Sovereign Debt Sustainability and Redistribution*

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February 2026

Abstract

This paper develops a theory of sovereign debt sustainability driven by the government's motive for redistribution. It studies a heterogeneous-agent small open economy in which redistribution relies on distortionary labor taxation and the government lacks commitment in its fiscal policies. Access to international credit markets lowers the cost of redistribution, while default into financial autarky raises it, generating an endogenous cost of default. Quantitatively, the model accounts for the buildup of Italy's external debt and the positive cross-country correlation between pre-tax income inequality and external debt. Optimal austerity is more gradual when distributional concerns are present.

Keywords: Inequality; Limited commitment; Optimal taxation; Redistribution; Sovereign debt

JEL Classifications: E62; F38; F41; H21; H23; H63

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*Disclaimer: The views expressed here are those of the author and not necessarily of the IMF. I thank V.V. Chari, Larry Jones, and Manuel Amador for their invaluable advice and guidance. I am grateful for comments from Fernando Arce, Cristina Arellano, Anmol Bhandari, Jarda Borovicka, Emine Boz, Jonathan Heathcote, Tim Kehoe, Ben Malin, Chris Phelan, Ignacio Presno, David Rahman, Paulina Restrepo-Echavarria, Aldo Rustichini, Jan Werner, the participants at the University of Minnesota, the Minneapolis Fed, the St. Louis Fed, NTA, Dehli School of Economics Winter School, CeMent, Midwest Macro, WAIFS, and RIDGE International Macro Workshop. This research was supported by the Center for Computational Research at the University at Buffalo.

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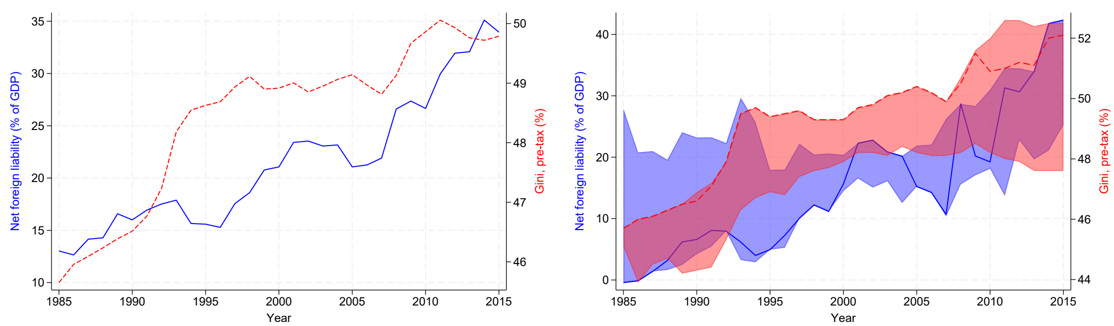
Introduction

Debt crises raise questions about the design of fiscal policy during economic downturns, when output contracts and external debt accumulates until constrained by repayment capacity. Austerity measures, through higher taxes or lower government spending, can expand repayment capacity, but they do so by imposing unequal burdens across residents. Governments therefore face a trade-off between debt sustainability and redistribution when setting fiscal policy. This trade-off has become increasingly salient as inequality and external debt have risen across many economies. Figure 1 documents this joint evolution for a broad set of countries with positive external debt.[1]



(a) GDP-weighted average (b) GDP-weighted interquartile range

Figure 1: Income Inequality and External Debt

Note: Figure 1 plots the time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) for countries with positive external debt on average in 1985-2015. Panel (a) plots the GDP-weighted average, and Panel (b) plots the GDP-weighted interquartile range. The left y-axis depicts the values in net foreign liablity-to-GDP (%), and the right y-axis depicts the values in pre-tax Gini (%) . See Appendix **??** for the list of countries. Sources: Lane and Milesi-Ferretti (2018), and Solt (2019).

Motivated by these observations, the paper studies how redistributive motives affect a government's incentives to sustain external debt and how inequality

[1]External debt is defined as the negative of the net foreign asset–to-GDP ratio from the External Wealth of Nations database of Lane and Milesi-Ferretti (2018). Inequality is measured by the pre-tax (market) Gini indices from the Standardized World Income Inequality Database (SWIID) of Solt (2019). Panel (a) reports the averages across countries that have positive external debt in 1985-2015. See Appendix **??** for the list of countries.

shapes the design of optimal austerity policies. To do so, it introduces redistributive concerns into a sovereign debt model with limited commitment, highlighting the trade-off between redistribution and debt sustainability.

The paper makes three main contributions. First, it develops a theory of external debt sustainability driven by redistributive motives. Costly redistribution generates an endogenous cost of default: default into financial autarky increases reliance on distortionary taxation, making redistribution more costly in autarky than under debt repayment. Second, the paper quantifies the redistributive cost of default by calibrating the model to Italy as an illustrative case.[2] Redistribution accounts for roughly 60 percent of Italy's long-run external debt, compared to only 12 percent explained by the standard aggregate insurance channel. The framework is also consistent with both the upward trend in Italian debt and the positive cross-country correlation between inequality and external debt. Third, the paper examines the design of austerity policies in the presence of inequality. The optimal adjustment to a negative productivity shock is gradual: the government initially expands borrowing and redistribution, then subsequently consolidates its budget by raising taxes and reducing redistribution.

The mechanism operates through the cost of redistribution, which arises from the need to use distortionary labor taxes to reallocate resources across agents. The government therefore faces a trade-off between redistributive benefits and efficiency costs, and the costs increase with inequality or stronger redistributive motives. Sustaining external debt mitigates these costs by allowing the government to substitute away from distortionary taxation. Impatient private agents borrow abroad without internalizing that their borrowing tightens the limited commitment constraint, creating an overborrowing externality. An optimal borrowing tax corrects this externality while also providing redistributive benefits, since high-skilled agents hold more foreign debt and thus bear a larger share of the tax burden. In contrast, default into financial autarky removes this instrument and forces the government to rely solely on distortionary taxes for redistribution. The cost of default associated with redistribution makes positive debt sustainability preferable.

[2] Italy has no history of sovereign default, but episodes such as the European sovereign debt crisis highlight the relevance of debt distress shaping fiscal constraints and policy trade-offs. The mechanism would apply to many other countries, including emerging markets.

Consequently, higher inequality implies a higher equilibrium level of sustainable debt. By raising the distortionary costs of redistribution under default, inequality strengthens incentives to repay and sustain higher debt. This channel maps inequality into external indebtedness in a way consistent with the observed comovement across countries.

The mechanism also has implications for overborrowing and macroprudential policy. Because private agents do not internalize how individual borrowing tightens the sovereign's limited commitment constraint, decentralized borrowing exceeds the socially optimal level, and macroprudential borrowing taxes improve debt sustainability while altering the distribution of tax burdens across agents.

The findings highlight the importance of inequality for sovereign borrowing and fiscal policy. The theoretical results imply that costly redistribution generates an endogenous cost of default, linking inequality directly to sustainable debt levels. The quantitative analysis demonstrates that this channel is quantitatively relevant in both Italy and cross-country data. The policy finding provides a rationale for more gradual austerity adjustments in unequal economies.

Related literature. This paper contributes to the sovereign debt literature with limited commitment. Seminal work emphasizes self-enforcing borrowing constraints and the role of default risk in international borrowing (Eaton and Gersovitz, 1981; Bulow and Rogoff, 1989; Aguiar and Amador, 2011; Broner and Ventura, 2011; Aguiar and Amador, 2014, 2016). Later work examines the quantitative implications of these models, highlighting volatile consumption, taxation, and fiscal policies (e.g., Kehoe and Perri, 2002; Cuadra et al., 2010; Pouzo and Presno, 2015; Arellano and Bai, 2016; Karantounias, 2018). This paper adds to the literature by introducing heterogeneity and redistributive concerns into a limited commitment framework, showing how costly redistribution shapes sustainable debt. The framework is closely related to Tran-Xuan (2023), who finds that front-loading redistribution is optimal in the presence of debt constraints.

The paper also relates to a growing literature linking inequality and sovereign default risk. Recent work shows that inequality affects sovereign risk premia, debt dynamics, and fiscal policy responses (Ferriere, 2015; Balke and Ravn, 2016; D'Erasmo and Mendoza, 2016, 2020; Dovis et al., 2016; Jeon and Kabukcuoglu, 2018; Bianchi et al., 2023; Tran-Xuan, 2023). The contribution here is to identify costly redistribution as a novel source of endogenous default costs, complementing previously studied mechanisms such as efficiency losses in production (Mendoza and Yue, 2012) or unemployment dynamics (Balke, 2017).

Finally, the paper connects to the public finance literature on redistribution and debt management (Barro, 1979; Lucas and Stokey, 1983; Chari et al., 1994; Bhandari et al., 2016, 2017; Jiang et al., 2022). Werning (2007) shows that debt is used to smooth redistribution costs over time, and Bhandari et al. (2017) emphasize the role of government debt as a safe asset that allows the extraction of monopoly rents. In contrast, this paper focuses on how external debt reduces the overall cost of redistribution. It also contributes to the policy debate by showing that redistribution concerns provide a rationale for more gradual fiscal adjustment during episodes of austerity.

Outline. The paper is organized as follows. Section 1 sets up the environment and competitive equilibrium. Section 2 defines the sustainable equilibrium and establishes the main theoretical result. Section 3 illustrates costly redistribution in an example economy. Section 4 presents the quantitative results and implications for austerity. Section 5 provides empirical evidence. Section 6 concludes.

1 A Model of Sovereign Debt and Inequality

This section sets up a small open economy model with aggregate uncertainty, heterogeneous agents, and a benevolent government. The competitive equilibrium is characterized by aggregate allocations and a time-invariant distribution of marginal utility shares.

1.1 Environment

A small open economy consists of a measure-one continuum of infinitely-lived agents differentiated by labor productivity types $(\theta^i)_{i\in I}$, which are publicly observable. The fraction of agents with productivity θ^i is π^i, where $(\pi^i)_{i\in I}$ and $(\theta^i)_{i\in I}$ are normalized such that $\sum_{i\in I}\pi^i = 1$ and $\sum_{i\in I}\pi^i\theta^i = 1$. All agents have the same discount factor β and the static utility $U(c,n)$ over consumption c and hours worked n. The utility of an agent with productivity θ^i (agent i) is

$$\sum_{t=0}^{\infty} \beta^t U(c_t^i, n_t^i) \tag{1}$$

Define the labor supply of agent i in efficiency units as $l^i = \theta^i n^i$. Then, the utility function can be rewritten as $\sum_{t=0}^{\infty} \beta^t U^i(c_t^i, l_t^i)$, where $U^i(c,l) = U\left(c, \frac{l}{\theta^i}\right)$. An individual allocation specifies consumption and labor in every period (c^i, l^i). The aggregate allocation is defined as $C \equiv \sum_{i\in I} \pi^i c^i$ and $L \equiv \sum_{i\in I} \pi^i l^i$.

There is a representative firm that uses labor to produce a single final good with the constant-returns-to-scale production function $F(L,t)$, where L is the aggregate labor.

Government spending $\{G_t\}_{t=0}^{\infty}$ is exogenous. In every period t, the government levies a lump-sum tax T_t, a marginal tax on labor income τ_t^n, and a marginal tax on the return on private saving τ_t^a. The tax rates are uniform across agents.

Domestic and international financial markets are competitive. Both the government and private agents have access to domestic and international financial markets. The exogenous risk-free international interest rate for borrowing is $\{r_t^*\}_{t=0}^{\infty}$, and $R_t^* = 1 + r_t^*$ denotes the gross risk-free interest rate in period t. Let $Q_t^* = \Pi_{\tau=1}^{t} \frac{1}{1+r_\tau^*}$ be the international price of one unit of period-t consumption in units of period-0 consumption, and normalize $Q_0^* = 1$.

1.2 Competitive Equilibrium

Private agents. Private agents have access to both domestic and international financial markets. Let q_{t+1}^d and q_{t+1}^f be the prices of one unit of domestic and foreign bonds in period $t+1$. The sequential budget constraint of agent i in period t is

$$c_t^i + q_{t+1}^d a_{t+1}^{d,i} + q_{t+1}^f a_{t+1}^{f,i} \leq (1-\tau_t^n) w_t l_t^i + (1-\tau_t^a) a_t^{d,i} + (1-\tau_t^a) a_t^{f,i} - T_t, \tag{2}$$

where $c_t^i, l_t^i, a_t^{d,i}, a_t^{f,i}$ denote the consumption, labor, and domestic and foreign bond holdings of agent i in period t, respectively. The savings tax τ^a applies to both domestic and foreign bond holdings of the private agents. The later analysis considers $-\tau^a$ as the subsidy on private savings, or tax on private borrowing.

No-Ponzi conditions for both bonds apply. No arbitrage implies that

$$q_t^d = q_t^f = \frac{1}{1 + r_t^*} \tag{3}$$

Define $a_t^i = a_t^{d,i} + a_t^{f,i}$ as the bond holdings of agent i. The aggregate private domestic asset and private foreign asset are $A^d = \sum_{i \in I} \pi^i a^{d,i}$, $A^f = \sum_{i \in I} \pi^i a^{f,i}$. Define the total private asset as $A = \sum_{i \in I} \pi^i a^i = A^d + A^f$. The later analysis considers $-A$ as the total private liability.

Representative firm. The firm takes as given the wage w_t and chooses aggregate labor L_t to maximize profit

$$\max_{L_t} \quad F(L_t, t) - w_t L_t,$$

which gives the following first-order condition:

$$w_t = F_L(L_t, t). \tag{4}$$

The firm's profit is zero in equilibrium because of the constant-returns-to-scale production function.

Government. Let B_t^d and B_t^f be the government's domestic and foreign bonds. The government's current budget constraint is

$$G_t + B_t^d + B_t^f \leq \tau_t^n w_t L_t + \tau_t^a \left(A_t^d + A_t^f \right) + T_t + q_{t+1}^d B_{t+1}^d + q_{t+1}^f B_{t+1}^f \tag{5}$$

Define $B_t^g = B_t^d + B_t^f$ as the total government liability. There is a no-Ponzi condition such that the present value of government debt is bounded below. Therefore, using equation (5), one can derive the present-value government's budget constraint as

$$\sum_{t=0}^{\infty} Q_t^* \left[\tau_t^n w_t L_t + \tau_t^a A_t + T_t - G_t \right] \geq B_0^g, \tag{6}$$

that is, the present-value of the government's primary balance is at least the initial value of total government debt.

Resource constraint. Define $B_t = B_t^f - A_t^f$ as the external liability of the economy, which includes both private agents' and the government's liabilities. The present-value aggregate resource constraint of the economy is

$$\sum_{t=0}^{\infty} Q_t^* \left[F(L_t, t) - C_t - G_t \right] \geq B_0, \tag{7}$$

that is, the present-value of the economy's net resources is at least the initial value of external debt.

Competitive equilibrium. Given the above equations, one can define the following competitive equilibrium with government policies.

Definition 1.1. Given initial external debt B_0 and individual private bond holdings $(a_0^i)_{i\in I}$, a competitive equilibrium with government policies for an open economy is individual agent's allocation $z^{H,i} = \left\{ \left(c_t^i, l_t^i, a_{t+1}^{d,i}, a_{t+1}^{f,i} \right) \right\}_{t=0}^{\infty}, \ \forall i \in I$, the representative firm's allocation $z^F = \{L_t\}_{t=0}^{\infty}$, prices $p = \left\{ q_t^d, q_t^f, w_t \right\}_{t=0}^{\infty}$, and government's policy $z^G = \left\{ \tau_t^n, \tau_t^a, T_t, B_t^d, B_t^f \right\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves individual i's problem that maximizes (1) subject to (2) and the no-Ponzi conditions of the agent's debts, (ii) given p and z^G, z^F solves the firm's problem, (iii) the government budget constraint (6) holds, (iv) the aggregate resource constraint (7) is satisfied, (v) the domestic bond market clears $B_t^d = A_t^d$, and (vi) p satisfies equations (3) and (4) given z^G.

1.3 Characterizing Competitive Equilibrium

Following Werning (2007), we show that the competitive equilibrium allocation can be characterized by a set of aggregate allocation and a time-invariant distribution of marginal utility. In equilibrium, the intra-temporal and inter-temporal rates of substitution are the same across agents, i.e., in each period t and for any individual i,

$$\begin{aligned} (1-\tau_t^n) w_t &= -\frac{U_l^i(c_t^i, l_t^i)}{U_c^i(c_t^i, l_t^i)} \\ 1-\tau_{t+1}^a &= \frac{U_c^i(c_t^i, l_t^i)}{\beta R_{t+1}^* U_c^i(c_{t+1}^i, l_{t+1}^i)}. \end{aligned}$$

Therefore, there exist a set of Negishi weights $\boldsymbol{\varphi} = (\varphi^i)_{i\in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i \varphi^i = 1$, such that individual allocation solves a static problem

$$\begin{aligned} V(C, L; \boldsymbol{\varphi}) \equiv \max_{(c^i, l^i)_{i\in I}} & \sum_{i\in I} \varphi^i \pi^i U^i(c^i, l^i) \\ s.t. \qquad & \sum_{i\in I} \pi^i c^i = C; \quad \sum_{i\in I} \pi^i l^i = L. \end{aligned}$$

This problem gives the allocation rule for individual i that is time-invariant and proportional to the aggregate allocation

$$h^i(C, L; \boldsymbol{\varphi}) = \left(c^i(C, L; \boldsymbol{\varphi}),\ l^i(C, L; \boldsymbol{\varphi})\right).$$

The intra-temporal and inter-temporal conditions become

$$(1 - \tau_t^n) w_t = -\frac{V_L(C_t, L_t; \boldsymbol{\varphi})}{V_C(C_t, L_t; \boldsymbol{\varphi})} \tag{8}$$

$$1 - \tau_{t+1}^a = \frac{V_C(C_t, L_t; \boldsymbol{\varphi})}{\beta R_{t+1}^* V_C(C_{t+1}, L_{t+1}; \boldsymbol{\varphi})}, \tag{9}$$

and the implementability constraint of individual i is

$$\sum_{t=0}^{\infty} \beta^t \left[V_C(C_t, L_t; \boldsymbol{\varphi}) c^i(C_t, L_t; \boldsymbol{\varphi}) + V_L(C_t, L_t; \boldsymbol{\varphi}) l^i(C_t, L_t; \boldsymbol{\varphi})\right] = V_C(C_0, L_0; \boldsymbol{\varphi}) \left(a_0^i - T\right), \tag{10}$$

where $T \equiv \sum_{t=0}^{\infty} \beta^t \frac{V_C(C_t, L_t; \boldsymbol{\varphi})}{V_C(C_0, L_0; \boldsymbol{\varphi})} T_t$ is the present value of lump-sum taxes.

The following proposition summarizes the competitive equilibrium characterization.

Proposition 1.1. *Given the initial external debt B_0 and individual bond holdings$(a_0^i)_{i\in I}$, an allocation $\{C_t, L_t\}_{t=0}^{\infty}$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if the resource constraint (7) holds, and there exist market weights $\boldsymbol{\varphi} = (\varphi^i)_{i\in I}$ and lump-sum tax T such that the implementability constraint (10) holds for all $i \in I$.*

Proof. See Appendix. □

2 Optimal Debt Sustainability

This section introduces a definition of debt sustainability for a government that cares about redistribution but lacks commitment in its fiscal policies. It then shows that, given certain assumptions, sustaining a positive level of external debt is optimal in the long run.

2.1 Sustainable Equilibrium

Two sources of time inconsistency arise in this environment. First, the government has an incentive to default on positive external debt to increase private consumption and leisure. Second, persistent wealth heterogeneity generates incentives for an inequality-averse government to expropriate and redistribute wealth. A sustainable debt policy is therefore defined as the ex-ante optimal, time-consistent policy under which the government has no incentive to default or renege on its tax commitments.

The government's objective is first specified, followed by a definition of the sustainable equilibrium in the absence of commitment. The resulting sustainable allocation then characterizes the sustainable debt policy.

Given a set of social welfare weights $\boldsymbol{\lambda} = (\lambda^i)_{i \in I}$, the government's objective is the weighted utility of all domestic agents

$$\sum_{i \in I} \lambda^i \pi^i \sum_{t=0}^{\infty} \beta^t U^i(c_t^i, l_t^i). \tag{11}$$

Following Chari and Kehoe (1990, 1993), the sustainable equilibrium is a subgame perfect equilibrium of a repeated game between the government, a continuum of domestic agents, and a continuum of external creditors. The sustainable equilibrium is characterized by the competitive equilibrium conditions described in Proposition 1.1 and the following sustainability constraint

$$\sum_{i \in I} \lambda^i \pi^i \sum_{k \geq t} \beta^{k-t} U^i(c_k^i, l_k^i) \geq \underline{U}_t, \ \forall t, \tag{12}$$

where $\underline{U}_t$ is the one-shot deviation value in which the government defaults on its debt and fully redistributes wealth among private agents.[3] The government then faces punishment imposed by private agents and external lenders for deviating from the contracted policies. As argued in Chari and Kehoe (1990), $\underline{U}_t$ is a value of a sustainable equilibrium. If it is the worst sustainable equilibrium value, then $\underline{U}_t$ represents the notion of the worst punishment, which will support the best sustainable equilibrium. The next subsection will focus on the case of financial autarky as the punishment. Constraint (12) imposes a limit on the utility, which endogenously determines a limit on external debt in every period.

Given the above setup, a sustainable allocation and a sustainable external debt policy are defined as follows.

Definition 2.1. A sustainable allocation $(\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*)$ maximizes the social welfare function (11) and satisfies the conditions in Proposition 1.1 and the sustainability constraint (12) for a given function $\underline{U}_t$. A sustainable external debt policy is $\{B_t^*\}_{t=0}^{\infty}$ that is consistent with the sustainable allocation in equilibrium.

The sustainable allocation solves the following contracting problem.

$$
\begin{aligned}
(P) \equiv \max_{\{C_t, L_t\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T} & \sum_{t=0}^{\infty} \beta^t U^P(C_t, L_t; \boldsymbol{\varphi}, \boldsymbol{\lambda}) \\
s.t. \qquad & \sum_{t=0}^{\infty} Q_t^* \left[F(L_t) - C_t - G_t\right] - B_0 \geq 0 \\
\forall i, \; & \sum_{t=0}^{\infty} \beta^t \left[V_C(t; \boldsymbol{\varphi}) c^i(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) l^i(t; \boldsymbol{\varphi})\right] \geq V_C(0; \boldsymbol{\varphi}) \left(a_0^i - T\right) \\
\forall t, \; & \sum_{k \geq t} \beta^{k-t} U^P(C_k, L_k; \boldsymbol{\varphi} \boldsymbol{\lambda}) \geq \underline{U}_t.
\end{aligned}
$$

where $U^P(C, L; \boldsymbol{\varphi}, \boldsymbol{\lambda}) = \sum_{i \in I} \lambda^i \pi^i U^i \left[c^i(C, L; \boldsymbol{\varphi}), l^i(C, L; \boldsymbol{\varphi})\right]$.

The first constraint is the resource constraint. The second constraint is the implementability constraint that takes into account the distortionary effect of the government's policies on individual decisions. The last constraint is the sustainability constraint due to the government's lack of commitment. Appendix C provides the characterization of this optimal contracting problem.

[3] See Appendix B for the formal setup of the sovereign game and its characterization.

2.2 Optimal Debt Policy

This subsection establishes the main theoretical result that the optimal policy is to sustain positive external debt in the long run. The proof proceeds under the following assumptions.

Assumption 1 (Separable Utility Function)**.** *The utility function satisfies (i) additive separability:* $U(c,n) = u(c) - v(n)$*, where (ii)* u, v *are twice differentiable with* $u^{'}, v^{'} > 0, u^{"} < 0$ *and* $v^{"} < 0$ *for all* $c > 0$ *and* $0 < n < \bar{n}$ *for some* $\bar{n}$*, and (iv) bounded marginal utilities and elasticities* $u^{'}, v^{'}, -u^{''}/u^{'}, v^{''}/v^{'}, -u^{''}c/u^{'}, v^{''}n/v^{'}$ *are bounded functions in* $(c,n) \in (\epsilon_c, \infty) \times (0, \epsilon_n)$ *for some* $\epsilon_c > 0$ *and* $\epsilon_n \in (0, \bar{n})$

The first assumption eliminates the cross-effects in marginal utilities between consumption and labor and allows the analysis to be separate in terms of the effects on consumption and labor supply. The second assumption is standard in that the utility functions satisfy monotonicity and concavity. The last assumption ensures that the marginal utilities and elasticities are well behaved as consumption becomes large or labor approaches zero. These assumptions hold for several preferences commonly used in macroeconomics, including the separable isoelastic preferences $U(c,n) = c^{1-\sigma}/(1-\sigma) - \omega n^{1+\nu}/(1+\nu)$ with $\sigma, \omega, \nu > 0$.

In addition, private agents are more impatient than the international financial markets in the limit, that is,

Assumption 2 (Impatience)**.** *There exists* $0 < \mathcal{M} < 1$ *and* $\mathcal{T}$ *such that for all* $t > \mathcal{T}$, $\beta R_t^* < \mathcal{M} < 1$.

Impatience implies a need for the country to accumulate external debt in the long run, as it is cheaper to borrow abroad.

For what follows, the deviation utility in the sustainable equilibrium is assumed to be the value of financial autarky, that is,

Assumption 3. $\underline{U}_t$ *is the value of the economy under financial autarky that is de-*

fined as

$$\begin{aligned}
\underline{U}_t \equiv \max_{\tau_t^n, T_t} & \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i U^i(c_t^i, l_t^i) \\
s.t. \qquad & c_t^i = (1 - \tau_t^n) l_t^i - T_t \\
& (1 - \tau_t^n) w_t = -\frac{U_l^i(c_t^i, l_t^i)}{U_c^i(c_t^i, l_t^i)} \\
& G_t \leq \tau_t^n L_t + T_t \\
& C_t + G_t \leq F(L_t, t) \\
& a_0^{i,d} = 0, \ \forall i \in I
\end{aligned}$$

Specifically, the punishment imposed by private agents and external lenders for a government deviation from its policies is financial autarky, under which all private and public debts are erased, and the government loses access to all financial markets permanently. Net supplies of domestic and external bonds are zero. The government can still levy labor and lump-sum taxes.

The assumption of financial autarky as the deviation value is consistent with the notion of worst sustainable punishment to support the best sustainable equilibrium, as discussed in Chari and Kehoe (1990, 1993) and subsection 2.1. First, financial autarky is a sustainable equilibrium. Second, it delivers the repeated worst static outcome of the subgame perfect equilibrium, making it a natural candidate for the punishment/deviation value.[4] Alternatively, one can relax the assumption by considering temporary exclusion or debt restructuring as part of the deviation. However, these extensions would improve the deviation value and depart from the worst punishment benchmark. Accordingly, the analysis in this paper is interpreted as characterizing the maximal debt sustainable in a renegotiation-proof, never-default equilibrium.

[4]Under certain cases, the repeated worst static outcome might not be the worst sustainable equilibrium given the dynamic time inconsistencies. See Chari and Kehoe (1993) for more details.

The value of financial autarky can be rewritten as

$$\begin{aligned}\underline{U}_t \equiv \max_{\{C_t,L_t\}_{t=0}^{\infty},\boldsymbol{\varphi},T} & \sum_{t=0}^{\infty}\beta^t U^P(C_t,L_t,\boldsymbol{\varphi};\boldsymbol{\lambda}) \\ s.t. \qquad & C_t + G_t \le F(L_t,z_t) \\ & \sum_{t=0}^{\infty}\beta^t\left[V_C(t;\boldsymbol{\varphi})c^i(t;\boldsymbol{\varphi}) + V_L(t;\boldsymbol{\varphi})l^i(t;\boldsymbol{\varphi})\right] \ge -V_C(0;\boldsymbol{\varphi})T,\end{aligned}$$

where T is the present value of lump-sum taxes.

Define the aggregate labor distortion as

$$\Omega = 1 + \frac{1}{F_L(L)}\frac{U_L^P(C,L,\boldsymbol{\varphi};\boldsymbol{\lambda})}{U_C^P(C,L,\boldsymbol{\varphi};\boldsymbol{\lambda})}$$

This measures the labor wedge of the aggregate allocation under the government's welfare objective. The following proposition establishes that the government is willing to sustain positive external debt in the long run if there is positive aggregate labor distortion in financial autarky. That is,

Proposition 2.1 (External Debt Sustainability)**.** *Suppose Assumptions 1–3 hold. If the aggregate labor distortion is positive in financial autarky and the steady state allocation exists, then the optimal external debt is positive in the long run.*

Proof. See Appendix. □

The proof relies on two properties of the optimal contract. First, given impatience, the government front-loads consumption and leisure by accumulating external debt. In the long run, the sustainability constraint binds infinitely often as the government reaches the maximal external debt. Second, deviating to financial autarky is never optimal. If autarky were optimal, the positive aggregate labor distortion under autarky would permit a profitable deviation: lowering future labor distortions without changing the continuation value allows the economy to produce more than its consumption in those periods. This deviation is possible by borrowing today and repaying the debt in the future. The extra unit of borrowing allows for more consumption today, and so welfare would be higher than under financial autarky. Combining the two properties implies that, in the long run, the government optimally finances positive external debt with taxes rather than defaulting into financial autarky.

2.3 Cost of Redistribution and Debt Sustainability

This subsection highlights the mechanism linking debt sustainability to the cost of redistribution. Proposition 2.1 shows that positive external debt is sustained because financial autarky is costly in the presence of distortionary redistribution. In autarky, the government must rely more heavily on labor taxation to transfer resources from high- to low-skilled households, as in Werning (2007), which depresses labor supply and aggregate output. Access to international financial markets mitigates these distortions, thereby making positive external debt sustainable in the long run.

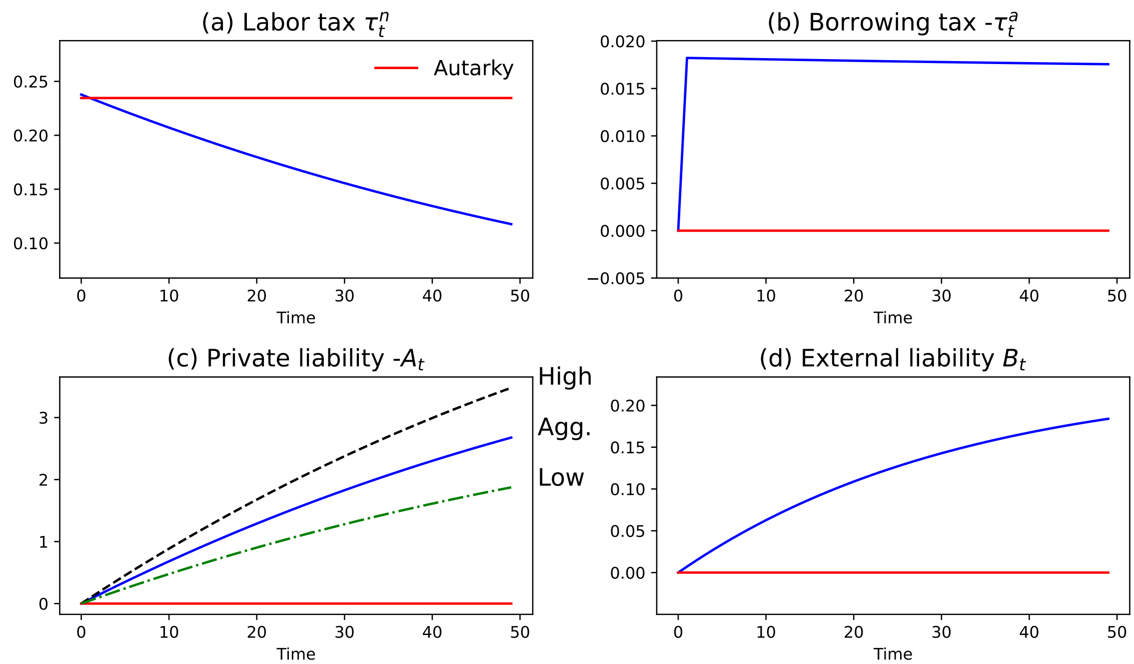


Figure 2: Time Paths of Aggregates: Equilibrium vs. Autarky

Note: Figure 2 plots the simulated time paths of aggregates in equilibrium and autarky. The implementation is that lump-sum taxes only occur in period 0. Panels (a) and (b) plot the optimal labor tax and borrowing tax. Panel (a) plots the total private debt. Panel (b) shows the economy's external liability position.

To illustrate this mechanism, first consider the economy under financial autarky. Figure 2 depicts the time paths of debt and taxes under autarky in red solid lines. The environment coincides with the closed-economy setup in Werning (2007). In this case, the optimal labor tax is strictly positive and constant over time, reflecting intratemporal tax smoothing and time-invariant trade-off between redistributive benefits and efficiency costs (Panel (a)). The optimal borrowing tax, the negative of the savings tax, is zero because there is no domestic saving (Panel (b)). There are no external asset positions for the private sector and the government, that is, $B_t^f = A_t^f = 0$. In addition, the domestic government bond is in zero net supply, so $B_t^d = A_t^d = 0$. Therefore, the total private liability and external liability are zero, $-A_t = B_t = 0$ (Panels (c) and (d)).

Now consider the economy under the optimal contract (Figure 2, non-red lines). Impatient private agents borrow abroad, resulting in an increase in private liability (Panel (c)) and a higher external liability position for the economy (Panel (d)). However, they do not internalize the effect of their borrowing on tightening the sustainability constraint. When this constraint binds, a tax on borrowing is optimal to correct the borrowing externality (Panel (b)). This tax also generates redistributive gains because high-skilled agents hold more debt abroad than lowly-skilled agents and thus bear a larger burden (Panel (a), dashed and dash-dotted lines). By mitigating the limited-commitment friction, the tax on borrowing serves as an additional redistributive instrument and allows the government to reduce distortionary labor taxation. Although the labor tax is initially higher than under financial autarky, it declines over time and eventually falls below the autarky level, increasing labor supply and expanding debt capacity (Panel (a)). In the long run, the government sustains external debt to maintain low labor distortions rather than defaulting into financial autarky, which would require higher labor distortions. The endogenous cost of default is driven by the difference between the labor tax under the optimal contract and under financial autarky, as depicted in Panel (a).

In summary, when redistribution comes with efficiency cost, the government has an incentive to sustain external debt, since access to international financial markets lowers the cost of redistribution. The cost of redistribution generates an endogenous mechanism for the cost of default. When inequality, or the government's motive for redistribution, is higher, the efficiency cost also increases as tax rates are higher. This makes financial autarky more costly and increasing the incentive to sustain external debt. Thus, the model predicts a positive correlation between inequality and sustainable debt levels.

2.4 Discussions of Model Ingredients

This subsection discusses the model ingredients that are important for the theoretical results, in particular, on the consequences of default, managing debt portfolios, and redistributive taxation.

Default consequences. The model assumes that when the government deviates from the optimally contracted policies, including defaulting on its debt, all outstanding private and public debts are wiped out. This assumption is similar to Broner and Ventura (2011), in which the government is unable to enforce external financial payments by the private sector. Default is therefore interpreted as a breakdown of the enforcement infrastructure supporting international financial contracts, both public and private, so that all external positions become valueless and the economy enters financial autarky. Related to Broner et al. (2010) and D'Erasmo and Mendoza (2016), default has redistributive consequences across private agents. The elimination of domestically held public debt is progressive, as richer households hold a larger share of government debt. In contrast, the cancellation of private external debt is regressive, since richer households are more indebted abroad than poorer households. Because private agents are net debtors on aggregate ($a^i < 0$), default reduces redistribution overall.

Managing debt portfolios. The model makes no sharp predictions about the relative quantities of debt held domestically and abroad for either private agents or the government. There is indeterminacy in the decentralization of the optimal allocation. Only total private liabilities, government liabilities, and the country's external liability position are uniquely pinned down. In particular, the amount of government debt issued domestically is indeterminate. An alternative decentralization is to assume that only the government has access to international financial markets. This assumption is reasonable for many developing countries in which private residents hold limited foreign positions, and most external borrowing and lending are undertaken by the government. In this case, private agents' bond holdings a^i consist solely of domestic government debt. The government borrows abroad and lends to the private sector over time.

Redistributive taxation. The model assumes affine taxes as the redistributive tool. Under this specification, the marginal labor tax τ^n captures the distortionary labor cost but also governs the progressiveness of the tax system (see Krueger and Perri (2011) for the analysis). Alternatively, following Heathcote et al. (2017) and Boar and Midrigan (2022), one could specify labor income taxes of the form $(wl^i) - \Lambda\,(wl^i)^{1-\tau^n}$, in which τ^n directly indexes the progressiveness of the marginal tax rates. In that case, higher values of τ^n generate stronger distortionary costs than under affine taxation, as labor supply distortions are concentrated among higher-skilled households. This higher cost of redistribution would further strengthen the debt-sustainability result, since access to external borrowing reduces reliance on distortionary taxation. This paper therefore focuses on affine taxes to isolate the distortionary cost of redistribution in a transparent way and also because affine taxation is shown to be nearly optimal in Boar and Midrigan (2022).

3 An Economy with Separable Isoelastic Preferences

Section 2 establishes that debt sustainability arises as the optimal policy when financial autarky exhibits aggregate labor distortions driven by redistributive motives. This section formally shows that, in an open-economy environment with separable isoelastic preferences, redistribution concerns generate strictly positive aggregate labor distortions under financial autarky.

Preferences. The utility of an agent with productivity θ^i over consumption $c^i \geq 0$ and efficiency-unit labor $l^i \geq 0$ is

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c^i\right)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right] \tag{13}$$

with $\sigma, \omega, \nu > 0$.

In this case, the allocation rule for individual i is time-invariant and proportional to the aggregate allocation

$$c_t^i = \psi_c^i C_t, \quad l_t^i = \psi_l^i L_t, \tag{14}$$

In addition, V and U^P inherit the separable and isoelastic properties,

$$V(C_t, L_t; \boldsymbol{\varphi}) = \Phi_C^V \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L_t^{1+\nu}}{1+\nu},$$
$$U^P(C, L; \boldsymbol{\varphi}, \boldsymbol{\lambda}) = \Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu},$$

where Φ_C^V, Φ_L^V depend on $\boldsymbol{\varphi}$, and Φ_C^P, Φ_L^P depend on $\boldsymbol{\varphi}$, $\boldsymbol{\lambda}$ (see Appendix C.1 for the formulas). The implementability constraint for individual i becomes

$$\sum_{t\geq 0} \beta^t \left(\Phi_C^V \psi_c^i C_t^{1-\sigma} - \Phi_L^V \psi_l^i L_t^{1+\nu} \right) = \Phi_C^V C_0^{-\sigma} \left(a_0^i - T \right), \tag{15}$$

The additional assumptions are

Assumption 4. *The welfare weights, skill distribution, and initial wealth distribution satisfy the following properties*

1. *Redistributive motive towards the low skills:* $\theta^i < \theta^j \iff \lambda^i > \lambda^j,\ \forall i, j \in I$
2. *Perfect correlation between skill and initial wealth:* $\theta^i < \theta^j \iff b_0^i < b_0^j,\ \forall i, j \in I$
3. *Elasticity of substitution is such that* $\sigma \geq 1$

The first assumption concerns the welfare weights: the government assigns relatively high weight to redistribution toward lower-skill, lower-income individuals, reflecting inequality aversion. The second assumption requires that the ordering of skill inequality coincides with that of initial wealth, so that lower-skill individuals are also endowed with lower initial wealth. Finally, the third assumption implies that the intratemporal elasticity of substitution is at least above the $\log$-preference benchmark. This assumption governs the direction of adjustment in optimal tax and debt policies in response to both intratemporal and intertemporal changes. Parameter values commonly used in quantitative macroeconomic analysis satisfy this assumption.

Given that the government cares about redistribution toward low-skilled agents, the following proposition shows that it is costly for the government to redistribute in financial autarky. That is,

Proposition 3.1. *Suppose Assumption 4 holds, then the aggregate labor distortion is positive in financial autarky.*

Proof. See Appendix. □

The key component that governs the aggregate labor distortion is the ratio between Pareto and Negishi weights λ^i/φ^i, which captures the extent to which the government's distributional preferences diverge from the market's equilibrium allocation of individual utilities. If the distributional preference agrees with the market distribution (i.e., $\lambda^i = \varphi^i, \forall i \in I$), then there is no aggregate labor distortion. This result occurs under two cases: the representative-agent case and the heterogeneous-agent case in which $\lambda^i = \varphi^{*i}, \forall i \in I$.

Proposition 3.2 (Zero aggregate labor distortion)**.** *The aggregate labor distortion in financial autarky is zero if either of the following cases holds:*

1. *There is no heterogeneity:* $\theta^i = \theta^j, a_0^i = a_0^j,\ \forall i, j \in I$.
2. *There exists* $\boldsymbol{\varphi}^*$ *such that* $\lambda^i = \varphi^{*i}, \forall i \in I$.

Proof. See Appendix. □

In the above cases, redistribution comes with no efficiency costs. Consequently, access to international financial markets yields no additional benefits. By contrast, default enables the government to raise consumption without financing the external debt. Ex ante, this implies that the sustainable level of debt is zero.

4 Quantitative Analysis

The quantitative analysis proceeds in three parts. First, it introduces a stochastic version of the model with state-contingent domestic and foreign bonds. In addition to redistribution motives, the government also values external debt for its insurance role against aggregate shocks, a channel well established in the literature (Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), Chatterjee and Eyigungor (2012), and many other papers). Second, the analysis quantifies the relative importance of redistribution and insurance for debt sustainability using Italian data. Finally, it characterizes the optimal austerity policies.

4.1 Stochastic Economy with State-Contingent Debt

Environment. A small open economy faces publicly observed aggregate shocks $s_t \in S$ in period t, where S is some finite set. Let $\Pr(s^t)$ denote the probability of any history $s^t = (s_0, s_1, ..., s_t)$, where $\Pr(s^{t+j}|s^t)$ denotes the probability conditional on history s^t, $j \geq 0$. Similarly, $\Pr(s_{t+1}|s^t)$ is the probability that period $t+1$'s state is s_{t+1}, conditional on history s^t. The individual utility is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i(c_t^i, l_t^i)$$

The production function in period t with history s^t is $F(L, s^t, t)$.

State-contingent bonds and budget constraints. Domestic and foreign bonds are state-contingent. The individual agent's budget constraint is

$$\begin{aligned} c_t^i + \sum_{s_{t+1}} q_{t+1}^d(s_{t+1}) a_{t+1}^{d,i}(s_{t+1}) + \sum_{s_{t+1}} q^f(s_{t+1}) a_{t+1}^{f,i}(s_{t+1}) \\ \leq (1-\tau_t^n) w_t l_t^i + (1-\tau_t^a) a_t^{d,i} + (1-\tau_t^a) a_t^{f,i} - T_t, \end{aligned}$$

and the government's budget constraint is

$$\begin{aligned} G_t + B_t^d + B_t^f \leq \tau_t^n w_t L_t + \tau_t^a \left(A_t^d + A_t^f\right) + T_t \\ + \sum_{s_{t+1}} q_{t+1}^d(s_{t+1}) B_{t+1}^d(s_{t+1}) + \sum_{s_{t+1}} q_{t+1}^f(s_{t+1}) B_{t+1}^f(s_{t+1}) \end{aligned}$$

Define $Q_t^* = \Pr(s^t)/(\Pi_{\tau=0}^t R_\tau^*)$ as the international price of one unit of consumption at history s^t in units of period-0 consumption. The optimal contracting problem becomes

$$\begin{aligned} (P) \equiv \max_{\{C_t, L_t\}, \boldsymbol{\varphi}, T} & \sum_{i \in I} \lambda^i \pi^i \mathbb{E}_0 \sum_{t \geq 0, s^t} \beta^t U^P(C_t, L_t; \boldsymbol{\varphi}, \boldsymbol{\lambda}) \\ s.t. \quad & \sum_{t=0}^{\infty} Q_t^* \left[F(L_t, s^t, t) - G_t - C_t\right] \geq B_0 \\ & \forall i,\ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[V_C(t; \boldsymbol{\varphi}) c^i(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) l^i(t; \boldsymbol{\varphi})\right] \geq V_C(0; \boldsymbol{\varphi}) \left(a_0^i - T\right) \\ & \forall t, \forall s^t,\ \sum_{i \in I} \lambda^i \pi^i \mathbb{E}_t \sum_{k \geq t} \beta^{k-t} U^P(C_k, L_k; \boldsymbol{\varphi}, \boldsymbol{\lambda}) \geq \underline{U}_t(s^t, t) \end{aligned}$$

4.2 Parametrization

The quantitative exercise assumes the following distributional and functional forms. The economy is populated by two types of agents with labor productivity $\{\theta^H, \theta^L\}$, where $\theta^H \geq \theta^L > 0$ and $\pi^H = \pi^L = 0.5$. The planner is utilitarian, i.e. $\lambda^H = \lambda^L$. The individual preference has the form of $U(c, n) = c^{1-\sigma}/(1-\sigma) - \omega n^{1+\nu}/(1+\nu)$ with $\sigma, \omega, \nu > 0$. The production function is linear in labor, i.e. $F(L, z) = zL$, where z is aggregate productivity. The aggregate shock is z_t that follows a logged $AR(1)$ process,

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t^z,\ \epsilon_t^z \sim \mathcal{N}(0, \sigma_z),$$

where ρ_z, σ_z are the autocorrelation and the residual standard deviation, respectively. The productivity process is discretized into a Markov chain using the Tauchen method with 31 evenly-spaced nodes. From now on, z is in place of s as the source of aggregate uncertainty. The government expenditure is constant over time and across histories: $G_t = \bar{g}$. The initial debt levels are $B_0 = 0$ and $a_0^H = a_0^L = 0$. The economy starts at the mean of the productivity distribution. The deviation utility $\underline{U}(z^t, t)$ is calculated as the closed-economy version of the model that starts with productivity z_t, zero external debt, and all domestic individuals start with the same initial wealth. $\underline{U}(z^t, t)$ varies with respect to the realized shock z_t.

With these assumptions, the model requires assigning values to the parameters of (i) the aggregate productivity process, ρ_z and σ_z; (ii) the cross-sectional wage ratio, θ^H/θ^L; (iii) the individual preferences, β, σ,ω, and ν; (iv) the government expenditure $\bar{g}$; and (v) the risk-free rate r^*. Table 1 summarizes the parameter values and targets from the calibration exercise.

A period in the model is one year. Output is the log-linear detrended real GDP series from 1985-2015. The auto-correlation of productivity,ρ_z, equals the auto-correlation of output. The wage ratio θ^H/θ^L is measured using the household-level data from the *Survey on Household Income and Wealth* (SHIW) conducted by the Bank of Italy.[5] The hourly wage is defined as total real compensation of employees, including fringe benefits, divided by total hours worked in a year.[6]

[5] See Appendix A.2 for details on the microeconomic data and sample selection.
[6] Real-data calculation uses CPI indices provided by the OECD.

Table 1: Parameters and Targets

Parameter	Description	Value	Target
Externally calibrated parameters			
r^*	Risk-free rate	0.017	Avg. real return on German bond
β	Discount factor	0.967	Avg. Italian real interest rate = 3.4%
σ	Intertemporal elasticity	1	Standard literature value
$1/\nu$	Labor elasticity	0.5	Standard literature value
ω	Labor utility weight	1	Standard literature value
θ^H/θ^L	Wage ratio	1.89	Mean top 50% wage / mean bottom 50% wage
ρ_z	Auto-corr. of prod.	0.927	Auto-corr. of log GDP
Internally calibrated parameters			
σ_z	Std. dev. of prod. res.	0.0205	Std. dev. log GDP
$\bar{g}$	Govt. spending	0.202	Avg. govt. consumption-to-GDP

Note: Table 1 describes the parameters, their values, and the targets in the calibration exercise. Statistics are annual. The risk-free rate and discount factor cover the period of 2002-2015. Wage ratio is the author's calculation from the household-level data set by *Survey on Household Income and Wealth* covering the period of 2002-2014. Auto-correlation and standard deviation of GDP and government final consumption cover the period of 1985-2015. Data sources: *Survey on Household Income and Wealth* (2014), Eurostat (2019), and The World Bank (2019)

Then θ^H/θ^L is set to match the time-average of the ratio of the mean wage of the top 50% of the wage distribution to the mean wage of the bottom 50% for the period of 2002 to 2014. The discount factor β is set so that the average real domestic interest rate is 3.4% for Italy from 2002 to 2015. The values for σ, ω, and ν follow the standard literature values. The risk-free rate is set to be the real rate of return on the German government bonds for the period 2002-2015 (these are secondary market returns, gross of tax, with around 10 years' residual maturity). The interest rate series starts at 2002 to isolate the effect of currency and exchange rate risks.[7]

The two remaining parameters, σ_z and $\bar{g}$, are selected to match (i) the standard deviation of logged output and (ii) the government's final consumption-to-GDP ratio for the period 1985-2015, using the simulated method of moments (SMM). Departing from the quantitative sovereign-debt literature, the analysis

[7]See Appendix A for more data descriptions and sources.

does not target the average external debt-to-output ratio, leaving it instead as a non-targeted moment.

4.3 Quantitative Results

Table 2 presents the quantitative results with moments for the data, the baseline, and alternative models. The first column reports the statistics from the data for Italy in the period of 1985-2015, except for external debt moments covering the period of 2002-2015. The second column reports the statistics from simulating the model and taking the long-run averages.[8] The calibration successfully matches the standard deviation of output and the government-consumption-output ratio for Italy.

External debt. The model is able to produce a quantitatively large amount of external debt-to-output, in consistency with the data. The model generates around 17% of external debt-to-output, compared to 24% of external debt-to-output ratio in the data. The model also matches the volatility of external debt-to-output ratio in the data. These features arise in the presence of a relatively high discount factor and without additional exogenous cost of default in terms of output or productivity loss.

Cyclical properties. Several cyclical features of the Italian data stand out. Consumption is as volatile as output and strongly procyclical, while net saving—defined as output minus total private and public consumption—is much less volatile and positively correlated with output.[9] The model closely matches these patterns. Although consumption and net saving are slightly more volatile than in the data, their cyclical correlations are similar. In contrast to the standard complete-markets model, the baseline framework generates realistic cyclical dynamics because occasionally binding borrowing constraints prevent full insurance across states and over time, even with state-contingent assets.

The redistributive motive for sustaining external debt is quantitatively important. Comparing the baseline model to a no-inequality specification ($\theta^H = \theta^L$)

[8]All model's moments are long-run averages of simulating the economy for 10500 periods and discarding the first 500 periods.

[9]As documented by Neumeyer and Perri (2005) for both advanced and emerging economies, these patterns are characteristic of international business cycles.

Table 2: Moments: Data, Baseline, and Alternative Models

	Data	Baseline	No inequality	Skill-dependent lump sum tax
Targeted moments				
Std. output (%)	5.3	5.3	5.4	5.4
Avg. govt. expenditure/output (%)	19	19	19	19
Non-targeted moments				
External debt property				
Avg. external debt/output (%)	24	17	2.9	2.9
Std. external debt/output (%)	2.7	2.1	0.45	0.45
Cyclical property				
Std. consumption / Std. output	1.0	1.2	1.2	1.1
Std. net savings/output (%)	1.5	1.8	1.8	1.7
Correlation with output (%)				
Consumption	97	95	95	95
Net savings/output	40	31	36	35

Note: Table 2 reports the non-targeted statistics of the data and the model. The first column reports data statistics which are across the period of 1985 to 2015, except for external debt moments covering the period of 2002 to 2015. The other columns report statistics coming from models' simulations for 10500 periods and excluding the first 500 periods. The no-inequality model corresponds to the case in which $\theta^H = \theta^L$. The skill-dependent-lump-sum-tax model corresponds to the case in which the government has access to skill-dependent lump-sum taxes $\left(T^i\right)_{i\in I}$. In the data, government expenditure is final government consumption excluding social transfers. Net saving is defined as output minus total private and government consumption in the data and the model. External debt is defined as the country's net financial liability in the data. For the second moments, output and consumption series are logged and linear detrended, and net saving and external debt ratio series are linear detrended.

shows that external debt falls from 17% to 2.9% of GDP, implying that the standard insurance channel explains only a small fraction of debt. Instead, roughly 60% of external debt is accounted for by the redistributive channel, whereby external borrowing reduces the distortionary cost of redistribution.

This mechanism is further illustrated by a model with skill-dependent lump-sum taxation. When redistribution can be implemented without labor distortions, both the level and volatility of external debt are substantially lower. By contrast, in the baseline model redistribution relies on distortionary labor taxation, strengthening the incentive to sustain external debt.

4.4 Effect of Inequality on External Debt Over Time

This subsection studies the model-implied relationship between income inequality and external debt using a comparative statics exercise for Italy over the subperiods 1985–2001 and 2002–2015. The earlier period is characterized by lower income inequality and lower external debt on average.

Table 3: Comparative Statics

Statistics	Data	Model
Targeted moment		
Δ Pre-tax Gini	3.0%	3.0%
Non-targeted moment		
Δ Extenal debt/output	14%	10%

Note: Table 3 reports the results of the comparative statics exercise. The first column reports the changes in the data statistics, computed as the average statistics of period 2002-2015 minus the average statistics of period 1985-2001. The second column reports the results from the model. The change in the model statistics is computed as the average statistic of a simulation for the model with the wage ratio equal to 1.89 minus the same statistic of the model with the wage ratio equal to 1.83.

The comparative statics exercise proceeds by feeding into the model the level of wage inequality observed in 1985–2001, holding all other parameters fixed, and computing the ergodic means of the pre-tax Gini coefficient and the external-debt-to-output ratio. Wage inequality in 1985–2001 is calibrated to match the observed change in the average pre-tax Gini coefficient between 1985–2001 and 2002–2015. Table 3 reports the results. Under this calibration, the model accounts for approximately 71 percent of the increase in Italy's external-debt-to-output ratio across the two periods, highlighting the quantitative importance of inequality-driven redistribution costs for debt accumulation in the long run.

4.5 Optimal Austerity Policies

This subsection studies optimal austerity policies in the presence of the government's redistribution concerns by measuring the responses of optimal policies to a negative productivity shock.[10]

[10] The analysis simulates 30,000 paths of the model over 2,050 periods. From periods 1 to 2050, aggregate productivity follows its underlying Markov chain, allowing the cross-sectional distri-

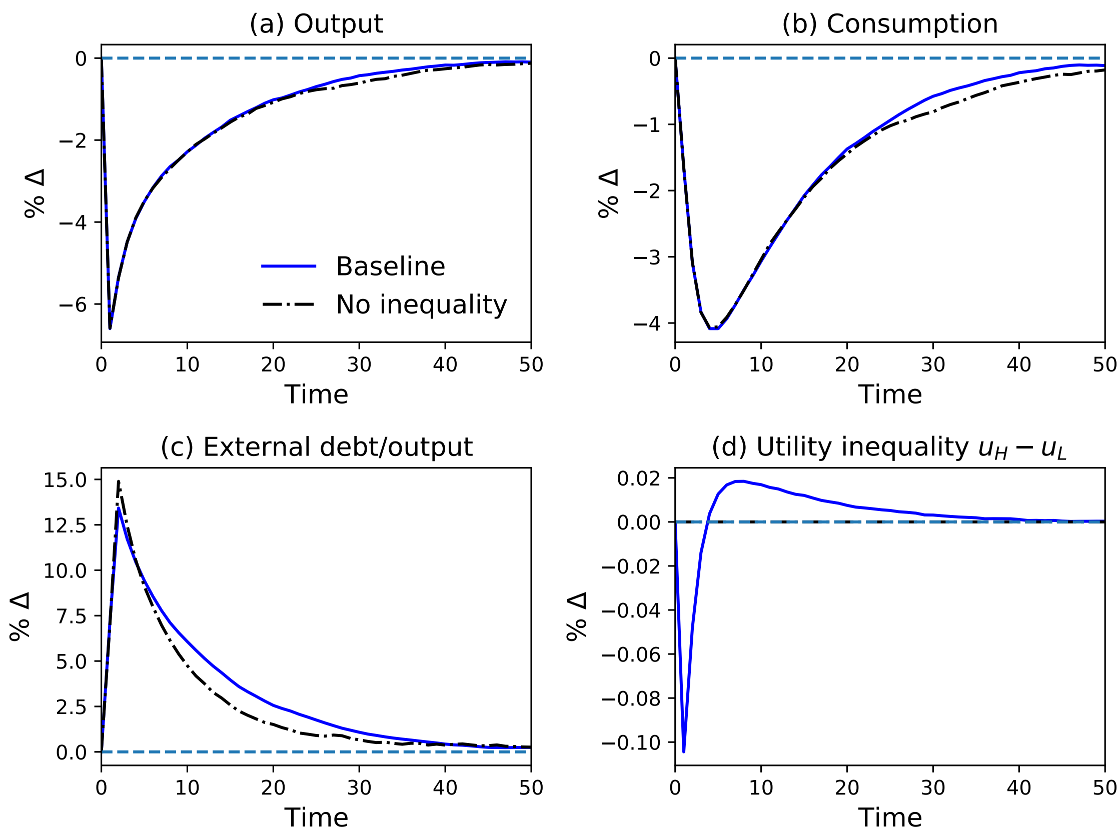


Figure 3: Impulse Responses

Note: Figure 3 plots the impulse responses of output, consumption, external debt-to-output, and utility inequality to a negative aggregate productivity drop in period 1. The impulse responses are averages across 30000 paths of the variables for the baseline and no-inequality models.

Figure 3 plots the impulse responses of output, consumption, external debt-to-output, and utility inequality $u^H - u^L$ for the baseline model and the model of no inequality ($\theta^H = \theta^L$). A negative productivity shock reduces both output and consumption, with a larger contraction in output. In response, external debt increases. Utility inequality initially declines before rising in subsequent periods, reflecting higher redistribution in the short run and lower redistribution in the long run. Intuitively, the negative shock lowers deviation utility, resulting in

bution of debt to converge to its ergodic distribution. In period 2051—normalized to 1 in the figures—aggregate productivity falls by one standard deviation of the innovation. From that period onward, productivity evolves according to the conditional Markov chain. Impulse responses report averages across simulated paths for periods 2050 through 2055.

the sustainability constraint being temporarily slack. This relaxation allows the government to accumulate external debt, reduce average tax rates, and expand redistribution. Over time, however, taxes must increase to service the accumulated debt, leading to a subsequent contraction in redistribution.

The no-inequality model generates similar dynamics for output and consumption. However, the external-debt-to-output ratio increases more sharply initially and declines more rapidly than in the baseline model. This pattern implies that, in the baseline economy, the government, despite borrowing less initially, pursues a more gradual fiscal consolidation by sustaining higher debt levels in later periods. This slower adjustment is achieved through reduced redistribution. Thus, the government's motive for redistribution results in a more gradual fiscal consolidation.

5 Empirical Evidence

Next is the assessment of whether the model's prediction linking inequality and external debt is supported empirically using a cross-country panel dataset. The theoretical and quantitative results imply that higher pre-tax income inequality—interpreted as a proxy for stronger redistributive motives—is associated with higher sustainable external debt levels.

Table 4 reports the estimates of the relationship between external indebtedness and inequality. The external indebtedness is measured as the negative of the net foreign asset–to-GDP ratio from the External Wealth of Nations database of Lane and Milesi-Ferretti (2018).[11] Inequality is measured by the pre-tax (market) Gini indices from the Standardized World Income Inequality Database (SWIID) of Solt (2019). The analysis focuses on countries that are frequently subject to debt crises (see Appendix A for the list of countries). Across specifications, the estimated coefficient is positive and statistically significant, consistent with the model's prediction that higher inequality is associated with higher external debt.

[11] The net foreign asset (NFA) position of a country is the value of the assets that country owns abroad, minus the value of the domestic assets owned by foreigners, adjusted for changes in valuation and exchange rates.

Table 4: Regression Result: Income Inequality and External Debt

	Net foreign liability-to-GDP (%)			
	(1)	(2)	(3)	(4)
Gini index, pre tax (%)	0.773***	0.472**	4.947***	5.456***
	(0.222)	(0.236)	(0.664)	(0.749)
GDP per capita (log)		-4.972**	-5.781	-1.608
		(1.952)	(4.093)	(7.211)
Real GDP per capita growth (%)		-1.311**	-1.371**	-1.427**
		(0.638)	(0.543)	(0.608)
Current account-to-GDP (%)		-2.061***	-1.090***	-1.100***
		(0.297)	(0.344)	(0.376)
Inflation (%)		0.013***	0.010***	0.010**
		(0.003)	(0.004)	(0.004)
Country fixed effect	No	No	Yes	Yes
Time fixed effect	No	No	No	Yes
No. Countries	30	30	30	30

Note: Table 4 reports the regression coefficients and standard errors in parenthesis of pre-tax Gini index (%) with respect to net foreign liability-to-GDP (%). Control variables are log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). * p<0.1, ** p< 0.05, *** p<0.001. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and The World Bank (2019).

6 Conclusion

This paper develops a theory of external debt sustainability in which redistribution requires distortionary taxation, and access to international financial markets alleviates these costs. Embedding redistributive motives into a sovereign debt framework with limited commitment, the analysis shows that costly redistribution generates a novel, endogenous cost of default. Higher inequality raises the distortionary cost of taxation, increases the cost of financial autarky, and therefore increases the incentives for countries to sustain higher external debt.

Quantitatively, the redistribution channel emerges as a central role for debt sustainability. In the case of Italy, the model attributes only 12 percent of long-run debt to the standard insurance channel, while the redistribution channel accounts for 60 percent.[12] The model is also consistent with the positive cross-country and time-series correlation between inequality and external debt.

[12]Even though the model was calibrated using Italian data, the mechanisms and underlying policy trade-offs apply to many countries, including emerging markets.

The analysis further highlights the implications for fiscal austerity. Optimal austerity policies are shaped by distributional concerns. Following a negative productivity shock, the government initially expands borrowing and redistribution and subsequently consolidates through higher taxes and reduced transfers. This result provides a rationale for more gradual fiscal adjustment in more unequal economies.

Relative to standard sovereign debt models, the framework abstracts from equilibrium default and instead highlights redistribution as a central determinant of debt sustainability. By identifying an endogenous cost of default arising from distortionary redistribution, the paper provides a new perspective on why unequal economies can sustain high levels of external debt. An important direction for future work is to allow for equilibrium default risk and alternative forms of debt crises.

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Appendix

A List of Countries

Argentina, Bolivia, Brazil, Cameroon, Colombia, Costa Rica, Ecuador, Egypt, El Salvador, Greece, Guatemala, Honduras, India, Indonesia, Italy, Ireland, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Peru, Philippines, Portugal, Spain, Sri Lanka, Thailand, Tunisia, Turkey, Venezuela.

B Sovereign Game

This section sets up the strategic sovereign game and define the sustainable equilibrium that is characterized by the sustainability constraints. Consider the general environment where the government's policy includes the decision to default on its debt δ, where $\delta \in \{0,1\}$ and $\delta = 0$ implies default.[13] The government's budget constraint becomes

$$G_t + \delta_t B_t^d + \delta_t B_t^f \leq \tau_t^n w_t L_t + \tau_t^a \left(A_t^d + A_t^f\right) + T_t + q_{t+1}^d B_{t+1}^d + q_{t+1}^f B_{t+1}^f.$$

As the government cannot commit to any of its policies, one can think that the government, domestic agents, and international lenders enter into a sovereign game in which they determine their actions sequentially. In every period, the state variable for the game is $\left\{B_t^d, B_t^f, \left(a_t^{i,d}, a_t^{i,f}\right)_{i\in I}\right\}$. The timing of the actions is as follows:

- Government chooses $z_t^G = \left(\tau_t^n, \tau_t^a, T_t, \delta_t, B_{t+1}^d, B_{t+1}^f\right) \in \Pi$ such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c_t^i, l_t^i, a_{t+1}^{i,d}, a_{t+1}^{i,f}\right)$ subject to their budget constraints, the representative firm produces output by choosing $z_t^F = (L_t)$, and the international lenders choose holdings of bonds $z_t^* = \left(B_{t+1}^f, A_{t+1}^f\right)$ given the interest rates r_t^*.

[13] Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its debt ($\delta_t = 1$).

Define $h^t = \left(h^{t-1}, z_t^G, \left(z_t^{H,i}\right)_{i\in I}, z_t^F, z_t^*, p\right) \in H^t$ as the history at the end of period t. Note that the history incorporates the government's policy, allocation, and prices. Define $h_p^t = \left(h^{t-1}, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period t. The government strategy is $\sigma_t^G : \ H^{t-1} \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i} : \ H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has the strategy $\sigma_t^F : \ H_p^t \to \mathbb{R}_+^2$, and the international lenders have the strategy $\sigma_t^* : H_p^t \to \mathbb{R}_+^2$. Prices are determined by the pricing rule: $p : \ H_p^t \to \mathbb{R}_+$.

Definition B.1 (Sustainable equilibrium)**.** A sustainable equilibrium is $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ such that (i) for all h^{t-1}, the policy z_t^G induced by the government strategy maximizes the weighted utility by $\boldsymbol{\lambda}$ subject to the government's budget constraint (6); (ii) for all h_p^t, the strategy induced policy $\left\{z_t^G\right\}_{t=0}^\infty$, allocation $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^\infty$, and prices $\{r_t^*\}_{t=0}^\infty$ constitute a competitive equilibrium with taxes.

The following focuses on characterizing a set of sustainable equilibria in which deviation triggers the worst payoff. In this case, the value of deviation is the worst equilibrium payoff.

Proposition B.1 (Sustainable equilibrium)**.** *An allocation and policy* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *is part of sustainable equilibrium if and only if (i) given* z^G*, there exist prices* p *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G, p\right\}$ *is a competitive equilibrium with taxes for an open economy; and (ii) for any* t*, there exists* $\underline{U}_t$ *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *satisfies the constraint*

$$\sum_{k=t}^{\infty} \beta^{k-t} \sum_{i\in I} \lambda^i \pi^i U^i \left(c_k^i, l_k^i\right) \geq \underline{U}_t. \tag{12}$$

Proof. Define $\underline{U}_t$ as the maximum discounted weighted utility for the private agents in period t when the government deviates. In period t, the private agents and the government can borrow abroad. In subsequent period $s > t$, the economy reverts to the worst equilibrium.

Suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an outcome of the sustainable equilibrium. Then by optimality, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes the weighted utility of the agents, satisfies the government budget constraint, and satisfies the international lender's problem at period 0. Thus, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is a competitive equilibrium with policies. For any period t and history h^{t-1}, an equilibrium strategy that has the government deviate in period t triggers reverting to the worst equilibrium in period $s > t$. Such a strategy must deliver a weighted utility value that is at least as high as the right-hand side of (12). So $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii).

Next, suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies conditions (i) and (ii). Let h^{t-1} be any history such that there is no deviation from z^G up until period t. Since $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t onward. Consider a deviation plan $\hat{\sigma}^G$ at period t that receives U_t^d in period t and U^{aut} for period $s > t$. Because the plan is constructed to maximize period-t utility, the right-hand side of (12) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. $\square$

C Characterizing Sustainable Allocation and Optimal Tax Policies

This section provides details on the characterization of the sustainable allocation and optimal tax policies. Section D will use this analysis to prove the propositions in the main text.

Let μ be the multiplier on the resource constraint, $\pi^i\eta^i$ be the multiplier on the implementability constraint for agent i, and $\beta^t\gamma_t$ be the multiplier on the aggregate debt constraint for period t. Define $\boldsymbol{\eta} = (\eta^i)_{i\in I}$ and rewrite the Lagrangian of the optimal contracting problem with a new pseudo-utility function that incorporates the implementability constraints,

$$\sum_{t=0}^{\infty}\beta^t U^W\left[t;\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\eta}\right] - V_C(0;\boldsymbol{\varphi})\sum_{i\in I}\pi^i\eta^i\left(a_0^i - T\right),$$

where

$$U^W\left[t;\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\eta}\right] \equiv \sum_{i\in I}\lambda^i\pi^i U^i(t;\boldsymbol{\varphi}) + \sum_{i\in I}\pi^i\eta^i\left[V_C(t;\boldsymbol{\varphi})h^{i,c}(t;\boldsymbol{\varphi}) + V_L(t;\boldsymbol{\varphi})h^{i,l}(t;\boldsymbol{\varphi})\right]$$

The first-order conditions are

$$U_C^W(t;\boldsymbol{\varphi}) + U_C^P(t;\boldsymbol{\varphi})\sum_{k=0}^{t}\gamma_k = \mu\frac{Q_t^*}{\beta^t} \tag{C.1}$$

$$U_L^W(t;\boldsymbol{\varphi}) + U_L^P(t;\boldsymbol{\varphi})\sum_{k=0}^{t}\gamma_k = \mu\frac{Q_t^*}{\beta^t}F_L(L_t) \tag{C.2}$$

$$\sum_i \pi^i\eta^i = 0 \tag{C.3}$$

In addition, there is the first-order condition with respect to the market weight φ^i. The necessary conditions to characterize the sustainable allocation are the first-order conditions, the aggregate resource constraint, the implementability constraints, and the aggregate debt constraints.

The optimal tax policies follow

$$\tau_t^n = 1 - \frac{U_C^W(t;\boldsymbol{\varphi}) + U_C^P(t;\boldsymbol{\varphi})\sum_{k=0}^{t}\gamma_k}{U_L^W(t;\boldsymbol{\varphi}) + U_L^P(t;\boldsymbol{\varphi})\sum_{k=0}^{t}\gamma_k}\frac{V_L(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})} \tag{C.4}$$

$$\tau_{t+1}^a = 1 - \frac{U_C^W(t+1;\boldsymbol{\varphi}) + U_C^P(t+1;\boldsymbol{\varphi})\sum_{k=0}^{t+1}\gamma_k}{U_C^W(t;\boldsymbol{\varphi}) + U_C^P(t;\boldsymbol{\varphi})\sum_{k=0}^{t}\gamma_k}\frac{V_C(t;\boldsymbol{\varphi})}{V_C(t+1;\boldsymbol{\varphi})} \tag{C.5}$$

C.1 Formulas for Separable Isoelastic Preferences

Consumption and labor shares ψ_c^i,ψ_l^i follow

$$\psi_c^i = \frac{\left(\varphi^i\right)^{1/\sigma}}{\sum_{i\in I}\pi^i(\varphi^i)^{1/\sigma}}, \quad \psi_l^i = \frac{\left(\theta^i\right)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}}{\sum_{i\in I}\pi^i(\theta^i)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}}. \tag{C.6}$$

Given equations (C.6), we have the followings:

$$\begin{aligned}
\Phi_C^V &= \left[\sum_i \pi^i(\varphi^i)^{1/\sigma}\right]^{\sigma}; & \Phi_L^V &= \omega\left[\sum_i \pi^i\left(\varphi^i\right)^{-1/\nu}(\theta^i)^{(1+\nu)/\nu}\right]^{-\nu} \\
\Phi_C^P &= \Phi_C^V\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_c^i; & \Phi_L^P &= \Phi_L^V\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_l^i \\
\Phi_C^W &= \Phi_C^V\sum_{i\in I}\pi^i\psi_c^i\left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]; & \Phi_L^W &= \Phi_L^V\sum_{i\in I}\pi^i\psi_l^i\left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right]
\end{aligned}$$

D Proofs

D.1 Proof of Proposition 1.1

Proof. ($\Rightarrow$) Let $\{C_t, L_t\}_{t=0}^{\infty}$ be an aggregate allocation of an open economy competitive equilibrium with government policies. Then by definition, $\{C_t, L_t\}_{t=0}^{\infty}$ satisfies aggregate resource constraint for every period. Moreover, given any market weights φ, $\{C_t, L_t\}_{t=0}^{\infty}$ satisfies

$$
\begin{aligned}
(1-\tau_t^n)w_t &= -\frac{V_L(C_t, L_t; \boldsymbol{\varphi})}{V_C(C_t, L_t; \boldsymbol{\varphi})} \\
1-\tau_{t+1}^a &= \frac{V_C(C_t, L_t; \boldsymbol{\varphi})}{\beta R_{t+1}^* V_C(C_{t+1}, L_{t+1}; \boldsymbol{\varphi})}
\end{aligned}
$$

Substituting for w_t into the budget constraint (2) and using $(c_t^i, l_t^i) = h^i(C_t, L_t; \boldsymbol{\varphi})$ gives the implementability constraint for each agent. Importantly, one can choose φ and T such that the individual implementability constraints hold with equality.

($\Leftarrow$) Given φ, T and an allocation $\{C_t, L_t\}_{t=0}^{\infty}$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\{w_t\}_{t=0}^{\infty}$ using firm's first-order conditions (4). $\{\tau_t^n\}_{t=0}^{\infty}$ can be calculated using the intratemporal condition (8), while one can choose $\left\{\tau_{t+1}^a\right\}_{t=0}^{\infty}$ that satisfy the intertemporal constraint (9). Define $\{Q_t^*\}_{t=0}^{\infty}$ as $Q_t^* = \Pi_{\tau=1}^t \frac{1}{1+r_\tau^*}$.

Rewriting the aggregate resource constraint using $F(L) = wL$ gives

$$
\begin{aligned}
&\sum_{t=0}^{\infty} Q_t^* \{C_t - (1-\tau_t^n)w_t L_t + T_t\} \\
&\quad + \sum_{t=0}^{\infty} Q_t^* [G_t - \tau_t^n w_t L_t - T_t] \leq -B_0
\end{aligned}
\tag{D.1}
$$

Aggregating up the agent's budget constraints implies

$$
C_t + \frac{1}{1+r_t^*}\left(A_{t+1}^d + A_{t+1}^f\right) = (1-\tau_t^n)w_t L_t + (1-\tau_t^a)\left(A_t^d + A_t^f\right) - T_t
$$

or

$$
C_t - (1-\tau_t^n)w_t L_t + T_t = (1-\tau_t^a)\left(A_t^d + A_t^f\right) - \frac{1}{1+r_t^*}\left(A_{t+1}^d + A_{t+1}^f\right)
$$

Substituting the last equation into (D.1) gives the government's budget constraint (6). Thus, $\{C_t, L_t\}_{t=0}^{\infty}$ is the aggregate allocation of the constructed competitive equilibrium with government policies. $\square$

D.2 Proof of Proposition 2.1

The proof proceeds in three steps. First, the sustainability constraint binds infinitely often in the long run. Second, the sustainable allocation is characterized by the steady-state level of debt that maximizes repayment capacity. Third, the financial autarkic allocation is never optimal at any point in time. Together, the results imply that the long-run sustainable debt corresponds to the maximum debt level and is strictly positive.

Before proceeding to the main proof, the following lemmas are useful. Consider the static problem

$$V(C, L; \boldsymbol{\varphi}) \equiv \max_{(c^i, l^i)_{i \in I}} \sum_{i \in I} \varphi^i \pi^i U^i(c^i, l^i)$$
$$s.t. \qquad \sum_{i \in I} \pi^i c^i = C; \quad \sum_{i \in I} \pi^i l^i = L.$$

The individual allocation is strictly increasing in the aggregate allocation.

Lemma D.1. *In equilibrium,* $\forall i \in I$*,* $c^i(C, L; \boldsymbol{\varphi}) = c^i(C; \boldsymbol{\varphi})$ *and is strictly increasing in* C*. Similarly,* $l^i(C, L; \boldsymbol{\varphi}) = l^i(L; \boldsymbol{\varphi})$ *and is strictly increasing in* L *.*

Proof. Let $\mu_c$and μ_l be the multipliers on the constraints. Then given assumption 1, the first-order condition with respect to $c^i$is $\varphi^i u'(c^i) = \mu_c$, which implies that $c^i = u'^{-1}(\mu_c/\varphi^i)$ and $C = \sum_i \pi^i c^i = \sum_i \pi^i u'^{-1}(\mu_c/\varphi^i)$. Since all of the functions are continuous, μ_c only depends on $C, \boldsymbol{\varphi}$. So c^i is only a function of $C, \boldsymbol{\varphi}$.

To show strict monotonicity, suppose $C_1 < C_2$ then $\sum_i \pi^i u'^{-1}\left(\frac{\mu_{c,1}}{\varphi^i}\right) < \sum_i \pi^i u'^{-1}\left(\frac{\mu_{c,2}}{\varphi^i}\right)$, which implies that $\mu_{c,1} > \mu_{c,2}$. u'^{-1} is strictly decreasing, so $c_1^i < c_2^i$.

Similarly, we can show that l^i only depends on $L, \boldsymbol{\varphi}$ and is strictly increasing in L. $\square$

This implies that U^P is strictly increasing in C and strictly decreasing in L.

I next show that the sustainable allocation features no immiseration.

Lemma D.2. *For any sustainable allocation* $\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*$*,* $\liminf_{t\to\infty} C_t^* > 0$ *and* $\limsup_{t\to\infty} L_t^* < \bar{L}$ *for some* $\bar{L}$*.*

Proof. Suppose, by contradiction that $\liminf_{t\to\infty} C_t^* \le 0$. Find $\epsilon > 0$ such that $\forall t$,

$$\sum_{s=t}^{\infty} \beta^{s-t} \left\{ U^P \left(C_s, L_s^*; \boldsymbol{\varphi}, \boldsymbol{\lambda} \right) \right\} \le M_U$$

with $C_t = \epsilon$ and $C_s = C_s^*,\ \forall s > t$. Such ϵ exists since the utility function is unbounded below. Because $\liminf_{t\to\infty} C_t^* \le 0$, there exists t_0 such that $C_{t_0}^* < \epsilon$. Then by monotonicity ,

$$\begin{aligned}
&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ U^P \left(C_s^*, L_s^*; \boldsymbol{\varphi}, \boldsymbol{\lambda} \right) \right\} \\
<&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu} \right\} \\
\le& M_U \\
\le& \underline{U}_{t_0}
\end{aligned}$$

which contradicts the aggregate debt constraint at t_0.

The similar argument applies to give $\limsup_{t\to\infty} L_t^* < \bar{L}$. $\square$

The marginal utilities from the optimal contracting problem are bounded.

Lemma D.3. *For any sustainable allocation* $\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*$*,* $U_C^W(t; \boldsymbol{\varphi})$ *and* $U_L^W(t; \boldsymbol{\varphi})$ *are bounded.*

Proof. The expressions can be written as

$$\begin{aligned}
U_C^W(t; \boldsymbol{\varphi}) &= \sum_i \pi^i \frac{\partial c_t^i}{\partial C_t} u'(c_t^i) \left\{ \lambda^i + \eta^i \varphi^i \left[\frac{u''(c_t^i) c_t^i}{u'(c_t^i)} + 1 \right] \right\} \\
U_L^W(t; \boldsymbol{\varphi}) &= -\sum_i \pi^i \frac{1}{\theta^i} v'(l_t^i/\theta^i) \frac{\partial l_t^i}{\partial L_t} \left\{ \lambda^i + \eta^i \varphi^i \left[\frac{1}{\theta^i} \frac{v''(l_t^i/\theta^i) l_t^i}{v'(l_t^i/\theta^i)} + 1 \right] \right\}
\end{aligned}$$

One can show that $\frac{\partial c_t^i}{\partial C_t} = -\frac{u'(c_t^i)}{\varphi^i u''(c_t^i)}$ and $\frac{\partial l_t^i}{\partial L_t} = -\frac{v'(l_t^i/\theta^i)}{\varphi^i \frac{1}{\theta^i} v''(l_t^i/\theta^i)}$ which are bounded given Assumption 1. The first-order condition of the optimal contracting problem with respect to φ^i gives that $\eta^i = \sum_{j\in I} \frac{\lambda^j}{\varphi^j} - \frac{\lambda^i}{\varphi^i}$ which is finite. Then $U_C^W(t; \boldsymbol{\varphi})$ and $U_L^W(t; \boldsymbol{\varphi})$ are bounded. $\square$

Now we proceed to the main proof.

Proof. **Step 1:** The sustainability constraint binds infinitely often in the long run. To show this, consider the rewritten first-order conditions of the contracting problem (C.1)-(C.2)

$$\frac{\beta^t}{Q_t^*}\left[U_C^W(t;\boldsymbol{\varphi})+U_C^P(t;\boldsymbol{\varphi})\sum_{k=0}^{t}\gamma_k\right]=\mu \tag{D.2}$$

$$\frac{\beta^t}{Q_t^*}\left[U_L^W(t;\boldsymbol{\varphi})+U_L^P(t;\boldsymbol{\varphi})\sum_{k=0}^{t}\gamma_k\right]=\mu F_L(L_t) \tag{D.3}$$

which implies

$$\frac{\beta^t}{Q_t^*}\left[U_C^W(t;\boldsymbol{\varphi})U_L^P(t;\boldsymbol{\varphi})-U_L^W(t;\boldsymbol{\varphi})U_C^P(t;\boldsymbol{\varphi})\right]=\mu\left[U_L^P(t;\boldsymbol{\varphi})-U_C^P(t;\boldsymbol{\varphi})F_L(L_t)\right] \tag{D.4}$$

Note that the resource constraint holds with equality and $\mu>0$ since U^P is strictly increasing in C and strictly decreasing in L. Then equation (D.4) implies that

$$\mu=\frac{\beta}{Q_1^*}\frac{U_C^W(1;\boldsymbol{\varphi})U_L^P(1;\boldsymbol{\varphi})-U_L^W(1;\boldsymbol{\varphi})U_C^P(1;\boldsymbol{\varphi})}{U_L^P(1;\boldsymbol{\varphi})-U_C^P(1;\boldsymbol{\varphi})F_L(L_1)}$$

which is a constant that only depends on the allocation at period $t=1$. Hence, μ is a positive constant for any period t. Then taking the limit of equation (D.2) as $t\to\infty$ gives

$$\lim_{t\to\infty}\frac{\beta^t}{Q_t^*}\left[U_C^W(t;\boldsymbol{\varphi})+U_C^P(t;\boldsymbol{\varphi})\sum_{k=0}^{t}\gamma_k\right]=\mu$$

Given assumption 2, Lemma D.3, and that μ is a positive constant, it must be that $\sum_{k=0}^{t}\gamma_k\to\infty$ as $t\to\infty$. This implies that $\gamma_t>0$ and the sustainability constraint binds infinitely often in the long run.

Step 2: The sustainable allocation $\{C_t^*,L_t^*\}_{t=0}^{\infty},\boldsymbol{\varphi}^*$ converges to the maximal steady-state debt allocation $\{C_{ss}^*,L_{ss}^*\}$that is the solution to

$$\max_{C,L}\left\{\frac{1+r^*}{r^*}\left(F(L)-C-G\right)\right\}$$
$$s.t.\quad U^P(C,L;\boldsymbol{\varphi}^*)\geq U^P(C_\infty^*,L_\infty^*;\boldsymbol{\varphi}^*)$$

where R^* and G are the associated steady state gross interest rate and fiscal expenditure. Let ψ_u be the multiplier on the utility constraint. Then $\{C^*_{ss}, L^*_{ss}\}$ satisfies first-order conditions

$$\begin{aligned}
\psi_u U^P_C(C^*_{ss}, L^*_{ss}; \boldsymbol{\varphi}) &= 1 \\
-\psi_u U^P_L(C^*_{ss}, L^*_{ss}; \boldsymbol{\varphi}) &= F_L(C^*_{ss}, L^*_{ss}) \\
\psi_u \left(U^P(C^*_{ss}, L^*_{ss}; \boldsymbol{\varphi}) - U^P(C^*_{\infty}, L^*_{\infty}, \boldsymbol{\varphi}^*) \right) &= 0
\end{aligned}$$

U^P is strictly increasing in C, so $\psi_u > 0$.

Define $\psi^t_u = \frac{\beta^t}{\mu Q^*_t} \sum_{k=0}^t \gamma_k$, then from equations (C.1) and (C.2), we have that the sustainable allocation follows

$$\begin{aligned}
1 - \psi^t_u U^P_C(C^*_t, L^*_t; \boldsymbol{\varphi}^*) &= \frac{\beta^t}{\mu Q^*_t} U^W_C(C^*_t, L^*_t; \boldsymbol{\varphi}^*) \\
F_L(L^*_t) + \psi^t_u U^P_L(C^*_t, L^*_t; \boldsymbol{\varphi}^*) &= -\frac{\beta^t}{\mu Q^*_t} U^W_L(C^*_t, L^*_t; \boldsymbol{\varphi}^*)
\end{aligned}$$

Taking the limit as $t \to \infty$ gives

$$\begin{aligned}
\lim_{t\to\infty} \left[1 - \psi^t_u U^P_C(C^*_t, L^*_t; \boldsymbol{\varphi}^*) \right] &= 0 \\
\lim_{t\to\infty} \left[F_L(t) + \psi^t_u U^P_L(t) \right] &= 0 \\
\lim_{t\to\infty} \left(U^P(C^*_{ss}, L^*_{ss}; \boldsymbol{\varphi}) - U^P(C^*_{\infty}, L^*_{\infty}, \boldsymbol{\varphi}^*) \right) &= 0
\end{aligned}$$

Therefore, the long-run sustainable allocation coincides with the maximal steady-state debt allocation.

Step 3: Financial autarkic allocation is never optimal at any point in time.

Suppose, by contradiction that the autarkic allocation $\{C^a_t, L^a_t, \varphi^a\}_{t=0}^{\infty}$ solves the planning problem from $t = \mathcal{T}$ onwards. Fix $\epsilon > 0$. Consider the allocation $\left\{\hat{C}_t, \hat{L}_t, \hat{\varphi}_t\right\}_{t=0}^{\infty}$ such that for $t < \mathcal{T}$,we have , $\hat{C}_t = C^*_t, \hat{L}_t = L^*_t, \hat{\varphi}_t = \varphi^*$ and $\hat{\varphi}_t = \varphi^a$ for $t \geq \mathcal{T}$

For period $\mathcal{T}$:$\hat{C}_{\mathcal{T}} = C^a_{\mathcal{T}} + \frac{\epsilon}{R^*} \left[F_L(L^a_{\mathcal{T}+1}) + \frac{U^P_L(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)}{U^P_C(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)} \right]$, $\hat{L}_{\mathcal{T}} = L^a_{\mathcal{T}}$

For period $\mathcal{T}+1$: $\hat{C}_{\mathcal{T}+1} = C^a_{\mathcal{T}+1} - \frac{U^P_L(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)}{U^P_C(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)}\epsilon$, $\hat{L}_{\mathcal{T}+1} = L^a_{\mathcal{T}+1} + \epsilon$

For period $t \geq \mathcal{T}+2$:$\hat{C}_t = C^a_t, \hat{L}_t = L^a_t$

First, $\left\{\hat{C}_t, \hat{L}_t, \hat{\varphi}_t\right\}_{t=0}^{\infty}$ is feasible because

$$
\begin{aligned}
&\sum_{t\geq 0} Q_t^* \left[F(\hat{L}_t) - \hat{C}_t - G_t\right] \\
=&F(\hat{L}_{\mathcal{T}}) - \hat{C}_{\mathcal{T}} - G_{\mathcal{T}} + \frac{1}{R_{t+1}^*}\left[F(\hat{L}_{\mathcal{T}+1}) - \hat{C}_{\mathcal{T}+1} - G_{\mathcal{T}+1}\right] \\
&+ \sum_{t<\mathcal{T}} Q_t^* \left[F(\hat{L}_t) - \hat{C}_t - G_t\right] + \sum_{t\geq \mathcal{T}+2}^{\infty} Q_t^* \left[F(\hat{L}_t) - \hat{C}_t - G_t\right] \\
=&\sum_{t<\mathcal{T}} \left(\frac{1}{R^*}\right)^t \left[F(\hat{L}_t) - \hat{C}_t - G_t\right] + \sum_{t\geq \mathcal{T}+2} \left(\frac{1}{R^*}\right)^t \left[F(L_t^a) - C_t^a - G_t\right] \\
&- \frac{\epsilon}{R^*}\left[F_L(L_1^a) + \frac{U_L^P(C_1^a, L_1^a)}{U_C^P(C_1^a, L_1^a)}\right] + \frac{1}{R^*}\left(F_L(L_1^a)\epsilon + \frac{U_L^P(C_1^a, L_1^a)}{U_C^P(C_1^a, L_1^a)}\epsilon\right) \\
=&\sum_{t<\mathcal{T}} \left(\frac{1}{R^*}\right)^t \left[F(\hat{L}_t) - \hat{C}_t - G_t\right] + \sum_{t\geq \mathcal{T}+2} \left(\frac{1}{R^*}\right)^t \left[F(L_t^a) - C_t^a - G_t\right] \\
\geq& B_0
\end{aligned}
$$

The last inequality is guaranteed by assumption that the autarkic allocation solves the planning problem from $t = \mathcal{T}$ onwards

$\left\{\hat{C}_t, \hat{L}_t, \hat{\varphi}_t\right\}_{t=0}^{\infty}$ is implementable in equilibrium since there exists a $\hat{T}$ such that $\forall i \in I$

$$
\sum_{t\geq 0} \beta^t \left[V_C(t;\boldsymbol{\varphi})c^i(t;\boldsymbol{\varphi}) + V_L(t;\boldsymbol{\varphi})l^i(t;\boldsymbol{\varphi})\right] = V_C(0;\boldsymbol{\varphi})\left(a_0^i - \hat{T}\right)
$$

The flow utilities for all period $t \geq \mathcal{T}+1$ do not change. That is,

$U_{\mathcal{T}+1}^P(\hat{C}_{\mathcal{T}+1}, \hat{L}_{\mathcal{T}+1}, \hat{\varphi}_{\mathcal{T}+1})$

$= U_{\mathcal{T}+1}^p(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a) + U_L^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)\epsilon - U_C^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a)\frac{U_L^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)}{U_C^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)}\epsilon$

$= U_{\mathcal{T}+1}^p(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)$ (for sufficiently small ϵ)

$U_t^P(\hat{C}_t, \hat{L}_t, \hat{\varphi}_t) = U_t^p(C_t^a, L_t^a, \varphi^a), \forall t \geq \mathcal{T}+2$

and the flow utility in period $\mathcal{T}$ increases. That is,

$$
\begin{aligned}
U_{\mathcal{T}}^P(\hat{C}_{\mathcal{T}}, \hat{L}_{\mathcal{T}}, \hat{\varphi}_{\mathcal{T}}) &= U_{\mathcal{T}}^p(C_{\mathcal{T}}^a, L_{\mathcal{T}}^a, \varphi^a) + U_{C,\mathcal{T}}^p(C_{\mathcal{T}}^a, L_{\mathcal{T}}^a, \varphi^a)\frac{\epsilon}{R^*}\left[F_L(L_{\mathcal{T}+1}^a) + \frac{U_L^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)}{U_C^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)}\right] \\
&> U_{\mathcal{T}}^p(C_{\mathcal{T}}^a, L_{\mathcal{T}}^a, \varphi^a)
\end{aligned}
$$

because $F_L(L_{\mathcal{T}+1}^a) + \frac{U_L^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)}{U_C^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)} > 0$ as the aggregate labor distortion in autarky is positive.

Thus, $V_0^P\left(\left\{\hat{C}_t, \hat{L}_t; \varphi^a\right\}_{t=0}^{\infty}\right) > V_0^P\left(\{C_t^a, L_t^a; \varphi^a\}_{t=0}^{\infty}\right)$, which contradicts

$\{C_t^a, L_t^a, \varphi^a\}_{t=0}^{\infty}$ being the optimal allocation. $\square$

D.3 Proof of Proposition 3.1

First, the following lemma must hold.

Lemma D.4. *cov* $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < 0$ *and cov* $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$

Proof. The first step is to show that for i and j such that $i \neq j$, $\theta^i > \theta^j \iff \varphi^i > \varphi^j$.

Suppose $\theta^i > \theta^j$ and $\varphi^i \leq \varphi^j$, then $\psi_l^i \leq \psi_l^j$. By the definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \leq \frac{\varphi^i}{\varphi^j} \leq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} > 1$, which is a contradiction.

Suppose $\varphi^i > \varphi^j$ and $\theta^i \leq \theta^j$, then $\psi_l^i > \psi_l^j$. By the definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} > \frac{\varphi^i}{\varphi^j} > 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \leq 1$, which is a contradiction.

Next, the individual implementability constraint is

$$\psi_c^i \Phi_C^V \sum_t \beta^t C_t^{1-\sigma} - \psi_l^i \Phi_L^V \sum_t \beta^t L_t^{1+\nu} = \Phi_C^V C_0^{-\sigma} \left(a_0^i - T\right)$$

or

$$\psi_c^i = \psi_l^i \frac{\Phi_L^V \sum_t \beta^t L_t^{1+\nu}}{\Phi_C^V \sum_t \beta^t C_t^{1-\sigma}} + \frac{\Phi_C^V C_0^{-\sigma} \left(a_0^i - T\right)}{\Phi_C^V \sum_t \beta^t C_t^{1-\sigma}}$$

By the definition of ψ_c^i, $\varphi^i > \varphi^j \iff \psi_c^i > \psi_c^j$, and by the assumption, $\theta^i > \theta^j \iff a_0^i > a_0^j$, which implies that $\theta^i > \theta^j \iff \psi_c^i > \psi_c^j \iff \psi_l^i > \psi_l^j$.

Thus, $\theta^i > \theta^j \iff \varphi^i > \varphi^j \iff \psi_c^i > \psi_c^j \iff \psi_l^i > \psi_l^j$.

In addition, $\theta^i > \theta^j \iff \lambda^i < \lambda^j$, which implies that

$$\psi_c^i > \psi_c^j \iff \frac{\lambda^i}{\varphi^i} < \frac{\lambda^j}{\varphi^j}$$
$$\psi_l^i > \psi_l^j \iff \frac{\lambda^i}{\varphi^i} < \frac{\lambda^j}{\varphi^j}$$

Hence, cov $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < 0$ and cov $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$. $\square$

Below is the main proof of the proposition.

Proof. In financial autarky, there exist a vector of market weights $\boldsymbol{\varphi}^a$, transfer T^a, and multiplier $\boldsymbol{\eta}^a$that satisfies the conditions in Proposition 1.1 such that

$$\begin{aligned}
\underline{U}_t \equiv & \max_{C_t, L_t, \boldsymbol{\varphi}^a, T^a} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^{W,a} \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^{W,a} \frac{L_t^{1+\nu}}{1+\nu} \right] - \Phi_C^{V,a} C_0^{-\sigma} \sum_i \pi^i \eta^{i,a} T^a \\
& s.t. \qquad C_t + G = F(L_t, t)
\end{aligned}$$

where $\beta^t \pi^i \eta^{i,a}$ is the Lagrange multiplier on the individual implementability constraint and $\Phi_C^{W,a}, \Phi_L^{W,a}$ follows the formulas in Appendix C.1.

The aggregate labor distortion is constant and equal to

$$\Omega^a = 1 - \frac{\Phi_C^W \Phi_L^P}{\Phi_L^W \Phi_C^P}$$

We have that

$$\begin{aligned}
\frac{\Phi_C^W}{\Phi_C^P} &= 1 + (1-\sigma) \left[\frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right]}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)} - 1 \right] \\
\frac{\Phi_L^W}{\Phi_L^P} &= 1 + (1+\nu) \left[\frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right]}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)} - 1 \right]
\end{aligned}$$

Lemma D.4 shows that $\text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right), \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$, which implies that

$$\frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right]}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)} - 1 > 0; \quad \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right]}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)} - 1 > 0$$

and given that $\sigma \geq 1$ and $\nu > 0$, we have $\frac{\Phi_C^W}{\Phi_C^P} < \frac{\Phi_L^W}{\Phi_L^P}$. Thus, $\Omega^a > 0$. □

D.4 Proof of Proposition 3.2

Proof. For both of the two cases, $\lambda^i = \varphi^i$ and $\eta^i = 0$. This implies that $\Phi_C^W = \Phi_C^P$ and $\Phi_L^W = \Phi_L^P$. The aggregate labor distortion in financial autarky is

$$\Omega^a = 1 - \frac{\Phi_C^W \Phi_L^P}{\Phi_L^W \Phi_C^P} = 0$$

□

Online Appendix

A Data

This section explains the data used for the calibration exercise in Section 4.

A.1 Macroeconomic Data Descriptions and Sources

Most data are annual series covering the 1985-2015 period. Some data samples cover the 2002-2015 period.

- Net foreign liability is the negative of net foreign asset (NFA) from the External Wealth of Nations Database, Lane and Milesi-Ferretti (2018)
- Net international investment position is the official international investment position (IIP) from the External Wealth of Nations Database, Lane and Milesi-Ferretti (2018)
- Pre-tax Gini Index the market Gini from the Standardized World Income Inequality Database, Solt (2019).
- GDP per capita is the constant 2010 US Dollar GDP per capita series from World Development Indicator Database, The World Bank (2019)
- GDP growth is the log difference of constant 2010 US Dollar GDP series from World Development Indicator Database, The World Bank (2019)
- Inflation is the annual inflation series measured by the GDP deflator from World Development Indicator Database, The World Bank (2019)
- Real GDP is GDP series in constant local currency units from World Development Indicator Database, The World Bank (2019)
- Real return on German bond is the interest rate on Gernman bond adjusted for inflation measured by the GDP deflator. The interest rate is the long-term interest rate for convergence purposes from the Eurostat Database (2019). These bonds have 10-year maturity and are denominated in Euro.

- Real interest rate is the lending interest rate adjusted for inflation as measured by the GDP deflator from World Development Indicator Database, The World Bank (2019)
- Italy's cross-sectional wage inequality is calculated from the micro-data by Jappelli and Pistaferri (2010) using Surveys of Household Income and Wealth conducted by the Bank of Italy for the period 1980-2006.
- Government consumption is the general government final consumption expenditure series from World Development Indicator Database, The World Bank (2019)
- Private consumption is the households and NPISHs final consumption expenditure series from World Development Indicator Database, The World Bank (2019)

A.2 Italian Household Survey

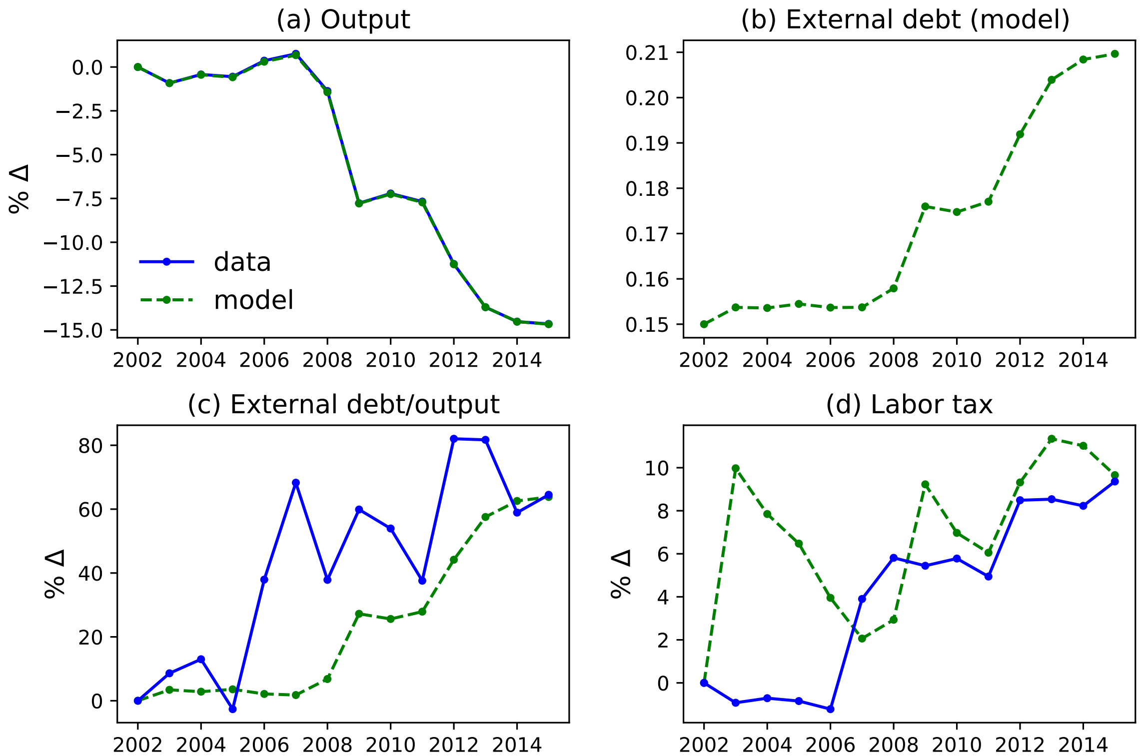
The analysis of wage inequality uses household-level data from the *Survey on Household Income and Wealth* (SHIW), conducted by the Bank of Italy. The SHIW includes cross-sectional and panel data on household's demographics, income, labor supply, consumption, and wealth. Jappelli and Pistaferri (2010) provide details on the survey design descriptions and data quality analysis. I use data for the period 1987 to 2014. In this period, the survey was conducted biennially, except for the period 1995 to 1998 with a 3-year interval.

The sample selection is followed. The original individual sample includes 329,446 units from 1987 to 2014. Given the focus on wage inequality and labor supply across households, I focus on heads of households between the age of 25 and 60. This selection criteria reduces the sample size to 71,621 units. To reduce the impact of outliers, I exclude observations with no hours worked, with negative income, or with hourly wage in the bottom 0.5% of the distribution. The final sample size is 67,176.

B Event Analysis

This section provides the additional analysis to Section 4: an event analysis for Italy in period of 2002 to 2015. The model is calibrated by feeding in a sequence of productivity shock realizations and the initial external debt-to-output ratio such that the simulated path of output matches the observed Italian output, and the change in the external debt-to-output ratio in the model coincides with that in the data over 2002–2015. The analysis then compares the relative change in the external debt-to-output ratio over time between the data and the model simulation, using 2002 as the benchmark year.

Figure 4: Italy's Recession: Data and Model



Note: The graph depicts the time paths of output, external debt, and external debt-to-output for the data and the model's simulation. Panel (a) plots the output path. Panel (b) plots the external debt paths of the model. Panel (c) plots external debt-to-output, and panel (d) plots the labor tax. The simulation uses a sequence of productivity shock realization such that the model's output matches the data output for Italy in 2002-2015. The initial external debt level is such that the model's external debt-to-output matches with the starting value in 2002 from the data. The benchmark period is 2002. Data sources: McDaniel (2007), Lane and Milesi-Ferretti (2018), and The World Bank (2019).

Figure 4 plots the exercise's results. Panel (a) plots the output paths of the data and the model. Panel (b) plots the time path of external debt in the model. Panel

(c) plots external debt-to-output time paths for both the data and the model. Panel (d) plots estimated labor taxes from McDaniel (2007) and the optimal labor taxes from the model. From 2011 to 2015, Italy's output has dropped by 7.6% below trend, accompanying with a 20% increase in external debt-to-output and a 4.2% increase in labor tax. In the model's simulation, the similar drop in output is associated with a 26% increase in external debt-to-output and a 3.5% increase in labor tax.

C Sensitivity Analysis

This section presents a sensitivity analysis to examine how optimal policies respond quantitatively to different model ingredients: government expenditure, aggregate uncertainty, and the discount factor. Table C.1 compares the moments from the baseline model with those from alternative specifications. The second column reports results from the no-government-expenditure case. Average external debt remains unchanged relative to the baseline, indicating that the level of exogenous government expenditure does not affect the optimal sustainable debt level. Higher government expenditure, however, is associated with larger second moments, with the exception of the consumption–output correlation.

Table C.1: Sensitivity Analysis

	Baseline	No govt. exp. ($g = 0$)	Deterministic ($\sigma_z = 0$)	Lower discount ($\beta = 0.95$)
Avg. external debt/output (%)	17	17	22	12
Std. external debt/output (%)	2.1	0.99	-	1.8
Std. consumption / Std. output	1.2	0.96	-	1.2
Std. net savings/output (%)	1.8	1.5	-	0.62
Correlation with output (%)				
Consumption	95	97	-	99
Net savings/output	31	29	-	-1.1

Note: Table C.1 reports the results for the sensitivity analysis. The moments are calculated from the model's simulation for 10500 periods and then excluding the first 500 periods. For the second moments, output and consumption series are logged and linear detrended, and net saving and external debt ratio series are linear detrended.

The third column of Table C.1 reports results for the deterministic case with no aggregate uncertainty. In this specification, the model sustains a higher level of debt than in the baseline. The intuition is that uncertainty in the baseline model generates a stronger precautionary motive, which limits overall external debt accumulation relative to the deterministic case.

The fourth column presents results for a lower discount factor compared to the baseline. In this case, the average external-debt-to-output ratio is lower. With a higher discount factor, the value of autarky is relatively smaller and the economy takes longer to reach the sustainability constraint, allowing the government to accumulate and sustain more debt. Finally, the lower-discount-factor model provides a poor match for the second moments of net savings in the data.

D Computational Algorithm

The computational algorithm is as follows.

1. Guess μ and φ. Compute η. Solve for the allocation following

   (a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ. Set initial guess for

   $$V(s_t, \mu_t, \Gamma_{t-1}) = \sum_{j \geq 0, s^t \subseteq s^{t+j}} \beta^j \Pr(s^{t+j}) \left[\Phi_C^P \frac{C(s^{t+j})^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^{t+j})^{1+\nu}}{1+\nu} \right]$$

   (b) Assume the constraint does not bind in s_t: $\gamma(s_t) = 0$. Solve for the allocation $C(s_t), L(s_t)$ using the first-order conditions

   $$\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1}\right] C(s_t)^{-\sigma} = \mu$$
   $$\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1}\right] L(s_t)^{\nu} = \mu F_L(s_t)$$

   (c) Since $\gamma(s_t) = 0$, compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$, then compute

$$
\begin{aligned}
A(s_t) &= \sum_{\tau=t}^{\infty}\beta^{\tau-t}\sum_{s^\tau\subseteq s^t} Pr(s^\tau)\left[\Phi_C^P\frac{C(s^\tau)^{1-\sigma}}{1-\sigma}-\Phi_L^P\frac{L(s^\tau)^{1+\nu}}{1+\nu}\right]\\
&= \Phi_C^P\frac{C(s_t)^{1-\sigma}}{1-\sigma}-\Phi_L^P\frac{L(s_t)^{1+\nu}}{1+\nu}\\
&+\beta\sum_{s_{t+1}}\Pr(s_{t+1}|s_t)V(s_{t+1},\mu_{t+1}=\beta R^*\mu_t,\Gamma_t=\beta R^*\Gamma_{t-1})
\end{aligned}
$$

(d) Check if $A(s_t)\geq \underline{U}(s_t)$. If it is, proceed to the next step. If not, solve for $C(s_t), L(s_t), \gamma(s_t)$ using these equations

$$
\begin{aligned}
\left[\mu_t\Phi_C^W+\Phi_C^V\left(\Gamma_{t-1}+\gamma(s_t)\right)\right]C(s_t)^{-\sigma} &= \mu\\
\left[\mu_t\Phi_L^W+\Phi_L^V\left(\Gamma_{t-1}+\gamma(s_t)\right)\right]L(s_t)^{\nu} &= \mu F_L(s_t)\\
\Phi_C^P\frac{C(s_t)^{1-\sigma}}{1-\sigma}-\Phi_L^P\frac{L(s_t)^{1+\nu}}{1+\nu}&\\
+\beta\sum_{s_{t+1}}\Pr(s_{t+1}|s_t)V\left(s_{t+1},\mu_{t+1}=\beta R^*\mu_t,\Gamma_t=\beta R^*\left(\Gamma_{t-1}+\gamma(s_t)\right)\right) &= \underline{U}(s_t)
\end{aligned}
$$

(e) Given $C(s_t), L(s_t), \gamma(s_t)$($\gamma$ can be zero or not), compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1},\mu_{t+1}=\beta R^*\mu_t,\Gamma_t=\Gamma_{t-1}+\gamma(s_t))$. Update the value function

$$
\begin{aligned}
V^{n+1}(s_t,\Gamma_{t-1}) &= \Phi_C^P\frac{C(s_t)^{1-\sigma}}{1-\sigma}-\Phi_L^P\frac{L(s_t)^{1+\nu}}{1+\nu}\\
&+\beta\sum_{s_{t+1}}\Pr(s_{t+1}|s_t)V^n\left(s_{t+1},\mu_{t+1}=\beta R^*\mu_t,\Gamma_t=\beta R^*\left(\Gamma_{t-1}+\gamma(s_t)\right)\right)
\end{aligned}
$$

2. Compute residuals to find μ and φ

$$
\begin{aligned}
r^\mu &= \sum_{t\geq 0} q_t\left[F(L_t)-G_t-C_t\right]-B_0\\
r^\varphi &= \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\Phi_C^V\left(\psi_c^i-\psi_c^j\right)C_t^{1-\sigma}-\Phi_L^V\left(\psi_l^i-\psi_l^j\right)L_t^{1+\nu}\right]
\end{aligned}
$$

3. Find μ and φ such that $r^\mu=0$ and $r^\varphi=0$.