

Homework Assignment 4

CS 430 Introduction to Algorithms
Fall Semester, 2014

Due: Wednesday, October 15

1. (a) Can the black-heights of nodes in a red-black tree be maintained as fields in the nodes of the tree without affecting the asymptotic performance of any of the red-black tree operations? Show how, or argue why not.
(b) Define the *red depth* of a node in a red-black tree as the number of red ancestors that the node has. Can the red depths of nodes in a red-black tree be maintained as fields in the nodes of the tree without affecting the asymptotic performance of any of the red-black tree operations? Show how, or argue why not.
2. Problem 16-1 on pages 446–447, adding
 - (d) i. As given on page 447, but use dynamic programming in its recursive formulation
ii. As given on page 447, but use dynamic programming in its iterative formulation
iii. Analyze the time required.
 - (e) Suppose that, in part (d), we add the restriction that each denomination can be used just once. Modify your algorithm to determine *if* making change for n cents is possible.
 - (f) For extra credit, prove that, with only two coins with relatively prime values a and b , the smallest value n for which change can be given for all values greater than or equal to n is $(a - 1)(b - 1)$.
3. India and Pakistan are to meet each other in the world championship of squash. The champion will be the first to win n matches in a series of $2n - 1$ matches. For any given match there is a fixed probability p that India will win, and hence a probability $q = 1 - p$ that Pakistan will win. Let P_{ij} be the probability that India will win the series given that they still need i more victories, whereas Pakistan needs j more victories for the championship. $P_{0j} = 1$, $1 \leq j \leq n$, because India needs no more victories to win. $P_{i0} = 0$, $1 \leq i \leq n$, as Pakistan cannot possibly win if India already has.
 - (a) Explain why $P_{ij} = pP_{i-1,j} + qP_{i,j-1}$.
 - (b) What is the value of P_{00} ?
 - (c) Devise and analyze an unmemorized dynamic programming algorithm that calculates P_{nn} , the probability that India will win the series.
 - (d) Devise and analyze a memoized $O(n^2)$ -time dynamic programming algorithm that calculates P_{nn} .