HOMEWORK ASSIGNMENT 1

1) Problem 2.3 - 3

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To prove: when n is a exact power of 2, the solution for the recurrence

$$T(n) = \begin{cases} 2 & \text{if } n = 2 \\ 2T(n/2) + n & \text{if } n = 2^k, \text{ for } k > 1 \end{cases}$$

is $T(n) = n \lg n$.

Solution:

when $n = 2$, $n \lg n = 2 \lg 2 = 2$

By Mathematical inductive hypothesis

$$T(n/2) = n/2 \lg n/2 \quad \text{--- (1)}$$

$$T(n) = 2T(n/2) + n$$

$$= 2(n/2)\lg(n/2) + n \quad (\text{From (1)})$$

$$= n(\lg n - 1) + n$$

$$= n \lg n - \cancel{n} + \cancel{n}$$

$$\boxed{T(n) = n \lg n}$$

Hence proved by mathematical induction when n is exact power of 2, the solution for recurrence is $T(n) = n \lg n$.

2) Problem 2.3 - 4

2) Problem 2·3–4

Insertion sort

To sort A[1...n], the procedure is to recursively sort A[1... n-1] and then to insert A[n] into the sorted array A [1... n-1]

Let T(n) be a function to sort A [1 ... n]

If there is only one element, it takes constant time $\Theta(1)$

If there is more than one element, we need to recursively sort A [1 ... n-1] which is represented by T[n-1], plus it takes $\Theta(n)$ to insert A[n]th element in the sorted array.

∴ The recurrence relation is.

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ T(n-1) + \Theta(n) & \text{if } n > 1 \end{cases}$$

The solution for this recurrence is $T(n) = \Theta(n^2)$, which is obvious from the algorithm that it needs nested loops (2 loops), to iterate over the entire array for comparisons and sorting accordingly.

3) Problem 2 - 3 (a)

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Horner's rule for evaluating a polynomial

$$P(x) = \sum_{k=0}^{n} a_k x^k$$

$$= a_0 + x(a_1 + x(a_2 + \cdots + x(a_{n-1} + x a_n)\cdots))$$

given the coeffecients $a_0, a_1, \cdots a_n$ and value for x

1. $y = 0$
2. for $i = n$ downto 0

   $y = a_i + x \cdot y$

(a) The running time of this code fragment for Horner's rule is $\Theta(n)$, since it takes only one for loop from n to 0 for evaluating a polynomial.

4. Problem 3-3 (a)

Order of growth for the following functions

$2^{\lg n}$, $(\lg n)^{\lg n}$, e^n, $4^{\lg n}$, $(n+1)!$, $\sqrt{\lg n}$

We can reduce some of the above functions by using their identities.

$2^{\lg n} = n$

$(\lg n)^{\lg n} = n^{\lg \lg n}$

$4^{\lg n} = n^2$

$(n+1)! = n! \times (n+1) = \Theta(n^{(n+1)/2} e^{-n}) \cdot (n+1)$

4th problem Continuation and 5th Problem 4 - 3 (a)

The exponential functions grows at a faster rate than the polynomial functions.

$\therefore$ The order of growth (In decesending order)

1) $(n+1)!$
2) e^n
3) $(\lg n)^{\lg n} = n^{\lg\lg n}$ [By the identities]
4) $4^{\lg n} = n^2$
5) $2^{\lg n} = n$
6) $\sqrt{\lg n}$

5. Problem 4-3 (a)

$$T(n) = 4T(n/3) + n\lg n.$$

(a) By Master theorem

$a = 4,\ b = 3,\ f(n) = n\lg n$

$n^{\log_b a} = n^{\log_3 4} = n^{(1.261)} = O(n^{1.261})$

Since $f(n) = \Omega(n^{\log_3 4 + \varepsilon})$, where $\varepsilon = 0.2$,

Case 3 Applies if we can show regularity condition holds for $f(n)$. For Large n, we have, $af(n/b) = 4(n/3)\lg(n/3) \leq (3/4)\, n\lg n = cf(n)$, for $c = 3/4$. Consequently by case 3, (as given in book), the solution to the recurrence is $T(n) = \Theta(n\lg n)$

5th problem continuation

By secondary recurrence

(b) $T(n) = 4T(n/3) + n \lg n$

$$T(n_i) = 4T(n_i) + n_i \lg n_i$$

$$n_i = n_i(1 + \lg n_i)$$

$$\therefore \cancel{n_i} = \cancel{n_i} + n_i \lg n_i$$

$$\therefore n_i \lg n_i = \langle 0 \rangle$$

$$\therefore T(n) = \theta(n \lg n)$$

Illinois Institute of Technology
Department of Computer Science

Honesty Pledge

CS 430 Introduction to Algorithms
Spring Semester, 2016

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All work I submit will be mine and mine alone.

I understand that all resources in print or on the web, aside from the text and class notes, used in solving the homework problems *must be explicitly cited.*

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