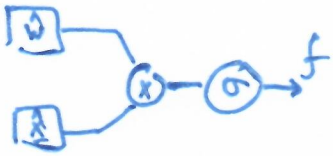
$$\underline{w} = \begin{bmatrix} 1 & -1 & -1 & 1 \\ -2 & 2 & 2 & -1 \\ 1 & 2 & 1 & -2 \end{bmatrix} \quad \underline{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad \sigma = \frac{1}{1+e^{-v}}$$

$$\underline{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

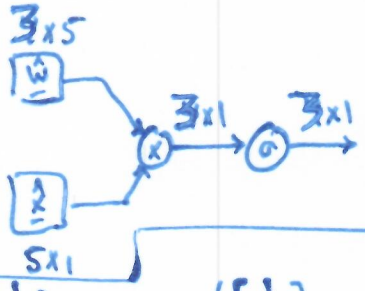
a) $f = \sigma(\underline{w} \cdot \underline{x} + \underline{b}) \Rightarrow f = \sigma(\hat{\underline{w}} \cdot \hat{\underline{x}})$; $\hat{\underline{w}} = (\underline{w} \mid \underline{b}) = \overbrace{\begin{bmatrix} 1 & -1 & -1 & 1 \\ -2 & 2 & 2 & -1 \\ 1 & 2 & 1 & -2 \end{bmatrix}}^{w} \overbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}^{b} \quad 3\times5$

$$\hat{\underline{x}} = \begin{pmatrix} \underline{x} \\ 1 \end{pmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} \quad 5\times1$$

b)



c)



d)

$$f = \sigma(\hat{\underline{w}} \cdot \hat{\underline{x}}) = \sigma\left(\begin{bmatrix} 1 & -1 & -1 & 1 & 1 \\ -2 & 2 & 2 & -1 & 2 \\ 1 & 2 & 1 & -2 & 3 \end{bmatrix}_{3\times5} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}_{5\times1}\right) = \sigma\left(\begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}_{3\times1}\right) = \begin{bmatrix} \sigma(1) \\ \sigma(6) \\ \sigma(2) \end{bmatrix} = \begin{bmatrix} 0.7 \\ 1.0 \\ 0.9 \end{bmatrix}$$

Note

$$\frac{d\sigma}{dx} = \sigma(1-\sigma) \quad (I)$$

e) $\frac{df}{d\hat{\underline{w}}} = ?$

$$\frac{df}{d\hat{\underline{w}}} = \frac{df}{du} \cdot \frac{du}{d\hat{\underline{w}}}$$

(1) $u = \hat{\underline{w}} \cdot \hat{\underline{x}} \rightarrow \frac{du}{d\hat{\underline{w}}} = 1 \cdot \hat{\underline{x}}^T$

(2) $\frac{df}{du} = \frac{d\sigma(u)}{du} \Rightarrow$ From (I) $\frac{df}{du} = \sigma(\underline{u})(1-\sigma(\underline{u}))$

$$\underline{u} = \hat{\underline{w}} \cdot \hat{\underline{x}} = \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix} \Rightarrow \frac{df}{du} = \sigma\begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix} \odot \left[1 - \sigma\begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix}\right] = \begin{bmatrix} 0.7 \\ 1.0 \\ 0.9 \end{bmatrix} \odot \begin{bmatrix} 1-0.7 \\ 1.0-1.0 \\ 1-0.9 \end{bmatrix} = \begin{bmatrix} 0.7\times0.3 \\ 1.0\times0 \\ 0.9\times0.1 \end{bmatrix}$$

$$\frac{df}{du} = \begin{bmatrix} 0.21 \\ 0 \\ 0.09 \end{bmatrix}$$

from (1) $\frac{d\hat{u}}{d\hat{w}} = 1 \cdot \hat{x}^T = [1\ 2\ 3\ 4]_{4\times1}$

and (2) $\frac{df}{du} = \begin{bmatrix} 0.21 \\ 0 \\ 0.09 \end{bmatrix}$

(f)

$$\frac{df}{d\hat{w}} = \frac{df}{du} \cdot \frac{du}{d\hat{w}} = \begin{bmatrix} 0.21 \\ 0 \\ 0.09 \end{bmatrix} [1\ 2\ 3\ 4]$$

$$\frac{df}{d\hat{w}} = \begin{bmatrix} 0.21 & 0.48 & 0.63 & 0.84 \\ 0 & 0 & 0 & 0 \\ 0.09 & 0.18 & 0.27 & 0.36 \end{bmatrix}$$

g) $\frac{df}{d\hat{x}} = \frac{du}{d\hat{x}} \cdot \frac{df}{du} = \hat{W}^T \cdot \frac{df}{dy} = \begin{bmatrix} 1 & -2 & 1 \\ -1 & 2 & 2 \\ -1 & 2 & 1 \\ 1 & -1 & -2 \end{bmatrix}_{4\times3} \begin{bmatrix} 0.21 \\ 0 \\ 0.09 \end{bmatrix}_{3\times1} = \begin{bmatrix} 0.21+0.09 \\ -0.21+0.18 \\ -0.21+0.09 \\ 0.2-0.18 \end{bmatrix}$

$$\frac{df}{d\underline{\hat{x}}} = \begin{bmatrix} 0.3 \\ -0.03 \\ -0.12 \\ 0.03 \end{bmatrix}$$