Optimal Redistributive Policy in Debt Constrained Economies*

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February 2023

Abstract

How should governments with a preference for redistribution design tax policies when facing limited borrowing? This paper studies optimal taxation in a small open economy with heterogeneous agents and endogenous debt constraints arising from the government's limited commitment to fiscal policies. The optimal labor tax decreases over time and is nonzero in the limit, and the optimal capital and domestic borrowing taxes are positive in the limit, deviating from the standard Ramsey tax results. The government's redistributive motive directly affects optimal tax levels, whereas binding debt constraints influence optimal tax dynamics. In the numerical analysis, a stronger redistributive preference requires greater initial tax distortions and a higher external debt level in the long run.

Keywords: Optimal taxation; Redistribution; Limited commitment; Sovereign debt
JEL Classifications: E62; F34; F38; H21; H23; H63

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*This is a revised version of the first chapter of my dissertation at the University of Minnesota. I thank the editor, Javier Bianchi, and two anonymous referees for their comments and suggestions. I thank V.V. Chari, Larry Jones, and Manuel Amador for their invaluable advice and guidance. I am grateful for comments from Fernando Arce, Anmol Bhandari, Ben Malin, Emily Moschini, Chris Phelan, David Rahman, Aldo Rustichini, and Jan Werner. I have also benefited from discussions with the seminar participants at the University of Minnesota, Federal Reserve Bank of Minneapolis, Federal Reserve Bank of Richmond, 2018 Midwest Macroeconomic Meetings, 2018 EEA Annual Congress, and 2019 North American Summer Meeting of the Econometric Society. I acknowledge the support from the 2017 AEA Summer Economics Fellowship.

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Introduction

How does a government with a desire for redistribution determine its tax policies while facing limited borrowing? Taxes are the main source of revenue that affects the government's ability to issue or repay debt. At the same time, distortionary taxes and transfers redistribute resources across agents, so the government's preferences for greater redistribution affect its willingness to raise revenue and influence its borrowing capacity. When borrowing is constrained, redistributing via taxes is limited. Empirical works on the recent European debt crisis have demonstrated how the government's redistributive motive and debt accumulation are closely related.[1] First, the rapid accumulation of external debt in the periods leading up to a crisis led to many countries facing such high costs of borrowing that they could not roll over their debt. Second, highly indebted countries such as Greece, Portugal, and Spain also experienced high levels of income dispersion. According to the European Union's Statistics on Income and Living Conditions, the Gini coefficients and S80/S20 income quintile share ratio in these countries were both higher than the EU-27 country average. Countries proposed different policy strategies to tackle the problems of constrained borrowing while maintaining the desired level of redistribution.

Motivated by these observations, this paper questions how both the government's redistributive motive and its limited ability to borrow affect optimal tax policies. The paper explores insights on the trade-off between redistribution and efficiency and studies how limited borrowing in the form of endogenously binding debt constraints restricts the government's ability to smooth taxes over time. While tax policies allow the government to achieve a desired level of redistribution, the government has an additional incentive to change taxes to relax debt constraints. Through tax policies, the government's redistributive motive influences its borrowing capacity.

I address these issues in a model of a small open economy with heterogeneous agents and the government's limited commitment. The model combines the sovereign debt framework of limited commitment (Aguiar et al. (2009) and Aguiar and Amador (2011, 2014, 2016)) with the Ramsey taxation framework (Chari et al. (1994), Chari and Kehoe (1999), and Werning (2007)). Domestic agents are impatient relative to the international intertemporal price of resources. Taxes are linear, consisting of distortionary and lump-sum components. The economy is subject to endogenous debt constraints arising from the government's lack of commitment in all policies. The debt constraints are such that the value of staying in contracted policies is weakly higher than the value of reneging. I consider a planner that chooses efficient allocation to maximize the social welfare but is subject to the distortionary costs of policies and debt constraints. I then study optimal policies that implement the efficient allocation.

I establish three theoretical results on optimal labor taxation under the case of separable isoelastic preferences. First, perfect labor tax smoothing occurs when debt constraints do not bind. When debt is not constrained, the government borrows as needed and sets optimal labor taxes to balance the trade-off between redistribution and efficiency, which depend on skill distribution, distributional preference, and elasticities that are constant over time.

Binding debt constraints, however, limit the government's ability to borrow and maintain constant labor tax distortions over time. I find that front-loading tax distortion is optimal when debt constraints are relevant. Specifically, in each period that the debt constraint binds, the optimal distortionary

[1] See Berg and Sachs (1988), Aizenman and Jinjarak (2012), Ferriere (2015), and Jeon and Kabukcuoglu (2018) for more empirical analysis on how high income inequality is correlated with high spreads and an increased likelihood of debt crises.

labor tax permanently decreases for all future periods, given that the government is inequality averse and preferences are highly concave in consumption. The key mechanism is back-loading efficiency: lower future labor distortions increase the current period's borrowing capacity. This mechanism is present because of the distortionary cost of policies. An inequality-averse government would like to levy distortionary labor taxes to redistribute resources toward low-income agents. Faced with the distortionary cost and debt constraints, the government front-loads tax distortions when debt constraints do not bind and reduces tax distortions when debt constraints bind.

The third result is that the optimal labor tax converges to a nonzero value when domestic agents are impatient and debt constraints are binding infinitely often in the long run. Moreover, the labor tax's level in the limit depends on heterogeneity and distributional preference. Back-loading efficiency implies that the optimal labor tax in the long run maximizes the economy's efficiency, which is affected by redistribution in this case. The effect of redistribution, captured by skill distribution and the government's distributional preference, gives a nontrivial value for the optimal labor tax in the limit. In a representative-agent framework such as in Aguiar and Amador (2016), redistribution does not affect the economy's maximum efficiency, so the optimal labor tax in the limit is zero.

Optimal capital controls in the form of taxes on the domestic sector's borrowing are positive when debt constraints bind. Impatient domestic agents borrow over time and do not internalize the effect of their individual borrowing decisions on the aggregate debt constraints. Therefore, it is optimal to levy positive taxes that discourage domestic borrowing when the debt constraint binds. Income inequality affects the distribution of domestic borrowing, whereas both income inequality and distributional preference affect the cost of binding debt constraints. Hence, the levels of the domestic borrowing taxes depend on heterogeneity and distributional preference. In the long run, the optimal tax on domestic borrowing only depends on the discount factor and the international interest rate.

In addition, the optimal capital tax is positive in the long run when the value of reneging increases in the current capital stock. Because the government can renege on its policies and expropriate capital holdings, the efficient outcome discourages capital accumulation.

I examine the effect of redistribution on optimal labor taxes and find that the main determinant of tax levels is the relative ratio between the government's distributional preference and equilibrium market distribution over individual utilities. If the distributional preference is the same as the equilibrium distribution, then the optimal labor tax is zero. The government sets nonzero labor taxes only when its distributional preference differs from the competitive equilibrium's outcomes that are driven by heterogeneity.

The numerical analysis illustrates optimal policies in a model of no capital and a utilitarian welfare function. The government levies a labor tax in the initial periods to redistribute resources toward poorer households. In the long run, however, the government sets a labor subsidy to encourage highly productive agents to work more to increase the economy's borrowing capacity. The optimal domestic borrowing tax is zero initially when debt constraints do not bind and becomes positive as debt constraints bind in the long run. External debt increases even after debt constraints are binding. Tax dynamics correspond to a reduction in the distortionary costs of policies whenever the debt constraint binds.

Optimal tax and debt policies respond to changes in income inequality. In a comparative statics exercise, I find that a more unequal economy corresponds to a higher labor tax for more redistribution in the beginning of time and a higher labor subsidy for more efficiency in the long run. Lump-sum transfers also increase with respect to income inequality. Higher income inequality also leads to higher taxes on domestic borrowing when debt constraints bind because the distribution of domestic borrowing is more unequal. In addition, a more unequal economy accumulates more external debt over time.

Related Literature. This paper contributes to the optimal taxation and debt management research of the public finance literature (e.g., Barro (1979), Lucas and Stokey (1983), Chari et al. (1994), Aiyagari and McGrattan (1998), Chari and Kehoe (1999), Aiyagari et al. (2002), and many other papers). The argument for labor tax smoothing often relies on the fact that the government can issue debt that is contingent to all states and is not constrained beyond the natural debt limit. In this paper, tax smoothing is not always optimal; the government's ability to smooth tax distortion is restricted by the international lenders' willingness to lend. The nonzero capital and domestic borrowing taxes are in contrast to the zero convergence of the capital tax from Judd (1985), Chamley (1986), Chari et al. (1994), and Straub and Werning (2020) because the endogenous debt limits depend on the capital stock, and the domestic agents do not internalize the effect of their borrowing on aggregate debt constraints.

The optimal taxation results are relevant to the current research in optimal fiscal policies with heterogeneity and the government's redistributive motive. Bhandari et al. (2017) and Werning (2007) both find that the government's redistributive motive has a significant impact on optimal policies. Werning (2007) develops the conditions for perfect tax smoothing with redistribution, while Bhandari et al. (2017) emphasize the impact of the initial distribution of heterogeneity on the optimal allocation and optimal debt in the long run. In this paper, I highlight the role of endogenous debt constraints in changing optimal tax dynamics, which results in imperfect tax smoothing. I also argue that the distribution of heterogeneity and the government's redistributive motive are important in determining the optimal debt level that is sustainable in the long run. The paper also relates to the work of Chien and Wen (2022), which studies optimal taxation with idiosyncratic risk and market incompleteness. In Chien and Wen (2022), tax smoothing is limited when upper debt limit is relevant, and debt accumulation is optimal for the government to provide liquidity for precautionary saving households. In this paper, the upper debt limit is endogenously determined by the government's ability to sustain debt, whereas debt accumulation is exogenously driven by domestic households' impatience.

I introduce heterogeneity and the redistributive effect of fiscal policies in the literature that studies the government's lack of commitment in both tax and debt policies. The volatile tax and government expenditures are similar to Cuadra et al. (2010) and Bauducco and Caprioli (2014). The theoretical finding of front-loading labor taxation when borrowing is tightened relates to the absence of tax smoothing in Pouzo and Presno (2022) and the quantitative result of Arellano and Bai (2016), in which higher tax distortion would make the country more likely to default. The result on front-loading tax distortion also relates to Karantounias (2018)'s "fear" mechanism that comes from the default risk that makes debt issuance expensive. In this paper, the threat of default is present when debt constraints are binding, which leads to lower tax distortions in future periods.

This paper contributes to research on the interaction between inequality and sovereign default risk (Ferriere (2015), Balke and Ravn (2016), D'Erasmo and Mendoza (2016), Dovis et al. (2016), Balke and Ravn (2016), Jeon and Kabukcuoglu (2018), Bianchi et al. (2019), Deng (2021), and others). I provide new theoretical findings on optimal taxation in a general framework of inequality and the government's lack of commitment. The paper establishes that front-loading labor tax distortions is optimal when redistribution is costly and borrowing is limited. The optimal capital control in the form of a tax on domestic borrowing is optimal when borrowing is constrained. Moreover, the government's distributional preference affects optimal tax levels only if it differs from the equilibrium market's distribution.

The dynamic environment in this paper is an extension of the one in Aguiar and Amador (2016), adding heterogeneity and distributional preference and allowing for richer tax systems. Aguiar and Amador (2016) find that the labor tax must go to zero in the long run as a result of front-loading efficient consumption and leisure allocation. In this paper, the tax limit can be any real value. More interestingly, when the effect of redistribution is turned off in the model, the labor tax is zero in the limit, consistent with their findings. The paper shows that the government's redistributive consideration, not heterogeneity, is the main source for the changes in tax levels.

The model qualitatively accounts for the positive correlation between income dispersion and the debt level that several empirical papers have documented. Berg and Sachs (1988) show that income dispersion is a key predictor of a country's probability of rescheduling debt and the bond spread in secondary markets. Aizenman and Jinjarak (2012) describe a negative correlation between income dispersion and the tax base and a positive correlation with sovereign debt. Recently, Ferriere (2015), Jeon and Kabukcuoglu (2018), and Deng (2021) also provide empirical evidence that rising income dispersion significantly increases sovereign default risk.

Outline. The paper is organized as follows. Section 1 describes the environment, the competitive equilibrium, and the government's lack-of-commitment problem. Section 2 characterizes the equilibrium. Section 3 formulates the efficiency problem, and section 4 derives the main results of the optimal taxation under separable isoelastic preferences. Section 5 analyzes the effect of redistribution on optimal taxes. Section 6 provides a numerical analysis of optimal policies and efficient allocation and demonstrates a comparative statics exercise with respect to changes in skill dispersion. Section 7 then concludes.

1 A Model of Redistribution and Limited Commitment

1.1 Environment

The model is a small open economy facing exogenous world interest rates $\{r_t^*\}_{t=0}^{\infty}$. Time is discrete. There is a measure-one continuum of infinitely lived domestic agents that differ by labor productivity types $\left(\theta^i\right)_{i\in I}$, which are publicly observable. The fraction of agents with productivity θ^i is π^i, where $\left(\pi^i\right)_{i\in I}$ is such that $\sum_{i\in I}\pi^i = 1$ and $\sum_{i\in I}\pi^i\theta^i = 1$. All agents have the same discount factor β and the static utility $U(c,n)$ over consumption c and hours worked n. The utility function of the agent with productivity θ^i over consumption $c_t^i \geq 0$ and efficiency units of labor $l_t^i \geq 0$ is

$$\sum_{t=0}^{\infty}\beta^t U^i(c_t^i, l_t^i), \tag{1}$$

where $U^i(c,l) = U\left(c, \frac{l}{\theta^i}\right)$.

In addition, there is a representative firm that uses both capital and labor to produce a single output good. The production function $F(K, L)$ is constant returns to scale, where K and L are aggregate capital and labor, respectively. Capital is depreciated at the δ rate each period, where $0 < \delta \leq 1$. The economy is subject to an exogenous sequence of government spending $\{G_t\}_{t=0}^{\infty}$. Both the government and private sector have access to domestic and international financial markets.

1.2 Competitive Equilibrium with Government Policies

In each period, government policies are issuances of domestic and international bonds, a lump-sum tax T_t, a marginal tax on labor income τ_t^n, a marginal tax on capital τ_t^k, and a marginal tax on domestic savings τ_t^d. The domestic savings tax τ_t^d is a residence-based tax on the returns of domestic agents' savings. Domestic agents pay this tax regardless of the source of the saving returns or its location. A negative domestic savings tax τ_t^d implies a positive tax on domestic borrowing. Prices faced by the domestic agents and the firm are the labor wage w_t, the return on capital r_t^k, and the return on domestic asset holdings r_t.

Domestic agents. An agent of type $i \in I$ faces the sequential budget constraint in period t,

$$\begin{aligned} c_t^i + k_{t+1}^i + b_{t+1}^{d,i} + b_{t+1}^{f,i} \leq (1-\tau_t^n) w_t l_t^i + \left\{1 + (1-\tau_t^d)\left[(1-\tau_t^k) r_t^k - \delta\right]\right\} k_t^i \\ + \left[1 + (1-\tau_t^d) r_t\right] b_t^{d,i} + \left[1 + (1-\tau_t^d) r_t^*\right] b_t^{f,i} - T_t, \end{aligned} \tag{2}$$

where $c_t^i, l_t^i, k_t^i, b_t^{d,i}, b_t^{f,i}$ denote, respectively, the consumption, efficiency unit labor, capital holdings, and domestic and international asset holdings, of agent i in period t.

No arbitrage implies that returns on domestic and international bonds are equal, $r_t = r_t^*$. Therefore, the domestic agent's budget constraint can be rewritten as

$$c_t^i + k_{t+1}^i + a_{t+1}^i \leq (1-\tau_t^n) w_t l_t^i + \left\{1 + (1-\tau_t^d)\left[(1-\tau_t^k) r_t^k - \delta\right]\right\} k_t^i + \left[1 + (1-\tau_t^d) r_t\right] a_t^i - T_t, \tag{3}$$

where $a^i = b^{d,i} + b^{f,i}$ is the net asset holdings of agent i.

No arbitrage between capital and assets implies that the after-tax return is equalized,

$$1 + (1-\tau_t^d)\left[(1-\tau_t^k) r_t^k - \delta\right] = 1 + (1-\tau_t^d) r_t,$$

which then follows that

$$(1-\tau_t^k) r_t^k = r_t + \delta = r_t^* + \delta.$$

Aggregate allocation. The aggregate consumption is $C_t = \sum_{i \in I} \pi^i c_t^i$. The aggregate labor is $L_t = \sum_{i \in I} \pi^i l_t^i$. The aggregate capital is $K_t = \sum_{i \in I} \pi^i k_t^i$. The aggregate asset of the domestic sector is $A_t = \sum_{i \in I} \pi^i a_t^i$.

Representative firm. The firm chooses the amount of capital and labor to maximize profits each period:

$$\max_{\{K_t, N_t\}} \quad F(K_t, L_t) - w_t L_t - r_t^k K_t,$$

which gives the following first-order conditions:

$$r_t^k = F_K(K_t, L_t) \tag{4}$$
$$w_t = F_L(K_t, L_t).$$

The firm's profits are zero in equilibrium because of the constant returns to scale production function.

Government. The government needs to finance an exogenous sequence of expenditures$\{G_t\}_{t=0}^{\infty}$. Let B_t^g be the government's borrowing from both domestic and international markets. The government's borrowing has a return of r_t. The government's budget constraint in each period is

$$G_t + (1 + r_t)B_t^g \leq \tau_t^n w_t L_t + \tau_t^k r_t K_t + \tau_t^d \left(r_t A_t + \left[(1 - \tau_t^k)r_t^k - \delta\right] K_t\right) + B_{t+1}^g + T_t.$$

The government also faces a no-Ponzi condition such that the present value of its borrowing is bounded from below.

Define q_t as the intertemporal price of a unit period-t consumption in terms of period-0 consumption units,

$$q_t = \Pi_{s=0}^t \frac{1}{1 + r_s}, \tag{5}$$

with $q_0 = 1$. Using $\{q_t\}_{t=0}^{\infty}$ as the relevant intertemporal price, one can write the government's present-value budget constraint as

$$\sum_{t=0}^{\infty} q_t \left\{G_t - \tau_t^n w_t L_t - \tau_t^k r_t K_t - \tau_t^d r_t (K_t + A_t) - T_t\right\} \leq -B_0^g. \tag{6}$$

Aggregate resource constraint. Using the domestic agents' budget constraints and the government's budget constraint, one can obtain an aggregate resource constraint in terms of the intertemporal prices and the economy's initial external debt level:

$$\sum_{t=0}^{\infty} q_t \left[F(K_t, L_t) + (1 - \delta)K_t - K_{t+1} - C_t - G_t\right] \geq B_0, \tag{7}$$

where $B_0 = B_0^g - A_0$ is the economy's initial external debt level.

I refer to $B_t = B_t^g - A_t$ as the net international liability position or the external debt of the economy.[2] To see why, note that A_t includes the domestic agents' net position in international assets plus domestic government bonds. The term B_t^g is the government liability that includes domestic government bonds and international government bonds. Therefore, B_t equals to the domestic sector and government's net international liabilities minus international lenders' net positions in domestic bonds, which is the net international liability of the economy.

Competitive equilibrium. Given the above equations, one can define the following competitive equilibrium with government policies.

[2]The term B_t is the negative of the net foreign asset position of the economy.

Definition 1. Given initial external debt B_0 and individual wealth positions $\left(\alpha_0^i\right)_{i\in I}$,[3] a competitive equilibrium with government policies for an open economy is domestic agents' allocation $z^{H,i} = \left\{c_t^i, l_t^i, k_{t+1}^i, a_{t+1}^i\right\}_{t=0}^{\infty}$, $\forall i \in I$, the firm's allocation $z^F = \{K_t, L_t\}_{t=0}^{\infty}$, prices $p = \left\{q_t, w_t, r_t, r_t^k\right\}_{t=0}^{\infty}$, and the government policy $z^G = \left\{\tau_t^n, \tau_t^k, \tau_t^d, T_t, B_{t+1}^g\right\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves agent i's problem that maximizes (1) subject to (3) and a no-Ponzi condition for the agent's debt value; (ii) given p and z^G, z^F solves the firm's problem, which implies the first-order conditions (4); (iii) the government's budget constraint (6) holds; (iv) the aggregate resource constraint (7) is satisfied; (v) the no-arbitrage condition holds, $(1-\tau_t^k)r_t^k = r_t + \delta = r_t^* + \delta$; and (vi) p satisfies (5) given z^G.

1.3 Government's Lack of Commitment

I consider the case in which the government is benevolent but lacks commitment in all tax and debt policies. The government sets policies to maximize the weighted discounted utility of all domestic agents, given by a set of social welfare weights $\boldsymbol{\lambda} = \left(\lambda^i\right)_{i\in I}$.[4] Nevertheless, in every period, the government cannot commit to future choices on debt repayments and taxes. I consider a dynamic game played by the government, domestic agents, and international lenders. If the government deviates from the contracted allocation at time t, such as defaulting on debt or expropriating capital, the government faces the worst punishment from the domestic agents and international lenders and receives a deviation utility $\underline{U}_t(K_t)$ that depends on the current aggregate capital level that it can expropriate. Following Chari and Kehoe (1990, 1993), the subgame perfect equilibrium of this dynamic game is characterized by the following limited commitment constraints:

$$\sum_{s=t}^{\infty} \beta^{s-t} \sum_{i\in I} \lambda^i \pi^i U^i\left(c_s^i, l_s^i\right) \geq \underline{U}_t(K_t),\ \forall t \tag{8}$$

Appendix A provides the details of the game and the subgame perfect equilibrium. Constraint (8) imposes an endogenous aggregate debt limit that depends on the government's current and future fiscal policies. Therefore, I refer to equation (8) as the aggregate debt constraint.

In general, the value of $\underline{U}_t$ depends on the assumptions on what punishment domestic households and international lenders impose after the government deviates. Equation (8) then characterizes an equilibrium of the dynamic game that is different from the subgame perfect equilibrium. For example, in the numerical analysis of Section 6, I assume that $\underline{U}_t$ is the value of financial autarky in which the country has no access to international financial markets. In that case, equation (8) characterizes the reverting-to-autarky equilibrium of the dynamic game.

2 Characterizing the Competitive Equilibrium

In equilibrium, the marginal rates of substitution between consumption and leisure are equal across agents. In addition, the marginal rates of substitution between consumption today and tomorrow are

[3] Define $\alpha_0^i \equiv \left[1 + (1-\tau_0^d)r_0\right](k_0^i + a_0^i)$

[4] In Section 3, I show that these weights characterize the set of efficient allocation that maximizes the lifetime equilibrium utility of one agent given that the equilibrium utilities of other agents are above feasible thresholds.

also equal across agents. That is, in each period $t \geq 0$, for all $i, j \in I$,

$$
\begin{aligned}
(1-\tau_t^n)w_t &= -\frac{U_l^i(c_t^i, l_t^i)}{U_c^i(c_t^i, l_t^i)} = -\frac{U_l^j(c_t^j, l_t^j)}{U_c^j(c_t^j, l_t^j)} \\
1+(1-\tau_t^d)r_t &= \frac{U_c^i(c_t^i, l_t^i)}{\beta U_c^i(c_{t+1}^i, l_{t+1}^i)} = \frac{U_c^j(c_t^j, l_t^j)}{\beta U_c^j(c_{t+1}^j, l_{t+1}^j)}.
\end{aligned}
$$

Given the aggregate allocation (C_t, L_t) in every period, there is an efficient assignment of individual allocation $\left(c_t^i, l_t^i\right)_{i\in I}$. Any inefficiencies arising from tax distortions are captured by the aggregate allocation. Therefore, the competitive equilibrium allocation can be completely characterized in terms of aggregates and a static rule for individual allocation. Following Werning (2007), I first analyze the static distortion problem and then look at the dynamics in aggregate levels.

Submarket analysis. For any equilibrium, there exist market weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i\in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i \varphi^i = 1$, such that individual assignments solve a static problem,

$$
\begin{aligned}
V^m(C, L; \boldsymbol{\varphi}) \equiv \max_{(c^i, l^i)_{i\in I}} & \sum_{i\in I} \varphi^i \pi^i U^i(c^i, l^i) \\
s.t. \qquad & \sum_{i\in I} \pi^i c^i = C; \quad \sum_{i\in I} \pi^i l^i = L.
\end{aligned}
$$

The market weights capture how individual allocation is determined given any aggregate allocation. This problem gives the policy functions for each agent i:

$$
h^i(C, L; \boldsymbol{\varphi}) = \left(h^{i,c}(C, L; \boldsymbol{\varphi}),\ h^{i,l}(C, L; \boldsymbol{\varphi})\right).
$$

A competitive equilibrium allocation then must satisfy $(c_t^i, l_t^i) = h^i(C_t, L_t; \boldsymbol{\varphi})$ for all i and t. The associated competitive equilibrium prices can be computed as if the economy were populated by a fictitious representative agent with utility function $V^m(C, L; \varphi)$. The envelope conditions of the static problem give

$$
(1-\tau_t^n)w_t = -\frac{V_L^m(C_t, L_t; \boldsymbol{\varphi})}{V_C^m(C_t, L_t; \boldsymbol{\varphi})} \tag{9}
$$

$$
1+r_{t+1} = \frac{V_C^m(C_t, L_t; \boldsymbol{\varphi})}{\beta V_C^m(C_{t+1}, L_{t+1}; \boldsymbol{\varphi})}. \tag{10}
$$

Furthermore, the present-value budget constraint for individual i can be written as

$$
\sum_{t=0}^{\infty} \beta^t \left[V_C^m(C_t, L_t; \boldsymbol{\varphi}) h^{i,c}(C_t, L_t; \boldsymbol{\varphi}) + V_L^m(C_t, L_t; \boldsymbol{\varphi}) h^{i,l}(C_t, L_t; \boldsymbol{\varphi})\right] = V_C^m(C_0, L_0; \boldsymbol{\varphi}) \left(\alpha_0^i - T\right), \tag{11}
$$

where T is the present value of lump-sum taxes, and α_0^i is the individual initial after-tax wealth.[5] Equation (11) is the individual implementability constraint.

The following proposition characterizes the competitive equilibrium with government policies.

[5] Define $T \equiv \sum_{t=0}^{\infty} \beta^t \frac{V_C^m(C_t, L_t; \boldsymbol{\varphi})}{V_C^m(C_0, L_0; \boldsymbol{\varphi})} T_t$.

Proposition 1. *Given initial individual wealth $\left\{\alpha_0^i\right\}_{i\in I}$ and external debt B_0, an allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ can be supported as an aggregate allocation of an open economy's competitive equilibrium with government policies if and only if the aggregate resource constraint (7) holds and there exist market weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i\in I}$ and a lump-sum tax T such that the implementability constraint (11) holds for all $i \in I$.*

Proof. See Appendix B.

3 Efficiency

This section formulates and characterizes the planning problem in terms of a Ramsey problem with additional aggregate debt constraints arising from the government's limited commitment.

3.1 Planning Problem

The equilibrium with limited commitment can be supported as a competitive equilibrium with government policies and aggregate debt constraints (8). Define the set $\mathcal{U}$ of attainable utilities $\left\{u^i\right\}_{i\in I}$ such that $u^i = \sum_{t=0}^{\infty}\beta^t U^i\left(c_t^i, l_t^i\right)$ for any such equilibrium allocation. Given Proposition 1, $\left\{u^i\right\}_{i\in I}$ is the individual lifetime utilities for any allocation $\{C_t, L_t, K_t\}_{t=0}^{\infty}$ and a vector of market weights $\boldsymbol{\varphi}$ such that the aggregate resource constraint and the implementability constraints for all $i \in I$ are satisfied. Specifically, $u^i = \sum_{t=0}^{\infty}\beta^t U^i\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]$. An efficient allocation is defined as one that reaches the northeastern frontier of $\mathcal{U}$ (i.e., one that maximizes the lifetime utility of one agent given that the utilities of other agents are above feasible thresholds). The necessary conditions can be derived by an alternative problem of maximizing a Pareto-weighted utility, where the Pareto weights are closely related to the feasible thresholds.[6]

Therefore, given social welfare weights $\boldsymbol{\lambda} = \left\{\lambda^i\right\}_{i\in I}$ and exogenous international interest rates $\{r_t^*\}_{t=0}^{\infty}$, an efficient allocation solves the following planning problem:

$$
\begin{aligned}
(P) \equiv \max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T} & \sum_{t=0}^{\infty}\beta^t \sum_{i\in I}\lambda^i \pi^i U^i\left[h^i(t;\boldsymbol{\varphi})\right] \\
s.t. \quad & \sum_{t=0}^{\infty} q_t\left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 \geq 0 \\
\forall i, \; & \sum_{t=0}^{\infty}\beta^t\left[V_C^m(t;\boldsymbol{\varphi})h^{i,c}(t;\boldsymbol{\varphi}) + V_L^m(t;\boldsymbol{\varphi})h^{i,l}(t;\boldsymbol{\varphi})\right] \geq V_C^m(0;\boldsymbol{\varphi})\left(\alpha_0^i - T\right) \\
\forall t, \; & \sum_{s=t}^{\infty}\sum_{i\in I}\beta^{s-t}\lambda^i\pi^i U^i\left[h^i(s;\boldsymbol{\varphi})\right] \geq \underline{U}_t(K_t).
\end{aligned}
$$

The first constraint is the resource constraint. The second constraint is the implementability constraint that takes into account the distortionary effect of government policies. The last constraint

[6]As the set of attainable utilities $\mathcal{U}$ might not be convex, an allocation that solves (P) might not attain the utilities in $\mathcal{U}$. The analysis focuses on the necessary conditions, as they are enough to develop the properties of the optimal taxes. The set of optimal taxes is a subset of the set of taxes that implement any allocation satisfying the necessary conditions for efficiency. Therefore, the optimal taxes also satisfy the attributes of taxes deriving from the necessary analysis. Park (2014) and Werning (2007) make a similar argument in their work.

is the aggregate debt constraint arising from the government's lack of commitment. The social welfare weights $\boldsymbol{\lambda} = \left\{\lambda^i\right\}_{i\in I}$ capture the government's distributional preference.

3.2 Characterizing the Efficient Allocation

Let μ be the multiplier on the resource constraint, $\pi^i\eta^i$ be the multiplier on the implementability constraint for agent i, and $\beta^t\gamma_t$ be the multiplier on the aggregate debt constraint for period t. Define $\boldsymbol{\eta} = \left(\eta^i\right)_{i\in I}$ and rewrite the Lagrangian of the planning problem with a new pseudo-utility function that incorporates the implementability constraints,

$$\sum_{t=0}^{\infty}\beta^t W\left[t;\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\eta}\right] - V_C^m(0;\boldsymbol{\varphi})\sum_{i\in I}\pi^i\eta^i\left(\alpha_0^i - T\right),$$

where

$$W\left[t;\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\eta}\right] \equiv \sum_{i\in I}\lambda^i\pi^i U^i\left[h^i(t;\boldsymbol{\varphi})\right] + \sum_{i\in I}\pi^i\eta^i\left[V_C^m(t;\boldsymbol{\varphi})h^{i,c}(t;\boldsymbol{\varphi}) + V_L^m(t;\boldsymbol{\varphi})h^{i,l}(t;\boldsymbol{\varphi})\right]$$

The first-order condition with respect to the lump-sum tax T is

$$\sum_i \pi^i\eta^i = 0. \tag{12}$$

Substituting in equation (12), the first-order conditions with respect to aggregate allocation C_t, L_t, K_{t+1} become

$$W_C(t) + \sum_{s=0}^{t}\gamma_s\frac{\partial V_C^m(t)}{\partial C_t} = \mu\frac{q_t}{\beta^t} \tag{13}$$

$$W_L(t) + \sum_{s=0}^{t}\gamma_s\frac{\partial V_C^m(t)}{\partial L_t} = \mu\frac{q_t}{\beta^t}F_L(K_t, L_t) \tag{14}$$

$$\mu q_{t+1}\left[1-\delta+F_K(K_{t+1},L_{t+1})\right] - \beta^{t+1}\gamma_{t+1}\underline{U}'_{t+1}(K_{t+1}) = \mu q_t. \tag{15}$$

In addition, there is the first-order condition with respect to the market weight φ^i.

The necessary conditions to characterize the efficient allocation are the first-order conditions of the planning problem, the aggregate resource constraint, the implementability constraints, and the aggregate debt constraints.

4 Optimal Taxation with Separable Isoelastic Preferences

This section presents the main optimal taxation results under separable isoelastic preferences. I examine how aggregate debt constraints and the government's preference for redistribution determine the optimal tax properties. I find that a binding debt constraint decreases all future labor taxes, and the optimal labor tax in the limit is a real constant that depends on the skill distribution and the government's distributional preference. The optimal domestic savings tax is negative when debt constraints bind, implying that taxing the domestic sector's borrowing is optimal. In addition, the optimal capital tax is positive when debt constraints bind.

Throughout this section, I consider that preferences are separable and isoelastic.

Assumption 1 (Separable isoelastic preference)**.** *The utility function* $U : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ *satisfies*

$$U(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \omega\frac{n^{1+\nu}}{1+\nu},$$

where $\sigma, \omega, \nu > 0$.

Given Assumption 1, individual consumption and efficient labor supply are time-independently proportional to the aggregates,

$$\begin{aligned} c_t^i &= h^{i,c}(C_t, L_t; \boldsymbol{\varphi}) = \psi_c^i C_t \\ l_t^i &= h^{i,l}(C_t, L_t; \boldsymbol{\varphi}) = \psi_l^i L_t, \end{aligned} \tag{16}$$

where

$$\psi_c^i = \frac{\left(\varphi^i\right)^{1/\sigma}}{\sum_{i\in I}\pi^i(\varphi^i)^{1/\sigma}}, \quad \psi_l^i = \frac{(\theta^i)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}}{\sum_{i\in I}\pi^i(\theta^i)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}}. \tag{17}$$

In addition, V^m and W inherit the separable and isoelastic properties from U,

$$\begin{aligned} V^m(C_t, L_t; \boldsymbol{\varphi}) &= \Phi_C^m \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^m \frac{L_t^{1+\nu}}{1+\nu} \\ W\left[C_t, L_t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] &= \Phi_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^W \frac{L_t^{1+\nu}}{1+\nu}, \end{aligned}$$

$\forall t$, and the planning objective is

$$\sum_{t=0}^{\infty}\beta^t\left(\Phi_C^P\frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P\frac{L_t^{1+\nu}}{1+\nu}\right),$$

where Φ_C^m, Φ_L^m depend on $\boldsymbol{\varphi}$, Φ_C^W, Φ_L^W depend on $\boldsymbol{\varphi}, \boldsymbol{\lambda}$, and $\boldsymbol{\eta}$, and Φ_C^P, Φ_L^P are functions of $\boldsymbol{\lambda}$ and $\boldsymbol{\varphi}$ (see Appendix B.1).

For later analysis, I also define the planning flow utility as

$$V^P(C_t, L_t; \boldsymbol{\varphi}) = \Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu}.$$

Planner's first-order conditions. The first-order conditions of the planning problem imply

$$F_L(K_t, L_t) = \frac{\left[\Phi_L^W + \Phi_L^P \sum_{s=0}^t \gamma_s\right] L_t^\nu}{\left[\Phi_C^W + \Phi_C^P \sum_{s=0}^t \gamma_s\right] C_t^{-\sigma}} \tag{18}$$

$$F_K(K_t, L_t) = r_t^* + \delta + \frac{\beta^t}{q_t}\frac{\gamma_t}{\mu}\underline{U}_t'(K_t) \tag{19}$$

and

$$C_t^{-\sigma} = \beta C_{t+1}^{-\sigma}(1 + r_{t+1}^*) \left[\frac{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t+1} \gamma_s}{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t} \gamma_s} \right]. \tag{20}$$

In addition, the first-order conditions for the market weights φ imply that

$$\eta^i = \sum_{j \in I} \frac{\lambda^j}{\varphi^j} - \frac{\lambda^i}{\varphi^i}$$

Implementation. To implement the efficient allocation as part of a competitive equilibrium with government policies, optimal taxes on labor, capital, and domestic savings must satisfy

$$(1 - \tau_t^n) F_L(K_t, L_t) = \frac{\Phi_L^m L_t^\nu}{\Phi_C^m C_t^{-\sigma}} \tag{21}$$

$$(1 - \tau_t^k) F_K(K_t, L_t) = r_t^* + \delta \tag{22}$$

$$C_t^{-\sigma} = \beta C_{t+1}^{-\sigma} \left(1 + (1 - \tau_{t+1}^d) r_{t+1}^*\right). \tag{23}$$

4.1 Optimal Labor Tax

The optimal labor tax follows from dividing equation (18) by equation (21):

$$\tau_t^n = 1 - \frac{\Phi_L^m}{\Phi_C^m} \left[\frac{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t} \gamma_s}{\Phi_L^W + \Phi_L^P \sum_{s=0}^{t} \gamma_s} \right], \tag{24}$$

The optimal labor tax depends on time-invariant components of the economy that are captured in Φ's and the time-varying sum of Lagrange multipliers $\{\gamma_t\}_t$ on aggregate debt constraints. Redistribution directly determines the levels of labor taxes via the Φ's, and aggregate debt constraints directly affect the labor tax dynamics via the γ_t's .[7] Furthermore, redistribution indirectly affects tax dynamics by influencing the timing of binding constraints. The presence of debt constraints changes the equilibrium outcomes φ and η that determine Φ's.

To examine these effects, I consider three cases: when the aggregate debt constraint does not bind, when it binds for the current period, and when it binds infinitely often.

Aggregate debt constraint does not bind. The optimal labor tax is constant when the aggregate debt constraint is not relevant. If the aggregate debt constraint never binds at period t, then $\gamma_s = 0$ for any period $s \leq t$, and $\sum_{s=0}^{t} \gamma_s = 0$. The optimal labor tax becomes

$$\tau_t^n = 1 - \frac{\Phi_L^m \Phi_C^W}{\Phi_C^m \Phi_L^W} \equiv \bar{\tau}^n.$$

[7] The special utility with constant elasticities makes the $\Phi' s$ constant over time. Therefore, I can separate the direct effect of binding debt constraints on labor tax dynamics.

This is the constant optimal labor tax formula found in Werning (2007) in which there are no aggregate debt constraints. By substituting in the definitions of $\Phi's$, we have that

$$\bar{\tau}^n = 1 - \frac{\sum_{i \in I} \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]}{\sum_{i \in I} \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right]}. \tag{25}$$

In addition, if the aggregate debt constraint does not bind at period t, then the optimal labor tax stays constant (i.e., $\tau_{t-1}^n = \tau_t^n$). The following proposition summarizes the case of perfect labor tax smoothing.

Proposition 2 (Perfect labor tax smoothing)**.** *Given Assumption 1, if the aggregate debt constraint does not bind at period t, then the optimal labor tax is constant: $\tau_t^n = \tau_{t-1}^n$. Moreover, if the aggregate debt constraint never binds, then the optimal labor tax is constant and equal to (25).*

Proof. The proof follows from equation (24). If the aggregate debt constraint does not bind at period t, $\gamma_t = 0$ and so $\sum_{s=0}^{t-1} \gamma_s = \sum_{s=0}^{t} \gamma_t$. If the aggregate debt constraint never binds, $\sum_{s=0}^{t} \gamma_s = 0$.

The intuition behind Proposition (2) is that the government uses lump-sum taxes for revenue needs and only levies distortionary labor taxes for redistribution. The labor distortion reflects the trade-off between the heterogeneity level, which is determined by the skill distribution, and the government's redistributive motive, which is represented by the social welfare weights. Because the skill distribution, welfare weights, and elasticities do not change over time, when borrowing is unconstrained, the government finds it optimal to keep the intratemporal distortion constant and borrow as needed. The unconstrained-debt level of the optimal labor tax is formulated by (25), which is a function of heterogeneity and the government's distributional preference.

Aggregate debt constraint binds for the current period. Binding debt constraints, however, limit the government's ability to borrow and maintain constant labor tax distortions over time. If the debt constraint binds in the current period, the debt constraint multiplier, which represents the shadow cost of borrowing, shows up in equation (24) and changes the dynamics of optimal labor taxes for all future periods. Any change in the labor tax distortion at any period t affects the economy's output in period t and in turn affects the government's ability to borrow not only in the current period but also in all of the previous periods $0 \leq s \leq t$. Therefore, to relax the debt constraint in the current period, the government finds it optimal to permanently change future taxes. The following proposition shows that under assumptions of inequality aversion, the binding debt constraint in the current period reduces optimal labor taxes in all future periods.

Proposition 3 (Front-loading labor distortion)**.** *Suppose Assumption 1 holds, and in addition, there is (i) an equal initial wealth distribution: $\alpha_0^i = \alpha_0^j, \ \forall i, j \in I$; (ii) a consumption-inequality aversion: $\sigma \geq 1$; and (iii) an inequality aversion: $\theta^i \geq \theta^j \iff \lambda^i \leq \lambda^j, \forall i, j \in I$. Then for any period t such that the debt constraint binds, the optimal labor tax decreases for all periods afterwards: $\tau_s^n \leq \tau_{t-1}^n, \ \forall s \geq t$.*

Proof. See Appendix B.

The mechanism behind Proposition (3) is back-loading efficiency, as in Aguiar and Amador (2016). Proposition 3 provides a formal proof of the conditions under which such mechanism exists. When the debt constraint binds in the current period, the government has an incentive to reduce not only current but also future labor distortions. By lowering future labor taxes, the government increases efficiency by encouraging more labor supply and output, which increases the current period's borrowing capacity and consumption.

The back-loading efficiency mechanism occurs because of the distortionary cost of policies. An inequality-averse government redistributes by levying distortionary labor taxes. When the debt constraint does not bind, the government redistributes with high labor taxes and faces high distortionary costs. When the debt constraint binds, it is necessary to reduce future labor taxes and increase efficiency to relax debt constraints. This model differs from Aguiar and Amador (2016) by the sources of distortionary costs. In Aguiar and Amador (2016), the cost comes from raising taxes to finance government expenditures. In this framework, the cost comes from raising taxes to redistribute. Absent the distortionary costs of redistribution, for example, if the government has access to type-dependent lump-sum taxes, the result goes away, and the optimal labor tax is zero for all periods.

The above discussion establishes that inequality aversion is key for decreasing optimal labor taxes when debt constraints bind. On the other hand, if the government is highly inequality loving such that in equilibrium $\theta^i \geq \theta^j \iff \lambda^i/\varphi^{*i} \geq \lambda^j/\varphi^{*j}, \forall i,j \in I$, then the optimal labor tax is permanently increasing when the debt constraint binds.[8] Because the government cares about high-income agents, it levies labor taxes lower than the efficient level when debt constraints do not bind. When debt constraints bind, increasing labor taxes helps to increase efficiency.[9]

Aggregate debt constraint binds infinitely often. I show that the optimal labor tax converges to a limit when the aggregate debt constraint binds infinitely often in the long run. This result happens when the domestic agents are impatient and the deviation utility is bounded below. That is,

Assumption 2 (Impatience). *There exists $M > 0$ and T such that $\forall t > T,\ \beta(1+r_t^*) < M < 1$.*

Assumption 3 (Bounded deviation utility). *$\underline{U}_t(\cdot)$ is bounded below; that is, there exists a finite real M_U such that $\inf_{K_t} \underline{U}_t(K_t) \geq M_U$.*

The argument follows Aguiar and Amador (2016) for the case of heterogeneous agents. Assumptions 2 and 3 imply that the debt constraint binds infinitely often and that the cumulative sum of multipliers $\sum_{s=0}^{t} \gamma_s$ diverges in the limit. To see why, consider the first-order condition of the planning problem with respect to efficient aggregate consumption (13) under separable isoelastic preferences:

$$\frac{\beta^t}{q_t}\left[\Phi_C^W + \Phi_C^m \left(\sum_{s=0}^{t} \gamma_s\right)\right] (C_t^*)^{-\sigma} = \mu. \tag{26}$$

In equilibrium, Φ_C^W, Φ_C^m are bounded, and $\mu > 0$. Assumption 2 of impatience implies that β^t/q_t converges to zero. If the deviation utility is bounded, aggregate consumption is bounded in the long run, and the economy features no immiseration:

Lemma 1 (No immiseration). *Suppose Assumptions 1 and 3 hold. Then for any efficient allocation $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$, $\liminf_{t\to\infty} C_t^* > 0$.*

[8] Following the proof of Proposition 3, we will have in this case that $\tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \geq 0$ and $\tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \geq 0$. Thus, if the debt constraint binds in period t, then $\tau_s^n \geq \tau_{t-1}^n,\ \forall s \geq t$.

[9] See Section 6.4 for the numerical illustration of optimal labor taxes under inequality-averse and inequality-loving distributional preferences.

Therefore, given equation (26), it must be that the cumulative sum of multipliers $\sum_{s=0}^{t}\gamma_s$ grows unbounded. Equation (24) then implies that

$$\lim_{t\to\infty}\tau_t^n = 1-\frac{\Phi_L^m\Phi_C^P}{\Phi_C^m\Phi_L^P}.$$

By substituting in the functions of $\Phi's$ in the above equation, I find that the optimal labor tax in the limit satisfies

$$\lim_{t\to\infty}\tau_t^n = 1-\frac{\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_c^i}{\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_l^i}. \tag{27}$$

The optimal labor tax in the limit depends on the distributional preference, $\{\lambda^i\}_{i\in I}$, and heterogeneity, captured by $\{\varphi^i\}_{i\in I}$ (utility shares),$\{\psi_c^i\}_{i\in I}$(consumption shares), and $\{\psi_l^i\}_{i\in I}$(labor shares). The labor tax limit's value differs from the unconstrained-debt labor tax level (25) by eliminating the terms associated with $\{\eta^i\}_{i\in I}$, which measure the distortionary cost of policies. Heterogeneity is constant over time and therefore does not alter the tax dynamics and the convergence result.

The following proposition characterizes the optimal labor tax in the limit.

Proposition 4 (Optimal labor tax in the limit)**.** *Given Assumptions 1–3, if an interior efficient allocation exists, then the optimal labor tax converges to a real constant given by (27) that depends on the skill distribution and the government's distributional preference. This result holds with or without the lump-sum transfers.*

Proof. See Appendix B.

In addition, if $\beta(1+r_t^*)=1$, then $\frac{\beta^t}{q_t}=1$ and Φ_C^W, which includes $\{\eta^i\}_{i\in I}$, remains relevant in equation (26) in the long run. Therefore, the optimal labor tax does not converge to formula (27). When domestic agents are patient, there is an incentive to save and the constraints become irrelevant in the long run. If there is enough saving such that the debt constraint never binds, then Proposition 2 gives perfect tax smoothing, and the optimal labor tax is given by (25).

The optimal labor tax in the limit is the level of labor distortion that delivers the maximum efficiency for the economy in the long run. In this case, the maximum efficiency depends on redistribution, given by skill dispersion and distributional preference. To see this, consider the following expenditure minimization problem for each period t:

$$\begin{aligned}(EM_t)\equiv \min_{C_s,L_s,K_{s+1}} &\sum_{s=t}^{\infty} q_s\left[C_s+G_s+K_{s+1}-F(K_s,L_s)-(1-\delta)K_s\right]\\ s.t.\quad &\sum_{s=t}^{\infty}\beta^{s-t}\left(\Phi_C^P\frac{C_s^{1-\sigma}}{1-\sigma}-\Phi_L^P\frac{L_s^{1+\nu}}{1+\nu}\right)=\underline{U}_t(K_t),\end{aligned}$$

which is the problem of minimizing the present value of resources needed for the planner to deliver $\underline{U}_t(K_t)$ as the promised utility at period t. The solution to this minimization problem gives the maximum efficiency for the economy and can be implemented with a labor tax of $1-\frac{\Phi_L^m\Phi_C^P}{\Phi_C^m\Phi_L^P}$. As the debt constraint binds in the long run, the planner stays in the contract by offering an allocation that delivers the promised utility $\underline{U}_t(K_t)$ in a less costly way and eventually reaches the allocation with minimal cost, which is the solution to (EM_∞). The optimal labor tax converges to the maximum efficiency level of $1-\frac{\Phi_L^m\Phi_C^P}{\Phi_C^m\Phi_L^P}$.

The limiting value of optimal labor tax (27) differs from the zero limiting value found in Aguiar and Amador (2016). The value differs because the model in Aguiar and Amador (2016), as a result of the representative-agent setup, does not have the effect of redistribution. In that case, the economy's maximum efficiency is determined by (EM_∞) with $\Phi_C^P = \Phi_L^P = \Phi_C^m = \Phi_L^m = 1$. The optimal labor tax is zero in the long run.

In addition, if the assumptions of inequality aversion and highly concave preferences in Proposition 3 hold, then it is optimal for the government to tax labor less in the limit than when the debt constraint never binds.

Corollary 1. *Given Assumptions 1–3 and the assumptions in Proposition 3, if an interior efficient allocation exists, then the optimal labor tax in the limit is weakly smaller than the optimal labor tax when the debt constraint never binds, that is,* $\bar{\tau}^n \geq \lim_{t\to\infty} \tau_t^n$.

Proof. The proof follows directly from Propositions 3 and 4. □

In summary, the optimal labor tax is constant when the debt constraint does not bind, decreases for all future periods when the debt constraint binds, and converges to a real constant in the long run as the debt constraint binds infinitely often due to the impatience of the domestic sector. The optimal labor tax in the limit is not necessarily zero and depends on the skill distribution and distributional preference.

4.2 Optimal Domestic Savings Tax (Optimal Capital Control)

Combining equations (20) and (23), the optimal domestic savings tax satisfies

$$\tau_t^d = 1 - \frac{(1+r_t^*)\left[\frac{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t} \gamma_s}{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t-1} \gamma_s}\right] - 1}{r_t^*}, \ \forall t \geq 1 \tag{28}$$

If the debt constraint does not bind in period t, then $\gamma_t = 0$ and $\tau_t^d = 0$, which means no domestic saving tax. If the debt constraint binds in period t, then $\gamma_t > 0$ and $\tau_t^d < 0$, which is a subsidy on domestic savings or a tax on domestic borrowing. In the long run, $\lim_{t\to\infty} \tau_t^d = \lim_{t\to\infty} -\frac{1-\beta(1+r_t^*)}{\beta r_t^*} < 0$. Proposition 5 summarizes the properties of domestic savings taxes.

Proposition 5. *Given Assumptions 2–3, if the interior efficient allocation exists, then* $\tau_t^d = 0$ *when the debt constraint does not bind in period* t, $\tau_t^d < 0$ *when the debt constraint binds in period* t, *and* $\lim_{t\to\infty} \tau_t^d < 0$.

Proof. The proof follows from the above discussion. □

In this framework, a tax on the domestic sector's savings is a form of capital control. Proposition 5 implies that capital control is optimal only when aggregate debt constraints are binding. Impatient domestic agents do not internalize the effect of their borrowing on tightening the aggregate debt constraints. The planner, on the other hand, internalizes that effect when choosing optimal allocation. If the debt constraint does not bind, the government should not tax the domestic sector's savings. If the debt constraint binds, to implement the efficient allocation, the government should tax the domestic sector's borrowing (alternatively, subsidize the domestic sector's savings) to discourage debt accumulation. In the long run, as the debt constraint binds infinitely often, it is optimal to tax the domestic sector's borrowing. In addition, the skill distribution and distributional preference affect the levels of capital controls via the terms Φ_C^W and Φ_C^P.

4.3 Optimal Capital Tax

Equations (19) and (22) imply that the optimal capital tax follows:

$$\tau_t^k = \frac{\frac{\beta^t}{q_t}\gamma_t \underline{U}_t'(K_t)}{F_K(K_t, L_t)}. \tag{29}$$

Suppose that the deviation utility $\underline{U}_t$ is strictly increasing in the capital stock K_t; that is, the higher the amount of capital that the government can expropriate, the higher the deviation utility is. If the debt constraint does not bind in period t, then $\gamma_t = 0$ and there is no tax on capital: $\tau_t^k = 0$. If the debt constraint binds in period t, then $\gamma_t > 0$ and it is optimal to tax capital: $\tau_t^k > 0$. In the long run, the optimal capital tax is positive as the debt constraint binds. Proposition 6 summarizes the properties of capital taxes.

Proposition 6. *Given Assumptions 2–3 and $\underline{U}_t' > 0$, if the interior efficient allocation exists, then $\tau_t^k = 0$ when the debt constraint does not bind in period t, $\tau_t^k > 0$ when the debt constraint binds in period t, and* $\lim_{t\to\infty} \tau_t^k > 0$.

Proof. The proof follows from the above discussion.

The efficient allocation features capital underinvestment when debt constraints are binding. The first-order condition (19) shows that $F_K(K_t, L_t) > r_t^* + \delta$ when $\gamma_t > 0$, where $r_t^* + \delta$ is the first-best interest rate. Because the government can expropriate capital and receive higher utility from reneging, the planner discourages capital accumulation by imposing positive capital taxes. This result is consistent with the debt overhang effect on capital investment, as in Aguiar et al. (2009). □

5 Effect of Redistribution

This section studies the effect of redistribution on the levels of optimal labor taxes at unconstrained borrowing and at the limit. Redistribution arises as the interaction between the skill distribution and the government's distributional preference. To analyze the effect, I rewrite the optimal labor tax formulas as

$$\bar{\tau}^n = 1 - \frac{\tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)} \tag{30}$$

$$\lim_{t\to\infty} \tau^n = 1 - \frac{\tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] + \tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] + \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}, \tag{31}$$

using the optimality condition $\eta^i = \sum_{j\in I} \pi^j \lambda^j/\varphi^j - \lambda^i/\varphi^i$ and definitions $\tilde{\mathbb{E}}\left[x^i\right] \equiv \sum_i \pi^i x^i$, $\tilde{cov}(x^i, y^i) \equiv \tilde{\mathbb{E}}\left[x^i y^i\right] - \tilde{\mathbb{E}}\left[x^i\right]\tilde{\mathbb{E}}\left[y^i\right]$. The optimal labor tax levels depend on the covariance between fractions of individual allocation to aggregates $\left(\psi_c^i, \psi_l^i\right)_{i\in I}$ and the weight ratios $\left(\lambda^i/\varphi^i\right)_{i\in I}$.

The key redistribution component that affects optimal labor tax levels is the ratio between Pareto and Negishi weights λ^i/φ^i, which measures the relative difference between the government's distributional preference and the equilibrium market distribution over individual utilities. If the distributional preference agrees with the market distribution (i.e., $\lambda^i = \varphi^i, \forall i \in I$), then equations (30) and (31) imply that the optimal labor tax is zero. This result happens under two cases: the representative-agent case and the heterogeneous-agent case in which the efficient allocation is such that $\lambda^i = \varphi^{*i}, \forall i \in I$.

Proposition 7 (Zero labor tax)**.** *The optimal labor tax is zero if either of the following cases holds:*

1. *There is no heterogeneity:* $\theta^i = \theta^j, a_0^i = a_0^j, \ \forall i, j \in I$.
2. *There exists an efficient allocation* $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ *such that* $\lambda^i = \varphi^{*i}, \forall i \in I$.

Proof. See Appendix B.

If there is no heterogeneity, there is no need for labor tax distortion for redistribution, and the government uses lump-sum taxes to finance its expenditures. If there is heterogeneity, the government's distributional preference changes optimal labor tax levels only when it deviates from the distribution determined by equilibrium markets. If the welfare weights are equal to the inverse of the marginal utilities, the desired redistribution is achieved through market forces, and the government does not need to levy distortionary taxes.

If there is a disagreement between distributional preference and equilibrium market distribution ($\lambda^i \neq \varphi^i$), then the optimal labor tax is nonzero. I discuss the conditions under which optimal labor taxes are positive or negative.

Signs of unconstrained-debt labor tax . If $\tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right), \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$, and $\sigma > 1$, then equation (30) implies that $\bar{\tau}^n > 0$. This result happens under the assumptions of Proposition (3). Intuitively, when the government is inequality averse, it puts higher relative weights λ^i/φ^i toward lower-income agents that have lower shares ψ_c^i, ψ_l^i of aggregate consumption and labor. The government has an incentive to redistribute toward lower income agents and levies labor taxes when the debt constraint never binds.

On the other hand, if $\tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right), \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) > 0$, and $\sigma > 1$, then equation (30) implies that $\bar{\tau}^n < 0$. This is the case in which the government is highly inequality loving such that the relative weights λ^i/φ^i are higher for higher-income agents who have higher shares ψ_c^i, ψ_l^i of aggregate consumption and labor. In this case, the government cares a lot about high-income agents and subsidizes labor supply when the debt constraint never binds.

Signs of labor tax in the limit. If $\tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)$, then equation (31) implies that $\lim_{t\to\infty} \tau_t^n > 0$. If $\tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) > \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)$, then equation (31) implies that $\lim_{t\to\infty} \tau_t^n < 0$, so there is a labor subsidy in the long run. To develop the intuition, consider the case in which the government is inequality averse and $\tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right), \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$. The redistributive benefit is higher when $\tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)$ is lower and $\tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)$ is higher. If $\tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)$, there is a redistributive benefit in the long run, so it is optimal to tax labor to reduce inequality. If $\tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) > \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)$, the redistributive benefit is low while increasing labor supply to relax the debt constraint is more beneficial, so it is optimal to subsidize labor in the long run.

Degree of back-loading. I define the degree of back-loading as $\frac{\lim_{t\to\infty}\tau_t^n-\bar{\tau}^n}{\bar{\tau}^n}$, which is the relative difference between the levels of optimal labor taxes at the limit and at the initial period when the debt constraint does not bind. In Aguiar and Amador (2016), the degree of back-loading is -1 because $\lim_{t\to\infty}\tau_t^n = 0$. The sign of the optimal labor tax in the limit determines how strong the degree of back-loading is compared to Aguiar and Amador (2016). If the government finds it optimal to tax labor in the long run ($\lim_{t\to\infty}\tau_t^n > 0$), then the degree of back-loading is mitigated. However, if it is optimal to subsidize labor in the long run ($\lim_{t\to\infty}\tau_t^n < 0$), then the degree of back-loading is amplified.

6 Numerical Analysis

This section considers the numerical analysis of the no-capital version of the model. I illustrate the dynamics of optimal taxes and the efficient allocation and explain the mechanism behind the dynamics. Lastly, I examine how optimal taxes respond to changes in skill heterogeneity that correspond to changes in the government's motive for redistribution.

6.1 Parameterization

Assumptions and Functional Forms. The economy consists of two types of domestic agents, denoted by $i=\{H,L\}$, with productivity levels $\theta^H\geq 1\geq\theta^L$. The two agents have equal population mass, $\pi^H=\pi^L$. I normalize the average productivity to $\sum_{i=H,L}\pi^i\theta^i=1$. The agent's preference is $U(c,n)=\log c-\omega\frac{n^{1+\nu}}{1+\nu}$. I assume that there is no capital. The production technology is linear in efficiency-unit labor: $F(L_t)=L_t$. The planner's objective is utilitarian: $\lambda^H=\lambda^L=1$.

I assume that if the government defaults, it defaults on all domestic and international debt. The government sets the tax rate on all returns on assets at 100% and chooses the labor tax. Domestic agents do not lend to the government, and international lenders do not lend. The labor tax is chosen endogenously by the government in default. Under this assumption, the deviation utility is the value of financial autarky, in which neither the domestic sector nor the government has access to international financial markets.

Under the no-capital assumption, the value of the deviation utility does not depend on the current capital stock. The deviation utility is then a constant finite $\underline{U}$ that is consistent with Assumption 3. This property implies that there exists a cutoff time period S in which for all periods before S, the debt constraint does not bind, and for all periods after S, the debt constraint binds. This property is captured in the following lemma.

Lemma 2. *If the debt constraint binds for a time period $t=S$, where S is finite, then it will bind for all time periods $t>S$.*

Proof. See Appendix B.

Parameter Values. Table 1 presents the parameter values used in the numerical analysis. The discount factor is set to $\beta=0.94$, and the international interest rate is constant over time with value $r_t^*=r^*=0.05$ so that $\beta(1+r^*)<1$. The preference parameters are $\omega=1$ and $\nu=2$ so that the elasticity of labor supply is 0.5. The government expenditure is constant for all periods and set as 0.2, which is 20% of the average productivity. All domestic agents' initial wealth positions are normalized to 0, and the economy starts with an initial net foreign asset position of 0.2, so $B_0 = -0.2$. I consider the case of relative skill dispersion $\theta^H/\theta^L = 2$.

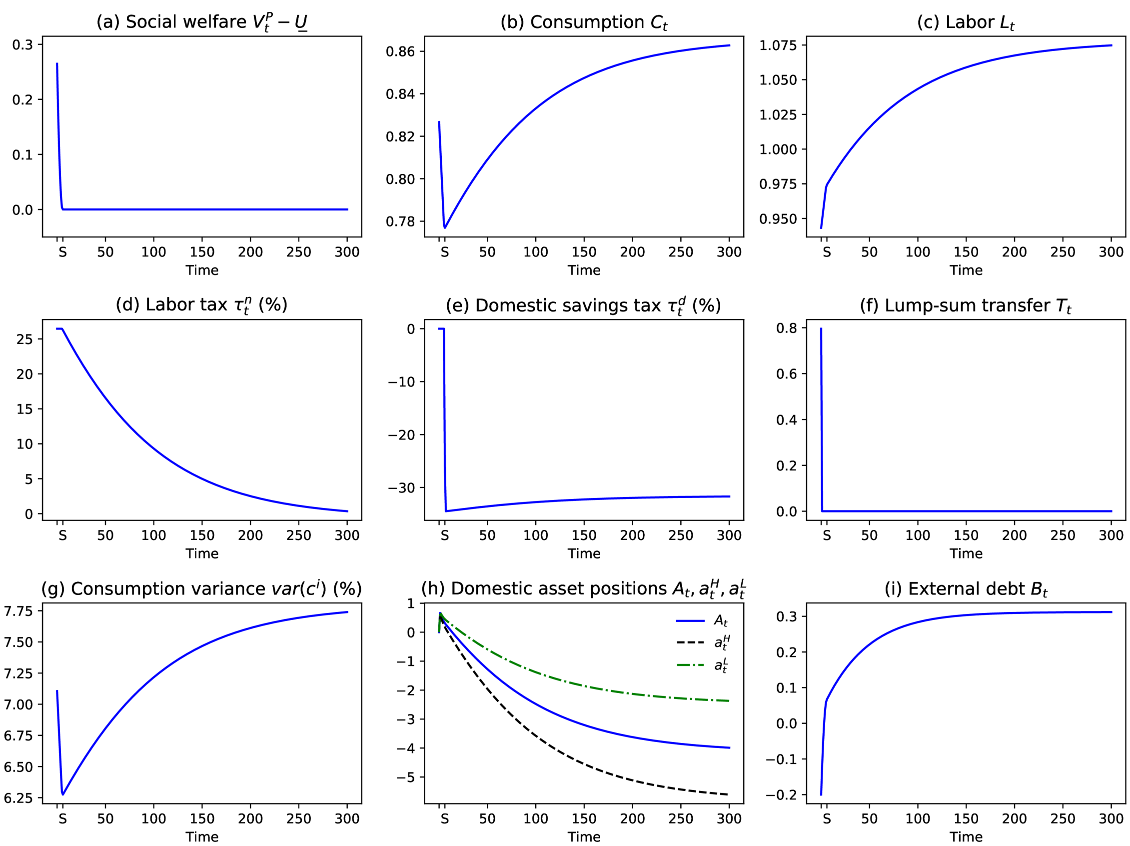
Table 1: Parameter values

Parameter	β	r^*	ω	ν	g	α_0^i	B_0	θ^H/θ^L
Value	0.94	0.05	1	2	0.2	0	-0.2	2

6.2 Dynamics of the Efficient Allocation and Optimal Policies

This subsection presents the short-run and long-run dynamics of the efficient allocation and the optimal policies that implemented them. Because of Ricardian equivalence, I consider a particular implementation of the efficient allocation where the planner gives the present-value lump-sum transfer only in period 0. Figure 1 depicts the time paths of aggregates. S is the first period in which the debt constraint binds.

Figure 1: Time paths of economic aggregates in the short run



Note: This figure plots the time paths of the social welfare, aggregate variables, and optimal policies from the model's simulation for 300 periods in the case of no capital and $\theta^H = 2\theta^L$. $S = 6$ is the first period in which the debt constraint binds. In the limit, $\lim_{t\to\infty} \tau_t^n = -0.59\%$ and $\lim_{t\to\infty} \tau_t^d = -31.8\%$.

Planner's utility and debt constraints. In Figure 1, Panel (a) plots the time path of the difference between the planner's continuation social welfare and the deviation utility, which are the two sides of the debt constraint. The planner's continuation social welfare decreases over time until it reaches the deviation utility $\underline{U}$ at period S and stays constant at $\underline{U}$ afterward. This feature implies that the debt constraint does not bind for all periods before S and binds for all periods after S.

Consumption and labor. Panels (b) and (c) of Figure 1 present the dynamics of aggregate consumption and labor. When debt constraints do not bind, there is front-loading of consumption and leisure. When debt constraints bind, aggregate consumption and labor increase over time until they converge to steady states.

Optimal taxes. Panels (d) and (e) of Figure 1 depict the optimal tax properties presented in Section 4. When the debt constraint does not bind, the optimal labor tax remains constant at a positive level, and the optimal domestic savings tax is zero. As the debt constraint starts binding, the labor tax decreases while the domestic savings tax becomes negative, implying a positive tax on the domestic sector's borrowing. In the limit, there are subsidies on labor income ($\lim_{t\to\infty}\tau_t^n = -0.59\%$) and taxes on the domestic sector's borrowing ($\lim_{t\to\infty}\tau_t^d = -\frac{1-\beta(1+r^*)}{\beta r^*} = -31.8\%$). As Panel (f) shows, there is a lump-sum transfer in the net present value in the initial period.

Redistribution. Panel (g) of Figure 1 plots the variance of consumption across domestic agents over time. When the debt constraint does not bind, the consumption variance decreases, implying that there is more redistribution over time. However, when the debt constraint binds, the consumption variance increases, implying that the government sacrifices redistribution as debt becomes more expensive.

Asset and debt policies. Impatient domestic agents borrow over time, as shown in Panel (h). Higher-income agents borrow more compared to lower-income agents because both types of agents borrow at similar fractions of their incomes over time. In Panel (i), the economy accumulates external debt quickly at the beginning of time. However, when the debt constraint hits, there is a slower accumulation of debt that eventually reaches its steady state, which is the maximum debt capacity of the economy. Panels (h) and (i) imply that the government is a net lender as it lends to the impatient domestic sector.

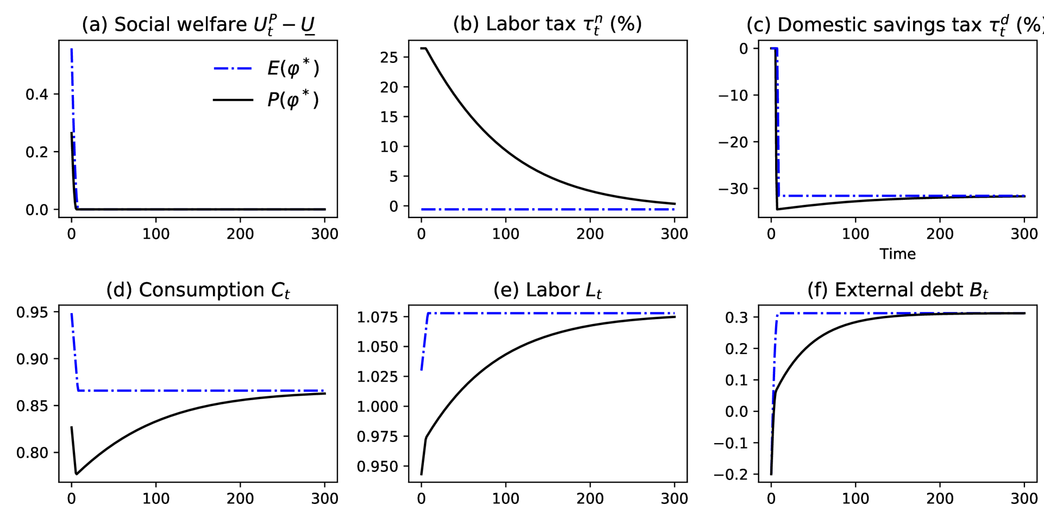
In summary, when there is no cost of borrowing, the government uses positive labor distortions and lump-sum transfers to redistribute resources among domestic agents. The high initial labor tax rate is because of the high benefit of redistribution. When debt constraints bind and borrowing is expensive, the government uses the tax on domestic borrowing for redistribution, as higher-income agents are more indebted than lower-income agents. This additional redistributive tool allows the government to reduce labor distortion over time and subsidize labor in the long run. In this case, the high-skilled agent is more productive than the average productivity level. A labor subsidy in the long run encourages her to produce more output, which increases the economy's borrowing capacity. The external debt level continues to rise even when debt constraints are binding.

6.3 Mechanism

I examine the mechanism behind the effect of binding debt constraints on the dynamics of efficient allocation and optimal policies. I consider the following relaxed planning problem in which the planner does not face the distortionary cost of redistribution but is subject to delivering the same distribution of individual outcomes, captured by the optimal $\boldsymbol{\varphi}^*$:

$$
\begin{aligned}
E(\boldsymbol{\varphi}^*) \equiv \max_{\{C_t, L_t\}_t} & \sum_{t=0}^{\infty} \beta^t \sum_{i\in I} \lambda^i \pi^i U^i \left[h^i(t;\boldsymbol{\varphi}^*)\right] \\
s.t. \quad & \sum_{t=0}^{\infty} q_t \left[L_t - C_t - G_t\right] - B_0 \geq 0 \\
\forall t, \; & \sum_{s=t}^{\infty} \sum_{i\in I} \beta^{s-t} \lambda^i \pi^i U^i \left[h^i(s;\boldsymbol{\varphi}^*)\right] \geq \underline{U}_t.
\end{aligned}
$$

Figure 2: Time paths of economic aggregates in the benchmark and relaxed planning problems

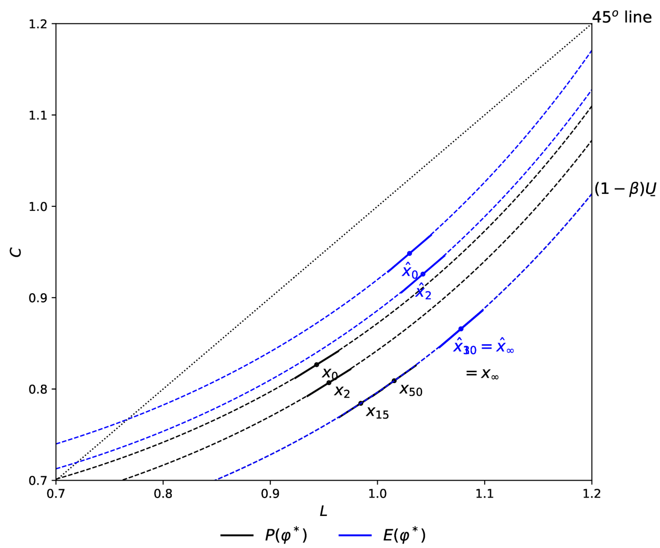


Note: This figure plots the time paths of the social welfare, aggregate variables, and optimal policies from the model simulations for 200 periods for the benchmark planning problem $P(\varphi^*)$ and the relaxed planning problem $E(\varphi^*)$.

Figure 2 compares the dynamics of aggregates between the benchmark problem $P(\boldsymbol{\varphi}^*)$ and the relaxed problem $E(\boldsymbol{\varphi}^*)$. The relaxed problem $E(\boldsymbol{\varphi}^*)$ delivers higher ex ante social welfare (Panel (a)). It takes longer for $E(\boldsymbol{\varphi}^*)$ to reach the debt constraint, but when the debt constraint binds, consumption, labor, taxes, and external debt stay constant. The implemented labor tax for the allocation of $E(\boldsymbol{\varphi}^*)$ is constant at a negative level. The solution of $P(\boldsymbol{\varphi}^*)$ converges to the solution of $E(\boldsymbol{\varphi}^*)$ in the long run. The relaxed problem $E(\boldsymbol{\varphi}^*)$ is the most efficient way to deliver $\boldsymbol{\varphi}^*$, while the benchmark problem $P(\boldsymbol{\varphi}^*)$ is the best way to deliver $\boldsymbol{\varphi}^*$, taking into account the distortionary cost of redistribution. The solution to $P(\boldsymbol{\varphi}^*)$ increases its efficiency every time the debt constraint binds and eventually reaches the most efficient outcome.

Figure 3 provides the trade-off path between aggregate consumption and aggregate labor of the benchmark and relaxed allocation over time. The dashed curves represent the intraperiod indifference curve of the planning utility with respect to aggregate consumption and labor. The benchmark and relaxed allocation for each period t are indexed by x_t and $\hat{x}_t$, respectively. The slope of each associated line is the marginal rate of substitution between consumption and labor at period t.[10] For the periods in which the debt constraints do not bind, the marginal rate of substitution remains constant, as the benchmark allocation drifts down the utility indifference curve, decreasing consumption and increasing labor. The decline in consumption and leisure reflects the impatience of domestic agents, while the constant marginal rate of substitution comes from the tax-smoothing argument. When the debt constraint starts binding, as illustrated before, staying at the same allocation is no longer sustainable. Therefore, the allocation moves along the autarkic utility indifference curve.

Figure 3: The consumption-labor trade-offs over time in the benchmark and relaxed planning problems



Note: This figure plots the trade-offs between aggregate consumption C and aggregate labor L over time for the benchmark planning problem $P(\varphi^*)$ and the relaxed planning problem $E(\varphi^*)$.

The benchmark allocation moves up along the flow autarkic utility indifference curve. In the benchmark planning problem, the marginal rate of substitution between consumption and labor starts at a low level, as the slope of the indifference curve at x_0 is less than one. The argument is that it is always better for the planner to redistribute by distorting intratemporal decisions instead of intertemporal decisions. The marginal rate of substitution is then lower than one because of the distortionary cost of redistribution. On the other hand, the relaxed allocation does not have to take this distortionary cost into account, so its allocation ($\hat{x}_t$) always has a slope of one, in which the slope of the indifference curve equals the slope of the aggregate resource constraint. Tax smoothing implies that at the end of the periods in which the debt constraints do not bind, the benchmark allocation's marginal rate of substitution has not changed and is less than one. Given the same promised utility, at the moment the debt constraint binds, suppose that the planner decreases one unit of labor. Then she can only decrease consumption by less than one unit, implying that the planner will need to take on more debt. If the planner instead increases one unit of labor, she will only need to increase consumption by less than one unit. Then the planner can

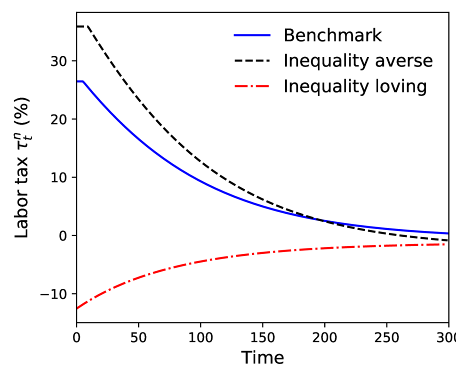
[10] The slope of the planning utility indifference curve is $\left.\frac{\Phi_L^P}{\Phi_C^P}\frac{L_t^{\nu}}{C_t^{-\sigma}}\right|_{u_t}$.

gain additional resources to pay back the existing debt. As a result, the benchmark allocation moves up along the indifference curve until it reaches the most efficient allocation with a slope of one.[11]

6.4 Optimal Labor Tax with Redistribution and Debt Constraints

I show how optimal labor taxes respond to the government's distributional preference by comparing optimal labor taxes in the benchmark to the inequality-averse model ($\sigma = 2$) and the inequality-loving model ($\lambda^H = 3\lambda^L$). Figure 4 shows that relative to the benchmark, when the government is more inequality averse, the unconstrained-debt level of optimal labor tax is higher because of the higher need for redistribution. It takes longer for the economy to hit the debt constraint. However, the government reduces labor tax more aggressively when the debt constraint binds, resulting in a lower labor tax in the limit. In contrast, when the government is highly inequality loving, it is optimal to subsidize labor initially and reduce the subsidy as debt constraints start binding. This case is the opposite of Proposition 3 as the optimal labor tax increases when debt constraints bind.

Figure 4: Optimal labor taxes and distributional preference

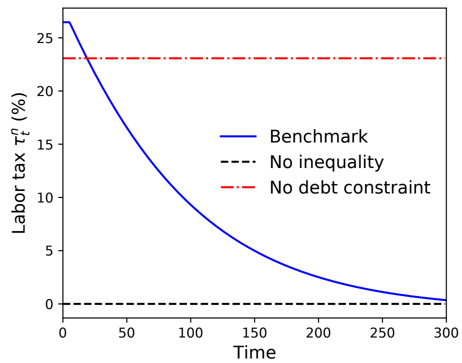


Note: The inequality-averse model corresponds to $\sigma = 2$. The inequality-loving model corresponds to $\lambda^H = 3\lambda^L$.

I next show that inequality and debt constraints affect the properties of optimal labor taxes. Figure 5 plots the time paths of optimal labor tax in the benchmark, no-inequality ($\theta^H = \theta^L$), and no-debt-constraint models. When there is no inequality, the government sets zero labor taxes for all periods, as shown by the first case of Proposition 7. The no-debt-constraint model corresponds to the open economy version of Werning (2007) in which the optimal labor tax is constant over time. The unconstrained-debt level of optimal labor tax $\bar{\tau}^n$ in the benchmark is higher than the optimal labor tax in the no-debt-constraint model. The presence of future binding debt constraints makes the government want to do more redistribution by raising labor taxes in the initial periods.

[11] In this example, one can show that $\frac{\Phi_L^P}{\Phi_C^P} < \frac{\Phi_L^W}{\Phi_C^W}$, which implies that the slope of the benchmark indifference curve is $\frac{\Phi_L^P L_t^\nu}{\Phi_C^P C_t^{-\sigma}} < \frac{\Phi_L^W L_t^\nu}{\Phi_C^W C_t^{-\sigma}} = 1$ for any period t such that debt constraints have not been binding before. In the long run, the slope of the benchmark indifference curve converges to $\lim_{t\to\infty} \frac{\Phi_C^P C_t^{-\sigma}}{\Phi_L^P L_t^\nu} = 1$.

Figure 5: Optimal labor taxes: benchmark, no inequality, and no debt constraint



Note: The no-inequality model corresponds to $\theta^H = \theta^L$.

6.5 Comparative Statics: Skill Dispersion

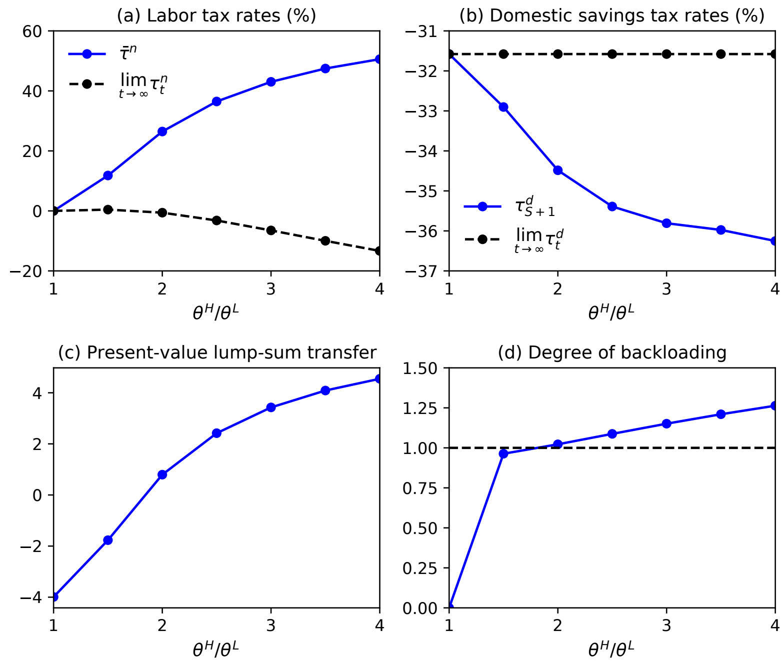
I demonstrate how optimal taxes and external debt change with respect to changes in skill dispersion. I find that higher skill dispersion, which represents a stronger redistributive motive of the government, requires greater labor tax distortions and lump-sum transfers initially but lower labor tax distortions in the long run. Higher skill dispersion also corresponds to higher taxes (capital controls) on the domestic sector's borrowing when debt constraints bind as well as higher external debt over time.

I consider the comparative statics exercise in which average productivity is normalized to $\sum_{i=H,L} \pi^i \theta^i = 1$ and the level of θ^H/θ^L changes from 1, the representative-agent case, to 4. Figure 6 illustrates the changes in optimal taxes with respect to the relative skill dispersion θ^H/θ^L. Panel (a) plots the labor tax rate in periods in which borrowing is unconstrained ($\bar{\tau}^n$) and at the limit ($\lim_{t\to\infty} \tau_t^n$). Panel (b) plots the domestic savings tax rate at period $S+1$ when τ^d takes the lowest value and τ^d the limit ($\lim_{t\to\infty} \tau_d^n$). Panel (c) plots present-value lump-sum transfers. Panel (d) plots the degree of back-loading, $\frac{\lim_{t\to\infty} \tau_t^n - \bar{\tau}^n}{\bar{\tau}^n}$, for each θ^H/θ^L level relative to the case of Aguiar and Amador (2016), which has a degree of back-loading equal to -1.

Optimal labor tax. Optimal labor tax rates at the debt-unconstrained level and at the limit respond differently to changes in the skill dispersion. As Panel (a) of Figure 6 shows, the debt-unconstrained level of optimal labor tax $\bar{\tau}^n$ increases with respect to the relative skill dispersion. As skill dispersion increases, the government increases its distributional preference toward low-income agents and levies higher labor tax rates when borrowing is not limited. However, the optimal labor tax in the limit $\lim_{t\to\infty} \tau_t^n$ decreases with respect to the relative skill dispersion. Moreover, $\lim_{t\to\infty} \tau_t^n$ is negative when there is skill heterogeneity, implying that it is optimal to subsidize labor in the long run. A higher level of skill dispersion implies that the highly productive agent becomes more productive. Increasing the labor subsidy in the long run can encourage greater output to increase the economy's borrowing capacity.

Although labor is subsidized in the long run, the government still achieves its redistributive purpose by combining the high initial labor tax rates and positive lump-sum transfers. Panel (c) of Figure 6 depicts that the present value of lump-sum transfers increases with respect to the relative skill dispersion.

Figure 6: Optimal labor taxes and lump-sum transfers by relative skill dispersion



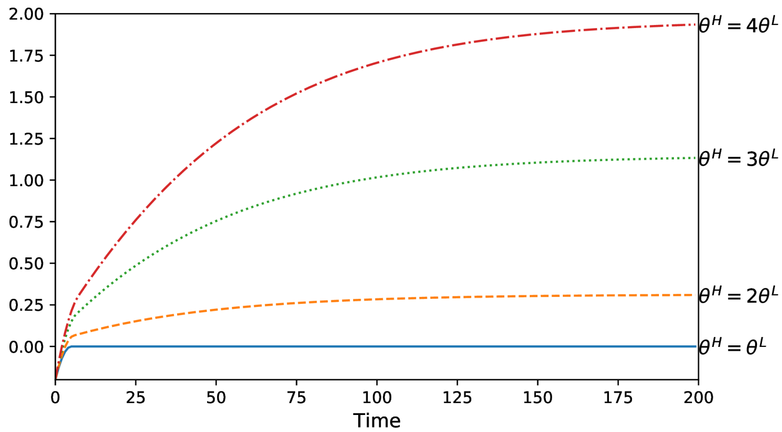
Note: This figure plots the optimal taxes for different levels of relative skill dispersion θ^H/θ^L. Average productivity is normalized to $\sum_{i=H,L} \pi^i\theta^i = 1$ for each case. Panel (a) plots the labor tax rate in periods in which borrowing is unconstrained ($\bar{\tau}^n$) and at the limit ($\lim_{t\to\infty} \tau_t^n$). Panel (b) plots the domestic savings tax rate at period $S+1$ when τ^d takes the lowest value and τ^d the limit ($\lim_{t\to\infty} \tau_d^n$). Panel (c) plots present-value lump-sum transfers. Panel (d) plots the degree of back-loading, $\frac{\lim_{t\to\infty} \tau_t^n - \bar{\tau}^n}{\bar{\tau}^n}$, for each relative skill dispersion level relative to the case of Aguiar and Amador (2016), which has a degree of back-loading equal to -1.

Optimal capital control. The optimal domestic savings tax also depends on the level of skill dispersion. As Panel (b) of Figure 6 shows, the minimum value of the optimal domestic savings tax decreases when skill dispersion increases. However, the optimal domestic savings tax converges to the same limiting value, $-\frac{1-\beta(1+r^*)}{\beta r^*}$, in the long run for all levels of skill dispersion. When the economy becomes more unequal, the optimal capital control in the form of taxes on domestic borrowing increases when debt constraints bind. A higher level of skill dispersion means that the highly productive, high-income agent has higher income and so borrows more. When debt constraints are tightened, it is optimal to discourage high-income households from borrowing by increasing taxes.

Degree of back-loading. In Panel (d) of Figure 6, the degree of back-loading increases as relative skill dispersion increases comparing to Aguiar and Amador (2016). For low levels of inequality (e.g., $\theta^H/\theta^L = 1.5$), the degree of back-loading is mitigated compared to Aguiar and Amador (2016). For high levels of inequality (e.g., $\theta^H/\theta^L \geq 2$), the degree of back-loading is amplified compared to Aguiar and Amador (2016).

External debt. The economy's external debt B_t increases with respect to skill dispersion, as shown in Figure 7. While all economies start with the same initial external debt position, an economy with a higher level of skill dispersion accumulates higher external debt over time. A highly dispersed economy wants to redistribute more by levying a higher labor tax rate during the periods in which the debt constraints have not been binding. The higher tax rate means that there is lower output, which is compensated by more borrowing.

Figure 7: Time paths of external debt by relative skill dispersion



Note: This figure plots the time paths of external debt B_t from the model simulations for 300 periods for different levels of relative skill dispersion θ^H/θ^L.

The higher debt capacity of the economy corresponds to the need to stabilize the higher debt level that the economy accumulates beforehand because of a higher redistributive motive. In addition, a higher level of skill dispersion is associated with a longer time of unconstrained borrowing. Since a highly dispersed economy has a greater redistributive motive, it is more costly to redistribute during periods of financial autarky. Therefore, it is optimal to prolong the periods in which the debt constraint does not bind, a time in which the government can redistribute the most.

7 Conclusion

This paper analyzes optimal taxation in a small open economy with the government's desire for redistribution and endogenous debt constraints. The country increases its external borrowing over time as a result of the impatience of the domestic agents, so debt constraints become relevant in the long run. Under separable isoelastic preferences, the optimal labor tax is constant when the debt constraint does not bind, decreases when the debt constraint binds, and converges to nonzero values in the limit that are associated with income inequality and the government's distributional preference. I also argue that capital controls in the form of taxes on the domestic sector's borrowing are optimal when borrowing is constrained. The optimal domestic borrowing tax and capital tax are positive in the long run.

The key mechanism for front-loading labor distortions is back-loading efficiency, and the government's desire for redistribution plays an important role for the mechanism to exist. Concerns for redistribution lead to the government's need to use distortionary taxes. When debt constraints bind, decreasing future labor distortions is optimal to increase efficiency. The result is the response to distortions and binding debt constraints. If the government can avoid the distortionary cost of redistribution, such as by having access to type-dependent lump-sum taxes, then there is no back-loading efficiency as the optimal labor distortions are zero. In addition, the optimal labor tax in the long run is nonzero and depends on both inequality and the government's distributional preference.

Redistribution has an effect on optimal policies. A government with a high redistributive motive wants to set high labor tax rates and accumulate a high level of debt when debt is unconstrained. When debt is constrained, the government continues sustaining higher debt by lowering labor tax rates and possibly subsidizing labor in the long run. In addition, capital controls in the form of domestic borrowing taxes increase when inequality or the government's redistributive motive increases.

In the model, the main source of income heterogeneity comes from skill dispersion. Other sources of heterogeneity, such as heterogeneous returns on investment in capital and wealth, are worth exploring in future research. In addition, enriching the tax system to nonlinear taxes can facilitate the study of optimal tax progressiveness in the presence of limited borrowing.

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A Sovereign Game

Before setting up the game, consider the general environment in which the government's policy includes the decision to default on debt $\{d_t\}_{t=0}^{\infty}$, where $d_t \in \{0, 1\}$ and $d_t = 0$ implies default.[12] The government's budget constraint becomes

$$G_t + (1 + r_t)B_t^g \leq \tau_t^n w_t L_t + \tau_t^k r_t K_t + \tau_t^d \left(r_t A_t + \left[(1 - \tau_t^k) r_t^k - \delta\right] K_t\right) + d_t B_{t+1}^g + T_t.$$

As the government cannot commit to any of its policies, one can think that the government, domestic agents, and international lenders enter into a sovereign game in which they determine their actions sequentially. In every period, the state variable for the game is $\left\{B_t^g, \left(k_t^i, a_t^i\right)_{i \in I}\right\}$. The timing of the actions is as follows:

- Government chooses $z_t^G = \left(\tau_t^n, \tau_t^d, \tau_t^k, T_t, d_t, B_{t+1}^g\right) \in \Pi$ such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c_t^i, l_t^i, k_{t+1}^i, a_{t+1}^i\right)$ subject to their budget constraints, the representative firm produces output by choosing $z_t^F = (K_t, L_t)$, and the international lenders choose holdings of bonds $z_t^* = \left(B_{t+1}^g, A_{t+1}\right)$ given the interest rates r_t^*.

Define $h^t = \left(h^{t-1}, z_t^G, \left(z_t^{H,i}\right)_{i \in I}, z_t^F, z_t^*, p\right) \in H^t$ as the history at the end of period t. Note that the history incorporates the government's policy, allocation, and prices. Define $h_p^t = \left(h^{t-1}, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period t. The government strategy is $\sigma_t^G : H^{t-1} \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i} : H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has the strategy $\sigma_t^F : H_p^t \to \mathbb{R}_+^2$, and the international lenders have the strategy $\sigma_t^* : H_p^t \to \mathbb{R}_+^2$. Prices are determined by the pricing rule: $p : H_p^t \to \mathbb{R}_+$.

Definition 2 (Sustainable equilibrium)**.** A sustainable equilibrium is $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ such that (i) for all h^{t-1}, the policy z_t^G induced by the government strategy maximizes the weighted utility by $\boldsymbol{\lambda}$ subject to the government's budget constraint (6); (ii) for all h_p^t, the strategy induced policy $\left\{z_t^G\right\}_{t=0}^{\infty}$, allocation $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^{\infty}$, and prices $\{r_t^*\}_{t=0}^{\infty}$ constitute a competitive equilibrium with taxes.

The following focuses on characterizing a set of sustainable equilibria in which deviation triggers the worst payoff. In this case, the value of deviation is the worst equilibrium payoff.

Proposition 8 (Sustainable equilibrium)**.** *An allocation and policy* $\left\{\left(z^{H,i}\right)_{i \in I}, z^F, z^G\right\}$ *can be part of sustainable equilibrium if and only if (i) given* z^G*, there exist prices* p *such that* $\left\{\left(z^{H,i}\right)_{i \in I}, z^F, z^G, p\right\}$ *is a competitive equilibrium with taxes for an open economy; and (ii) for any* t*, there exists* $\underline{U}_t(\cdot)$ *such that* $\left\{\left(z^{H,i}\right)_{i \in I}, z^F, z^G\right\}$ *satisfies the constraint*

$$\sum_{s=t}^{\infty} \beta^{s-t} \sum_{i \in I} \lambda^i \pi^i U^i\left(c_s^i, l_s^i\right) \geq \underline{U}_t(K_t). \tag{8}$$

[12] Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its international debt ($d_t = 1$).

Proof. Define $\underline{U}_t(K_t)$ as the maximum discounted weighted utility for the domestic agents in period t when the government deviates. In period t, the domestic agents and the government can borrow abroad. In subsequent period $s > t$, the economy reverts to the worst equilibrium.

Suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an outcome of the sustainable equilibrium. Then by the optimal problems of the government, domestic agents, and international lenders, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes the weighted utility of the agents, satisfies the government budget constraint, and satisfies the international lender's problem at period 0. Thus, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is a competitive equilibrium with policies. For any period t and history h^{t-1}, an equilibrium strategy that has the government deviate in period t triggers reverting to the worst equilibrium in period $s > t$. Such a strategy must deliver a weighted utility value that is at least as high as the right-hand side of (8). So $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii).

Next, suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies conditions (i) and (ii). Let h^{t-1} be any history such that there is no deviation from z^G up until period t. Since $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t onward. Consider a deviation plan $\hat{\sigma}^G$at period t that receives $U_t^d(K_t)$ in period t and U^{aut} for period $s > t$. Because the plan is constructed to maximize period-t utility at K_t, the right-hand side of (8) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. □

B Formulas and Proofs

B.1 Formulas for separable isoelastic preference

Given the formulas for ψ_c^i and ψ_l^i in (17), we have the followings:

$$\Phi_C^m = \left[\sum_i \pi^i (\varphi^i)^{1/\sigma}\right]^\sigma; \qquad \Phi_L^m = \omega\left[\sum_i \pi^i \left(\varphi^i\right)^{-1/\nu}\left(\theta^i\right)^{(1+\nu)/\nu}\right]^{-\nu}$$

$$\Phi_C^W = \Phi_C^m \sum_{i\in I}\pi^i\psi_c^i\left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]; \qquad \Phi_L^W = \Phi_L^m \sum_{i\in I}\pi^i\psi_l^i\left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right]$$

$$\Phi_C^P = \Phi_C^m \sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_c^i; \qquad \Phi_L^P = \Phi_L^m \sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_l^i$$

B.2 Proof of Proposition 1

Proof. $(\Rightarrow)$ Let $\{C_t, L_t, K_{t+1}\}_{t=0}^\infty$ be an aggregate allocation of an open economy competitive equilibrium with government policies. Then by definition, $\{C_t, L_t, K_t\}_{t=0}^\infty$ satisfies aggregate resource constraint for every period. Moreover, given any market weights $\boldsymbol{\varphi}$, $\{C_t, L_t, K_{t+1}\}_{t=0}^\infty$ satisfies

$$(1-\tau_t^n)w_t = -\frac{V_L^m(C_t, L_t; \boldsymbol{\varphi})}{V_C^m(C_t, L_t; \boldsymbol{\varphi})}$$

$$1 + (1-\tau_{t+1}^d)r_{t+1}^* = \frac{V_C^m(C_t, L_t; \boldsymbol{\varphi})}{\beta V_C^m(C_{t+1}, L_{t+1}; \boldsymbol{\varphi})}$$

Substituting for w_t into the budget constraint (3) and using $(c_t^i, l_t^i) = h^i(C_t, L_t; \boldsymbol{\varphi})$ gives the implementability constraint for each agent. Importantly, one can choose $\boldsymbol{\varphi}$ and T such that the individual implementability constraints hold with equality.

($\Leftarrow$) Given $\boldsymbol{\varphi}$, T and an allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\{w_t, r_t^k\}_{t=0}^{\infty}$ using firm's first-order conditions (4). $\{\tau_t^n\}_{t=0}^{\infty}$ can be calculated using the intratemporal condition (9), while one can choose $\{\tau_{t+1}^d\}_{t=0}^{\infty}$ that satisfy the intertemporal constraint (10). The tax on capital $\{\tau_t^k\}_{t=0}^{\infty}$ can be derived from $(1-\tau_t^k)r_t^k = r_t^* + \delta$. Define $\{q_t\}_{t=0}^{\infty}$ by (5).

Rewriting the aggregate resource constraint using $F(K, L) = wL + rK$ gives

$$\sum_{t=0}^{\infty} q_t \left\{C_t + K_{t+1} - (1-\tau_t^n)w_t L_t - \left[1 + (1-\tau_t^k)r_t^k - \delta\right] K_t + T_t\right\} \\ + \sum_{t=0}^{\infty} q_t \left[G_t - \tau_t^k r_t K_t - \tau_t^n w_t L_t - T_t\right] \leq -B_0 \tag{B.1}$$

Aggregating up the agent's budget constraints implies

$$C_t + K_{t+1} + A_{t+1} = (1-\tau_t^n)w_t L_t + \left[1 + (1-\tau_t^k)r_t^k - \delta\right] K_t + (1 + r_t) A_t - T_t$$

or

$$C_t + K_{t+1} - (1-\tau_t^n)w_t L_t - \left[1 + (1-\tau_t^k)r_t^k - \delta\right] K_t + T_t = (1 + r_t) A_t - A_{t+1}$$

Substituting the last equation into (B.1) gives the government's budget constraint (6). Thus, $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ is the aggregate allocation of the constructed competitive equilibrium with government policies. □

B.3 Proof of Proposition 3

Proof. Rewrite the optimal labor tax formulas as

$$\tau_t^n = 1 - \frac{\Phi_L^m \Phi_C^W + \Phi_L^m \Phi_C^P \sum_{s=0}^{t} \gamma_s}{\Phi_C^m \Phi_L^W + \Phi_C^m \Phi_L^P \sum_{s=0}^{t} \gamma_s} \tag{B.2}$$

By definitions,

$$\frac{\Phi_L^m \Phi_C^W}{\Phi_C^m \Phi_L^W} = \frac{\sum_i \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]}{\sum_i \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right]} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

and

$$\frac{\Phi_L^m \Phi_C^P}{\Phi_C^m \Phi_L^P} = \frac{\tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] + \tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] + \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

using the optimal conditions $\eta^i = \sum_j \pi^j \lambda^j/\varphi^j - \lambda^i/\varphi^i$, and the definitions $\tilde{\mathbb{E}}\left[x^i\right] \equiv \sum_i \pi^i x^i$, $\tilde{\text{cov}}(x^i, y^i) \equiv \tilde{\mathbb{E}}\left[x^i y^i\right] - \tilde{\mathbb{E}}\left[x^i\right] \tilde{\mathbb{E}}\left[y^i\right]$.

Lemma 3. *$\tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ and $\tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$*

Proof. Given that $\alpha_0^i = \alpha_0, \forall i \in I$, the individual implementability constraints can be rewritten as

$$\psi_c^i \Phi_C^m \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma} - \psi_l^i \Phi_L^m \sum_{t=0}^{\infty} \beta^t L_t^{1+\nu} = \Phi_C^m C_0^{-\sigma} \left(\alpha_0 - T\right)$$

or

$$\psi_c^i = \psi_l^i \frac{\Phi_L^m \sum_{t=0}^{\infty} \beta^t L_t^{1+\nu}}{\Phi_C^m \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma}} + \frac{\Phi_C^m C_0^{-\sigma} \left(\alpha_0 - T\right)}{\Phi_C^m \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma}}$$

which implies that $\psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$. By definition of ψ_c^i, $\varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$.

The next step is to show that $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j$.

Suppose $\theta^i \geq \theta^j$ and $\varphi^i < \varphi^j$, then $\psi_l^i < \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < \frac{\varphi^i}{\varphi^j} < 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq 1$, which is a contradiction.

Suppose $\varphi^i \geq \varphi^j$ and $\theta^i < \theta^j$, then $\psi_l^i \geq \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq \frac{\varphi^i}{\varphi^j} \geq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < 1$, which is a contradiction.

Thus, $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$. In addition, $\theta^i \geq \theta^j \iff \lambda^i \leq \lambda^j$, which implies that

$$\psi_c^i \geq \psi_c^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$
$$\psi_l^i \geq \psi_l^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$

Hence, $\tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ and $\tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$. ☐

Lemma 3 and $\sigma \geq 1, \nu > 0$ imply that $\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq \tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] + \tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)$ and $\tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \geq \tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] + \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \geq 0$. Therefore, $\frac{\Phi_L^m \Phi_C^W}{\Phi_C^m \Phi_L^W} \leq \frac{\Phi_L^m \Phi_C^P}{\Phi_C^m \Phi_L^P}$.

Suppose that the debt constraint binds at period t, then $\gamma_t > 0$, which leads to $\sum_{s=0}^{t} \gamma_s > \sum_{s=0}^{t-1} \gamma_s$. Applying equation (B.2) gives $\tau_t^n \leq \tau_{t-1}^n$. ☐

B.4 Proof of Lemma 1

Proof. Given an efficient allocation $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}$, suppose, by contradiction that $\liminf_{t\to\infty} C_t^* \leq 0$. Find $\epsilon > 0$ such that $\forall t$,

$$\sum_{s=t}^{\infty} \beta^{s-t} \left\{ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{\left(L_s^*\right)^{1+\nu}}{1+\nu} \right\} \leq M_U$$

with $C_t = \epsilon$ and $C_s = C_s^*, \ \forall s > t$. Such ϵ exists since the utility function is unbounded below. Because $\liminf_{t\to\infty} C_t^* \leq 0$, there exists t_0 such that $C_{t_0}^* < \epsilon$. Then by monotonicity,

$$
\begin{aligned}
&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \Phi_C^P \frac{(C_s^*)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu} \right\} \\
<&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu} \right\} \\
\leq& M_U \\
\leq& \underline{U}_{t_0}(K_{t_0}^*)
\end{aligned}
$$

which contradicts the aggregate debt constraint at t_0. $\square$

B.5 Proof of Proposition 4

Proof. Let $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exists $\boldsymbol{\lambda}$ such that $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). Define

$$
A_C = \sum_{i\in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_c^i, \quad A_L = \sum_{i\in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_l^i \tag{B.3}
$$

where ψ_c^i, ψ_l^i are defined by equations (17) using $\boldsymbol{\varphi}^*$. First, one can show that A_C and A_L are positive and bounded:

Lemma 4. *Given an interior allocation,* $0 < A_C < \infty$ *and* $0 < A_L < \infty$

Proof. Interior allocation means that for any i, $c_t^i, l_t^i > 0, \ \forall t$. This implies that $\psi_c^i, \psi_l^i > 0$. By (17), $\boldsymbol{\varphi}^{*i} > 0$.

For all $i, \ \pi^i > 0, \lambda^i \geq 0$ and since $\sum_{i\in I} \pi^i \lambda^i = 1$, there exists at least an i such that $\lambda^i > 0$. Given that $\psi_c^i, \psi_l^i > 0, \ \forall i$, it must be that $A_C, A_L > 0$.

Since $\sum_{i\in I} \pi^i \boldsymbol{\varphi}^{*i} = 1 < \infty$ and $\forall i, \ \pi^i, \boldsymbol{\varphi}^{*i} > 0$, it must be that $\boldsymbol{\varphi}^{*i} < \infty$. So by definition, $\psi_c^i, \psi_l^i < \infty$. Moreover, $\boldsymbol{\varphi}^{*i} > 0$ implies that $\lambda^i / \boldsymbol{\varphi}^{*i} < \infty$. Then by definition, $A_C, A_L < \infty$. $\square$

Define (P^M) the same problem as (P) with the restriction that $(C_t, L_t) = (C_t^*, L_t^*), \forall t > M$, $\boldsymbol{\varphi} = \boldsymbol{\varphi}^*$, $T = T^*$, and $K_t = K_t^*, \forall t$. Note that $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}$ is a solution to (P^M), and (P^M) has a finite number of constraints. By a Lagrangian theorem in Luenberger (1969), there exists non-negative, not identically zero vector $\left\{r^M, \mu^M, \eta^{M,1}, \ldots, \eta^{M,I}, \gamma_0^M, \ldots, \gamma_M^M\right\}$ such that the first-order and complementarity conditions hold, i.e. $\forall t \geq 1$

$$
\frac{\beta^t}{q_t} \left\{ r^M A_C + \sum_i \pi^i \eta^{M,i} (1-\sigma) \psi_c^i + \sum_{s=0}^{t} \gamma_s^M A_C \right\} \Phi_C^m C_t^{-\sigma} = \mu^M \tag{B.4}
$$

$$
\frac{\beta^t}{q_t} \left\{ r^M A_L + \sum_i \pi^i \eta^{M,i} (1+\nu) \psi_l^i + \sum_{s=0}^{t} \gamma_s^M A_L \right\} \Phi_L^m L_t^{\nu} = \mu^M F_L(K_t, L_t) \tag{B.5}
$$

Since the allocation is interior and $A_C, A_L > 0$, one can rewrite the first-order conditions as

$$\frac{\beta^t}{q_t}\left\{r^M A_C + \sum_i \pi^i \eta^{M,i}(1-\sigma)\psi_c^i + \sum_{s=0}^{t}\gamma_s^M A_C\right\}\Phi_C^m C_t^{-\sigma} = \mu^M$$
$$\frac{\beta^t}{q_t}\left\{r^M A_C + \sum_i \pi^i \eta^{M,i}(1+\nu)\psi_l^i \frac{A_C}{A_L} + \sum_{s=0}^{t}\gamma_s^M A_C\right\}\Phi_C^m C_t^{-\sigma} = \mu^M \frac{A_C}{A_L} F_L(K_t, L_t)\frac{\Phi_C^m C_t^{-\sigma}}{\Phi_L^m L_t^{\nu}}$$

Subtracting the first from the second line gives

$$\frac{\beta^t}{q_t}\left\{\Phi_C^m \sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right]\right\}C_t^{-\sigma} = \mu^M\left[\frac{A_C}{A_L}F_L(K_t, L_t)\frac{\Phi_C^m C_t^{-\sigma}}{\Phi_L^m L_t^{\nu}} - 1\right] \tag{B.6}$$

The following lemma shows that the resource constraint binds for any sub-problem (P^M) and $M \geq 1$.

Lemma 5. *In any sub-problem (P^M) with $M \geq 1$, $\mu^M > 0$*

Proof. Suppose, by contradiction, that $\mu^M = 0$ so the resource constraint does not bind. Consider allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ which is the solution to (P^M). Then there exists $\epsilon > 0$ such that

$$\sum_{t=0}^{\infty} q_t\left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 - \epsilon \geq 0$$

Define $\left\{\hat{L}_t\right\}_{t=0}^{\infty}$ where $\hat{L}_1 < L_1$ such that $F(K_1, \hat{L}_1) = F(K_1, L_1) - \epsilon/q_1$, and $\hat{L}_t = L_t,\ \forall t > 1$. The allocation $\left\{C_t, \hat{L}_t, K_{t+1}\right\}_{t=0}^{\infty}$ satisfies the resource constraint and because of the preference's strict monotonicity, $\left\{C_t, \hat{L}_t, K_{t+1}\right\}_{t=0}^{\infty}$ also satisfies the implementability constraints and the aggregate debt constraints. However,

$$\sum_{t=0}^{\infty}\sum_{i\in I}\beta^t \lambda^i \pi^i U^i\left[h^i(C_t, \hat{L}_t; \boldsymbol{\varphi})\right] > \sum_{t=0}^{\infty}\sum_{i\in I}\beta^t \lambda^i \pi^i U^i\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]$$

which contradicts $\{C_t, L_t, K_t\}_{t=0}^{\infty}$ being optimal solution for (P^M). $\square$

By Lemma 5 and interior allocation, we can rewrite equation (B.6) as

$$\frac{\Phi_C^m}{\mu^M}\sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right] = \frac{q_t}{\beta^t}C_t^{\sigma}\left[\frac{A_C}{A_L}F_L(K_t, L_t)\frac{\Phi_C^m C_t^{-\sigma}}{\Phi_L^m L_t^{\nu}} - 1\right]$$

Specifically, for any $M \geq 1$,

$$\frac{\Phi_C^m}{\mu^M}\sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right] = \frac{q_1}{\beta}\left(C_1^*\right)^{\sigma}\left[\frac{A_C}{A_L}F_L(1)\frac{\Phi_C^m \left(C_1^*\right)^{-\sigma}}{\Phi_L^m \left(L_1^*\right)^{\nu}} - 1\right]$$

Note that the left-hand side is a function of (C_1^*, L_1^*, K_1^*) , which implies that there exists a constant κ such that $\forall M \geq 1$,

$$\frac{\Phi_C^m}{\mu^M}\sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right] = \kappa$$

Hence, (B.6) can be rewritten as

$$\frac{\beta^t}{q_t} C_t^{-\sigma} \kappa = \frac{A_C}{A_L} F_L(K_t, L_t) \frac{\Phi_C^m C_t^{-\sigma}}{\Phi_L^m L_t^\nu} - 1$$

Note that $\lim_{t\to\infty} \beta^t/q_t = 0$ and $C_t^{-\sigma}$ is bounded by Lemma 1, so taking the limit on both sides gives

$$\lim_{t\to\infty} \frac{A_C}{A_L} F_L(K_t, L_t) \frac{\Phi_C^m C_t^{-\sigma}}{\Phi_L^m L_t^\nu} = 1$$

Hence, given the definition of τ_t^n and the fact that A_C,A_L are bounded,

$$\lim_{t\to\infty} \tau_t^n = \lim_{t\to\infty} \left[1 - \frac{\Phi_L^m L_t^\nu}{\Phi_C^m C_t^{-\sigma}} \frac{1}{F_L(K_t, L_t)}\right] = 1 - \frac{A_C}{A_L}$$

□

B.6 Proof of Proposition 7

Proof. $\lambda^i = \boldsymbol{\varphi}^{*i}, \ \forall i \in I$ implies that $A_C = 1$ and $A_L = 1$. Therefore, $\{C_t^*, L_t^*\}_{t=0}^\infty, \boldsymbol{\varphi}^*, T^*$ solves

$$\begin{aligned}
\max_{\{C_t, L_t, K_{t+1}\}_{t=0}^\infty, \boldsymbol{\varphi}, T} \quad & \sum_{t=0}^\infty \beta^t V(C_t, L_t; \boldsymbol{\varphi}) \\
s.t. \quad & \sum_{t=0}^\infty q_t \left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 \geq 0 \\
& \sum_{t=0}^\infty \beta^t \left[V_C(t; \boldsymbol{\varphi}) h^{i,c}(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) h^{i,l}(t; \boldsymbol{\varphi})\right] \geq V_C(0; \boldsymbol{\varphi}) \left(a_0^i - T\right) \\
& \sum_{s=t}^\infty \beta^{s-t} V(C_t, L_t; \boldsymbol{\varphi}) \geq \underline{U}_t(K_t)
\end{aligned}$$

To implement $\{C_t^*, L_t^*\}_{t=0}^\infty, \boldsymbol{\varphi}^*, T^*$ given the specified tax system, by (27), it must be that

$$\lim_{t\to\infty} \tau_t^n = 0$$

□

B.7 Proof of Lemma 2

Proof. Note that the sustainability constraint is rewritten as $\forall t$,

$$\sum_{s=t}^\infty \beta^{s-t} \left\{\Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s^{1+\nu}}{1+\nu}\right\} \geq \underline{U}$$

Define $u_t = \Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu}$. Then the proof is similar to Lemma 2 in Aguiar and Amador (2016). □

C Numerical Appendix

This section explains the calculation of deviation utility and the numerical algorithm that solves the model in in Section 6.

C.1 Deviation Utility

The deviation utility $\underline{U}$ is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes where there is no access to international financial markets. Given that output is equal to the total efficiency-unit labor supply, one has

$$\begin{aligned}
\underline{U} \equiv & \max_{c_t^i, l_t^i, \tau_t^n, T_t} \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i U^i(c_t^i, l_t^i) \\
s.t. \qquad & c_t^i + b_{t+1}^{i,d} = (1-\tau_t^n) l_t^i - T_t + (1+r_t) b_t^{i,d} \\
& G_t + (1+r_t) B_t^d \leq \tau_t^n L_t + T_t + B_{t+1}^d
\end{aligned}$$

There exist a vector of market weights $\hat{\boldsymbol{\varphi}}$ such that

$$\begin{aligned}
\underline{U} \equiv & \max_{C_t, L_t, \hat{\boldsymbol{\varphi}}, T} \sum_{t=0}^{\infty} \beta^t \left[\hat{\Phi}_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \hat{\Phi}_L^W \frac{L_t^{1+\nu}}{1+\nu} \right] \\
s.t. \qquad & C_t + G_t \leq L_t
\end{aligned}$$

where $\hat{\psi}_c^i, \hat{\psi}_l^i, \hat{\Phi}_C^V, \hat{\Phi}_L^V, \hat{\Phi}_C^W, \hat{\Phi}_L^W$ are calculated using $\hat{\boldsymbol{\varphi}}$.

C.2 Algorithm

State variables: μ, Γ

1. Guess μ and φ. Compute η.
   (a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ
   Initial guess of the expectation $V(\mu_t, \Gamma_{t-1}) = \sum_{s=t}^{\infty} \beta^{\tau-t} \left[\Phi_C^P \frac{C_s{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s{}^{1+\nu}}{1+\nu} \right]$.
   (b) Assume the constraint does not bind in t: $\gamma_t = 0$. Solve for the allocation C_t, L_t using the following first-order conditions

$$\begin{aligned}
\left[\mu_t \Phi_C^W + \Phi_C^m \Gamma_{t-1} \right] C_t{}^{-\sigma} &= \mu \\
\left[\mu_t \Phi_L^W + \Phi_L^m \Gamma_{t-1} \right] L_t{}^{\nu} &= \mu
\end{aligned}$$

   (c) Compute $V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$, then compute

$$\begin{aligned}
A_t &= \sum_{s=t}^{\infty} \beta^{\tau-t} \left[\Phi_C^P \frac{C_s{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s{}^{1+\nu}}{1+\nu} \right] \\
&= \Phi_C^P \frac{C_t{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t{}^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1}, \Gamma_t)
\end{aligned}$$

(d) Check if $A_t \geq \underline{U}_t$. If it is, proceed to the next step. If not, solve for C_t, L_t, γ_t using these optimality equations

$$\begin{aligned}
\left[\mu_t \Phi_C^W + \Phi_C^m \Gamma_{t-1}\right] {C_t}^{-\sigma} &= \mu \\
\left[\mu_t \Phi_L^W + \Phi_L^m \Gamma_{t-1}\right] {L_t}^{\nu} &= \mu \\
\Phi_C^P \frac{{C_t}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{{L_t}^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^*(\Gamma_{t-1} + \gamma_t)) &= \underline{U}_t
\end{aligned}$$

(e) Given C_t, L_t, γ_t (γ_t can be zero or not), compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^*(\Gamma_{t-1} + \gamma_t))$. Update the value function

$$V^{n+1}(s_t, \Gamma_{t-1}) = \Phi_C^P \frac{{C_t}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{{L_t}^{1+\nu}}{1+\nu} + \beta V^n(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^*(\Gamma_{t-1} + \gamma_t))$$

2. Compute residuals to find μ and φ

$$\begin{aligned}
r^{\mu} &= \sum_{t=0}^{\infty} q_t \left[L_t - G_t - C_t\right] - B_0 \\
r_{ij}^{\varphi} &= \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^m \left(\psi_c^i - \psi_c^j\right) {C_t}^{1-\sigma} - \Phi_L^m \left(\psi_l^i - \psi_l^j\right) {L_t}^{1+\nu}\right]
\end{aligned}$$

$$r = (r^{\mu})^2 + \sum_{i,j} (r_{ij}^{\varphi})^2 \tag{C.1}$$

3. Find μ and φ such that (C.1) is minimized using a Nelder-Mead algorithm.