Math 180A HW2

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Problem 4.

$$\begin{align}
P(A^c|B) &= \frac{P(A^c \cap B)}{P(B)} \tag{1}\\
&= \frac{P(B) - P(A \cap B)}{P(B)} \tag{2}\\
&= 1 - \frac{P(A \cap B)}{P(B)} \tag{3}\\
&= 1 - P(A|B) \tag{4}
\end{align}$$

From (1) to (2), note that $P(B) = P(A \cap B) + P(A^c \cap B)$, thus $P(A^c \cap B) = P(B) - P(A \cap B)$.

Problem 5.

(a) $\Omega = \{(head, i) : 1 \leq i \leq 4, i \in \mathbb{Z}\} \cup \{(tail, i) : 1 \leq i \leq 6, i \in \mathbb{Z}\}$. $P = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ for head or $P = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$ for tail.

(b) Let A_1 and A_2 be the event with die roll 1 and 2 respectively, B_1 and B_2 be the coin event with head and tail respectively. Note that A_1, A_2 and B_1, B_2 are disjoint, and $B_1 \cup B_2 = \Omega_{coin}$.

$$\begin{align}
P(A_1 \cup A_2) &= P(A_1) + P(A_2) \tag{5}\\
&= (P(B_1)P(A_1|B_1) + P(B_2)P(A_1|B_2)) + (P(B_1)P(A_2|B_1) + P(B_2)P(A_2|B_2)) \tag{6}\\
&= (\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6}) + (\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{6}) \tag{7}\\
&= \frac{5}{12} \tag{8}
\end{align}$$

Problem 6. Let A be the event of scoring two points, B_1 be the event of making the shot, B_2 be the event of missing the shot and not geeting fouled, B_3 be the event of missing the shot and getting fouled. Note that B_1, B_2, and B_3 are disjoint, and $B_1 \cup B_2 \cup B_3 = \Omega$.

$$\begin{align}
P(A) &= P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3) \tag{9}\\
&= 50\% \times 100\% + 25\% \times 0\% + 25\% \times 77\% \tag{10}\\
&= 69.25\% \tag{11}
\end{align}$$

Problem 7.

(a) Let A be the event of the contestant hitting the bullseye on their first shot, B be the event that the contestant is a Merry Man.

$$\begin{aligned}
P(B|A) &= \frac{P(A \cap B)}{P(A)} && (12) \\
&= \frac{P(B)P(A|B)}{P(B)P(A|B) + P(B^c)P(A|B^c)} && (13) \\
&= \frac{0.25 \times 0.9}{0.25 \times 0.9 + 0.75 \times 0.1} && (14) \\
&= 0.75 && (15)
\end{aligned}$$

(b) Let A be the event that the chosen contestant is a Merry Man, B be the event that the first shot missed, C be the event that the second shot missed.

$$\begin{aligned}
P(C|B) &= \frac{P(B \cap C)}{P(B)} && (16) \\
&= \frac{P((B \cap C) \cap A) \cup ((B \cap C) \cap A^c)}{P(A)P(B|A) + P(A^c)P(B|A^c)} && (17) \\
&= \frac{P(C \cap B \cap A) + P(C \cap B \cap A^c)}{P(A)P(B|A) + P(A^c)P(B|A^c)} && (18) \\
&= \frac{P(A)P(C \cap B|A) + P(A^c)P(C \cap B|A^c)}{P(A)P(B|A) + P(A^c)P(B|A^c)} && (19) \\
&= \frac{0.25 \times 0.1 \times 0.1 + 0.75 \times 0.9 \times 0.9}{0.25 \times 0.1 + 0.75 \times 0.9} && (20) \\
&= \frac{61}{70} && (21)
\end{aligned}$$

Note that from (17) to (18), $B \cup C \cup A$ and $C \cup B \cup A^c$ are disjoint.