

Union/Find Problem (ch 21)

Disjoint sets

make set (x)

union (x, y)

find (x)

{x}

{...x, ...} \cup {...y, ...}

what set is x in?

each set will have a "representative"
(name of the set)

{a, b, c, d, e}

find(a) \rightarrow b

(c. 1950s)

History

FORTRAN

INTEGER

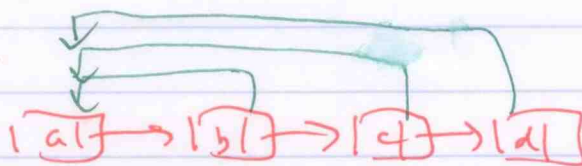
A(100, 100), B(100, 100)

EQUIVANCE A, B

SIMPLE APPROACH

set \rightarrow list

{a, b, c, d}



where is c?

find(c) \rightarrow a

set representative is first item in list

Amortized cost

of sequence of

finds, make sets, unions

$n < m$

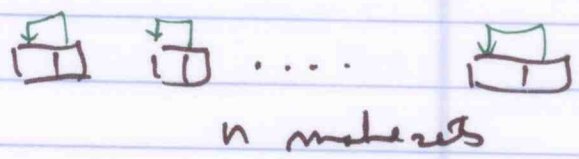
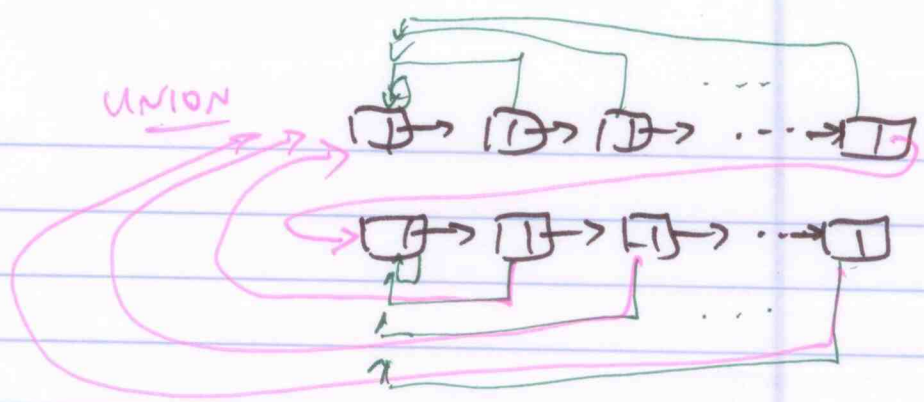
$n-1$

2n-1 ops.

$\Theta(n)$

$0, 0, 0, \dots, 0_m$

~~$\Theta(n)$~~



Union 1		# of pointers that change 1 1
Union 2		2 1
Union 3		3 1
⋮		
Union n-1		n-1, 1
		$\frac{1}{\Theta(n)}$
		$\Theta(n^2)$ changes

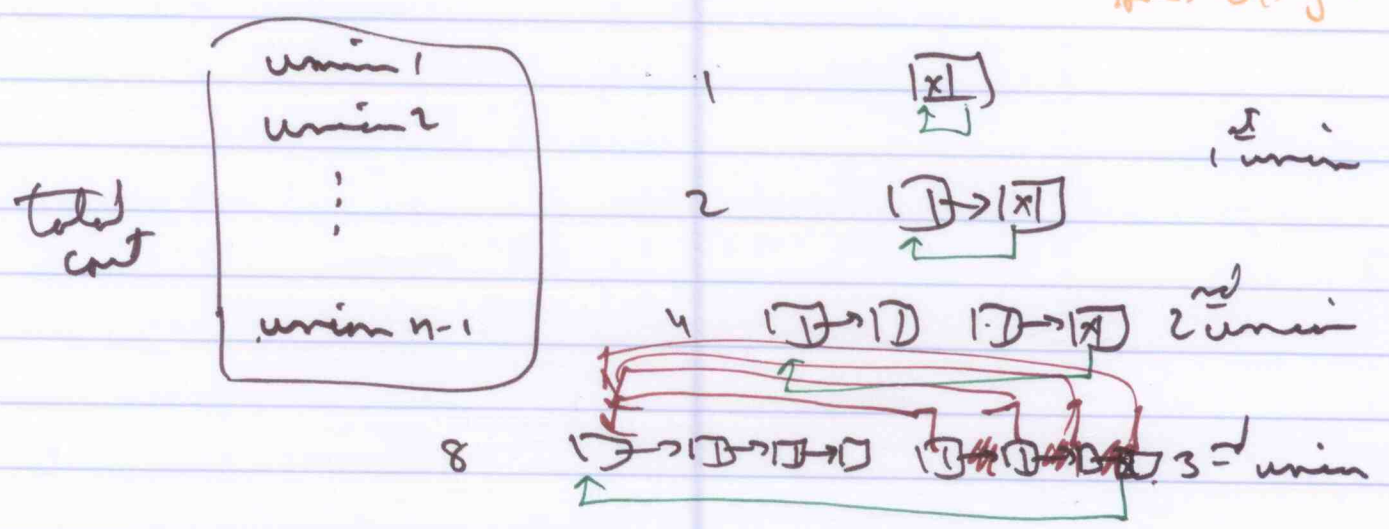
$\{0, 0_2 \dots 0_m\}$

n subsets $O(1)$ each \leftarrow
 $\leq n-1$ unions $\leftarrow O(n \log n)$
finds $O(1)$ each \leftarrow

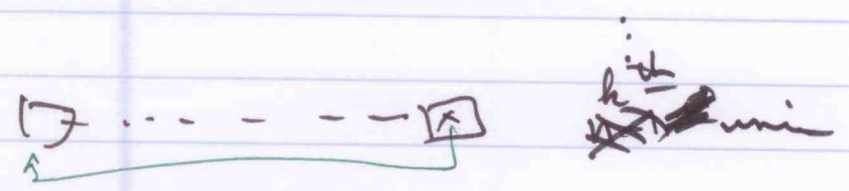
How large can the cost be of the (at most) $n-1$ unions?

Union Rule large sets absorb smaller sets

UNION BY SIZE
 \Downarrow
A.C. $O(\log n)$

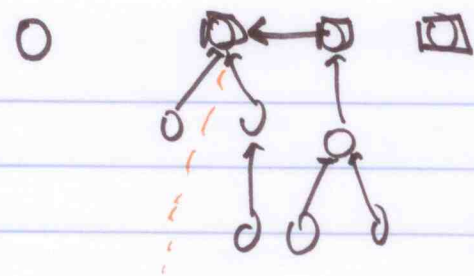


$$n \geq 2^k$$
$$\Downarrow$$
$$\log n \geq k$$



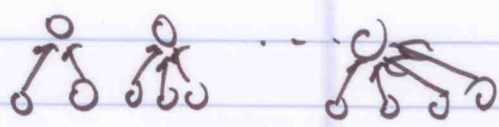
h pointer changes for x

at most $\log n$ pointer changes for x per element



subset $O(1)$
union $O(1)$
find - length of alt

$\alpha(n)$



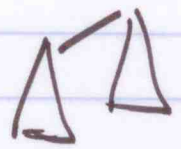
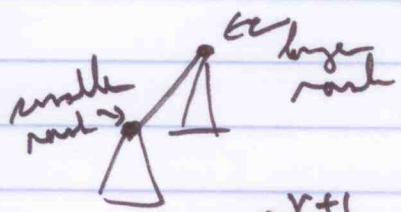
union by ~~size~~ ^{rank}

o_1, o_2, \dots, o_m n subsets

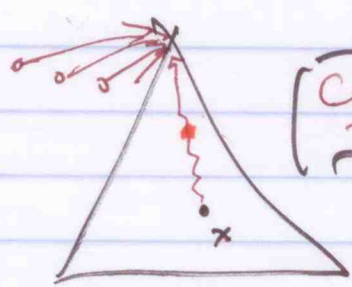
rank \rightarrow subset $O(1)$ union find $O(1)$

what's deepest alt in the trees?

$\lg n$



A.C. $\Theta(\lg n)$



COMPRESS THE PATH

Conclusion A.C. is $O(\alpha(n))$ $\alpha \lg \dots \lg \lg n < 1$

how many \lg 's? $\lg^* n$

$$A_h(j) = \begin{cases} j+1 & h=0 \\ A_{h-1}^{(j+1)}(j) & h \geq 1 \end{cases}$$

$j=1$ $j=2$ $j=3$ $j=4$ $j=5$ $j=6$ $j=7$ \dots
 $A_0(j)$ 2 3 4 5 6 7 8 \dots $A_0(j) = j+1$
 $A_1(j)$ 3 5 7 9 11 13 15 \dots $A_1(j) = 2j+1$
 $A_2(j)$ 7 23 63 159 383 1023 2047 \dots $2^{j+1}(j+1) - 1$
 $A_3(j)$ 2047 $> 10^{10^8}$
 $A_4(j)$ $>> 10^{19728}$

$\alpha(n)$ = # of rows down in col 1 before you get a value bigger than n

$$\alpha(n) \leq 4 \quad \text{for all } n \leq \boxed{}$$

lower bond

$$|g^*_n|$$