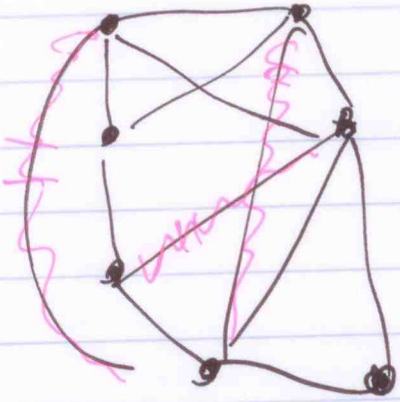
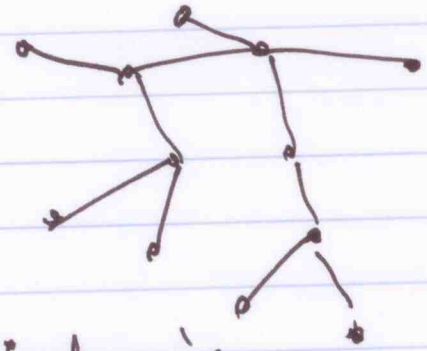
MINIMUM

Spanning Trees (on undirected graphs)



weighted
connected
acyclic
undirected
} Tree



Simple Greedy techniques

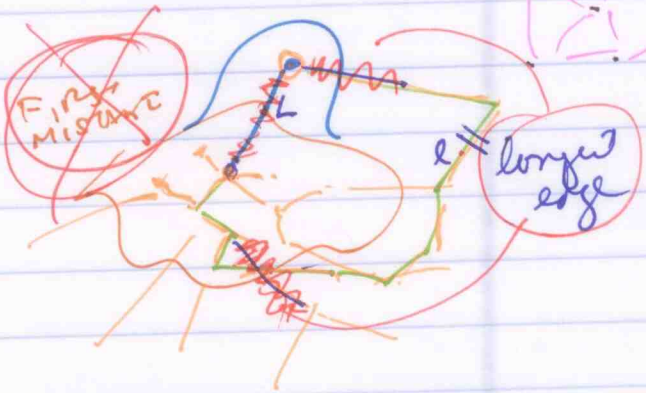
Prim's Alg ←
Kruskal's Alg

vertex outside
closest to a
vertex inside

How ?
How costly

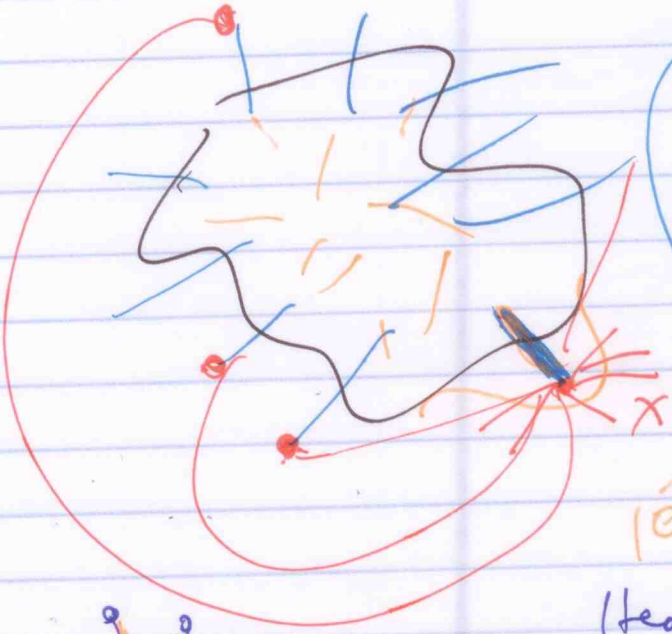
Partial MST

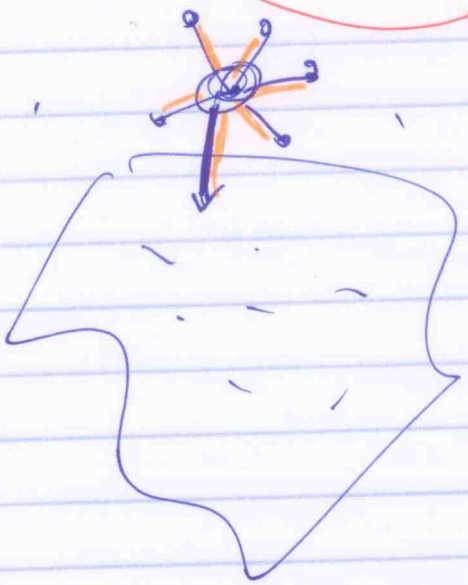
Why done
get a
MINIMUM
S.T.



$\ell \not< L$ No

$\ell \leq L$





$\Theta(n \log n)$

Find cheapest?

How to bring it inside?

VERTICES

Store edges — Find heap

Extract min

Decrease key for every vertex adjacent to x

$|E|$

Heap maintains cheapest connection for every vertex outside P.M.S.T.

$\Theta(|V| \log |V|)$

$\Theta(|E|)$

$\Theta(|E| + |V| \log |V|)$

Kruskal's Alg

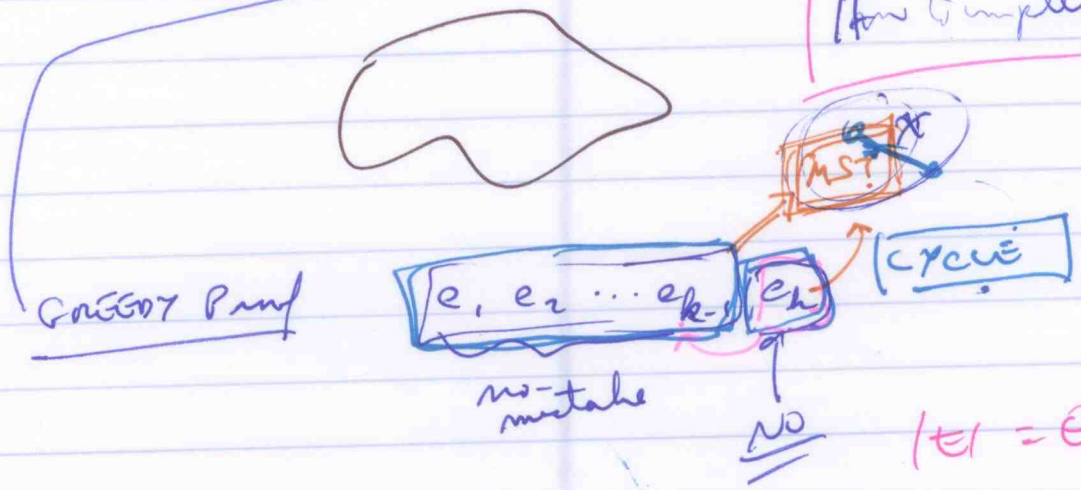
Add edges in increasing order by length, provided that edge doesn't cause a cycle.

$\Theta(|E| \log |V|)$ → $\Theta(|E| \log |E|)$ SORT edges

$e_1 \le e_2 \le e_3 \le \cdots \le e_{|E|}$

Why is this an MST?

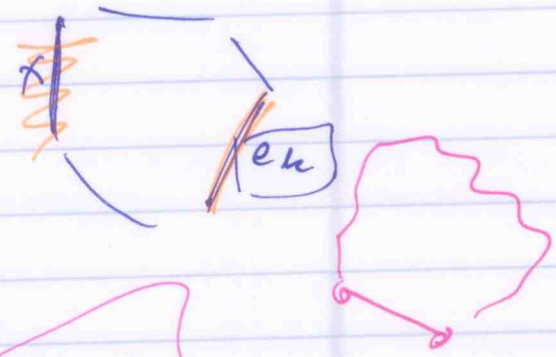
How to implement?

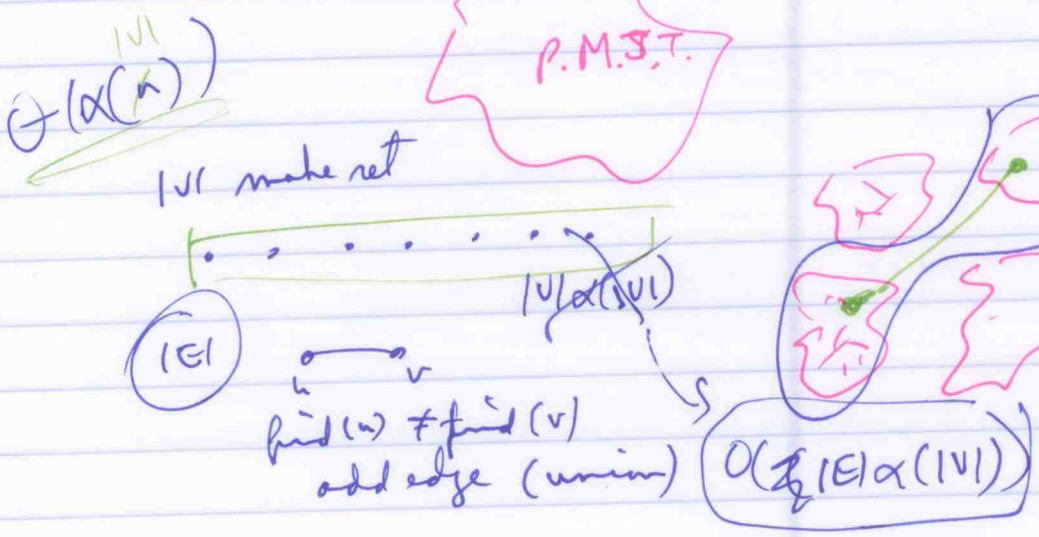


CONTRADICTION Proof

$|E| = \Theta(|V|^2)$

$\log |E| = \Theta(\log |V|)$

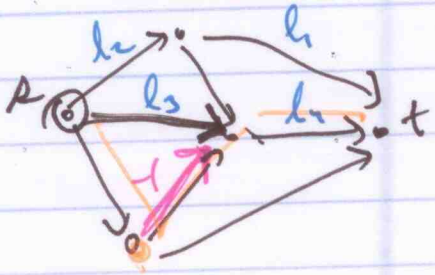




UNION/FIND

$\Theta(|E| \log |V|)$

Single Source Shortest Paths



DIJKSTRA'S ALG

Non neg. edge lengths

BFS

PRIORITY QUEUE

o o o

EXTRACT MIN

DECREASE KEY

(per edge

$O(|E|)$

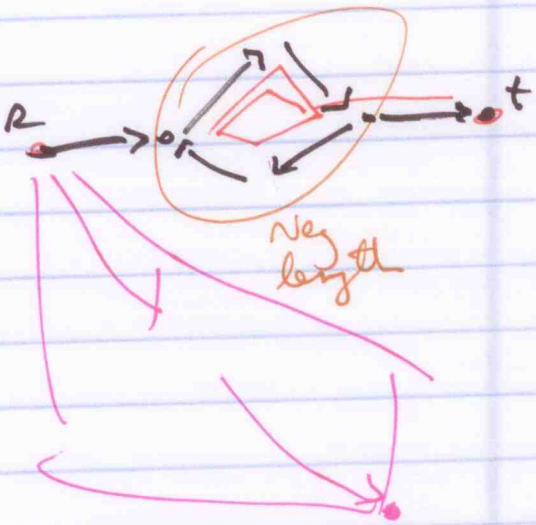
+

$O(|V| \log |V|)$

$O(|V| + |E|)$

$n \quad \pi(n)$

(2) ~~~ $\pi(n)$ — (n)



Neg cycles ⟹ No S.P.

"RELAXATION"