AMORTIZED ANALYSIS

$$\underbrace{o_1, o_2 \cdots o_n}_{\text{total cost}} = \sum_{i=1}^{n} cost(o_i)$$

$$\sum_{i=1}^{n} \text{amortized cost}(o_i)$$

I. Stack w/ multipop
II. Counter w/ bit change
III. Dynamic Tables
~ Hash Tables

1) Aggregate method
2) Accounting method
3) Potential method – bank acct ≥ 0

Φ(data structure) = $ in the bank

NOT IMPLEMENTED
INSTRUMENT
WE CONTROL

III Dynamic Tables

sequence of insertions
- $O(1)$ simple
- $\Theta(n)$ table full – needs expansion, copy all n items to bigger space

~~hourglass deletions~~

When table is full & we get an insertion
DOUBLE TABLE SIZE

Insertion #	Work
1	1
2	1 + 1
3	1 + 2
4	1
5	1 + 4
6	1
7	1
8	1
9	1 + 8
⋮	⋮
n	

Work = insertion of new item + copying

insertion of new item: n

copying: $1 + 2 + 4 + 8 + \cdots + 2^{\lg n} = 2^{1+\lg n} - 1 \approx 2n$

$n + 2n = 3n$

AGGREGATE

$o_1, o_2 \cdots o_n$

$$\sum_{i=1}^{n} \text{cost}(o_i) = 3n$$

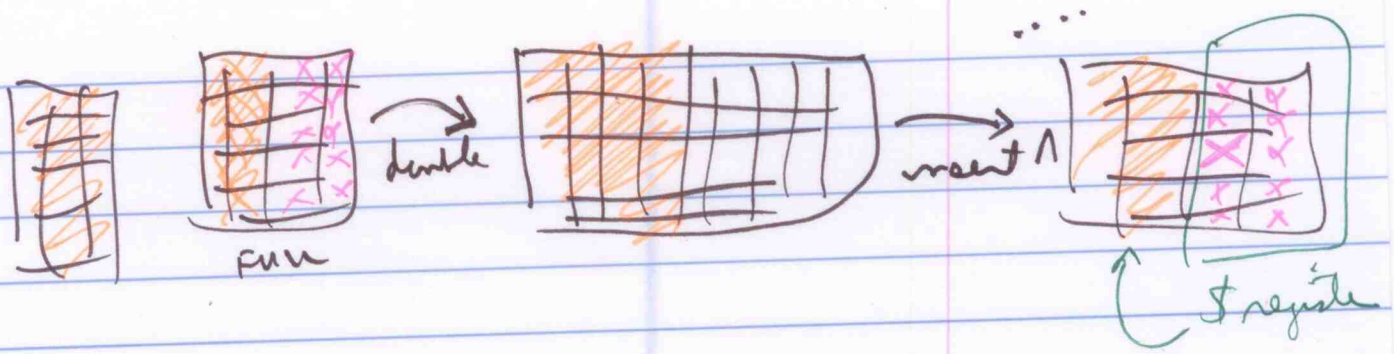
AVG cost per op = 3

ACCOUNTANT

charge customer \$3/insertion

- \$1 → employee
- \$1 → tape it to the element (pays for future move)
- \$1 → cash register

elements w/o \$1 taped ≤ \$ in register



$$\Phi(\text{table}) = 2 \cdot \text{num} - \text{size}$$

P.I.D.2O.MA.

$\Phi = aN + bS$

$N = S \rightarrow = N$

$N = S/2 \quad = 0$

ACTUAL COST = AMORTIZED COST + (Bank right before) − (Bank right after)

pay employee

charge customer

$\Delta\Phi$

AMORT COST $\hat{c}_i$ = ACTUAL COST c_i − $\Delta\Phi$

$\Delta\Phi$ = Bank right after − Bank right before

$$\hat{c}_i = c_i + \Phi(\text{after } o_i) - \Phi(\text{before } o_i)$$

Φ(Table) = 2 · number of elts − size of Table

$\hat{c}_i = c_i + \Phi(\text{after}) - \Phi(\text{before})$

n : $2n = 2(n-1)$: $n-1$ = 2

amortized cost of i^{th} insertion

Case 1 Simple insertion (room in table)

actual cost = 1

$\Delta\Phi = (2n - 2) - [2(n-1) - 2]$

$= 2n - 2 - 2n + 2 + 2$

$= 2$

$A.C = 1 + 2 = 3$

Case 2 Doubling insertion (no room)

actual cost $n-1$ copies + 1 insertion

$\Delta\Phi = 2n - (2(n-1)) - [2(n-1) - (n-1)]$

$= 2n - 2n + 2 - 2n + 2 + n - 1$

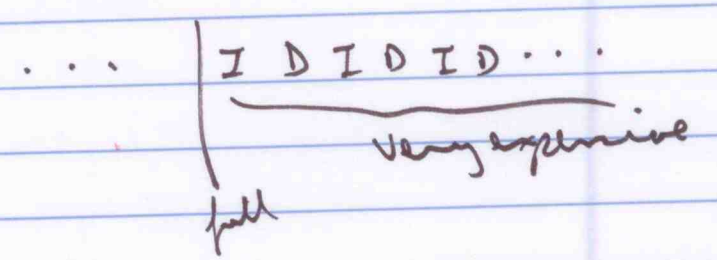
$3 - n$

$+ = 3$

Slow deletions:

full table → double or insertion

~~half full~~ ¼ full → halve on deletion



$$\hat{c}_i = c_i + \Delta\Phi$$