ASSIGNMENT-9

ALGORITHMS

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1 Use the weight on pg 4 of Prof graph coloring slides to prove that 3-coloring a planar graph is NP-complete. Remember, to prove NP-completeness you must prove NP-hardness & that the problem is in the ~~class~~ class NP.

Solution: We know the reduction 3-COL $\leq_p$ PLANAR-3-COL, or construct a new graph G' from input graph G such that G is 3-colorable $\Leftrightarrow$ G' is planar 3-colorable.

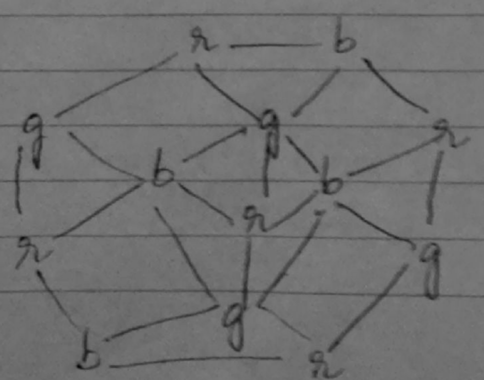
To construct G', we replace all edges crossings in G. If an edge in G is crossed by multiple other edges the gadgets that replace these crossings need to be linked together at the edges. This propagates the fact that the nodes at either end of the edge must be different colors.

It is easy to see that G' is planar 3-colorable, & it also easy to see that removing edges from such a graph gives G. This reduction time runs in polynomial time, & thus PLANAR-3-COL is NP-complete.

* NP-hard

This can be proved by proving that it can be polynomially reduced from an NP-complete problem.

Arbitrary 3-coloring graph problem is NP complete.
3 coloring graph $\leq_p$ planar 3-coloring graph



Walking through the arbitrary graph & finding edges which cross over each other & reach crossing over egdge.

Since the walking of edges & vertices ~~takes~~ takes polynomial time only we can reduce
arbitrary 3-colouring graph $\leq_p$ planar 3-color graph

Hence, planar-3 color graph is NP-hard

Hence, planar-3 color graph is NP-complete

* 3-color planar graph is in close NP

We can verify the solution given for 3-color planar graph in polynomial time. The solution can be verified by walking through all edges in graph & checking whether each edge in graph has its cornered vertices having different color. BFS or DFS search can be used for this.
Since either search takes polynomial time we can verify in polynomial time.

Hence 3-color planar graph is NP-close

KAVYA SHAMSUNDARA

(2) Ex 34.5-7 on pg 1101
The longest simple-cycle problem is the problem of determining a simple cycle (no repeated vertices) of max. length in a graph. Formulate a related decision problem, & show that the decision problem is NP-complete

Solution: We first define the decision version of the longest-simple-cycle problem as below:

LONGEST SIMPLE CYCLE (G,k): Given an undirected graph G & an integer k, does G has a simple cycle of length atleast k.

Now, we will show that LONGESTSIMPLEGRAPH (G,k) is NP-Complete. We will reduce a known NP-Complete problem, namely HAMCYCLE (H) problem for this proof. The HAMCYCLE (H) is defined as below:

HAMCYCLE (H): Given an undirected graph H does H has a Hamiltonian cycle.

LONGESTSIMPLECYCLE(G,k) ∈ NP:
Proof: Let (G,k) be an instance of LONGESTSIMPLECYCLE. Given a certificate of proof 'y', which is a sequence of vertices, we can simply scan through the graph G in polynomial time to verify that 'y' is a cycle, no vertex in 'y' appears more than once, & the length of 'y' is 'k' or higher.

HAMCYCLE (H) $\leq_P$ LONGEST SIMPLE CYCLE (G,k):
Proof: From an instance of HAMCYCLE (H), we construct an instance LONGESTSIMPLECYCLE (H, |V|). The Instrumentation is trivial & it can be done in constant (polynomial) time. Now we claim,

KAVYA SHAMSUNDARA

the graph H(V,E) has a Hamiltonian cycle, if & only if the length of its longest simple cycle is equal to |V|.

The claim is correct, because if H(V,E) has a Hamiltonian cycle, that cycle is a simple path of length |V|. On the other hand, if H does not have a Hamiltonian cycle, the length of the longest simple in H must be strictly less than |V|.

The above proofs conclude that the LONGESTSIMPLECYCLE is NP-Complete.