Recitation Note - CS430 Fall 2014

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- This is my personal note for the recitation, and it may contain some error. Please let me know if you find one (or more).
- I do not guarantee I will prepare a note for every recitation.
- 'lg' stands for the logarithm of base 2, and 'ln' of base $e \approx 2.718$ (per textbook).

1 Growth rates and notations

1.1

Determine whether the following notations are correct.

1. $2n^3 + 100n^2 = O(n^3)$
2. $2n^3 + 100n^2 = O(n^4)$
3. $2n^3 + 100n^2 = O(n^2)$
4. $2n^3 + 100n^2 = \Omega(n^3)$
5. $2n^3 + 100n^2 = \Omega(n^4)$
6. $2n^3 + 100n^2 = \Omega(n^2)$
7. $2n^3 + 100n^2 = \Theta(n^3)$
8. $2n^3 + 100n^2 = \Theta(n^4)$
9. $2n^3 + 100n^2 = \Theta(n^2)$

1.2

Order the following lists of functions by their big-theta notations. Also, mark all functions of the same growth rate.

$$\lg(\lg n) \qquad \lg(n^2) \qquad (\lg n)^2$$

$$2^{\lg n} \qquad (\sqrt{2})^{\lg n} \qquad n^2 \qquad n^n \qquad n \lg n$$

$$(n+1)! \qquad n! \qquad n^n \qquad 2^n$$

$$\lg n \qquad \ln n \qquad \log_{10} n$$

2 Recurrence

2.1

Sequence	Annihilator
$\left\langle \alpha \right\rangle$	$\mathbf{E} - 1$
$\left\langle \alpha a^i \right\rangle$	$\mathbf{E} - a$
$\left\langle \alpha a^i + \beta b^i \right\rangle$	$(\mathbf{E} - a)(\mathbf{E} - b)$
$\left\langle \alpha_0 a_0^i + \alpha_1 a_1^i + \cdots + \alpha_n a_n^i \right\rangle$	$(\mathbf{E} - a_0)(\mathbf{E} - a_1) \cdots (\mathbf{E} - a_n)$
$\left\langle \alpha i + \beta \right\rangle$	$(\mathbf{E} - 1)^2$
$\left\langle (\alpha i + \beta) a^i \right\rangle$	$(\mathbf{E} - a)^2$
$\left\langle (\alpha i + \beta) a^i + \gamma b^i \right\rangle$	$(\mathbf{E} - a)^2 (\mathbf{E} - b)$
$\left\langle (\alpha_0 + \alpha_1 i + \cdots \alpha_{n-1} i^{n-1}) a^i \right\rangle$	$(\mathbf{E} - a)^n$

If $\mathbf{X}$ annihilates $\langle a_i \rangle$, then $\mathbf{X}$ also annihilates $c\langle a_i \rangle$ for any constant c.

If $\mathbf{X}$ annihilates $\langle a_i \rangle$ and $\mathbf{Y}$ annihilates $\langle b_i \rangle$, then $\mathbf{XY}$ annihilates $\langle a_i \rangle \pm \langle b_i \rangle$.

Figure 1: Table of Annihilators, from lecture note Aug27.

Find all solutions of the recurrence relation $T(n) = 2T(n-1) + 2n^2$ using operator methods (annihilator).

2.2

The Master Theorem. The recurrence $T(n) = aT(n/b) + f(n)$ can be solved as follows.

- If $af(n/b)/f(n) < 1$, then $T(n) = \Theta(f(n))$.
- If $af(n/b)/f(n) > 1$, then $T(n) = \Theta(n^{\log_b a})$.
- If $af(n/b)/f(n) = 1$, then $T(n) = \Theta(f(n) \log_b n)$.
- If none of these three cases apply, you're on your own.

Figure 2: Master's Theorem, from lecture note Aug27.

Find growth rate of the function $T(n)$ defined via recurrence relation $T(n) = 2T(n/2) + n$ using Master's Theorem.

2.3

Find all solutions of the recurrence relation $T(n) = 2T(n/2) + n$ using the secondary recurrences.

3 Answers

1.1

Correct: 1, 2, 4, 6, 7. Incorrect: 3, 5, 8, 9.

1.2

**In non-decreasing order.

$$\lg(\lg n), \quad \lg(n^2) = 2\lg n, \quad (\lg n)^2$$
$$(\sqrt{2})^{\lg n} = n^{0.5}, \quad 2^{\lg n} = n, \quad n\lg n, n^2, \quad n^n$$
$$2^n, \quad n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1, \quad (n+1)! = (n+1)n!, \quad n^n = n\cdot n\cdots n\cdot n$$
$$\lg n, \quad \ln n = \tfrac{\lg n}{\lg e}, \quad \log_{10} n = \tfrac{\lg n}{\lg 10} \text{ have the same growth rate.}$$

2.1

Annihilator of homogeneous part: $(E-2)$, and non-homogeneous part: $(E-1)^3 \Rightarrow$ The annihilator of the relation is $(E-2)(E-1)^3 \Rightarrow$ All solutions containing unknown constants are:

$$k_1 2^n + k_2 + k_3 n + k_4 n^2$$

2.2

$a = 2, b = 2, f(n) = n \Rightarrow af(n/b)/f(n) = 2(n/2)/n = 1 \Rightarrow T(n) = \Theta(n\log_2 n) = \Theta(n\log n)$.

2.3

Let $n = t_i, n/2 = t_{i-1}$, then $t_i = 2t_{i-1}$. Annihilator for t_i is $(E-2)$, which implies $t_i = k2^i$.

Denote $T(t_i) = F(i)$. Then, the original recurrence relation becomes $F(i) = 2F(i-1) + n$, where $n = t_i = k2^i$. This can be trivially solved by operator methods.