 5.2.12

$$
L(\theta) = \prod_{i\in[1,n]} \frac{2y_i}{\theta^2}.
$$

We realize that $L(\theta)$ is a strictly decreasing function for $\theta \geq 0$. Thus, $L(\theta)$ is maximized when $\theta \to 0$. However, notice that $0 \leq y \leq \theta$, so $L(\theta)$ is maximized when $\theta = max(y_i)$ for $i \in [1, n]$.

Hence, we can write $\hat{\theta}_{MLE} = max(y_i)$.

Problem 5.2.15

$$
\begin{aligned}
L(\alpha, \beta) &= \prod_{i\in[1,n]} \alpha\beta y_i^{\beta-1} e^{-\alpha y_i^{\beta}} \\
l(\alpha, \beta) &= \sum_{i\in[1,n]} \ln\alpha + \ln\beta + (\beta - 1)\ln y_i - \alpha y_i^{\beta}.
\end{aligned}
$$

Then, we differentiate $l(\alpha, \beta)$ with respect to α and set it equal to 0,

$$
\begin{aligned}
\frac{\partial}{\partial\alpha} l(\alpha, \beta) &= \sum_{i\in[1,n]} \frac{1}{\alpha} - y_i^{\beta} = 0 \\
\frac{n}{\alpha} &= \sum_{i\in[1,n]} y_i^{\beta} \\
\hat{\alpha}_{MLE} &= \frac{n}{\sum\limits_{i\in[1,n]} y_i^{\beta}}.
\end{aligned}
$$

Problem 5.3.1 $\bar{y} \sim N(107.5, \frac{15}{\sqrt{50}})$.

$$\begin{aligned} CI &= \bar{y} \pm z_{\alpha/2}\frac{15}{\sqrt{50}} \\ CI &= 107.9 \pm 1.96\frac{15}{\sqrt{50}} \\ CI &\approx 107.9 \pm 4.16. \end{aligned}$$

Hence, the 95% confidence interval is $(103.74, 112.06)$.

Problem 5.3.2 Let X_i be the FEV_1/VC ratio of the ith subject. Then,

$$\begin{aligned} \bar{X} &= \frac{1}{19}\sum_{i=1}^{19} x_i \\ &= \frac{1}{19}(0.61 + \cdots + 0.85 + 0.87) \\ &\approx 0.766. \end{aligned}$$

Then, we can compute the standard error of $\bar{X}$,

$$\begin{aligned} \sigma_{\bar{X}} &= \frac{\sigma}{\sqrt{n}} \\ &= \frac{0.09}{\sqrt{19}} \\ &\approx 0.0206. \end{aligned}$$

Hence, the 95% confidencen interval is

$$\begin{aligned} CI &= \bar{X} \pm z_{\alpha/2}\sigma_{\bar{X}} \\ &= 0.766 \pm 1.96 \times 0.0206 \\ &\approx 0.766 \pm 0.0404 \\ &= (0.726, 0.806). \end{aligned}$$

Therefore, the norm 0.80 still falls within our 95% confidence interval, and it is believable that the exposure has not particular effect.