Math 109 HW3

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Problem 6.5

(i)

Proposition 1. $A \subseteq B \Leftrightarrow A \cup B = B$.

Proof. $(\Rightarrow; A \cup B \subseteq B)$ $\forall x \in A \cup B, x \in B$ because $A \subseteq B$.

$(\Rightarrow; B \subseteq A \cup B)$ By definition, $\forall y \in B, y \in B \cup S$ for any arbitrary set S. Therefore, $B \subseteq A \cup B$.

Since $A \cup B \subseteq B$ and $B \subseteq A \cup B$, $A \cup B = B$, and $(\Rightarrow)$ is proved.

$(\Leftarrow)$ By definition, $\forall z \in A, z \in A \cup S$ for any arbitrary set S, which means $A \subseteq A \cup S$. Hence, $A \subseteq A \cup B$, which is equivalent to $A \subseteq B$. □

(ii)

Proposition 2. $A \subseteq B \Leftrightarrow A \cap B = A$.

Proof. $(\Rightarrow; A \cap B \subseteq A)$ By definition, $\forall x \in A \cap B, x \in A$, thus $A \cap B \subseteq A$.

$(\Rightarrow; A \subseteq A \cap B)$ $\forall y \in A, x \in A \cap B$ because $A \subseteq B$.

Since $A \cap B \subseteq A$ and $A \subseteq A \cap B$, $A \cap B = A$, and $(\Rightarrow)$ is proved.

$(\Leftarrow)$ By definition, $(B \cap S) \subseteq B$ for any arbitrary set S. Hence, $A = A \cap B \subseteq B$. □

Problem 6.6

Proposition 3. *If $A \cap B \subseteq C$ and $x \in B$, then $x \notin A - C$.*

Proof. Assume to the contrary that if $A \cap B \subseteq C$ and $x \in B$, then $x \in A - C$. It means that $x \in A$ and $x \notin C$. Since $A \cap B \subseteq C$, $x \notin C \Rightarrow x \notin A \cap B$. We know $x \in A$ and $x \notin A \cap B$, therefore, $x \in A \cap B^c$. It means $x \in B^c \Rightarrow x \notin B$, which contradicts that $x \in B$. □

Problem 6.7

Proposition 4. *For subsets of a universal set U, $A \subseteq B$ if and only if $B^c \subseteq A^c$.*

Proof. $A \subseteq B$ means that for an arbitrary x, if $x \in A$, then $x \in B$. Logically, it is equivalent to its contrapositive, which states for an arbitrary x, if $x \notin B$, then $x \notin A$. $x \notin B$ can be written as $x \in B^c$, and $x \notin A$ can be written as $x \in A^c$. Therefore, the entire statement can be rewritten as for an arbitrary x, if $x \in B^c$, then $x \in A^c$, which is the definition of $B^c \subseteq A^c$. □

Problem 7.1

(i) $m = \mathbb{Z}^+$

(ii) $m = \{1\}$

(iii) $m = \mathbb{Z}^+$

(iv) $n = \emptyset$

Problem 7.2

Problem 7.4

Problem 7.7

Page 115 Problem 4

Page 117 Problem 13