Introduction To Algorithms
CS430

Fall 2015
HomeWork 8
Due 24th November

1. A depth-first forest classifies the edges of a graph into tree, back, forward, and cross edges. A breadth-first tree can also be used to classify the edges reachable from the source of the search into the same four categories.

   (a) Prove that in a breadth-first search of an undirected graph, the following properties hold:

      i. There are no back edges and no forward edges.
      ii. For each tree edge (u, v) , we have $v.d = u.d + 1$.
      iii. For each cross edge (u, v), we have $v.d = u.d$ or $v.d = u.d + 1$.

   (b) Prove that in a breadth-first search of a directed graph, the following properties hold:

      i. There are no forward edges.
      ii. For each tree edge (u, v) , we have $v.d = u.d + 1$.
      iii. For each cross edge (u, v), we have $v.d \leq u.d + 1$..
      iv. For each back edge (u, v), we have $0 \leq v.d \leq u.d$

   (20 pts)

2. Let $G = (u, v)$ be a directed graph in which each vertex $u \in V$ is labeled with a unique integer $L(u)$ from the set $\{1, 2, ..., |V|\}$. For each vertex $u \in V$, let $R(u) = \{v \in V : u \rightarrow v\}$ be the set of vertices that are reachable from u. Define $min(u)$ to be the vertex in $R(u)$ whose label is minimum, i.e., $min(u)$ is the vertex v such that $L(v) = min\{L(w) : w \in R(u)\}$. Give an $O(V + E)$-time algorithm that computes $min(u)$ for all vertices $u \in V$. (20 pts)

3. An Eulerian tour in an undirected graph is a cycle that is allowed to pass through each edge exactly once and can use a vertex multiple number of times.

   (a) Show that a graph has an eulerian tour if and only if all vertices are of even degree.

   (b) Design an algorithm to determine the eulerian tour in a given graph.

   (20pts)

4. Suppose that all edge weights in a graph are integers in the range from 1 to $|V|$. How fast can you make Prim's algorithm run? What if the edge weights are integers in the range from 1 to W for some constant W ? (20 pts)

5. Let G be an undirected graph. Prove that if all edge weights are distinct then the graph has a unique minimum spanning tree. (20pts)