Homework Assignment 4 (Corrected)

CS 430 Introduction to Algorithms
Spring Semester, 2018

Due: Friday, February 23

1. Define the *red depth* of a node in a red-black tree as the number of red ancestors that the node has. Can the red depths of nodes in a red-black tree be maintained as fields in the nodes of the tree without affecting the asymptotic performance of any of the red-black tree operations? Show how, or argue why not.

2. India and Pakistan are to meet each other in the world championship of squash. The champion will be the first to win n matches in a series of $2n-1$ matches. For any given match there is a fixed probability p that India will win, and hence a probability $q = 1 - p$ that Pakistan will win. Let P_{ij} be the probability that India will win the series given that they still need i more victories, whereas Pakistan needs j more victories for the championship. $P_{0j} = 1$, $1 \le j \le n$, because India needs no more victories to win. $P_{i0} = 0$, $1 \le i \le n$, as India cannot possibly win if Pakistan already has.

   (a) Explain why $P_{ij} = pP_{i-1,j} + qP_{i,j-1}$.

   (b) What is the value of P_{00}?

   (c) Devise and analyze an unmemoized dynamic programming algorithm that calculates P_{nn}, the probability that India will win the series.

   (d) Devise and analyze a memoized $O(n^2)$-time dynamic programming algorithm that calculates P_{nn}.

3. Problem 16-1 on pages 446–447, adding

   (d) i. As given on page 447, but use dynamic programming in its recursive formulation

        ii. As given on page 447, but use dynamic programming in its iterative formulation

        iii. Analyze the time required.

   (e) Suppose that, in part (d), we add the restriction that each denomination can be used just once. Modify your algorithm to determine *if* making change for n cents is possible.