ASSIGNMENT 9

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Homework Assignment 9.

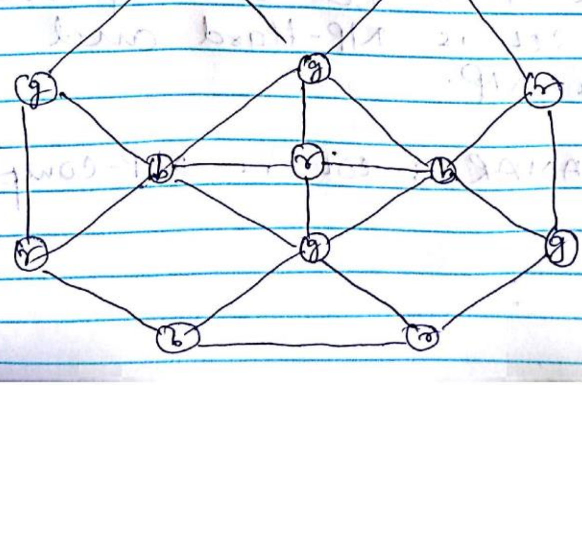
1. Prove that 3-coloring a planar graph is NP-complete.

Solution :-

We can show that a certificate of this problem could be verified in polynomial time by checking the graph's 3-coloring. Therefore PLANAR-3-COL $\in$ NP.

Further, we prove this by reduction that 3-COL $\leq_p$ PLANAR-3-COL or construct a new graph $\hat{G}$ from the input graph G such that G is 3-colorable $\Leftrightarrow$ $\hat{G}$ is planar-3-colorable.

$\hat{G}$ construction :- we replace all edge crossings in G with the below gadget



r → red
b → Blue
g → green.

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Properties of gadget:-

(1) Every valid 3-coloring of it has opposite corners the same color.

(2) Any such coloring of the corners extends to a 3-coloring of the entire gadget.

We can show the above properties by enumerating the gadget's possible 3-colorings.

If an edge in G is crossed by multiple other edges, the gadgets that replace those crossings need to be linked together at the edges. This propagates the fact that the nodes at either end of the ~~edo~~ edge must be different colors.

Now we can see that $\hat{G}$ is planar 3-colorable and if we remove the gadge from $\hat{G}$ we get the graph G.

This reduction runs in polynomial time and thus we have proved that PLANAR 3-COL is NP-Hard and also in the class NP.

Hence PLANAR 3-COL is NP-complete.

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2) Longest Simple Cycle

Solution:- Let us first define a decision problem of Longest Simple Cycle:-

Given an undirected graph $G=(V,E)$ and integer K, does G has a Longest simple cycle of length atleast K.

Now, we will know that $LSC(G,K) \in NP$ by reducing a known NP-complete problem namely HAMILTON-CYCLE(H). The HAMILTON-CYCLE(H) is defined as - Given an undirected graph H does H has a Hamiltonian Cycle.

∴ Prove that HAMILTON CYCLE $\leq_p$ LONGEST-SIMLE CYCLE

Proof:- Let ~~the~~ (G, K) be an Longest Simple cycle.
Given a Yes-certificate, with list of vertices which forms the LSC we can traverse through the graph G in polynomial time to verify that yes there is a cycle and no vertex appear more than once and length is atleast K.

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Now from an instance of HAMILTON-CYCLE(H) we construct an instance of Longest simple cycle(H, |V|).
This construction can be done in constant (polynomial) time.
We claim that the graph H(V,E) has a Hamiltonian cycle, if and only if the length of its longest simple cycle is at least equal to |V|.
This claim is correct because if H(V,E) has a Hamiltonian cycle, that cycle is a simple path of length |V|.
On the other hand, if H does not have a Hamiltonian cycle, the length of the longest simple in H must be strictly less than |V|.

Hence we have proved that the LONGESTSIMLECYCLE is NP-Complete.

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