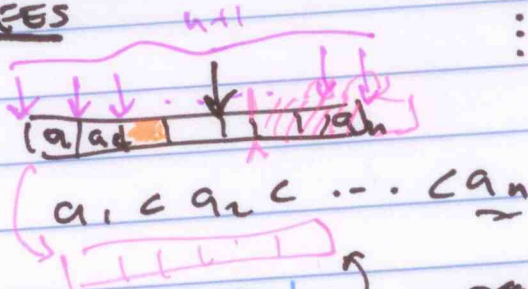


BINARY SEARCH TREES

$O(\lg n)$ INSERTION
DELETION
SEARCH

MAX
MIN
SUCCESSOR
PREDECESSOR

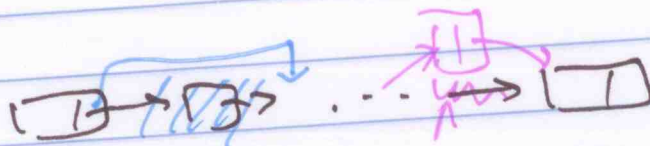


BINARY SEARCH $\Theta(\lg n)$

COMPROMISE

UPDATE — DELETION/INSERTION $\Theta(n)$

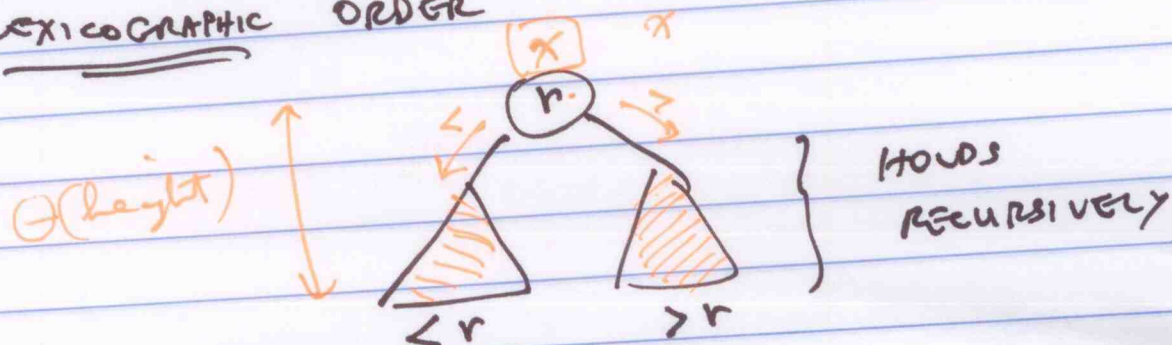
(LAZY — MARK IT DELETED)



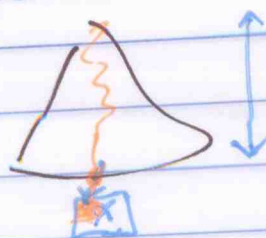
UPDATE — EASY $\Theta(1)$

SEARCH — LINEAR $\Theta(n)$

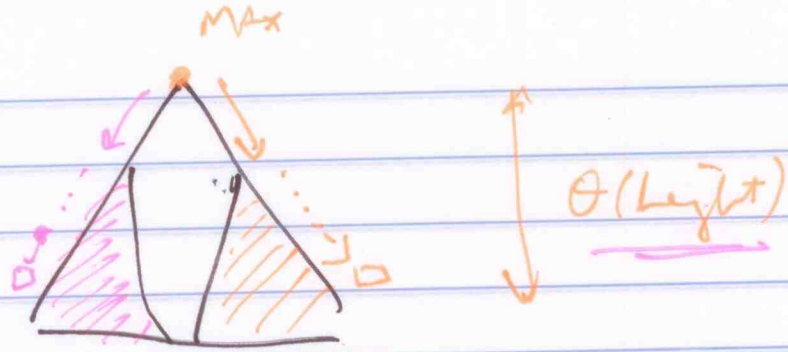
LEXICOGRAPHIC ORDER



INSERTION — $\Theta(\text{height})$



MAX/MIN

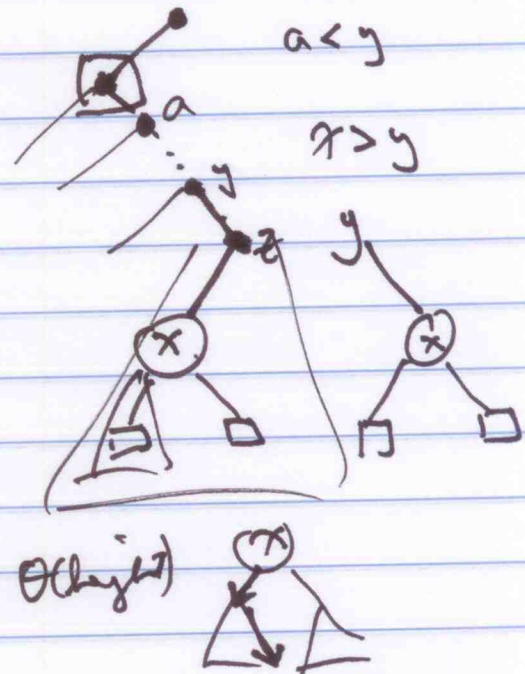
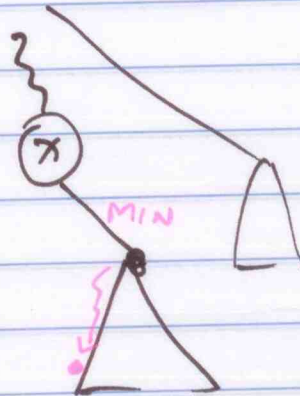
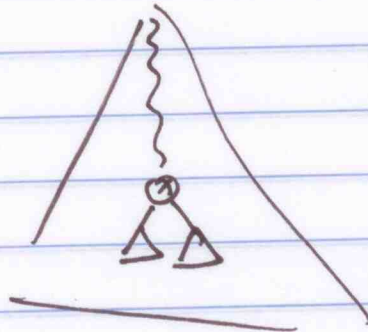


SUCC/PRED

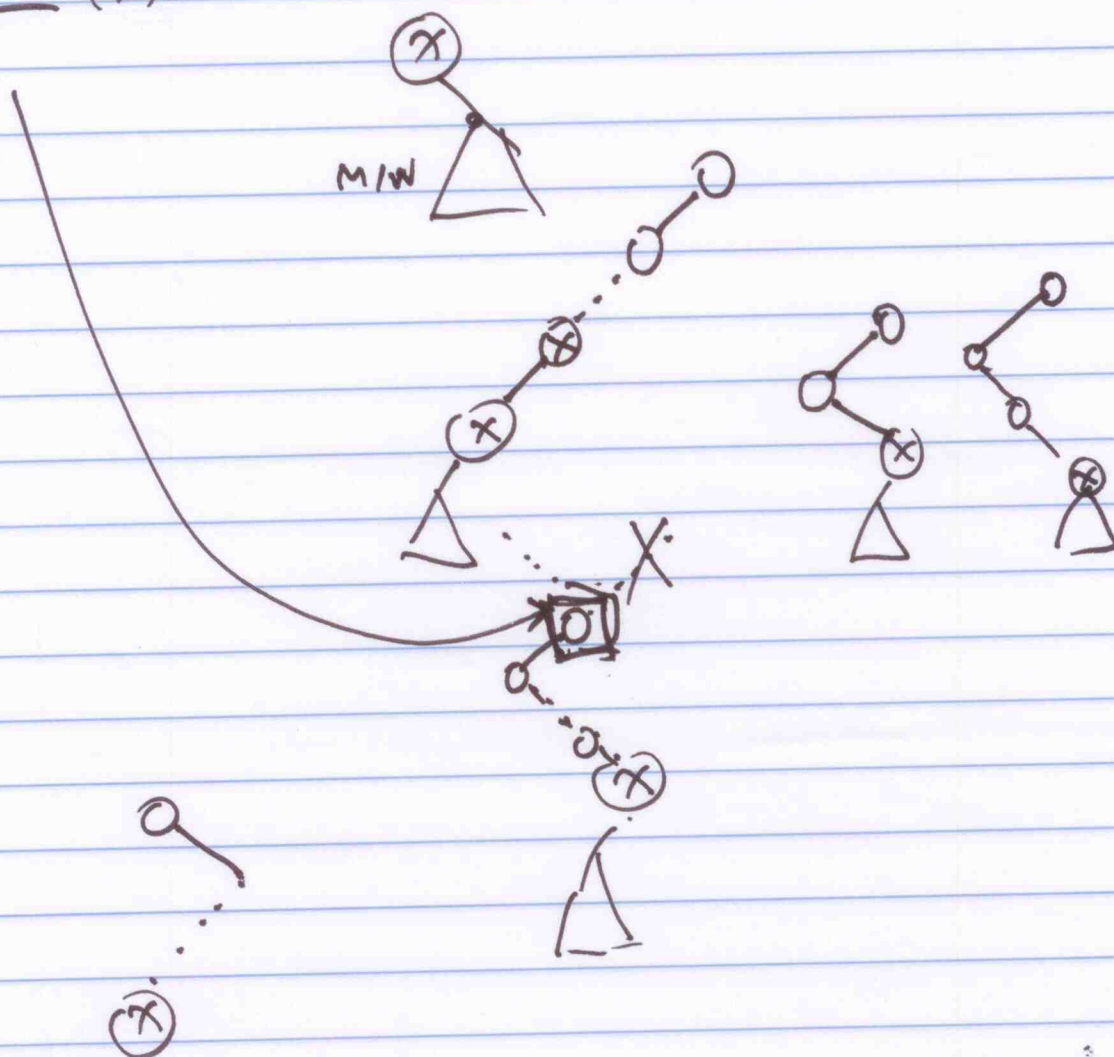
x

SEARCH FOR x

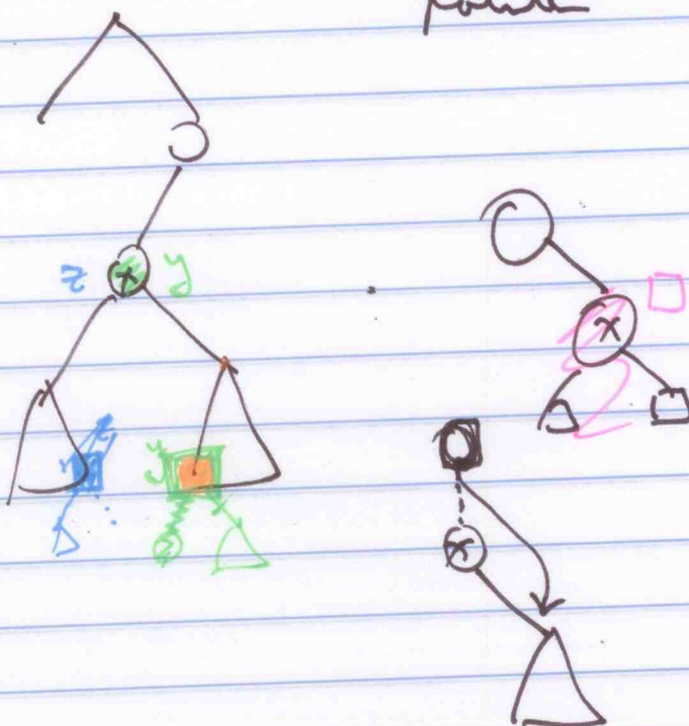
least elt
 $> x$



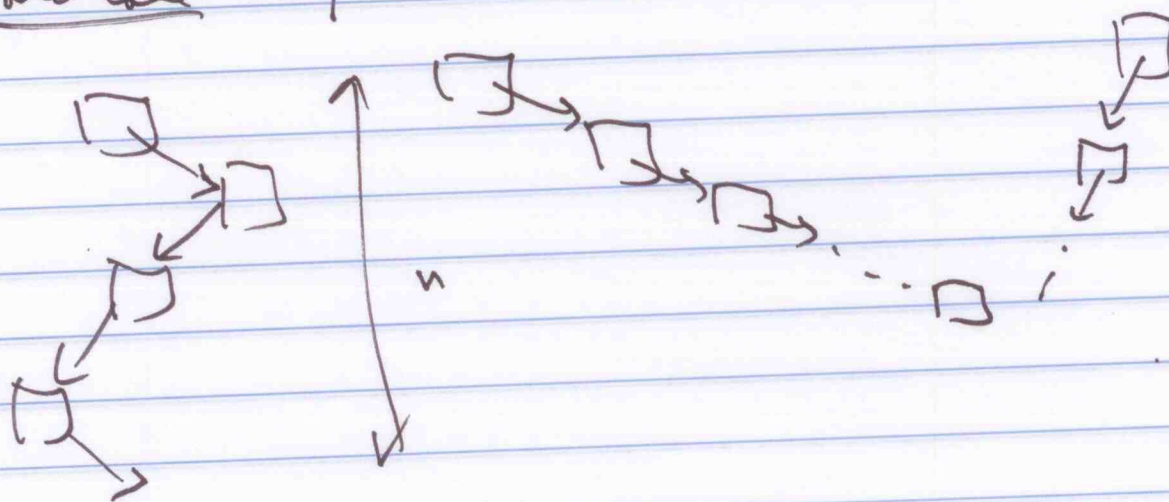
Succ(x)



DELETION - easy if x has 0 or 1 children
 - change parent's pointer

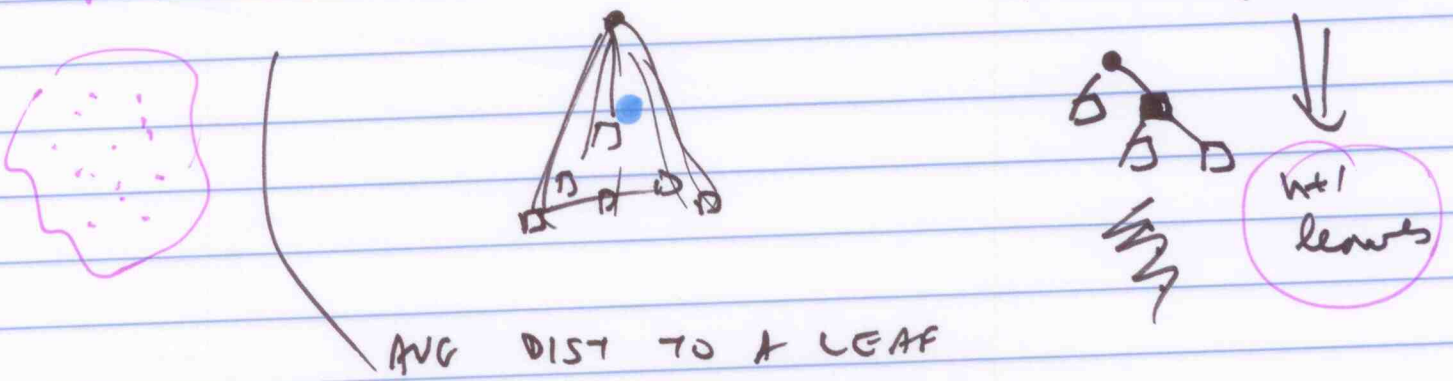


Worst case req insertion and/or deletions - $\Theta(n)$



§12.4 - READ "LIGHTLY"
Pror 12-3 - $\Theta(\lg n)$ \uparrow \downarrow $c \lg n$ \rightarrow $c \lg n$ such that $c \lg \frac{2e}{c} = 1$
 $\Rightarrow c \approx 4.3$

UNSUCCESSFUL
AVG SEARCH TIME after n random insertion and deletion
AVG all trees w.e.
 n items (internal)



$$\frac{1}{\# \text{leaves}} \sum_{\text{leaves } l} \text{depth}(l)$$

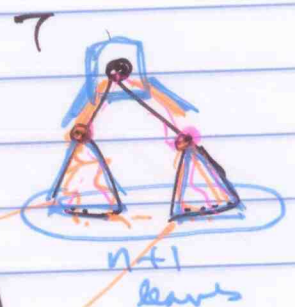
$$\frac{1}{n+1} \left[\sum_{\text{leaves } l} \text{depth}(l) \right]$$

E_n



$$EPL(T) = \sum_{\text{leaves } l \in T} \text{depth}(l)$$

$$E_{h-1} = EPL(\text{left}(T)) + EPL(\text{right}(T)) + n+1$$



$$IPL(T) = EPL(\text{left}(T)) + I(\text{right}(T)) + n-1$$

□

$$\begin{aligned} EPL(\text{empty tree}) &= 0 \\ IPL(\text{empty tree}) &= 0 \end{aligned}$$

$$EPL(T) = IPL(T) + 2n$$

6

$$E_n = \sum_{h=1}^n (E_h + E_{n-h} + n+1) \underbrace{P_n(h^{\text{th}} \text{ is root})}_{1/n}$$

$$E_n = n+1 + \frac{2}{n} \sum_{h=0}^{n-1} E_h$$

QUICKSORT

$$E_n = \cancel{2n+1} H_n = 2n(H_n + O(1))$$

$$\frac{E_n}{n+1} \approx \boxed{2H_n} = \underline{\underline{\Theta(\lg n)}}$$

AVG UNSUCC

$$I_n = 2n(H_n - 1) + O(1)$$

$$\frac{I_n}{n}$$

AVG SUCC

