Math 110 HW2

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Problem 1.

Suppose $U := \{(x, x, 3x) : x \in \mathbb{R}\}$ and $W := \{(x, -x, -3x) : x \in \mathbb{R}\}$.

(a)

Proposition 1. *U and W are subspaces of $\mathbb{R}^3$.*

(b) Describe $U + W$ using symbols.

(c) Describe $U + W$ without symbols.

Problem 2.

Suppose $\mathbb{F} = \mathbb{R}$ or $\mathbb{F} = \mathbb{C}$ and let

$$U = \{(x, y, x + y, -y, -x) \in \mathbb{F}^5 : x, y \in \mathbb{F}\}.$$

Find three subspaces W_1, W_2, W_3 of $\mathbb{F}^5$, none of which equals $\{0\}$, such that $\mathbb{F}^5 = U \oplus W_1 \oplus W_2 \oplus W_3$.

Problem 3.

Proposition 2. *Let V be a vector space over $\mathbb{F}$. Suppose that $1 + 1 \neq 0$ in $\mathbb{F}$ and the list v_1, v_2, v_3, v_4 is linearly independent in V. Then the list $v_1 - v_2, v_1 + v_2, v_3 - v_2, v_4 - v_1$ is also linearly independent in $\mathbb{V}$.*

Problem 4.

Does the statement of Problem 3 still hold if we replace "linearly independent" by "a basis"?

Problem 5.

Proposition 3. *The space $\mathbb{R}^{[0,1]}$ is infinite-dimensional.*