Solution to Homework Assignment 3 (CS 430)

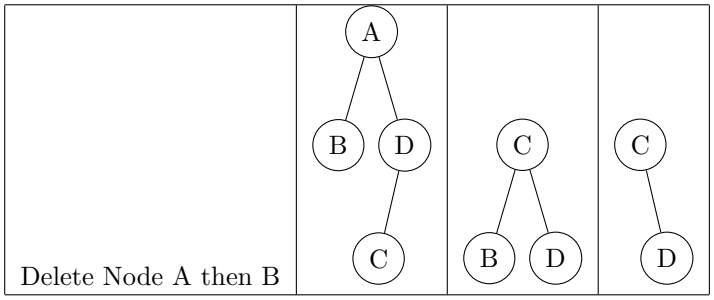
Saptarshi Chatterjee
CWID: A20413922

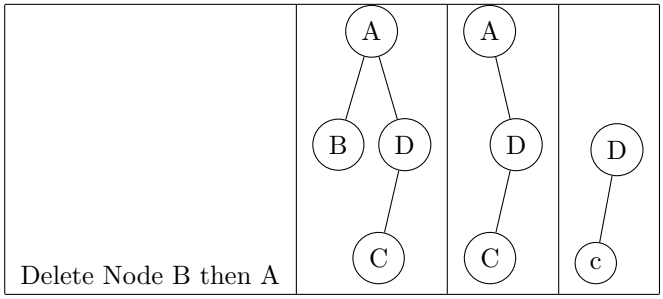
February 15, 2018

1 Problem 12.3-4 on page 299. Is the operation of deletion "commutative" in the sense that deleting x and then y from a binary search tree leaves the same tree as deleting y and then x? Argue why it is or give a counterexample.

Answer -

Delete is not commutative. Proof by contradiction . The example below produces different trees when we change deletion order. Hence they are not commutative.





2 Problem 13.1-6 on page 312. What is the largest possible number of internal nodes in a red-black tree with black- height k? What is the smallest possible number?

Answer -

Case 1 (largest possible number of internal nodes).
Black height is constant . i.e. k. Height of the tree will be maximum if there is 1 red node between every 2 black nodes that makes tree of height $2k + 1$.
$\therefore$ total nodes = N = $2^{2k+1} - 1$

Case 2 (smallest possible number of internal nodes).
Height of the tree will be minimum if there is no red tree at all. Height of such tree will be $k + 1$.
$\therefore$ total nodes = N = $2^{k+1} - 1$

3 Problem 13.3-4 on page 322 . Professor Teach is concerned that RB-INSERT-FIXUP might set T:nil:color to RED, in which case the test in line 1 would not cause the loop to terminate when z is the root. Show that the professor's concern is unfounded by arguing that RB- INSERT-FIXUP never sets T:nil:color to RED

Answer -

i) Algorithm given in text Page 316 modifies color of a node to RED only in lines 7 and 13 and it changes color of the grand parent of the given node z. If execution reached 7th/13th line, then Z.p.color is RED . If Z.p.color is RED then it cannot be Root as Root is always Black . So root must be at least 1 level up of Z.p $\therefore$ Z.p.p exists then T:nil:color will not be set to RED. It might set Root to RED , If depth of the tree is 2 . But that's handled in line 16.

iii) As the algo never directly tries to change color of z to RED, instead tries to change color of the grand parents of z. As the BLACK leaf nodes never will have any child or grand parents , changing their color is out of question.

4 Problem 13.4-6 on page 330 . Professors Skelton and Baron are concerned that at the start of case 1 of RB-DELETE-FIXUP, the node x:p might not be black. If the professors are correct, then lines 5 - 6 are wrong. Show that x:p must be black at the start of case 1, so that the professors have nothing to worry about.

Answer -

Algorithm given in text Page 326. At line 3, the right sibling of x is set to w . Now at line 4 we are checking if w.color is RED, and then proceeding to line 5-6. If line 5-6 gets executed, that implies the parent of w cannot be red. Because

only problem in the tree is double black node x , rest of the tree should hold all the property of Red-Black tree . So Both w and W's parent cant be red . It will violet property 4 . So x:p must be black

5 Problem 14.2-2 on page 347. Can we maintain the black-heights of nodes in a red-black tree as attributes in the nodes of the tree without affecting the asymptotic performance of any of the red- black tree operations? Show how, or argue why not. How about maintaining the depths of nodes?

Answer - Theorem 14.1 (Augmenting a red-black tree) Page 346 -

Let f be an attribute that augments a red-black tree T of n nodes, and suppose that the value of f for each node x depends on only the information in nodes x, x:left, and x:right, possibly including x:left:f and x:right:f. Then, we can maintain the values of f in all nodes of T during insertion and deletion without asymptotically affecting the O.lg n/ performance of these operations.

Because the black-height of a node can be computed from the information at the node (Own color) and its It's children's Black height , so according to the above theorem it would not affect the asymptotic performance of any of the red-black tree operations.

The same is not true for depth of the nodes . If we delete the root of a tree then we could end up updating the depths of all nodes, affecting the asymptotic performance

References

[1] Cornell Univ Solution
$http://www.cs.cornell.edu/courses/cs409/2000SP/Homework/hw03Solution.htm$

[2] Github Solution
$https://github.com/gzc/CLRS/blob/master/C13-Red-Black-Trees/13.4.md$