Math 104 HW2

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Exercise 4.1

For each set below that is bounded above, list three upper bounds for the set. Otherwise write "NOT BOUNDED ABOVE".

(a) [0,1]
(c) {2,7}
(e) $\{\frac{1}{n} : n \in \mathbb{N}\}$
(g) [0,1] ∪ [2,3]
(i) $\cap_{n=1}^{\infty} \left[\frac{-1}{n}, 1 + \frac{1}{n}\right]$
(k) $\{n + \frac{(-1)^n}{n} : n \in \mathbb{N}\}$
(m) $\{r \in \mathbb{Q} : r^2 < 4\}$
(o) $\{x \in \mathbb{R} : x < 0\}$
(q) {0, 1, 2, 4, 8, 16}
(s) $\{\frac{1}{n} : n \in \mathbb{N}$ *and* n *is prime*$\}$
(u) $\{x^2 : x \in \mathbb{R}\}$
(w) $\{sin\left(\frac{n\pi}{3}\right) : n \in \mathbb{N}\}$

Exercise 4.2

Repeat Exercise 4.1 for lower bounds.

Exercise 4.8

Let S and T be nonempty subsets of R with the following property: $s \leq t$ for all $s \in S$ and $t \in T$.

(a) Oberserve that S is bounded above and T is bounded below.

(b)

Proposition 1. *Sup* $S \leq$ *inf* T.

(c) Give an exampe of such sets S and T where $S \cap T$ is nonempty.

(d) Give an example of sets S and T where sup S = inf T and $S \cap T$ is an empty set.

Exercise 4.14

Let A and B be nonempty bounded subsets of $\mathbb{R}$, and let $A+B$ be the set of all sums $a+b$ where $a \in A$ and $b \in B$.

(a)

Proposition 2. *Sup* $(A+B) = sup\ A + sup\ B$. *Hint: To show sup* A + *sup* $B \leq sup\ (A+B)$, *show that for each* $b \in B$, *sup* $(A+B) - b$ *is an upper bound for* A, *hence sup Ale sup* $(A+B) - b$. *Then show sup* $(A+B)$ - *sup* A *is an upper bound for* B.

(b)

Proposition 3. *Inf* $(A+B) = inf\ A + inf\ B$.

Exercise 4.16

Proposition 4. *Sup* $\{r \in \mathbb{Q} : r < a\} = a$ *for each* $a \in \mathbb{R}$.

Exercise 5.5

Proposition 5. *Inf* $S \leq sup\ S$ *for every nonempty subset of* $\mathbb{R}$. *Consider both bounded and unbounded sets.*