

Binary Search Balanced Trees

HEIGHT

$\Theta(\lg n)$ for n items

Constant

$\left\{ \begin{array}{l} \text{search} \\ \text{insert} \\ \text{delete} \\ \text{read/write} \\ \text{max/min} \end{array} \right\} \Theta(\text{height})$

"Perfect" binary tree (min EPC)

$\rightarrow \text{height } \lceil \lg n \rceil$

Examples

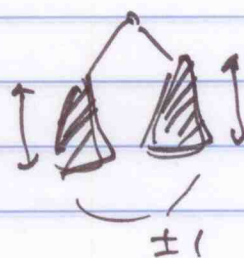
Height-Balanced (AVL)

Weight-Balanced (BB)

3-2 (2-3)

B-trees (external memory)

:

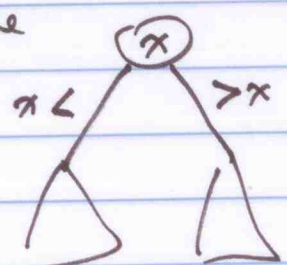


SEEK

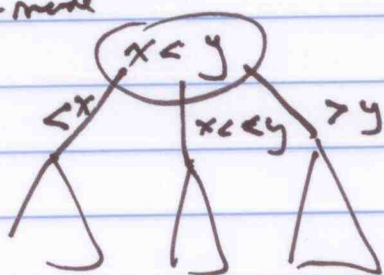
Red-Black

2-3-4 Trees (RB in disguise)

2-node



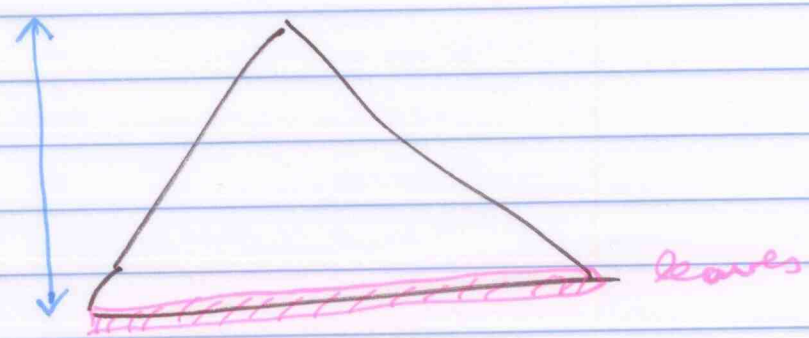
3-node



4-node



All leaves are at same depth from root



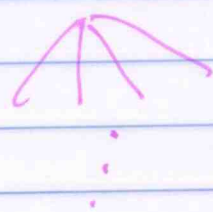
n items

only of 2-nodes
 TALLEST



$\log n$

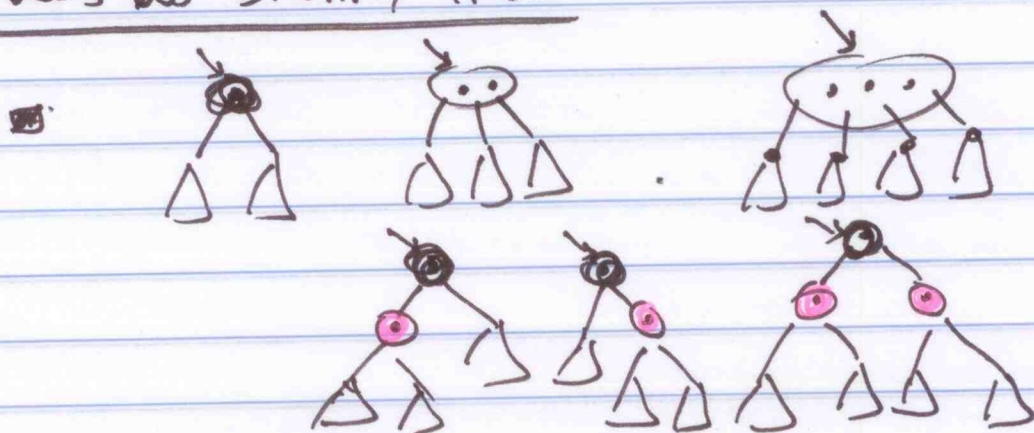
SHORTEST only 4-nodes



$\log_4 n = \frac{1}{2} \log n$

$$\frac{1}{2} \log n \leq \text{height} \leq \log n \Rightarrow \text{search } \underline{\underline{\Theta(\log n)}}$$

2-3-4 Trees as BINARY TREES



Red/Black Trees

- 1) No two red nodes in a row (parent & child)
- 2) Root is black
- 3) Every node is red or black
- 4) Leaves are black
- 5) Paths from root to any leaf have the same number of black nodes

$$\left. \begin{array}{l} \text{Black height} \\ \frac{1}{2} h_n \leq \end{array} \right\} \leq h_n$$

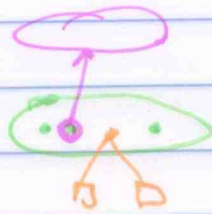
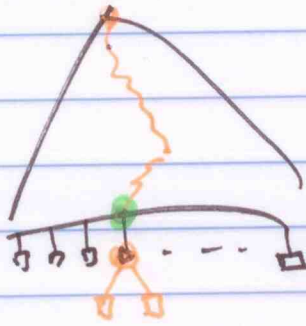
$$\text{height(RB tree)} \leq 2 \text{ black height} \leq 2 h_n$$

Search, max, min, succ, pred

$$\Theta(h_n)$$

Insert/Delete

Insert in B-T tree



Rotation

RIGHT ROT

LEFT ROT

