Recitation Note - CS430 Fall 2014

Taeho Jung
Illinois Institute of Technology

Nov 21, 2014

- I do not guarantee I will prepare a note for every recitation.
- This note is partially based on [1]. Please refer to it for more examples on the polynomial reduction.

1 Basic NPC problems

1. SAT-3 problem
2. Partition
3. Vertex cover
4. Hamiltonian circuit

Partition: given a set of numbers, find a partition of them such that the sum of them are equal.

Vertex cover: Given an undirected graph $G(V, E)$, determine if there exists a subset of V of size k which covers G.

2 Partition $\leq_P$ Subset sum

Subset sum: decide whether a subset of given integers is equal to k

Given an instance of Partition, add all the numbers to get the sum S. Then, Partition is reduced to the subset sum problem where $k = S/2$. That is, if anyone can solve this subset sum problem, he can also solve the original Partition problem.

3 Partition $\leq_P$ Knapsack

Given an instance of Partition, we set $V = C = S/2$ where S is the sum of all numbers in the Partition. Then, if anyone can solve the Knapsack with those V and C, he can also solve the Partition problem.

4 Partition $\leq_P$ Bin packing

Bin packing: Given items with sizes $s_1, \cdots, s_n \in (0, 1]$, pack them into k bins where each bin is of size 1.

Given an instance of partition, first normalize the numbers in the instance so that all numbers $n_1, \cdots n_n$ are between (0,1]. Then, add all the numbers to get the sum S. If anyone can pack the items of sizes

$n_1, \cdots, n_n$ into two bins of size $S/2$ (S is the sum of $n_1, \cdots, n_n$), he can also solve the original partition problem.

5 Vertex Cover $\leq_P$ Clique

Clique: Given an undirected graph $G(V, E)$, determine there is a complete sub-graph of size k.

Given an instance of Vertex cover for cover size k', complement the graph G' in the instance of vertex cover to achieve G (*i.e.,* add if there was no edge, and remove if there is an edge). Then, if anyone can solve the Clique problem in G for $k = |V| - k'$ (complement graph of G' in Vertex cover), he also solves the vertex cover in G' by choosing the complement set of vertices.

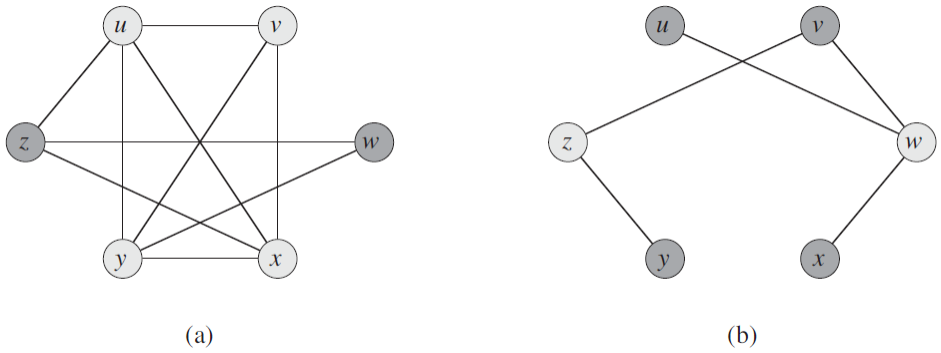

Figure 1: (a) is G in Clique, (b) is G' in Vertex cover

6 Partition $\leq_P$ Scheduling

Scheduling: Given a number of tasks with duration π_i, number of total processors p, and the time limit T, determine whether there exists a scheduling not exceeding T.

Given an instance of Partition where it has numbers n_i's, we set $\pi_i = n_i$, and let $p = 2$, and $T = S/2$ where S is the sum of all numbers n_i. Then, if anyone can solve this Scheduling problem having 2 processors, he also solves the original Partition problem.

References

[1] "Np-completeness reduction," http://tka4.org/materials/study/9%20sem/3.%20Teoria%20igr/%D0%A2%D0%B8%D0%B3%D1%80%D1%8B%203%20%D0%BF%D0%BE%D1%82%D0%BE%D0%BA/%D0%A2%D0%B5%D0%BE%D1%80%D0%B8%D1%8F%20%D1%81%D0%BB%D0%BE%D0%B6%D0%BD%D0%BE%D1%81%D1%82%D0%B8%20%D0%BD%D0%B0%20%D0%B0%D0%BD%D0%B3%D0%BB%D0%B8%D0%B9%D1%81%D0%BA%D0%BE%D0%BC.pdf.