

HOMEWORK ASSIGNMENT 7

Problem 1

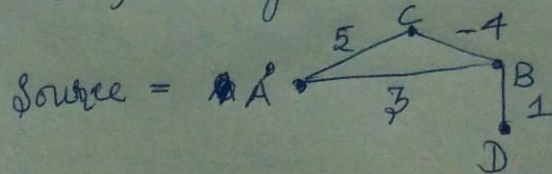
Homework Assignment 7

- 1) * Modified version of Dijkstra's algorithm that works for negative edge weights, as there are no negative cycles.

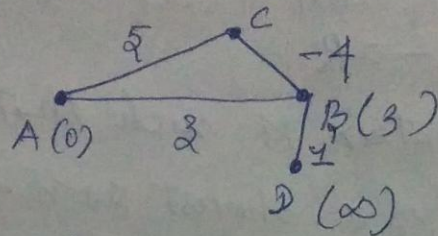
Solution:

```
Dijkstra( $G, s$ )  
for each  $v \in V$   
     $d[v] \leftarrow \infty$   
 $d[s] \leftarrow 0$   
 $S \leftarrow \emptyset$   
 $Q \leftarrow V$   
while  $Q \neq \emptyset$   
     $u \leftarrow \text{ExtractMin}(Q)$   
     $S \leftarrow S \cup \{u\}$   
    for each  $v \in \text{Adj}[u]$   
        if  $d[v] > d[u] + w(u, v)$   
            decreasekey( $v, d[u] + w(u, v)$ )
```

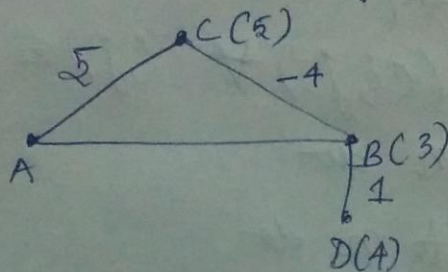
- * Dijkstra's algorithm does not work if the graph has negative weight edges, i.e. it might return incorrect result, as following example shows.



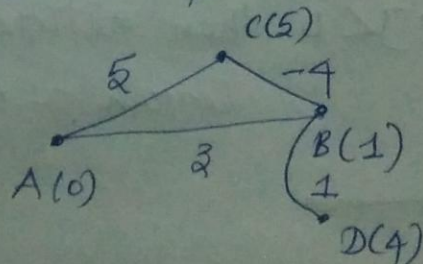
- First source A is popped out of the queue & edges (A, C) & (A, B) are relaxed.



- Now B is on top of min-heap queue Q . We now pop B & relax the only edge out of B (i.e., (B, D)) giving D distance $3+1=4$



- Now D is on top of min-heap & we remove D. There are no edges going out of D to relax. The only vertex now left in the queue is C. we remove C & relax the edge (C, B) . Since $5-4=1 < 3$, distance to B must be updated to 1.



- The Algorithm now terminates because the queue is empty. However, the path $A \rightarrow C \rightarrow B \rightarrow D$ has total length $5 + (-4) + 1 = 2 < 4$; thus the length of the shortest path from source A to D was not correctly evaluated.

Problem 2

② Floyd-Warshall Algorithm for negative cycles

Input: A digraph G with $V(G) = \{1, \dots, n\}$
& weights $c: E(G) \rightarrow \mathbb{R}$

Output: An $n \times n$ matrix M such that $M[i, j]$
contains the length of a shortest path from vertex i
to vertex j .

$$M[i, j] := \infty \quad \forall i \neq j.$$

$$M[i, i] := 0 \quad \forall i$$

$$M[i, j] := c(i, j) \quad \forall (i, j) \in E(G)$$

for $i := 1$ to n do

 for $j := 1$ to n do

 for $k := 1$ to n do

 if $M[j, k] > M[j, i] + M[i, k]$

 then $M[j, k] := M[j, i] + M[i, k]$

for $i := 1$ to n do

 if $M[i, i] < 0$ then return (graph
 contains a negative cycle)