Pine Age 1 Analysis

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Introduction

This paper aims to analyze a game that is often implemented as a reward system and try to find out the best strategy to maximize the reward. The rules are as follow:

1. There are in total 12 rounds, and at the end of each round, 250,000 tokens (T) will be distributed to the players.
2. In each round, all players will bet a certain amount of T into a pool that is independent of the 250,000T pool. All the betted T by players are unredeemable.
3. The amount of T that each participant receives at the end of each round is proportional to the amount that they betted in the pool. For example, if player 1 betted 10T while others betted 40T in total, player 1 will receive $\frac{10}{10+40} = \frac{1}{5}$ of the 250,000T, which is 50,000T.
4. The betting pool is cumulative. For example, if player 1 betted 10T and others betted 40T in first round, then player 1 betted 20T and others betted 20T in second round, player 1 will receive $\frac{1}{5}$ of 250,000T in first round and $\frac{10+20}{10+40+20+20} = \frac{1}{3}$ of 250,000T in second round.
5. The profit that a player gains at the end of i-th round can be formulated as:

$$T_{i,profit} = 250{,}000 \times \frac{\sum_{k=1}^{i} T_{k,player\ bet}}{\sum_{k=1}^{i} T_{k,total\ bet}} - \sum_{k=1}^{i} T_{k,player\ bet} \tag{1}$$

$$= \sum_{k=1}^{i} T_{k,player\ bet} \times \left(\frac{250{,}000}{\sum_{k=1}^{i} T_{k,total\ bet}} - 1 \right). \tag{2}$$

The total profit can be formulated as:

$$T_{total\ profit} = \sum_{n=1}^{12} \sum_{k=1}^{n} T_{k,player\ bet} \times \left(\frac{250{,}000}{\sum_{k=1}^{i} T_{k,total\ bet}} - 1 \right). \tag{3}$$

Objectives

This section defines the criteria for the "best" strategy. A good strategy should follow the criteria:

1. Maximizes $T_{total\ profit}$ / Minimizes $T_{total\ loss}$.

Possible Approaches

1. Model the game with Monte Carlo method.
2. Model the game with recurrence equation and optimize each round's bet.
3. Model the game with multi-variable equation and optimize with gradient descent.

Tentative Analysis (Aggresive Player)

We can categorize all players into two groups. The first group of players start playing since the first round, while the second group do not join the first round.

Since the game is cumulative, it is natural for the players from first group to bet all their T in first round.

Assumptions (Conservative Player)

1. The game will be analyzed from My perspective as one of the player.
2. The price of T is constant throughout the entire event. Therefore, we will use T as the unit of account for this paper.
3. I and every player never want a loss by the end of ith epoch.
4. I and every player always want to maximize their T_{reward} for each epoch.
5. Let mb be my bet, tb be total bet, ob be others' bet.

First Epoch

Assume there are more than one player in this game, then $T_{tb} > T_{mb}$. Therefore, to maximize T_{reward}, it is natural for me and every player to burn more T to increase T_{mb} thus $\frac{T_{mb}}{T_{tb}}$.

For me to not suffer a loss by the end of the first epoch, it means

$$T_{reward} = 250{,}000 \times \frac{T_{mb}}{T_{tb}} \geq T_{mb} \tag{4}$$

$$\frac{250{,}000}{T_{tb}} \geq 1 \tag{5}$$

Since me and every player always wants to burn more T, T_{tb} will continue to rise until it converges to 250,000. Therefore, we can assume that $T_{1,tb} = 250{,}000$ and $T_{mb} = 250{,}000 - T_{ob}$.

Second Epoch

To not suffer a loss by the end of the second epoch, it means

$$250{,}000 \times \frac{T_{1,mb}}{T_{1,tb}} + 250{,}000 \times \frac{T_{1,mb} + T_{2,mb}}{T_{1,tb} + T_{2,tb}} \geq T_{1,mb} + T_{2,mb} \tag{6}$$

$$T_{1,mb} + 250{,}000 \times \frac{T_{1,mb} + T_{2,mb}}{250{,}000 + T_{2,tb}} \geq T_{1,mb} + T_{2,mb} \tag{7}$$

$$250{,}000 \times \frac{T_{1,mb} + T_{2,mb}}{250{,}000 + T_{2,tb}} \geq T_{2,mb} \tag{8}$$

$$250{,}000 \times T_{1,mb} + 250{,}000 \times T_{2,mb} \geq 250{,}000 \times T_{2,mb} + T_{2,mb} \times T_{2,tb} \tag{9}$$

$$T_{2,mb} \leq \frac{250{,}000 \times T_{1,mb}}{T_{2,tb}} \tag{10}$$

$$T_{2,mb} \leq \frac{250{,}000 \times T_{1,mb}}{T_{2,mb} + T_{ob}} \tag{11}$$

$$(T_{2,mb})^2 + T_{2,ob} \times T_{2,mb} - 250{,}000 \times T_{1,mb} \leq 0 \tag{12}$$