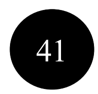
Problem 1: 10/10	Problem 2: 10/10	Problem 3: 5/5

Homework 5 – Red Black Trees

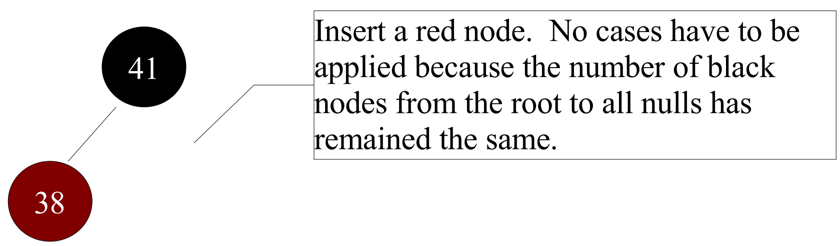
Assigned: Thursday October 21, 2010 *Due: Monday October 24, 2010 by 11:59pm*

1. *(10 points)* Show the red-black trees after successively inserting the keys 41, 38, 31, 12, 19, 8 into and initially empty red-black tree.

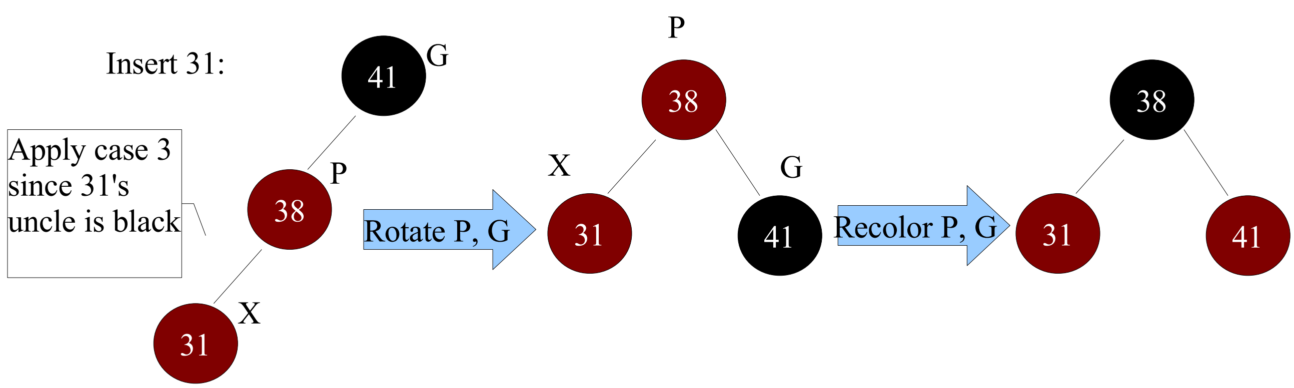
Insert 41:



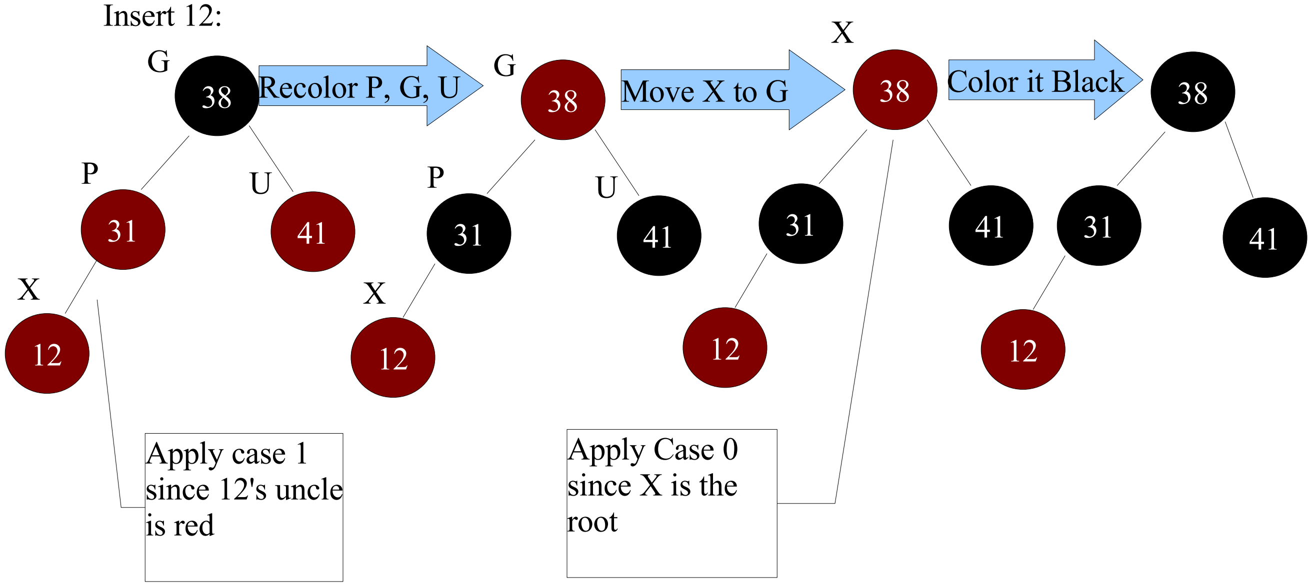
Insert 38:



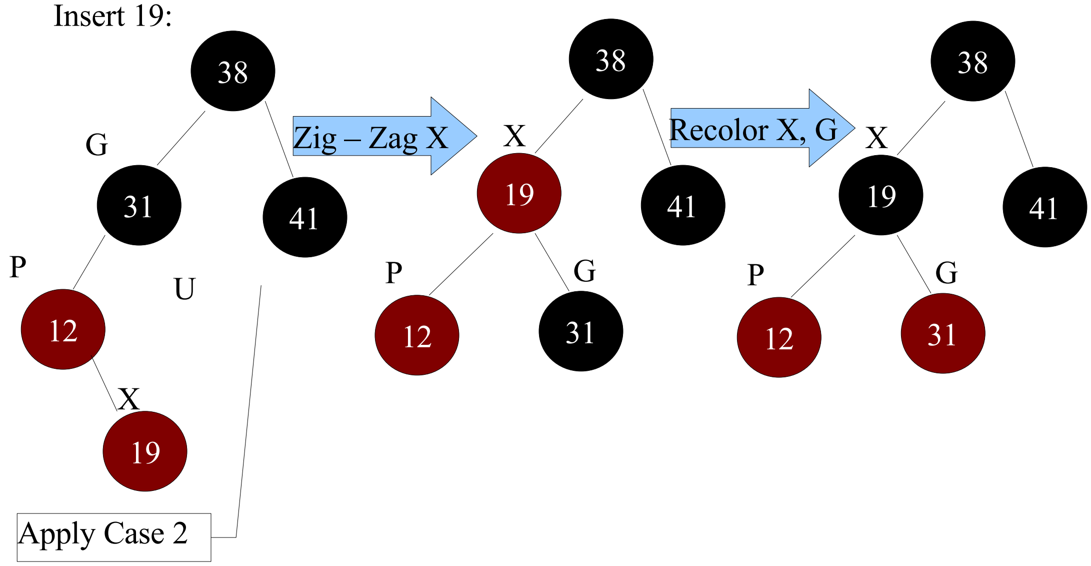
Insert 31:



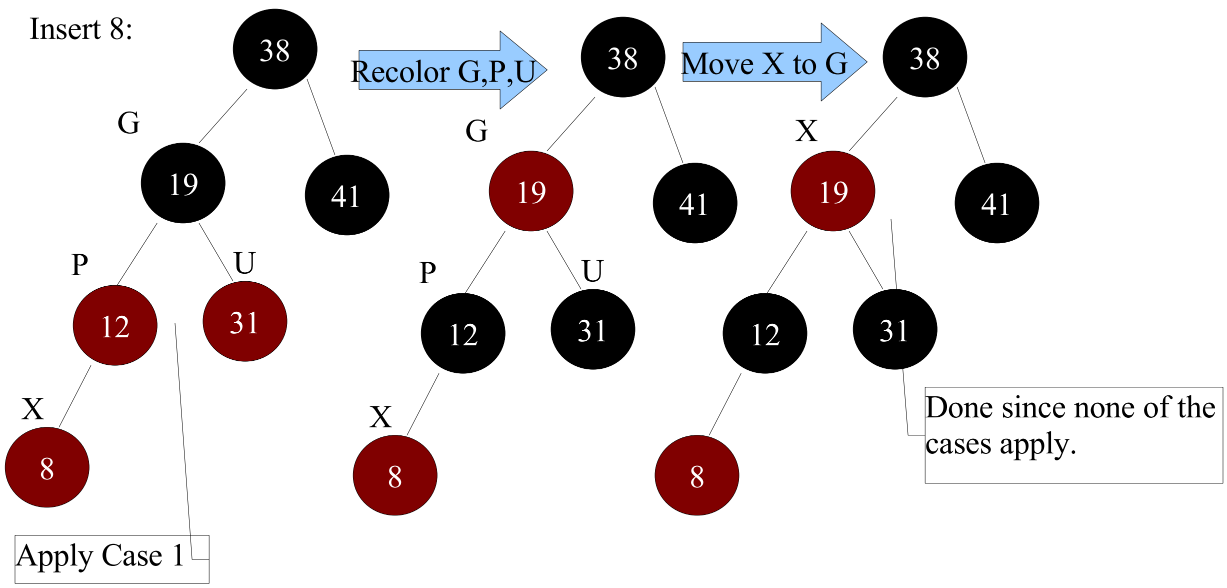
Insert 12:



Insert 19:



Insert 8:



Grading Rubric:

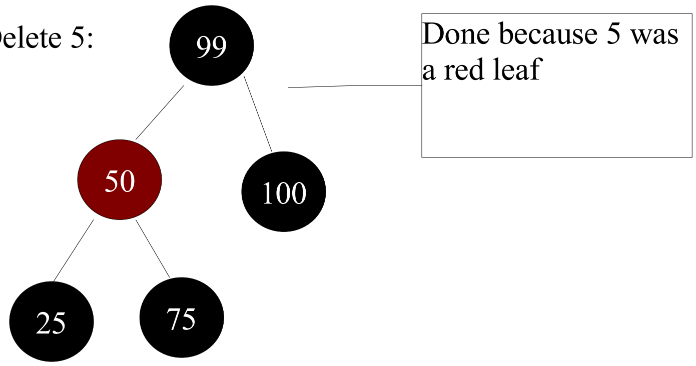
Grade each insertion by itself. The first 2 insertions of 41, 38 are worth 1 point each. Grade all other insertions being worth 2 points.

To keep grading simple each step has to have the correct tree after that insertion. If they have messed up a step the following steps can not be right, so direct them to the solution.

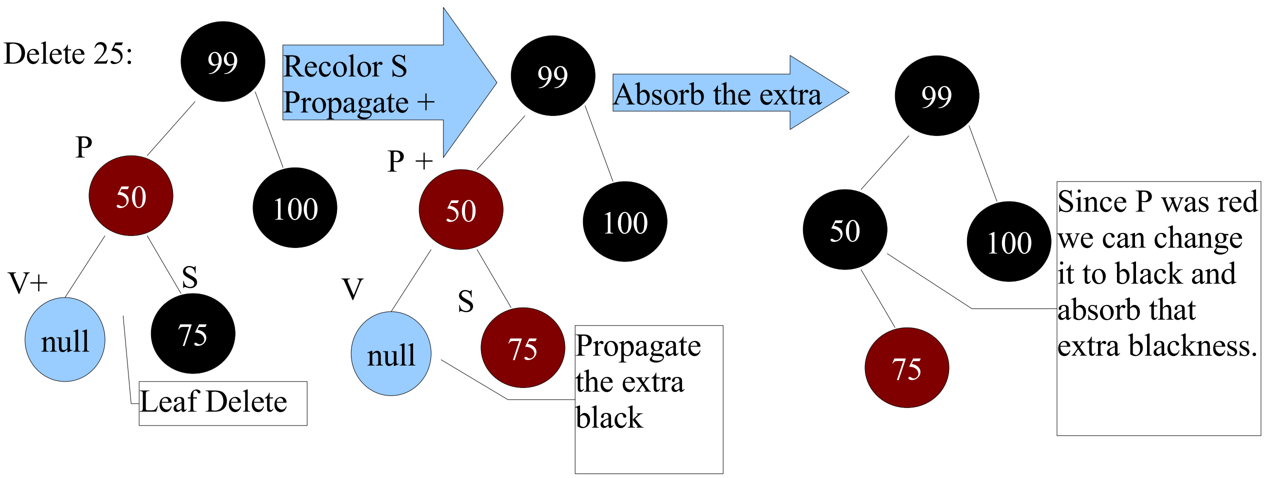
There is a total of 10 points that come from this problem.

2. (10 points) Show the red-black trees that result from the successive deletion of the keys in the order 5, 25, 50, 75, 99.

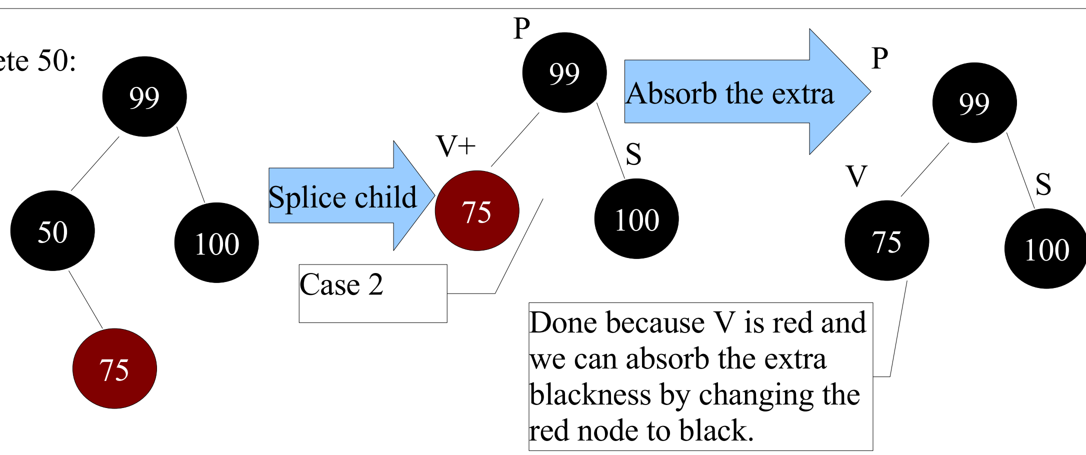
Delete 5:



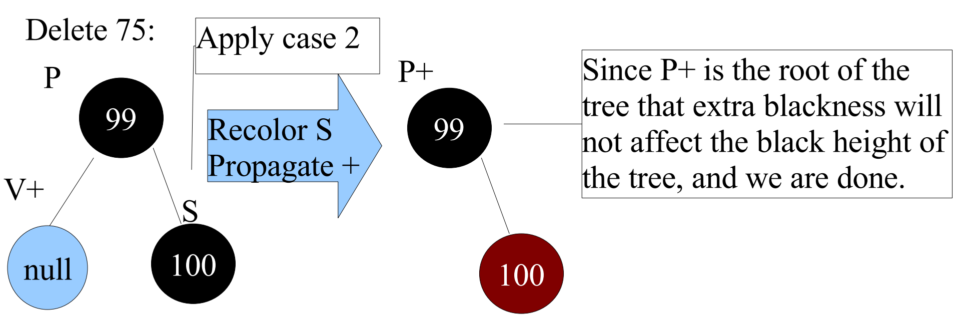
Delete 25:



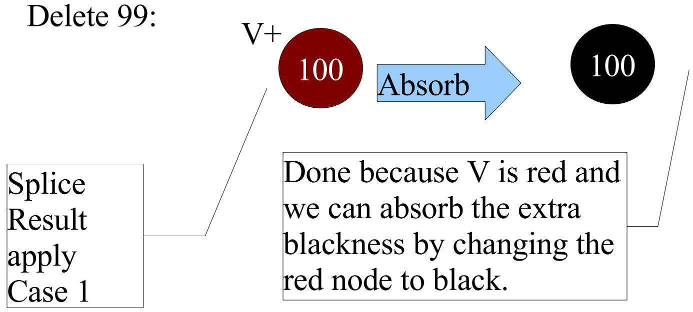
Delete 50:



Delete 75:



Delete 99:



Grading Rubric:

Each deletion is worth 2 points.

To keep grading simple each step has to have the correct tree after the deletion is completed. If they have messed up a step the following steps can not be right, so direct them to the solution.

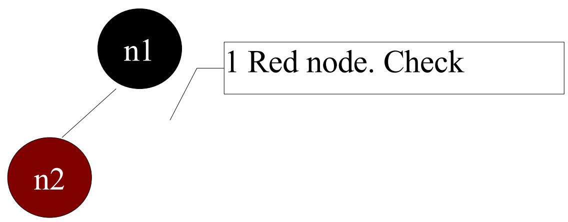
There is a total of 10 points that come from this problem.

3. (5 points) Consider a red-black tree formed by inserting n nodes into an initially empty red-black tree. Argue that if $n > 1$, the tree has at least one red node. Your argument must be in the form of a proof by induction. There are two cases you will need to take into consideration when making your inductive step. The trivial case when Red Node $N+1$ is inserted as a child of a black node does not need to be solved.

Proof by Induction:

Base Case:

N = 2



Inductive Hypothesis: Assume that for a red-black tree of n nodes, where $1 < n \leq N$, there exists at least 1 red node.

Inductive Step: Prove that for a red-black tree of $N+1$ nodes there exists at least 1 red node.

There are 2 cases we have to of interest on the $N+1$ th insertion.

Case 1: Red Node $N+1$ is inserted as a child of a black node. This is the base case which we proved to be true already. This is the trivial case.

Case 2: Red Node $N+1$ is inserted as a child of a red node. We must look at the insertion cases that have X as a child of a red node P.

Insertion Case 1: X remains red after the recoloring of P, G, U. It also remains the same color after we move reference X to Grandparent of X. Thus we have atleast 1 red node.

Insertion Case 2: The parent P of X remains red after the Zig-Zag rotation of X about P and then G. P remains red after recoloring X and G. Thus we have atlease 1 red node.

Insertion Case 3: X remains red after the rotation of P about G. X remains red after the recoloring of P and G. Thus we have atleast 1 red node.

Insertion Case 0: Not included since $n! = 1$, Insertion Cases 2 & 3 are terminating conditions, and Insertion Case 1 still produces 1 red node.

Q.E.D.

Please read the question before Grading.

5 points to a solid proof. Must show for all cases.
4 points to an attempt at a proof but has not proven all cases.
3 points to not attempt at a formal proof but still looks at some required cases.
0 points for no real attempt at the problem.