



CS 412 Intro. to Data Mining

Chapter 6. Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

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Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts 
- Efficient Pattern Mining Methods
- Pattern Evaluation
- Summary

Pattern Discovery: Basic Concepts

- What Is Pattern Discovery? Why Is It Important?

- Basic Concepts: Frequent Patterns and Association Rules

- Compressed Representation: Closed Patterns and Max-Patterns

What Is Pattern Discovery?

- **What are patterns?** គំរូ pattern
- **Patterns:** A set of items, subsequences, or substructures that occur frequently together (or strongly correlated) in a data set
- Patterns represent **intrinsic** and **important properties** of datasets
ព័ត៌មានវិទ្យាត្រឹម
- **Pattern discovery:** Uncovering patterns from massive data sets
- Motivation examples:
 - What products were often purchased together?
 - What are the subsequent purchases after buying an iPad?
 - What code segments likely contain copy-and-paste bugs?
 - What word sequences likely form phrases in this corpus?

Pattern Discovery: Why Is It Important?

- Finding inherent regularities in a data set
- Foundation for many essential data mining tasks
 - Association, correlation, and causality analysis
 - Mining sequential, structural (e.g., sub-graph) patterns
 - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
 - Classification: Discriminative pattern-based analysis
 - Cluster analysis: Pattern-based subspace clustering
- Broad applications
 - Market basket analysis, cross-marketing, catalog design, sale campaign analysis, Web log analysis, biological sequence analysis

Basic Concepts: k-Itemsets and Their Supports

- **Itemset**: A set of one or more items
- **k-itemset**: $X = \{x_1, \dots, x_k\}$
 - Ex. {Beer, Nuts, Diaper} is a 3-itemset
- **(absolute) support (count)** of X , $\text{sup}\{X\}$:
Frequency or the number of occurrences of an itemset X
 - Ex. $\text{sup}\{\text{Beer}\} = 3$
 - Ex. $\text{sup}\{\text{Diaper}\} = 4$
 - Ex. $\text{sup}\{\text{Beer, Diaper}\} = 3$
 - Ex. $\text{sup}\{\text{Beer, Eggs}\} = 1$

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk

- **(relative) support**, $s\{X\}$: The fraction of transactions that contains X (i.e., the probability that a transaction contains X)
 - Ex. $s\{\text{Beer}\} = 3/5 = 60\%$
 - Ex. $s\{\text{Diaper}\} = 4/5 = 80\%$
 - Ex. $s\{\text{Beer, Eggs}\} = 1/5 = 20\%$

Basic Concepts: Frequent Itemsets (Patterns)

- An itemset (or a pattern) X is *frequent* if the support of X is no less than a *minsup* threshold σ
- Let $\sigma = 50\%$ (σ : *minsup* threshold)
For the given 5-transaction dataset
 - All the frequent 1-itemsets:
 - Beer: 3/5 (60%); Nuts: 3/5 (60%)
 - Diaper: 4/5 (80%); Eggs: 3/5 (60%)
 - All the frequent 2-itemsets:
 - {Beer, Diaper}: 3/5 (60%)
 - All the frequent 3-itemsets?
 - None

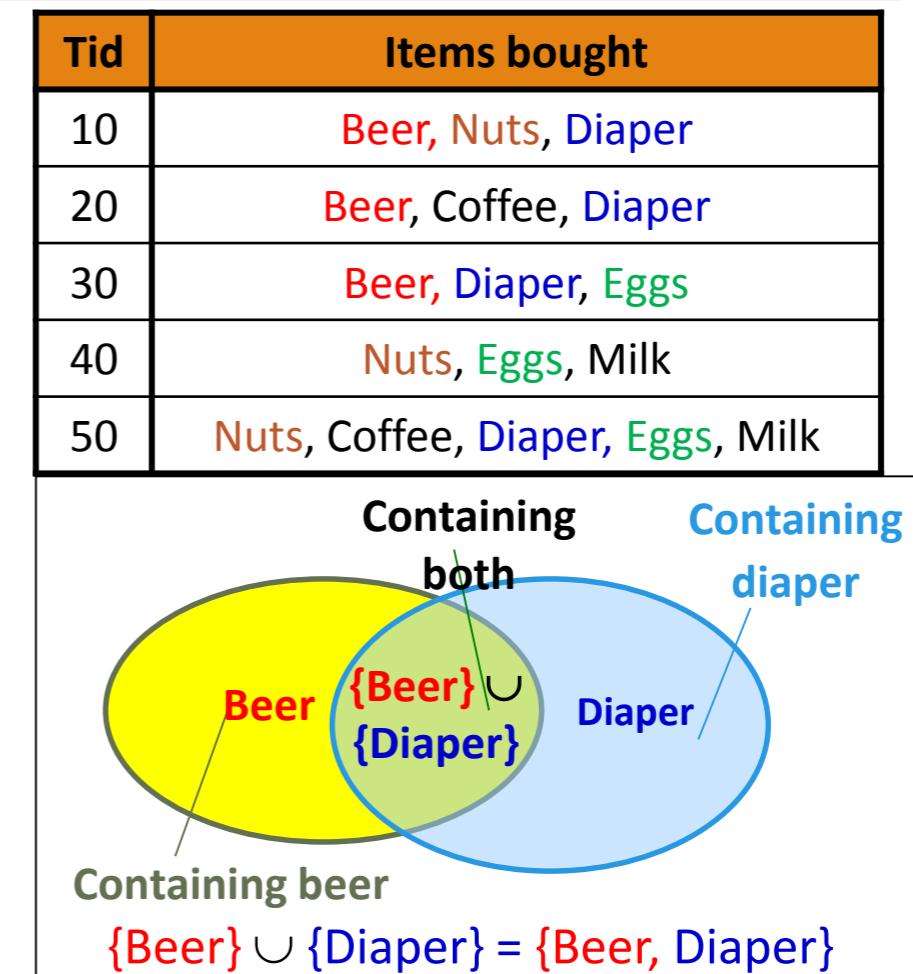


Tid	Items bought
10	Beer, Nuts, Diaper
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50	Nuts, Coffee, Diaper, Eggs, Milk

- Why do these itemsets (shown on the left) form the complete set of frequent k-itemsets (patterns) for any k ?
- Observation:** We may need an efficient method to mine a complete set of frequent patterns

From Frequent Itemsets to Association Rules

- Comparing with itemsets, rules can be more telling
 - Ex. *Diaper → Beer*
 - *Buying diapers may likely lead to buying beers*
- How strong is this rule? (support, confidence)
 - Measuring association rules: $X \rightarrow Y(s, c)$
 - Both X and Y are itemsets
 - **Support**, s : The probability that a transaction contains $X \cup Y$
 - Ex. $s\{\text{Diaper, Beer}\} = 3/5 = 0.6$ (i.e., 60%)
 - **Confidence**, c : The *conditional probability* that a transaction containing X also contains Y
 - Calculation: $c = \text{sup}(X \cup Y) / \text{sup}(X)$
 - Ex. $c = \text{sup}\{\text{Diaper, Beer}\}/\text{sup}\{\text{Diaper}\} = 3/4 = 0.75$



Note: $X \cup Y$: the union of two itemsets
 ■ The set contains both X and Y

Mining Frequent Itemsets and Association Rules

- Association rule mining
 - Given two thresholds: $minsup$, $minconf$
 - Find **all** of the rules, $X \rightarrow Y$ (s, c)
 - such that, $s \geq minsup$ and $c \geq minconf$
- Let $minsup = 50\%$
 - Freq. 1-itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3
 - Freq. 2-itemsets: {Beer, Diaper}: 3
- Let $minconf = 50\%$
 - $Beer \rightarrow Diaper$ (60%, 100%)
 - $Diaper \rightarrow Beer$ (60%, 75%)



Tid	Items bought
10	Beer, Nuts, Diaper
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- Observations:
 - Mining association rules and mining frequent patterns are very close problems
 - Scalable methods are needed for mining large datasets

(Q: Are these all rules?)

Challenge: There Are Too Many Frequent Patterns!

- A long pattern contains a combinatorial number of sub-patterns
- How many frequent itemsets does the following TDB₁ contain?

- TDB₁: T₁: {a₁, ..., a₅₀}; T₂: {a₁, ..., a₁₀₀}
- Assuming (absolute) *minsup* = 1
- Let's have a try

1-itemsets: {a₁} : 2, {a₂} : 2, ..., {a₅₀} : 2, {a₅₁} : 1, ..., {a₁₀₀} : 1,

2-itemsets: {a₁, a₂} : 2, ..., {a₁, a₅₀} : 2, {a₁, a₅₁} : 1, ..., ..., {a₉₉, a₁₀₀} : 1,

..., ..., ..., ...

99-itemsets: {a₁, a₂, ..., a₉₉} : 1, ..., {a₂, a₃, ..., a₁₀₀} : 1

100-itemset: {a₁, a₂, ..., a₁₀₀} : 1

- The total number of frequent itemsets:

$$\binom{100}{1} + \binom{100}{2} + \binom{100}{3} + \dots + \binom{100}{100} = 2^{100} - 1$$

A too huge set for any
one to compute or store!

Expressing Patterns in Compressed Form: Closed Patterns

- How to handle such a challenge?
- Solution 1: **Closed patterns**: A pattern (itemset) X is **closed** if X is *frequent*, and there exists *no super-pattern* $Y \supset X$, *with the same support* as X
 - Let Transaction DB TDB_1 : $T_1: \{a_1, \dots, a_{50}\}$; $T_2: \{a_1, \dots, a_{100}\}$
 - Suppose $minsup = 1$. How many closed patterns does TDB_1 contain?
 - Two: $P_1: \{\{a_1, \dots, a_{50}\}: 2\}$; $P_2: \{\{a_1, \dots, a_{100}\}: 1\}$
 - **Closed pattern** is a **lossless compression** of frequent patterns
 - Reduces the # of patterns but does not lose the support information!
 - You will still be able to say: $\{\{a_2, \dots, a_{40}\}: 2\}$, $\{\{a_5, a_{51}\}: 1\}$

Expressing Patterns in Compressed Form: Max-Patterns

- Solution 2: **Max-patterns**: A pattern X is a **max-pattern** if X is frequent and there exists no frequent super-pattern $Y \supset X$
- Difference from close-patterns?
 - Do not care the real support of the sub-patterns of a max-pattern
 - Let Transaction DB TDB_1 : $T_1: \{a_1, \dots, a_{50}\}$; $T_2: \{a_1, \dots, a_{100}\}$
 - Suppose $minsup = 1$. How many max-patterns does TDB_1 contain?
 - One: $P: \{a_1, \dots, a_{100}\}: 1$
- **Max-pattern is a lossy compression!**
 - We only know $\{a_1, \dots, a_{40}\}$ is frequent
 - But we do not know the real support of $\{a_1, \dots, a_{40}\}, \dots$, any more!
- Thus in many applications, mining close-patterns is more desirable than mining max-patterns

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Efficient Pattern Mining Methods

- The Downward Closure Property of Frequent Patterns
- The Apriori Algorithm
- Extensions or Improvements of Apriori
- Mining Frequent Patterns by Exploring Vertical Data Format
- FP-Growth: A Frequent Pattern-Growth Approach
- Mining Closed Patterns

The Downward Closure Property of Frequent Patterns

- Observation: From TDB₁: T₁: {a₁, ..., a₅₀}; T₂: {a₁, ..., a₁₀₀}
 - We get a frequent itemset: {a₁, ..., a₅₀}
 - Also, its subsets are all frequent: {a₁}, {a₂}, ..., {a₅₀}, {a₁, a₂}, ..., {a₁, ..., a₄₉}, ...
 - There must be some hidden relationships among frequent patterns!
- The **downward closure (also called “Apriori”)** property of frequent patterns
 - If {beer, diaper, nuts} is frequent, so is {beer, diaper}
 - Every transaction containing {beer, diaper, nuts} also contains {beer, diaper}
 - Apriori: Any subset of a frequent itemset must be frequent
- Efficient mining methodology
 - If **any subset of an itemset S** is infrequent, then there is no chance for S to be frequent—why do we even have to consider S!?  A sharp knife for pruning!

Apriori Pruning and Scalable Mining Methods

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not even be generated! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Scalable mining Methods: Three major approaches
 - Level-wise, join-based approach: Apriori (Agrawal & Srikant@VLDB'94)
 - Vertical data format approach: Eclat (Zaki, Parthasarathy, Ogihara, Li @KDD'97)
 - Frequent pattern projection and growth: FPgrowth (Han, Pei, Yin @SIGMOD'00)

Apriori: A Candidate Generation & Test Approach

- Outline of Apriori (level-wise, candidate generation and test)
 - Initially, scan DB once to get frequent 1-itemset
 - **Repeat**
 - Generate length-($k+1$) candidate itemsets from length- k frequent itemsets
 - Test the candidates against DB to find frequent $(k+1)$ -itemsets
 - Set $k := k + 1$
 - **Until** no frequent or candidate set can be generated
 - Return all the frequent itemsets derived

The Apriori Algorithm (Pseudo-Code)

C_k : Candidate itemset of size k

F_k : Frequent itemset of size k

$K := 1;$

$F_k := \{\text{frequent items}\}; \quad // \text{frequent 1-itemset}$

While ($F_k \neq \emptyset$) **do {** $\quad // \text{when } F_k \text{ is non-empty}$

$C_{k+1} := \text{candidates generated from } F_k; \quad // \text{candidate generation}$

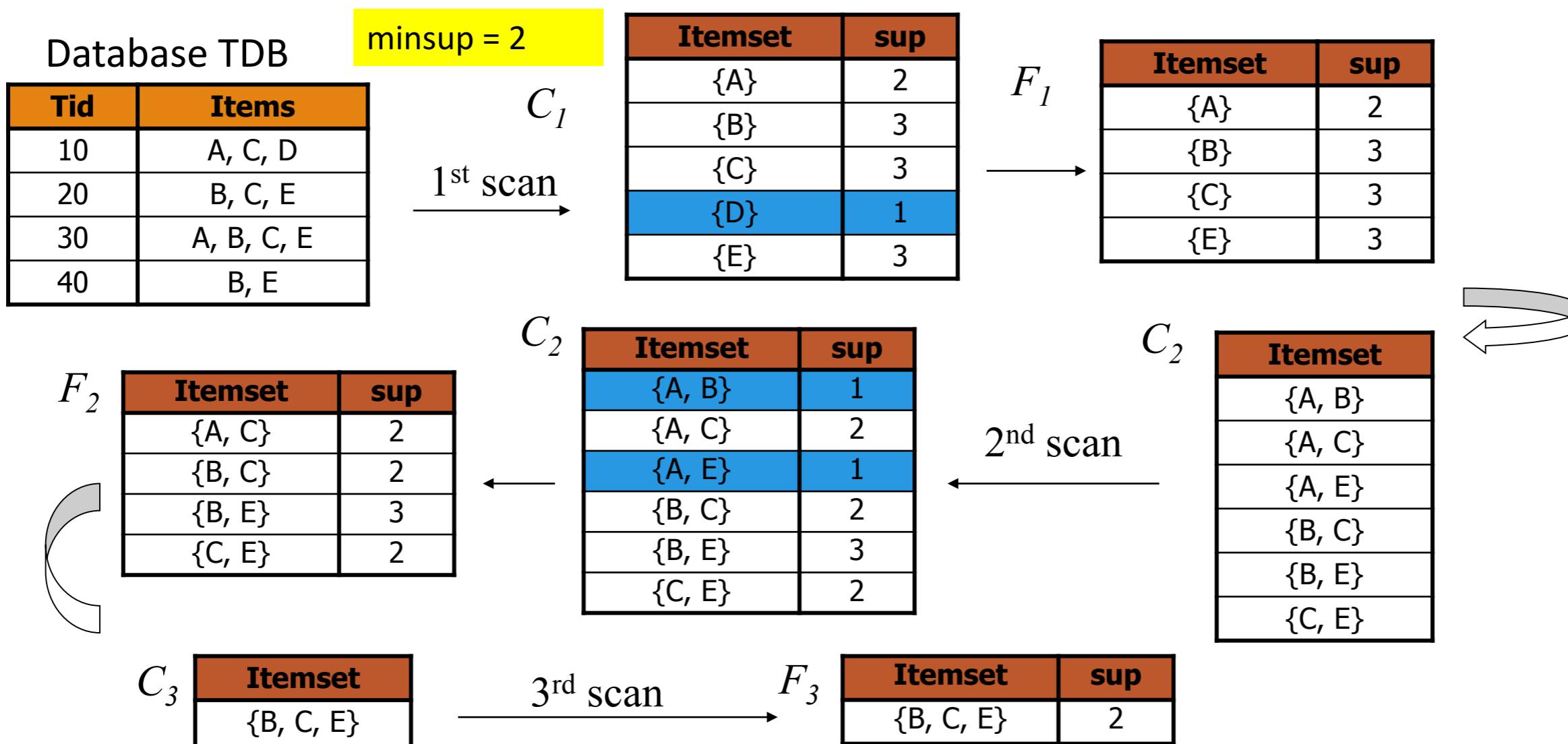
Derive F_{k+1} by counting candidates in C_{k+1} with respect to TDB at minsup ;

$k := k + 1$

}

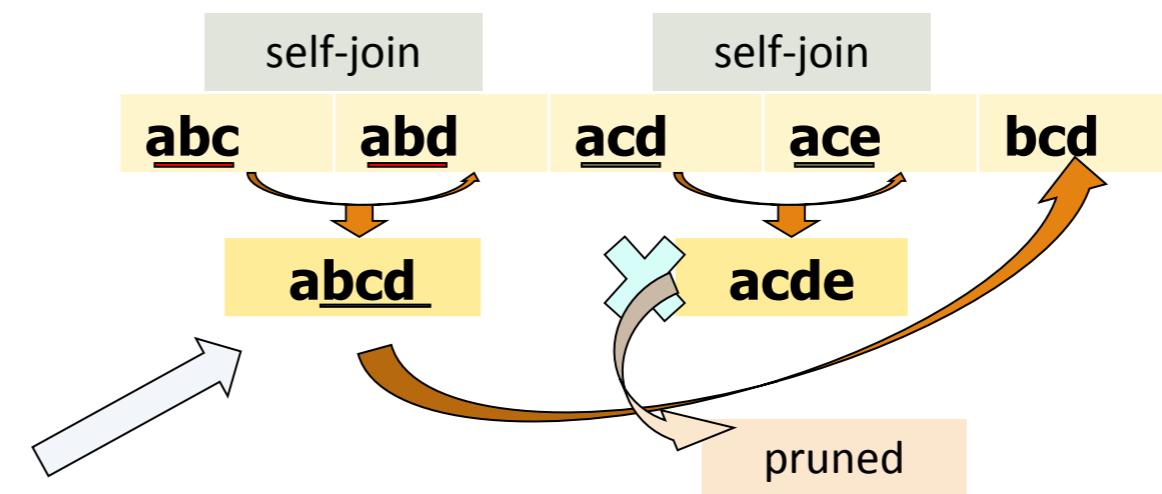
return $\cup_k F_k \quad // \text{return } F_k \text{ generated at each level}$

The Apriori Algorithm—An Example



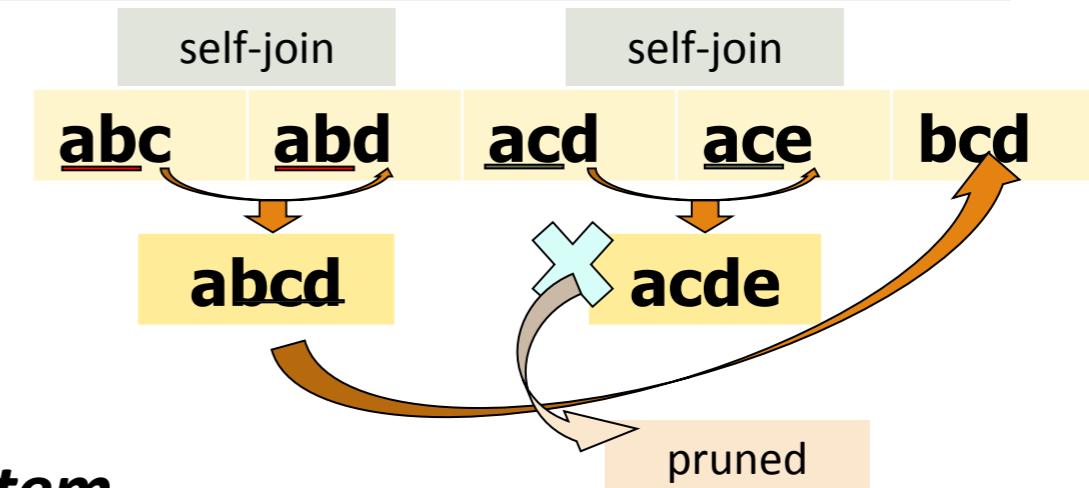
Apriori: Implementation Tricks

- How to generate candidates?
 - Step 1: self-joining F_k
 - Step 2: pruning
- Example of candidate-generation
 - $F_3 = \{abc, abd, acd, ace, bcd\}$
 - Self-joining: $F_3 * F_3$
 - $abcd$ from abc and abd
 - $acde$ from acd and ace
 - Pruning:
 - $acde$ is removed because ade is not in F_3
 - $C_4 = \{abcd\}$



Candidate Generation: An SQL Implementation

- Suppose the items in F_{k-1} are listed in an order
- Step 1: self-joining F_{k-1} insert into C_k
select $p.item_1, p.item_2, \dots, p.item_{k-1}, q.item_{k-1}$
from F_{k-1} as p, F_{k-1} as q
where $p.item_1 = q.item_1, \dots, p.item_{k-2} = q.item_{k-2}, p.item_{k-1} < q.item_{k-1}$
- Step 2: pruning
for all ***itemsets c in C_k*** do
for all ***(k-1)-subsets s of c*** do
 if (s is not in F_{k-1}) then delete c from C_k



Apriori: Improvements and Alternatives

- Reduce passes of transaction database scans
 - Partitioning (e.g., Savasere, et al., 1995)  To be discussed in subsequent slides
 - Dynamic itemset counting (Brin, et al., 1997)
- Shrink the number of candidates
 - Hashing (e.g., DHP: Park, et al., 1995)  To be discussed in subsequent slides
 - Pruning by support lower bounding (e.g., Bayardo 1998)
 - Sampling (e.g., Toivonen, 1996)
- Exploring special data structures
 - Tree projection (Agarwal, et al., 2001)
 - H-miner (Pei, et al., 2001)
 - Hypocube decomposition (e.g., LCM: Uno, et al., 2004)

Partitioning: Scan Database Only Twice

- Theorem: Any itemset that is potentially frequent in TDB must be frequent in at least one of the partitions of TDB

Here is the proof!

$$\begin{matrix} \boxed{} & + & \boxed{} & \dots & \dots & + & \boxed{} \\ \text{TDB}_1 & + & \text{TDB}_2 & + & \dots & + & \text{TDB}_k \\ \text{sup}_1(X) < \sigma|\text{TDB}_1| & & \text{sup}_2(X) < \sigma|\text{TDB}_2| & & \dots & & \text{sup}_k(X) < \sigma|\text{TDB}_k| \\ & & & & & & = \text{sup}(X) < \sigma|\text{TDB}| \end{matrix}$$

- Method: Scan DB twice (A. Savasere, E. Omiecinski and S. Navathe, VLDB'95)
 - Scan 1: Partition database so that each partition can fit in main memory (why?)
 - Mine local frequent patterns in this partition
 - Scan 2: Consolidate global frequent patterns
 - Find global frequent itemset candidates (those frequent in at least one partition)
 - Find the true frequency of those candidates, by scanning TDB_i one more time

Direct Hashing and Pruning (DHP)

- DHP (Direct Hashing and Pruning): (J. Park, M. Chen, and P. Yu, SIGMOD'95)
- Hashing: Different itemsets may have the same hash value: $v = \text{hash}(\text{itemset})$
- 1st scan: When counting the 1-itemset, hash 2-itemset to calculate the bucket count
- Observation: A k -itemset cannot be frequent if its corresponding hashing bucket count is below the minsup threshold
- Example: At the 1st scan of TDB, count 1-itemset, and
 - Hash 2-itemsets in the transaction to its bucket
 - {ab, ad, ce}
 - {bd, be, de}
 - ...
 - At the end of the first scan,
 - if $\text{minsup} = 80$, remove ab, ad, ce, since $\text{count}\{\text{ab, ad, ce}\} < 80$

Itemsets	Count
{ab, ad, ce}	35
{bd, be, de}	298
.....	...
{yz, qs, wt}	58

Hash Table

Exploring Vertical Data Format: ECLAT

- ECLAT (Equivalence Class Transformation): A depth-first search algorithm using set intersection [Zaki et al. @KDD'97]
- Tid-List: List of transaction-ids containing an itemset
- Vertical format: $t(e) = \{T_{10}, T_{20}, T_{30}\}$; $t(a) = \{T_{10}, T_{20}\}$; $t(ce) = \{T_{10}, T_{20}\}$
- Properties of Tid-Lists
 - $t(X) = t(Y)$: X and Y always happen together (e.g., $t(ac) = t(d)$)
 - $t(X) \subset t(Y)$: transaction having X always has Y (e.g., $t(ac) \subset t(ce)$)
- Deriving frequent patterns based on vertical intersections
- Using **diffset** to accelerate mining
 - Only keep track of differences of tids
 - $t(e) = \{T_{10}, T_{20}, T_{30}\}$, $t(ce) = \{T_{10}, T_{30}\} \rightarrow \text{Diffset } (ce, e) = \{T_{20}\}$

A transaction DB in Horizontal Data Format

Tid	Itemset
10	a, c, d, e
20	a, b, e
30	b, c, e

The transaction DB in Vertical Data Format

Item	TidList
a	10, 20
b	20, 30
c	10, 30
d	10
e	10, 20, 30

Why Mining Frequent Patterns by Pattern Growth?

- ❑ Apriori: A *breadth-first search* mining algorithm
 - ❑ First find the complete set of frequent k-itemsets
 - ❑ Then derive frequent (k+1)-itemset candidates
 - ❑ Scan DB again to find true frequent (k+1)-itemsets
- ❑ Motivation for a different mining methodology
 - ❑ Can we develop a *depth-first search* mining algorithm?
 - ❑ For a frequent itemset ρ , can subsequent search be confined to only those transactions that contain ρ ?
- ❑ Such thinking leads to a frequent pattern growth approach:
 - ❑ FP-Growth (J. Han, J. Pei, Y. Yin, “Mining Frequent Patterns without Candidate Generation,” SIGMOD 2000)

Example: Construct FP-tree from a Transaction DB

TID	Items in the Transaction	Ordered, frequent itemlist
100	{f, a, c, d, g, i, m, p}	f, c, a, m, p
200	{a, b, c, f, l, m, o}	f, c, a, b, m
300	{b, f, h, j, o, w}	f, b
400	{b, c, k, s, p}	c, b, p
500	{a, f, c, e, l, p, m, n}	f, c, a, m, p

After inserting the 1st frequent Itemlist: "f, c, a, m, p"

1. Scan DB once, find single item frequent pattern:

Let min_support = 3

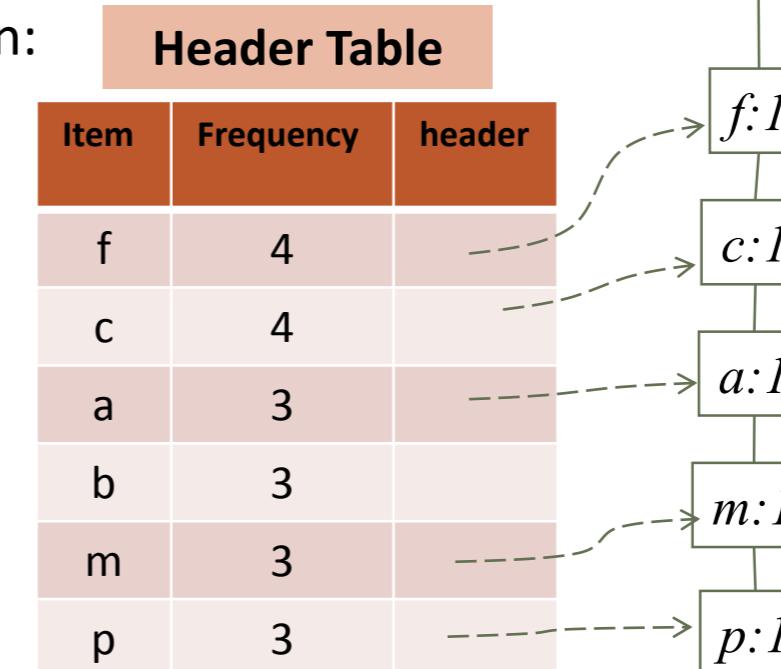
f:4, a:3, c:4, b:3, m:3, p:3

2. Sort frequent items in frequency descending order, f-list

F-list = f-c-a-b-m-p

3. Scan DB again, construct FP-tree

- The frequent itemlist of each transaction is inserted as a branch, with shared sub-branches merged, counts accumulated



Example: Construct FP-tree from a Transaction DB

TID	Items in the Transaction	Ordered, frequent itemlist
100	{f, a, c, d, g, i, m, p}	f, c, a, m, p
200	{a, b, c, f, l, m, o}	f, c, a, b, m
300	{b, f, h, j, o, w}	f, b
400	{b, c, k, s, p}	c, b, p
500	{a, f, c, e, l, p, m, n}	f, c, a, m, p

After inserting the 2nd frequent itemlist "f, c, a, b, m"

1. Scan DB once, find single item frequent pattern:

Let min_support = 3

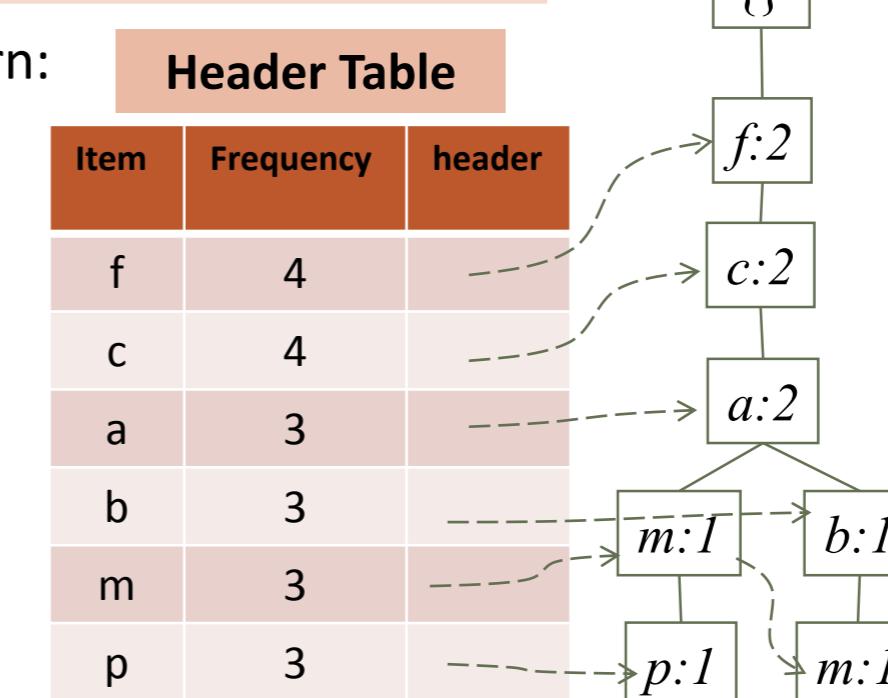
f:4, a:3, c:4, b:3, m:3, p:3

2. Sort frequent items in frequency descending order, f-list

F-list = f-c-a-b-m-p

3. Scan DB again, construct FP-tree

- The frequent itemlist of each transaction is inserted as a branch, with shared sub-branches merged, counts accumulated



Example: Construct FP-tree from a Transaction DB

TID	Items in the Transaction	Ordered, frequent itemlist
100	{f, a, c, d, g, i, m, p}	f, c, a, m, p
200	{a, b, c, f, l, m, o}	f, c, a, b, m
300	{b, f, h, j, o, w}	f, b
400	{b, c, k, s, p}	c, b, p
500	{a, f, c, e, l, p, m, n}	f, c, a, m, p

After inserting all the frequent itemlists

- Scan DB once, find single item frequent pattern:

Let min_support = 3

f:4, a:3, c:4, b:3, m:3, p:3

- Sort frequent items in frequency descending order, f-list

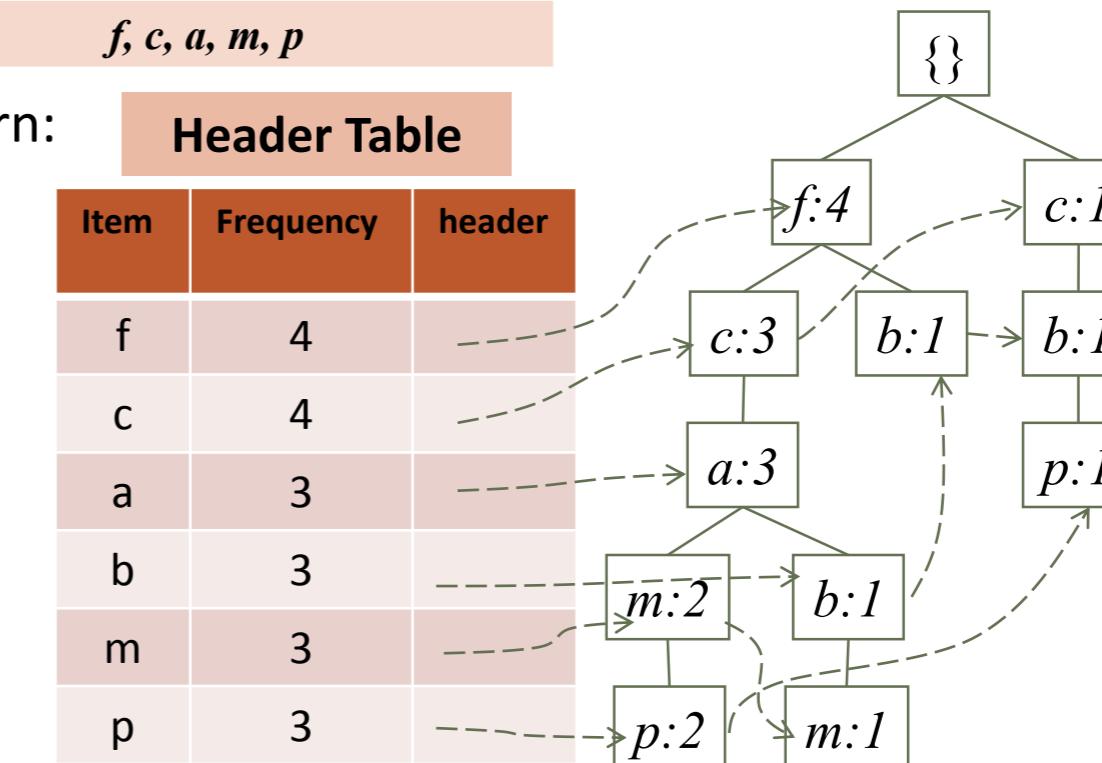
F-list = f-c-a-b-m-p

- Scan DB again, construct FP-tree

- The frequent itemlist of each transaction is inserted as a branch, with shared sub-branches merged, counts accumulated

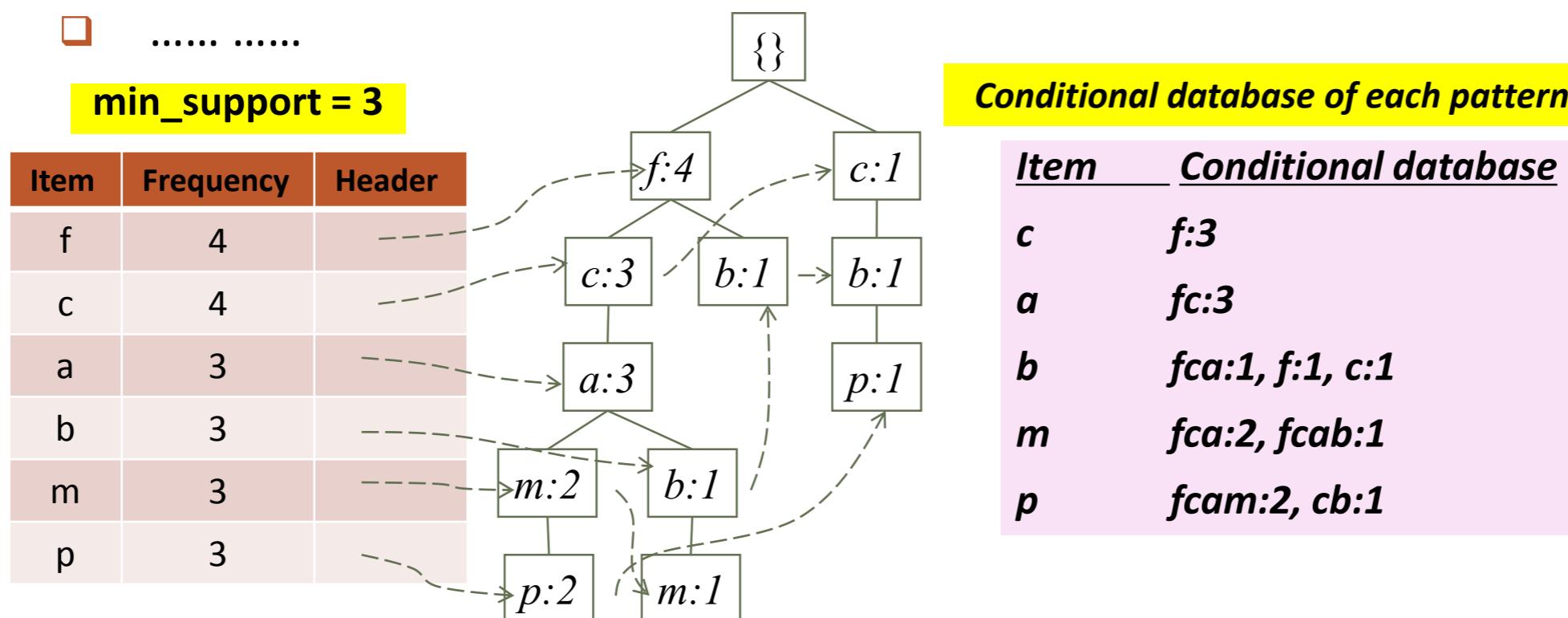
Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



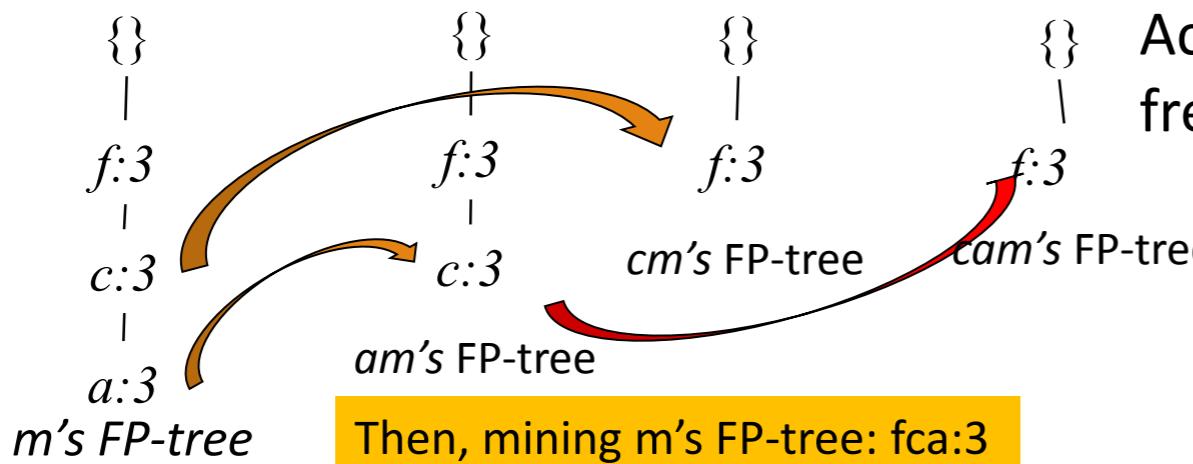
Mining FP-Tree: Divide and Conquer Based on Patterns and Data

- Pattern mining can be partitioned according to current patterns
- Patterns containing p : p 's conditional database: $fcam:2, cb:1$
- p 's conditional database (i.e., the database under the condition that p exists):
 - *transformed prefix paths* of item p
- Patterns having m but no p : m 's conditional database: $fca:2, fcab:1$
-



Mine Each Conditional Database Recursively

min_support = 3	
Conditional Data Bases	
item	cond. data base
c	f:3
a	fc:3
b	fca:1, f:1, c:1
m	fca:2, fcab:1
p	fcam:2, cb:1



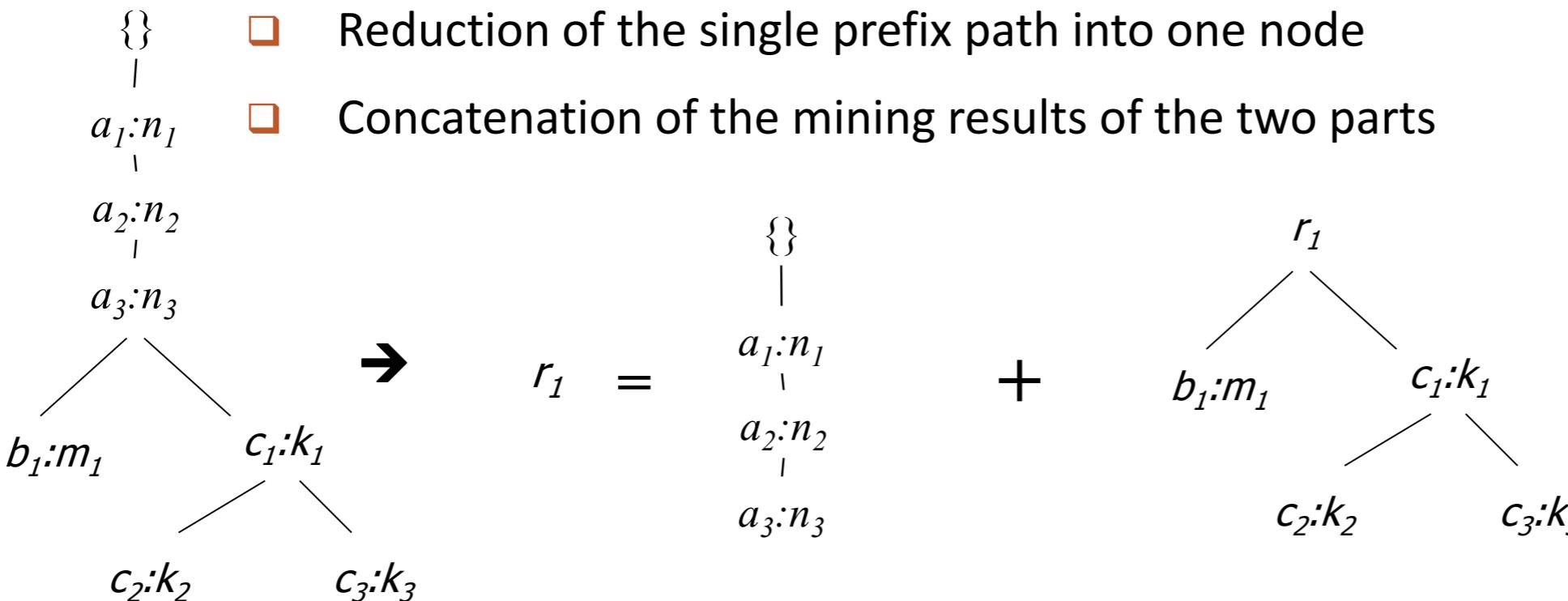
- For each conditional database
 - Mine single-item patterns
 - Construct its FP-tree & mine it
 - p's conditional DB: $\text{fcam}:2, \text{cb}:1 \rightarrow c: 3$
 - m's conditional DB: $\text{fca}:2, \text{fcab}:1 \rightarrow \text{fca}: 3$
 - b's conditional DB: $\text{fca}:1, \text{f}:1, \text{c}:1 \rightarrow \emptyset$

Actually, for single branch FP-tree, all the frequent patterns can be generated in one shot

m: 3
fm: 3, cm: 3, am: 3
fcm: 3, fam:3, cam: 3
fcam: 3

A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts

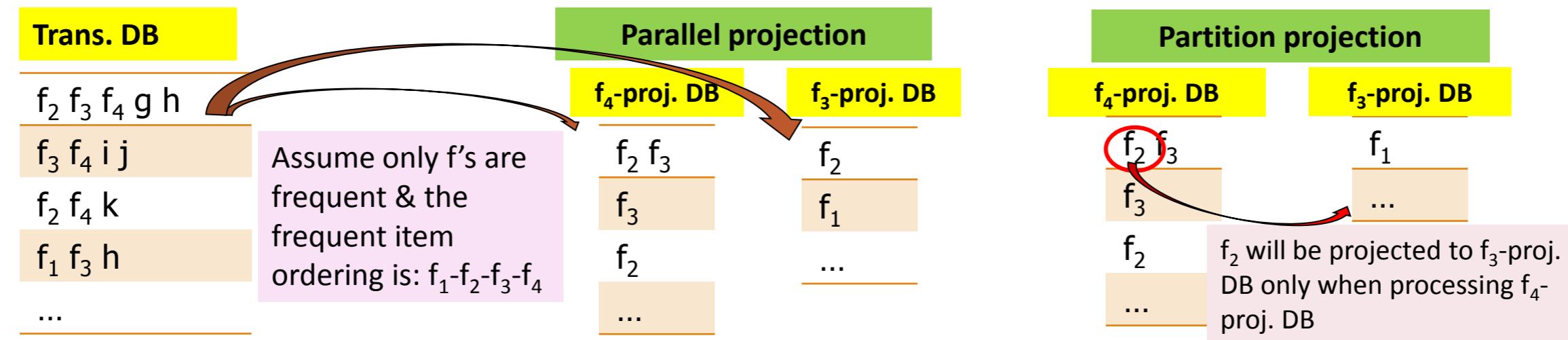


FPGrowth: Mining Frequent Patterns by Pattern Growth

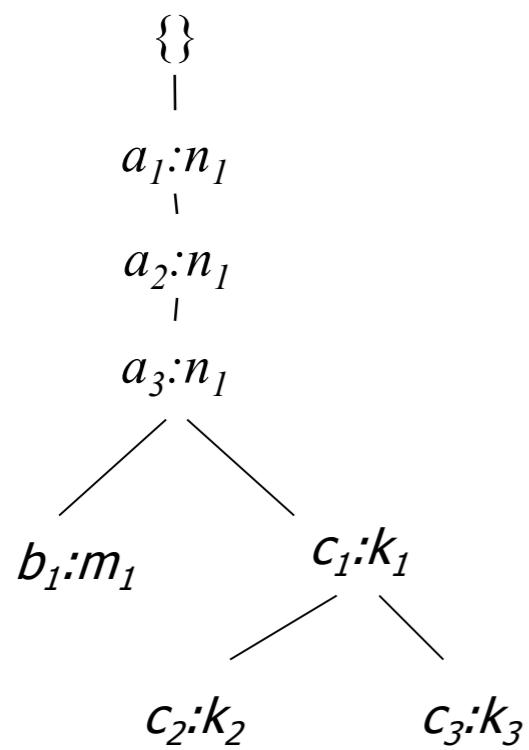
- Essence of frequent pattern growth (FPGrowth) methodology
 - Find frequent single items and partition the database based on each such single item pattern
 - Recursively grow frequent patterns by doing the above for each *partitioned database* (also called the pattern's *conditional database*)
 - To facilitate efficient processing, an efficient data structure, FP-tree, can be constructed
- Mining becomes
 - Recursively construct and mine (conditional) FP-trees
 - Until the resulting FP-tree is empty, or until it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

Scaling FP-growth by Item-Based Data Projection

- What if FP-tree cannot fit in memory?—Do not construct FP-tree
 - “Project” the database based on frequent single items
 - Construct & mine FP-tree for each projected DB
- Parallel projection vs. partition projection
 - Parallel projection: Project the DB on each frequent item
 - Space costly, all partitions can be processed in parallel
 - Partition projection: Partition the DB in order
 - Passing the unprocessed parts to subsequent partitions



CLOSET+: Mining Closed Itemsets by Pattern-Growth



- Efficient, *direct* mining of closed itemsets
- Intuition:
 - If an FP-tree contains a single branch as shown left
 - “ a_1, a_2, a_3 ” should be merged
- Itemset merging: If Y appears in every occurrence of X, then Y is merged with X
 - d -proj. db: {acef, acf} \rightarrow acfd-proj. db: {e}
- Final closed itemset: acfd:2
- There are many other tricks developed
- For details, see J. Wang, et al., “CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets”, KDD'03

TID	Items
1	acdef
2	abe
3	cefg
4	acdf

Let minsupport = 2
 a:3, c:3, d:2, e:3, f:3
 F-List: a-c-e-f-d

Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Efficient Pattern Mining Methods
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Pattern Evaluation

- Limitation of the Support-Confidence Framework
- Interestingness Measures: Lift and χ^2
- Null-Invariant Measures
- Comparison of Interestingness Measures

How to Judge if a Rule/Pattern Is Interesting?

- Pattern-mining will generate a large set of patterns/rules
 - Not all the generated patterns/rules are interesting
- Interestingness measures: Objective vs. subjective
 - Objective interestingness measures
 - Support, confidence, correlation, ...
 - Subjective interestingness measures:
 - Different users may judge interestingness differently
 - Let a user specify
 - Query-based: Relevant to a user's particular request
 - Judge against one's knowledge-base
 - unexpected, freshness, timeliness

Limitation of the Support-Confidence Framework

- Are s and c interesting in association rules: “ $A \Rightarrow B$ ” [s, c]?
- Example: Suppose one school may have the following statistics on # of students who may play basketball and/or eat cereal:

	play-basketball	not play-basketball	sum (row)
eat-cereal	400	350	750
not eat-cereal	200	50	250
sum(col.)	600	400	1000

2-way contingency table

- Association rule mining may generate the following:
 - $play\text{-}basketball \Rightarrow eat\text{-}cereal$ [40%, 66.7%] (higher s & c)
 - But this strong association rule is misleading: The overall % of students eating cereal is $75\% > 66.7\%$, a more telling rule:
 - $\neg play\text{-}basketball \Rightarrow eat\text{-}cereal$ [35%, 87.5%] (high s & c)

Interestingness Measure: Lift

- Measure of dependent/correlated events: **lift**

$$lift(B, C) = \frac{c(B \rightarrow C)}{s(C)} = \frac{s(B \cup C)}{s(B) \times s(C)}$$

- Lift(B, C) may tell how B and C are correlated

- Lift(B, C) = 1: B and C are independent
- > 1 : positively correlated
- < 1 : negatively correlated

- For our example,

$$lift(B, C) = \frac{400/1000}{600/1000 \times 750/1000} = 0.89$$

$$lift(B, \neg C) = \frac{200/1000}{600/1000 \times 250/1000} = 1.33$$

- Thus, B and C are negatively correlated since $lift(B, C) < 1$;
- B and $\neg C$ are positively correlated since $lift(B, \neg C) > 1$

Lift is more telling than s & c

	B	$\neg B$	Σ_{row}
C	400	350	750
$\neg C$	200	50	250
$\Sigma_{\text{col.}}$	600	400	1000

Interestingness Measure: χ^2

- Another measure to test correlated events: χ^2

$$\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$$

- For the table on the right,

$$\chi^2 = \frac{(400 - 450)^2}{450} + \frac{(350 - 300)^2}{300} + \frac{(200 - 150)^2}{150} + \frac{(50 - 100)^2}{100} = 55.56$$

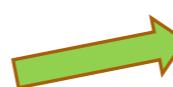
	B	$\neg B$	Σ_{row}
C	400 (450)	350 (300)	750
$\neg C$	200 (150)	50 (100)	250
Σ_{col}	600	400	1000

Expected value
Observed value

- By consulting a table of critical values of the χ^2 distribution, one can conclude that the chance for B and C to be independent is very low (< 0.01)
- χ^2 -test shows B and C are negatively correlated since the expected value is 450 but the observed is only 400
- Thus, χ^2 is also more telling than the support-confidence framework

Lift and χ^2 : Are They Always Good Measures?

- Null transactions: Transactions that contain neither B nor C
- Let's examine the new dataset D
 - BC (100) is much rarer than B¬C (1000) and ¬BC (1000), but there are many ¬B¬C (100000)
 - Unlikely B & C will happen together!
 - But, Lift(B, C) = 8.44 >> 1 (Lift shows B and C are strongly positively correlated!)
 - $\chi^2 = 670$: Observed(BC) >> expected value (11.85)
 - Too many null transactions may “spoil the soup”!*



	B	$\neg B$	Σ_{row}
C	100	1000	1100
$\neg C$	1000	100000	101000
$\Sigma_{\text{col.}}$	1100	101000	102100

A yellow callout box labeled "null transactions" points to the cell containing 101000 in the $\Sigma_{\text{col.}}$ row.

Contingency table with expected values added			
	B	$\neg B$	Σ_{row}
C	100 (11.85)	1000	1100
$\neg C$	1000 (988.15)	100000	101000
$\Sigma_{\text{col.}}$	1100	101000	102100

Interestingness Measures & Null-Invariance

- *Null invariance*: Value does not change with the # of null-transactions
- A few interestingness measures: Some are null invariant

Measure	Definition	Range	Null-Invariant?
$\chi^2(A, B)$	$\sum_{i,j} \frac{(e(a_i, b_j) - o(a_i, b_j))^2}{e(a_i, b_j)}$	$[0, \infty]$	No
$Lift(A, B)$	$\frac{s(A \cup B)}{s(A) \times s(B)}$	$[0, \infty]$	No
$Allconf(A, B)$	$\frac{s(A \cup B)}{\max\{s(A), s(B)\}}$	$[0, 1]$	Yes
$Jaccard(A, B)$	$\frac{s(A \cup B)}{s(A) + s(B) - s(A \cup B)}$	$[0, 1]$	Yes
$Cosine(A, B)$	$\frac{s(A \cup B)}{\sqrt{s(A) \times s(B)}}$	$[0, 1]$	Yes
$Kulczynski(A, B)$	$\frac{1}{2} \left(\frac{s(A \cup B)}{s(A)} + \frac{s(A \cup B)}{s(B)} \right)$	$[0, 1]$	Yes
$MaxConf(A, B)$	$\max\left\{\frac{s(A \cup B)}{s(A)}, \frac{s(A \cup B)}{s(B)}\right\}$	$[0, 1]$	Yes

χ^2 and lift are not null-invariant

Jaccard, consine, AllConf, MaxConf, and Kulczynski are null-invariant measures

Null Invariance: An Important Property

- Why is null invariance crucial for the analysis of massive transaction data?
 - Many transactions may contain neither milk nor coffee!

milk vs. coffee contingency table

	<i>milk</i>	$\neg\text{milk}$	Σ_{row}
<i>coffee</i>	<i>mc</i>	$\neg\text{mc}$	<i>c</i>
$\neg\text{coffee}$	<i>m</i> $\neg\text{c}$	$\neg\text{m}$ $\neg\text{c}$	$\neg\text{c}$
Σ_{col}	<i>m</i>	$\neg\text{m}$	Σ

- Lift and χ^2 are not null-invariant: not good to evaluate data that contain too many or too few null transactions!
- Many measures are not null-invariant!

Null-transactions
w.r.t. m and c

Data set	<i>mc</i>	$\neg\text{mc}$	<i>m</i> $\neg\text{c}$	$\neg\text{m}$ $\neg\text{c}$	χ^2	<i>Lift</i>
D_1	10,000	1,000	1,000	100,000	90557	9.26
D_2	10,000	1,000	1,000	100	0	1
D_3	100	1,000	1,000	100,000	670	8.44
D_4	1,000	1,000	1,000	100,000	24740	25.75
D_5	1,000	100	10,000	100,000	8173	9.18
D_6	1,000	10	100,000	100,000	965	1.97

Comparison of Null-Invariant Measures

- Not all null-invariant measures are created equal
- Which one is better?
 - $D_4 - D_6$ differentiate the null-invariant measures
 - Kulc (Kulczynski 1927) holds firm and is in balance of both directional implications

2-variable contingency table

	<i>milk</i>	$\neg\text{milk}$	Σ_{row}
<i>coffee</i>	<i>mc</i>	$\neg mc$	<i>c</i>
$\neg coffee$	<i>m</i> $\neg c$	$\neg m$ $\neg c$	$\neg c$
Σ_{col}	<i>m</i>	$\neg m$	Σ

All 5 are null-invariant

Data set	<i>mc</i>	$\neg mc$	<i>m</i> $\neg c$	$\neg m$ $\neg c$	<i>AllConf</i>	Jaccard	Cosine	Kulc	MaxConf
D_1	10,000	1,000	1,000	100,000	0.91	0.83	0.91	0.91	0.91
D_2	10,000	1,000	1,000	100	0.91	0.83	0.91	0.91	0.91
D_3	100	1,000	1,000	100,000	0.09	0.05	0.09	0.09	0.09
D_4	1,000	1,000	1,000	100,000	0.5	0.33	0.5	0.5	0.5
D_5	1,000	100	10,000	100,000	0.09	0.09	0.29	0.5	0.91
D_6	1,000	10	100,000	100,000	0.01	0.01	0.10	0.5	0.99

Subtle: They disagree on those cases

Analysis of DBLP Coauthor Relationships

- DBLP: Computer science research publication bibliographic database
 - > 3.8 million entries on authors, paper, venue, year, and other information

ID	Author A	Author B	$s(A \cup B)$	$s(A)$	$s(B)$	Jaccard	Cosine	Kulc
1	Hans-Peter Kriegel	Martin Ester	28	146	54	0.163 (2)	0.315 (7)	0.355 (9)
2	Michael Carey	Miron Livny	26	104	58	0.191 (1)	0.335 (4)	0.349 (10)
3	Hans-Peter Kriegel	Joerg Sander	24	146	36	0.152 (3)	0.331 (5)	0.416 (8)
4	Christos Faloutsos	Spiros Papadimitriou	20	162	26	0.119 (7)	0.308 (10)	0.446 (7)
5	Hans-Peter Kriegel	Martin Pfeifle	18	146	18	0.123 (6)	0.351 (2)	0.562 (2)
6	Hector Garcia-Molina	Wilbert Labio	16	144	18	0.110 (9)	0.314 (8)	0.500 (4)
7	Divyakant Agrawal	Wang Hsiung	16	120	16	0.133 (5)	0.365 (1)	0.567 (1)
8	Elke Rundensteiner	Murali Mani	16	104	20	0.148 (4)	0.351 (3)	0.477 (6)
9	Divyakant Agrawal	Oliver Po	12	120	12	0.100 (10)	0.316 (6)	0.550 (3)
10	Gerhard Weikum	Martin Theobald	12	106	14	0.111 (8)	0.312 (9)	0.485 (5)

Advisor-advisee relation: Kulc: high, Jaccard: low, cosine: middle

- Which pairs of authors are strongly related?
 - Use Kulc to find Advisor-advisee, close collaborators

Imbalance Ratio with Kulczynski Measure

- IR (Imbalance Ratio): measure the imbalance of two itemsets A and B in rule implications:

$$IR(A, B) = \frac{|s(A) - s(B)|}{s(A) + s(B) - s(A \cup B)}$$

- Kulczynski and Imbalance Ratio (IR) together present a clear picture for all the three datasets D_4 through D_6
 - D_4 is neutral & balanced; D_5 is neutral but imbalanced
 - D_6 is neutral but very imbalanced

Data set	<i>mc</i>	$\neg mc$	<i>m</i> $\neg c$	$\neg m$ $\neg c$	Jaccard	<i>Cosine</i>	<i>Kulc</i>	IR
D_1	10,000	1,000	1,000	100,000	0.83	0.91	0.91	0
D_2	10,000	1,000	1,000	100	0.83	0.91	0.91	0
D_3	100	1,000	1,000	100,000	0.05	0.09	0.09	0
D_4	1,000	1,000	1,000	100,000	0.33	0.5	0.5	0
D_5	1,000	100	10,000	100,000	0.09	0.29	0.5	0.89
D_6	1,000	10	100,000	100,000	0.01	0.10	0.5	0.99

What Measures to Choose for Effective Pattern Evaluation?

- Null value cases are predominant in many large datasets
 - Neither milk nor coffee is in most of the baskets; neither Mike nor Jim is an author in most of the papers;
- *Null-invariance* is an important property
- Lift, χ^2 and cosine are good measures if null transactions are not predominant
 - Otherwise, *Kulczynski + Imbalance Ratio* should be used to judge the interestingness of a pattern
- Exercise: Mining research collaborations from research bibliographic data
 - Find a group of frequent collaborators from research bibliographic data (e.g., DBLP)
 - Can you find the likely advisor-advisee relationship and during which years such a relationship happened?
 - Ref.: C. Wang, J. Han, Y. Jia, J. Tang, D. Zhang, Y. Yu, and J. Guo, "Mining Advisor-Advisee Relationships from Research Publication Networks", KDD'10

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Summary

- ❑ Basic Concepts
 - ❑ What Is Pattern Discovery? Why Is It Important?
 - ❑ Basic Concepts: Frequent Patterns and Association Rules
 - ❑ Compressed Representation: Closed Patterns and Max-Patterns
- ❑ Efficient Pattern Mining Methods
 - ❑ The Downward Closure Property of Frequent Patterns
 - ❑ The Apriori Algorithm
 - ❑ Extensions or Improvements of Apriori
 - ❑ Mining Frequent Patterns by Exploring Vertical Data Format
 - ❑ FP-Growth: A Frequent Pattern-Growth Approach
 - ❑ Mining Closed Patterns
- ❑ Pattern Evaluation
 - ❑ Interestingness Measures in Pattern Mining
 - ❑ Interestingness Measures: Lift and χ^2
 - ❑ Null-Invariant Measures
 - ❑ Comparison of Interestingness Measures

Recommended Readings (Basic Concepts)

- R. Agrawal, T. Imielinski, and A. Swami, “Mining association rules between sets of items in large databases”, in Proc. of SIGMOD'93
- R. J. Bayardo, “Efficiently mining long patterns from databases”, in Proc. of SIGMOD'98
- N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal, “Discovering frequent closed itemsets for association rules”, in Proc. of ICDT'99
- J. Han, H. Cheng, D. Xin, and X. Yan, “Frequent Pattern Mining: Current Status and Future Directions”, Data Mining and Knowledge Discovery, 15(1): 55-86, 2007

Recommended Readings (Efficient Pattern Mining Methods)

- R. Agrawal and R. Srikant, “Fast algorithms for mining association rules”, VLDB'94
- A. Savasere, E. Omiecinski, and S. Navathe, “An efficient algorithm for mining association rules in large databases”, VLDB'95
- J. S. Park, M. S. Chen, and P. S. Yu, “An effective hash-based algorithm for mining association rules”, SIGMOD'95
- S. Sarawagi, S. Thomas, and R. Agrawal, “Integrating association rule mining with relational database systems: Alternatives and implications”, SIGMOD'98
- M. J. Zaki, S. Parthasarathy, M. Ogihsara, and W. Li, “Parallel algorithm for discovery of association rules”, Data Mining and Knowledge Discovery, 1997
- J. Han, J. Pei, and Y. Yin, “Mining frequent patterns without candidate generation”, SIGMOD'00
- M. J. Zaki and Hsiao, “CHARM: An Efficient Algorithm for Closed Itemset Mining”, SDM'02
- J. Wang, J. Han, and J. Pei, “CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets”, KDD'03
- C. C. Aggarwal, M.A., Bhuiyan, M. A. Hasan, “Frequent Pattern Mining Algorithms: A Survey”, in Aggarwal and Han (eds.): Frequent Pattern Mining, Springer, 2014

Recommended Readings (Pattern Evaluation)

- C. C. Aggarwal and P. S. Yu. A New Framework for Itemset Generation. PODS'98
- S. Brin, R. Motwani, and C. Silverstein. Beyond market basket: Generalizing association rules to correlations. SIGMOD'97
- M. Klemettinen, H. Mannila, P. Ronkainen, H. Toivonen, and A. I. Verkamo. Finding interesting rules from large sets of discovered association rules. CIKM'94
- E. Omiecinski. Alternative Interest Measures for Mining Associations. TKDE'03
- P.-N. Tan, V. Kumar, and J. Srivastava. Selecting the Right Interestingness Measure for Association Patterns. KDD'02
- T. Wu, Y. Chen and J. Han, Re-Examination of Interestingness Measures in Pattern Mining: A Unified Framework, Data Mining and Knowledge Discovery, 21(3):371-397, 2010

