GREEDY METHODS

$F_1\ F_2 \cdots F_n$ files
$L_1\ L_2 \cdots L_n$ lengths – time to read file
$P_1\ P_2 \cdots P_n$

Store on tape – read sequentially

$F_1 \cdots F_{k-1} | F_k$

$L_1 + L_2 + \cdots + L_{k-1} + L_k$

cost(k) = cost of reading file F_k

$$= \sum_{i=1}^{k} L_i$$

Avg cost when all files equally likely, prob(F_k) = $1/n$

$$E(\text{cost}) = \sum_{k=1}^{n} \text{cost}(k) \cdot \text{prob of file } k$$

$$= \sum_{k=1}^{n} \frac{1}{n} \sum_{i=1}^{k} L_i$$

$$= \frac{1}{n} \sum_{k=1}^{n} \sum_{i=1}^{k} L_i \quad \text{minimize}$$

Greedy choice – shortest file first – OPT

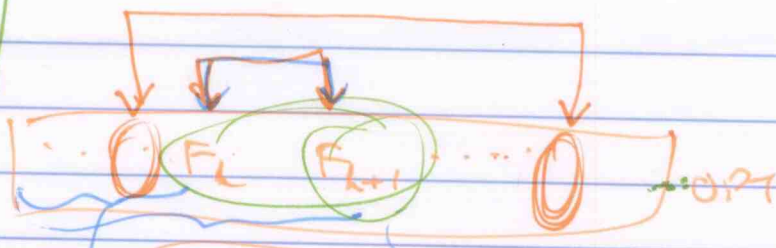
$L_{\pi(1)} \leq L_{\pi(2)} \leq \cdots \leq L_{\pi(n)}$

$P_{\pi(1)} \quad P_{\pi(2)} \quad P_{\pi(n)}$

π = a permutation of $1 \cdots n$

$F_{\pi(1)} \cdots F_{\pi(n)}$

Suppose not



$L_k > L_{k+1}$

p_k p_{k+1}

old cost $F_k = \Sigma$

$F_{k+1} = \ldots + L_{k+1}$

$L_k p_{k+1} - L_{k+1} p_k$

$L_{k+1} p_k - L_k p_{k+1} < 0$

$p_k L_{k+1} - p_{k+1} L_k$

after swap F_k = old cost + L_{k+1} p_{k+1}

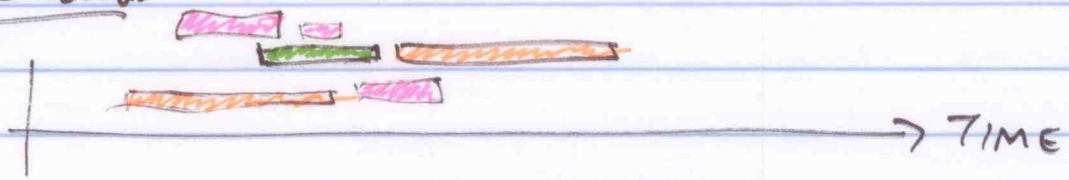
F_{k+1} = old cost − L_k p_{k+1}

old $\overset{\text{avg}}{\text{cost}}$ $L_{k+1} - L_k < 0$

F_k F_{k+1}

Δ avg cost is negative — contradiction

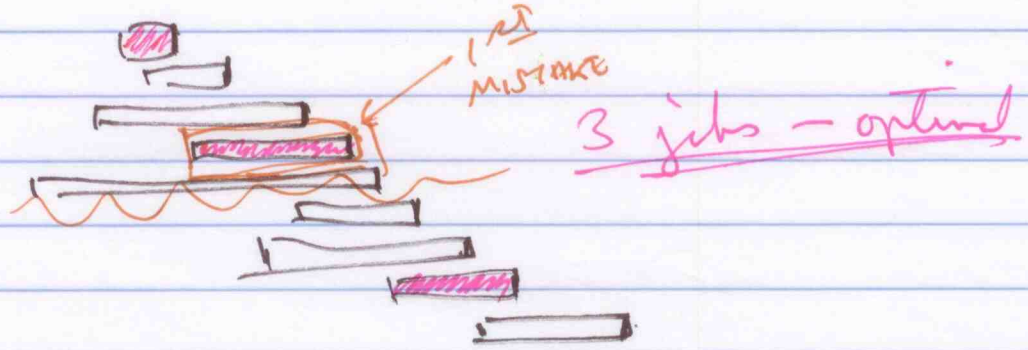
Activity selection



Schedule MOST jobs

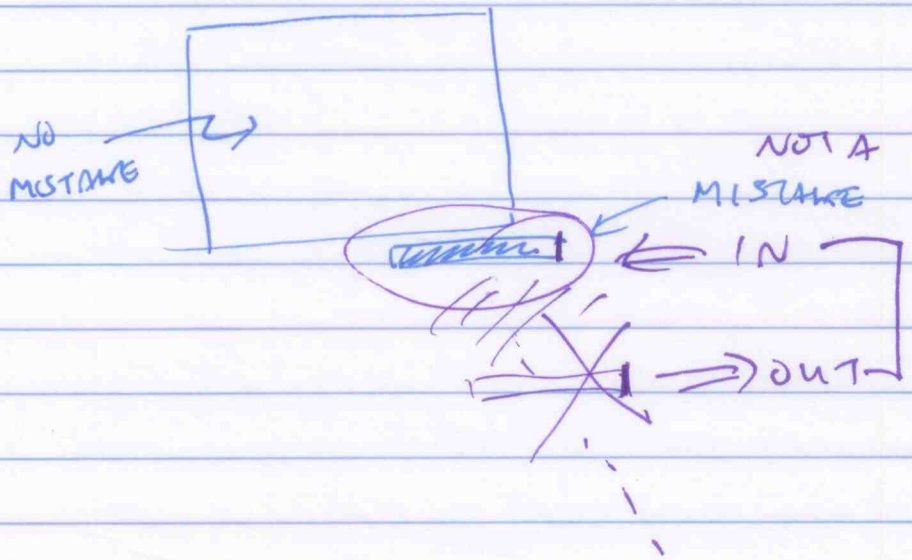
Apply GREED

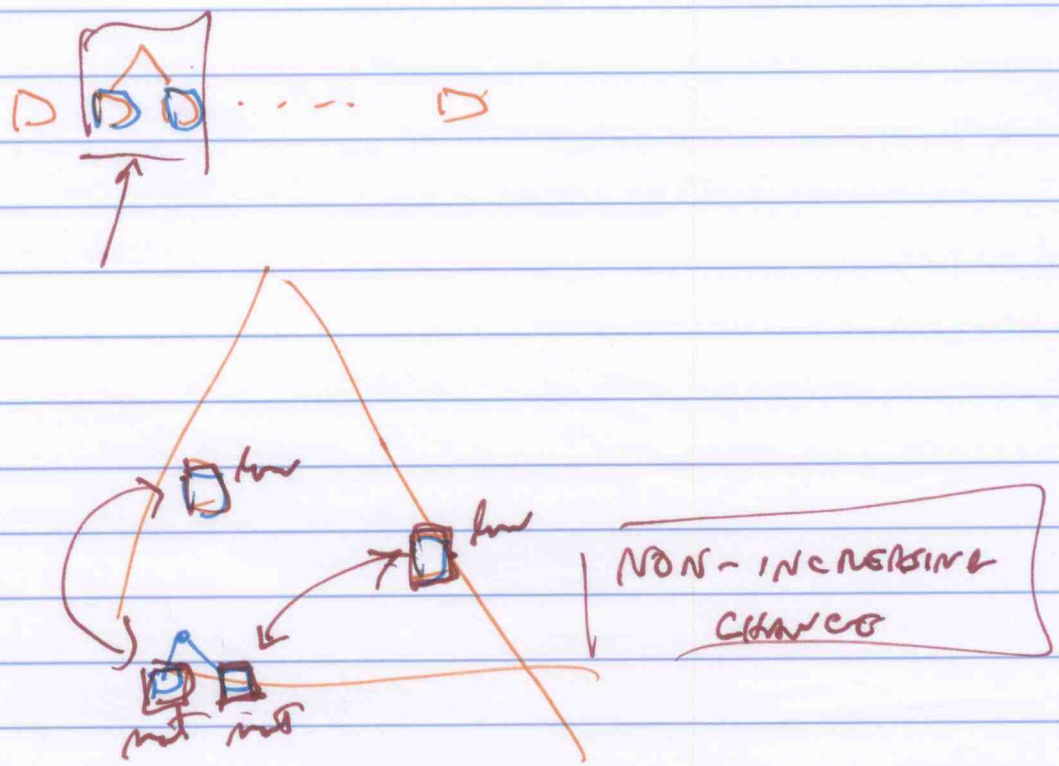
SORT BY END TIME

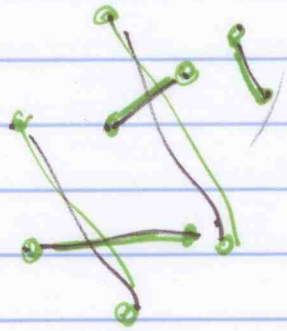


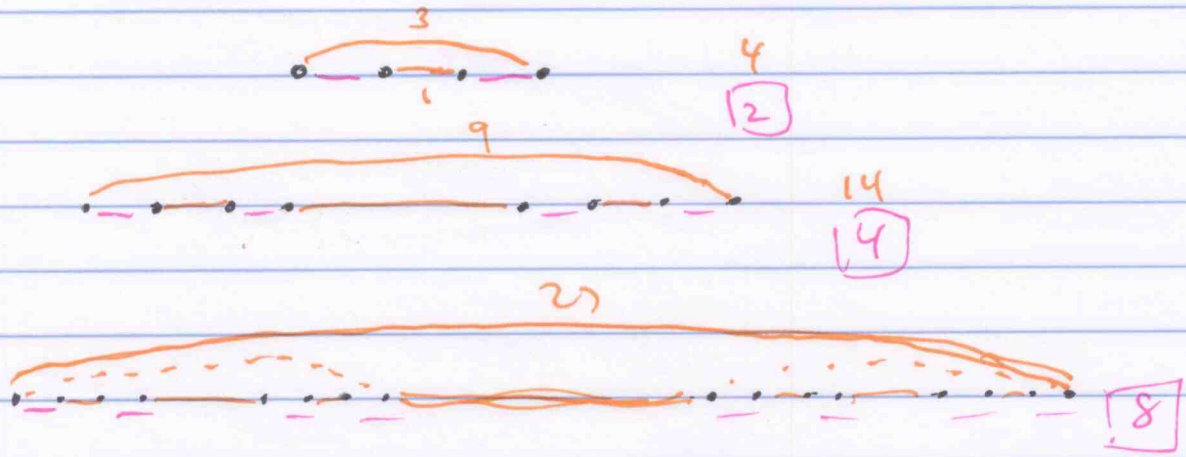
P/ is by contradiction – NOT OPT

FIRST MISTAKE – OMISSION
COMISSION




NON-INCREASING
CHANGES





Given 2^n points, how bad can the greedy alg. be?

$OPT_n = 2^{n-1}$ $\quad OPT_1 = 1$

$GREEDY_n = 2GREEDY_{n-1} - 2L_{n-1} + L_n + L_{n-1}$

$= 2GREEDY_{n-1} + L_n - L_{n-1}$

$L_n - L_{n-1} = 3^n - 3^{n-1}$

$GREEDY_n = 2GREEDY_{n-1} + 2 \cdot 3^{n-1}$

$(G-2)(G-3)$

$\sim 3^n$

$\frac{GREEDY}{OPT} = \Theta\left(\frac{N^{\log \frac{3}{2}}}{2^n}\right) \approx 0.58\ldots$