Math 180A HW2

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Problem 4.

$$
\begin{aligned}
P(A^c|B) &= \frac{P(A^c \cap B)}{P(B)} && (1)\\
&= \frac{P(B) - P(A \cap B)}{P(B)} && (2)\\
&= 1 - \frac{P(A \cap B)}{P(B)} && (3)\\
&= 1 - P(A|B) && (4)
\end{aligned}
$$

From (1) to (2), note that $P(B) = P(A \cap B) + P(A^c \cap B)$, thus $P(A^c \cap B) = P(B) - P(A \cap B)$.

Problem 5.

(a) $\Omega_{coin} = \{head, tail\}, P_{coin} = \frac{1}{2}$; $\Omega_{4die} = \{1, 2, 3, 4\}, P_{4die} = \frac{1}{4}$; $\Omega_{6die} = \{1, 2, 3, 4, 5, 6\}, P_{6die} = \frac{1}{6}$

(b) Let A_1 and A_2 be the event with die roll 1 and 2 respectively, B_1 and B_2 be the coin event with head and tail respectively. Note that A_1, A_2 and B_1, B_2 are disjoint, and $B_1 \cup B_2 = \Omega_{coin}$.

$$
\begin{aligned}
P(A_1 \cup A_2) &= P(A_1) + P(A_2) && (5)\\
&= (P(B_1)P(A_1|B_1) + P(B_2)P(A_1|B_2)) + (P(B_1)P(A_2|B_1) + P(B_2)P(A_2|B_2)) && (6)\\
&= (\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6}) + (\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6}) && (7)\\
&= \frac{5}{12} && (8)
\end{aligned}
$$

Problem 6. Let A be the event of scoring two points, B_1 be the event of making the shot, B_2 be the event of missing the shot and not geeting fouled, B_3 be the event of missing the shot and getting fouled. Note that B_1, B_2, and B_3 are pairwise disjoint, and $B_1 \cup B_2 \cup B_3 = \Omega$.

$$
\begin{aligned}
P(A) &= P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3) && (9)\\
&= 50\% \times 100\% + 25\% \times 0\% + 25\% \times 77\% && (10)\\
&= 69.25\% && (11)
\end{aligned}
$$

Problem 7.

(a) Let A be the event of the contestant hitting the bullseye on their first shot, B be the event that the contestant is a Merry Man.

$$
\begin{align}
P(B|A) &= \frac{P(A \cap B)}{P(A)} \tag{12} \\
&= \frac{P(B)P(A|B)}{P(B)P(A|B) + P(B^c)P(A|B^c)} \tag{13} \\
&= \frac{0.25 \times 0.9}{0.25 \times 0.9 + 0.75 \times 0.1} \tag{14} \\
&= 0.75 \tag{15}
\end{align}
$$

(b) Let A be the event that the chosen contestant is a villager, B be the event that the chosen contestant is a Merry Man, C be the event of the contestant missing the second shot. Note that A and B are disjoint, and $A + B = \Omega$.

Note that for both villager and Merry Man, the first and second shot are independent, thus $P(C|B) = 0.1$ and $P(C|A) = 0.9$ is always true.

$$
\begin{align}
P(C) &= P(A)P(C|A) + P(B)P(C|B) \tag{16} \\
&= 0.75 \times 0.9 + 0.25 \times 0.1 \tag{17} \\
&= 0.7 \tag{18}
\end{align}
$$