Illinois Institute of Technology
Department of Computer Science

Homework Assignment 2

CS 430 Introduction to Algorithms
Spring Semester, 2017

Due: Wednesday, February 1

1. (a) Let $\Pi = (\pi_1, \pi_2, \ldots, \pi_n)$ be a random permutation of $\{1, 2, \ldots, n\}$. What is the expected value of

$$\frac{1}{n} \sum_{i=1}^{n} |\pi_i - i|?$$

   (b) Explain why this value is the average distance that an item will move during sorting.

   (c) What can be concluded about a sorting algorithm (such as insertion) that performs only adjacent interchanges?

2. Problem 6.4-3 on page 160.

3. Problem 7-4 on page 188. Add a part (d), as follows: Determine the *average stack depth* for Tail-Recursive-Quicksort, assuming that all permutations are equally likely. (*Hint*: You need an analysis that is somewhat similar to that done in class on January 25.)

4. Problem 8-3(a) on page 206.

5. CLRS3 has code for RANDOMIZED-SELECTION on page 216. A careless CS 430 student implemented the code, but omitted the "-1" on line 8, typing q instead of $q - 1$.

   (a) Does the corrupted code still work (that is, correctly find the ith smallest element) always, sometimes, or never? Explain.

   (b) Analyze the worst-case running time of the corrupted code.

   (c) Analyze the best-case running time of the corrupted code.

   (d) Analyze the average-case running time of the corrupted code. (*Hint*: Be careful that you do not get snagged by the pitfall described in the middle of page 86 of CLRS3.)

   (e) There is (should be, if you did it correctly) something strange about your answer to the previous part; what is strange and how do you explain it?