

Sovereign Debt Sustainability and Redistribution*

Monica Tran-Xuan[†]

October 2022

Abstract

This paper proposes a theory of sovereign debt sustainability based on the government's motive for redistribution. I study a small open economy model with income inequality, distortionary taxes, and endogenous borrowing constraints because the government lacks commitment. Having access to external financing allows the government to reduce the distortionary cost of redistribution by redistributing wealth via domestic financial markets. Therefore, default is endogenously costly in the presence of inequality. Quantitatively, the theory accounts for the external debt's recent buildup in Italy and is consistent with the positive correlation between pre-tax income inequality and external debt across countries. The optimal austerity policies are increasing external borrowing and redistribution in the short run while reducing redistribution to repay debt in the long run.

Keywords: Inequality; Limited commitment; Optimal taxation; Redistribution; Sovereign debt

JEL Classifications: E62; F38; F41; H21; H23; H63

*This is a revised version of the second chapter of my dissertation at the University of Minnesota. I thank V.V. Chari, Larry Jones, and Manuel Amador for their invaluable advice and guidance. I am grateful for comments from Fernando Arce, Cristina Arellano, David Argente, Anmol Bhandari, Jarda Borovicka, Amy Handlan, Jonathan Heathcote, Tim Kehoe, Ben Malin, Chris Phelan, David Rahman, Paulina Restrepo-Echavarria, Aldo Rustichini, Jan Werner, the participants at the University of Minnesota, the Federal Reserve Bank of Minneapolis, the Federal Reserve Bank of St. Louis, NTA, Dehli School of Economics Winter School, and CeMent. This research is supported by the Center for Computational Research at the University at Buffalo.

[†]Department of Economics, University at Buffalo. 425 Fronczak Hall, Buffalo NY 14260. Email: monica@buffalo.edu

Introduction

The recent European debt crises have prompted intense policy debates on the design of fiscal policies during severe economic downturns, in which output is low and external debt rapidly increases until it is constrained by the country's inability to repay. Austerity policies such as increasing tax revenue or reducing government expenditure provide more resources to repay debt but have unequal effects on domestic residents.¹ Therefore, appropriate policy design requires understanding the interaction between a government's commitment to debt repayment and its commitment to maintain a level of redistribution. Two key questions arise: How does a government's redistributive goals affect its incentive to repay debt? Given the answer to the first question, how does one design optimal austerity policies taking their distributional consequences into account?

To address these questions, I study a small open economy model in which redistribution comes with an efficiency cost in terms of labor tax distortion and the government faces endogenous borrowing constraints due to its lack of commitment. Given these constraints, the cost of financial autarky endogenously determines the sustainable level of external debt. I examine the trade-off between the government's desire for redistribution and its ability to sustain external debt and evaluate optimal responses of fiscal policies to aggregate shocks in the presence of inequality.

I develop a theory of external debt sustainability based on the government's motive for redistribution. When the government can borrow externally, it levies low distortionary labor taxes by redistributing wealth to domestic households via domestic financial markets. Default resulting in financial autarky is endogenously costly because, without external borrowing, redistribution requires high distortionary taxes, which reduce the economy's efficiency. Therefore, the government is willing to repay its external obligations in order to continue having access to external financial markets. I show that this theory can quantitatively account for the recent buildup of external debt in Italy and is consistent with the positive correlation between pre-tax income inequality and external debt across countries and over time. The optimal austerity policies responding to a negative productivity shock are increasing external borrowing and increasing redistribution in the short run. In the long run, it's optimal to reduce redistribution to repay debt.

As an empirical motivation, I first document the cross-country and time series properties of pre-tax income inequality and external debt using two multi-country panel data sets on inequality and balance of payments. First, highly indebted countries also experience high levels of pre-tax income inequality. Second, the increase in the country's net financial outflows coin-

¹The United Kingdom and Ireland implemented expenditure cuts, while Greece, Italy, Portugal, and Spain implemented both tax increases and expenditure cuts for their austerity plans. Most of these plans include cuts in public services, pension programs, and education programs. [Monastiriotis \(2011\)](#) argues that, in Greece, the prolonged fiscal consolidation has exacerbated regional disparities and imbalances. [Leventi and Matsaganis \(2016\)](#) use a micro-simulation model to assess the distributional effects of austerity policies and find that such policies have led to higher poverty and after-tax income inequality, worsening the adverse distributional effects of the recession. [Brinca, Homem Ferreira, Franco, Holter, and Malafray \(2019\)](#) show that fiscal consolidations are more recessive when income inequality is higher.

cides with an increase in aggregate pre-tax income inequality. The regression estimation shows that this positive relationship is robust to changes in output levels and output growth.² These facts point to a connection between a government's redistributive goals and its external debt management.

The model features a continuum of domestic agents that are impatient and differ by labor productivity types. The aggregate shocks are in aggregate productivity and government spending.³ Domestic and external credit markets consist of state-contingent assets. The tax system has lump-sum taxes as well as marginal taxes that are distortionary to individual labor supply and saving decisions. The government cares about all domestic agents, assigning individual welfare weights that represent its distributional preference. The government lacks commitment in all tax and debt policies.

Concerns for redistribution rationalize the need for distortionary taxation. Since all domestic agents face the same tax rates, the government redistributes resources by levying a positive labor tax alongside a lump-sum transfer. In this way, highly-skilled, high-income agents bear a larger tax burden than low-skilled, low-income ones. Alternatively, the government can use a tax on domestic borrowing and a lump-sum transfer, which implies that the highly indebted agents will pay more taxes than the less-indebted agents. In this environment, levels of tax distortions represent the cost of redistribution.⁴

I consider the sustainable equilibrium that is characterized by self-enforcing borrowing constraints due to the government's lack of commitment, as in [Aguiar and Amador \(2014, 2016\)](#). The government sets an ex ante set of policies such that for any given time period and history, if the government deviates from its equilibrium policies, it triggers a punishment to financial autarky, in which the government is permanently excluded from external credit markets. At the beginning of financial autarky, the government defaults on both domestic and external debt and fully redistributes wealth among domestic agents.⁵ The self-enforcing constraints are such that the continuation of value of staying in equilibrium has to be at least the value of financial autarky. These constraints impose endogenous limits on the economy's external borrowing. The equilibrium allocation determines the amount of external debt that the government can sustain without defaulting.

The main theoretical finding is that the government finds it optimal to sustain positive external debt in equilibrium. This result holds given that the government is inequality averse, initial

²The end of this section provides an overview of other papers that document similar and related empirical patterns. These papers include [Berg and Sachs \(1988\)](#), [Aizenman and Jinjark \(2012\)](#), [Jeon, and Kabukcuoglu \(2018\)](#), and [Ferriere \(2015\)](#) which use different measures and estimation techniques.

³Later on when I match the model to the data, government spending is measured as the government's final consumption of public goods, excluding spending on social welfare programs.

⁴This is because the government can raise lump-sum taxes to finance expenditures and debt repayment without distorting the domestic agents' decisions. [Werning \(2007\)](#) provides a similar intuition. The presence of lump-sum taxes removes the revenue purposes of distortionary taxation.

⁵This is equivalent to the assumption of nondiscriminatory defaults on domestic or external lenders. See, for example, [D'Erasmus and Mendoza \(2016\)](#) for a similar assumption. For an example on discriminatory defaults, see [Gonzalez-Aguado \(2018\)](#).

wealth inequality is positively correlated with skill inequality, and the intratemporal elasticity of substitution is at least above the log preference, and the elasticity of labor supply is at most one. Debt sustainability is optimal because it is costly for the government to default and faces financial autarky.⁶ Default is costly not only because the government cannot use debt to smooth consumption over the business cycles, but also because redistribution is more distortionary and less efficient in financial autarky than in the contract. The distributive cost of default is endogenous and novel to the literature, in contrast to the standard exogenous cost of default in terms of output or productivity losses.

Having access to external financing allows the government to redistribute more than in financial autarky. In the contract, future declines in labor taxes improve the economy's efficiency by encouraging higher output and allowing the government to borrow more in the present. This additional unit of resource implies higher present consumption and so higher welfare than financial autarky. Furthermore, the contract exhibits a lower level of redistribution than in financial autarky at the same amount tax distortion in the long run.

The theory is quantitatively consistent with salient features of the data. Using Italy's data, I calibrate the model to match key macroeconomic statistics and average cross-sectional wage inequality and find that the model accounts for the average level and upward trend of the external debt-to-output ratio in Italy for the period 2002-2015, while also being consistent with key business cycle statistics. The model also accounts for the positive association between pre-tax income inequality and external debt across countries. Specifically, I perform a regression analysis on a sample of simulated economies differentiated only by wage inequalities and find a statistically significant and positive correlation of pre-tax income Gini index and external debt-to-output. Moreover, a counterfactual exercise for Italy during the periods 1985-2001 and 2002-2015 exhibits that an increase in the underlying wage inequality that matches the increase in income inequality can account for 71% of the increase in the average debt-to-output ratio. These findings are consistent with the theory's prediction that a higher level of inequality, or a higher redistributive motive, corresponds to a higher cost of default, which results in a higher sustainable debt level.

To quantify the role of the government's redistributive motive and the associated distortionary cost, I compare the baseline model to a model of no inequality and a model with skill-dependent lump-sum taxes. I find that in the absence of inequality and the concern for redistribution, the no-inequality model quantitatively generates much smaller amount of external debt comparing to the data. In skill-dependent-lump-sum-tax model, the government can achieve perfect redistribution without distortionary cost. Default is not as costly as before, and so the government does not sustain large amount of external debt.

I then study optimal austerity in the presence of inequality through the lens of optimal policy response to shocks. Following a negative innovation of the productivity shock, external debt

⁶In some works in the literature, default means not repaying debt. In this model, I use the term default as the case in which the government does not honor any terms of the contract, either in debt repayment or taxes.

increases while utility differences among agents decline initially and increase in subsequent periods. More external borrowing allows higher transfers to individuals and more redistribution, while higher taxes and lower redistribution are needed in the future to repay debt. In contrast, the representative agent model only borrows a small amount in response to the negative productivity shock.

Endogenous borrowing constraints are essential to the theory because they link the government's external borrowing decisions to its concern for redistribution and tax distortions through the value of financial autarky. If the borrowing constraint is exogenous, given impatience, the only optimal policy is to borrow up to the exogenous debt limit, regardless of the desired level of redistribution or inequality.

The model builds on state-contingent financial markets and features no equilibrium default, in contrast to the sovereign default model.⁷ However, the threat of default still affects the optimal allocation, as in [Thomas and Worrall \(1988\)](#) and [Kehoe and Levine \(1993\)](#). The endogenous borrowing constraints imply that there is imperfect insurance against aggregate risk. These constraints also endogenously determine the optimal debt portfolio, in contrast to the incomplete framework in which the risk-free bond is the only financial asset. Furthermore, defaults in reality often accompany a non-zero net capital flow as a country often goes through a lengthy process of renegotiation and haircuts.⁸ This framework embeds part of the default procedure into self-enforcing borrowing constraints, instead of assuming zero net capital flows, as in the standard sovereign default model.⁹

I last show that the amount of exogenous government expenditure does not quantitatively affect the external debt level. Aggregate uncertainty leads to precautionary saving that reduces the external debt level. Lower discount factor corresponds to lower external debt level because it implies a higher value of financial autarky and a shorter time period it takes for the economy to accumulate debt before reaching the borrowing constraints.

Related literature. This paper builds on the sovereign debt literature of limited commitment and state-contingent asset market that follows [Aguar and Amador \(2011, 2014\)](#). I present the quantitative predictions of this type of model, including highly volatile consumption and fiscal policies, which are similar with the findings in [Kehoe and Perri \(2002\)](#) for a two-country international real business cycle and [Bauducco and Caprioli \(2014\)](#) with two-sided limited commitment. I introduce heterogeneity and redistributive effect of fiscal policies in the literature that studies the government's lack of commitment in both tax and debt policies. The volatile tax and government expenditures are similar to [Cuadra, Sanchez, and Sapriza \(2010\)](#). Tax smoothing is

⁷See, for example, [Eaton and Gersovitz \(1981\)](#).

⁸Standard & Poor's defines default as the failure to meet a principal or interest payment on the due date contained in the original terms of a debt issue. This definition covers both missed payments (breach of contract) and distressed debt restructurings that involve losses for creditors. This is the standard default definition used in the literature (e.g., [Reinhart and Rogoff \(2009\)](#)). See [Ams, Baqir, Gelpert, and Trebesch \(2018\)](#) for a discussion on the pros and cons of different definitions of default.

⁹See [Restrepo-Echavarria \(2019\)](#) for a discussion on these issues.

absent in my model, similar to findings in [Pouzo and Presno \(2015\)](#) and [Arellano and Bai \(2016\)](#), in which higher tax distortion would make the country more likely to default.

This paper contributes to the literature of endogenous cost of default in sovereign debt model beyond insurance motive. In [Mendoza and Yue \(2012\)](#), it is the efficiency loss in production as default prevents the final good producers to finance the purchase of imports, which only have imperfect substitutes at home. [Balke \(2017\)](#) shows how default limits the supply of bank's loans that firms use to finance vacancies and wages. Therefore, default leads to a large increase in unemployment, which is endogenously costly. In this paper, distortionary taxation needed for redistribution plays an important role in determining the cost of default. Default is costly because of the high and volatile labor distortions need to redistribute in financial autarky.

This paper relates to the optimal taxation and debt management research of the public finance literature ([Barro \(1979\)](#), [Lucas and Stokey \(1983\)](#), [Chari, Christiano, and Kehoe \(1994\)](#), [Aiyagari and McGrattan \(1998\)](#), [Chari and Kehoe \(1999\)](#), [Aiyagari, Marcet, Sargent, and Seppälä \(2002\)](#), and many other papers). I find the optimal policy by characterizing the best allocation of any tax-distorted equilibrium, i.e., the primal approach. I relax the assumption on the government's commitment to policies in many papers that study trade-off between redistribution and debt management. I build upon the framework of optimal taxation with redistribution in [Werning \(2007\)](#), in which perfect tax smoothing occurs under the same assumptions. I show how the quantitatively high cost of default of redistribution lead to the high positive debt level in the long run, in contrast to [Bhandari, Evans, Golosov, and Sargent \(2016\)](#)'s finding that the average long-run on optimal public debt is not positive. Similar to [Bhandari, Evans, Golosov, and Sargent \(2017\)](#) which emphasize the importance of the distribution of initial asset holdings, I find that it affects the long-run debt repayment capacity via the equilibrium level of redistribution.

Several recent research studies the interaction between inequality and external debt. [Ferreire \(2015\)](#) assumes one-period commitment to tax progressivity and finds a related result that a more progressive economy finds a higher cost of default because redistribution is more distortionary in autarky. I focus on the long-run trade-off between inequality and external debt, in contrast to [Dovis, Golosov, and Shourideh \(2016\)](#) that argued how this trade-off endogenized the dynamic cycles of fiscal policies over time. The model features redistributive consequences of domestic defaults, as emphasized by [D'Erasmus and Mendoza \(2016, 2020\)](#). I incorporate this insight into linking redistributive incentives and the cost of default, which in turn affects the sustainable level of external debt. [Balke and Ravn \(2016\)](#) studies tax and debt policies from a Markov-perfect equilibrium with inequality through unemployment. [Deng \(2021\)](#) studies how inequality and migration affect sovereign default risk in a Markov-perfect equilibrium. This paper focuses on the sustainable equilibrium characterized by self-enforcing constraints. The equilibrium determines the amount of external debt that the government can sustain without defaulting.

Other papers have documented a positive relationship between income inequality and sovereign debt. [Berg and Sachs \(1988\)](#) show that income inequality is a key predictor of a coun-

try's probability of rescheduling debt and the bond spread in secondary markets. [Aizenman and Jinjark \(2012\)](#) describe a negative correlation between income dispersion and the tax base and a positive correlation with sovereign debt. [Jeon and Kabukcuoglu \(2018\)](#), [Ferriere \(2015\)](#), and [Deng \(2021\)](#) also provide evidence that rising income dispersion significantly increases sovereign default risk. This paper extends the panel analysis to a more recent data set on income inequality and provides a theory that can collectively account for the increase in debt and income dispersion.

Outline. The paper is organized as follows. Section 1 documents the relationship between income inequality and external debt. Section 2 describes the environment and sets up the competitive equilibrium. Section 3 formulates the planning problem and the main theoretical results of optimal debt sustainability. Section 4 provides a quantitative analysis and analyzes optimal austerity policies. Section 5 provides a sensitivity analysis, and Section 6 concludes.

1 Empirical Motivation

This section presents the empirical relationship between income inequality and external debt. I document that income inequality is positively correlated with external debt in both the cross section and time series. To measure a country's external indebtedness, I use the negative of the net foreign asset-to-GDP ratio from the External Wealth of Nations Database of [Lane and Milesi-Ferretti \(2018\)](#).¹⁰ The database contains data on foreign assets and foreign liabilities for a large sample of countries for the period 1970-2015. For income inequality, I use pre-tax (market) Gini indices from Standardized World Income Inequality Database (SWIID) of [Solt \(2019\)](#) that covers from 1960 to 2018.

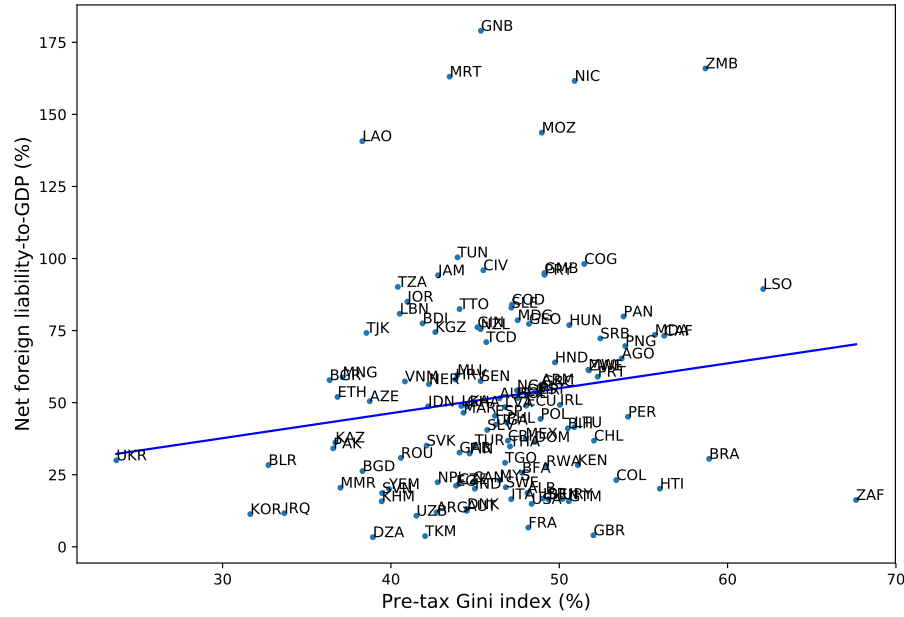
Fact 1: Countries with high pre-tax income inequality also experience high external debt levels. Figure 1 plots averages across the period of 1985-2015 of net foreign liability-to-GDP and pre-tax Gini index for 120 countries.¹¹ The figure establishes a positive correlation between income inequality and external debt across countries.

Table 1 reports the regression results of net foreign liability-to-GDP on pre-tax Gini index, reported as averages across 1985-2015. The results show a statistically significant positive correlation between income inequality and external debt.

¹⁰The net foreign asset (NFA) position of a country is the value of the assets that country owns abroad, minus the value of the domestic assets owned by foreigners, adjusted for changes in valuation and exchange rates. A different measure of a country's external position is the net international investment position (NIIP), which is the difference between a country's stock of foreign assets and foreigner's stock of that country's assets. See Appendix A.4 for the estimation using NIIP.

¹¹Given the focus on external debt and inequality, I study countries that have the population in 1985 over one million people and the net foreign liability above zero. See Appendix A.3 for the list of those countries.

Figure 1: Income inequality and external debt across countries



Note: The graph shows the 1985-2015 time averages of net financial liability-to-GDP and pre-tax Gini index for all countries in the data set. Sources: [Lane and Milesi-Ferretti \(2018\)](#), [Solt \(2019\)](#), and [The World Bank \(2019\)](#).

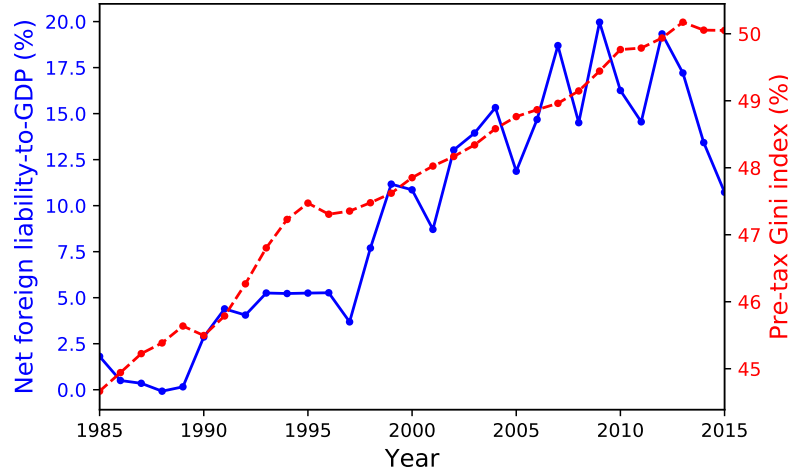
Table 1: Regression analysis of income inequality and external debt

	Dependent Variable: Net foreign liability-to-GDP (%) Averages across 1985-2015	
	(1)	(2)
Gini index, pre tax (%)	0.865* (0.521)	0.968** (0.487)
Controls	No	Yes
No. Countries	120	120

Note: The table shows the regression coefficient and standard error in parenthesis of pre-tax Gini index (%) with respect to net foreign liability-to-GDP (%). Control variables are log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). *, ** represent significant levels of 10% and 5%, respectively. Sources: [Lane and Milesi-Ferretti \(2018\)](#), [Solt \(2019\)](#), and [The World Bank \(2019\)](#).

Fact 2: Increases in pre-tax income inequality coincides with increases in external debt in the long run for most countries. Figure 2 plots the GDP-weighted average of net financial liability-to-GDP and pre-tax Gini index for countries in the European Union. Over time, there are increasing trends in both income inequality and external debt. Income inequality has been rising over time across countries ([Alvaredo et al. \(2018\)](#)). At the same time, external debt is also increasing

Figure 2: Time series of income inequality and external debt



Note: The graph shows the GDP-weighted average of net financial liability-to-GDP and pre-tax Gini index for countries in the European Union from 1985 to 2015. Sources: [Lane and Milesi-Ferretti \(2018\)](#) and [Solt \(2019\)](#).

across many countries. The 2007-2009 financial crises contributed to the increase in borrowing across countries, particularly across European ones.¹²

2 A Model of Sovereign Debt and Inequality

In this section, I setup the model of a small open economy with aggregate uncertainty, heterogeneous agents, and a benevolent government. I define the competitive equilibrium given government policies and show that the competitive equilibrium can be characterized by a set of aggregate allocation and a time-invariant distribution of marginal utility shares.

2.1 Environment

A small open economy faces publicly observed aggregate shocks $s_t \in S$ in period t , where S is some finite set. Let $\Pr(s^t)$ denote the probability of any history $s^t = (s_0, s_1, \dots, s_t)$, where $\Pr(s^{t+j}|s^t)$ denotes the probability conditional on history s^t , $j \geq 0$. Similarly, $\Pr(s_{t+1}|s^t)$ is the probability period $t+1$'s state is s_{t+1} , conditional on history s^t . When it does not cause confusion, I use x_t to denote a random variable with a time t for all s^t .

There is a measure-one continuum of infinitely-lived agents different by labor productivity types $(\theta^i)_{i \in I}$, which are publicly observable. The fraction of agents with productivity θ^i is π^i , where $(\pi^i)_{i \in I}$ and $(\theta^i)_{i \in I}$ are normalized such that $\sum_{i \in I} \pi^i = 1$ and $\sum_{i \in I} \pi^i \theta^i = 1$. All agents have the same discount factor β and the static utility $U(c, n)$ over consumption c and hours worked n .

¹²[Reinhart and Rogoff \(2010\)](#) reported large increases in public debt across countries, especially in the period 2007-2009. External debt levels were particularly high among European countries.

The utility of agent with productivity θ^i over consumption $c_t^i \geq 0$ and efficiency-unit labor $l_t^i \geq 0$ is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right] \quad (1)$$

where $\sigma, \nu > 0$.

In addition, there is a representative firm that uses labor to produce a single final good. The production function in period t with history s^t is $F(L, s^t, t)$, constant returns to scale, where L is aggregate labor (in efficiency units). The economy is subject to an exogenous sequence of government spending $\{G_t\}_{t=0, s^t \in S^t}^{\infty}$. Both the production function and government expenditures depend on the time period t , capturing deterministic changes such as growth, and the history s^t , capturing the uncertainty impact.

An individual allocation specifies consumption and labor in every period after every history for each agent i : $\{(c_t^i, l_t^i)\}_t$. The aggregates are denoted by $C_t \equiv \sum_{i \in I} \pi^i c_t^i$ and $L_t \equiv \sum_{i \in I} \pi^i l_t^i$.

The exogenous risk-free international interest rate for borrowing is r^* . Both the domestic and international financial markets are competitive. The government can issue domestic debt from a full set of state-contingent bonds, which can be traded across agents. The government also have access to a full set of state-contingent external bonds. Let $R^* = 1 + r^*$ denote the gross risk-free interest rate. Define $Q_t(s_{t+1}) = \Pr(s_{t+1}|s^t)/R^*$ as the international price of one unit of consumption at state s_{t+1} in period $t+1$, conditional on history s^t , in units of consumption at history s^t . Similarly, $q_t = \Pr(s^t)/(R^*)^t$ is the international price of one unit of consumption at history s^t in units of consumption at s^0 . Let normalize $q_0 = 1$.¹³ Note that $q_{t+1} = Q_t(s_{t+1})q_t$. I assume that only the government can borrow abroad.¹⁴

2.2 Competitive Equilibrium

In every period t and history s^t , the government can issue both domestic and foreign bonds and impose a lump-sum tax T_t , a marginal tax on labor income τ_t^n , and a tax on the return of domestic saving τ_t^d . The firm and agents face the labor wage $w(s^t)$.

Domestic agent. Individual agent of type $i \in I$ faces the sequential budget constraint

$$c_t^i + \sum_{s_{t+1}|s_t} Q_t^d(s_{t+1}) b_{t+1}^{d,i} \leq (1 - \tau_t^n) w_t l_t^i + (1 - \tau_t^d) b_t^{d,i} - T_t, \quad (2)$$

¹³This normalization is without loss of generality since the initial level of external debt is fixed.

¹⁴In the data, domestic residents often hold a very small amount of foreign assets, so most models assume that they do not have access to the external credit market. In this environment, the set up is equivalent to the case where the domestic agents can save abroad with the bond price $Q^*(s^t)$, but then face a residence-based tax $\tau^d(s^t)$. External debt will be the net foreign liability of both the private and public sectors, instead of only the public sector here. I choose this particular set up so that it is more straightforward to characterize the strategic game later on in Appendix B.

where $c_t^i, l_t^i, b_t^{d,i}$ denote the consumption, labor, and domestic bond holding of agent i in period t and history s^t , respectively. $Q_t^d(s_{t+1})$ is the price of one unit of domestic asset for realization s_{t+1} in period $t + 1$ given history s^t .

Representative firm. The firm chooses aggregate labor to maximize profit

$$\max_{\{L_t\}_t} F(L_t, s^t, t) - w_t L_t,$$

which gives the following first-order condition

$$w_t = F_L(L_t, s^t, t). \quad (3)$$

The firm's profit is zero in equilibrium because of the constant-return-to-scale production function.

Government. There is an exogenous government expenditure $\{G_t\}_{t=0, s^t \in S^t}^\infty$. Given the one-period state-contingent domestic bond B_t^d and external bond B_t , the government's budget constraint is

$$G_t + (1 - \tau_t^d)B_t^d + B_t \leq \tau_t^n w_t L_t + \sum_{s_{t+1}|s^t} Q_t^d(s_{t+1})B_{t+1}^d + \sum_{s_{t+1}|s^t} Q_t(s_{t+1})B_{t+1} + T_t,$$

where $B_t^d = \sum_{i \in I} \pi^i b_t^{d,i}$ is aggregate domestic bond, and B_t is the amount of the government's external debt. There is a no-Ponzi condition such that the present value of external debt is bounded below.

The government's present-value budget constraint is

$$\sum_{t \geq 0, s^t} q_t \left\{ G_t - \tau_t^n w_t L_t - T_t + \sum_{s_{t+1}|s^t} Q_t(s_{t+1})B_{t+1}^d - (1 - \tau_t^d)B_t^d \right\} \leq B_0. \quad (4)$$

Resource constraint. Using the agent's budget constraints and government's budget constraint, one can obtain a present-value resource constraint in terms of the inter-temporal international prices and the initial external debt,

$$\sum_{t \geq 0, s^t} q_t [F(L_t, s^t, t) - G_t - C_t] \geq B_0. \quad (5)$$

Competitive equilibrium. Given the above equations, one can define the following competitive equilibrium with government policies.

Definition 2.1. Given initial external debt B_0 and individual individual bond positions $(b_0^{i,d})_{i \in I}$, a competitive equilibrium with government policies for an open economy is individual agent's

allocation $z^{H,i} = \left\{ (c_t^i, l_t^i, b_t^{i,d}) \right\}_{t=0}^{\infty}$, $\forall i \in I$, the representative firm's allocation $z^F = \{L_t\}_{t=0}^{\infty}$, prices $p = \{q_t, w_t, Q^d(s_{t+1}|s^t)\}_{t=0}^{\infty}$, and government's policy $z^G = \{\tau_t^n, \tau_t^d, T_t, B_t^d, B_t\}_{t=0}^{\infty}$ such that (i) given p and z^G , $z^{H,i}$ solves individual i 's problem that maximizes (1) subject to (2) and a no-Ponzi condition of agent's debt value, (ii) given p and z^G , z^F solves firm's problem, (iii) the government budget constraint (4) holds, (iv) the aggregate resource constraint (5) is satisfied, (iv) the domestic bond market clears $B_t^d = \sum_{i \in I} \pi^i b_t^{d,i}$, and (v) p satisfies $q_t = \Pr(s^t)/(R^*)^t$ and equation (3) given z^G .

2.3 Characterizing Competitive Equilibrium

In equilibrium, the intra-temporal and inter-temporal rates of substitution are the same across agents, i.e. in each period t and each history s^t , for any individual i ,

$$(1 - \tau_t^n)w_t = -\frac{1}{\theta^i} \frac{(l_t^i/\theta^i)^\nu}{(c_t^i)^{-\sigma}},$$

$$\frac{Q_t^d(s_{t+1})}{1 - \tau_t^d} = \beta \Pr(s_{t+1}|s^t) \frac{(c_{t+1}^i)^{-\sigma}}{(c_t^i)^{-\sigma}}.$$

Given the aggregate allocation (C_t, L_t) , there is an time-invariant efficient assignment of individual allocation $(c_t^i, l_t^i)_{i \in I}$ due to the equal marginal rates of substitution between consumption and labor. Any inefficiencies due to tax distortions are captured by the aggregate allocation. This property allows the competitive equilibrium allocation to be characterized in terms of aggregates and a static rule for individual allocation.

For any equilibrium, there exist a set of Neghishi weights $\varphi = (\varphi^i)_{i \in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i \varphi^i = 1$, such that individual allocation solve a static problem

$$V(C, L; \varphi) \equiv \max_{(c^i, l^i)_{i \in I}} \sum_{i \in I} \varphi^i \pi^i \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right]$$

$$s.t. \quad \sum_{i \in I} \pi^i c^i = C; \quad \sum_{i \in I} \pi^i l^i = L$$

This problem gives the allocation rule for individual i that is time-invariant proportional to the aggregate allocation

$$c_t^i = \psi_c^i C_t, \quad l_t^i = \psi_l^i L_t, \tag{6}$$

where

$$\psi_c^i = \frac{(\varphi^i)^{1/\sigma}}{\sum_{i \in I} \pi^i (\varphi^i)^{1/\sigma}}, \quad \psi_l^i = \frac{(\theta^i)^{\frac{1+\nu}{\nu}} (\varphi^i)^{-1/\nu}}{\sum_{i \in I} \pi^i (\theta^i)^{\frac{1+\nu}{\nu}} (\varphi^i)^{-1/\nu}}. \tag{7}$$

In addition, V inherits the separable and isoelastic properties,

$$V(C_t, L_t; \varphi) = \Phi_C^V \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L_t^{1+\nu}}{1+\nu},$$

where Φ_C^V, Φ_L^V depend on φ . (see Appendix D.1). The envelope conditions of the static problem give

$$(1 - \tau_t^n)w_t = \frac{\Phi_L^V L_t^\nu}{\Phi_C^V C_t^{-\sigma}}, \quad (8)$$

$$\frac{Q_t^d(s_{t+1})}{1 - \tau_t^d} = \beta \Pr(s_{t+1}|s^t) \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}. \quad (9)$$

Furthermore, the present-value budget constraint for individual i can be written as

$$\mathbb{E}_0 \sum_{t \geq 0} \beta^t \left(\Phi_C^V \psi_c^i C_t^{1-\sigma} - \Phi_L^V \psi_l^i L_t^{1+\nu} \right) = \Phi_C^V C_0^{-\sigma} (b_0^i - T), \quad (10)$$

where T is the present-value of lump-sum taxes.¹⁵ Equation (10) is the individual implementability constraint.

Then I have the following proposition in which the competitive equilibrium can be characterized by a set of aggregate allocation and a time-invariant distribution of marginal utility shares.

Proposition 2.1. *Given the initial external debt B_0 and individual bond holdings $\{b_0^i\}_{i \in I}$, an allocation $\{C_t, L_t\}_{t=0}^\infty$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if the resource constraint (5) holds, and there exist market weights $\varphi = (\varphi^i)_{i \in I}$ and lump-sum tax T such that the implementability constraint (10) holds for all $i \in I$.*

Proof. See Appendix. □

3 Optimal Debt Sustainability

3.1 Sustainable Equilibrium

I define the problem of a benevolent government that cares about redistribution but lacks commitment in both tax and debt policies. The solution to this problem is the sustainable allocation in which the government does not default on its debt.

Given a set of social welfare weights $\lambda = (\lambda^i)_{i \in I}$, the government's objective is the weighted utility of all domestic agents

$$^{15}T \equiv \sum_{t=0}^\infty \beta^t \sum_{s^t \in S^t} \frac{V_C[h^i(C(s^t), L(s^t)); \varphi]}{V_C[h^i(C(s_0), L(s_0)); \varphi]} T(s^t)$$

$$\sum_{i \in I} \lambda^i \pi^i \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right]. \quad (11)$$

However, in every period and history node, the government cannot commit to future choices on repayments of debt and taxes. Following [Chari, and Kehoe \(1990, 1993\)](#), the policies are determined in a repeated game between the government, a continuum of domestic agents, and a continuum of international creditors. The sustainable equilibrium supported by trigger strategies to autarky is characterized by the competitive equilibrium conditions described in [Proposition 2.1](#) and the following self-enforcing constraint

$$\sum_{i \in I} \lambda^i \pi^i \mathbb{E}_t \sum_{k \geq t} \beta^{k-t} \left[\frac{(c_k^i)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_k^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right] \geq \underline{U}_t(s^t, t), \quad \forall t, \forall s^t, \quad (12)$$

where $\underline{U}(s^t, t)$ is the one-shot deviation value in which the government defaults on both domestic and external debt and fully redistributes wealth among domestic agents.¹⁶ The government is then in financial autarky, in which it has no access to external financial markets. $\underline{U}(s^t, t)$ is the value associated with an allocation of a closed economy where the initial states are realized s_t at period t and history s^t , the initial wealth inequality among agents are equal, and the net supplies of domestic and international bonds are zero.

The self-enforcing constraint captures the time-inconsistency of the government policies. If there is a positive net external debt, the government has an incentive to default externally to increase domestic consumption and leisure. In addition, in every history node, there is a non-degenerate distribution of wealth across the domestic agents. The inequality-averse government will also have an incentive to expropriate all the wealth and equally redistribute it. The self-enforcing constraint imposes a limit on the utility, which endogenously determines a limit on external debt for every period and history. These constraints act as endogenous borrowing constraints.

Given the above set-up, a sustainable allocation is defined as follows

Definition 3.1. A sustainable allocation $(\{C_t, L_t\}_{t=0}^{\infty}, \varphi)$ maximizes the social welfare function [\(11\)](#) and satisfies the conditions in [Proposition 2.1](#) and the self-enforcing borrowing constraint [\(12\)](#)

The objective of the government can be rewritten in terms of aggregate allocation and the Negishi weights

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} \right),$$

where Φ_C^P, Φ_L^P are functions of λ and φ (see [Appendix D.1](#)).

¹⁶See [Appendix B](#) for the formal set up of the sovereign game and its equilibrium characterization.

The sustainable allocation is part of the solution to the following planning problem

$$\begin{aligned}
(P) \equiv & \max_{\{C_t, L_t\}_t, \varphi, T} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} \right) \\
s.t. & \sum_{t \geq 0, s^t} q_t \left[F(L_t, s^t, t) - G_t - C_t \right] \geq B_0 \\
& \forall i, \mathbb{E}_0 \sum_{t \geq 0} \beta^t \left(\Phi_C^V \psi_c^i C_t^{1-\sigma} - \Phi_L^V \psi_l^i L_t^{1+\nu} \right) = \Phi_C^V C_0^{-\sigma} (b_0^i - T) \\
& \forall t, \forall s^t, \sum_{i \in I} \lambda^i \pi^i \mathbb{E}_t \sum_{k \geq t} \beta^{k-t} \left[\Phi_C^P \frac{C_k^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_k^{1+\nu}}{1+\nu} \right] \geq \underline{U}_t(s^t, t).
\end{aligned}$$

The first constraint is the resource constraint. The second constraint is the implementability constraints that take into account the distortionary effect of the government's policies on individual decisions. The last constraint is the borrowing constraint due to the government's lack of commitment. Domestic agents do not directly internalize the effect of their borrowing decisions on these borrowing constraints. The government, on the other hand, has to consider these constraints when choosing optimal allocation and policies. Therefore, the borrowing constraints indirectly affect domestic borrowing choices via the government's decision on domestic saving taxes.

Appendix C provides the characterization of the sustainable allocation and optimal tax policies that implement the sustainable allocation in competitive equilibrium.

3.2 Optimal Debt Policy

I now argue that the optimal policy is to sustain positive external debt in equilibrium under the following assumptions

Assumption 1 (Impatience). *There exists $0 < \mathcal{M} < 1$ such that $\beta R^* < \mathcal{M} < 1$.*

Assumption 2. *The welfare weights, skill distribution, and initial wealth satisfy the following properties*

1. *Redistributive motive towards the low skills: $\theta^i < \theta^j \iff \lambda^i > \lambda^j, \forall i, j \in I$*
2. *Perfect correlation between skill and initial wealth: $\theta^i < \theta^j \iff b_0^i < b_0^j, \forall i, j \in I$*
3. *Elasticities of substitution and labor supply are such that $\sigma \geq 1$ and $1/\nu \leq 1$*

The first assumption implies that the domestic agents are impatient, and therefore, there is a need for debt accumulation. The country will accumulate debt over time, as the international interest rate is lower than the domestic intertemporal rate. For the second set of assumptions, the first part is on the welfare weights which states that the government has a high motive of redistribution towards the lower skill, lower income individuals. The government is inequality

averse. The second part makes sure that the direction of inequality in skill is the same as the direction of inequality in initial wealth, meaning that lower skill individuals start off with lower initial wealth endowment. The last part implies that the intratemporal elasticity of substitution is at least above the log preference, and the elasticity of labor supply is at most 1. This assumption determines the direction of change in optimal tax and debt policies in response to intratemporal and intertemporal changes.

Given that the government cares about redistribution towards low skilled workers, I argue that it is costly for the government to redistribute in financial autarky. That is,

Proposition 3.1. *Suppose Assumptions 1–2 hold, then the optimal labor tax is positive in financial autarky.*

Proof. See Appendix. □

Because of this cost of efficiency in autarky, the government is willing to sustain external debt rather than defaulting. That is,

Proposition 3.2 (Sustainable Debt). *Suppose Assumptions 1–2 hold, then it is optimal for the government to sustain positive external debt in equilibrium.*

Proof. See Appendix. □

The proof relies on the argument that it is not optimal for the government follows financial autarky. Given that there is positive labor distortion in financial autarky, one can construct a deviation from the financial autarkic allocation in which lowering the labor distortion in future periods without changing the continuation value allows the economy to produce more than its consumption in those periods. This deviation implies that the government can borrow today and repay the debt in the future. The extra unit of borrowing allows for more consumption, and so the deviation gives higher welfare than financial autarky.

3.3 Mechanism

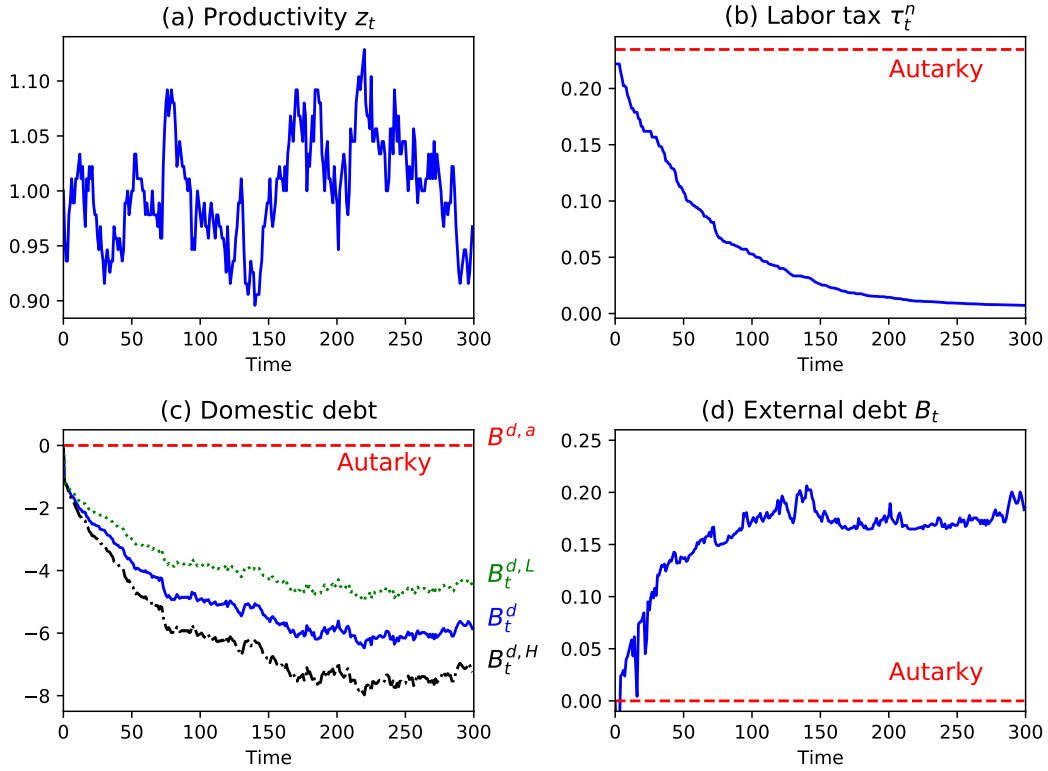
In this framework, the government is willing to sustain external debt in equilibrium instead of defaulting because of two reasons: insurance and redistribution. The former comes from the fact that without external credit markets, the government cannot insure itself against aggregate fluctuations. The insurance incentive is standard in the literature ([Eaton and Gersovitz \(1981\)](#), [Aguiar and Gopinath \(2006\)](#), [Arellano \(2008\)](#), [Chatterjee and Eyigungor \(2012\)](#), and many other papers). In this paper, I argue that the government also wants to sustain external debt because it is less costly to redistribute when the government has access to external credit markets.

What entails the redistributive benefit of external debt repayment? By repaying debt and having access to external credit markets, the government can redistribute resources via domestic credit markets. This additional redistributive tool allows the government to distort labor supply less. However, if the government defaults and stays in financial autarky, it can only rely on

labor taxes for redistribution. As a result, the labor distortion is high in autarky. In other words, external debt repayment allows the government to reduce the efficiency cost of redistribution.

Figure 3 plots simulated time paths of aggregate productivity, labor tax, domestic debt, and external debt in equilibrium and autarky for a two-agent economy.¹⁷ Panel (a) plots the aggregate productivity path, which is the same for both equilibrium and autarky. Panel (b) plots the optimal labor taxes over time. Panel (c) plots the aggregate and individual domestic debts. Note that these are domestic debts issued by the government, so a negative value means that the domestic agents are indebted. Panel (d) plots the external debt levels.

Figure 3: Simulated time paths of aggregates in equilibrium and autarky



Note: The graph plots the simulated time paths of optimal policies and aggregates of the government's problem in equilibrium and autarky. The implementation is that lump-sum taxes only occur in period 0. Panel (a) and (b) plot the realized aggregate productivity path and the optimal labor tax, respectively. Panel (c) plots the aggregate and individual domestic debt holdings. In autarky, all agents hold zero domestic debt. Panel (d)

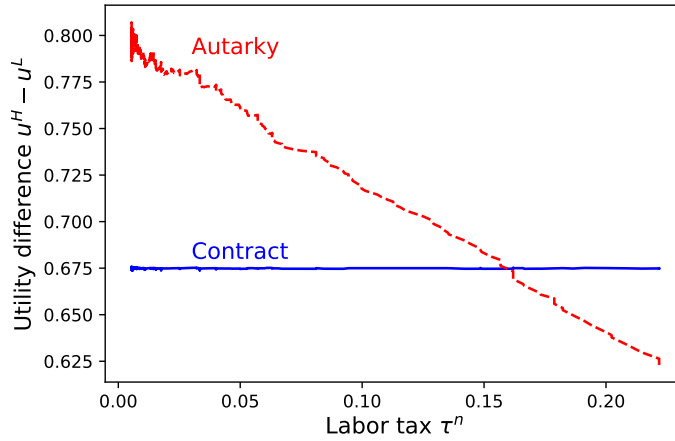
In equilibrium, the high-income agents borrow more over time and become net domestic debtors in the long run. This feature implies that the domestic credit market is a channel for redistributing wealth from high-income agents to low-income agents. As the government borrows more externally, it redistributes resources via a combination of increasing wealth redistribution via domestic credit markets and decreasing labor taxes over time. In contrast, when in autarky, domestic agents net domestic debt holdings are zero. The government can only rely labor taxes

¹⁷For details on the underlying assumptions and parameter values, see Section 4.

for redistribution. The optimal labor tax in autarky is constant at a higher level than the one in equilibrium. In the long run, the government wants to sustain debt because defaulting creates an adverse distributive effect in which all domestic wealth are equalized across agents, so it implicitly transfers more resources to the high income agents.

Efficiency cost of redistribution in the long run. Figure 4 plots the individual utility difference with respect to a given level of labor tax in the contract and in autarky after the government defaults in period 500. Initially, when labor tax is high, the utility difference in the contract is higher than in autarky. However, in the long run, when the optimal labor tax is low, the utility inequality in the contract is lower than in autarky. Therefore, in the long run, to achieve the same level of redistribution, autarky requires a higher cost of efficiency.

Figure 4: Labor tax and utility difference



Note: The graph depicts the level of utility difference $u^H - u^L$ for a given level of labor taxes τ^n in the contract and in financial autarky after the government defaults in period 500. The contract line comes from the simulated data for 1500 periods. The autarky line comes from solving the autarkic allocation given each level of labor tax.

4 Quantitative Analysis

In the previous section, I argue that the government's incentive to sustain debt depends not only on the insurance benefit but also the redistributive benefit of having access to external credit markets. In this section, I quantify these two channels that affect debt sustainability using calibrated parameters that match Italian data. I also show that the model is quantitatively consistent with the positive correlation between income inequality and external debt in the cross section and over time as described in the empirical motivation.

4.1 Parameterization

For the quantitative exercise, I assume the following distributional and functional forms. The economy is populated by two types of agents with labor productivity $\{\theta^H, \theta^L\}$, where $\theta^H \geq \theta^L > 0$ and $\pi^H = \pi^L = 0.5$. The planner is utilitarian, i.e. $\lambda^H = \lambda^L$. The individual preference has the form of

$$U^i(c, l) = \log c - \frac{l^{1+\nu}}{1+\nu}$$

The production function is linear in labor, i.e. $F(L, z) = zL$, where z is the aggregate productivity. The aggregate shock is z_t that follows a logged $AR(1)$ process,

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t^z, \quad \epsilon_t^z \sim \mathcal{N}(0, \sigma_z),$$

where ρ_z, σ_z are the auto-correlation and the residual standard deviation, respectively. I discretize the productivity process into a Markov chain using Tauchen method with 31 evenly-spaced nodes. From now on, I will use z in place of s as the source of aggregate uncertainty. The government expenditure is constant over time and across histories: $G_t = \bar{g}$. The initial debt levels are $B_0 = 0$ and $b_0^{H,d} = b_0^{L,d} = 0$. The economy starts at the mean of the productivity distribution. The deviation utility $\underline{U}(z^t, t)$ is calculated as the closed-economy version of the model that starts with productivity z_t , zero external debt, and all domestic individuals start with the same initial wealth. $\underline{U}(z^t, t)$ varies with respect to the realized shock z_t .

With these assumptions, the model requires giving values to the parameters of (i) the aggregate productivity process, ρ_z and σ_z ; (ii) the cross-sectional wage ratio, θ^H/θ^L ; (iii) the individual preference, β and ν ; (iv) the government expenditure \bar{g} ; and (v) the risk-free rate r^* .

A period in the model is one year. For output, I use the logged and linear detrended real GDP series from 1985-2015. I set the auto-correlation of productivity, ρ_z , equals to the auto-correlation of output, which is 0.928. To measure the wage ratio θ^H/θ^L , I use the household-level data from the *Survey on Household Income and Wealth* (SHIW) conducted by the Bank of Italy.¹⁸ Hourly wage is defined as total real compensation of employees, including fringe benefits, divided by total hours worked in a year.¹⁹ For each year in the database, I calculate the ratio of the mean wage of the top 50% of the wage distribution to the mean wage of the bottom 50%. Then θ^H/θ^L is set to 1.89, which is the time-average of these wage ratios for the period of 2002 to 2014. The discount factor β is set to 0.967 so that the average real domestic interest rate is 3.4% for Italy from 2002 to 2015. I choose $\nu = 2$ so that the elasticity of labor supply is 0.5, a standard value in the literature. The risk-free rate is set at 0.017, which is the real rate of return on the German government bonds for the period 2002-2015 (these are secondary market returns, gross of tax, with around 10 years' residual maturity). The interest rate series start at 2002 to isolate the effect of currency and exchange rate risks.²⁰

¹⁸See Appendix A.2 for details on the microeconomic data and sample selection.

¹⁹Real data calculation uses CPI indices provided by the OECD.

²⁰See Appendix A for more data descriptions and sources

The two remaining parameters, σ_z and \bar{g} , are selected to match (i) the standard deviation of logged output and (ii) the government's final consumption-to-GDP ratio for the period 1985-2015. I use the simulated method of moments (SMM). Departing from the quantitative literature on sovereign debt, I do not target the average external debt-to-output ratio but instead leave it as one of the non-targeted moments.²¹

Table 2 summarizes the parameter values and targets from the calibration exercise.

Table 2: Parameters and Targets

Parameter	Description	Value	Target
<i>Externally calibrated parameters</i>			
r^*	Risk-free rate	0.017	Avg. real return on German bond
β	Discount factor	0.967	Avg. Italian real interest rate = 3.4%
$1/\nu$	Labor elasticity	0.5	Standard literature value
θ^H/θ^L	Wage ratio	1.89	Mean top 50% wage / mean bottom 50% wage
ρ_z	Auto-corr. of prod.	0.927	Auto-corr. of log GDP
<i>Internally calibrated parameters</i>			
σ_z	Std. dev. of prod. res.	0.0205	Std. dev. log GDP
\bar{g}	Govt. spending	0.202	Avg. govt. consumption-to-GDP

Note: The table describes the parameters, their values, and the targets in the calibration exercise. Statistics are annual. The risk-free rate and discount factor cover the period of 2002-2015. Wage ratio is the author's calculation from the household-level data set by *Survey on Household Income and Wealth* covering the period of 2002-2014. Auto-correlation and standard deviation of GDP and government final consumption cover the period of 1985-2015. Data sources: *Survey on Household Income and Wealth* (2014), Eurostat (2019), and [The World Bank](#) (2019)

4.2 Moments

Table 3 shows the moments for the data, baseline, and alternative models. The first column reports the statistics from the data for Italy in the period of 1985-2015, except for external debt moments covering the period of 2002-2015. The second column reports the statistics from simulating the model and taking the long-run averages.²² The calibration successfully matches the standard deviation of output and the government consumption-output ratio for Italy.

²¹See Section ?? for the results of non-targeted moments. Alternatively, the discount factor β can be used to target the debt-to-output ratio.

²²All model's moments are long-run averages of simulating the economy for 10500 periods and discarding the first 500 periods.

Table 3: Moments: Data, Baseline, and Alternative Models

	Data	Baseline	No inequality	Skill-dependent lump sum tax
<i>Targeted moments</i>				
Std. output (%)	5.3	5.3	5.4	5.4
Avg. govt. expenditure/output (%)	19	19	19	19
<i>Non-targeted moments</i>				
<i>External debt property</i>				
Avg. external debt/output (%)	24	17	2.9	2.9
Std. external debt/output (%)	2.7	2.1	0.45	0.45
<i>Cyclical property</i>				
Std. consumption / Std. output	1.0	1.2	1.2	1.1
Std. net savings/output (%)	1.5	1.8	1.8	1.7
<i>Correlation with output (%)</i>				
Consumption	97	95	95	95
Net savings/output	40	31	36	35

Note: This table reports the non-targeted statistics of the data and the model. The first column reports data statistics which are across the period of 1985 to 2015, except for external debt moments covering the period of 2002 to 2015. The other columns report statistics coming from models' simulations for 10500 periods and excluding the first 500 periods. The no-inequality model corresponds to the case in which $\theta^H = \theta^L$. The skill-dependent-lump-sum-tax model corresponds to the case in which the government has access to skill-dependent lump-sum taxes $(T^i)_{i \in I}$. In the data, government expenditure is final government consumption excluding social transfers. Net saving is defined as output minus total private and government consumption in the data and the model. External debt is defined as the country's net financial liability in the data. For the second moments, output and consumption series are logged and linear detrended, and net saving and external debt ratio series are linear detrended.

External debt. Most importantly, the model is able to produce a quantitatively large amount of external debt-to-output, in consistence with the data. The model generates around 17% of external debt-to-output ratio, average at the ergodic distribution, comparing to 24% of external debt-to-output ratio in the data. The model also matches the volatility of external debt-to-output ratio in the data. These features are with the presence of a relatively high discount factor (0.969) and no additional exogenous cost of default in terms of output/productivity loss.

Cyclical properties. Several cyclical features of the Italian data stand out. First, consumption is as volatile as output and is highly correlated with output. Net saving, defined as output minus the total private and public consumption, only has a volatility of more than a quarter of the volatility of output and a positive correlation with output that is around 40%.²³ The model matches closely the quantitative patterns of the data. The volatility of consumption and net savings relative to output are slightly higher in the model than in the data. Both model consumption and net saving are pro-cyclical with similar correlation levels as in the data. The model is able

²³Neumeyer and Perri (2005) reported key business cycle statistics for both advanced and emerging market economies.

to generate realistic cyclical patterns of the data, in contrast to the standard model of complete markets. The main reason is that, even with state-contingent assets, the occasionally binding borrowing constraints lead to an imperfect insurance across states and time periods.

4.3 Redistributive Benefit and Distortionary Cost

The redistributive benefit to sustain external debt is quantitatively significant and driven by the distortionary cost of redistribution. To show these features, I compare the baseline model to the two alternative models of no inequality and skill-dependent lump-sum taxes.

The no-inequality model corresponds to the case in which $\theta^H = \theta^L$, and its moments are reported in the third column of Table 3. The no-inequality model generates lower level of external debt, only 2.9% of GDP, comparing to 17% of GDP in the baseline model. This feature implies that the insurance benefit to sustain external debt, in which having access to international financial markets allow the government to insure itself against aggregate uncertainty, only accounts for quantitatively small level of external debt. On the other hand, the redistributive benefit to sustain external debt, in which having access to international financial markets allow the government to redistribute at lower distortionary costs, accounts for quantitatively large level of external debt. The no-inequality model also has lower external debt volatility comparing to the baseline model.

To highlight the role of the distortionary cost of redistribution, I consider the skill-dependent-lump-sum-tax model in which the government has access to lump-sum taxes that are dependent on household's skill levels. In this alternative model, the government can achieve perfect redistribution without levying positive distortionary taxes by taxing or transferring specific amount of resources to each type of households. The fourth column of Table 3 reports the moments of this model. Comparing to the baseline model, the skill-dependent-lump-sum-tax model gives lower level of external debt-to-output ratio and a lower external debt volatility. In skill-dependent-lump-sum-tax model, the government redistributes by lump-sum taxing the highly skilled households and lump-sum transferring resources to lowly skilled households.²⁴ In the baseline model, the government can only redistribute by levying positive distortionary taxes.

4.4 Cross-Country Implication

In this subsection, I measure how well the model can account for the cross-country correlation between inequality and external debt. Table 4 shows the estimation results of the correlation between pre-tax Gini index and net foreign liability-to-GDP from the data and the model. The data values are from the second column of Table 1, robust to other controls. The model values come from the regression on the model's simulated data. Given the calibrated parameters, I

²⁴The present value-lump-sum tax-to-output ratio for the highly skilled households is 13, and the present-value-lump-sum-transfer-to-output ratio for the lowly skilled is 15.

solve different versions of the model differentiated only by wage inequality and compute the pre-tax Gini indices, long-run averages of external debt-to-output ratios, and output per capita. I then estimate the regression $NFL_i = \beta_0 + \beta_1 \text{Gini}_i + \beta_2 \log \text{GDP per capita}_i + \epsilon_i$ and report $\hat{\beta}_1$ and its standard error.²⁵

Table 4: Cross-country estimation: Data and Model

	Dependent Variable: Net foreign liability-to-GDP (%)	
	Data: 1985-2015	Model
Gini index, pre tax (%)	0.968** (0.487)	2.42*** (0.0674)
Controls	Yes	Yes
No. Observations	120	30

Note: The table describes the cross-country estimation of the coefficient of pre-tax Gini index (%) on net foreign liability-to-GDP (%) in the data and in the model. The model estimation comes from simulated data of 30 different versions of the model that are differentiated by wage ratios.

The model produces a positive and statistically significant coefficient of the pre-tax Gini index. The coefficients imply that a one percent increase in the pre-tax Gini index corresponds to a 0.968% increase in net foreign liability-to-GDP in the data, comparing to a 2.42% increase in the model. The data coefficient is lower than the model coefficient because there are additional factors in the data that potentially dampen the debt sustainability channel: incomplete markets, default on path, and endogenous human capital investment.

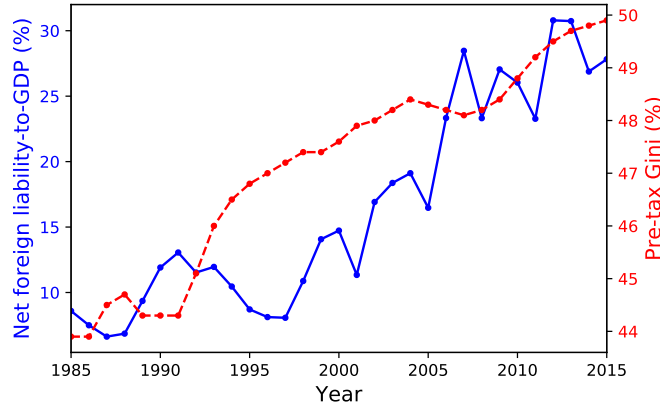
4.5 Over-Time Implication

I then examine the model's implication on the effect of income inequality on external debt over time. I conduct a comparative statics exercise in the case of Italy for two time periods of 1985-2001 and 2002-2015. Figure 5 plots the time series of net foreign liability-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. On average, in the period of 1985-2001, Italy has a lower levels of income inequality and external debt comparing to the period of 2002-2015.

The comparative statics exercise is as follows. I feed into the model a value of wage inequality for the period 1985-2001 and keep other parameter values fixed. I compute ergodic means of pre-tax Gini index and external-debt-to-output ratios. The 1985-2001 value of wage inequality is such that the change in the average pre-tax Gini income from 1985-2001 to 2002-2015 is the same as the change in the data. Table 5 reports the results of the policy experiment. Given the targeted increase in the pre-tax Gini indices in Italy from 1985-2001 to 2002-2015, the model can account for about 71% of the increase in the external debt-to-output ratio.

²⁵In the model, average output growth rates and inflation are zero, so I omit them as control variables. Since the regression uses the ergodic means of the model as variables, country and time fixed effects are redundant.

Figure 5: Income inequality and external debt in Italy



Notes: The graph shows the time series of net foreign liability-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. The left y-axis depicts the values in net foreign liability-to-GDP (%), and the right y-axis depicts the values in pre-tax Gini (%). Sources: Lane and Milesi-Ferretti (2018), and Solt (2019).

Table 5: Comparative statics results for periods 1985-2001 and 2002-2015

Statistics	Data	Model
<i>Targeted</i>		
Δ Pre-tax Gini	3.0%	3.0%
<i>Non-targeted</i>		
Δ External debt/Y	14%	10%

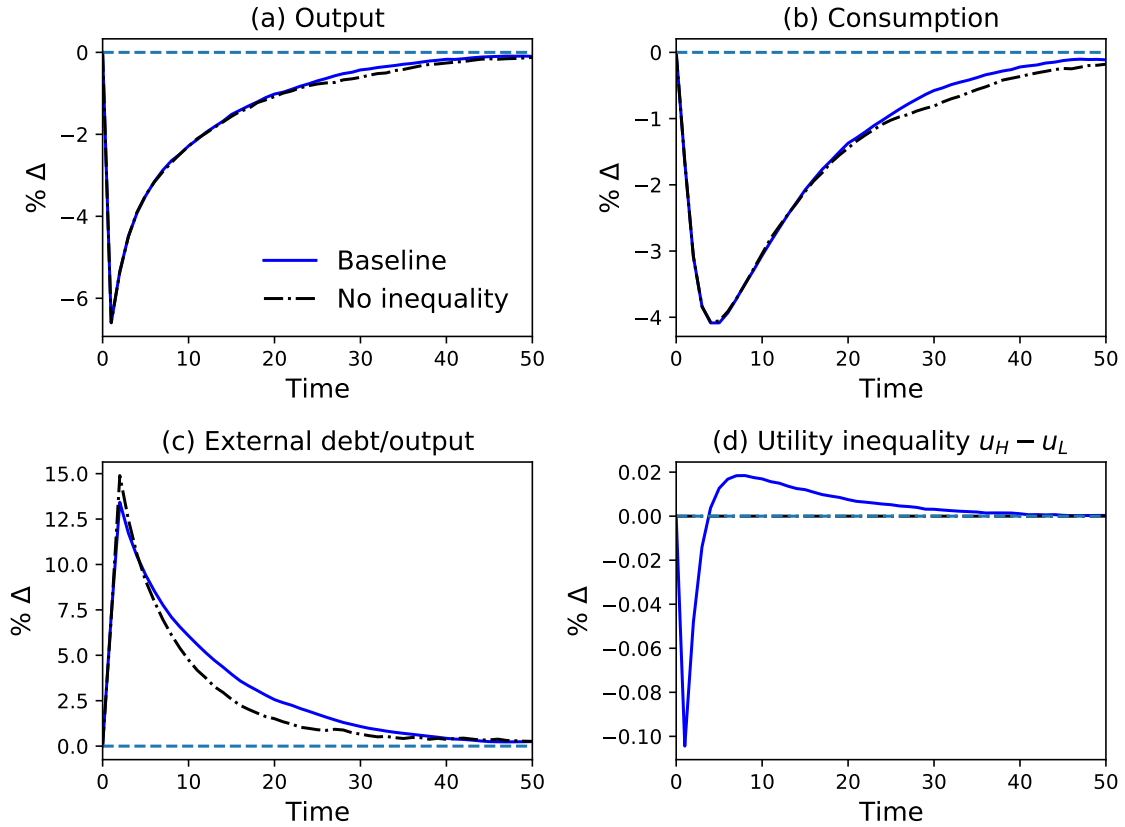
Notes: The table reports the results of the comparative statics exercise. The first column reports the changes in the data statistics, computed as the average statistics of period 2002-2015 minus the average statistics of period 1985-2001. The second column reports the results from the model. The change in the model statistics is computed as the average statistic of a simulation for the model with the wage ratio equal to 1.89 minus the same statistic of the model with the wage ratio equal to 1.83.

4.6 Optimal Austerity Policies

In this subsection, I study optimal austerity policies in the presence of inequality by measuring the responses of policies to a negative productivity shock. I simulate 30000 paths for the model for 2050 periods. From periods 1 to 2050, the aggregate productivity shock follows its underlying Markov chain so that the cross-sectional distribution of debt converges to the ergodic distribution. In period 2051, normalized to 1 in the plots, the aggregate productivity drops by one standard deviation of the error term. From period 2051 on, the aggregate productivity shock follows the conditional Markov chain. The impulse responses plot the average, across 30000 paths, of the variables from period 2050 to 2055.

Figure 6 plots the impulse responses of output, consumption, external debt-to-output, and

Figure 6: Impulse responses to a negative productivity shock



Notes: The graph shows the impulse responses across 30000 paths of the variables for the baseline and no-inequality models. Panel (a), (b), (c), (d) plot the responses of output, consumption, external debt-to-output, and utility inequality to a negative aggregate productivity drop in period 1, respectively.

utility inequality $u^H - u^L$, for the baseline model and the model of no inequality ($\theta^H = \theta^L$). A decline in productivity growth leads to declines in both output and consumption with a higher drop in output. External debt-to-output increases in response to a low productivity. The utility inequality initially decreases then increases subsequently, implying that there is high redistribution in the short run and low redistribution in the long run. Intuitively, a negative shock leads to a reduction in the deviation utility, and so the borrowing constraint becomes non-binding. The non-binding constraint allows the government to accumulate external debt, temporarily decreases average tax rates, and increases redistribution. In future periods, the government raises taxes to repay the debt and reduces redistribution.

The no-inequality model has similar responses in terms of output and consumption over time. However, the external debt-to-output increases more initially and decreases faster over time comparing to the baseline model. This implies that in the baseline model with inequality the government, despite borrowing less initially, is able to sustain higher debt in the future periods.

5 Sensitivity Analysis

This section discusses how the optimal policies quantitatively respond to different model ingredients: government expenditure, aggregate uncertainty, and discount factor. Table 6 compares the moments from the baseline model to other alternative models. The second column reports the moments from the no-government-expenditure model in which $g = 0$. Average external debt-to-output ratio stays the same as the baseline model. This result implies that the amount of exogenous government expenditure does not quantitatively affect the optimal sustainable debt level. Higher government expenditure coincides with higher second moments except for correlation of consumption with output.

Table 6: Role of government expenditure, aggregate uncertainty, and discount factor

	Baseline	No govt. exp. ($g = 0$)	Deterministic ($\sigma_z = 0$)	Lower discount ($\beta = 0.95$)
Avg. external debt/output (%)	17	17	22	12
Std. external debt/output (%)	2.1	0.99	-	1.8
Std. consumption / Std. output	1.2	0.96	-	1.2
Std. net savings/output (%)	1.8	1.5	-	0.62
<i>Correlation with output (%)</i>				
Consumption	95	97	-	99
Net savings/output	31	29	-	-1.1

Notes: The moments are calculated from the model's simulation for 10500 periods and then excluding the first 500 periods. For the second moments, output and consumption series are logged and linear detrended, and net saving and external debt ratio series are linear detrended.

The third column of Table 6 considers the deterministic case in which there is no aggregate uncertainty. The deterministic model generates higher level of sustainable debt than the baseline model. An explanation is that uncertainty in the baseline model implies a higher precautionary motive than the deterministic case, which leads to the lower overall external debt accumulation. The fourth column describes the moments of the model with lower discount factor ($\beta = 0.95$) than the baseline model. The average external debt-to-output ratio is lower when the discount factor is lower. This is because a higher discount factor implies a lower value of autarky and a longer time period it takes for the economy to reach the borrowing constraints in the baseline economy. Therefore, it allows the government to accumulate and sustain a higher external debt level. In addition, the lower-discount model poorly matches the second moments of net savings in the data.

6 Conclusion

This paper proposes a theory of external debt sustainability that comes from the motive for redistribution of the government. I introduce the government's redistributive concern and distortionary taxation into a sovereign debt framework. I analyze the interaction between distortionary and distributive effect of fiscal policies and the government's lack of commitment. The endogenous borrowing constraints arise from the government's lack of commitment, and become relevant in the long run due to the domestic agents' impatience. Redistribution comes with the cost of tax distortions.

The paper's theoretical contribution is the effect of redistribution on optimal debt sustainability. Given that the government wants to redistribute towards lower skilled lower income households, initial wealth inequality is positively correlated with skill inequality, and conditions on the intratemporal and intertemporal elasticities, it is optimal for the government to sustain positive external debt. Default leading to financial autarky is costly not only because the government cannot use debt to smooth consumption over the business cycles, but also because redistribution is more distortionary and less efficient in financial autarky than in the contract. The distributive cost of default is endogenous and novel to the literature.

The quantitative contribution is showing that the effect of redistribution is quantitatively significant to match observed debt levels in the data. The result contributes to the ongoing literature on endogenous default costs in sovereign debt models. The quantitative analysis shows that by taking into account the redistributive cost of default, the model can quantitatively account for 71% of the long-run average external debt-to-output, while the insurance cost of default alone can only accounts for 12% the long-run average external debt-to-output. The key factor driving these results is the distortionary cost of redistribution. Furthermore, the theory is quantitatively consistent with the cross-country and time-series correlation between income inequality and external debt in the data.

The model has implications on optimal austerity policies in the presence of inequality. Estimations from the model's simulation points out that a negative productivity shock leads to an increase in external debt and a temporary decrease in borrowing taxes, while labor taxes remain unchanged. The average tax-to-income ratio initially decreases for all agents and more for high-income agents, and utility difference decreases. In the future, the government raises average taxes and reduces redistribution to repay debt.

Future extensions of the model can incorporate equilibrium defaults and various types of debt crises.

References

- Aguiar, Mark and Gita Gopinath**, “Defaultable debt, interest rates and the current account,” *Journal of International Economics*, 2006, 69 (1), 64–83.
- **and Manuel Amador**, “Growth in the Shadow of Expropriation,” *Quarterly Journal of Economics*, 2011, 126 (2), 651–697.
- **and —**, “Sovereign Debt,” *Handbook of International Economics*, 2014, 4, 647–87.
- **and —**, “Fiscal policy in debt constrained economies,” *Journal of Economic Theory*, 2016, 161, 37–75.
- Aiyagari, S. Rao, Albert Marcet, Thomas J. Sargent, and Juha Seppälä**, “Optimal taxation without state-contingent debt,” *Journal of Political Economy*, 2002, 110 (6), 1220–1254.
- **and Ellen R. McGrattan**, “The optimum quantity of debt,” *Journal of Monetary Economics*, 1998, 42 (3), 447–469.
- Aizenman, Joshua and Yothin Jinjarak**, “Income inequality, tax base and sovereign spreads,” *FinanzArchiv: Public Finance Analysis*, 2012, 68 (4), 431–444.
- Alvaredo, Facundo, Lucas Chancel, Thomas Piketty, Emmanuel Saez, and Gabriel Zucman**, *World inequality report 2018*, Belknap Press, 2018.
- Ams, Julianne, Reza Baqir, Anna Gelpern, and Christoph Trebesch**, “Sovereign Default,” 2018.
- Arellano, Cristina**, “Default risk and income fluctuations in emerging economies,” *American Economic Review*, 2008, 98 (3), 690–712.
- **and Yan Bai**, “Fiscal austerity during debt crises,” *Economic Theory*, 2016, pp. 1–17.
- Balke, Neele**, “The Employment Cost of Sovereign Default,” 2017.
- **and Morten O. Ravn**, “Time-Consistent Fiscal Policy in a Debt Crisis,” 2016.
- Barro, Robert J.**, “On the determination of the public debt,” *Journal of Political Economy*, 1979, 87 (5), 940–71.
- Bauducco, Sofia and Francesco Caprioli**, “Optimal fiscal policy in a small open economy with limited commitment,” *Journal of International Economics*, 2014, 93 (2), 302–315.
- Berg, Andrew and Jeffrey Sachs**, “The debt crisis structural explanations of country performance,” *Journal of Development Economics*, 1988, 29 (3), 271–306.
- Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J. Sargent**, “Fiscal policy and debt management with incomplete markets,” *Quarterly Journal of Economics*, 2016, 132 (2), 617–663.

- , —, —, and **Thomas J Sargent**, “Public debt in economies with heterogeneous agents,” *Journal of Monetary Economics*, 2017, 91, 39–51.
- Brinca, Pedro, Miguel Homem Ferreira, Francesco A Franco, Hans Aasnes Holter, and Laurence Malafry**, “Fiscal consolidation programs and income inequality,” *Available at SSRN 3071357*, 2019.
- Chari, Varadarajan V. and Patrick J. Kehoe**, “Sustainable plans,” *Journal of Political Economy*, 1990, pp. 783–802.
- and —, “Sustainable plans and debt,” *Journal of Economic Theory*, 1993, 61 (2), 230–261.
- and —, “Optimal fiscal and monetary policy,” *Handbook of Macroeconomics*, 1999, 1, 1671–1745.
- , **Lawrence J. Christiano, and Patrick J. Kehoe**, “Optimal fiscal policy in a business cycle model,” *Journal of Political Economy*, 1994, 102 (4), 617–652.
- Chatterjee, Satyajit and Burcu Eyigungor**, “Maturity, indebtedness, and default risk,” *American Economic Review*, 2012, 102 (6), 2674–99.
- Cuadra, Gabriel, Juan M. Sanchez, and Horacio Saprizza**, “Fiscal policy and default risk in emerging markets,” *Review of Economic Dynamics*, 2010, 13 (2), 452 – 469.
- Deng, Minjie**, “Inequality, Taxation, and Sovereign Default Risk,” 2021.
- D’Erasmus, Pablo and Enrique G. Mendoza**, “Distributional incentives in an equilibrium model of domestic sovereign default,” *Journal of the European Economic Association*, 2016, 14 (1), 7–44.
- and —, “History remembered: Optimal sovereign default on domestic and external debt,” *Journal of Monetary Economics*, 2020.
- Dovis, Alessandro, Mikhail Golosov, and Ali Shourideh**, “Political Economy of Sovereign Debt: A Theory of Cycles of Populism and Austerity,” 2016.
- Eaton, Jonathan and Mark Gersovitz**, “Debt with potential repudiation: Theoretical and empirical analysis,” *Review of Economic Studies*, 1981, 48 (2), 289–309.
- Ferriere, Axelle**, “Sovereign Default, Inequality, and Progressive Taxation,” 2015.
- Gonzalez-Aguado, Eugenia**, “Financial Development and Vulnerability to External Shocks: The Role of Sovereign Debt Composition,” 2018.
- Jappelli, Tullio and Luigi Pistaferri**, “Does Consumption Inequality Track Income Inequality in Italy?,” *Review of Economic Dynamics*, 2010, 13 (1), 133–153.

- Jeon, Kiyoung and Zeynep Kabukcuoglu**, “Income Inequality and Sovereign Default,” *Journal of Economic Dynamics and Control*, 2018, 95 (C), 211–232.
- Kehoe, Patrick J. and Fabrizio Perri**, “International business cycles with endogenous incomplete markets,” *Econometrica*, 2002, 70 (3), 907–928.
- Kehoe, Timothy J. and David K. Levine**, “Debt-constrained asset markets,” *Review of Economic Studies*, 1993, 60 (4), 865–888.
- Lane, Philip R. and Gian Maria Milesi-Ferretti**, “The External Wealth of Nations Revisited: International Financial Integration in the Aftermath of the Global Financial Crisis,” *IMF Economic Review*, March 2018, 66 (1), 189–222.
- Leventi, Chrysa and Manos Matsaganis**, “Estimating the distributional impact of the Greek crisis (2009-2014),” 2016, (1312).
- Lucas, Robert E. and Nancy L. Stokey**, “Optimal fiscal and monetary policy in an economy without capital,” *Journal of Monetary Economics*, 1983, 12 (1), 55–93.
- Marcet, Albert and Ramon Marimon**, “Recursive contracts,” *Econometrica*, 2019.
- McDaniel, Cara**, “Average tax rates on consumption, investment, labor and capital in the OECD 1950-2003,” *Arizona State University mimeo*, 2007.
- Mendoza, Enrique G. and Vivian Z. Yue**, “A general equilibrium model of sovereign default and business cycles,” *Quarterly Journal of Economics*, 2012, 127 (2), 889–946.
- Monastiriotis, Vassilis**, “Making geographical sense of the Greek austerity measures: compositional effects and long-run implications,” *Cambridge Journal of Regions, Economy and Society*, 10 2011, 4 (3), 323–337.
- Neumeyer, Pablo and Fabrizio Perri**, “Business cycles in emerging economies: the role of interest rates,” *Journal of Monetary Economics*, 2005, 52 (2), 345–380.
- Pouzo, Demian and Ignacio Presno**, “Optimal taxation with endogenous default under incomplete markets,” 2015.
- Reinhart, Carmen M. and Kenneth S. Rogoff**, *This time is different: Eight centuries of financial folly*, Princeton University Press, 2009.
- and —, “Growth in a Time of Debt,” *American Economic Review*, 2010, 100 (2), 573–78.
- Restrepo-Echavarria, Paulina**, “Endogenous Borrowing Constraints and Stagnation in Latin America,” *Journal of Economic Dynamics and Control*, 2019, p. 103774.
- Solt, Frederick**, “Measuring income inequality across countries and over time: The standardized world income inequality database,” 2019.

The World Bank, “World development indicators database,” 2019.

Thomas, Jonathan and Tim Worrall, “Self-enforcing wage contracts,” *Review of Economic Studies*, 1988, 55 (4), 541–554.

Tran-Xuan, Monica, “Optimal Redistributive Policy in Debt Constrained Economies,” 2021.

Werning, Iván, “Optimal Fiscal Policy with Redistribution,” *Quarterly Journal of Economics*, 08 2007, 122 (3), 925–967.

Appendices

A Empirical Appendix

A.1 Macroeconomic Data Descriptions and Sources

Most data are annual series covering the 1985-2015 period. Some data samples cover the 2002-2015 period.

- Net foreign liability is the negative of net foreign asset (NFA) from the External Wealth of Nations Database, [Lane and Milesi-Ferretti \(2018\)](#)
- Net international investment position is the official international investment position (IIP) from the External Wealth of Nations Database, [Lane and Milesi-Ferretti \(2018\)](#)
- Pre-tax Gini Index the market Gini from the Standardized World Income Inequality Database, [Solt \(2019\)](#).
- GDP per capita is the constant 2010 US Dollar GDP per capita series from World Development Indicator Database, [The World Bank \(2019\)](#)
- GDP growth is the log difference of constant 2010 US Dollar GDP series from World Development Indicator Database, [The World Bank \(2019\)](#)
- Inflation is the annual inflation series measured by the GDP deflator from World Development Indicator Database, [The World Bank \(2019\)](#)
- Real GDP is GDP series in constant local currency units from World Development Indicator Database, [The World Bank \(2019\)](#)
- Real return on German bond is the interest rate on German bond adjusted for inflation measured by the GDP deflator. The interest rate is the long-term interest rate for convergence purposes from the Eurostat Database (2019). These bonds have 10-year maturity and are denominated in Euro.
- Real interest rate is the lending interest rate adjusted for inflation as measured by the GDP deflator from World Development Indicator Database, [The World Bank \(2019\)](#)
- Italy's cross-sectional wage inequality is calculated from the micro-data by [Jappelli and Pistaferri \(2010\)](#) using Surveys of Household Income and Wealth conducted by the Bank of Italy for the period 1980-2006.
- Government consumption is the general government final consumption expenditure series from World Development Indicator Database, [The World Bank \(2019\)](#)
- Private consumption is the households and NPISHs final consumption expenditure series from World Development Indicator Database, [The World Bank \(2019\)](#)

A.2 Italian Household Survey

For the analysis of wage inequality, I use household-level data from the *Survey on Household Income and Wealth* (SHIW), conducted by the Bank of Italy. The SHIW includes cross-sectional and panel data on household's demographics, income, labor supply, consumption, and wealth. [Jappelli and Pistaferri \(2010\)](#) provide details on the survey design descriptions and data quality analysis. I use data for the period 1987 to 2014. In this period, the survey was conducted biennially, except for the period 1995 to 1998 with a 3-year interval.

The sample selection is followed. The original individual sample includes 329,446 units from 1987 to 2014. Given the focus on wage inequality and labor supply across households, I focus on heads of households between the age of 25 and 60. This selection criteria reduces the sample size to 71,621 units. To reduce the impact of outliers, I exclude observations with no hours worked, with negative income, or with hourly wage in the bottom 0.5% of the distribution. The final sample size is 67,176.

A.3 Lists of Countries

Albania, Algeria, Angola, Argentina, Armenia, Australia, Austria, Azerbaijan, Bangladesh, Belarus, Benin, Bolivia, Bosnia and Herzegovina, Brazil, Bulgaria, Burkina Faso, Burundi, Cambodia, Cameroon, Canada, Central African Republic, Chad, Chile, Colombia, Congo, Costa Rica, Côte d'Ivoire, Croatia, Czech Republic, Dem. Rep. Congo, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Estonia, Ethiopia, Finland, France, Gabon, Gambia, Georgia, Ghana, Greece, Guatemala, Guinea, Guinea-Bissau, Haiti, Honduras, Hungary, India, Indonesia, Iraq, Ireland, Israel, Italy, Jamaica, Jordan, Kazakhstan, Kenya, Korea, Kyrgyz Republic, Lao, Latvia, Lebanon, Lesotho, Lithuania, Madagascar, Malawi, Malaysia, Mali, Mauritania, Mexico, Moldova, Mongolia, Morocco, Mozambique, Myanmar, New Zealand, Nicaragua, Niger, Nigeria, Norway, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Rwanda, Senegal, Serbia, Sierra Leone, Slovakia, Slovenia, South Africa, Spain, Sri Lanka, Sweden, Tajikistan, Tanzania, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, Turkmenistan, Uganda, Ukraine, United Kingdom, United States, Uruguay, Uzbekistan, Vietnam, Yemen, Zambia, Zimbabwe.

A.4 Net International Investment Position

This subsection provides the estimation using the negative of net international investment position an alternative definition of a country's external indebtedness.

Table 7 reports the regression results. High income inequality levels are correlated with high external debt positions, though the coefficients are not statistically significant.

Table 7: Regression analysis of income inequality and external debt

	Dependent Variable: Net international liability position-to-GDP (%) Time periods: 1985-2015	
	(1)	(2)
Gini index, pre tax (%)	0.2681 (0.7528)	0.1755 (0.7707)
Country fixed effects	Yes	Yes
Time fixed effects	Yes	Yes
Controls	No	Yes
No. Countries	137	137
No. Observations	2028	2028

Note: The table describes the panel regression results using all countries in the data set. Both columns show the regression coefficient and standard error in parenthesis of pre-tax Gini index (%) with respect to net international liability position-to-GDP (%). Control variables are log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). Both regressions have country and time fixed effects. All standard errors are clustered. *, **, *** represent significant levels of 10%, 5%, and 1%, respectively. Sources: [Lane and Milesi-Ferretti \(2018\)](#), [Solt \(2019\)](#), and [The World Bank \(2019\)](#).

B Sovereign Game

In this section, I set up the strategic sovereign game and define the sustainable equilibrium that is characterized by the self-enforcing constraints. All histories are written out explicitly to illustrate the strategic interaction between the government, private agents, and international lenders.

I consider the general environment where the government's policy includes the decision to default on external bond $\{\delta(s^t)\}$, where $\delta \in \{0, 1\}$ and $\delta = 0$ implies default.²⁶ The government's budget constraint becomes

$$G(s^t) + (1 - \tau^d(s^t))B^d(s^t) + \delta(s^t)B(s^t) \leq \tau_t^n(s^t)w(s^t)L(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) + \sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t)B(s^{t+1}) + T(s^t)$$

The price of international debt takes into account the probability of default is

$$Q(s_{t+1}|s^t) = \frac{\Pr(s_{t+1}|s^t)\delta(s_{t+1}|s^t)}{1 + r^*}$$

As the government cannot commit to any of its policies, one can think that the government, private agents, and international lenders enter in a sovereign game where they determine

²⁶Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its foreign debt ($d_t = 1$).

their actions sequentially. In every period and every history, the state variable for the game is $\left\{ B(s^t), \left(b^{i,d}(s^t) \right)_{i \in I} \right\}$. The timing of the actions is as follows.

- Aggregate shock s_t is realized
- Government chooses $z_t^G = \left(\tau^n(s^t), \tau^d(s^t), T(s^t), \delta(s^t), B(s_{t+1}, s^t), B^d(s_{t+1}, s^t) \right) \in \Pi$ such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c^i(s^t), l^i(s^t), b^{d,i}(s_{t+1}, s^t) \right)$ subject to their budget constraints, the representative firm produce output by choosing $z_t^F = L(s^t)$, and the international lenders choose holdings of government's bonds $z_t^* = B(s_{t+1}, s^t)$.

Define $h^t = \left(h^{t-1}, z_{t-1}^G, \left(z_{t-1}^{H,i} \right)_{i \in I}, z_{t-1}^F, z_t^*, s_t \right) \in H^t$ as the history after shock s_t is realized. Note that the history incorporates the government's policy, allocation and prices. Define $h_p^t = \left(h^t, z_t^G \right) \in H_p^t$ as the history after the government announce its policies at period t . The government strategy is $\sigma_t^G : H^t \rightarrow \Pi$. The individual agent's strategy is $\sigma_t^{H,i} : H_p^t \rightarrow \mathbb{R}_+^3 \times \mathbb{R}$. The firm has strategy $\sigma_t^F : H_p^t \rightarrow \mathbb{R}_+^2$, and the international lenders have strategy $\sigma_t^* : H_p^t \rightarrow \mathbb{R}_+$.

Definition B.1 (Sustainable equilibrium). A sustainable equilibrium is $(\sigma^G, \sigma^H, \sigma^F, \sigma^*)$ such that (i) for all h^t , the policy z_t^G induced by the government strategy maximizes the socially weighted utility given λ subject to the government's budget constraint (4) (ii) for all h_p^t , the strategy induced policy $\{z_t^G\}_{t=0}^\infty$, allocation $\{z_t^{H,i}, z_t^F, z_t^*\}_{t=0}^\infty$, and prices $\{Q_t\}_{t=0}^\infty$ constitute a competitive equilibrium with taxes.

The following focuses on characterizing a set of sustainable equilibrium in which deviation triggers autarky, where there is no domestic and foreign borrowing. In this case, the value of deviation includes the autarkic payoff.

By definitions, autarky is a sustainable equilibrium. Given that the domestic agents do not save/invest, the representative firm produces only with labor, and the international creditors do not lend, the government finds it optimal to default on its external debt, set saving and capital taxes such that the after-tax gross returns on domestic bonds and capital are zero, and set the labor tax such that it maximizes the socially weighted utility. Given the government defaulting and fully taxing all returns from domestic savings and capital, international creditors do not want to lend, agents do not save or invest in capital, and output is produced only by labor. Lastly, given that the government will be in autarky in the future, it is optimal in the current period for the government to also follow the autarkic strategies.

Reverting to autarky equilibrium is defined as a sustainable equilibrium of the above game such that following any government's deviation from the promised plans, the economy reverts to autarky. One can characterize the equilibrium as follows.

Proposition B.1 (Reverting to autarky equilibrium). *An allocation and policy $\left\{ \left(z^{H,i} \right)_{i \in I}, z^F, z^G \right\}$ can be supported by reverting to autarky equilibrium if and only if (i) given z^G , there exist prices p*

such that $\left\{ \left(z^{H,i} \right)_{i \in I}, z^F, z^G, p \right\}$ is a competitive equilibrium with taxes for an open economy, and
(ii) for any t and any s^t , there exists $\underline{U}(s^t, t)$ such that $\left\{ \left(z^{H,i} \right)_{i \in I}, z^F, z^G \right\}$ satisfies the constraint

$$\sum_{i \in I} \lambda^i \pi^i \sum_{k \geq t, s^t \subseteq s^k} \beta^{k-t} \Pr(s^k | s^t) U^i(c^i(s^k), l^i(s^k)) \geq \underline{U}(s^t, t) \quad (12)$$

Proof. Define $\underline{U}(s^t, t)$ as the maximum discounted weighted utility for the agents in period t , history s^t , when the government deviates. At period t and history s^t , the government taxes all domestic wealth ($\tau^d(s^t) = 1$) and redistributes equally across agents, and the government defaults on the external debt. In subsequent period $k > t$, the economy reverts to financial autarky where agents do not save in domestic bonds, and the government is excluded from international lending. This economy ensembles a neoclassical growth closed economy that has an initial aggregate state s_t , distortionary taxation on labor, and equal initial wealth across individuals.

Suppose $\left\{ \left(z^{H,i} \right)_{i \in I}, z^F, z^G \right\}$ is an outcome of the reverting to autarky equilibrium. Then by the optimal problems of the government, agents, and foreign lenders, $\left\{ \left(z^{H,i} \right)_{i \in I}, z^F, z^G \right\}$ maximizes the weighted utility of the agents, satisfies government budget constraint and foreign lender's problem at period 0. Thus, $\left\{ \left(z^{H,i} \right)_{i \in I}, z^F, z^G \right\}$ is an open-economy tax-distorted competitive equilibrium. For any period t and history h^t , an equilibrium strategy that has the government deviates in period t triggers reverting to autarky in period $k > t$. Such strategy must deliver the weighted value at least as high as the right-hand side of (12). So $\left\{ \left(z^{H,i} \right)_{i \in I}, z^F, z^G \right\}$ satisfies condition (ii).

Next, suppose $\left\{ \left(z^{H,i} \right)_{i \in I}, z^F, z^G \right\}$ satisfies conditions (i) and (ii). Let h^t be any history such that there is no deviation from z^G up until period t and history s^t . Since $\left\{ \left(z^{H,i} \right)_{i \in I}, z^F, z^G \right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t and history s^t onward. Consider a deviation plan $\hat{\sigma}^G$ at period t that receives $U^d(s_t, t)$ in period t and $U^{aut}(s_t)$ for the subsequent period $k > t$. Because the plan is constructed to maximize the utility in period t , the right-hand side of (12) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{ \left(z^{H,i} \right)_{i \in I}, z^F, z^G \right\}$ satisfies condition (ii), the original no-deviation plan is optimal. \square

Proposition B.1 can be extended to the general characterization of sustainable equilibrium, as in [Chari and Kehoe \(1990\)](#).

C Characterizing Sustainable Allocation and Optimal Tax Policies

This section provides details on the characterization of the sustainable allocation and optimal tax policies. Section D will use this analysis to prove the propositions in the main text.²⁷

Let μ be the multiplier on the resource constraint, $\pi^i \eta^i$ be the multiplier on the implementability constraint for agent i , and $\beta^t \Pr(s^t) \gamma(s^t)$ be the multiplier on the aggregate debt constraint for period t . Define $\eta = (\eta^i)_{i \in I}$ and rewrite the Larangian of the planning problem with a new pseudo-utility function that incorporates the implementability constraints:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^W \frac{L(s^t)^{1+\nu}}{1+\nu} \right] - \Phi_C^V C_0^{-\sigma} \sum_{i \in I} \pi^i \eta^i (b_0^i - T)$$

where Φ_C^W, Φ_L^W depend on φ, λ , and η (see Appendix D.1).

The first-order conditions of the planning problem for any period $t \geq 1$ can be summarized as

$$F_L(L_t, s^t, t) = \frac{\left\{ \Phi_L^W + \Phi_L^P \sum_{k=0, s^k \subseteq s^t}^t \gamma_k \right\} L_t^\nu}{\left\{ \Phi_C^W + \Phi_C^P \sum_{k=0, s^k \subseteq s^t}^t \gamma_k \right\} C_t^{-\sigma}} \quad (\text{C.1})$$

and

$$Q(s_{t+1}|s^t) = \beta \Pr(s_{t+1}|s^t) \frac{C_{t+1}^{-\sigma} \Phi_C^W + \Phi_C^P \sum_{k=0, s^k \subseteq s^t}^{t+1} \gamma_k}{C_t^{-\sigma} \Phi_C^W + \Phi_C^P \sum_{k=0, s^k \subseteq s^t}^t \gamma_k} \quad (\text{C.2})$$

The optimal tax policies follow

$$\tau_t^n = 1 - \frac{1}{F_L(L_t, s^t, t)} \frac{\Phi_L^V L_t^\nu}{\Phi_C^V C_t^{-\sigma}} \quad (\text{C.3})$$

$$\frac{Q_t^d(s_{t+1})}{1 - \tau_{t+1}^d} = \beta \Pr(s_{t+1}|s^t) \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \quad (\text{C.4})$$

D Formulas and Proofs

D.1 Formulas

Given the formulas for ψ_c^i and ψ_l^i in (??), we have the followings:

$$\begin{aligned} \Phi_C^V &= \left[\sum_i \pi^i (\varphi^i)^{1/\sigma} \right]^\sigma; & \Phi_L^V &= \omega \left[\sum_i \pi^i (\varphi^i)^{-1/\nu} (\theta^i)^{(1+\nu)/\nu} \right]^{-\nu} \\ \Phi_C^P &= \Phi_C^V \sum_{i \in I} \pi^i \frac{\lambda^i}{\varphi^i} \psi_c^i; & \Phi_L^P &= \Phi_L^V \sum_{i \in I} \pi^i \frac{\lambda^i}{\varphi^i} \psi_l^i \\ \Phi_C^W &= \Phi_C^V \sum_{i \in I} \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i \right]; & \Phi_L^W &= \Phi_L^V \sum_{i \in I} \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i \right] \end{aligned}$$

²⁷This analysis is similar to [Tran-Xuan \(2021\)](#) in an economy with aggregate uncertainty.

D.2 Proof of Proposition 2.1

Proof. (\Rightarrow) Let $\{(C_t, L_t)\}_{t=0}^{\infty}$ be an aggregate allocation of an open economy competitive equilibrium with taxes. Then by definition, $\{(C_t, L_t)\}_{t=0}^{\infty}$ satisfies the aggregate resource constraint. Moreover, given any market weights φ , (C_t, L_t) satisfies

$$(1 - \tau_t^n)w_t = \frac{\Phi_L^V L_t^\nu}{\Phi_C^V C_t^{-\sigma}}$$

$$\frac{Q_t^d(s_{t+1})}{1 - \tau_t^d} = \beta \Pr(s_{t+1}|s^t) \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$$

Substituting for w_t into the budget constraint (2), and using $c_t^i = \psi_c^i C_t$, $l_t^i = \psi_l^i L_t$, gives the implementability constraint for each agent. Importantly, choose φ and T such that the individual implementability constraints hold.

(\Leftarrow) Given φ , T and an allocation $\{(C_t, L_t)\}_{t=0}^{\infty}$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\{w_t\}_{t=0}^{\infty}$ using the firm's first-order condition (3). $\{\tau_t^n\}_{t=0}^{\infty}$ can be calculated using the intra-temporal condition (8), and choosing $\{Q_t^d(s_{t+1}), \tau_t^d\}_{t=0}^{\infty}$ to satisfy the inter-temporal constraint (9).²⁸ Define $\{q_t\}_{t=0}^{\infty}$ by $q_t = \Pr(s^t)/(R^*)^t$.

Rewriting the aggregate resource constraint using $F(L) = wL$ gives

$$\sum_{t \geq 0, s^t} q_t \{C_t - (1 - \tau_t^n)w_t L_t + T_t\} + \sum_{t \geq 0, s^t} q_t [G_t - \tau_t^n w_t L_t - T_t] \leq -B_0 \quad (\text{D.1})$$

Aggregating up the agent's budget constraints implies

$$C_t + \sum_{s_{t+1}} Q_t^d(s_{t+1}) B_{t+1}^d = (1 - \tau_t^n)w_t L_t + (1 - \tau_t^d)B_t^d - T_t$$

or

$$C_t - (1 - \tau_t^n)w_t L_t + T_t = (1 - \tau_t^d)B_t^d - \sum_{s_{t+1}} Q_t^d(s_{t+1}) B_{t+1}^d$$

Substituting the last equation into (D.1) gives the government's budget constraint (4). Thus, $\{(C_t, L_t)\}_{t=0}^{\infty}$ is the aggregate allocation of the constructed competitive equilibrium with government policies. \square

²⁸There is indeterminacy in setting the price on domestic debt and the domestic savings tax. Any combination of $\{Q_t^d(s_{t+1}), \tau_t^d\}$ that satisfies the intertemporal constraint is part of the competitive equilibrium. Later on, I consider a particular implementation in which $Q^d = Q$, the price of external debt. τ^d then captures the difference between the domestic and external prices.

D.3 Proof of Proposition 3.1

First, I show that the following lemma must hold.

Lemma D.1. $\text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < 0$ and $\text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$

Proof. The first step is to show that for i and j such that $i \neq j$, $\theta^i > \theta^j \iff \varphi^i > \varphi^j$.

Suppose $\theta^i > \theta^j$ and $\varphi^i \leq \varphi^j$, then $\psi_l^i \leq \psi_l^j$. By the definitions of ψ_l , $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \leq \frac{\varphi^i}{\varphi^j} \leq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} > 1$, which is a contradiction.

Suppose $\varphi^i > \varphi^j$ and $\theta^i \leq \theta^j$, then $\psi_l^i > \psi_l^j$. By the definitions of ψ_l , $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} > \frac{\varphi^i}{\varphi^j} > 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \leq 1$, which is a contradiction.

Next, the individual implementability constraint is

$$\psi_c^i \Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma} - \psi_l^i \Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu} = \Phi_C^V C(s_0)^{-\sigma} (a^i(s_0) - T)$$

or

$$\psi_c^i = \psi_l^i \frac{\Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu}}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}} + \frac{\Phi_C^V C_0^{-\sigma} (b^i(s_0) - T)}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}}$$

By the definition of ψ_c^i , $\varphi^i > \varphi^j \iff \psi_c^i > \psi_c^j$, and by the assumption, $\theta^i > \theta^j \iff b^i(s_0) > b^j(s_0)$, which implies that $\theta^i > \theta^j \iff \psi_c^i > \psi_c^j \iff \psi_l^i > \psi_l^j$.

Thus, $\theta^i > \theta^j \iff \varphi^i > \varphi^j \iff \psi_c^i > \psi_c^j \iff \psi_l^i > \psi_l^j$.

In addition, $\theta^i > \theta^j \iff \lambda^i < \lambda^j$, which implies that

$$\begin{aligned} \psi_c^i > \psi_c^j &\iff \frac{\lambda^i}{\varphi^i} < \frac{\lambda^j}{\varphi^j} \\ \psi_l^i > \psi_l^j &\iff \frac{\lambda^i}{\varphi^i} < \frac{\lambda^j}{\varphi^j} \end{aligned}$$

Hence, $\text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < 0$ and $\text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$. □

Now I proceed to the main proof of the proposition.

Proof. In financial autarky, there exist a vector of market weights φ^a , transfer T^a , and multiplier η^a that satisfies the conditions in Proposition 2.1 such that

$$\begin{aligned} \underline{U}(z) &\equiv \max_{C_t, L_t, \varphi^a, T^a} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^{W,a} \log C_t - \Phi_L^{W,a} \frac{L_t^{1+\nu}}{1+\nu} \right] - \Phi_C^{V,a} C_0^{-\sigma} \sum_i \pi^i \eta^{i,a} T^a \\ \text{s.t.} \quad &C_t + G = z_t L_t \\ &z_0 = z \end{aligned}$$

where $\beta^t \pi^i \eta^{i,a}$ is the Lagrange multiplier on the individual implementability constraint and $\Phi_C^{W,a}, \Phi_L^{W,a}$ follows the formulas in Appendix D.1.

The optimal labor tax in autarky is constant over time and is equal to

$$\tau^{n,a} = 1 - \frac{\hat{\Phi}_L^V \hat{\Phi}_C^W}{\hat{\Phi}_C^V \hat{\Phi}_L^W} = 1 - \frac{\mathbb{E} \left[\frac{\lambda^i}{\varphi^i} \right] + \sigma \text{cov} \left(\psi_c^i, \frac{\lambda^i}{\varphi^i} \right)}{\mathbb{E} \left[\frac{\lambda^i}{\varphi^i} \right] - \nu \text{cov} \left(\psi_l^i, \frac{\lambda^i}{\varphi^i} \right)}$$

Lemma D.1 shows that $\text{cov} \left(\psi_c^i, \frac{\lambda^i}{\varphi^i} \right), \text{cov} \left(\psi_l^i, \frac{\lambda^i}{\varphi^i} \right) < 0$, which implies that

$$\mathbb{E} \left[\frac{\lambda^i}{\varphi^i} \right] + \sigma \text{cov} \left(\psi_c^i, \frac{\lambda^i}{\varphi^i} \right) < \mathbb{E} \left[\frac{\lambda^i}{\varphi^i} \right] - \nu \text{cov} \left(\psi_l^i, \frac{\lambda^i}{\varphi^i} \right)$$

or

$$\frac{\mathbb{E} \left[\frac{\lambda^i}{\varphi^i} \right] + \sigma \text{cov} \left(\psi_c^i, \frac{\lambda^i}{\varphi^i} \right)}{\mathbb{E} \left[\frac{\lambda^i}{\varphi^i} \right] - \nu \text{cov} \left(\psi_l^i, \frac{\lambda^i}{\varphi^i} \right)} < 1.$$

Thus, $\tau^{n,a} > 0$ □

D.4 Proof of Proposition 3.2

Proof. Suppose, by contradiction that the autarkic allocation $\{C_t^a, L_t^a, \varphi^a\}_{t=0}^\infty$ solves the planning problem given s_0 . Then the planning value at period 0 is $V_0^P(\{C_t^a, L_t^a, \varphi^a\}_{t=0}^\infty) = \underline{U}(z_0)$.

Fix $\epsilon > 0$. Consider the allocation $\{\hat{C}_t, \hat{L}_t, \varphi^a\}_{t=0}^\infty$ starting at s_0 and s_1 such that $\hat{C}_0 = C_0^a + \frac{\epsilon}{R^*} \mathbb{E}_{s_1|s_0} \left[F_L(L_1^a) - \frac{\Phi_L^{P,a}(L_1^a)^\nu}{\Phi_C^{P,a}(C_1^a)^{-\sigma}} \right]$, $\hat{L}_0 = L_0^a$, $\hat{C}_1 = C_1^a + \frac{\Phi_L^{P,a}(L_1^a)^\nu}{\Phi_C^{P,a}(C_1^a)^{-\sigma}} \epsilon$, $\hat{L}_1 = L_1^a + \epsilon$, and for any $t \geq 2$, $\hat{C}_t = C_t^a$, $\hat{L}_t = L_t^a$.

First, $\{\hat{C}_t, \hat{L}_t, \varphi^a\}_{t=0}^\infty$ is feasible because

$$\begin{aligned} & \sum_{t \geq 0} \mathbb{E}_0 \left(\frac{1}{R^*} \right)^t [F(\hat{L}_t) - \hat{C}_t - G_t] \\ &= F(\hat{L}_0) - \hat{C}_0 - G_0 + \frac{1}{R^*} \mathbb{E}_{s_1|s_0} [F(\hat{L}_1) - \hat{C}_1 - G_1] + \sum_{s_1} \mathbb{E}_{s_1|s_1} \sum_{t \geq 2} \left(\frac{1}{R^*} \right)^t [F(\hat{L}_t) - \hat{C}_t - G_t] \\ &= \sum_{t \geq 0} \mathbb{E}_0 \left(\frac{1}{R^*} \right)^t [F(L_t^a) - C_t^a - G_t] - \frac{\epsilon}{R^*} \mathbb{E}_{s_1|s_0} \left[F_L(L_1^a) - \frac{\Phi_L^{P,a}(L_1^a)^\nu}{\Phi_C^{P,a}(C_1^a)^{-\sigma}} \right] + \frac{1}{R^*} \mathbb{E}_{s_1|s_0} \left(F_L(L_1^a) \epsilon - \frac{\Phi_L^{P,a}(L_1^a)^\nu}{\Phi_C^{P,a}(C_1^a)^{-\sigma}} \epsilon \right) \\ &= \sum_{t \geq 0} \mathbb{E}_0 \left(\frac{1}{R^*} \right)^t [F(L_t^a) - C_t^a - G_t] \\ &\geq B_0 \end{aligned}$$

Second, $\{\hat{C}_t, \hat{L}_t, \varphi^a\}_{t=0}^\infty$ is implementable in equilibrium since there exists a \hat{T} such that $\forall i \in I$

$$\sum_{t \geq 0, s^t \in S^t} \beta^t \left[\Phi_C^{V,a} \psi_c^{i,a} \hat{C}_t^{1-\sigma} - \Phi_L^{V,a} \psi_l^{i,a} \hat{L}_t^{1+\nu} \right] \geq \Phi_C^{V,a} \hat{C}_0^{-\sigma} (b^i(s^0) - \hat{T})$$

The flow utilities for all period $t \geq 1$ do not change. That is,

$$\begin{aligned}
u_1^P(\hat{C}_1, \hat{L}_1, \varphi^a) &= \Phi_C^{P,a} \frac{(\hat{C}_1)^{1-\sigma}}{1-\sigma} - \Phi_L^{P,a} \frac{(\hat{L}_1)^{1+\nu}}{1+\nu} = \Phi_C^{P,a} \frac{(C_1^a)^{1-\sigma}}{1-\sigma} + \Phi_C^{P,a} (C_1^a)^{-\sigma} \frac{\Phi_L^{P,a} (L_1^a)^\nu}{\Phi_C^{P,a} (C_1^a)^{-\sigma}} \epsilon - \\
&\Phi_L^{P,a} \frac{(L_1^a)^{1+\nu}}{1+\nu} - \Phi_L^{P,a} (L_1^a)^\nu \epsilon = u_1^p(C_1^a, L_1^a, \varphi^a) \\
u_t^P(\hat{C}_t, \hat{L}_t, \varphi^a) &= u_t^p(C_t^a, L_t^a, \varphi^a), \forall t \geq 2 \\
&\text{and the flow utility in period 0 increases. That is,}
\end{aligned}$$

$$\begin{aligned}
u_0^P(\hat{C}_0, \hat{L}_0, \varphi^a) &= \Phi_C^{P,a} \frac{(\hat{C}_0)^{1-\sigma}}{1-\sigma} - \Phi_L^{P,a} \frac{(\hat{L}_0)^{1+\nu}}{1+\nu} \\
&= \Phi_C^{P,a} \frac{(C_0^a)^{1-\sigma}}{1-\sigma} + \Phi_C^{P,a} (C_0^a)^{-\sigma} \frac{\epsilon}{R^*} \mathbb{E}_{s_1|s_0} \left[F_L(L_1^a) - \frac{\Phi_L^{P,a} (L_1^a)^\nu}{\Phi_C^{P,a} (C_1^a)^{-\sigma}} \right] - \Phi_L^{P,a} \frac{(L_0^a)^{1+\nu}}{1+\nu} \\
&= u_0^p(C_0^a, L_0^a, \varphi^a) + \Phi_C^{P,a} (C_0^a)^{-\sigma} \frac{\epsilon}{R^*} \mathbb{E}_{s_1|s_0} \left[F_L(L_1^a) - \frac{\Phi_L^{P,a} (L_1^a)^\nu}{\Phi_C^{P,a} (C_1^a)^{-\sigma}} \right]
\end{aligned}$$

Lemma D.1 implies that $\frac{\Phi_L^{P,a} (L_1^a)^\nu}{\Phi_C^{P,a} (C_1^a)^{-\sigma}} < \frac{\Phi_L^{W,a} (L_1^a)^\nu}{\Phi_C^{W,a} (C_1^a)^{-\sigma}} = F_L(L_1^a), \forall s_1$, so $u_0^P(\hat{C}_0, \hat{L}_0, \varphi^a) > u_0^p(C_0^a, L_0^a, \varphi^a)$. Thus, $V_0^P(\{\hat{C}_t, \hat{L}_t, \varphi^a\}_{t=0}^\infty) > V_0^P(\{C_t^a, L_t^a, \varphi^a\}_{t=0}^\infty)$, which contradicts $\{C_t^a, L_t^a, \varphi^a\}_{t=0}^\infty$ being the optimal allocation. \square

E Quantitative Appendix

This section provides additional details that is implemented in Section 4.

E.1 Deviation Utility

The deviation utility $\underline{U}(z)$ is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes of a closed economy where the government does not issue both domestic and external debts.

$$\underline{U}(z) \equiv \max_{c_t^i, l_t^i, \tau_t^n, T} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right]$$

$$\begin{aligned}
s.t. \quad & C_t + G = z_t L_t \\
& c_t^i + \sum_{s_{t+1}} Q_t(s_{t+1}) b_{t+1}^{d,i} = (1 - \tau_t^n) z_t l_t^i + b_t^{d,i} - T_t \\
& (1 - \tau_t^n) z_t = -\frac{1}{\theta^i} \frac{\omega \left(\frac{l_t^i}{\theta^i} \right)^\nu}{(c_t^i)^{-\sigma}} \\
& Q_t(s_{t+1}) = \beta \Pr(s^{t+1} | s^t) \frac{(c_{t+1}^i)^{-\sigma}}{(c_t^i)^{-\sigma}} \\
& b_0^{d,i} = b_0^{d,j}, \forall i, j \in I \\
& z_0 = z
\end{aligned}$$

There exist a vector of market weights φ^a , transfer T^a , and multiplier η^a that satisfies the conditions in Proposition 2.1 such that

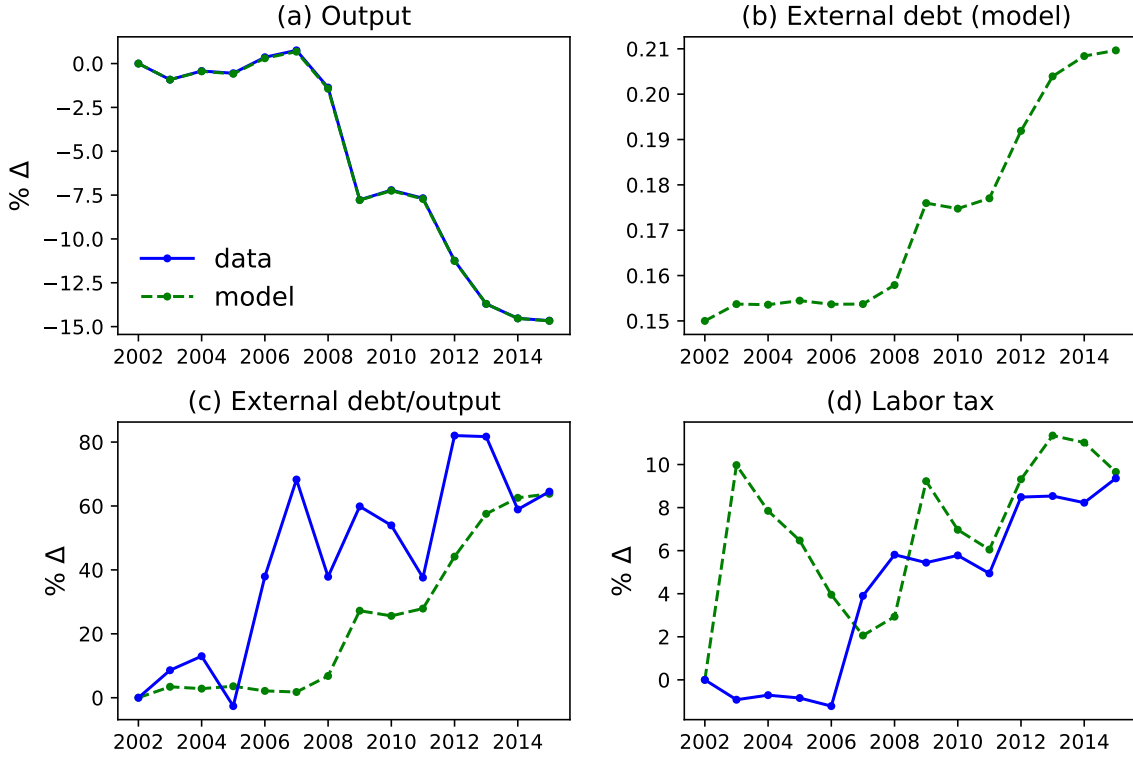
$$\begin{aligned}
\underline{U}(z) \equiv & \max_{C_t, L_t, \varphi^a, T^a} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^{W,a} \log C_t - \Phi_L^{W,a} \frac{L_t^{1+\nu}}{1+\nu} \right] - \Phi_C^{V,a} C_0^{-\sigma} \sum_i \pi^i \eta^{i,a} T^a \\
s.t. \quad & C_t + G = z_t L_t \\
& z_0 = z
\end{aligned}$$

where $\beta^t \pi^i \eta^{i,a}$ is the Lagrange multiplier on the individual implementability constraint and $\Phi_C^{W,a}, \Phi_L^{W,a}$ follows the formulas in Appendix D.1.

E.2 Event Analysis

I conduct an event analysis for Italy in period of 2002 to 2015. I feed into the model a sequence of productivity shock realizations and the initial external debt-to-output ratio such that the model's sequence of outputs matches the sequence of Italian output and the change in external debt-to-output ratio is the same both in the model and the data from 2002 to 2015. I then compare the relative change of external debt-to-output in the data and in the model's simulation over time, given that 2002 is the benchmark year. Figure 7 plots the exercise's results. Panel (a) plots the output paths of the data and the model. Panel (b) plots the time path of external debt in the model. Panel (c) plots external debt-to-output time paths for both the data and the model. Panel (d) plots estimated labor taxes from [McDaniel \(2007\)](#) and the optimal labor taxes from the model. From 2011 to 2015, Italy's output has dropped by 7.6% below trend, accompanying with a 20% increase in external debt-to-output and a 4.2% increase in labor tax. In the model's simulation, the similar drop in output is associated with a 26% increase in external debt-to-output and a 3.5% increase in labor tax.

Figure 7: Italy's Recession: Data and Model



Note: The graph depicts the time paths of output, external debt, and external debt-to-output for the data and the model's simulation. Panel (a) plots the output path. Panel (b) plots the external debt paths of the model. Panel (c) plots external debt-to-output, and panel (d) plots the labor tax. The simulation uses a sequence of productivity shock realization such that the model's output matches the data output for Italy in 2002-2015. The initial external debt level is such that the model's external debt-to-output matches with the starting value in 2002 from the data. The benchmark period is 2002. Data sources: [McDaniel \(2007\)](#), [Lane and Milesi-Ferretti \(2018\)](#), and [The World Bank \(2019\)](#).

E.3 Computational Algorithm

Given the forward-looking borrowing constraints, I implement the recursive formulation developed by [Marcet and Marimon \(2019\)](#). The key co-state variable is the discounted sum of the Lagrange multipliers on the borrowing constraints: $\Gamma_t = (\beta R^*)^t \sum_{k \leq t} \gamma_k$. Γ_t reflects the marginal benefit of relaxing the current and previous borrowing constraints from increasing one unit of utility in period t and history s^t (either by giving more consumption or leisure). In the long run, Γ corresponds to the amount of external debt that can be sustained in equilibrium. Impatience implies that in the long run, there exists a positive component in the price of borrowing coming from the lack of commitment. If there exists an ergodic distribution of the efficient allocation, then Γ will also follow an ergodic distribution. This property allows the computational algorithm using Γ as one of the state variables to converge.

1. Guess μ and φ . Compute η .

(a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ

Initial guess for $V(s_t, \mu_t, \Gamma_{t-1}) = \sum_{j \geq 0, s^t \subseteq s^{t+j}} \beta^j \Pr(s^{t+j}) \left[\Phi_C^P \frac{C(s^{t+j})^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^{t+j})^{1+\nu}}{1+\nu} \right]$.

- (b) Assume the constraint does not bind in s_t : $\gamma(s_t) = 0$. Solve for the allocation $C(s_t), L(s_t)$ using the first-order conditions

$$\begin{aligned} [\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1}] C(s_t)^{-\sigma} &= \mu \\ [\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1}] L(s_t)^\nu &= \mu F_L(s_t) \end{aligned}$$

- (c) Since $\gamma(s_t) = 0$, compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t . Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$ (interpolating the expectation), then compute

$$\begin{aligned} A(s_t) &= \sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^\tau \subseteq s^t} Pr(s^\tau) \left[\Phi_C^P \frac{C(s^\tau)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^\tau)^{1+\nu}}{1+\nu} \right] \\ &= \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1}) \end{aligned}$$

- (d) Check if $A(s_t) \geq \underline{U}(s_t)$. If it is, proceed to the next step. If not, solve for $C(s_t), L(s_t), \gamma(s_t)$ using these equations

$$\begin{aligned} [\mu_t \Phi_C^W + \Phi_C^V (\Gamma_{t-1} + \gamma(s_t))] C(s_t)^{-\sigma} &= \mu \\ [\mu_t \Phi_L^W + \Phi_L^V (\Gamma_{t-1} + \gamma(s_t))] L(s_t)^\nu &= \mu F_L(s_t) \\ \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} \\ + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* (\Gamma_{t-1} + \gamma(s_t))) &= \underline{U}(s_t) \end{aligned}$$

- (e) Given $C(s_t), L(s_t), \gamma(s_t)$ (γ can be zero or not), compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t . Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \Gamma_{t-1} + \gamma(s_t))$. Update the value function

$$\begin{aligned} V^{n+1}(s_t, \Gamma_{t-1}) &= \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} \\ &+ \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V^n(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* (\Gamma_{t-1} + \gamma(s_t))) \end{aligned}$$

2. Compute residuals to find μ and φ

$$\begin{aligned} r^\mu &= \sum_{t \geq 0} q_t [F(L_t) - G_t - C_t] - B_0 \\ r^\varphi &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^V (\psi_c^i - \psi_c^j) C_t^{1-\sigma} - \Phi_L^V (\psi_l^i - \psi_l^j) L_t^{1+\nu} \right] \end{aligned}$$

3. Find μ and φ such that $r^\mu = 0$ and $r^\varphi = 0$.