HOMEWORK ASSIGNMENT 8

Problem 1

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1. Solution

We know the reduction 3-COL $\leq_p$ PLANAR-3-COL or construct a new graph G' from input graph G, such that G is 3-colorable $\Leftrightarrow$ G' is planar 3-colorable.

To construct G' we replace all edges crossing in G. If an edge in G is crossed by multiple other edges the gadgets that replace these crossings need to be linked together at the edges. This propogates the fact that nodes at either end of the edge must be different colours

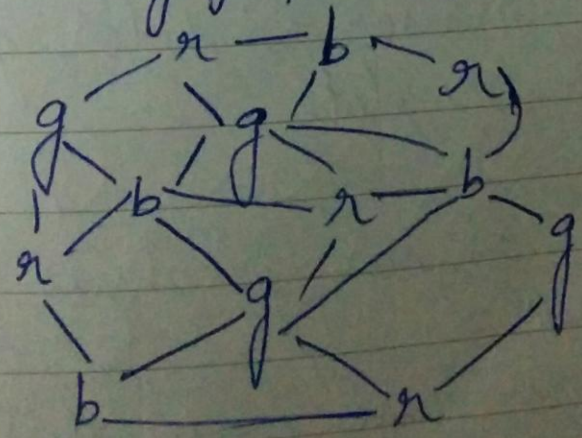
It is easy to see that G' is planar 3-colorable, & it is also to see that removing edges from such a graph gives G. This reduction time runs in polynomial time & thus PLANAR-3-COL is NP-complete.

* NP-hard.

This can be proved by proving that it can be polynomially reduced from an NP-complete problem.

Arbitrary 3-coloring graph problem is NP complete.

3 coloring graph $\leq_p$ planar 3-coloring graph.



walking through the arbitrary graph & finding edges which cross over each other is reaching ~~Cros~~ Crossing over edge.

Since the walking of edges & vertices takes polynomial time only, we can reduce
arbitrary 3-coloring graph $\leq_p$ planar 3-color graph.

Hence, planar-3 color graph is NP-hard.

Hence, planar-3 color graph is NP-complete.

* 3-color planar graph is in close NP.
we can verify that the solution given for 3-color planar graph in polynomial time. The

Problem 2

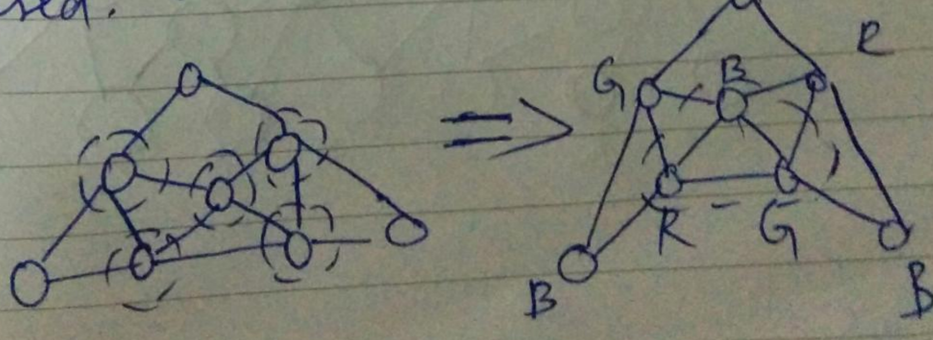
solution can be verified by walking through all edges in graph & checking whether each edge in graph has its cornered vertices having different color. BFS or DFS search can be used for this.

Since either search takes polynomial time, we can verify in polynomial time

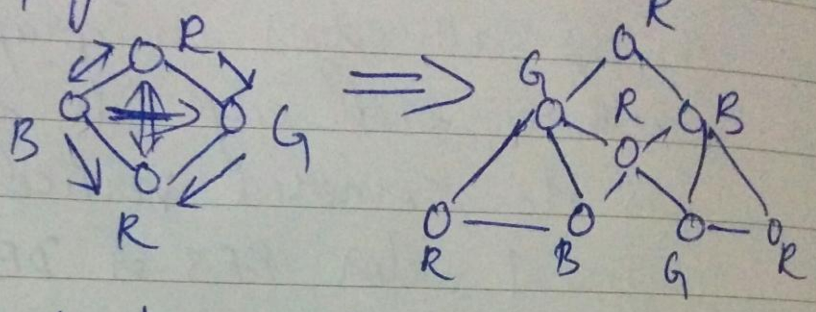
Hence 3-color planar graph is NP-close.

2. Graph 3 colorability for maximum degree 4 graphs

Proof: "Degree Reduction" of gadgets is used.



3 Colorability constraint propagation



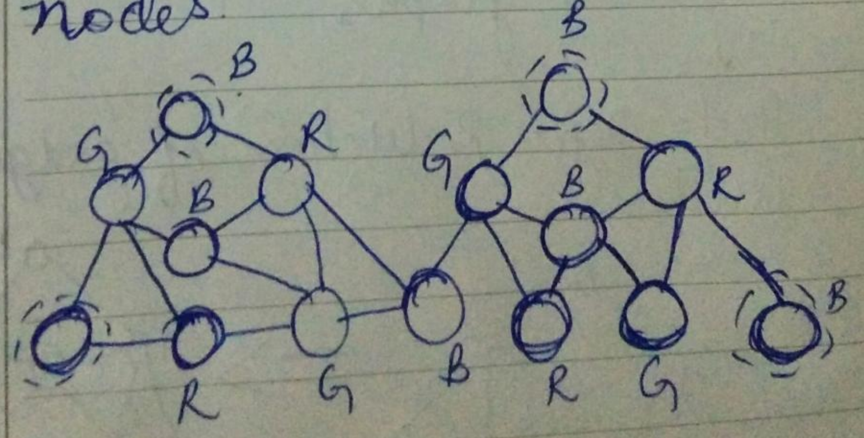

Gadget properties

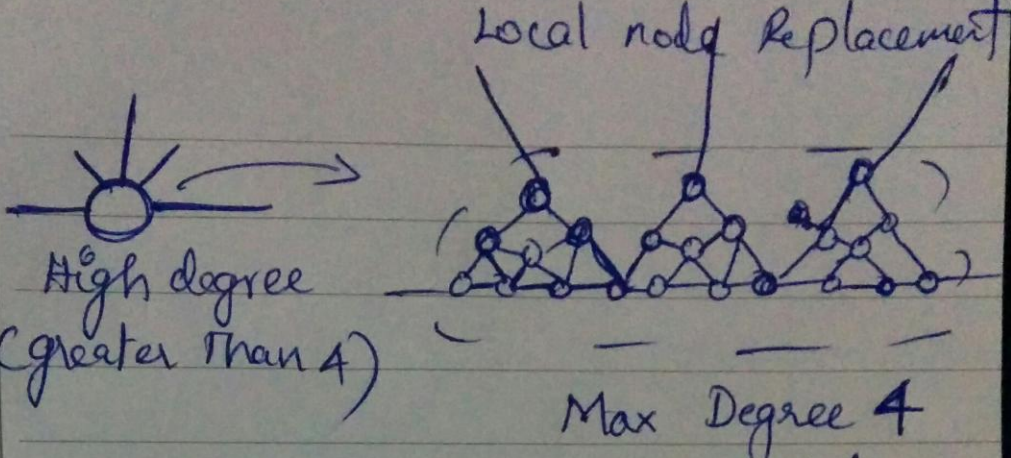
a) Gadget has max-degree of 4

b) Gadget is 3 colorable but not 2-colorable.

c) In any 3-coloring all corners get the same color.

Combine the gadgets into super nodes.






Properties

* Super-nodes has the same property of gadgets.

Super Nodes are used as "fan-out" components to reduce all node degrees to 4 or less.

Once the graph of nodes with degree 4 is 3-colored. it will be NP-complete provided the onle original graph ~~is~~ can be 3-colored. NP completeness can be proved from the problem 1, since it will take polynomial time to traverse the entire graph.