Redistribution, Sovereign Debt, and Optimal Taxation

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Abstract

This paper examines the interaction between a country's concern for redistribution and its external indebtedness. I document a positive correlation between income inequality and external debt across countries and time periods. I propose a model, consistent with empirical patterns, of small open economy in which the government has a redistributive motive and lacks commitment, and taxes are distortionary. Domestic and external credit markets are state-contingent, and the government faces endogenous borrowing constraints. Theoretically, when borrowing constraints bind, the government increases its ability to repay debt by lowering labor taxes and levying taxes on domestic borrowing. Default is endogenously costly because labor distortion needed to redistribute is higher and more volatile in financial autarky than in the contract. I calibrate the model using Italy's data and show that the model accounts for (i) average and volatility of Italy's external debt-to-output ratio, (ii) cross-sectional positive correlation between pre-tax income inequality and external debt (iii) increase in external debt-to-output given an increase in Italy's income inequality from 1985-2001 to 2002-2015. I study optimal austerity by estimating responses of fiscal policies and inequality to a negative productivity shock. The findings are that the government increases its external borrowing and redistributes less resources towards low-income residents, while relative consumption inequality remains unchanged.

Keywords: Redistribution; Inequality; Sovereign debt; Optimal taxation; Limited commitment

JEL Classifications: F34; F38; H21; H23; H63

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1 Introduction

In the recent European debt crises, there have been intense policy debates on the design of fiscal policies during severe economic downturns. Austerity policies such as increasing tax revenue or reducing government's expenditure help reduce debt levels, but have unequal consequences across domestic residents[1]. Such policy designs often require a thorough understanding about the interaction between a government's concern for redistribution and its commitment to debt repayment.

This paper aims to fill in the knowledge gap by answering three key questions: first, how does limited borrowing affect a government's ability to redistribute? Second, how does a government's redistributive motive affect its incentive to repay debt? Third, given the answers to the first two questions, how do we design optimal austerity policies given their redistributive consequences?

In addressing these questions, this paper develops a theoretical and quantitative analysis of a small open economy framework with the government's redistributive motive, the government's lack of commitment, and distortionary taxes. The theoretical findings show how optimal taxation, which are the government's redistributive tools, respond to limited borrowing. The framework also provides a mechanism in which the government's redistributive motive influences the opportunity cost of default and hence its repaying incentive. I calibrate the model to Italy's data and show that this mechanism can quantitatively account for the empirical relationship between income inequality and external debt. Lastly, I study optimal austerity policies by quantifying the responses of fiscal policies and inequality to a negative productivity shock, such as in the case of a recession.

The paper first documents the empirical properties of income inequality and external debt by using two novel panel data sets on inequality and balance of payments. I show that a high pre-tax income inequality is associated with a high external debt-to-output both across countries and over time. This relationship still holds after controlling for output levels and output growth. The panel estimation implies a positive and statistically significant effect of pre-tax income inequality on external debt-to-output.

The model is a small open economy in which domestic agents are impatient and differ by labor productivity types. The economy faces aggregate uncertainty in terms of shocks to the

[1] The United Kingdom and Ireland implemented expenditure cuts, while Greece, Italy, Portugal, and Spain implemented both tax increases and expenditure cuts for their austerity plans. Most of these plans include cuts in public services, pension, and education programs. Monastiriotis (2011) argued that the Greek prolonged fiscal consolidation exacerbated the regional disparities and imbalances. Leventi and Matsaganis (2016) used micro-simulation model to assess distributional effects of austerity policies. They found that such policies have led to higher poverty and after-tax income inequality, worsening the adverse distributional effects of the recession.

aggregate productivity and government spending. Both domestic and external credit markets are state-contingent. Domestic agents only have access to the domestic credit market, while the government has access to both markets[2]. Tax policies include marginal taxes on labor, marginal taxes on domestic saving, and lump-sum taxes that do not depend on individual income levels. The government has lack of commitment in all fiscal policies and cares about the social welfare of domestic agents, which captures its redistributive motive.

Concerns for redistribution rationalize the need for distortionary taxation. Since all domestic agents face the same tax rates, a government that has a redistributive motive towards the low-income agents may find it optimal to levy a positive labor tax and a lump-sum transfer. In this way, high-income agents bear higher tax burden than low-income ones. All government's revenue needed to finance expenditures or debt repayment does not come from distortionary taxes but instead from lump-sum taxes. Therefore, the levels of tax distortions represent the equilibrium cost of redistribution.

The government's lack of commitment endogenously imposes limits on the economy's external borrowing. The government chooses its policies sequentially to maximize the social welfare. However, the government cannot commit to future choices on repayments of debt and taxes. The policies are determined in a repeated game between the government, domestic agents, and international creditors. If the government deviates from the contract, it triggers punishment to financial autarky, in which there is no future access to the domestic and external credit markets. The feasible set of policies is determined by the endogenous borrowing constraints, in which the continuation value of staying in the contract has to be at least the value of financial autarky.

The impatience of the domestic agents means that they would want to borrow. Since they only have access to the domestic market, the government also acts as a financial intermediary between the domestic agents and the international lenders. The need for borrowing leads to the country run up debt and eventually hit the borrowing constraints. However, an infinitesimal domestic agent does not internalize the fact that as she borrows more, the borrowing constraints get tightened. When the borrowing constraint binds, the government can use the tax on domestic borrowing to make the domestic agent's inter-temporal substitution constraint take into account the additional cost of debt.

In contrast to the sovereign default model that has incomplete market and equilibrium default, this framework builds on the complete state-contingent market and features no equilibrium default. However, the threat of default affects the optimal allocation, similar to Thomas and Worrall (1988) and Kehoe and Levine (1993). The endogenous borrowing

[2] This assumption on market structure does not affect the main results. Allowing domestic agents to access to the external credit market only changes the interpretation of external debt.

constraints imply that there is imperfect insurance against the aggregate risk. These constraints also endogenously determine the optimal portfolio of debt, in contrast to the incomplete framework where there is only the risk-free bond. Furthermore, defaults in reality often accompany with a non-zero net capital flow as a country often goes through a lengthy process of renegotiation and haircuts. Instead of zero net capital flows as in the sovereign default model, this framework embeds part of the default procedure into self-enforcing borrowing constraints.

The answer to the first question lies within the main theoretical result, which establishes how optimal taxation dynamically respond to borrowing constraints. This is an extension of the results in Tran Xuan (2019). When the borrowing constraints do not bind, the marginal labor tax does not change, and domestic borrowing taxes are zero. Nevertheless, when the borrowing constraints bind, all future labor taxes weakly decrease, and domestic borrowing taxes are positive. Intuitively, without borrowing constraints, it is optimal to redistribute by distorting intratemporal margins (labor taxes). The optimal labor taxes balance the marginal benefit of redistribution and the marginal cost of distortion. Since borrowing is not costly, the government can use debt to smooth out the labor distortions[3]. When borrowing constraints bind, distorting intertemporal margins (borrowing taxes) become beneficial because it aligns the intertemporal interests of the government and the domestic agents. It turns out that the optimal borrowing taxes have a redistributive benefit. When the borrowing constraint binds, the government can maintain the same redistribution as before by using more borrowing taxes and less labor taxes. Declines in labor distortions improve the economy's efficiency and allow the government to sustain its external debt.

The government's redistributive motive affects its repayment incentive via changing the cost of default. This an endogenous component of cost of default that is novel to the literature. The main idea is that redistribution is more costly in financial autarky than in the contract, so the government is more willing to repay its debt. First, note that if the government defaults, it does so both domestically and externally. It turns out that domestic default creates an adverse distributive effect by transferring more resources to the high-income agents. Moreover, the labor distortion needed for redistribution in financial autarky is higher and more volatile than the labor distortion in the contract. Access to both domestic and external credit markets allows the government to use less labor distortion by redistributing via borrowing taxes, and to use debt to smooth out distortions over aggregate fluctuations.

[3]See Lucas and Stokey (1983) and Werning (2007) for more details on the labor tax smoothing. Both frameworks have state-contingent asset markets but do not have the borrowing constraints because the government have commitment.

I quantify the effect of the government's concern for redistribution on its sovereign debt policies in the case of Italy. I calibrate the model to match the average cross-sectional wage inequality and show that the model accounts for average and volatility of external debt-to-output ratio in Italy without generating any counterfactual assumptions and business cycle statistics.

The theory predicts that a higher inequality, or a higher redistributive motive, corresponds to a higher cost of default. It is then optimal for the government to sustain a higher debt level. In a cross-sectional simulation, I show that the model produces a positive association between pre-tax income inequality and external debt across countries. Specifically, I run a regression analysis on a sample of simulated economies differentiated only by wage inequalities and find a statistically significant and positive effect of the pre-tax income inequality on the external debt-to-output. To validate the model's time series prediction, I conduct a comparative statics exercise for Italy in the periods of 1985-2001 and 2002-2015. I show that an increase in the underlying wage inequality that matches the increase in income inequality can account for 93% of the increase in the average debt-to-output ratio.

This paper studies optimal austerity policies by showing how fiscal policies and inequality optimally respond to a negative innovation of the productivity shock. There is an increase in external debt. While labor tax remains unchanged, there is an increase in domestic borrowing tax in the period right after the productivity reduction. In terms of implications on redistribution, I show two different measures. The first measure is the relative consumption inequality, which remains unchanged across periods. The second measure is the average tax-to-income across individuals, capturing the amount of resources that households give to the government as percentages of their incomes. A negative productivity shock leads to decreases in average tax rates among agents, with a higher reduction for high-income agents. This measure implies that there is less redistribution towards low-income agents.

Lastly, this paper discusses the roles of different model ingredients. Heterogeneity plays an important role since it determines the optimal level of redistribution and so the optimal level of external debt. Distortionary taxation is also a key assumption. When the government has access to fully income-dependent lump-sum transfer, financial autarky is not costly because the government can achieve a perfect redistribution without any tax distortions, so the government is more likely to default on a high debt level.

Related literature. This paper contributes to the sovereign debt literature. Seminal papers study sovereign debt in the limited commitment environment include Eaton and Gersovitz (1981),Aguiar and Gopinath (2007), Arellano (2008), Aguiar and Amador (2011), and Aguiar and Amador (2014). There are a few recent works on optimal policies in the Eaton-Gersovitz-Arellano framework in which the government sometimes defaults in equilibrium and the spread of the sovereign debt depends on the probability of default. Cuadra et al. (2010) show how the sovereign default model of incomplete market and endogenous fiscal policy generates optimal pro-cyclical policy. I introduce distributive effect of fiscal policies in a complete market framework and finds that the distortionary labor tax drifts down over time as the borrowing constraint is tightened. Pouzo and Presno (2015) study the optimal fiscal policies in a representative-agent closed economy with distortionary taxes and defaultable government bonds. One of their main results that is similar to this paper is that the government's lack of commitment hinders the ability to use debt to smooth taxes. Arellano and Bai (2016) quantitatively find that in the case of the government committing to the tax policy, higher tax distortion would have made the country more likely to default, and hence it is not optimal. I find that without commitment to the tax policy, the government finds it optimal to reduce the tax distortion when the borrowing constraint binds. Karantounias (2018) also studies optimal time-consistent tax policy in a representative-agent model with defaultable domestic government bond and shows how the two incentives of "greed" and "fear" determine the optimal back-loading or front-loading tax distortions. In this paper, with the interest rate is exogenously given and the binding borrowing constraints, the optimal policy is to front load the tax distortion.

The framework of limited commitment has been explored by Kehoe and Perri (2002) with two-country international real business cycle and Bauducco and Caprioli (2014) with optimal fiscal policy. I add heterogeneous agents to this framework and study the implication on debt sustainability.

Endogenous cost of default beyond insurance motive has been explored by Mendoza and Yue (2012), which is the efficiency loss in production as default prevents the final good producers to finance the purchase of imports, which only have imperfect substitutes at home. Balke (2017) shows how default limits the supply of bank's loans that firms use to finance vacancies and wages. Therefore, default leads to a large increase in unemployment, which is endogenously costly. In this paper, distortionary taxation plays an important role in determining the cost of default. Default is costly because of the government cannot redistribute as much, and that they do not have access to the inter-temporal tax on labor to mitigate the labor distortion.

This paper finds optimal policy by characterizing the best allocation of any tax-distorted equilibrium, i.e. the primal approach as in the public finance literature (Aiyagari and McGrattan (1998), Aiyagari et al. (2002), Barro (1979) Chari et al. (1994), Chari and Kehoe (1999),Lucas and Stokey (1983), and many other papers). The argument for labor tax smoothing in these papers relies on the fact that the government can issue debt that is contingent to all states and is not constrained (in a sense of beyond the natural debt limit). In this paper, tax smoothing is not always optimal; the government's ability to smooth tax distortion is restricted by the willingness to lend by the international lenders.

Bhandari et al. (2016),Bhandari et al. (2017) and Werning (2007) study optimal taxation with heterogeneity and find that redistribution had significant impact on optimal policies. This paper relaxes their assumption on the government's commitment to policies. Bhandari et al. (2016) establishes the existence of the ergodic distribution of debt in the long run and finds that the average long-run on optimal public debt is not positive. In contrast, this paper shows the quantitatively high cost of default of redistribution and the high positive debt level in the long run. Bhandari et al. (2017) emphasizes the impact of the distribution of initial asset holdings on optimal allocation. In this paper, the initial distribution of after-tax net asset holdings matters in determining the equilibrium redistribution across agents, which indirectly affect the long-run repaying capacity. Werning (2007) develops the conditions for perfect tax smoothing under heterogeneity. Here, I establish that labor tax smoothing only occurs when the borrowing constraint does not bind. Long-run binding borrowing constraints then alters the dynamic of the labor tax, resulting in imperfect tax smoothing.

Several recent papers addressed the trade-off between redistribution and external debt. Ferriere (2015) shows that committing (one-period ahead) to the tax progressivity reduces the incentive for the government to default. In this paper, I extend the lack of commitment to both tax and debt policies and finds a related result that a more progressive economy finds a higher cost of default since redistribution is more distortionary in autarky. D'Erasmo and Mendoza (2016) focus on how redistribution incentives affected defaults on domestic debt. They asserted that equilibrium with debt could be supported only when the government was politically biased towards bond holders. Dovis et al. (2016) argues how the interaction between inequality and debt endogenized the dynamic cycles of debt, taxes, and transfers over time. Balke and Ravn (2016) studies time-consistent fiscal policy in a sovereign debt model à la Eaton and Gersovitz (1981) with inequality through unemployment. They find that austerity policies are optimal during debt crises since they reduce the default premium, which was correlated with debt issuance, and increased access to international lending market. This paper instead emphasizes on the endogenous borrowing constraints arising from a self-enforcing contracting problem among the government, international lenders, and domestic agents. The paper features the endogenous binding debt constraints, in which austerity policies might not be optimal if they generate more distortion. The main result is that taking into account the distributitive and distortionary effect of fiscal policies, the model can quantitatively sustain a high level of debt.

Other papers have documented the positive relationship between the income dispersion and sovereign debt. Berg and Sachs (1988) shows that income dispersion was a key predictor of a country's probability of rescheduling debt and the bond spread in secondary markets. Aizenman and Jinjarak (2012) describes a negative correlation between income dispersion and the tax base and a positive correlation with sovereign debt. Jeon et al. (2014) and Ferriere (2015) also provide evidence of rising income dispersion significantly increases sovereign default risk. This paper extends the panel analysis to a more recent data set on income inequalityprovides a theory that can account for the increase in debt and income dispersion together.

Outline. The paper is organized as follows. Section 2 documents the relationship between income inequality and external debt. Section 3 describes the environment and sets up the competitive equilibrium. Section 4 formulates the planning problem and the main theoretical results. Section 5 provides the quantitative analysis and analyzes the optimal austerity. Section 6 discusses the assumptions and robustness. Section 7 then concludes.

2 Empirical Motivation

This section presents the empirical relationship between income inequality and external debt. I document that income inequality is positively correlated with external debt in both the cross section and time series. To measure a country's external indebtedness, I use the net financial liability-to-GDP measure from the External Wealth of Nations Database of Lane and Milesi-Ferretti (2018). The database contains data on foreign assets and foreign liabilities for a large sample of countries for the period 1970-2015. For income inequality, I use pre-tax (market) Gini indices from Standardized World Income Inequality Database (SWIID) of Solt (2019). This database contains 169 countries and covers from 1960 to 2018.

2.1 Cross-Sectional Properties

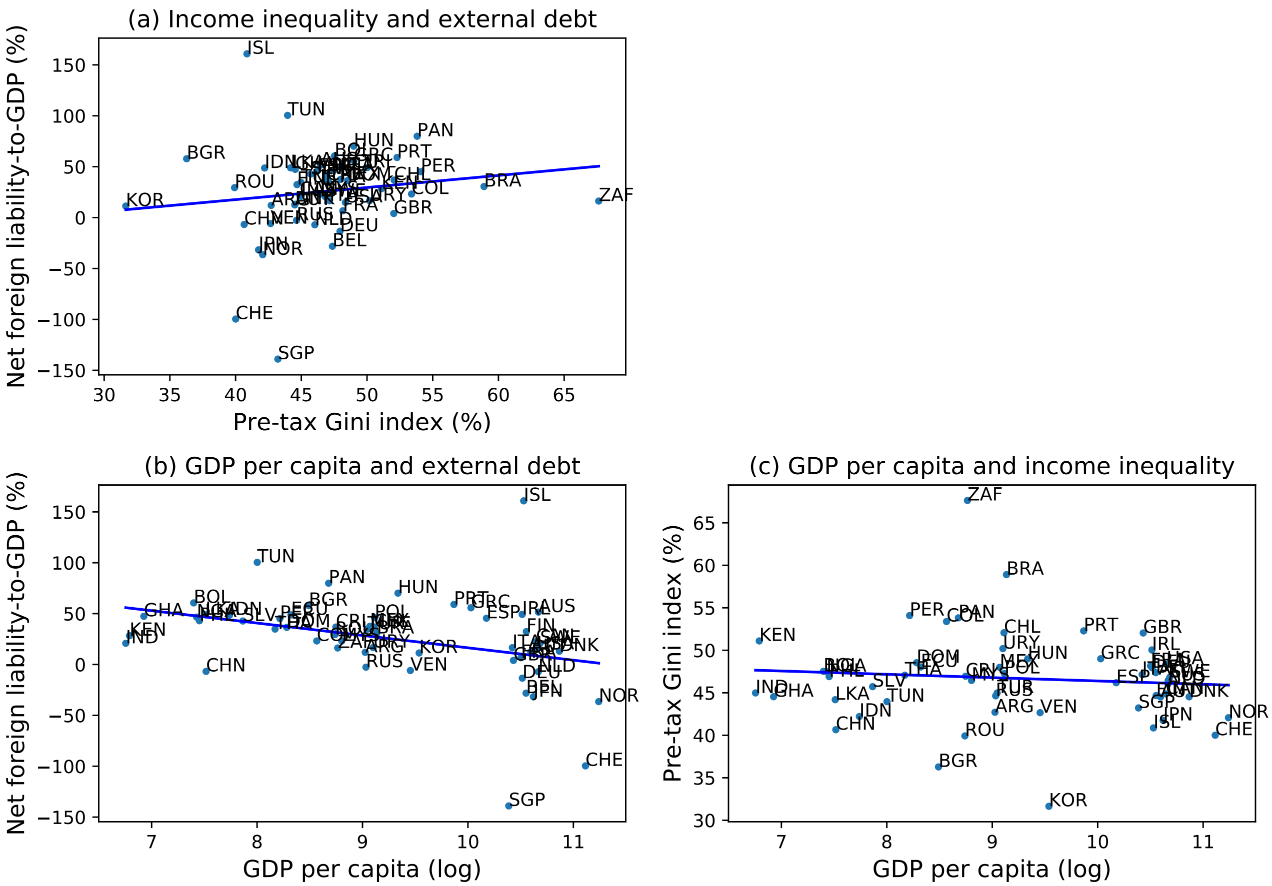
In this subsection, I show the cross-sectional properties in a selected sample of advanced and emerging market economies[4].

Figure 1 plots averages across the time period of 1985-2015 for net financial liability-to-GDP, pre-tax Gini index, and GDP per capita, which is log of the constant 2010 US Dollars GDP per capita series in the World Development Indicator Database. Panel (a) establishes a positive relationship between income inequality and external debt across countries. One

[4]See appendix F.2 for the list of advanced and emerging market economies. Appendix ?? shows similar graphs for the whole dataset.

important factor that accounts for the observed levels in both income inequality and external debt across countries is GDP per capita. Panel (b) and (c) show the relationship of net-foreign liability-to-GDP and pre-tax Gini index with respect to GDP per capita. Countries with high GDP per capita tend to have lower net foreign liability position and lower pre-tax income inequality.

Figure 1: Income inequality, external debt, and GDP per capita across countries



Note: The graph shows the 1985-2015 time averages of net financial liability-to-GDP, pre-tax Gini index, and GDP per capita in constant 2010 US Dollars for advanced and emerging market economies. Panel (a) plots averages of pre-tax Gini index (%) and net foreign liability-to-GDP (%). Panel (b) plots averages of log of GDP per capita and net foreign liability-to-GDP (%). Panel (c) plots averages of log of GDP per capita and pre-tax Gini index (%). Sources: World Development Indicator Database (2019), Lane and Milesi-Ferretti (2018), and Solt (2019).

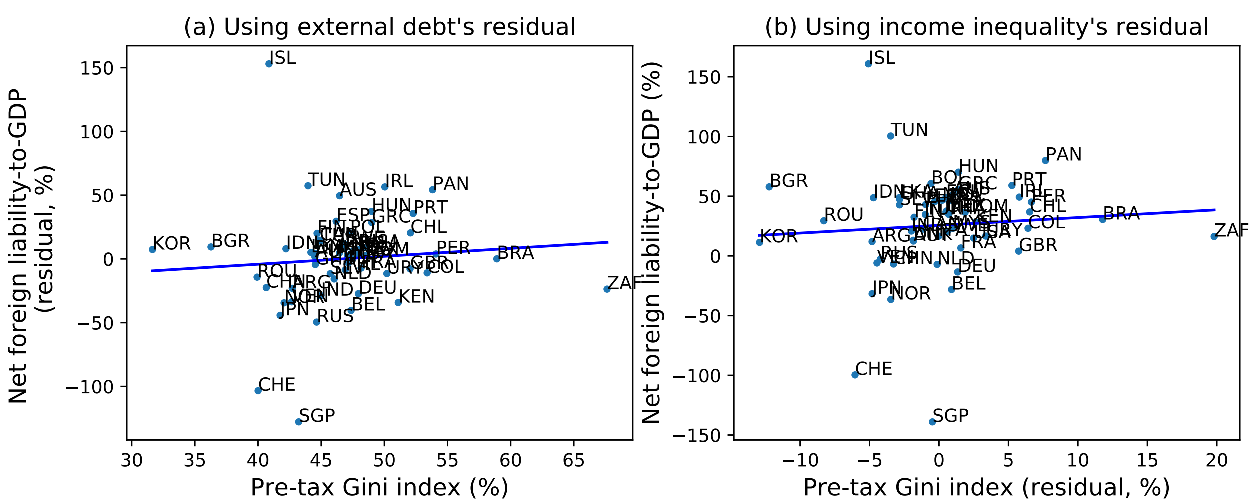
I show next that the positive correlation between pre-tax Gini index and net foreign liability-to-GDP is robust with respect to other factors such as GDP per capita and GDP growth rates. In other words, increases in the pre-tax Gini index can account for increases in net foreign liability-to-GDP, excluding from differences in GDP per capita and GDP growth rates. The exercises are as follows. For a given country in the sample, I calculate average statistics across the time period of 1985-2015. I consider two regressions of net foreign liability-to-GDP and pre-tax Gini index on log GDP per capita and GDP growth rates:

$$\text{Net foreign liability-to-GDP}_i = \beta_0 + \beta_1 \text{GDP per capita}_i + \beta_3 \text{GDP growth}_i + \epsilon_i^{nfl} \quad (1)$$

$$\text{Pre-tax Gini Index}_i = \beta_0 + \beta_1 \text{GDP per capita}_i + \beta_3 \text{GDP growth}_i + \epsilon_i^{gini} \quad (2)$$

Panel (a) of Figure 2 plots the residuals ϵ_i^{nfl} of equation (1) with respect to the pre-tax Gini index. Panel (b) plots the net foreign liability-to-GDP with respect to the residuals ϵ_i^{gini} of equation (2). The positive trend in Panel (a) implies that a higher pre-tax Gini index is associated with a higher net foreign liability-to-GDP that do not come from GDP per capita or GDP growth rates. The positive trend in Panel (b) implies that a higher pre-tax Gini index that is not due to different GDP per capita or GDP growth levels is correlated with a higher net foreign liability-to-GDP level.

Figure 2: Cross-sectional relationship between income inequality and external debt

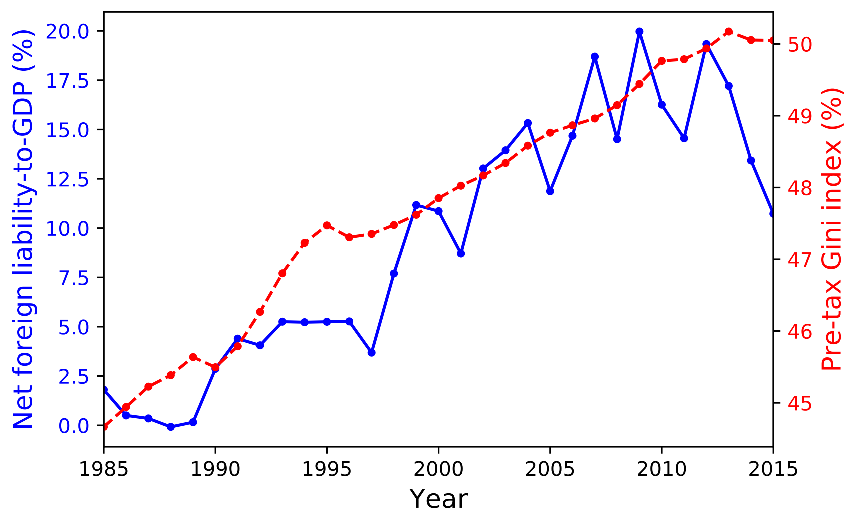


Note: Panel (a) plots the residuals ϵ_i^{nfl} (in percentage) of equation (1) and the pre-tax Gini index (%). Panel (b) plots the net foreign liability-to-GDP (%) and the residuals ϵ_i^{gini} (in percentage) of equation (2). The sample includes advanced and emerging market economies. Sources: World Development Indicator Database (2019), Lane and Milesi-Ferretti (2018), and Solt (2019).

2.2 Time Series Properties

Income inequality has been rising over time across countries (Alvaredo et al. (2018)). At the same time, external debt is also increasing across many countries. The 2007-2009 financial

Figure 3: Time series of income inequality and external debt



Note: The graph shows the GDP-weighted average of net financial liability-to-GDP and pre-tax Gini index for countries in the European Union from 1985 to 2015. Sources: Lane and Milesi-Ferretti (2018) and Solt (2019).

crises contributed to the increase in borrowing across countries, particularly across European ones[5]. Figure 3 plots the GDP-weighted average of net financial liability-to-GDP and pre-tax Gini index for countries in the European Union. Over time, there are increasing trends in both income inequality and external debt.

2.3 Estimation

To estimate the effect of income inequality on external debt, I use the following specification

$$\begin{aligned}\text{Net foreign liability-to-GDP}_{i,t} &= \alpha_0 + \alpha_1 \text{Gini}_{i,t} + \alpha_2 \text{GDP per capita}_{i,t} \\ &+ \alpha_3 \text{GDP growth}_{i,t} + \alpha_4 \text{Inflation}_{i,t} + u_i + z_t + \epsilon_{i,t} \qquad (3)\end{aligned}$$

where GDP per capita is the log of real GDP per capita series in constant 2010 US Dollars and inflation is calculated from GDP deflators. Both GDP per capita and inflation series are from the World Development Database[6].

Table 1 shows the regression results for the time period of 1985-2015. The first column presents the regression result of equation (3) without control variables. The second column presents the full regression. Clustered standard errors are in parentheses. The correlation

[5] Reinhart and Rogoff (2010) reported large increases in public debt across countries, especially in the period 2007-2009. External debt levels were particularly high among European countries.

[6] Appendix F.2 lists all countries considered in the regression.

between inequality and external debt is positive and statistically significant, taking into account country and time fixed effects, and other determinants on external debt levels: GDP per capita, GDP per capita growth, and inflation rates. GDP per capita, GDP growth, and inflation rate are negatively correlated with net foreign liability-to-GDP, but the effect of GDP growth is not statistically significant.

Table 1: Regression analysis of income inequality and external debt

	Dependent Variable: Net foreign liability-to-GDP (%)	
	Time periods: 1985-2015	
	(1)	(2)
Gini index, pre tax (%)	1.4367***	1.3272***
	(0.4150)	(0.4576)
GDP per capita (log)		-21.741***
		(7.0379)
GDP growth (%)		-0.3330
		(0.2802)
Inflation (%)		-0.0083**
		(0.0043)
Country fixed effects	Yes	Yes
Time fixed effects	Yes	Yes
No. Countries	179	176
No. Observations	39324	3848

Note: The table describes the panel regression results using all countries in the data set. The first column shows the regression coefficient and standard error in parenthesis of pre-tax Gini index (%) with respect to net foreign liability-to-GDP (%). The second column shows the regression coefficients and standard errors in parentheses that include other control variables: log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). Both regressions have country and time fixed effects. All standard errors are clustered. *,**,*** represent significant levels of 10%, 5%, and 1%, respectively. Sources: World Development Indicator Database (2019), Lane and Milesi-Ferretti (2018), and Solt (2019).

3 Model

This section describes the main framework and sets up the competitive equilibrium given the government's policies. The competitive equilibrium can be characterized by a set of aggregate allocation and a distribution of marginal utility shares.

3.1 Environment

A small open economy faces publicly observed aggregate shocks $s_t \in S$ in period t, where S is some finite set. Let $\Pr(s^t)$ denote the probability of any history $s^t = (s_0, s_1, ..., s^t)$, where

$\Pr(s^{t+j}|s^t)$ denotes the probability conditional on history s^t, $j \geq 0$. Similarly, $\Pr(s_{t+1}|s^t)$ is the probability period $t+1$'s state is s_{t+1}, conditional on history s^t. The exogenous risk-free international interest rate for borrowing is r^*. There is a measure-one continuum of infinitely-lived agents different by labor productivity types $(\theta^i)_{i\in I}$, which are publicly observable. The fraction of agents with productivity θ^i is π^i, where $(\pi^i)_{i\in I}$ is normalized such that $\sum_{i\in I}\pi^i\theta^i = 1$. All agents have the same discount factor β and the static utility $U(c,n)$ over consumption c and hours worked n. The utility of agent with productivity θ^i over consumption $c_t^i \geq 0$ and efficiency-unit labor $l_t^i \geq 0$ is

$$\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0 U^i(c_t^i, l_t^i) \tag{4}$$

where $U^i(c,l) = U\left(c, \frac{l}{\theta^i}\right)$.

In addition, there is a representative firm that uses labor to produce a single final good. The production function in period t with history s^t is$F(L, s^t, t)$, constant return to scale, where L is the aggregate labor. The economy is subject to an exogenous sequence of government spending $\{G_t(s^t)\}_{t=0,s^t\in S^t}^{\infty}$. Both the production function and government expenditures depend on the time period t, capturing deterministic changes such as growth, and the history s^t, capturing the uncertainty impact.

An allocation specifies consumption and labor in every period after every history: $\{c^i(s^t), l^i(s^t)\}$. The aggregates are denoted by $C(s^t) \equiv \sum_{i\in I}\pi^i c^i(s^t)$ and $L(s^t) \equiv \sum_{i\in I}\pi^i l^i(s^t)$.

Both the domestic and international financial markets are competitive. The government can issue domestic debt from a full set of state-contingent bonds, which can be traded across agents. The government also have access to a full set of state-contingent external bonds. Let $R^* = 1 + r^*$ denote the gross risk-free interest rate. Define $Q(s_{t+1}|s^t) = \Pr(s_{t+1}|s^t)/R^*$ as the international price of one unit of consumption at state s_{t+1} in period $t+1$, conditional on history s^t, in units of consumption at history s^t. Similarly, $q(s^t) = \Pr(s^t)/(R^*)^t$ is the international price of one unit of consumption at history s^t in units of consumption at s^0. Let normalize $q(s^0) = 1$[7]. Note that $q(s^{t+1}) = Q(s_{t+1}|s^t)q(s^t)$. Assume only the government can borrow abroad[8].

[7]This normalization is without loss of generality since the initial level of external debt is fixed.

[8]In the data, domestic residents often hold a very small amount of foreign assets, so most models assume that they do not have access to the external credit market. This set up is equivalent to the set-up where the domestic agents can save abroad with the bond price $Q^*(s^t)$, but then face a residence-based tax $\tau^d(s^t)$. External debt will be the net foreign liability of both the private and public sectors, instead of only the public sector here. I choose this particular set up so that it is more straightforward to characterize the strategic game later on in Appendix B.

3.2 Competitive Equilibrium

In every period and history s^t, the government can issue both domestic and foreign bonds, impose a lump-sum tax $T(s^t)$, a marginal tax on labor income $\tau^n(s^t)$ and levies a tax on the return of domestic saving $\tau^d(s^t)$. Both the firm and agents face the labor wage $w(s^t)$.

Domestic Agent. Individual agent of type $i \in I$ faces the sequential budget constraint in period t and history s^t

$$c^i(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t) b^{d,i}(s^{t+1}) \leq (1-\tau^n(s^t))w(s^t)l^i(s^t) + (1-\tau^d(s^t))b^{d,i}(s^t) - T(s^t) \tag{5}$$

where $c^i(s^t), l^i(s^t), b^{d,i}(s^t)$ denote the consumption, labor, and domestic bond holding of agent i in period t, history s^t, respectively. $Q^d(s_{t+1}|s^t)$ is the price of one unit of domestic asset for realization s_{t+1} in period $t+1$ given history s^t.

Representative Firm. The firm chooses the amount of capital and labor to maximize profit in each history node s^t

$$\max_{\{L(s^t)\}} \quad F\left(L(s^t), s^t, t\right) - w(s^t)L(s^t)$$

which gives the first-order condition

$$w(s^t) = F_L\left(L(s^t), s^t, t\right) \tag{6}$$

The firm's profit is zero in equilibrium because of the constant-return-to-scale production function.

Government. There is an exogenous government expenditure $\{G_t(s^t)\}_{t=0,s^t\in S^t}^{\infty}$. Given the one-period state-contingent domestic bond $B^d(s^t)$ and external bond $B(s^t)$, the government's budget constraint in each history node s^t is

$$\begin{aligned}
&G(s^t) + (1-\tau^d(s^t))B^d(s^t) + B(s^t) \\
\leq &\tau^n(s^t)w(s^t)L(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) + \sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t)B(s^{t+1}) + T(s^t)
\end{aligned}$$

where $B^d(s^t) = \sum_{i\in I} \pi^i b^{d,i}(s^t)$ is the aggregate domestic bond, and $B(s^t)$ is the amount of the government's external debt. There is a no-Ponzi condition such that the present value of external debt is bounded below.

The government's present-value budget constraint is

$$\sum_{t\geq 0, s^t\in S^t} q(s^t)\left\{G(s^t) - \tau^n(s^t)w(s^t)L(s^t) - T(s^t)\right. \tag{7}$$

$$\left. + \sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t)B^d(s^{t+1}) - (1-\tau^d(s^t))B^d(s^t)\right\} \leq B(s^0)$$

Resource constraint. Using the agent's budget constraints and government's budget constraint, one can obtain a present-value resource constraint in terms of the inter-temporal international prices and the initial external debt,

$$\sum_{t\geq 0, s^t\in S^t} q(s^t)\left[F(L(s^t), s^t, t) - G(s^t) - C(s^t)\right] \geq B(s^0) \tag{8}$$

Competitive equilibrium. Given the above equations, one can define the following competitive equilibrium with taxes.

Definition 3.1. Given initial external debt $B(s^0)$ and individual individual bond positions $\left(b^{i,d}(s^0)\right)_{i\in I}$, a competitive equilibrium with taxes for an open economy is individual agent's allocation $z^{H,i} = \left\{c^i(s^t), l^i(s^t), b^{i,d}(s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$, $\forall i \in I$, the representative firm's allocation $z^F = \{L(s^t)\}_{t=0,s^t}^{\infty}$, prices $p = \left\{q(s^t), w(s^t), Q^d(s_{t+1}|s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$, and government's policy $z^G = \left\{\tau^n(s^t), \tau^d(s^t), \ T(s^t), B^d(s^t), B(s^t)\right\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves individual i's problem that maximizes (4) subject to (5) and a no-Ponzi condition of agent's debt value, (ii) given p and z^G, z^F solves firm's problem, (iii) the government budget constraint (7) holds, (iv) the aggregate resource constraint (8) is satisfied, (iv) the domestic bond market clears $B^d(s^t) = \sum_{i\in I}\pi^i b^{d,i}(s^t)$, and (v) p satisfies $q(s^t) = \Pr(s^t)/(R^*)^t$ and equation (6) given z^G.

3.3 Characterizing Equilibrium

In equilibrium, the intra-temporal and inter-temporal rates of substitution are the same across agents, i.e. in each period t and each history s^t, for any individual i,

$$(1-\tau^n(s^t))w(s^t) = -\frac{U_l^i(c^i(s^t), l^i(s^t))}{U_c^i(c^i(s^t), l^i(s^t))}$$

$$\frac{Q^d(s_{t+1}|s^t)}{1-\tau^d(s^{t+1})} = \beta\Pr(s^{t+1}|s^t)\frac{U_c^i(c^i(s^{t+1}), l^i(s^{t+1}))}{U_c^i(c^i(s^t), l^i(s^t))}$$

Given the aggregate allocation $(C(s^t), L(s^t))$, there is an efficient assignment of individual allocation $(c^i(s^t), l^i(s^t))_{i\in I}$ due to the equal marginal rates of substitution between consumption and labor. Moreover, the efficient assignment needs to be the same across time, because of the equal marginal rates of substitution of future to current consumption. Any inefficiencies due to tax distortions are captured by the aggregate allocation. This property allows the competitive equilibrium allocation to be characterized in terms of aggregates and a static rule for individual allocation.

For any equilibrium, there exist a set of Neghishi (market) weights $\boldsymbol{\varphi} = (\varphi^i)_{i\in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i \varphi^i = 1$, such that individual allocation solve a static problem

$$
\begin{aligned}
V(C, L; \boldsymbol{\varphi}) \equiv \max_{(c^i, l^i)_{i\in I}} & \sum_{i\in I} \varphi^i \pi^i U^i(c^i, l^i) \\
s.t. \qquad & \sum_{i\in I} \pi^i c^i = C; \quad \sum_{i\in I} \pi^i l^i = L
\end{aligned}
$$

This problem gives the policy functions for each individual i

$$
h^i(C, L; \boldsymbol{\varphi}) = \left(h^{i,c}(C, L; \boldsymbol{\varphi}),\ h^{i,l}(C, L; \boldsymbol{\varphi})\right)
$$

A competitive equilibrium allocation must satisfy $(c^i(s^t), l^i(s^t)) = h^i(C(s^t), L(s^t); \boldsymbol{\varphi})$ for all i and s^t. The associate competitive equilibrium prices can be computed as if the economy were populated by a fictitious representative agent with utility function $V(C, L; \varphi)$. The envelope conditions of the static problem give

$$
(1 - \tau^n(s^t))w(s^t) = -\frac{V_L\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]} \tag{9}
$$

$$
\frac{Q^d(s_{t+1}|s^t)}{1 - \tau^d(s^{t+1})} = \beta \Pr(s^{t+1}|s^t)\frac{V_C\left[h^i(C(s^{t+1}), L(s^{t+1}); \boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]} \tag{10}
$$

Furthermore, the present-value budget constraint for individual i can be written as

$$
\begin{aligned}
\sum_{t\geq 0, s^t\in S^t} \beta^t \Pr(s^t) \Big[& V_C(C(s^t), L(s^t); \boldsymbol{\varphi}) h^{i,c}(C(s^t), L(s^t); \boldsymbol{\varphi}) \\
& + V_L(C(s^t), L(s^t); \boldsymbol{\varphi}) h^{i,l}(C(s^t), L(s^t); \boldsymbol{\varphi}) \Big] = V_C(C(s^0), L(s^0); \boldsymbol{\varphi}) \left(b^i(s^0) - T\right)
\end{aligned} \tag{11}
$$

where T is the present-value of lump-sum taxes[9]. Equation (11) is the individual implementability

[9] $T \equiv \sum_{t=0}^{\infty} \beta^t \sum_{s^t\in S^t} \frac{V_C\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s_0), L(s_0); \boldsymbol{\varphi})\right]} T(s^t)$

constraint.

One has the following characterization proposition.

Proposition 3.1. *Given the initial external debt $B(s^0)$ and individual bond holdings$\{b^i(s_0)\}_{i\in I}$,an allocation $\{C(s^t), L(s^t), K(s^t)\}_{t=0,s^t\in S^t}^{\infty}$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if the resource constraint (8) holds, and there exist market weights $\boldsymbol{\varphi} = (\varphi^i)_{i\in I}$ and lump-sum tax T such that the implementability constraint (11) holds for all $i \in I$.*

4 A Planning Problem

This section characterizes the planning problem of a benevolent government that cares about redistribution but lacks commitment in both tax and debt policies. The main theoretical result entails how limited borrowing affects optimal taxes and hence the government's redistributive ability.

4.1 Lack of Commitment

The government cares about all residents in the country, and its objective is the social welfare

$$\sum_{i\in I} \lambda^i \pi^i \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 U^i \left(c^i(s^t), l^i(s^t)\right), \tag{12}$$

given by a set of social welfare weights $\boldsymbol{\lambda} = (\lambda^i)_{i\in I}$. The government chooses its policies sequentially. In every period and history node, the government cannot commit to future choices on repayments of debt and taxes. Following Chari and Kehoe (1990, 1993), the policies are determined in a repeated game between the government, a continuum of domestic agents, and a continuum of international creditors. The subgame perfect equilibrium supported by trigger strategies to autarky is characterized by the competitive equilibrium conditions described in Proposition 3.1 and the following self-enforcing constraint

$$\sum_{i\in I} \lambda^i \pi^i \sum_{k\geq t, s^t\subseteq s^{t+j}} \beta^{k-t} \Pr(s^k|s^t) U^i(c^i(s^k), l^i(s^k)) \geq \underline{U}(s^t, t),\ \forall t, \forall s^t \tag{13}$$

$\underline{U}(s^t, t)$ is the one-shot deviation value in which the government fully redistributes wealth among the domestic agents and default on external debt. The government is then in financial autarky, in which it it has no access to the domestic and international financial markets. $\underline{U}(s^t, t)$ is the value associated with an allocation of a closed economy where the initial states are realized s_t at period t and history s^t, the initial wealth inequality among agents are equal, and the net supplies of domestic and international bonds are zero.

The self-enforcing constraint captures the time-inconsistency of the government's policies. If there is a positive net external debt, the government has an incentive to default externally to increase domestic consumption and leisure. In addition, in every history node, there is a non-degenerate distribution of wealth across the domestic agents. The inequality-averse government will also have an incentive to expropriate all the wealth and equally redistribute it.

The self-enforcing constraint imposes a limit on the utility, which endogenously determines a limit on external debt for every period and history. These constraints act as endogenous borrowing constraints.

4.2 Efficient Allocation

Given the above set-up, an efficient allocation is defined as follows

Definition 4.1. An efficient allocation $\{C(s^t), L(s^t)\}, \boldsymbol{\varphi}$ maximizes the social welfare function (12) and satisfies the conditions in Proposition 3.1 and the self-enforcing constraint (13)

The efficient allocation is part of the solution to a planning problem

$$(P) \equiv \max_{\{C(s^t),L(s^t)\},\boldsymbol{\varphi},T} \sum_{i\in I} \lambda^i \pi^i \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 \left[h^i(s^t;\boldsymbol{\varphi})\right]$$

$$\begin{aligned}
s.t. \qquad & \sum_{t\geq 0, s^t\in S^t} q(s^t)\left[F(L(s^t), s^t, t) - G(s^t) - C(s^t)\right] \geq B(s^0) \\
\forall i, \ & \sum_{t\geq 0, s^t\in S^t} \beta^t \left[V_C(s^t;\boldsymbol{\varphi})h^{i,c}(s^t;\boldsymbol{\varphi}) + V_L(s^t;\boldsymbol{\varphi})h^{i,l}(s^t;\boldsymbol{\varphi})\right] = V_C(s_0;\boldsymbol{\varphi})\left(b^i(s^0) - T\right) \\
\forall t, \forall s^t, \ & \sum_{i\in I} \lambda^i \pi^i \sum_{k\geq t, s^t\subseteq s^{t+j}} \beta^{k-t} \Pr(s^k|s^t) U^i \left[h^i(s^k;\boldsymbol{\varphi})\right] \geq \underline{U}(s^t,t) \geq \underline{U}(s^t,t)
\end{aligned}$$

where $(s^t;\boldsymbol{\varphi}) \equiv (C(s^t), L(s^t);\boldsymbol{\varphi})$ for notation convenience.

The first constraint is the resource constraint. The second constraint presents the private-sector equilibrium conditions. Basically, it takes into account the distortionary effects of the government's policies. The last constraint is the borrowing constraint due to the government's lack of commitment. Domestic agents do not directly internalize the effect of their borrowing decisions on these borrowing constraints. The government, on the other hand, has to consider these constraints when choosing optimal allocation and policies.

Therefore, borrowing constraints affect domestic borrowing choices via the government's decision on saving taxes.

4.3 Optimal Taxation and Borrowing Constraints

I derive optimal policies that implements the efficient allocation. Optimal labor and saving taxes reflect the trade-off between redistribution and efficiency. I show that this trade-off is dynamically changed over time in the presence of borrowing constraints.

I make the following assumptions

Assumption 1 (Separable isoelastic utility)**.** *The individual preference follows*

$$U^i\left(c,l\right)=\frac{c^{1-\sigma}}{1-\sigma}-\omega\frac{\left(\frac{l}{\theta^i}\right)^{1+\nu}}{1+\nu}$$

Assumption 2. *The welfare weights, skill distribution, and initial wealth satisfy the following properties*

1. *Redistribution motive towards the low skills:* $\theta^i\leq\theta^j\iff\lambda^i\geq\lambda^j,\ \forall i,j\in I$
2. *Perfect correlation between skill and initial wealth:* $\theta^i\leq\theta^j\iff b^i(s^0)\leq b^j(s^0),\ \forall i,j\in I$
3. *High elasticity of substitution:* $\sigma\geq1$

The first assumption implies that the individual preference is separable between consumption and labor, and the elasticities of substitution are constant across periods and histories. For the second set of assumptions, the first part is on the welfare weights, meaning that the planner has a high motive of redistribution towards the lower skill, lower income individuals. The second part makes sure that there direction of inequality in skill is the same as the direction of inequality in initial wealth, meaning that lower skill individuals start off with lower initial wealth. The last assumption implies that the intra-temporal elasticity of substitution is at least above log preference. This assumption determines the direction of change in the optimal tax rate in response to inter-temporal changes.

Using the solution to the planning problem, I establish the main theoretical result on the optimal labor tax,

Proposition 4.1. *For any period* $\mathcal{T}$*, history* $s^{\mathcal{T}}$ *, and* $\forall s^{\mathcal{T}-1}\subseteq s^{\mathcal{T}}$*,*

1. *If the borrowing constraint does not bind at* $s^{\mathcal{T}}$*, the optimal labor tax does not change, i.e.* $\tau^n(s^{\mathcal{T}})=\tau^n(s^{\mathcal{T}-1})$*, and the optimal saving tax is zero, i.e.* $\tau^d(s^{\mathcal{T}})=0$

2. *If the borrowing constraint binds at $s^{\mathcal{T}}$, the optimal labor tax is weakly decreasing in the future, i.e.* $\tau^n(s^t) \leq \tau^n(s^{\mathcal{T}-1})$, $\forall t \geq \mathcal{T}, \forall s^{\mathcal{T}} \subseteq s^t$, *and there is optimal saving subsidy, i.e.* $\tau^d(s^{\mathcal{T}}) < 0$

The proposition first implies that given non-binding borrowing constraints, optimal labor taxes do not change over time and across histories, and optimal saving taxes are zero. The labor tax result is related to the standard tax smoothing argument as in Lucas and Stokey (1983). Since the borrowing constraint does not bind, it is optimal to use debt to smooth out the distortionary cost across all states. Zero saving taxes, on the other hand, are related to the idea of no intertemporal distortion in Judd (1985) and Chamley (1986).

When the borrowing constraint binds, all future labor tax rates weakly decrease. The tax smoothing property implies that a one-time binding borrowing constraint does not decrease the labor tax in one current period but the reduction is spread out to all tax rates in the subsequent periods. In addition, the government finds it optimal to use a saving subsidy to discourage the impatient domestic agents from over-borrowing.

Intuitively, distortionary taxation is a mechanism for redistribution. A positive marginal labor tax and lump-sum rebate to all agents imply that the higher skilled, higher income individuals pay more taxes than the lower skilled, lower income individuals. Therefore, a planner that cares about redistribution towards low income agents will find it necessary to levy a positive labor tax. Optimal labor taxes balance the marginal benefit of redistribution and the marginal cost of distortion. The determinants of inequality and distributive motive do not vary over time and are independent of the aggregate shocks. When borrowing is not costly (non-binding borrowing constraints), both the distributive benefit and the distortionary cost do not vary, so it is optimal to keep labor taxes unchanged. When borrowing is limited (binding constraints), distortion becomes more costly because it reduces the amount of resources that the economy can use to repay debt. A decrease in distortionary labor tax is then optimal to increase the repaying capacity and relax borrowing constraints.

The forward-looking borrowing constraints induce a backward-looking effect of the optimal policies. Suppose that the labor tax τ^n decreases in node $s^{\mathcal{T}}$, then efficiency (e.g. aggregate output Y) is higher in node $s^{\mathcal{T}}$. Not only this increase in efficiency allows the economy to repay more debt in $s^{\mathcal{T}}$, but also to repay more debt in any previous histories $s^t \subseteq s^{\mathcal{T}}$ where $0 \leq t \leq \mathcal{T}$. As a result, a lower labor tax in the future relaxes all of the past borrowing constraints. Therefore, instead of a one-time large decrease in the tax rate when the borrowing constraint binds, a permanent small decrease in all future tax rates will relax more borrowing constraints.

Optimal saving taxes, on the other hand, relate to intertemporal distortions. When borrowing constraints do not bind, it is optimal not to distort the intertemporal margins[10]. When borrowing constraints bind, saving subsidies make the domestic agents internalize the additional cost of borrowing that comes from the binding constraints.

Formally, the proof relies on the property that the only component in the optimal taxes that varies across time periods and histories is the sum of all Larange multipliers on the borrowing constraints (γ) up to node $s^{\mathcal{T}}$, i.e. $\sum_{k=0.s^k \subseteq s^{\mathcal{T}}}^{\mathcal{T}} \gamma(s^k)$. If the borrowing constraint does not bind ($\gamma(s^{\mathcal{T}}) = 0$), the sum stays constant, and so labor taxes remain the same as before, and saving taxes are zero in $s^{\mathcal{T}}$. However, if borrowing constraints bind ($\gamma(s^{\mathcal{T}}) > 0$), the sum increases, which leads to a permanent decline in labor taxes. Differences in today's sum at $s^{\mathcal{T}}$ and yesterday's sum at $s^{\mathcal{T}-1}$ leads to a tax in domestic borrowing's return at $s^{\mathcal{T}}$.

Proposition 4.1 implies how limited borrowing affect the government's redistributive policies. When borrowing constraints do not bind, it is optimal to distort the intratemporal margins and not the intertemporal margins. Hence, the government optimally redistribute using labor taxes and levies no borrowing taxes. However, when borrowing constraints bind, distorting the intertemporal margins is beneficial since it aligns the domestic agents' interests with the government's interests. It turns out that the domestic borrowing taxes also redistribute resources among agents. The government optimally redistributes via more borrowing taxes and less labor taxes, which reduces the overall cost of distortion in the economy.

5 Quantitative Analysis

The previous section provides a theoretical foundation on how borrowing constraints affect optimal taxation and the government's ability to redistribute. This section shows the impact of the government's concern for redistribution on its incentive to repay debt and how that implies the positive correlation between income inequality and external debt in the cross section and over time periods. The benchmark calibration uses Italy's data. Lastly, the section presents optimal austerity policies, specifically the fiscal and redistributive implications of a negative productivity shock.

Throughout this section, there are the following assumptions on the domestic discount factor and deviation utility

Assumption 3 (Impatience)**.** *There exists* $0 < \mathcal{M} < 1$ *such that* $\beta R^* < \mathcal{M} < 1$.

Assumption 4. $\underline{U}$ *is bounded below, i.e. there exists a finite real* M_U *such that* $\inf_{s^t,t} \underline{U}(s^t, t) \geq M_U$.

[10] This is an insight from the public finance literature, found in Judd (1985), Chamley (1986), Chari and Kehoe (1999), Werning (2007), and many other papers.

5.1 Computation

Given the foward-looking borrowing constraints, I implement the recursive formulation developed by Marcet and Marimon (2019). Appendix E.2 provides more details on the computational algorithm. The key co-state variable is the discounted sum of the Larange multipliers on the borrowing constraints: $\Gamma(s^t) = (\beta R^*)^t \sum_{s^k \subseteq s^t} \gamma(s^k)$. Given that the domestic agents are impatient, the borrowing constraint will bind infinitely often in the long run. It turns out that Γ is bounded, i.e.

Proposition 5.1. *Suppose Assumptions 1, 3, and 4 hold, if an interior efficient allocation exisits, then* $\lim_{t\to\infty} \Gamma(s^t) > 0$.

$\Gamma(s^t)$ reflects the marginal benefit of relaxing the current and previous borrowing constraints from increasing one unit of utility in period t and history s^t (either by giving more consumption or leisure). In the long run, Γ corresponds to the amount of external debt that can be sustained in equilibrium. Impatience implies that in the long run, there exists a positive component in the price of borrowing coming from the lack of commitment. If there exists an ergodic distribution of the efficient allocation, then Γ will also follows an ergodic distribution. This property allows the computational algorithm using Γ as one of the state variables to converge.

5.2 Calibration

For the quantitative exercise, I assume the following distributional and functional forms. The economy is populated by two types of agents with labor productivity $\{\theta^H, \theta^L\}$, where $\theta^H \geq \theta^L > 0$ and $\pi^H = \pi^L = 0.5$. The planner is utilitarian, i.e. $\lambda^H = \lambda^L$. The individual preference has the form of

$$U^i(c,l) = \log c - \frac{l^{1+\nu}}{1+\nu}$$

The production function is linear in labor, i.e. $F(L,z) = zL$, where z is the aggregate productivity. The aggregate shock is z_t that follows a logged $AR(1)$ process,

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t^z,\ \epsilon_t^z \sim \mathcal{N}(0, \sigma_z),$$

where ρ_z, σ_z are the auto-correlation and the residual standard deviation, respectively. I discretize the productivity process into a Markov chain using Tauchen's method with 31 evenly-spaced nodes. Let $s_t = z_t$. The government expenditure is constant over time and across histories: $G(s^t, t) = \bar{g}$. The initial debt levels are $B(s^0) = 0$ and $b^{H,d}(s^0) = b^{L,d}(s^0) = 0$, where s^0 is the mean of the productivity distribution. The deviation utility $\underline{U}(s^t)$ is calculated as the closed-economy version of the model that starts with productivity z_t, zero external debt, and all domestic individuals start with the same initial wealth. $\underline{U}(s^t, t)$ varies with respect to the realized shock $s_t = z_t$.

With these assumptions, the model requires giving values to the parameters of (i) the aggregate productivity process, ρ_z and σ_z; (ii) the cross-sectional wage ratio, θ^H/θ^L; (iii) the individual preference, β and ν; (iv) the government expenditure $\bar{g}$; and (v) the risk-free rate r^*.

A period in the model is one year. For output, I use the logged and linear detrended real GDP series from 1985-2015. I set the auto-correlation of productivity,ρ_z, equals to the auto-correlation of output, which is 0.928. To calculate the wage ratio θ^H/θ^L, I use the data on cross-sectional inequality by Jappelli and Pistaferri (2010). For each year in the database, I calculate the ratio of the mean wage of the top 50% of the wage distribution to the mean wage of the bottom 50%. Then θ^H/θ^L is set to 1.9475, which is the time-average of these wage ratios for the period 2002-2006. The discount factor β is set to 0.967 so that the average real domestic interest rate is 3.4% for Italy from 2002 to 2015. I choose $\nu = 2$ so that the elasticity of labor supply is 0.5, a standard value in the literature. The risk-free rate is set at 0.017, which is the real rate of return on the German government bonds for the period 2002-2015 (these are secondary market returns, gross of tax, with around 10 years' residual maturity). The interest rate series start at 2002 to isolate the effect of currency and exchange rate risks[11].

The two remaining parameters, σ_z and $\bar{g}$, are selected to match (i) the standard deviation of logged output and (ii) the government's final consumption-to-GDP ratio for the period 1985-2015. I use the simulated method of moments (SMM). Departing from the quantitative literature on sovereign debt, I do not target the average external debt-to-output ratio but instead leave it as one of the non-targeted moments[12].

Table 2 summarizes the parameter values and targets from the calibration exercise.

5.3 Calibration Results

Table 3 shows the moment matching exercise of the model and the data. The first column reports the statistics from the data for Italy in the period of 1985-2015. The second

[11]See Appendix F for more data descriptions and sources

[12]See Section 5.3 for the results of non-targeted moments. Alternatively, the discount factor β can be used to target the debt-to-output ratio.

Table 2: Calibrated Parameters and Targets

Parameter	Description	Value	Target
Externally calibrated parameters			
r^*	Risk-free rate	0.017	Avg. real return on German bond
β	Discount factor	0.967	Avg. Italian real interest rate = 3.4%
$1/\nu$	Labor elasticity	0.5	Standard literature value
θ^H/θ^L	Wage ratio	1.9475	Mean top 50% wage / mean bottom 50% wage
ρ_z	Auto-corr. of prod.	0.927	Auto-corr. of log GDP
Internally calibrated parameters			
σ_z	Std. dev. of prod. res.	0.019	Std. dev. log GDP
$\bar{g}$	Govt. spending	0.202	Avg. govt. consumption-to-GDP

Note: The table describes the parameters, their values, and the targets in the calibration exercise. Statistics are annual. The risk-free rate and discount factor cover the period of 2002-2015. Wage ratio is the author's calculation from the cross-sectional data set by Jappelli and Pistaferri (2010), covering the period of 2002-2006. Auto-correlation and standard deviation of GDP and government final consumption cover the period of 1985-2015. Data sources: Jappelli and Pistaferri (2010), Eurostat (2019), and World Development Indicator Database (2019).

column reports the statistics from simulating the model and taking the long-run averages[13]. The calibration successfully matches the standard deviation of output and the government consumption-output ratio for Italy.

Table 3: Targeted Statistics: Data and Model

Statistics	Data: 1985-2015	Model
Std. dev. log GDP	0.053	0.053
Avg. govt. consumption-to-GDP	0.19	0.19

Note: The table describes the targeted statistics from the calibration exercise. The first column reports data statistics which are across the period of 1985-2015. The second column reports the model statistics which come from the model's simulation for 10500 periods and excluding the first 500 periods.

Table 4 reports the non-targeted statistics of the model comparing to the data. The first column is from the Italian data, and the second column is from the model. The key cyclical

[13] All model statistics are long-run averages of simulating the economy for 10500 periods and discarding the first 500 periods.

properties are the volatility and correlation with respect to output of consumption and net saving ratio. I consider net savings as the amount of output minus the total consumption. In the model, net saving is the net amount of resources used to repay external debt in every period.

Table 4: Non-targeted Statistics: Data and Model

Statistics	Data	Model
Cyclical property		
std (C) / std (Y)	1.0	1.2
std (NS/Y) / std (Y)	0.29	0.34
corr (C,Y)	0.97	0.94
corr (NS/Y,Y)	0.40	0.30
External debt property[a]		
Mean external debt/Y	0.24	0.21
Std. dev. external debt/Y	0.027	0.022

[a]Sample period: 2002-2015.

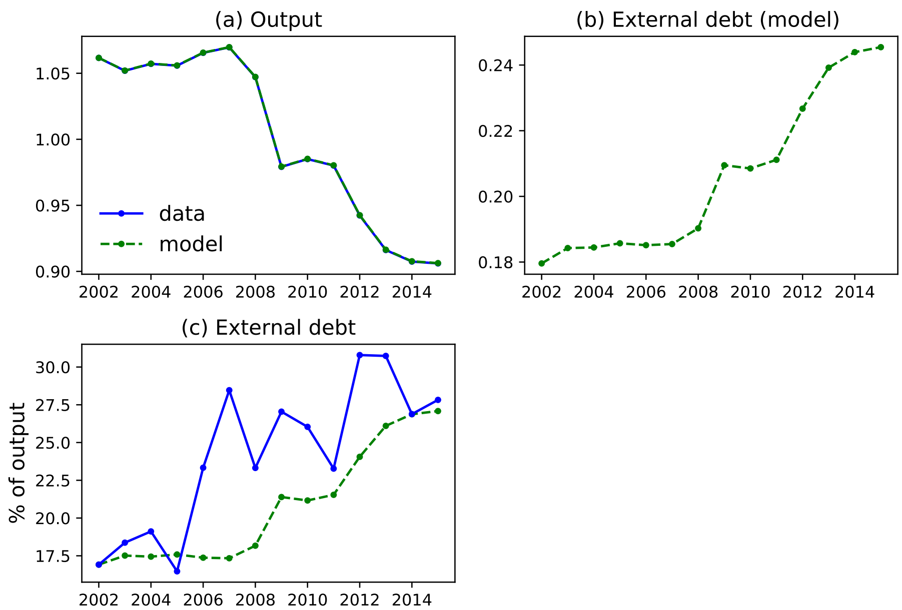
Note: This table reports the non-targeted statistics of the data and the model. The first column reports data statistics which are across the period of 1985-2015, unless specified. The second column reports the model statistics which come from the model's simulation for 10500 periods and excluding the first 500 periods. Net saving (NS) is defined as output minus total private and government consumption in the data and the model. External debt is defined as the country's net financial liability in the data. For the second moments, output and consumption series are logged and linear detrended. Net saving and external debt ratio series is linear detrended.

Several cyclical features of the Italian data stand out. First, consumption is as volatile as output and is highly correlated with output. Net saving only has a volatility of more than a quarter of the volatility of output, and has a positive correlation with output that is around 40%[14]. The model correctly gets the qualitative patterns of the data. The volatility of consumption and net savings relative to output are slightly higher in the model than in the data. Both model consumption and net saving are pro-cyclical with similar correlation levels as in the data. The model is able to generate realistic cyclical patterns of the data, in contrast to the standard model of complete markets. The main reason is that, even with state-contingent assets, the occasionally binding borrowing constraints lead to an imperfect insurance across states and time periods.

[14]Neumeyer and Perri (2005) reported key business cycle statistics for both advanced and emerging market economies.

External debt. In the data, external debt is defined as the net foreign liability position, as reported by Lane and Milesi-Ferretti (2018)'s External Wealth of Nation Database. The model explains well both the first and second moments of external debt for Italy from 2002 to 2015. The model generates on average around 21% of external debt-to-output ratio, comparing to 24% of net foreign liability-to-output ratio in the data. This model feature is with a relatively high discount factor (0.969) with respect to the literature. The model also matches the volatility of external debt-to-output ratio in the data.

Figure 4: Italy's Recession: Data and Model



Note: The graph depicts the time paths of output, external debt, and external debt-to-output for the data and the model's simulation. Panel (a) plots the output path. Panel (b) plots the external debt paths of the model. Panel (c) plots external debt-to-output.The simulation uses a sequence of productivity shock realization such that the model's output matches the data output for Italy in 2002-2015. The initial external debt level is such that the model's external debt-to-output matches with the starting value in 2002 from the data. Data sources: Lane and Milesi-Ferretti (2018) and World Development Indicator Database (2019)

Event analysis. I now conduct an event analysis for Italy in period of 2002-2015. I feed into the model a sequence of productivity shock realizations such that the model's outputs matches ones in Italy from 2002 to 2015. I simulate a time path of external debt in the model given that the initial external debt-to-output is the data value in 2002 . I then compare the evolution of external debt-to-output in the data and in the model's simulation over time. Figure 4 plots the exercise's results. Panel (a) plots the output paths of the data and the model. Panel (b) plots the time path of external debt in the model. Panel (c) plots external debt-to-output time paths for both the data and the model. From 2011 to 2015, Italy's output has dropped by 7.4% below trend, while external debt-to-output has increased by 4.6%. In the model's simulation, external debt has increased by 3.4%, which leads to a 5.5% increase in external debt-to-output.

5.4 Model Mechanics

This subsection explains the mechanism on how the government's concerns for redistribution affect its incentive to repay debt. The first part analyzes dynamics of optimal policies, which provides insights on the properties of redistributive policies that allow the government to sustain debt. The second part studies the cost of default and its interaction with the government's redistributive motive. This interaction is the key explanation for the positive association between income inequality and external debt.

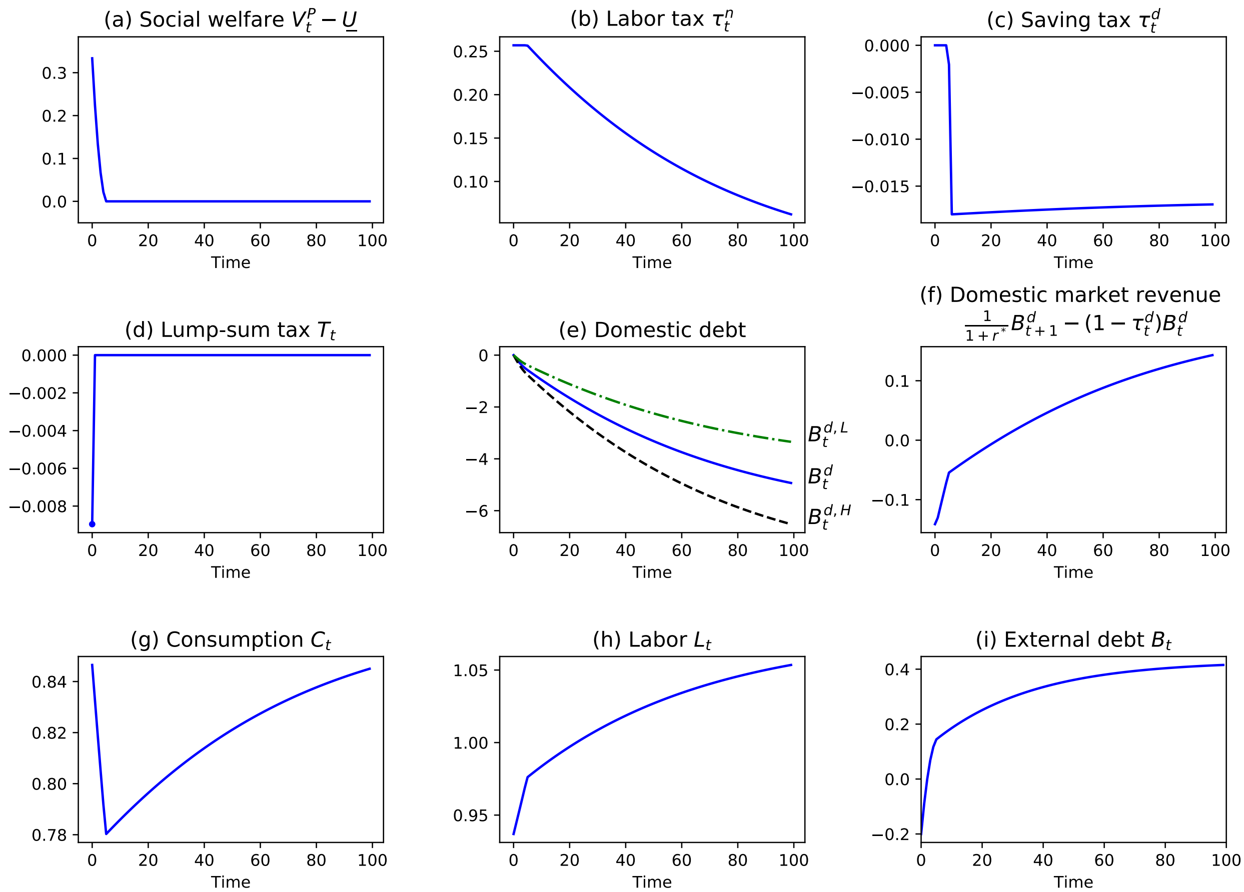
5.4.1 Dynamics of optimal policies

I first consider a special case of the model that faces a deterministic path of productivity: $z_t = \bar{z} = 1, \forall t$, and the economy starts with an initial external asset position: $B_0 = -0.2$. The other parameter values are the same as in Table 2. Without shocks, when the borrowing constraint binds, all future borrowing constraint will also bind. This framework can clearly show the effect of the binding borrowing constraint on optimal policies, in which there is a region where the borrowing constraint does not bind, and a region where it binds. The Ricardian equivalence implies that there is indeterminacy between lump-sum taxes and domestic debt holdings. Therefore, a particular implementation is that the government only levies the present-value of lump-sum taxes in the initial period. These lump-sum taxes determine the effective initial wealth positions of domestic agents.

Figure 5 depicts the aggregate time paths of this special case. Panel (a) plots the difference between the social welfare and deviation utility over time. Panel (b) and (c) describes the optimal labor tax and saving tax. Panel (d) depicts the lump-sum taxes. Panel (e) plots the time paths of total and individual domestic debt. Panel (f) plots the net revenue that the government collects from participating in the domestic credit market, $\frac{1}{1+r^*}B^d_{t+1} - (1-\tau^d_t)B^d_t$. Panel (g) and (h) plot aggregate consumption and labor, respectively. Panel (f) describes the time path of the external debt.

The borrowing constraint does not bind when $V^P > \underline{U}$, and binds when $V^P = \underline{U}$. Due to impatience, the social welfare decreases over time until it reaches the deviation utility value. Optimal taxes follow the properties in Proposition 4.1. Optimal labor taxes are positive and constant, and saving taxes are zero when borrowing constraints do not bind. However, optimal labor taxes permanently decrease as the borrowing constraints start

Figure 5: Time paths of aggregates in the special case: $z_t = 1$ and $B_0 = -0.2$



Note: The graph plots the deterministic time paths of optimal policies and aggregates from the planning problem in which $z_t = \bar{z} = 1, \forall t$ and $B_0 = -0.2$. The implementation is that lump-sum taxes only occur in period 0. Panel (a) plots the difference between the social welfare and deviation utility. Panel (b) and (c) describes the optimal labor tax and saving tax. Panel (d) depicts the lump-sum taxes. Panel (e) plots the time paths of total and individual domestic debt. Panel (f) plots the net government's revenue of domestic market. Panel (g) and (h) plot aggregate consumption and labor, respectively. Panel (f) describes the time path of the external debt.

binding, and saving taxes become negative, imply taxes on domestic borrowing. In present value terms, the planner gives a small lump-sum transfer (negative lump-sum tax) to all residents. Domestic debt decreases over time, implying that agents are borrowing. Since all agents borrow at the same fraction of their income, high-income agents borrow more than low-income agents and are net debtors. The government acts as an financial intermediary between the domestic agents and international lenders. Panel (f) describes the net resources that the government receives from the domestic credit market. In the beginning of time, the government gives resources to domestic agents. The government collects taxes on labor income in return. When borrowing constraints bind, the government uses borrowing taxes ($\tau^d < 0$) and start collecting net revenue from the domestic credit market. As labor taxes decline over time, there is less labor tax revenue. Impatience implies that the planner front loads consumption and leisure when borrowing constraints do not bind. When borrowing constraints bind, declining labor taxes encourage increases in labor and output, while saving subsidies encourage back-loading consumption.

The similar dynamics occur in the baseline model with a stochastic aggregate productivity. Figure 6 depicts the time paths of the aggregates and policies of the baseline model for a simulation. In this case, the borrowing constraint is occasionally binding, generating the fluctuations in the efficient allocation and policies. Both the social welfare V^P and aggregate consumption C are highly correlated with the productivity shock z. Over time, since the borrowing constraint binds more often, both aggregate consumption and labor go up, as predicted by the deterministic case. The labor tax drifts down, while the domestic market revenue increases. The external debt increases over time and eventually reaches an ergodic distribution.

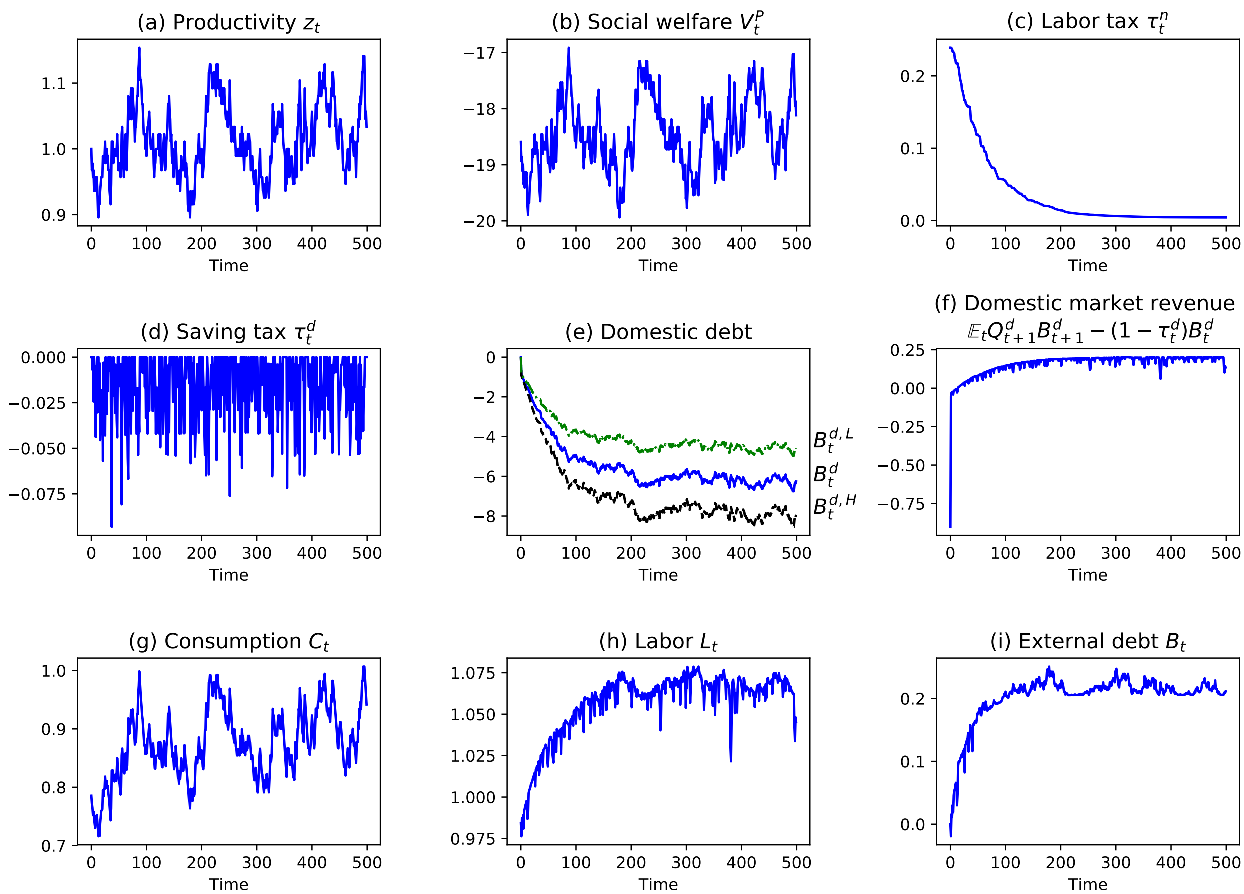
Figure 6 points out the changes in the government's redistributive policies over time. Since high-income agents borrow more than low-income ones, borrowing taxes act as an additional redistributive policy when borrowing constraints bind. The government redistributes via taxes on borrowing instead of labor taxes, which increases the efficiency gain and allows the government to sustain the existing level of debt.

5.4.2 Cost of default

The cost of default determines the equilibrium level of external debt. The more costly is to default, the more likely that the government is willing to repay debt, and so the higher level of external debt is.

The cost of default is the opportunity cost of not having access to domestic and external credit markets. In this framework, there are two main components: insurance and redistributive costs. The former comes from the fact that without credit markets, the government cannot

Figure 6: Simulated time paths of aggregates in the baseline model



Note: The graph plots the simulated time paths of optimal policies and aggregates of the planning problem for the baseline model. The implementation is that lump-sum taxes only occur in period 0. Panel (a) plots the difference between the social welfare and deviation utility. Panel (b) and (c) describes the optimal labor tax and saving tax. Panel (d) depicts the lump-sum taxes. Panel (e) plots the time paths of total and individual domestic debt. Panel (f) plots the net government's revenue of domestic market. Panel (g) and (h) plot aggregate consumption and labor, respectively. Panel (f) describes the time path of the external debt.

insure itself against aggregate fluctuations. The insurance cost is present in many representative-agent models in the literature (Eaton and Gersovitz (1981), Aguiar and Gopinath (2007), Arellano (2008), Chatterjee and Eyigungor (2012), and many other papers). This paper introduces the redistributive cost, which is novel to the literature. The main idea is that redistribution is more costly in financial autarky than in the contract.

The government's redistributive motive affects its repayment incentive via changing the cost of default. This an endogenous component of the cost of default that is novel to the literature. The main idea is that redistribution is more costly in financial autarky than in the contract, so the government is more willing to repay its debt.

What entails the redistributive cost of default, or the benefit of repayment? First, note that if the government defaults, it does so both domestically and externally. It turns out that domestic default creates an adverse distributive effect. Figure 6 shows that the high-income agents are net domestic debtors in the long run. Domestic default erases the distribution of domestic wealth and so implicitly transfer more resources to the high-income agents. Furthermore, the labor distortion needed for redistribution in financial autarky is higher and more volatile than the labor distortion in the contract. Access to both domestic and external credit markets allows the government to use less labor distortion by redistributing via borrowing taxes when borrowing constraints bind, and is able to smooth out the distortions over aggregate fluctuations.

Figure 7 shows optimal labor taxes in different scenarios. Panel (a) plots the path of productivity shock in the long run, starting at period 500. Panel (b) and (c) plot the baseline simulation of optimal labor taxes in contract and in autarky, respectively. The autarky case means that the government defaults at period 500 and faces financial autarky onward. Panel (d) and (e) are the analogs of panel (b) and (c) for the one-agent's model.

In the one-agent model, there is no need for redistribution, so the labor taxes are zero across time periods and histories. If the government defaults and goes into financial autarky, labor taxes remain zero. However, in the baseline model with heterogeneous agents, there are differences in labor taxes between the contract and autarky. The labor tax in autarky is higher and more volatile than the labor tax in the contract. These properties lead to financial autarky, or default, be more costly than repayment.

Quantitatively, the welfare cost of insurance is trivial, so the amount of external debt that the one-agent model can sustain is quantitatively small[15]. When the government has concerns for redistribution, the redistributive cost of default arise endogenously. This cost

[15] Lucas (1987) pointed out that the cost of eliminating business cycles is quantitatively small for the standard neoclassical growth model. In the sovereign debt framework, Aguiar and Gopinath (2007), Arellano (2008), and Chatterjee and Eyigungor (2012) have shown that in order to match the observed debt levels in the data without counterfactual discount factors, they need to impose additional output losses in default.

Figure 7: Labor distortion in contract and in autarky

Note: The graph describes the time paths of productivity shock and optimal labor taxes in contract and in financial autarky for the baseline model and the one-agent model. The autarky case is when the government defaults at period 500 and is permanently excluded from all credit markets.

turns out to be quantitatively large enough to account for the observed external debt levels. Given the calibrated parameters, the long-run average external debt-to-output is 2.8% in the one-agent model, while it is 21% in the heterogeneous-agent model. These results imply that the insurance cost accounts for 13% , and the redistributive cost accounts for 87% of the long-run average external debt-to-output.

Higher income inequality is correlated with higher external debt because of the higher redistributive cost of default. Given the government's redistributive preference towards low-income agents, a higher income inequality implies a higher motive for redistribution and a larger labor tax distortion in financial autarky, making it more costly to default. Therefore, the government is willing to sustain a higher amount of external debt.

In the next subsections, I estimate the effect of income inequality on external debt in the cross section and over time.

5.5 Cross-Sectional Estimation

Table 5 shows the estimation results of the correlation between pre-tax Gini index and net foreign liability-to-GDP from the model and the data. The data values are from the second column of Table 1, robust to country, time, and other controls. The model values come from the regression on the model's simulated data. Given the calibrated parameters, I solve different versions of the model differentiated only by wage inequality and compute the pre-tax Gini indices, long-run averages of external debt-to-output ratios, and output per capita. I then estimate the regression $\text{NFL}_i = \beta_0 + \beta_1 \text{Gini}_i + \beta_2 \log \text{GDP per capita}_i + \epsilon_i$ and report $\hat{\beta}_1$ and its standard error[16].

Table 5: Cross-sectional estimation: Data and Model

	Dependent Variable: Net foreign liability-to-GDP (%)	
	Data: 1985-2015	Model
Gini index, pre tax (%)	1.3272***	2.4233***
	(0.4576)	(0.0674)
Controls	Yes	Yes
No. Observations	3848	30

Note: The table describes the cross-sectional estimation of the coefficient of pre-tax Gini index (%) on net foreign liability-to-GDP (%) in the data and in the model. Details on data estimation are from Table 1. The model estimation comes from simulated data of 30 different versions of the model that are differentiated by wage ratios.

The model produces a positive and statistically significant coefficient of the pre-tax Gini index. The coefficients imply that a one percent increase in the pre-tax Gini index corresponds to a 1.3272% increase in net foreign liability-to-GDP in the data, comparing to a 2.4233% increase in the model.

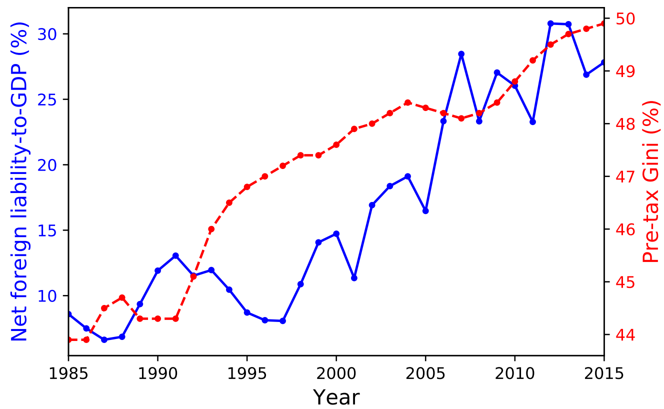
5.6 Time Series Analysis

This subsection estimates the effect of income inequality on external debt over time. I conduct a comparative statics exercise in the case of Italy for two time periods of 1985-2001 and 2002-2015. Figure 8 plots the time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. On average, in the period of 1985-2001, Italy has a lower levels of income inequality and external debt comparing to the period of 2002-2015.

The comparative statics exercise is as follows. I feed into the model a value of wage inequality for the period 1985-2001 and keep other parameter values fixed. I compute ergodic means of pre-tax Gini index and external-debt-to-output ratios. The 1985-2001 value of wage inequality is such that the change in the average pre-tax Gini income from 1985-2001 to 2002-2015 is the same as the change in the data. Table 6 reports the results of the policy experiment. Given the targeted increase in the pre-tax Gini indices in Italy from 1985-2001

[16]In the model, average output growth rates and inflation are zero, so I omit them as control variables. Since the regression uses the ergodic means of the model as variables, country and time fixed effects are redundant.

Figure 8: Income inequality and external debt in Italy



Notes: The graph shows the time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. The left y-axis depicts the values in net foreign liablity-to-GDP (%), and the right y-axis depicts the values in pre-tax Gini (%) . Sources: Lane and Milesi-Ferretti (2018), and Solt (2019).

to 2002-2015, the model can account for 93% of the increase in the external debt-to-output ratio.

Table 6: Inequality and external debt across time periods: 1985-2001 and 2002-2015

Statistics	Data[a]	Model[b]
Targeted		
Δ Pre-tax Gini	3.0%	3.0%
Non-targeted		
Δ Extenal debt/Y	14%	13%

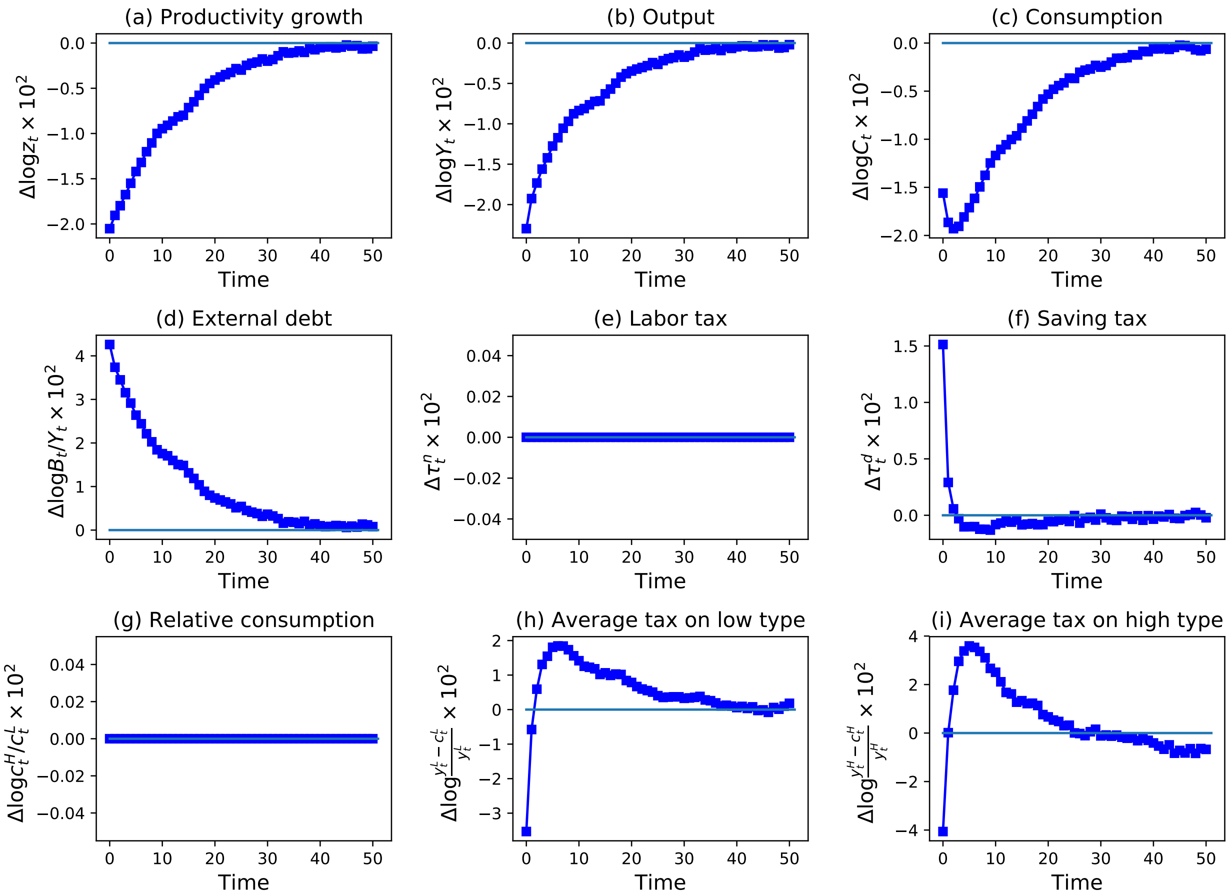
[a]The change in the data statistics is computed as the average statistic of period 2002-2015 minus the average statistic of 1985-2001

[b]The change in the model statistics is computed as the average statistic of a simulation for the model with the wage ratio equal to 1.9475 minus the same statistic of the model with the wage ratio equal to 1.73

5.7 Optimal Austerity

This section analyzes optimal austerity policies in the context of policies responses to a negative productivity shock in the presence of inequality.

Figure 9: Impulse response functions to a negative productivity shock



Notes: The graph shows the impulse response functions $\sigma_z \times IRF_\tau$ computed by local projection methods as in Jordà (2005). Panel (a) plots the productivity growth response. Panel (b) and (c) plot the responses of output and consumption, respectively. Panel (d), (e), and (f) plot the responses of fiscal policies: external debt, labor, and saving taxes, respectively. Panel (g), (h), and (i) show the responses of redistribution: relative consumption between high- and low-income agents and average tax-to-income ratios across agents.

Figure 9 plots the impulse response functions of aggregate variables and tax policies with respect to a one standard deviation decline in productivity growth, computed via local projection[17]. Panel (a) plots the path of productivity growth given the negative innovation shock occurring in period 0. There are three groups of responses: aggregates, fiscal policies, and redistribution. Panel (b) and (c) plot the first group of responses of

[17] The approach to calculate impulse respond functions is econometrically equivalent to the approach of Jordà (2005). I simulate the economy for 10500 periods with aggregate productivity shocks and exclude the first 500 periods. I the calculate the realized time series of shocks to productivity, ϵ_t^z. To compute the response of a variable X to the shock ϵ_t^z, I run the OLS regressions $\Delta \log X_t = \alpha + \beta_k \epsilon_{t+k}^z + \eta_t$ to get the estimated $\hat{\beta}_k$. The horizontal τ IRF is then $IRF_\tau = \sum_{k=0}^{\tau} \hat{\beta}_t$. The effect of one standard deviation shock to ϵ_t^z is the responses $\sigma_z \times IRF_\tau$. Since labor and saving taxes can be zero or negative, the dependent variable in the OLS regressions are ΔX_t instead of $\Delta \log X_t$. To my best knowledge, Mongey (2019) is the first paper that applies this computational technique.

output and consumption, respectively. For fiscal policies, Panel (d), (e), and (f) plot the responses of external debt, labor taxes, and saving taxes. Lastly, Panel (g), (h), and (i) plot the responses in relative consumption and average tax-to-income across households, which are measures of redistribution[18].

A decline in productivity growth leads to declines in both output and consumption with a higher drop in output. External debt-to-output increases in response to a low productivity. Labor taxes remains unchanged, while there is a sharp increase in saving taxes in the first period, accompanying with a decline. Note that the optimal saving taxes are negative in the long run. The initial increase in saving taxes comes from the non-binding borrowing constraints in the first few periods after a negative shock[19]. However, as borrowing constraints bind in the future, it is then optimal to increase borrowing taxes, or reducing saving taxes. In terms of redistribution, the relative consumption across agents remain unchanged. Initially, average tax rates decrease for both agents, and decrease more for high-income agents. Average taxes then increase and increase more for high-income agents. This implies that following a negative shock, the government redistributes less towards low-income agents. Nevertheless, over time, the redistribution shifts towards low-income agents.

6 Discussions

6.1 Role of Aggregate Uncertainty and Heterogeneity

I want to show how the two main ingredients: aggregate uncertainty and heterogeneity affect the optimal policies. Table 7 describes the long-run statistics and optimal policies for different cases of the model and data. The first column reports the long run statistics in the data. The other columns specify the statistics from model simulations and the optimal tax policies that supported the efficient allocation.

[18] Relative consumption is defined as the ratio of consumption between high-income agents and low-income agents. Average tax-to-income ratio is defined as total amount of taxes/total income for individuals. The formula is $\frac{y^i - c^i}{y^i}$ for an individual i with income y^i and consumption c^i

[19] Proposition 4.1 shows that $\tau^d = 0$ when borrowing constraints do not bind. So τ^d goes from a negative number to zero initially.

Table 7: Role of Aggregate Uncertainty and Heterogeneity

	Data	Baseline	One agent	No shock	One agent & no shock
Long-run statistics					
std (C) / std (Y)	1.0	1.2	1.2	–	–
Mean B/Y	0.24	0.21	0.028	0.25	0.0
Std. dev. B/Y	0.027	0.022	0.082	–	–
Tax policies in the model					
Initial τ^n	–	0.25	0.0	0.25	0.0
Mean LR τ^n	–	0.0042	0.0	0.0029	0.0
Lump-sum tax T_0/Y_0	–	0.87	5.8	0.6	5.7

Notes: The table reports long-run statistics and tax policies for data and different cases of the model. The first column reports data values. The second column reports the baseline values. The third column presents the results of the representative-agent case. The fourth column shows the results of the deterministic case. The last column shows the results of the deterministic and representative-agent case.

6.2 Role of Distortionary Taxation

The previous section has shown how the redistributive tax policy comes with a cost of distortion, and by front-loading the distortion, the economy sustains a high debt. I now consider a scenario in which the planner has access to skill-specific lump-sum transfer so that the planner achieves perfect redistribution without any distortion. I show that the need to use distortionary taxation as a redistributive tool makes the government be willing to sustain highly positive debt in the long run. Table 8 reports the external debt and tax policies of the baseline model using linear taxes comparing to the alternative framework with lump-sum tax depending on income.

The alternative framework quantitatively generates a much smaller amount of debt with a higher volatility than the baseline model. Across all periods, the labor tax is zero, since all of the redistribution is done via the type-dependent lump-sum taxes. In present-value terms, the government taxes the high-income agents and transfers to the low-income agents.

6.3 Role of Government Consumption

This subsection shows the role of the government consumption, G_t in the optimal tax and debt policies.

Table 8: Role of Distortionary Taxation

	Baseline	Skill-dependent lump-sum tax
External debt		
Mean B/Y	0.21	0.029
Std. dev. B/Y	0.022	0.082
Tax		
Initial τ^n	0.25	0.0
Mean LR τ^n	0.0042	0.0
Lump-sum tax T_0/Y_0	0.88	5.7, high type = 20, low type =−8.6

Notes: The table reports long-run statistics and tax policies for the baseline and the case in which the government has access to fully skill-dependent lump-sum taxes, $T^i, \forall i \in I$. The last row and column reports the average lump-sum tax, as well as the individual lump-sum taxes.

Table 9: Role of Government Consumption

	Baseline	No g
External debt		
Mean B/Y	0.21	0.20
Std. B/Y	0.022	0.011
Policies		
Initial τ^n	0.25	0.21
Mean LR τ^n	0.0042	−0.016
Lump-sum tax T_0/Y_0	0.88	4.4

Notes: The table reports long-run statistics and tax policies for the baseline and the case in which $g = 0$.

7 Conclusion

This paper studies the interaction between a country's concern for redistribution and its external indebtedness. I introduce the government's motive for redistribution and distortionary taxes into the sovereign debt framework and analyze the interaction between distortionary and distributive effect of fiscal policies and the government's lack of commitment. The endogenous borrowing constraints arise from the government's lack of commitment, and become relevant in the long run due to impatience. Tax policies are both distortionary and lump-sum, and redistribution comes with the cost of tax distortions.

The paper's theoretical contribution is the effect of borrowing constraints on optimal taxation in the presence of inequality. While the inequality determines the optimal level of taxes and redistribution, the borrowing constraints determine the dynamics of taxes. The main conclusion is that labor taxes are unchanged, and borrowing taxes are zero with non-binding borrowing constraints. Binding borrowing constraints lead to permanent declines in labor taxes and positive borrowing taxes to discourage domestic borrowing. Borrowing taxes have a redistributive benefit, which allows the government to distort labor less and increases the economy's efficiency.

The quantitative analysis suggests that the government's redistributive motive plays an important role in determining the equilibrium level of debt. This channel comes from the additional cost of redistribution during financial autarky. The result contributes to the ongoing literature on endogenous default costs in sovereign debt models. The redistributive cost of default quantitatively accounts for 87% of the long-run average external debt-to-output, while the insurance cost of default only accounts for 13%. Another contribution of the paper is that the theory can account for the cross-sectional and time-series relationship between income inequality and external debt.

The model has implications on optimal austerity policies in the presence of inequality. Estimations from the model's long-run simulation points out that a negative productivity shock leads to an increase in external debt and a temporary decrease in borrowing taxes, while labor taxes remain unchanged. In terms of redistribution, relative consumption is unchanged, while average tax-to-income decreases for both agents and more for high-income agents. This result indicates that the government redistributes less resources towards the low-income agents.

One important aspect that this model cannot speak to is the implication of inequality and concern for redistribution on default risk and sovereign spread. In future work, I extend the framework to incorporate incomplete markets and equilibrium defaults.

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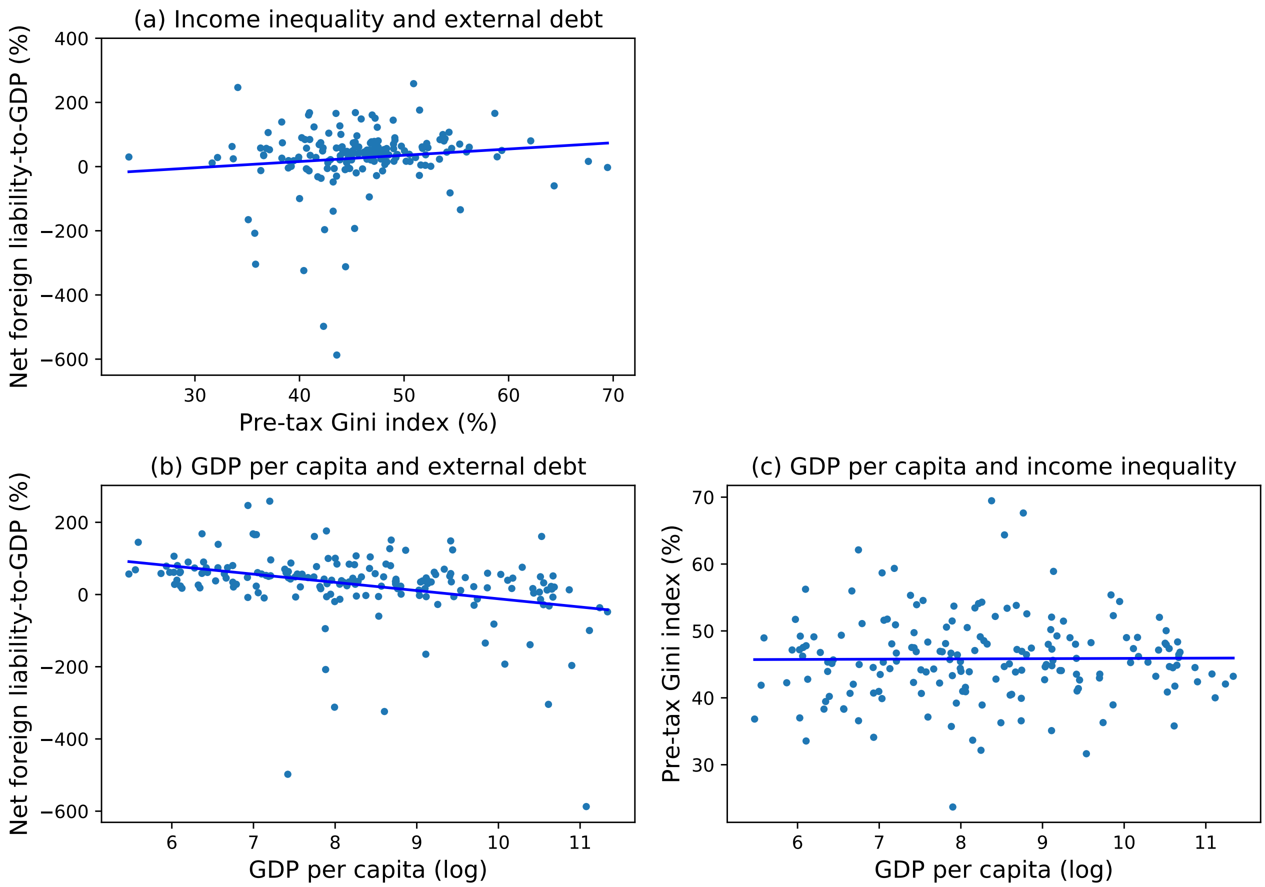
A Empirical Appendix

This subsection presents the analogs of Figure 1 and Figure 2 for all countries in the dataset.

Figure 10 shows the 1985-2015 time averages of net financial liability-to-GDP, pre-tax Gini index, and GDP per capita in constant 2010 US Dollars for all countries in the data set. I omit country's labels for visual purposes. The pre-tax Gini index is positively correlated with the net foreign liability-to-GDP. The GDP per capita negatively associates with the net foreign liability-to-GDP, while it does not have a strong correlation with the pre-tax Gini index.

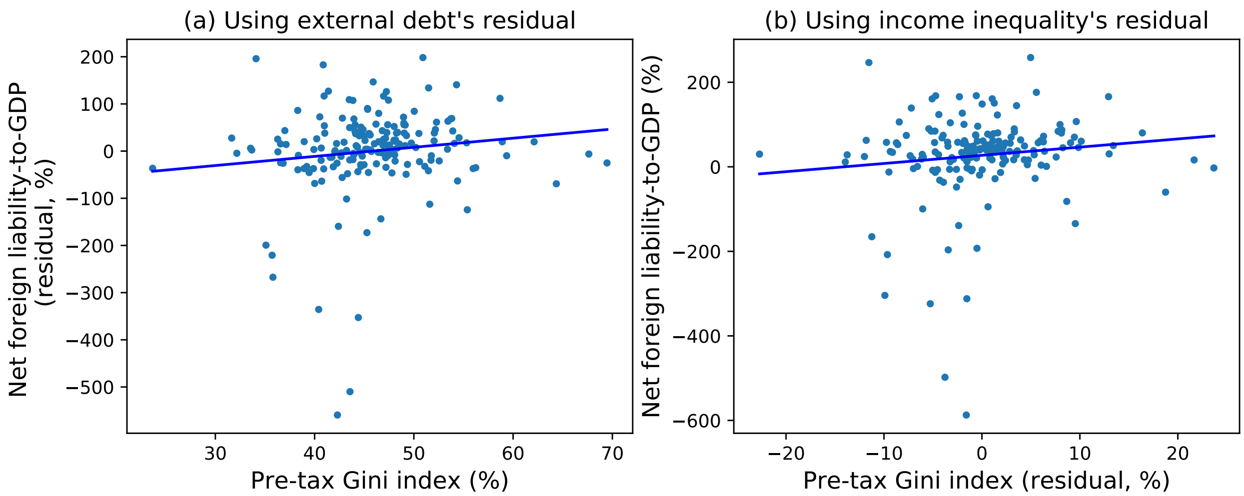
Figure 10: Income inequality, external debt, and GDP per capita across countries



Note: The graph shows the 1985-2015 time averages of net financial liability-to-GDP, pre-tax Gini index, and GDP per capita in constant 2010 US Dollars for all economies. Panel (a) plots averages of pre-tax Gini index (%) and net foreign liability-to-GDP (%). Panel (b) plots averages of log of GDP per capita and net foreign liability-to-GDP (%). Panel (c) plots averages of log of GDP per capita and pre-tax Gini index (%). Sources: World Development Indicator Database (2019), Lane and Milesi-Ferretti (2018), and Solt (2019).

Figure 11 shows the cross-sectional relationship between income inequality and external debt controlling for other common factors, for all countries in the sample. Panel (a) plots the residuals ϵ_i^{nfl} (in percentage) of equation (1) and the pre-tax Gini index (%). Panel (b) plots the net foreign liability-to-GDP (%) and the residuals ϵ_i^{gini} (in percentage) of equation (2). Both panels show a positive correlation between the two main variables in the cross section.

Figure 11: Cross-sectional relationship between income inequality and external debt



Note: Panel (a) plots the residuals ϵ_i^{nfl} (in percentage) of equation (1) and the pre-tax Gini index (%). Panel (b) plots the net foreign liability-to-GDP (%) and the residuals ϵ_i^{gini} (in percentage) of equation (2). The sample includes all economies in the dataset. Sources: World Development Indicator Database (2019), Lane and Milesi-Ferretti (2018), and Solt (2019).

B Sovereign Game

Before setting up the game, consider the general environment where the government's policy includes the decision to default on external bond $\{\delta(s^t)\}$, where $\delta \in \{0, 1\}$ and $\delta = 0$ implies default[20]. The government's budget constraint becomes

$$\begin{aligned} &G(s^t) + (1 - \tau^d(s^t))B^d(s^t) + \delta(s^t)B(s^t) \\ \leq &\tau^n(s^t)w(s^t)L(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) + \sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t)B(s^{t+1}) + T(s^t) \end{aligned}$$

The price of international debt takes into account the probability of default is

$$Q(s_{t+1}|s^t) = \frac{\Pr(s_{t+1}|s^t)\delta(s_{t+1}|s^t)}{1 + r^*}$$

[20]Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its foreign debt ($d_t = 1$).

As the government cannot commit to any of its policies, one can think that the government, private agents, and international lenders enter in a sovereign game where they determine their actions sequentially. In every period and every history, the state variable for the game is $\left\{B(s^t), \left(b^{i,d}(s^t)\right)_{i\in I}\right\}$. The timing of the actions is as follows.

- Aggregate shock s_t is realized
- Government chooses $z_t^G = \left(\tau^n(s^t), \tau^d(s^t), T(s^t), \delta(s^t), B(s_{t+1}, s^t), B^d(s_{t+1}, s^t)\right) \in \Pi$such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c^i(s^t), l^i(s^t), b^{d,i}(s_{t+1}, s^t)\right)$ subject to their budget constraints, the representative firm produce output by choosing $z_t^F = L(s^t)$, and the international lenders choose holdings of government's bonds $z_t^* = B(s_{t+1}, s^t)$.

Define $h^t = \left(h^{t-1}, z_{t-1}^G, \left(z_{t-1}^{H,i}\right)_{i\in I}, z_{t-1}^F, z_t^*, s_t\right) \in H^t$ as the history after shock s_t is realized. Note that the history incorporates the government's policy, allocation and prices. Define $h_p^t = \left(h^t, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period t. The government strategy is $\sigma_t^G : H^t \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i} : H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has strategy $\sigma_t^F : H_p^t \to \mathbb{R}_+^2$, and the international lenders have strategy $\sigma_t^* : H_p^t \to \mathbb{R}_+$.

Definition B.1 (Sustainable equilibrium)**.** A sustainable equilibrium is $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ such that (i) for all h^t, the policy z_t^G induced by the government strategy maximizes the socially weighted utility given λ subject to the government's budget constraint (7) (ii) for all h_p^t, the strategy induced policy $\left\{z_t^G\right\}_{t=0}^{\infty}$, allocation $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^{\infty}$, and prices $\{Q_t, \}_{t=0}^{\infty}$ constitute a competitive equilibrium with taxes.

The following focuses on characterizing a set of sustainable equilibrium in which deviation triggers autarky, where there is no domestic and foreign borrowing. In this case, the value of deviation includes the autarkic payoff.

By definitions, autarky is a sustainable equilibrium. Given that the domestic agents do not save/invest, the representative firm produces only with labor, and the international creditors do not lend, the government finds it optimal to default on its external debt, set saving and capital taxes such that the after-tax gross returns on domestic bonds and capital are zero, and set the labor tax such that it maximizes the socially weighted utility. Given the government defaulting and fully taxing all returns from domestic savings and capital, international creditors do not want to lend, agents do not save or invest in capital, and output is produced only by labor. Lastly, given that the government will be in autarky in the future, it is optimal in the current period for the government to also follow the autarkic strategies.

Reverting to autarky equilibrium is defined as a sustainable equilibrium of the above game such that following any government's deviation from the promised plans, the economy reverts to autarky. One can characterize the equilibrium as follows.

Proposition B.1 (Reverting to autarky equilibrium)**.** *An allocation and policy* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *can be supported by reverting to autarky equilibrium if and only if (i) given* z^G*, there exist prices* p *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G, p\right\}$ *is a competitive equilibrium with taxes for an open economy, and (ii) for any* t *and any* s^t*, there exists* $\underline{U}(s^t, t)$ *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *satisfies the constraint*

$$\sum_{i\in I}\lambda^i\pi^i\sum_{k\geq t, s^t\subseteq s^k}\beta^{k-t}\Pr(s^k|s^t)U^i(c^i(s^k), l^i(s^k)) \geq \underline{U}(s^t, t) \tag{13}$$

Proof. Define $\underline{U}(s^t, t)$ as the maximum discounted weighted utility for the agents in period t, history s^t, when the government deviates. At period t and history s^t, the government taxes all domestic wealth ($\tau^d(s^t) = 1$) and redistributes equally across agents, and the government defaults on the external debt. In subsequent period $k > t$, the economy reverts to financial autarky where agents do not save in domestic bonds, and the government is excluded from international lending. This economy ensembles a neoclassical growth closed economy that has an initial aggregate state s_t, distortionary taxation on labor, and equal initial wealth across individuals.

Suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an outcome of the reverting to autarky equilibrium. Then by the optimal problems of the government, agents, and foreign lenders, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes the weighted utility of the agents, satisfies government budget constraint and foreign lender's problem at period 0. Thus, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an open-economy tax-distorted competitive equilibrium. For any period t and history h^t, an equilibrium strategy that has the government deviates in period t triggers reverting to autarky in period $k > t$. Such strategy must deliver the weighted value at least as high as the right-hand side of (13). So $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii).

Next, suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies conditions (i) and (ii). Let h^t be any history such that there is no deviation from z^G up until period t and history s^t. Since $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t and history s^t onward. Consider a deviation plan $\hat{\sigma}^G$at period t that receives $U^d(s_t, t)$ in period t and $U^{aut}(s_t)$ for the subsequent period $k > t$. Because the plan is constructed to maximize the utility in period t, the right-hand side of (13) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. $\square$

Proposition B.1 can be extended to the general characterization of sustainable equilibrium, as in Chari and Kehoe (1990).

C Separable Isoelastic Utility Case

This section provides details on the characterization of the efficient allocation and optimal policies given that the individual utility is

$$U^i\left(c,l\right)=\frac{c^{1-\sigma}}{1-\sigma}-\omega\frac{\left(\frac{l}{\theta^i}\right)^{1+\nu}}{1+\nu}$$

The analysis is an extension to Tran Xuan (2019).

C.1 Equilibrium Characterization

Individual consumption and efficient labor supply are time- and history-independently proportional to the aggregates:

$$\begin{aligned} c^i(s^t) &= h^{i,c}(C(s^t),L(s^t);\boldsymbol{\varphi})=\psi_c^i C(s^t) \\ l^i(s^t) &= h^{i,l}(C(s^t),L(s^t);\boldsymbol{\varphi})=\psi_l^i L(s^t) \end{aligned} \tag{B.1}$$

where

$$\psi_c^i=\frac{\left(\varphi^i\right)^{1/\sigma}}{\sum_{i\in I}\pi^i(\varphi^i)^{1/\sigma}},\quad \psi_l^i=\frac{\left(\theta^i\right)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}}{\sum_{i\in I}\pi^i(\theta^i)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}} \tag{B.2}$$

C.2 Planning Problem

Let μ be the multiplier on the resource constraint, $\pi^i\eta^i$ be the multiplier on the implementability constraint for agent i, and $\beta^t\Pr(s^t)\gamma(s^t)$ be the multiplier on the aggregate debt constraint for period t. Define $\boldsymbol{\eta}=\left(\eta^i\right)_{i\in I}$ and rewrite the Larangian of the planning problem with a new pseudo-utility function that incorporates the implementability constraints:

$$\sum_{t=0}^{\infty}\beta^t W\left[s^t;\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\eta}\right]-V_C(s_0;\boldsymbol{\varphi})\sum_{i\in I}\pi^i\eta^i\left(b^i(s^0)-T\right)$$

where

$$W\left[s^t;\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\eta}\right] \equiv \sum_{i\in I}\lambda^i\pi^iU^i\left[h^i(s^t;\boldsymbol{\varphi})\right] + \sum_{i\in I}\pi^i\eta^i\left[V_C(s^t;\boldsymbol{\varphi})h^{i,c}(s^t;\boldsymbol{\varphi}) + V_L(s^t;\boldsymbol{\varphi})h^{i,l}(s^t;\boldsymbol{\varphi})\right]$$

Then V and W inherit the separable and isoelastic properties from U, i.e. $\forall t, \forall s^t$,

$$\begin{aligned} V(C(s^t),L(s^t);\boldsymbol{\varphi}) &= \Phi_C^V\frac{C(s^t)^{1-\sigma}}{1-\sigma} - \Phi_L^V\frac{L(s^t)^{1+\nu}}{1+\nu} \\ W\left[C(s^t),L(s^t);\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\eta}\right] &= \Phi_C^W\frac{C(s^t)^{1-\sigma}}{1-\sigma} - \Phi_L^W\frac{L(s^t)^{1+\nu}}{1+\nu} \end{aligned}$$

and the social welfare is

$$\sum_{t\geq 0, s^t\in S^t}\beta^t\Pr(s^t)\left(\Phi_C^P\frac{C(s^t)^{1-\sigma}}{1-\sigma} - \Phi_L^P\frac{L(s^t)^{1+\nu}}{1+\nu}\right)$$

where

$$\begin{aligned} \Phi_C^V &= \left[\sum_i \pi^i(\varphi^i)^{1/\sigma}\right]^\sigma; & \Phi_L^V &= \omega\left[\sum_i \pi^i\left(\varphi^i\right)^{-1/\nu}(\theta^i)^{(1+\nu)/\nu}\right]^{-\nu} \\ \Phi_C^W &= \Phi_C^V\sum_{i\in I}\pi^i\psi_c^i\left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]; & \Phi_L^W &= \Phi_L^V\sum_{i\in I}\pi^i\psi_l^i\left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right] \\ \Phi_C^P &= \Phi_C^V\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_c^i; & \Phi_L^P &= \Phi_L^V\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_l^i \end{aligned}$$

C.3 Optimal Policy

The first-order conditions of the planning problem for any period $t \geq 1$ can be summarized as

$$F_L(s^t,t) = \frac{\left\{\Phi_L^W + \Phi_L^P\sum_{k=0,s^k\subseteq s^t}^t\gamma(s^k)\right\}L(s^t)^\nu}{\left\{\Phi_C^W + \Phi_C^P\sum_{k=0,s^k\subseteq s^t}^t\gamma(s^k)\right\}C(s^t)^{-\sigma}} \tag{B.3}$$

and

$$Q(s_{t+1}|s^t) = \beta\Pr(s^{t+1}|s^t)\frac{C(s^{t+1})^{-\sigma}}{C(s^t)^{-\sigma}}\frac{\Phi_C^W + \Phi_C^P\sum_{k=0,s^k\subseteq s^{t+1}}^{t+1}\gamma(s^k)}{\Phi_C^W + \Phi_C^P\sum_{k=0,s^k\subseteq s^t}^t\gamma(s^k)} \tag{B.4}$$

The optimal policies follow

$$\tau^n(s^t) = 1 - \frac{1}{F_L(s^t,t)}\frac{\Phi_L^V L(s^t)^\nu}{\Phi_C^V C(s^t)^{-\sigma}} \tag{B.5}$$

$$\frac{Q^d(s_{t+1}|s^t)}{1-\tau^d(s^{t+1})} = \beta\Pr(s^{t+1}|s^t)\frac{C(s^{t+1})^{-\sigma}}{C(s^t)^{-\sigma}} \tag{B.6}$$

D Proofs

D.1 Proof of Proposition 3.1

Proof. ($\Rightarrow$) Let $\{C(s^t), L(s^t)\}_{t=0,s^t\in S^t}^{\infty}$ be an aggregate allocation of an open economy competitive equilibrium with taxes. Then by definition, $\{C(s^t), L(s^t)\}$ satisfies aggregate resource constraint for every period. Moreover, given any market weights $\boldsymbol{\varphi}$, $\{C(s^t), L(s^t)\}$ satisfies

$$(1-\tau^n(s^t))w(s^t) = -\frac{V_L\left[h^i(C(s^t), L(s^t);\boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t), L(s^t);\boldsymbol{\varphi})\right]}$$
$$\frac{Q^d(s_{t+1}|s^t)}{1-\tau^d(s^{t+1})} = \beta\Pr(s^{t+1}|s^t)\frac{V_C\left[h^i(C(s^{t+1}), L(s^{t+1});\boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t), L(s^t);\boldsymbol{\varphi})\right]}$$

Substituting for $w(s^t)$ into the budget constraint (5), and using $(c^i(s^t), l^i(s^t)) = h^i(C(s^t), L(s^t);\boldsymbol{\varphi})$ gives the implementability constraint for each agent. Importantly, choose $\boldsymbol{\varphi}$ and T such that the individual implementability constraints hold with equality.

($\Leftarrow$) Given $\boldsymbol{\varphi}$, T and an allocation $\{C(s^t), L(s^t)\}$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\{w(s^t)\}$ using the firm's first-order condition (6). $\{\tau^n(s^t)\}$ can be calculated using the intra-temporal condition (9), and choosing $\left\{Q^d(s^t)\right\}$ to satisfy the inter-temporal constraint (10). Define $\{q(s^t)\}$ by $q(s^t) = \Pr(s^t)/(R^*)^t$.

Rewriting the aggregate resource constraint using $F(L) = wL$ gives

$$\begin{aligned}&\sum_{t\geq 0, s^t\in S^t} q(s^t)\left\{C(s^t) - (1-\tau^n(s^t))w(s^t)L(s^t) + T(s^t)\right\}\\ &+\sum_{t\geq 0, s^t\in S^t} q(s^t)\left[G(s^t,t) - \tau^n(s^t)w(s^t)L(s^t) - T(s^t)\right] \leq -B(s^0)\end{aligned} \tag{C.1}$$

Aggregating up the agent's budget constraints implies

$$C(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) = (1-\tau^n(s^t))w(s^t)L(s^t) + (1-\tau^d(s^t))B^d(s^t) - T(s^t)$$

or

$$C(s^t) - (1-\tau^n(s^t))w(s^t)L(s^t) + T(s^t) = (1-\tau^d(s^t))B^d(s^t) - \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1})$$

Substituting the last equation into (C.1) gives the government's budget constraint (7). Thus, $\{C(s^t), L(s^t)\}$ is the aggregate allocation of the constructed competitive equilibrium with taxes. $\square$

D.2 Proof of Proposition 4.1

Proof. Given equations (B.3) and (B.5), the optimal labor tax is

$$\tau^n(s^t) = 1 - \frac{\Phi_L^V \Phi_C^W + \Phi_L^V \Phi_C^P \sum_{k=0, s^k \subseteq s^t}^{t} \gamma(s^k)}{\Phi_C^V \Phi_L^W + \Phi_C^V \Phi_L^P \sum_{k=0, s^k \subseteq s^t}^{t} \gamma(s^k)} \tag{C.2}$$

Given equations (B.4) and (B.6), the optimal saving tax is[21]

$$\tau^d(s^t) = \frac{\Phi_C^W + \Phi_C^P \sum_{k=0, s^k \subseteq s^{t-1}}^{t-1} \gamma(s^k)}{\Phi_C^W + \Phi_C^P \sum_{k=0, s^k \subseteq s^t}^{t} \gamma(s^k)} - 1 \tag{C.3}$$

Suppose that the borrowing constraint does not bind at period $\mathcal{T}$ and history $s^{\mathcal{T}}$, then $\gamma(s^{\mathcal{T}}) = 0$, which implies $\tau^n(s^{\mathcal{T}}) = \tau^n(s^{\mathcal{T}-1})$, and $\tau^d(s^t) = 0$

To prove the second part of the proposition, I first show that $\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} \leq \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$. By definitions,

$$\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} = \frac{\sum_i \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]}{\sum_i \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right]} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

and

$$\frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

using the optimal conditions $\eta^i = \sum_j \pi^j \lambda^j / \varphi^j - \lambda^i / \varphi^i$, and the definitions $\mathbb{E}\left[x^i\right] \equiv \sum_i \pi^i x^i$, $\text{cov}(x^i, y^i) \equiv \mathbb{E}\left[x^i y^i\right] - \mathbb{E}\left[x^i\right] \mathbb{E}\left[y^i\right]$.

Lemma D.1. *cov* $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ *and cov* $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$

Proof. The first step is to show that $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j$.

Suppose $\theta^i \geq \theta^j$ and $\varphi^i < \varphi^j$, then $\psi_l^i < \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < \frac{\varphi^i}{\varphi^j} < 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq 1$, which is a contradiction.

Suppose $\varphi^i \geq \varphi^j$ and $\theta^i < \theta^j$, then $\psi_l^i \geq \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq \frac{\varphi^i}{\varphi^j} \geq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < 1$, which is a contradiction.

[21] There is an indeterminacy between Q^d and τ^d. Here I assume a particular implementation where $Q^d(s_{t+1}|s^t) = \frac{\Pr(s_{t+1}|s^t)}{1+r^*} = Q(s_{t+1}|s^t)$, that is the price of the domestic debt is the same as the price of the external debt. The government uses τ^d to manipulate the domestic stochastic discount factor.

Next, the individual implementability constraint is

$$\psi_c^i \Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma} - \psi_l^i \Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu} = \Phi_C^V C(s_0)^{-\sigma} \left(a^i(s_0) - T\right)$$

or

$$\psi_c^i = \psi_l^i \frac{\Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu}}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}} + \frac{\Phi_C^V C_0^{-\sigma} \left(a^i(s_0) - T\right)}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}}$$

By definition of ψ_c^i, $\varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j$, and by assumption, $\theta^i \geq \theta^j \iff a^i(s_0) \geq a^j(s_0)$, which implies that $\theta^i \geq \theta^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$.

Thus, $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$.

In addition, $\theta^i \geq \theta^j \iff \lambda^i \leq \lambda^j$, which implies that

$$\psi_c^i \geq \psi_c^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$
$$\psi_l^i \geq \psi_l^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$

Hence, $\operatorname{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ and $\operatorname{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$. ∎

Lemma D.1 and $\sigma \geq 1, \nu > 0$ imply that $\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} \leq \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$.

Suppose that the borrowing constraint binds at period $\mathcal{T}$ and history $s^{\mathcal{T}}$, then $\gamma(s^{\mathcal{T}}) > 0$, which leads to $\sum_{\tau=0,s^\tau}^{\mathcal{T}} \gamma(s^\tau) > \sum_{\tau=0,s^\tau}^{\mathcal{T}-1} \gamma(s^\tau)$. Applying equation (C.2) gives $\tau^n(s^{\mathcal{T}}) \leq \tau^n(s^{\mathcal{T}-1})$. In addition, equation (C.3) implies that $\tau^d(s^{\mathcal{T}}) < 0$. ∎

D.3 Proof of Proposition 5.1

Proof. Let $\{C^*(s^t), L^*(s^t), K^*(s^t)\}_{t,s^t}, \boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exists $\boldsymbol{\lambda}$ such that $\{C^*(s^t), L^*(s^t), K^*(s^t)\}_{t,s^t}, \boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). Define

$$A_C = \sum_{i \in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_c^i, \quad A_L = \sum_{i \in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_l^i \tag{C.4}$$

where ψ_c^i, ψ_l^i are defined by equations (B.2) using $\boldsymbol{\varphi}^*$. First, one can show that A_C and A_L are positive and bounded:

Lemma D.2. *Given an interior allocation,* $0 < A_C < \infty$ *and* $0 < A_L < \infty$

Proof. Interior allocation means that for any i, $c_t^i, l_t^i > 0, \ \forall t$. This implies that $\psi_c^i, \psi_l^i > 0$. By (B.2), $\boldsymbol{\varphi}^{*i} > 0$.

For all i, $\pi^i > 0, \lambda^i \geq 0$ and since $\sum_{i\in I} \pi^i \lambda^i = 1$, there exists at least an i such that $\lambda^i > 0$. Given that $\psi_c^i, \psi_l^i > 0$, $\forall i$, it must be that $A_C, A_L > 0$.

Since $\sum_{i\in I} \pi^i \boldsymbol{\varphi}^{*i} = 1 < \infty$ and $\forall i$, $\pi^i, \boldsymbol{\varphi}^{*i} > 0$, it must be that $\boldsymbol{\varphi}^{*i} < \infty$. So by definition, $\psi_c^i, \psi_l^i < \infty$. Moreover, $\boldsymbol{\varphi}^{*i} > 0$ implies that $\lambda^i / \boldsymbol{\varphi}^{*i} < \infty$. Then by definition, $A_C, A_L < \infty$. $\square$

For any M and s^M, define (P^{s^M}) the same problem as (P) with the restriction that $(C(s^t), L(s^t)) = (C^*(s^t), L^*(s^t))$, $\forall t > M$, $s^t \supset s^M$, $\boldsymbol{\varphi} = \boldsymbol{\varphi}^*$, $T = T^*$, and $K_t = K_t^*, \forall t$. Note that $\{C^*(s^t), L^*(s^t), K^*(s^t)\}$ is a solution to (P^{s^M}), and (P^{s^M}) has a finite number of constraints. By a Lagrangian theorem in Luenberger (1969), there exists non-negative, not-identically zero vector $\left\{r^{s^M}, \mu^{s^M}, \eta^{s^M,1}, \ldots, \eta^{s^M,I}, \gamma^{s^M}(s^0), \ldots, \gamma^{s^M}(s^M)\right\}$ such that the first-order and complementarity conditions hold for $t \in \{1, .., M\}, s^t \subseteq s^M$, i.e.

$$(\beta R^*)^t \left\{ r^{s^M} A_C + \sum_i \pi^i \eta^{s^M,i}(1-\sigma)\psi_c^i + \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) A_C \right\} \Phi_C^V C(s^t)^{-\sigma} = \mu^{s^M} \tag{C.5}$$

$$(\beta R^*)^t \left\{ r^{s^M} A_L + \sum_i \pi^i \eta^{s^M,i}(1+\nu)\psi_l^i + \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) A_L \right\} \Phi_L^V L(s^t)^{\nu} = \mu^{s^M} F_L(K(s^t), L(s^t), s^t) \tag{C.6}$$

Equation (C.5) can be rewritten as

$$(\beta R^*)^t \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) = \frac{\mu^{s^M}}{A_C \Phi_C^V C(s^t)^{-\sigma}} - (\beta R^*)^t \left[r^{s^M} + \frac{1}{A_C} \sum_i \pi^i \eta^{s^M,i}(1-\sigma)\psi_c^i \right] \tag{C.7}$$

The following lemma shows that μ^{s^M} and $C(s^t)^{-\sigma}$ are always positive for the sub-problem (P^{s^M}) for any $M \geq 1$ and any s^M.

Lemma D.3. *In the sub-problem (P^{s^M}) for any $M \geq 1$ and s^M, $\mu^{s^M} > 0$*

Proof. Suppose, by contradiction, that $\mu^{s^M} = 0$ so the resource constraint does not bind. Consider allocation $\{C(s^t), L(s^t), K(s^t)\}$ which is the solution to (P^{s^M}). Then there exists $\epsilon > 0$ such that

$$\sum_{t\geq 0, s^t} q(s^t) \left[F(L(s^t), s^t, t) - G(s^t, t) - C(s^t) \right] - B(s^0) - \epsilon \geq 0$$

Define $\left\{\hat{L}(s^t)\right\}$ such that for a fixed s^1, $\hat{L}(s^1) < L(s^1)$ such that $F(\hat{L}(s^1), s^1, 1) = F(L(s^1), s^1, 1) - \epsilon / q(s^1)$, and $\hat{L}(s^t) = L(s^t)$, $\forall t > 1, \forall s^t$. The allocation $\left\{C(s^t), \hat{L}(s^t)\right\}$ satisfies the resource constraint and because of the preference's strict monotonicity, $\left\{C(s^t), \hat{L}(s^t)\right\}$ also satisfies the implementability constraints and the aggregate debt constraints. However,

$$\sum_{i\in I}\lambda^i\pi^i\sum_{t\geq 0,s^t}\beta^t\Pr(s^t)U^i\left[h^i(C(s^t),\hat{L}(s^t);\varphi)\right] > \sum_{i\in I}\lambda^i\pi^i\sum_{t\geq 0,s^t}\beta^t\Pr(s^t)U^i\left[h^i(C(s^t),L(s^t);\varphi)\right]$$

which contradicts $\{(C(s^t), L(s^t))_{s^t}\}_{t=0}^{\infty}$ being optimal solution for (P^{s^M}). □

The consumption path is bounded below by zero in the long run, i.e.

Lemma D.4 (No immiseration)**.** *Suppose Assumptions 1 and 4 hold, then for any efficient allocation* $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$, $\liminf_{t\to\infty} C_t^* > 0$.

Proof. Given an efficient allocation $\{C^*(s^t), L^*(s^t)\}$, suppose, by contradiction that for a sequence of shocks $\{s_0, ..., s_t, ...\}$, $\liminf_{t\to\infty} C^*(s^t) \leq 0$. Find $\epsilon > 0$ such that $\forall t, \forall s^t$,

$$\sum_{k=t}^{\infty}\beta^{\tau-t}\sum_{s^t\subseteq s^k}Pr(s^\tau)\left[\Phi_C^V\frac{C(s^k)^{1-\sigma}}{1-\sigma}-\Phi_L^V\frac{L^*(s^k)^{1+\nu}}{1+\nu}\right]\leq M_U$$

with $C(s^t) = \epsilon$ and $C(s^k) = C^*(s^k)$, $\forall k > t,\ s^t \subset s^k$. Such ϵ exists since the utility function is unbounded below. Because $\liminf_{t\to\infty} C^*(s^t) \leq 0$, there exists a t_0 such that $C^*(s^{t_0}) < \epsilon$. Then by monotonicity,

$$\begin{aligned}&\sum_{k=t_0}^{\infty}\beta^{\tau-t_0}\sum_{s^{t_0}\subseteq s^k}\Pr(s^k)\left[\Phi_C^V\frac{C^*(s^k)^{1-\sigma}}{1-\sigma}-\Phi_L^V\frac{L^*(s^k)^{1+\nu}}{1+\nu}\right]\\ <&\sum_{k=t_0}^{\infty}\beta^{\tau-t_0}\sum_{s^{t_0}\subseteq k}\Pr(s^k)\left[\Phi_C^V\frac{C(s^k)^{1-\sigma}}{1-\sigma}-\Phi_L^V\frac{L^*(s^k)^{1+\nu}}{1+\nu}\right]\\ \leq& M_U\\ \leq& \underline{U}(s^{t_0}, t_0)\end{aligned}$$

which contradicts the aggregate debt constraint at s^{t_0}. □

Taking the limit on both sides of equation (C.7) gives

$$\begin{aligned}\lim_{t\to\infty}(\beta R^*)^t\sum_{k=0}^{t}\sum_{s^k\subseteq s^M}\gamma^{s^M}(s^k) &= \lim_{t\to\infty}\left\{\frac{\mu^{s^M}}{A_C\Phi_C^V C(s^t)^{-\sigma}}-(\beta R^*)^t\left[r^{s^M}+\frac{1}{A_C}\sum_i\pi^i\eta^{s^M,i}(1-\sigma)\psi_c^i\right]\right\}\\ &= \lim_{t\to\infty}\frac{\mu^{s^M}}{A_C\Phi_C^V C(s^t)^{-\sigma}}\\ &> 0\end{aligned}$$

$\square$

E Computational Appendix

This section provides additional details that is implemented in Section 5 .

E.1 Deviation Utility

The deviation utility $\underline{U}(z)$ is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes of a closed economy where the government does not issue both domestic and external debts.

$$\underline{U}(z) \equiv \max_{c^i(s^t),l^i(s^t),\tau^n(s^t),T(s^t)} \sum_{t=0}^{\infty} \beta^t \sum_{i\in I} \lambda^i \pi^i \mathbb{E}_z U^i(c^i(s^t), l^i(s^t))$$

$$\begin{aligned}
s.t. \qquad & C(s^t) + G = z(s^t)L(s^t) \\
& c^i(s^t) + \sum_{s_{t+1}} Q(s_{t+1}|s^t) b^{d,i}(s^{t+1}) = (1-\tau^n(s^t))z(s^t)l^i(s^t) + b^{d,i}(s^t) - T(s^t) \\
& (1-\tau^n(s^t))z(s^t) = -\frac{U^i_l(c^i(s^t), l^i(s^t)}{U^i_c(c^i(s^t), l^i(s^t)} \\
& Q(s_{t+1}|s^t) = \beta \Pr(s^{t+1}|s^t) \frac{U^i_c(c^i(s^{t+1}), l^i(s^{t+1}))}{U^i_c(c^i(s^t), l^i(s^t))} \\
& b^{d,i}(s^0) = b^{d,j}(s^0), \ \forall t \geq 1, \sum_{i\in I} b^{d,i}(s^t) = 0 \\
& z(s^0) = z
\end{aligned}$$

There exist a vector of market weights $\hat{\boldsymbol{\varphi}}$ that satisfies the conditions in Proposition 3.1 such that

$$\begin{aligned}
\underline{U}(z) \equiv \max_{C(s^t),L(s^t),\hat{\boldsymbol{\varphi}},T} & \sum_{t=0}^{\infty} \beta^t \mathbb{E}_z \left[\hat{\Phi}^W_C \log C(s^t) - \hat{\Phi}^W_L \frac{L(s^t)^{1+\nu}}{1+\nu} \right] \\
s.t. \qquad & C(s^t) + G = z(s^t)L(s^t) \\
& z(s^0) = z
\end{aligned}$$

where $\hat{\psi}^i_c, \hat{\psi}^i_l, \hat{\Phi}^V_C, \hat{\Phi}^V_L, \hat{\Phi}^W_C, \hat{\Phi}^W_L$ are calculated using $\hat{\boldsymbol{\varphi}}$ (see Appendix C for the formulas).

E.2 Computational Algorithm

1. Guess μ and φ. Compute η.

   (a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ
   Initial guess for $V(s_t, \mu_t, \Gamma_{t-1}) = \sum_{j\geq 0, s^t \subseteq s^{t+j}} \beta^j \Pr(s^{t+j}) \left[\Phi_C^P \frac{C(s^{t+j})^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^{t+j})^{1+\nu}}{1+\nu}\right]$.

   (b) Assume the constraint does not bind in s_t: $\gamma(s_t) = 0$. Solve for the allocation $C(s_t), L(s_t)$ using the first-order conditions

$$\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1}\right] C(s_t)^{-\sigma} = \mu$$
$$\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1}\right] L(s_t)^{\nu} = \mu F_L(s_t)$$

   (c) Since $\gamma(s_t) = 0$, compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$ (interpolating the expectation), then compute

$$\begin{aligned} A(s_t) &= \sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^\tau \subseteq s^t} Pr(s^\tau) \left[\Phi_C^P \frac{C(s^\tau)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^\tau)^{1+\nu}}{1+\nu}\right] \\ &= \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1}) \end{aligned}$$

   (d) Check if $A(s_t) \geq \underline{U}(s_t)$. If it is, proceed to the next step. If not, solve for $C(s_t), L(s_t), \gamma(s_t)$ using these equations

$$\begin{aligned} \left[\mu_t \Phi_C^W + \Phi_C^V \left(\Gamma_{t-1} + \gamma(s_t)\right)\right] C(s_t)^{-\sigma} &= \mu \\ \left[\mu_t \Phi_L^W + \Phi_L^V \left(\Gamma_{t-1} + \gamma(s_t)\right)\right] L(s_t)^{\nu} &= \mu F_L(s_t) \\ \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} & \\ +\beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V\left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t)\right)\right) &= \underline{U}(s_t) \end{aligned}$$

   (e) Given $C(s_t), L(s_t), \gamma(s_t)$($\gamma$ can be zero or not), compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \Gamma_{t-1} + \gamma(s_t))$. Update the value function

$$\begin{aligned} V^{n+1}(s_t, \Gamma_{t-1}) &= \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} \\ &\quad + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V^n\left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t)\right)\right) \end{aligned}$$

2. Compute residuals to find μ and φ

$$r^{\mu} = \sum_{t \geq 0, s^t \in S^t} q^*(s^t) \left[F(L(s^t), s^t) - G(s^t) - C(s^t) \right] - B(s^0)$$

$$r^{\varphi} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^V \left(\psi_c^i - \psi_c^j \right) C(s^t)^{1-\sigma} - \Phi_L^V \left(\psi_l^i - \psi_l^j \right) L(s^t)^{1+\nu} \right]$$

3. Find μ and φ such that $r^{\mu} = 0$ and $r^{\varphi} = 0$.

F Data

F.1 Data sources

Most data are annual series covering the 1985-2015 period. Some variable samples cover the 2002-2015 period.

- Net foreign liability: negative of net foreign asset (NFA) from the External Wealth of Nations Database, Lane and Milesi-Ferretti (2018)
- Pre-tax Gini Index: Market Gini from the Standardized World Income Inequality Database, Solt (2019).
- GDP per capita: GDP per capita (Constant 2010 US Dollars) series from World Development Indicator Database (2019)

F.2 Lists of countries

- List of all countries in the data set:

Afghanistan, Albania, Algeria, Angola, Antigua and Barbuda, Argentina, Armenia, Australia, Austria, Azerbaijan, Bahrain, Bangladesh, Barbados, Belarus, Belgium, Belize, Benin, Bhutan, Bolivia, Bosnia and Herzegovina, Botswana, Brazil, Bulgaria, Burkina Faso, Burundi, Cabo Verde, Cambodia, Cameroon, Canada, Central African Republic, Chad, Chile, China, Colombia, Comoros, Congo, Costa Rica, Côte d'Ivoire, Croatia, Cyprus, Czech Republic, Dem. Rep. Congo, Denmark, Dominica, Dominican Republic, Ecuador, Egypt, Arab Rep., El Salvador, Equatorial Guinea, Estonia, Ethiopia, Fiji, Finland, France, Gabon, Gambia, The, Georgia, Germany, Ghana, Greece, Grenada, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, Hong Kong SAR, China, Hungary, Iceland, India, Indonesia, Iran, Islamic Rep., Iraq, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kazakhstan, Kenya, Kiribati, Korea, Kosovo, Kuwait, Kyrgyz Republic, Lao PDR, Latvia, Lebanon, Lesotho, Libya, Lithuania, Luxembourg, Madagascar, Malawi, Malaysia, Maldives, Mali, Malta, Mauritania, Mauritius, Mexico, Micronesia, Fed. Sts., Moldova, Mongolia, Montenegro, Morocco, Mozambique, Myanmar, Namibia, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Oman, Pakistan, Palau, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Qatar, Romania, Russian Federation, Rwanda, Samoa, Sao Tome and Principe, Saudi Arabia, Senegal, Serbia, Seychelles, Sierra Leone, Singapore, Slovak Republic, Slovenia, Solomon Islands, South Africa, South Sudan, Spain, Sri Lanka, St. Kitts and Nevis, St. Lucia, Sudan, Suriname, Sweden, Switzerland, Tajikistan, Tanzania, Thailand, Timor-Leste, Togo, Tonga, Trinidad and Tobago, Tunisia, Turkey, Turkmenistan, Tuvalu, Uganda, Ukraine, United Arab Emirates, United Kingdom, United States, Uruguay, Uzbekistan, Vanuatu, Venezuela, RB, Vietnam, Yemen, Rep., Zambia, Zimbabwe.

- List of advanced and emerging market economies:

Argentina, Australia, Austria, Belgium, Bolivia, Brazil, Bulgaria, Canada, Chile, China, Colombia, Costa Rica, Denmark, Dominican Republic, Ecuador, El Salvador, Finland, France, Germany, Ghana, Greece, Hungary, Iceland, India, Indonesia, Ireland, Italy, Japan, Kenya, Korea, Malaysia, Mexico, Netherlands, Nigeria, Norway, Panama, Peru, Philippines, Poland, Portugal, Romania, Russian Federation, Singapore, South Africa, Spain, Sri Lanka, Sweden, Switzerland, Thailand, Tunisia, Turkey, United Kingdom, United States, Uruguay, Venezuela.