HOMEWORK ASSIGNMENT 9

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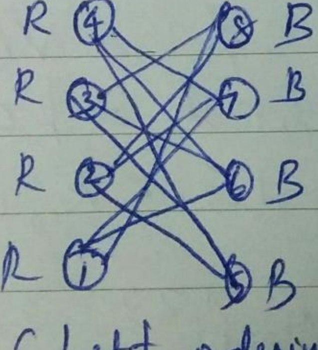
1. Solution:

Greedy coloring of a graph does not approximate the optimal coloring to within any constant ratio.

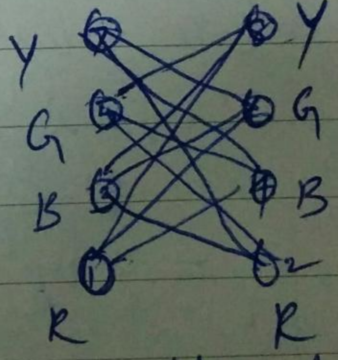
* The Algorithm does not give the lowest k for which, there exists k-coloring, but tries to find a reasonable coloring while still being reasonably expensive.

* The Algorithm is as follows. Consider the vertices in a specific order $v_1, \ldots, v_n$ and assign to v_i, the smallest available color not used by v_i's neighbour $v_1, \ldots, v_{i-1}$, adding fresh color if needed. This Algorithm finds a reasonable coloring and is $O(|V|+|E|)$

* The problem is that it does not find the optimal k for which the graph is k-Colorable. In some cases k can be as high $n/2$ when the optimal k would be 2. This is true with the following graph (which can be extended to any even number of vertices), where the left ordering leads to coloring using only 2 colors, & the right ordering leads to 4 colors



(Left ordering)



(Right ordering)

* Based upon the ordering, the k-Colorable graph, the approximation with respect to optimal coloring changes & it does not fit in within any constant ratio, since the ratio changes based on

the ordeing of the vertices

2 Solution

For any graph $G = (V, E)$ there is an ordering of the vertices such that the greedy Algorithm yeilds an optimal coloring.

Let $c: V(G) \longrightarrow [k]$ with $k = \chi(G)$ be an optimal coloring of the vertices of G. Consider the color classes $C_i = \{v \in V(G) \mid c(v) = i\}$ and let $k_i = |C_1 \cup C_2 \cup \dots \cup C_i|$ for $1 \leq i \leq k$, we have $C_i = \{v_{k_{i-1}+1}, v_{k_{i-1}+2}, \dots, v_{k_i}\}$,

i.e, the ordering is built of blocks of consecutive vertices belonging to one color class. Given this ordering, the greedy Algorithm will

Color the graph with $\chi(G)$ colors.

where $\chi(G) \rightarrow$ smallest number of colors needed to color a graph G. (also referred to as Chromatic number)

3. Solution:
Optimal ordering of the Vertices of a graph is NP-complete

* The greedy coloring Algorithm can be used to find optimal colorings in polynomial time, by choosing the vertex ordering to be the reverse of a perfect elimination ordering for the graph. The perfect orderable graphs generalizes this property & its NP-hard to find a perfect ordering of these graphs.

* Perfectly orderable graph have vertices, which are ordered in such a way that a greedy coloring Algorithm with that ordering optimally colors every induced Subgraphs of the given graph. Testing of graph which is perfectly orderable is NP- complete.