GRAPH ALGORITHMS (CS 330)
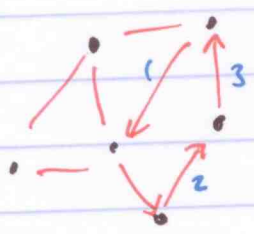

vertices

edges

V E $G = (V, E)$

directed →
undirected —

weighted edges have "length"

labeled — edges
        ↘ vertices

edges — capacity

"Structural" — EULER PATH ——— Depth First Search
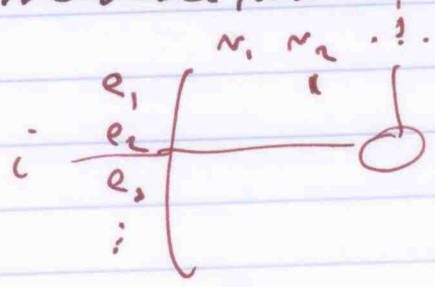
Optimization — shortest path
              min cycle   Breadth First Search
              max cycle

Representation

1) [Adjacency structures] $\Theta(|V| + |E|)$

2) Adjacency matrix $\Theta(|V|^2)$

THEORY (3) Incidence matrix — [NP-hardness]

$$\begin{array}{c} & N_1 \ N_2 \ \cdots \ N_n \\ \begin{array}{c} e_1 \\ i \quad e_2 \\ e_3 \\ \vdots \end{array} \left(\quad \right) \end{array}$$

vertex j is an
endpoint of edge i

$\Theta(|V||E|)$

Adjacency Structure

one list / vertex (vertices)

└ list of edges

list of vertices

Reverse all edges

Graph $\Longrightarrow$ New Graph w/ all edges reversed

$\Theta(|V| + |E|)$

$\Theta(|V|^2) = \binom{|V|}{2}$

$|V|$ vertices

max # of edges

SIMPLE LINEAR { No parallel edges

No self loops

vertices

self loop

$G = (V, E)$

how much space?

each edge twice

$\Theta(|V|)$ $\Theta(|E|)$

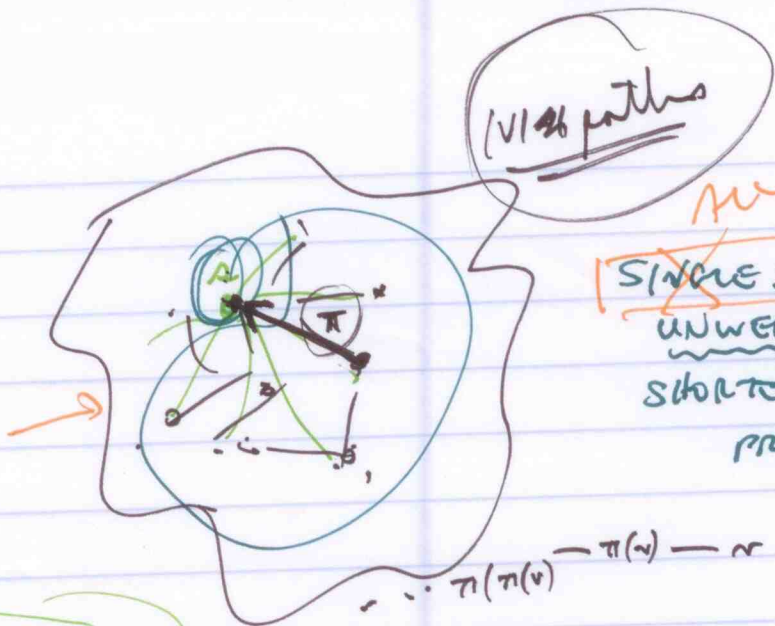$\Theta(|V| + |E|)$

edges

Adjacency Matrix
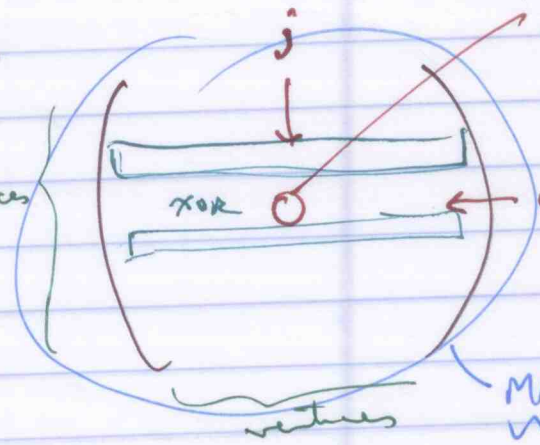
$A_{ij} = \begin{cases} 1 & \text{if edge from vertex } i \text{ to vertex } j \\ 0 & \text{if not} \end{cases}$
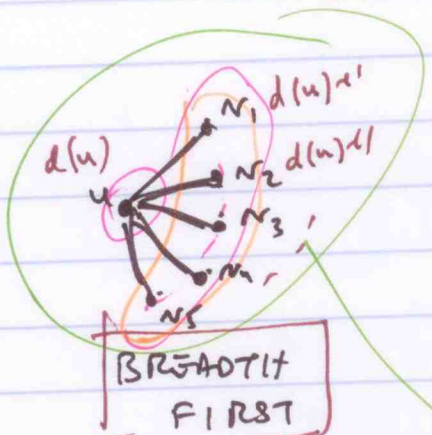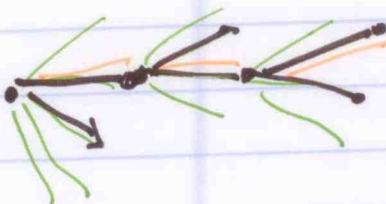
j

XOR

i

$|V| \times |V|$

$\Theta(|V|^2)$

MATRIX

vertices

$A^1 =$

vertices

$A + A^2 + A^3 + \cdots$

$A^{\#}$

Explore
Graph

$|V| \neq 6$ paths

SINGLE SOURCE
UNWEIGHTED
SHORTEST PATH
PROBLEM

from all
s

$\pi(\pi(v)) \longrightarrow \pi(v) \longrightarrow v$

$d(u)$
u
$N_1 \; d(u)+1$
$N_2 \; d(u)+1$
N_3
N_4
N_5

BREADTH FIRST

DEPTH FIRST

FIRST MISTAKE

GREEDY ALG

store future
vertices
in a QUEUE

$\Theta(|E|)$
$\Theta(|V|)$

"timestamps d
are length of the shortest
path from s

# of edges

$\sum$ length of edges

while Q not empty do

$u \; N_1 \; N_2 \; N_3 \; N_4 \; N_5$

u's
neighbours
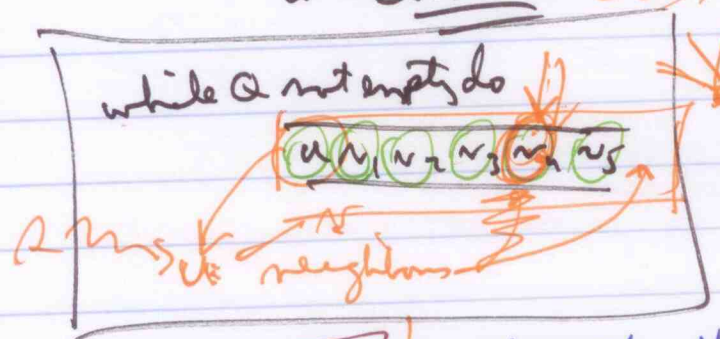
VIRGIN   White — initially all vertices are white

LIVE NON-VIRGIN Gray ⟸ ON the Queue

DEAD — Black — when they're removed from Q

$|E|$
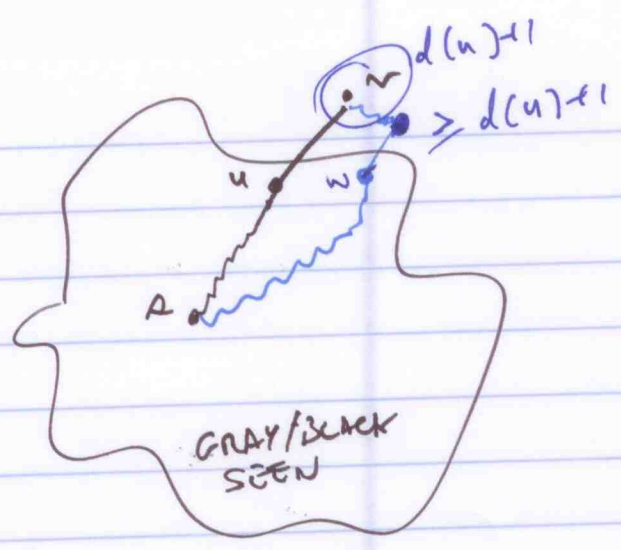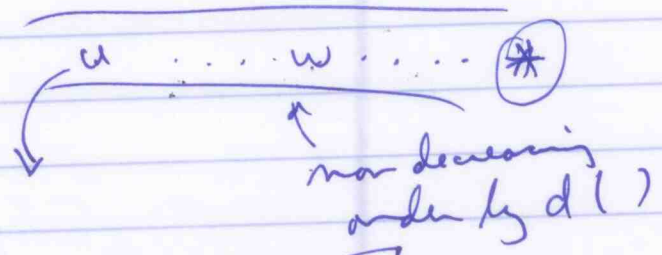
$\Theta(|V|+|E|)$

$\times \Theta(|V|)$ $\Theta(|V|^2 + |V||E|)$ $(|E||V| + |V|)$

thm 22.5

$d(v)+1$

$\geq d(u)+1$

u   w

A

GRAY/BLACK
SEEN

UNSEEN
(WHITE)

u . . . . . w . . . . . ⊛

non decreasing
order by $d()$

$\boxed{d(u) \leq d(w)}$

$d(u)+1$