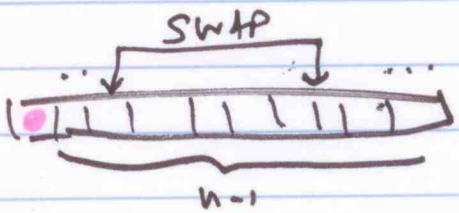
More on Quicksort

Data moves?



$\lfloor n/2 \rfloor$

$E(n) = \lfloor n/2 \rfloor + E(\lfloor n/2 \rfloor) + E(\lceil n/2 \rceil)$

$E(0) = 0$

$E(1) = 0$

$E'(n) = n/2 + 2E'(n/2)$

$n = 2^k$

$e_k = \frac{1}{2}2^k + 2e_{k-1}$

$(E-2)(E-2) = (E-2)^2$

$\Rightarrow e_k = \Theta(k\,2^k)$

$\Rightarrow E'(n) \approx E(n)$ (Why?)

$\hookrightarrow = \Theta(n \lg n)$

$E(n) = \sum_{i=0}^{n-1} \nu(i)$

HW: Prove by induction

0	00000
1	00001
2	00010
⋮	⋮
n-1	⋯ (lg n)

$\nu(i)$ = # of 1's in binary expansion of i

$\frac{1}{n}\sum_{i=0}^{n-1} \nu(i) \approx \frac{1}{2}\lg n$

$\Rightarrow E(n) \approx \frac{n \lg n}{2}$

$= \Theta(n \lg n)$

Expected stack depth?

$$S(n) = 1 + \frac{1}{n}\sum_{i=0}^{n-1} S(i)$$

$$T(n) = \Theta(n) + \frac{2}{n}\sum_{i=0}^{n-1} T(i)$$

$2nH_n$

$S(n)$ = expected stack depth when

$\sum$ prob(i) × stack depth(i)

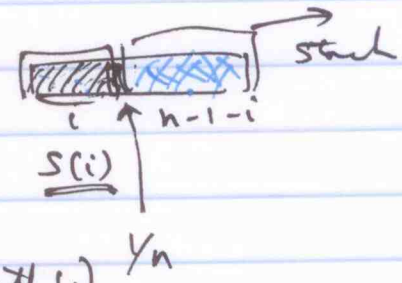
$S(n) = \Theta(\log n)$

$H_n = \ln n + O(1)$

Q( )

Q( )

Q( )

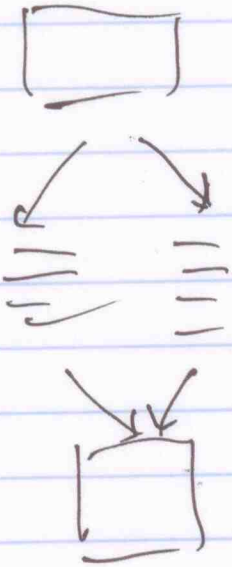


① Median of 3 randomly chosen #s.

② Contrd. recursive depth

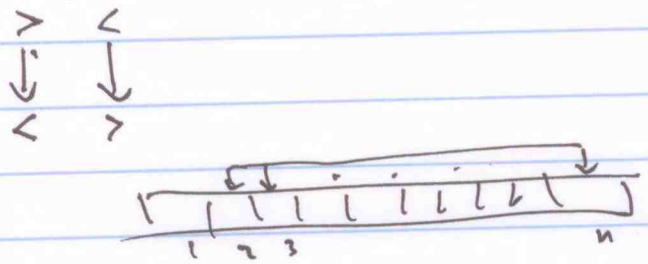
Divide + Conquer – Merge Sort

$$T(n) = \Theta(n) + 2T(n/2) \Rightarrow T(n) = \Theta(n \log n)$$



Selection – Given n elts, find k^{th} largest.

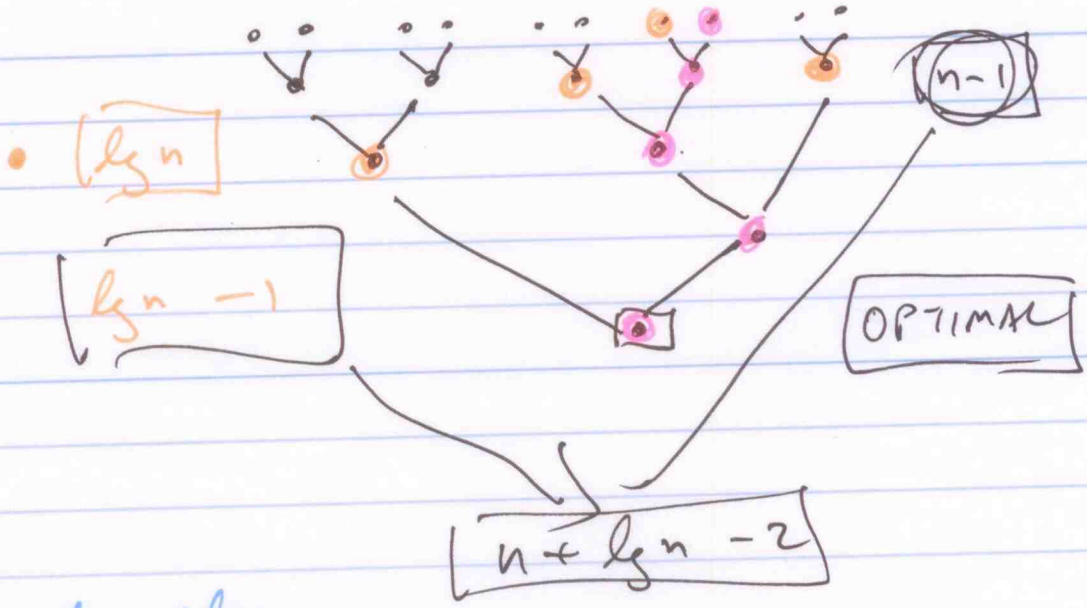
$k=1$ (largest) (same as $k=n$, smallest)



$n-1$

$k=2$ (2^{nd} largest) $n-1+n-2$ $\boxed{2n-3}$

$n+\lg n$



$k=3$ $n+\lg n+\lg n$

$\vdots$

k $n+\underbrace{\lg n+\lg n+\cdots+\lg n}_{k-1} \Rightarrow \boxed{\Theta(n+k\lg n)}$

$= \Theta(n)$ $k = \begin{cases} O(1) \\ n-O(1) \end{cases}$

MEDIAN

$k=n/2 \longrightarrow \Theta(n\lg n)$ SORTING!

SELECTION IN LINEAR TIME

$k=1$ OK
$k=2$ OK
$k=O(1)$, $k=n-O(1)$ } OK

DIVIDE & CONQUER

$$T(n) = \Theta(1) + T(n/2)$$

$\Theta(n)$

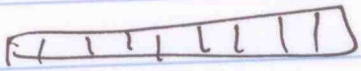
$O(n)$

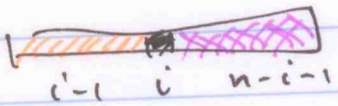
$$T(n) = \Theta(n) + T(n/2)$$

$n = 2^h \Rightarrow t_h = 2^h + t_{h-1}$

$(\in -2)$ $(\in -1)$ $\quad \alpha 2^h + \beta$

$$T(n) = \Theta(n)$$





h^{th} smallest

$h < i$

$h > i$

$h = i$
DONE

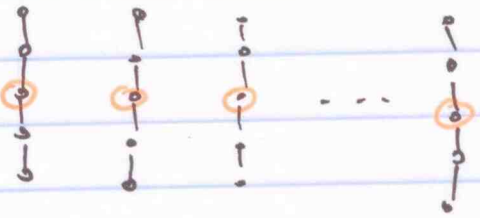
AVG $\quad c(n) = \Theta(n) + \frac{1}{n}\sum_{i=0}^{n-1} c(i) \Rightarrow c(n) = \Theta(n)$

WORST CASE $\quad$ BAD

n elts

n/5

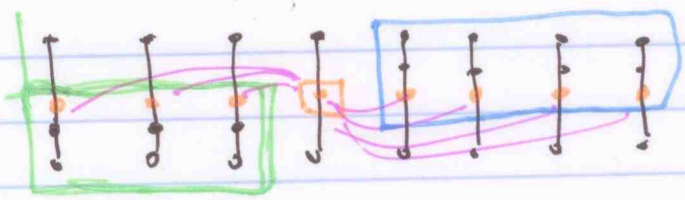
$\Theta(n)$



$\frac{n}{5} \times O(1)$

n/5 medians ↷

find median of



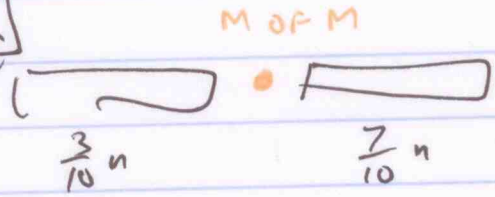
n/10 groups

3 elts/group < M of M.

$\frac{3}{10} n <$ M of M.

$\frac{3}{10} n >$ M of M

$T(n)$ = Time to find k^{th} largest of n



$$T(n) \leq T(n/5) + \Theta(n) + T\left(\tfrac{7}{10} n\right)$$

$$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7}{10}n\right) + \Theta(n)$$

$T(\alpha n) + T(\beta n)$

$\alpha + \beta < 1$

Guess: $T(n) \le kn$ for some k

Pf by induction

$$kn \overset{?}{\ge} k\frac{n}{5} + k\frac{7n}{10} + cn$$

$$kn \le \frac{9}{10}kn + cn$$

$$\frac{k}{10} \le c$$

choose $k = 10c$.

$$Q(n) = \Theta(n) + \Theta(n) + 2Q(n/2)$$

↑ median ↑ partition

$$Q(n) = \Theta(n) + 2Q(n/2)$$

$$\Rightarrow Q(n) = \Theta(n \lg n)$$