Optimal Redistributive Policy in Debt Constrained Economies

Monica Tran Xuan*

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Abstract

This paper studies optimal taxation in an open economy. The government has a redistributive motive and faces self-enforcing debt constraints that arise from the limited commitment of the government. Redistributive policies are proportional taxes on labor and domestic saving. The standard Ramsey results of labor tax smoothing and a zero capital tax in the limit no longer hold. Instead, optimal labor taxes decrease over time and eventually converge to a non-zero limit, and the optimal capital tax is positive in the limit. The efficient contract features front-loading distortion and back-loading efficiency, allowing the government to borrow more in the future. The model's numerical exercise shows that a stronger redistributive motive requires greater tax distortions at the beginning of time as well as a higher external debt level in the long run.

Keywords: Optimal taxation; Redistribution; Limited commitment; Sovereign debt
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*Department of Economics, University of Minnesota and Federal Reserve Bank of Minneapolis. Email: tranx888@umn.edu. The author thanks V.V. Chari, Larry Jones, and Manuel Amador for their invaluable advice and guidance. The author is grateful for comments from Anmol Bhandari, Javier Bianchi, Chris Phelan, David Rahman, Aldo Rustichini, Jan Werner, Fernando Arce, Job Boerma, Emily Moschini, Amy Handlan, the seminar participants at University of Minnesota, the Federal Reserve Bank of Minneapolis, the Federal Reserve Bank of Richmond, 2018 Midwest Macroeconomic Meetings, 2018 Annual Congress of the European Economic Association, and 2019 North American Summer Meeting of the Econometric Society. The author also acknowledges the support from the AEA Summer Economics Fellowship. The views express therein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis and the Federal Reserve System.

1 Introduction

What are the optimal tax and debt policies for an economy that faces limited borrowing but has a motive for redistribution? Taxes are the main source of the government's revenue that affects the government's ability to issue or repay debt. At the same time, distortionary taxes and transfers redistribute resources across agents under social preferences, so preferences for greater redistribution significantly affect the government's willingness to raise revenue and influence its borrowing capacity. On the other hand, when borrowing is constrained, the government's ability to redistribute is limited.

Empirical works on the recent European debt crisis have demonstrated how the redistributive motive and debt accumulation are closely related. First, the rapid accumulation of external debt in the periods leading up to a crisis led to many countries facing such high costs of borrowing that they could not roll over their debt. Second, highly indebted countries, such as Greece, Portugal, and Spain, also experienced high levels of income dispersion. According to the European Union's statistics on Income and Living Conditions, the Gini coefficients and S80/S20 income quintile share ratio in these countries were both higher than the EU-27 country average. Countries proposed different policy strategies to tackle the problems of constrained borrowing while maintaining the desired level of redistribution.

Motivated by these observations, this paper proposes a small open economy with heterogeneous agents and a government's lack of commitment. In each period, a benevolent government, without commitment, chooses both a level of debt going forward and a system of labor and capital income taxes. Borrowing is limited by the aggregate borrowing constraints. Taxes do not depend on income. The objective function of the government features a desire to redistribute. The government has the same discount factor as the domestic agents, which are less patient than the international lenders.

Borrowing constraints arise from a game between domestic agents, international lenders, and the government. When the government deviates from the contract, it faces a punishment imposed by domestic and international agents, which corresponds to a value of deviation. I assume the punishment to be financial autarky, where the government cannot have access to both domestic and international lending markets. In each period, the subgame perfect equilibrium can be characterized by self-enforcing constraints. These constraints are such that the future discounted welfare of the whole economy must be higher than or equal to the utility the government receives from deviation in each period. The constraints endogenously set the aggregate debt limits that the government faces when choosing the optimal policies.

Impatience implies that the intertemporal discounting rate of future utilities is lower than the international intertemporal price of resources. This assumption represents a weaker domestic financial market or political friction, so it provides an ex-ante motive for debt accumulation, and so the borrowing constraint will be binding in the future.

The redistributive concern is a natural rationale for distortionary taxation. For example, a government that wants to redistribute more toward lower-income agents will levy marginal labor taxes and lump-sum transfers. In this case, higher-income individuals will bear a higher tax burden.

Binding debt constraints impose an additional distortion on intratemporal substitution between consumption and leisure, encourage less accumulation of capital and less domestic borrowing. Redistributive concerns affect tax levels via the trade-off between equity and efficiency, whereas debt constraints affect tax dynamics. The optimal labor tax is constant during periods of unconstrained borrowing but falls permanently every time borrowing is limited, eventually converging to a non-zero limit that depends on the level of heterogeneity and social distribution. The contract features front-loading distortion and back-loading efficiency. This property allows the economy to increase its borrowing capacity even when debt constraints bind and thereby accumulate more debt. These results hold with separable isoelastic preference, impatience, and bounded deviation utility.

In addition, capital accumulation increases the level of capital stock that the government can expropriate when deviating from the contract ex post, which in turn increases the deviation utility that the government can receive and tightens the debt constraints. Domestic borrowing, on the other hand, reduces the level of future utility, which also tightens the debt constraints. Therefore, in order to relax the debt constraints, taxes on capital and domestic borrowing are positive in the long run. The equilibrium level of capital is lower than the one would arise in the case with full commitment, featuring the problem of debt overhang.

Intuitively, nonbinding debt constraints entail no borrowing cost. In this case, the optimal choice for the government is to use debt to smooth finances over time and keep labor distortions equal to the level that maximizes the redistributive benefit. When debt constraints bind, there is an additional motive to increase social welfare and relax the debt constraints. The government then finds it optimal to adjust tax rates to lower the cost of delivering the promised utility. An increase in the current period's efficiency relaxes the debt constraints for all previous periods. Anticipating this effect, the government finds it beneficial to back-load the increase in efficiency, which implies a front-loading distortion.

In the long run, the efficient allocation minimizes the cost of delivering the deviation utility and maximizes the net payment to the international financial markets. The labor tax converges to a limit associated with the maximal sustainable debt level of the whole economy. Skill distribution and social preferences influence both sides of the debt constraints: the value of staying in the contract and the value of deviating. Therefore, they affect the endogenous level of the maximum aggregate debt, which in turn determines the tax limit. In this way, primitives determine the long-run steady state of the economy under optimal taxation.

With high elasticity of substitution and redistributive motive towards low-skilled agents, the optimal labor tax drifts downward over time as the debt constraints bind, i.e. the tax rates in any periods after the debt constraint binds are weakly lower than the tax rates in the periods before the debt constraint binds. This result implies that, when the willingness to lend of the international lenders is high (as the debt constraints do not bind), the government would like to redistribute via a high labor tax rate. When the willingness to lend decreases (as the debt constraints bind), the government reduces distortions (lower tax rates) to repay the debt.

The paper applies the sub-market analysis in Werning (2007) to a small open economy with limited commitment. The competitive equilibrium generates an efficient assignment of individual allocation, which is captured by a set of market–Negishi weights that determine individual utility shares. This analysis provides a key insight of the redistributive effect on the optimal tax limit. Without heterogeneity or redistributive motive, the model collapses to the representative setting as in Aguiar and Amador (2016), which proved that the labor tax limit is zero. When there is heterogeneity, the labor tax is zero as long as the equilibrium Negishi weights are equal to the social weights. The redistributive motive affects optimal distortions only when it can deviate from the implied distribution of the competitive equilibrium.

The numerical exercise simulates the dynamics of allocation, taxes, and debt of the model. It shows that the government would find it optimal to front-load labor distortion when borrowing is not limited (using high positive labor tax rates). When the borrowing is tightened, the government discourages domestic borrowing by taxing it. It turns out that the higher-income agents borrow more over time comparing to the lower-income agents, so the borrowing taxes are redistributive. The government redistributes more via the borrowing taxes and less via labor taxes until it reaches the highest efficiency level where it does not face any distortionary cost. The limit represents the maximum debt capacity of the economy. The key feature is that the government dynamically substitutes between the intratemporal and intertemporal distortions.

The paper provides a comparative static analysis of the tax rates and external debt with respect to the skill dispersion, measured as the ratio of individual labor productivity levels. Increasing in relative skill dispersion increases the labor tax when the debt constraints do not bind, and decreases labor tax limits. A higher skill-dispersion means a higher motive for redistribution, translating into a higher level of tax before the debt constraint binds. When reaching the debt constraints, the economy faces a higher distortion. In order to relax the debt constraints, the planner needs to reduce the higher efficiency cost. Consequently, the labor tax limit declines when the skill dispersion rises. When the debt constraints start binding, the return on domestic savings is higher the higher the skill dispersion is, but eventually converge to the steady-state non-zero rates. The external debt levels also change with respect to skill dispersion, in which for the higher the skill dispersion, the longer the periods of unconstrained borrowing, and the higher external debt positions in the long run. The main reason is that it is more costly for the higher dispersed economy to redistribute in financial autarky. Redistribution now leads to high debt accumulation before reaching the constraints, which leads to a higher debt later on to sustain the allocation.

Extending the analysis to separable preferences, a few results change. First, the labor tax is not constant even when the debt constraints do not bind. Instead, it changes with respect to the time-varying elasticities. The long-run convergence property of the labor tax is similar to the case of constant elasticities. If the economy converges to a steady state, the optimal labor tax also converges to a real constant. However, if steady-state allocation does not exist, the long-run optimal labor tax fluctuates in a bounded region. Similar to the case of separable isoelastic preference, this region corresponds to the region of the maximum debt capacity of the economy.

Literature Review. Several empirical papers have documented the correlation between the income dispersion and sovereign debt. Specifically, Berg and Sachs (1988) showed that income dispersion was a key predictor of a country's probability of rescheduling debt and the bond spread in secondary markets. Aizenman and Jinjarak (2012) described a negative correlation between income dispersion and the tax base and a positive correlation with sovereign debt. Recently, Ferriere (2015) and Jeon et al. (2014) also provided empirical evidence of rising income dispersion significantly increases sovereign default risk. This paper provides a theoretical model that qualitatively accounts for the positive relationship between income dispersion and debt level.

This paper finds optimal policy by characterizing the best allocation of any tax-distorted debt constrained equilibrium, i.e. the primal approach as in the public finance literature (Barro (1979), Lucas and Stokey (1983), Chari et al. (1994), Aiyagari and McGrattan (1998), Chari and Kehoe (1999), Aiyagari et al. (2002), and many other papers). The argument for labor tax smoothing in these papers relies on the fact that the government can issue debt that is contingent to all states and is not constrained (in a sense of beyond the natural debt limit). In this paper, tax smoothing is not always optimal; the government's ability to smooth tax distortion is restricted by the willingness to lend by the international lenders. The non-zero capital and domestic borrowing taxes are contrast to the zero convergence of the capital tax from Judd (1985), Chamley (1986), Chari et al. (1994), and Straub and Werning (2014), because here the capital stock and domestic borrowing affect the debt constraints have an externality in influencing the debt constraints.

The paper also contributes to the sovereign debt literature. Seminal papers studied sovereign debt in the lack of commitment environment include Eaton and Gersovitz (1981), Aguiar and Gopinath (2007) Arellano (2008), Aguiar et al. (2009), and Aguiar and Amador (2011). There are a few recent works on optimal policies in the Eaton-Gersovitz-Arellano framework (Ferriere (2015) and Arellano and Bai (2016)), in which the government sometimes defaults in equilibrium and the spread of the sovereign debt depends on the probability of default. They quantitatively found that in the case of the government committing to the tax policy, fiscal austerity (in terms of raising tax rates), distortionary affect of the taxes would have made the country more likely to default, and hence it is not optimal. This paper instead studies the self-enforcing debt contract as in Aguiar and Amador (2011) and Aguiar and Amador (2014), and allows the government not to commit to any tax and debt policies.

The dynamic environment in this paper is an extension to one in Aguiar and Amador (2016), adding heterogeneity, distribution motive, and allowing for richer tax systems. Aguiar and Amador (2016) found that labor tax must go to zero in the long run as a result of front-loading efficient consumption and leisure allocation. In this paper, the tax limit can be any real value. More interestingly, when turning off the redistributive effect in the model, the limit of labor tax is zero, consistent with their findings. The paper shows that redistributive consideration is the main source for the changes in optimal policies.

Work in optimal taxation with heterogeneity and redistributive motive includes Bhandari et al. (2017) and Werning (2007), which both found that redistribution had significant impact on optimal policies. This paper instead explores redistribution in a small open economy setting and endogenous aggregate debt constraints. Werning (2007) developed the conditions for perfect tax smoothing, while Bhandari et al. (2017) emphasized the impact of the distribution of initial asset holdings on optimal allocation. The framework and analysis in this paper are closely related to Werning (2007). The finding is that labor tax smoothing only occurs when the debt constraint does not bind. Long-run binding debt constraints then alters the dynamic of the labor tax, resulting in imperfect tax smoothing. Moreover, the initial distribution of after-tax net asset holdings matters in determining the private market shares across agents, which indirectly affect the labor tax limit.

Several recent papers addressed the trade-off between redistribution and external debt. D'Erasmo and Mendoza (2016) focused on how redistributive incentives affected defaults on domestic debt. They asserted that equilibrium with debt could be supported only when the government was politically biased towards bond holders. Ferriere (2015) showed how modifying tax progressiveness could mitigate the cost of default. Dovis et al. (2016) argued how the interaction between inequality and debt endogenized the dynamic cycles of debt, taxes, and transfers over time. Balke and Ravn (2016) studied time-consistent fiscal policy in a sovereign debt model à la Eaton and Gersovitz (1981) with inequality through unemployment. They found that austerity policies were optimal during debt crises since they reduced default premium, which was correlated with debt issuance, and increased access to international lending market. This paper instead emphasizes on the enforcement constraints arising from a self-enforcing contracting problem among the government, international lenders, and domestic agents. These constraints restrict the present value of future social welfare, which act like endogenous debt constraints. The paper features the long-run binding debt constraints, in which austerity policies might not be optimal if they generate more distortion. The analysis here is more general with any distributive preference, instead of an utilitarian social welfare function as in these papers. The general redistributive motive determines both the level and dynamic of the taxes.

Outline. The paper is organized as follows. Section 2 describes the environment, the competitive equilibrium, and the government's lack of commitment problem. Section 3 characterizes the equilibrium. Section 4 formulates the efficiency problem, while section 5 derives the main results of the optimal policies. Section 6 analyzes the effect of redistribution on optimal taxes. Section 7 provides the numerical exercise, explaining the dynamics of the efficient allocation as well as the comparative statics, and Section 8 generalizes the optimal tax formulas with separable preferences. Section 9 then concludes.

2 Model

2.1 Environment

A small open economy faces exogenous world interest rates $\{r_t^*\}_{t=0}^{\infty}$. There is a measure-one continuum of infinitely-lived agents different by labor productivity types $(\theta^i)_{i\in I}$, which are publicly observable. The fraction of agents with productivity θ^i is π^i, where $(\pi^i)_{i\in I}$ is normalized such that $\sum_{i\in I}\pi^i\theta^i = 1$. All agents have the same discount factor β and the static utility $U(c, n)$ over consumption c and hours worked n. The utility of agent with productivity θ^i over consumption $c_t^i \geq 0$ and efficient labor $l_t^i \geq 0$ is

$$\sum_{t=0}^{\infty} \beta^t U^i(c_t^i, l_t^i) \tag{1}$$

where $U^i(c, l) = U\left(c, \frac{l}{\theta^i}\right)$.

In addition, there is a representative firm that uses both capital and labor to produce a single output good. The production function $F(K, L)$ is constant return to scale, where K and L are respectively the aggregate capital and labor. The economy is subject to an exogenous sequence of government spending $\{G_t\}_{t=0}^{\infty}$. In each period, the government issues domestic and foreign bonds, imposes a lump-sum tax T_t, a marginal tax on labor income τ_t^n, a marginal tax on capital income τ_t^k, and set the return on domestic bond r_t. Assume that only the government can borrow abroad[1].

2.2 Equilibrium

Consider the set of prices facing by agents and firm: w_t the labor wage, r_t^k the return on capital, and δ the capital depreciation rate.

Agents. Agent of type $i \in I$ faces the sequential budget constraint in period t,

$$c_t^i + k_{t+1}^i + b_{t+1}^{d,i} \leq (1-\tau_t^n) w_t l_t^i + \left[1 + (1-\tau_t^k) r_t^k - \delta\right] k_t^i + (1+r_t)\, b_t^{d,i} - T_t \tag{2}$$

where $c_t^i, l_t^i, k_t^i, b_t^{d,i}$ denote the consumption, effective labor, capital holding, and domestic bond holding of agent i in period t, respectively.

Moreover, no arbitrage exists such that the after-tax return is the same when investing in capital or in domestic bonds:

$$1 + (1-\tau_t^k) r_t^k - \delta = 1 + r_t$$

which implies $(1-\tau_t^k) r_t^k = r_t + \delta$.

Representative Firm. The firm chooses the amount of capital and labor to maximize profit each period:

$$\max_{\{K_t, N_t\}} \quad F(K_t, L_t) - w_t L_t - r_t^k K_t$$

which gives the first-order conditions:

$$\begin{aligned} r_t^k &= F_K(K_t, L_t) \\ w_t &= F_L(K_t, L_t) \end{aligned} \tag{3}$$

[1] This assumption is standard in the sovereign debt literature, based on the empirical observation that domestic households often hold a very small amount of foreign assets.

Note that profits are zero in equilibrium because of the constant return to scale production function.

Government. The government needs to finance an exogenous expenditure $\{G_t\}_{t=0}^{\infty}$. The government sells one-period bond B_t^d to domestic agents and B_{t+1} to the international lenders at a price Q_{t+1}. The government's budget constraint in each period is

$$G_t + (1 + r_t)B_t^d + B_t \leq \tau_t^n w_t L_t + \tau_t^k r_t^k K_t + B_{t+1}^d + Q_{t+1}B_{t+1} + T_t$$

where $L_t = \sum_{i \in I} \pi^i l_t^i$ is the aggregate labor, $K_t = \sum_{i \in I} \pi^i k_t^i$ is the aggregate capital, $B_t^d = \sum_{i \in I} \pi^i b_t^{d,i}$ is the aggregate domestic bond, and B_t is the amount of the government's external debt. The government faces a no-Ponzi condition such that the present value of external debt is bounded below.

Define q_t as international price of a unit period-t consumption in terms of period-0 consumption units:

$$q_t = \Pi_{s=0}^{t} \frac{1}{1 + r_s^*} \tag{4}$$

Optimality of the risk-neutral international lenders' problem gives $Q_t = \frac{1}{1+r_t^*}$. Using $\{q_t\}_{t=0}^{\infty}$ as the relevant intertemporal price, one can write the government's present-value budget constraint as

$$\sum_{t=0}^{\infty} q_t \left\{G_t - \tau_t^n w_t L_t - \tau_t^k r_t^k K_t + (1 + r_t)\, B_t^d - B_{t+1}^d - T_t\right\} \leq -B_0 \tag{5}$$

with normalizing $1 + r_0^* = 1$ [2]

Aggregate resource constraint. In a small open economy, markets do not have to clear in every period. However, using the agent's budget constraints and government's budget constraint, one can obtain an aggregate resource constraint in terms of the intertemporal prices and the initial external debt:

$$\sum_{t=0}^{\infty} q_t \left[F(K_t, L_t) + (1 - \delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 \geq 0 \tag{6}$$

Competitive equilibrium. Given the above equations, one can define the following competitive equilibrium with taxes.

[2] This assumption is without loss of generality to fix the initial level of external debt.

Definition 2.1. Given initial external debt B_0 and individual wealth positions $(a_0^i)_{i\in I}$[3], a competitive equilibrium with taxes for an open economy, is agent's allocation $z^{H,i} = \left\{c_t^i, l_t^i, k_{t+1}^i, b_{t+1}^{i,d}\right\}_{t=0}^{\infty}$, $\forall i \in I$, the firm's allocation $z^F = \{K_t, L_t\}_{t=0}^{\infty}$, prices $p = \left\{q_t, Q_t, w_t, r_t, r_t^k\right\}_{t=0}^{\infty}$, and government's policy $z^G = \left\{\tau_t^n, \tau_t^k, T_t, r_t, B_{t+1}^d, B_{t+1}\right\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves agent i's problem that maximizes (1) subject to (2) and a no-Ponzi condition of agent's debt value, (ii) given p and z^G, z^F solves firm's problem, which implies the first-order conditions (3), (iii) government budget constraint (5) holds, (iv) aggregate resource constraint (6) is satisfied, $\sum_{i\in I} b_t^{d,i} = B_t^d$, (iv) no arbitrage condition $(1-\tau_t^k)r_t^k = r_t + \delta$, and (v) p satisfies (4) given z^G.

2.3 Lack of commitment

Assume that the government is benevolent in that its objective is the weighted discounted utility of all agents in the economy, given by a set of social welfare weights $\boldsymbol{\lambda} = (\lambda^i)_{i\in I}$[4]. The government enters into contracts with private agents and foreign creditors that specify allocation of consumption, capital, labor, domestic and foreign bonds. Nevertheless, in every period, the government can drop its external and domestic obligations, change the tax schedules, and expropriate all capital holdings. When deviating from the contracted allocation, the government faces the punishment from private agents and foreign lenders. The government then receives a deviation utility $\underline{U}_t(K_t)$ that depends on the current aggregate capital level that it can expropriate. The limited commitment implies that there exists a lower bound on future discounted aggregate utility. Specifically, for all $t \geq 0$,

$$\sum_{s=t}^{\infty} \beta^{s-t} \sum_{i\in I} \lambda^i \pi^i U^i\left(c_s^i, l_s^i\right) \geq \underline{U}_t(K_t) \tag{7}$$

Following Chari and Kehoe (1990, 1993), this sustainability constraint is a characterization of a sub-game perfect equilibrium of a dynamic sovereign game between the government, private agents, and foreign creditors. The equilibrium sustains risk-free debt and no default on path. Appendix A provides the details of the game and the equilibrium definition. In general, the set of sustainable equilibrium payoffs of this sovereign game can be supported by trigger strategies to the equilibrium that has the worst payoff. Due to the complication in characterizing the worst equilibrium of this dynamic game, this paper uses autarky, in which there are no international and domestic financial markets, as the punishment for deviation. This assumption does not change the main results of the paper. The reverting-to-autarky

[3] $a_0^i \equiv \left[1 + (1-\tau_0^k)r_0^k - \delta\right] k_0^i + (1+r_0)\, b_0^{d,i}$

[4] For the relationship between the welfare weights and the utility frontier, see Section 4.

equilibrium is characterized by the constraint (7) where $\underline{U}_t(K_t)$ incorporates the autarky value[5]. It is a constraint on aggregate allocation such that private agents do not directly take into account when solving their problems. The constraint imposes endogenous limits on the aggregate debt levels over time.

3 Characterizing Equilibrium

In equilibrium, because all agents have the same preference, facing the same tax rates, earn the same wage on their efficient labor units, and have the same return on savings, the intratemporal and intertemporal conditions are the same across agents, i.e. in each period t, for all i,

$$
\begin{aligned}
(1-\tau_t^n)w_t &= -\frac{U_l^i(c_t^i, l_t^i)}{U_c^i(c_t^i, l_t^i)} \\
1 + r_{t+1} &= \frac{U_c^i(c_t^i, l_t^i)}{\beta U_c^i(c_{t+1}^i, l_{t+1}^i)}
\end{aligned}
$$

Given the aggregate allocation (C_t, L_t) in every period, there is an efficient assignment of individual allocation $(c_t^i, l_t^i)_{i\in I}$ due to the equal marginal rates of substitution between consumption and efficient labor. Moreover, because of the equal marginal rates of substitution of future to current consumption, the efficient assignment needs to be the same across time. Any inefficiencies due to tax distortions are captured by the aggregate allocation. Werning (2007) incorporated these properties of the equilibrium allocation into first analyzing the static distortion problem, then looking at the dynamics in aggregate levels. This method allows for aggregation such that the competitive equilibrium allocation can be completely characterized in terms of aggregates and a static rule for individual allocation.

Sub-market analysis. For any equilibrium, there exist market weights $\boldsymbol{\varphi} = (\varphi^i)_{i\in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i \varphi^i = 1$, such that individual assignments solve a static problem.

$$
\begin{aligned}
V(C, L; \boldsymbol{\varphi}) \equiv \max_{(c^i, l^i)_{i\in I}} & \sum_{i\in I} \varphi^i \pi^i U^i(c^i, l^i) \\
s.t. \qquad & \sum_{i\in I} \pi^i c^i = C; \quad \sum_{i\in I} \pi^i l^i = L
\end{aligned}
$$

[5] Many papers have pointed out the problem in charaterizing the worst equilibrium in this type of dynamic games as it might not be the repeated worst static Nash equilibrium. Instead, they made the same assumption of using autarky as the worst equilibrium (see Chari and Kehoe (1993); Dovis et al. (2016)).

The market weights capture how individual allocation are determined given any aggregate allocation. This problem gives the policy functions for each agent i:

$$h^i(C, L; \boldsymbol{\varphi}) = \left(h^{i,c}(C, L; \boldsymbol{\varphi}),\ h^{i,l}(C, L; \boldsymbol{\varphi})\right)$$

A competitive equilibrium allocation must satisfy: $(c_t^i, l_t^i) = h^i(C_t, L_t; \boldsymbol{\varphi})$ for all i and t. The associate competitive equilibrium prices can be computed as if the economy were populated by a fictitious representative agent with utility function $V(C, L; \varphi)$. The envelope conditions of the static problem give

$$(1 - \tau_t^n)w_t = -\frac{V_L\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]}{V_C\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]} \tag{8}$$

$$1 + r_{t+1} = \frac{V_C\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]}{\beta V_C\left[h^i(C_{t+1}, L_{t+1}; \boldsymbol{\varphi})\right]} \tag{9}$$

Furthermore, the present-value budget constraint for individual i can be written as

$$\sum_{t=0}^{\infty} \beta^t \left[V_C(C_t, L_t; \boldsymbol{\varphi}) h^{i,c}(C_t, L_t; \boldsymbol{\varphi}) + V_L(C_t, L_t; \boldsymbol{\varphi}) h^{i,l}(C_t, L_t; \boldsymbol{\varphi})\right] = V_C(C_0, L_0; \boldsymbol{\varphi})\left(a_0^i - T\right) \tag{10}$$

where T is the present-value of lump-sum taxes[6], and a_0^i is the individual initial after-tax wealth. Equation (10) is the individual implementability constraint.

Now one has the following characterization proposition.

Proposition 3.1. *Given initial individual wealth $\{a_0^i\}_{i \in I}$ and external debt B_0, an allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if aggregate resource constraint (6) holds, and there exist market weights $\boldsymbol{\varphi} = (\varphi^i)_{i \in I}$ and lump-sum tax T such that the implementability constraint (10) holds for all $i \in I$.*

4 Efficiency

This section formulates the planning problem in terms of a Ramsey problem with the additional sustainability constraints induced by the limited commitment. It follows the primal approach in public finance to characterize the best equilibrium allocation and derive the optimal policies.

[6] $T \equiv \sum_{t=0}^{\infty} \beta^t \frac{V_C\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]}{V_C\left[h^i(C_0, L_0; \boldsymbol{\varphi})\right]} T_t$

4.1 Planning problem

The set of equilibrium with limited commitment can be supported as a competitive equilibrium with taxes and the sustainability constraint (7). Define the set $\mathcal{U}$ of attainable utilities $\{u^i\}_{i\in I}$ such that $u^i = \sum_{t=0}^{\infty} \beta^t U^i\left(c_t^i, l_t^i\right)$ for any such equilibrium allocation. Given Proposition 3.1, $\{u^i\}_{i\in I}$ is the individual lifetime utilities for any allocation $\{C_t, L_t, K_t\}_{t=0}^{\infty}$ and a vector of market weights $\boldsymbol{\varphi}$ such that the aggregate resource constraint and the implementability constraint all $i \in I$ are satisfied. Specifically, $u^i = \sum_{t=0}^{\infty} \beta^t U^i\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]$. An efficient allocation is defined as one that reaches the northeastern frontier of $\mathcal{U}$, i.e. maximizing lifetime utility of one agent given that the utilities of other agents are above feasible thresholds. The necessary conditions can be derived by an alternative problem of maximizing a Pareto-weighted utility, where the Pareto weights are closely related to the feasible thresholds[7].

Therefore, given the Pareto weights $\boldsymbol{\lambda} = \{\lambda^i\}_{i\in I}$ and exogenous international interest rates $\{r_t^*\}_{t=0}^{\infty}$, an efficient allocation maximizes the weighted utility subject to the aggregate resource constraint, the individual implementability constraints, and each-period sustainability constraint. The planning problem is formulated as

$$
\begin{aligned}
(P) \equiv \max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T} & \sum_{t=0}^{\infty} \beta^t \sum_{i\in I} \lambda^i \pi^i U^i\left[h^i(t; \boldsymbol{\varphi})\right] \\
s.t. \qquad & \sum_{t=0}^{\infty} q_t\left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 \geq 0 \\
\forall i, \ & \sum_{t=0}^{\infty} \beta^t\left[V_C(t; \boldsymbol{\varphi}) h^{i,c}(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) h^{i,l}(t; \boldsymbol{\varphi})\right] \geq V_C(0; \boldsymbol{\varphi})\left(a_0^i - T\right) \\
\forall t, \ & \sum_{s=t}^{\infty} \sum_{i\in I} \beta^{s-t} \lambda^i \pi^i U^i\left[h^i(s; \boldsymbol{\varphi})\right] \geq \underline{U}_t(K_t)
\end{aligned}
$$

4.2 Characterizing efficient allocation

Let μ be the multiplier on the resource constraint, $\pi^i \eta^i$ be the multiplier on the implementability constraint for agent i, and $\beta^t \gamma_t$ be the multiplier on the aggregate debt constraint for period t. Define $\boldsymbol{\eta} = (\eta^i)_{i\in I}$ and rewrite the Larangian of the planning problem with a

[7] As the set of attainable utilities $\mathcal{U}$ might not be convex, an allocation that solves (P) might not attain the utilities in $\mathcal{U}$. The analysis focuses on the necessary conditions, as they are enough to develop the properties of the optimal taxes. The set of optimal taxes is a subset of the set of taxes that implement any allocation satisfying the necessary conditions for efficiency. Therefore, the optimal taxes also satisfy the attributes of taxes deriving from the necessary analysis. Park (2014); Werning (2007) made the similar argument in their work.

new pseudo-utility function that incorporates the implementability constraints:

$$\sum_{t=0}^{\infty} \beta^t W\left[t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] - V_C(0; \boldsymbol{\varphi}) \sum_{i \in I} \pi^i \eta^i \left(a_0^i - T\right)$$

where

$$W\left[t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] \equiv \sum_{i \in I} \lambda^i \pi^i U^i \left[h^i(t; \boldsymbol{\varphi})\right] + \sum_{i \in I} \pi^i \eta^i \left[V_C(t; \boldsymbol{\varphi}) h^{i,c}(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) h^{i,l}(t; \boldsymbol{\varphi})\right]$$

The necessary conditions to characterize the set of efficient allocation are the first-order conditions of the planning problem, the aggregate resource constraint, the sustainability constraints, and the implementability constraints.

5 Optimal Taxation

This section provides the main optimal taxation results. The work emphasizes on the case of separable isoelastic preferences. The optimal taxes are derived such that the efficient allocation can be implemented as an allocation of a competitive equilibrium with taxes. A key assumption throughout this section is the impatience of private agents with respect to the international intertemporal interest rates that the country faces when borrowing abroad, i.e.

Assumption 1 (Impatience)**.** *There exists* $M > 0$ *and* T *such that* $\forall t > T,\ \beta(1 + r_t^*) < M < 1$.

Consider the following functional form of the utility:

Assumption 2 (Separable isoelastic preference)**.** *The utility function* $U : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ *satisfies*

$$U(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \omega \frac{n^{1+\nu}}{1+\nu}$$

for $\sigma, \omega, \nu > 0$.

Given that the preference is separable and isoelastic, individual consumption and efficient labor supply are time-independently proportional to the aggregates:

$$\begin{aligned} c_t^i &= h^{i,c}(C_t, L_t; \boldsymbol{\varphi}) = \psi_c^i C_t \\ l_t^i &= h^{i,l}(C_t, L_t; \boldsymbol{\varphi}) = \psi_l^i L_t \end{aligned} \tag{11}$$

where

$$\psi_c^i = \frac{(\varphi^i)^{1/\sigma}}{\sum_{i\in I} \pi^i (\varphi^i)^{1/\sigma}}, \quad \psi_l^i = \frac{(\theta^i)^{\frac{1+\nu}{\nu}} (\varphi^i)^{-1/\nu}}{\sum_{i\in I} \pi^i (\theta^i)^{\frac{1+\nu}{\nu}} (\varphi^i)^{-1/\nu}} \tag{12}$$

Then V, W inherit the separable and isoelastic properties from U, i.e. $\forall t$,

$$V(C_t, L_t; \boldsymbol{\varphi}) = \Phi_C^V \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L_t^{1+\nu}}{1+\nu}$$
$$W\left[C_t, L_t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] = \Phi_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^W \frac{L_t^{1+\nu}}{1+\nu}$$

and the objective is

$$\sum_{t=0}^{\infty} \beta^t \left(\Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} \right)$$

where Φ_C^V, Φ_L^V depend on $\boldsymbol{\varphi}$, Φ_C^W, Φ_L^W depend on $\boldsymbol{\varphi}, \boldsymbol{\lambda}$, and $\boldsymbol{\eta}$, and Φ_C^P, Φ_L^P are functions of $\boldsymbol{\lambda}$ and $\boldsymbol{\varphi}$ (see Appendix B.1).

The first-order conditions of the planning problem for any period $t \geq 1$ can be summarized as

$$F_L(K_t, L_t) = \frac{\left\{\Phi_L^W + \Phi_L^P \sum_{s=0}^t \gamma_s\right\} L_t^\nu}{\left\{\Phi_C^W + \Phi_C^P \sum_{s=0}^t \gamma_s\right\} C_t^{-\sigma}} \tag{13}$$

$$F_K(K_t, L_t) = r_t^* + \delta + \frac{\beta^t}{q_t} \frac{\gamma_t}{\mu} \underline{U}_t'(K_t) \tag{14}$$

and

$$C_t^{-\sigma} = \beta C_{t+1}^{-\sigma} (1 + r_{t+1}^*) \left[\frac{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t+1} \gamma_s}{\Phi_C^W + \Phi_C^P \sum_{s=0}^t \gamma_s} \right] \tag{15}$$

To implement the efficient allocation, optimal taxes are derived such that the competitive equilibrium allocation satisfies the first-order conditions of the planning problem. From the competitive equilibrium characterization, the taxes on labor, capital and saving return must satisfy

$$(1 - \tau_t^n) F_L(K_t, L_t) = \frac{\Phi_L^V L_t^\nu}{\Phi_C^V C_t^{-\sigma}} \tag{16}$$

$$(1 - \tau_t^k) F_K(K_t, L_t) = r_t + \delta \tag{17}$$

$$C_t^{-\sigma} = \beta \left(1 + r_{t+1}\right) C_{t+1}^{-\sigma} \tag{18}$$

5.1 Labor income tax

Dividing equation (13) by equation (16) gives

$$\tau_t^n = 1 - \frac{\Phi_L^V}{\Phi_C^V}\left[\frac{\Phi_C^W + \Phi_C^P \sum_{s=0}^t \gamma_s}{\Phi_L^W + \Phi_L^P \sum_{s=0}^t \gamma_s}\right] \tag{19}$$

which is the equation for optimal labor income tax. Note that the time-variant component of the tax is the sum of the Larange multipliers on the debt constraints, reflecting how limited borrowing influences the dynamic of the taxes. When the aggregate debt constraints do not bind, i.e. $\gamma_s = 0,\ \forall s \leq t$, the tax on labor income becomes

$$\tau_t^n = 1 - \frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} \equiv \bar{\tau}^n \tag{20}$$

which is a time-independent constant. The labor tax is constant when the debt constraint is not relevant. The intuition is that distortionary tax is a mechanism for redistribution. The distortion reflects the trade-off between the dispersion level, which is determined by the skill distribution, and the redistribution motive, which is from the social welfare weights. Because the skill distribution and welfare weights do not change, and borrowing is unconstrained, the government finds it optimal keep the intratemporal distortion constant and borrow as needed to finance expenditure. The unconstrained optimal level is formulated by (20), which indirectly depends on skill distribution and Pareto weights (see Appendix for the formulas of Φ's).

On the other hand, binding debt constraint limits the government's ability to borrow and to smooth taxes over time. When the debt constraint binds, the cumulative sum of debt-constraint multipliers show up in the optimal labor tax formula. Given that private agents are impatient, the country's aggregate debt increases over time. In the environment without debt constraints, or debt constraints never bind, the Ramsey allocation features immiseration in the long run such that the marginal utility of consumption is growing without bound. Such scenario happens when the deviation utility is unbounded below, i.e. the value of deviating is low enough such that the government will always commit to the contract. However, if the deviation utility is bounded below, the full-commitment Ramsey allocation cannot be supported. There is no immiseration in the long run as the future utility is always bounded below. This no immiseration result is common in many models of limited commitment.

Note that no immiseration also means that the debt constraint eventually binds, and the multiplier γ_t increases over time. In the long run, the cumulative sum of multipliers will diverge[8]. Given equation (19), it must be that $\lim_{t\to\infty}\tau_t^n = 1-\frac{\Phi_L^V\Phi_C^P}{\Phi_C^V\Phi_L^P}$, or by substituting in the definitions,

$$\lim_{t\to\infty}\tau_t^n = 1-\frac{\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_c^i}{\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_l^i}, \tag{21}$$

which is a different level from the unconstrained distortion level. The tax limit similarly depends on redistributive preference, which comes from $\{\lambda^i\}_{i\in I}$, and inequality, captured by $\{\varphi^i\}_{i\in I}$ (utility shares),$\{\psi_c^i\}_{i\in I}$(consumption shares), and $\{\psi_l^i\}_{i\in I}$(labor shares).

Formally, consider the following assumption on deviation utility.

Assumption 3. *$\underline{U}_t(\cdot)$ is bounded below, i.e. there exists a finite real M_U such that* $\inf_{K_t}\underline{U}_t(K_t)\geq M_U$.

Given this assumption, the consumption path is bounded below by zero in the long run, i.e.

Lemma 5.1 (No immiseration)**.** *Suppose assumptions 2 and 3 hold, then for any efficient allocation $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$,* $\liminf_{t\to\infty} C_t^* > 0$.

The following proposition characterizes the optimal tax on labor income in an economy facing debt constraints and distributive concern.

Proposition 5.1 (Optimal labor tax)**.** *Given assumption 2, if an efficient allocation exists and debt constraint does not bind, there is constant labor tax given by (20). Moreover, if assumptions 1 and 3 also hold, and an interior efficient allocation exists, then the optimal labor tax converges to a real constant given by (21) that depends on skill distribution and redistributive preference. These results hold with or without the lump-sum transfers.*

Redistributive motive and limited borrowing

Both the redistributive motive and the limited borrowing determine the optimal level of distortion in the economy, expressed in equation (19). The intuition is best explained in the case with lump-sum transfers. The optimal tax then depends on the marginal benefit of redistribution and the marginal cost of distortion. When borrowing is not constrained, it is optimal to set the marginal cost equal to the marginal benefit, which is constant over time. Therefore, optimal tax rates do not change during the periods that debt constraints do not bind. As debt constraint binds, there is an additional benefit of relaxing the constraints. The marginal cost of distortion is equal to the net marginal benefit of redistribution and

[8]Intuitively, optimal allocation must satisfies $\beta^t/q_t\left(\Phi_C^W+\Phi_C^P\sum_{s=0}^t\gamma_s\right)C_t^{-\sigma}=\mu$. No immiseration implies that $C_t^{-\sigma}$ is bounded below. Since $\mu>0$, as $\beta^t/q_t\to 0$, it must be that $\sum_{s=0}^t\gamma_s\to\infty$

relaxing debt constraints. The following Proposition shows that this marginal cost of distortion is decreasing over time as the debt constraint binds, in an environment of only skill heterogeneity, high consumption-inequality aversion, measured as the relative risk aversion σ, and high redistributive motive towards the low-skilled agents.

Proposition 5.2. *Given assumptions 1–3, and additionally suppose that there are (i) equal initial wealth distribution: $a_0^i = a_0^j$, $\forall i, j \in I$, (iii) high consumption-inequality aversion: $\sigma \geq 1$, and (iv) redistributive motive towards the low types: $\theta^i \geq \theta^j \iff \lambda^i \leq \lambda^j$, $\forall i, j \in I$, then for any period t such that the debt constraint binds, the optimal labor tax decreases, i.e. $\tau_t^n \leq \tau_{t-1}^n$. Moreover, $\tau_s^n \leq \tau_{t-1}^n$, $\forall s \geq t$.*

This Proposition points out that it requires a lower labor tax rate not only at the period the debt constraint binds, but also at any periods afterwards. The optimal labor tax drifts downward over time as the debt constraints bind, and given Proposition 5.1, it will eventually converge to a limit. As a result, the optimal labor tax before the debt constraint binds is at least as high as the optimal limit, i.e.

Corollary 5.1. *Given the assumptions of Proposition 5.2, $\bar{\tau}^n \geq \lim_{t\to\infty} \tau_t^n$.*

Intuitively, a government that has high redistributive motive would like to keep a high labor tax. However, labor taxes weakly decrease whenever these constraints bind. A lower tax rate, in return, encourages more labor supply, output, and relaxes the debt constraints by increasing its borrowing capacity.

In general, this efficiency motive is a driver for the dynamic in the optimal tax rates. Consider the following expenditure minimization problem for each period t

$$(EM_t) \equiv \min_{C_s, L_s, K_{s+1}} \sum_{s=t}^{\infty} q_s \left[C_s + G_s + K_{s+1} - F(K_s, L_s) - (1-\delta)K_s\right]$$
$$s.t. \quad \sum_{s=t}^{\infty} \beta^{s-t} \left(\Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s^{1+\nu}}{1+\nu} \right) = \underline{U}_t(K_t)$$

which is the problem of minimizing the present value of the resource cost to deliver $\underline{U}_t(K_t)$ as the planner's promised utility at period t. The solution to this minimization problem can be implemented with the labor tax $\tau_s^n = 1 - \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$ for $s \geq t$. Note that this equation is exactly the labor tax limit. The optimal labor tax formula (19) incorporates the solution to the Ramsey planner when debt is unconstrained (Φ^W's) and as debt constraint binds, part of the solution to this expenditure minimization problem shows up (Φ^P's) with respect to the tightness of the constraints ($\sum_{s\leq t} \gamma_s$). When there is unlimited borrowing, the planner should set the tax rate to achieve the most redistributive allocation, which is the allocation that the planner would choose if she never runs into the debt constraints. However, when debt constraint binds, the planners want the international lenders to continue the contract by offering an allocation such that it delivers the promised utility $\underline{U}_t(K_t)$ in a less costly way. As the debt constraint binds in the long run, the optimal allocation continues to lower the delivering cost and eventually reaches the allocation with minimal cost, which is the solution to (EM_∞). The labor tax starts of with the most redistributive level for the country, which is $1 - \frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W}$ and gradually converges to the most efficient level , i.e. $1 - \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$.

In the case without lump-sum transfers, besides the redistributive benefit, the marginal tax rate is also set to meet the budgetary needs of the government. This additional need only changes the optimal tax levels, but not the dynamics. The limited borrowing still requires the distortion to be front-loaded.

5.2 Capital income and saving taxes

Combining equations (15) and (18), the optimal domestic return satisfies

$$1 + r_t = (1 + r_t^*) \frac{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t} \gamma_s}{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t-1} \gamma_s} \tag{22}$$

Note that when the sustainability constraint is not relevant, i.e. $\gamma_s = 0 \ \forall s \leq t$, it is optimal to set the domestic interest rates equal to the exogenous foreign interest rates. However, as the sustainability constraints start binding, equation (22) implies that $r_t > r_t^*$, which implies a saving subsidy. As the economy reaches its debt limits, shown as the binding sustainability constraints, the government has incentive to subsidize more on saving, or tax more on borrowing, to discourage private agents to accumulate debt.

Moreover, the first-order conditions (14) and (17) give

$$\tau_t^k = 1 - \frac{r_t + \delta}{r_t^* + \delta + \frac{\beta^t}{q_t} \frac{\gamma_t}{\mu} \underline{U}_t'(K_t)} \tag{23}$$

If $\underline{U}_t'$ is positive[9], the above equation implies that there is a higher tax on capital income when debt constraints bind. A greater capital tax reflects that there is capital underinvestment of the efficient allocation. Indeed, the first-order condition (14) shows that $F_K(K_t, L_t) > r_t^* + \delta$ when $\gamma_t > 0$, where $r_t^* + \delta$ is the first-best interest rate. Because the government can expropriate more capital and receive higher utility from reneging, the optimal contract discourages capital accumulation, which can be implemented by imposing

[9] The higher the amount of capital the government can expropriate, the higher the deviation utility. Proposition A.1 proves a case when it is true.

more tax on capital income.

6 Redistributive Effect

This section analyzes how redistributive motive affects the optimal labor tax. Redistribution not only influences the tax levels from the trade-off between equity and efficiency, but also the tax dynamics from the interaction with the debt constraints. Suppose that there is no heterogeneity, then the problem becomes the standard representative Ramsey problem in a small open economy. If the government can use lump-sum taxes, there is no need for distortion, and the optimal labor tax will be zero. If the government can only impose distortionary taxes, the model collapses to the representative setting as in Aguiar and Amador (2016), where the zero labor tax in the limit is optimal.

Proposition 6.1 (No heterogeneity)**.** *Suppose that* $\theta^i = \theta^j, a_0^i = a_0^j, \ \forall i, j \in I$. *Then there is zero labor tax in the long run. This result holds with or without lump-sum transfers.*

Proof. Follow from equation (21) with $\varphi^i = \psi_c^i = \psi_l^i = 1, \ \forall i \in I$. □

While the market weights determine how the competitive market chooses individual shares of utility, the Pareto weights regulate the social shares of utility. Any agent has an exogenous Pareto weight based on the government's distributive preference, and a market weight that depends on her relative skill and initial wealth. This is due to skill and initial wealth positions determine individual budget constraints, which gives individual implementability constraints in the planning problem. An interesting case is when the vector of market weights is equal to the vector of Pareto weights ($\boldsymbol{\psi} = \boldsymbol{\lambda}$). This implies that there is no distributive effect, because the government, as a planner, distributes aggregate utility exactly the same way as the competitive market does. In this situation, (21) turns out to be $\lim_{t\to\infty} \tau_t^n = 0$, that is, the optimal labor tax converges to zero. These results are summarized in the followings.

Proposition 6.2. *There exists an efficient allocation* $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ *such that for all i,* $\lambda^i = \varphi^{*i}$. *Such an allocation can be implemented with a zero labor tax in the long run.*

Generally, changing the welfare weights, which are a measure for redistributive motive, affects the social utility, and so the debt constraints[10]. Proposition 6.2 shows that redistributive motive also has effect on the levels of optimal policies, yet only when it can deviate from

[10] The welfare weights determine the discounted future utility and the deviation utility, derived in the Appendix, so they influence both sides of the sustainability constraints.

the distribution rising from the competitive equilibrium markets. This is the special case in which the welfare weights equal to the inverse of marginal utilities. Therefore, the efficient allocation is the non-distorted competitive equilibrium allocation (see . The redistributive motive, not heterogeneity, is the source of differences in optimal policies comparing to the representative-agent setting.

7 Numerical Exercise

This section illustrates the theoretical results by a numerical exercise with endogenous debt constraints and redistribution. There is no capital. Assume that production is linear in effective labor, i.e. $F(L_t) = L_t$. The deviation utility is a constant finite $\underline{U}$ so that it is consistent with Assumption 3.

Lemma 7.1. *If the sustainability constraint binds for some finite S , then it will bind for all $t > S$.*

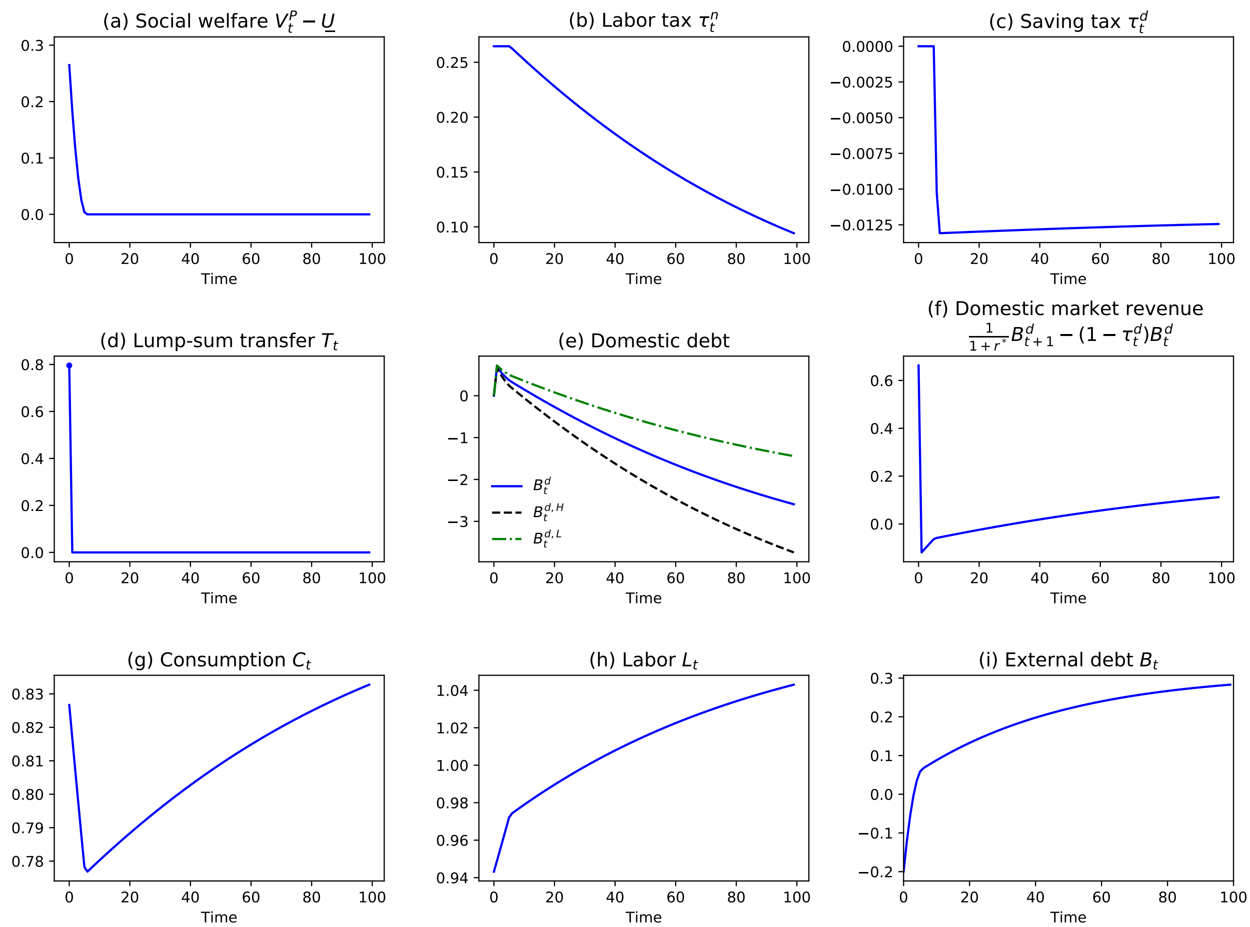
It must be true that the socially weighted utility, $\sum_{i\in I} \lambda^i \pi^i U^i[h^i(C_t, L_t; \boldsymbol{\varphi})]$ is equal to across all period $t > S$ that the debt constraint binds. Combining this feature and the planner's first-order conditions solves the efficient allocation at each period after the constraints bind, then the allocation at period $t < S$ can be derived by solving backwards.

Consider a parametric economy consisting of two types of agents, denoted $I = \{H, L\}$, where $\theta^H \geq 1 \geq \theta^L$. Let $\pi^H = \pi^L = 0.5$, which implies that $\theta^H = 2 - \theta^L$. The two types have zero initial wealth positions, i.e. $a_0^i = 0,\ \forall i$. Let $U(c, n) = \log c - \omega \frac{n^{1+\nu}}{1+\nu},\ \omega = 1,\ \nu = 2$ so that the Frisch elasticity of labor supply is 0.5. Assign $\beta = 0.94$ and $r_t^* = r^* = 0.05$ so that $\beta(1 + r^*) < 1$. The planner is utilitarian: $\lambda^H = \lambda^L = 1$. The government expenditure is constant across time and set to be 20 percent of the average productivity. The economy starts with an initial external asset position of 20%. $\underline{U}$ is the value of autarky, which is calculated as the maximal utility attained from a tax-distorted competitive equilibrium of the economy with no domestic and international credit markets.

7.1 Dynamics of policies and allocation

Figure 1 depicts the time paths of optimal policies and efficient allocation when the relative skill dispersion is such that $\theta^H = 2\theta^L$. Figure 6 expands the time periods to show the long-run properties. Time is the horizontal axis. Panel (a), (b), and (c) plot the planner's utility relative to the deviation utility, the labor tax, and the domestic saving tax, respectively. The planner's utility is decreasing over time until it reaches the deviation utility and stays constant, as the debt constraint binds. When the debt constraint does not bind, the labor tax starts at a positive level and constant, while the saving tax is zero. As the debt constraint starts binding, labor tax decreases while saving is subsidized, implying a positive tax on borrowing. Figure 6 shows that in the long run, there is labor subsidy and borrowing tax.

Figure 1: Time paths of economic aggregates when $\theta^H = 3\theta^L$



Due to Ricardian equivalence, I consider a particular implementation of the efficient allocation where the planner gives the present-value lump-sum transfer only in period 0. Panel (d) depicts the lump-sum transfer over time. Given this implementation, panel (e) plots the aggregate and individual domestic debt levels. Because of impatience, domestic agents borrow over time, and the planner acts as an intermediary that borrows abroad and lends to domestic agents. However, when debt constraints bind, the planner levies taxes on borrowing. In net, the planner collects revenue from the domestic market, as decribed in panel (f).

Panel (g) and (h) plot the dynamics of the aggregate consumption and labor. When debt constraints do not bind, there is front-loading consumption and leisure. When debt constraints bind, given the positive tax on borrowing and the decreasing labor tax, aggregate consumption and labor increase over time. Panel (f) shows the path of external debt B_t. The economy accumulates external debt quickly in the beginning of time. However, when the debt constraint hits, there is a slower accumulation of debt that eventually reaches its steady state, which is the maximum debt capacity of the economy.

One important point from panel (e) is that the higher-income agents borrow more over time. This is because all agents borrow at similar fractions of their income over time. Therefore, when the planner uses the marginal tax on borrowing, she also redistributes more resources towards the lower-income agents, as the higher-income agents pay higher taxes on borrowing. When there is no cost of borrowing, the planner uses the labor distortion to redistribute. The high initial labor tax rate reflects the redistributive motive, and that the marginal benefit of redistribution is high. When hitting debt constraints, the planner can use taxes on borrowing to redistribute. This allows a lower labor distortion until subsidy in the long run. In this case, the high-skilled agent is more productive than the average productivity. A lower labor tax encourages her to produce more output, which increases the economy's ability to repay.

7.2 Discussion of the optimal properties

Why do allocation, taxes, and debt change even when debt constraint binds? To answer this question, consider the following planning problem that does not incur any distortionary cost of redistribution but subject to delivering the same distribution of individual outcomes, which is fixing the optimal $\boldsymbol{\varphi}^*$,

$$\begin{aligned} E(\boldsymbol{\varphi}^*) \equiv \max_{\{C_t, L_t\}_t} & \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i U^i \left[h^i(t; \boldsymbol{\varphi}^*) \right] \\ s.t. \qquad & \sum_{t=0}^{\infty} q_t \left[L_t + -C_t - G_t \right] - B_0 \geq 0 \\ \forall t, & \sum_{s=t}^{\infty} \sum_{i \in I} \beta^{s-t} \lambda^i \pi^i U^i \left[h^i(s; \boldsymbol{\varphi}^*) \right] \geq \underline{U}_t \end{aligned}$$

Figure 2 compares the dynamics of aggregates between the benchmark problem $P(\boldsymbol{\varphi}^*)$ and the relaxed problem $E(\boldsymbol{\varphi}^*)$. $E(\boldsymbol{\varphi}^*)$ delivers a higher ex-ante welfare (panel (a)). It takes longer for $E(\boldsymbol{\varphi}^*)$ to reach the debt constraint (see Figure **??** for a shorter time frame), but when the debt constraint binds, consumption, labor, taxes, and external debt stay constant. The implemented labor tax for the allocation of $E(\boldsymbol{\varphi}^*)$ is constant at the negative level. It is important to note that the solution of $P(\boldsymbol{\varphi}^*)$ converges to the solution of $E(\boldsymbol{\varphi}^*)$ in the long run. $E(\boldsymbol{\varphi}^*)$ is the most efficient way to deliver $\boldsymbol{\varphi}^*$, while $P(\boldsymbol{\varphi}^*)$ is the best way to deliver $\boldsymbol{\varphi}^*$ taking into account the distortionary cost of redistribution. The solution to $P(\boldsymbol{\varphi}^*)$ increases its efficiency every time the debt constraint binds and eventually reaches the most efficient outcome.

Figure 2: Time paths of benchmark and relaxed economic aggregates when $\theta^H = 3\theta^L$

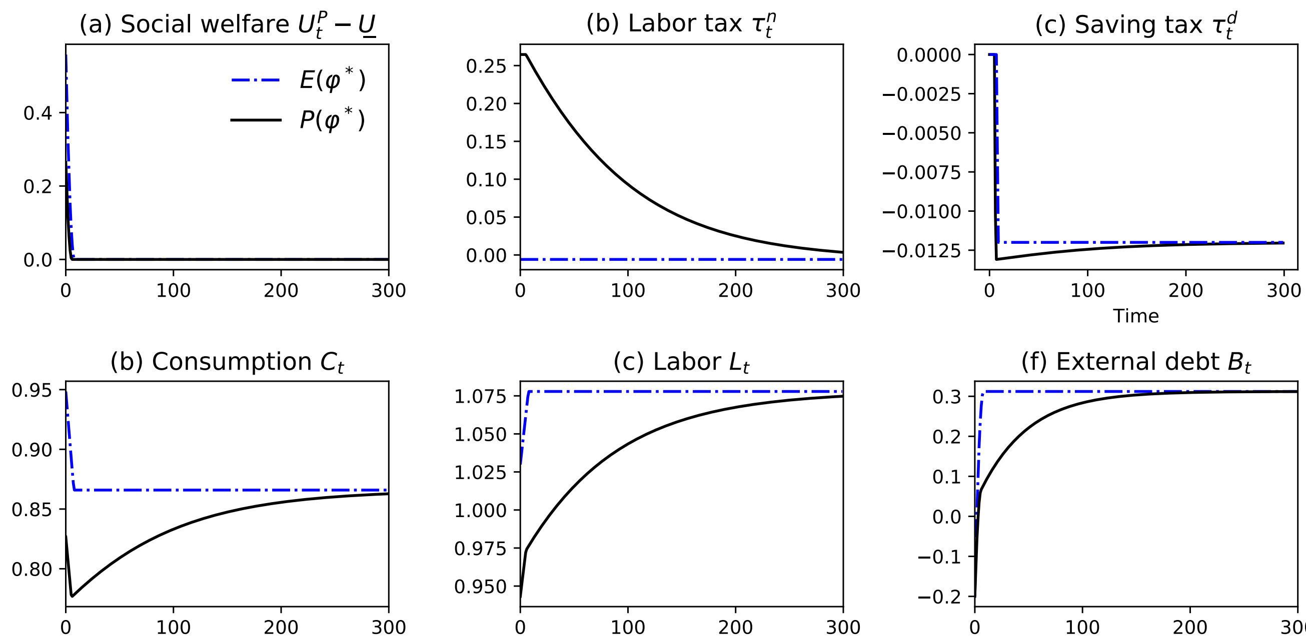
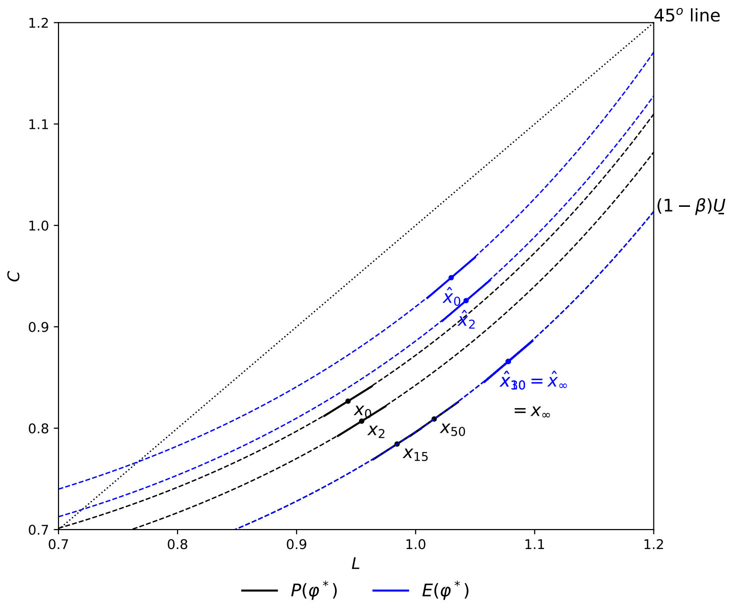


Figure 3 provides the trade-off path of the planning and relaxed allocation over time. The dash curves represent the intra-period indifference curve of the planning utility with respect to the aggregate consumption and labor. The planning and alternative allocation for each period t are indexed by x_t and $\hat{x}_t$, respectively. The slope of each associated line is the marginal rate of substitution at period t[11]. For the periods that the debt constraints do not bind, the marginal rate of substitution remains constant, as the planning allocation drifts down the utility indifference curve, decreasing consumption and increasing labor. The decline in consumption and leisure reflects the impatience of domestic agents, while the constant marginal rate of substitution comes from the tax smoothing argument. When the debt constraint starts binding, as illustrated before, it is not sustainable to stay at the same allocation. Therefore, the allocation moves along the autarkic utility indifference curve.

The question is in which direction the efficient allocation moves along the flow autarkic utility indifference curve. It turns out that the planning allocation goes up over time. In the planning problem, the marginal rate of substitution between consumption and labor starts

[11]The slope of the planning utility indifference curve is $\frac{\Phi_C^P}{\Phi_L^P}\frac{C_t^{-\sigma}}{L_t^{\nu}}\Big|_{u_t}$

Figure 3: The trade-off path of the efficient allocation when $\theta^H = 3\theta^L$



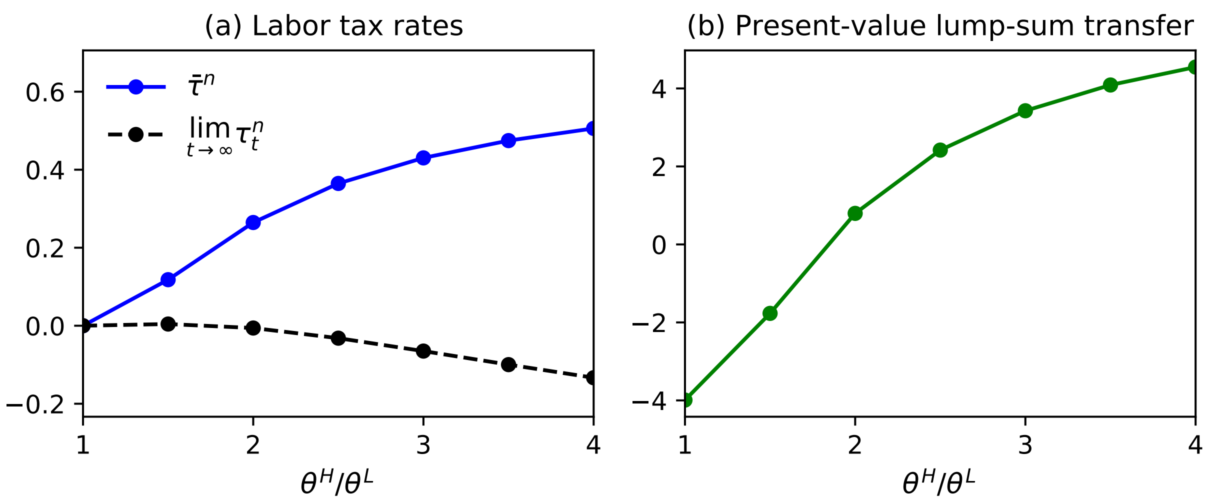
at a low level, as the slope of the indifference curve at x_0 is less than one. The argument is that it is always better for the planner to redistribute by distorting intratemporal decisions instead of intertemporal decisions. The marginal rate of substitution is then lower than one because of the distortionary cost of redistribution. On the other hand, the relaxed allocation does not have to take into account this distortionary cost, so its allocation ($\hat{x}_t$) always has a slope of one, in which the slope of the indifference curve equals to slope of the aggregate resource constraint. Tax smoothing implies that at the end of the periods that the debt constraints do not bind, the planning allocation's marginal rate of substitution has not changed and is less than one. Given the same promised utility, at the moment the debt constraint binds, suppose that the planner decreases one unit of labor, then she can only decrease consumption by less than one unit, implying that the planner will need to take more debt. If the planner instead increases one unit of labor, she will only need to increase consumption by less than one unit. Then the planner can gain additional resources to pay back the existing debt. As a result, the planning allocation moves up along the indifference curve, until it reaches the most efficient allocation with a slope of one[12].

[12] In this example, one can show that $\frac{\Phi_C^W}{\Phi_L^W} > \frac{\Phi_C^P}{\Phi_L^P}$, which implies that $\frac{\Phi_C^P C_t^{-\sigma}}{\Phi_L^P L_t^\nu} < \frac{\Phi_C^W C_t^{-\sigma}}{\Phi_L^W L_t^\nu} = 1$ for any period t such that the debt constraints have not binded before. In the long run, $\lim_{t\to\infty} \frac{\Phi_C^P C_t^{-\sigma}}{\Phi_L^P L_t^\nu} = 1$.

7.3 Comparative statics: skill dispersion

Figure 4 illustrates the changes, with respect to the relative skill dispersion θ^H/θ^L, of the optimal labor and lump-sum taxes. Panel (a) plots the labor tax rate in periods where borrowing is unconstrained ($\bar{\tau}^n$) and at the limit ($\lim_{t\to\infty}\tau_t^n$), for the utilitarian planner. Panel (b) depicts the present-value of lump-sum tax. When there is no heterogeneity ($\theta^H = \theta^L$), the problem collapses to a Ramsey's problem of a representative-agent small open economy. Due to the presence of lump-sum tax, it is optimal to have zero labor distortion in all periods. As the skill dispersion increases, the government increases its motive for redistribution. While debt constraints do not bind, it is optimal to levy higher tax rates. Therefore, as shown in panel (a),$\bar{\tau}^n$ increases with θ^H/θ^L.

Figure 4: Relative skill dispersion and tax rates

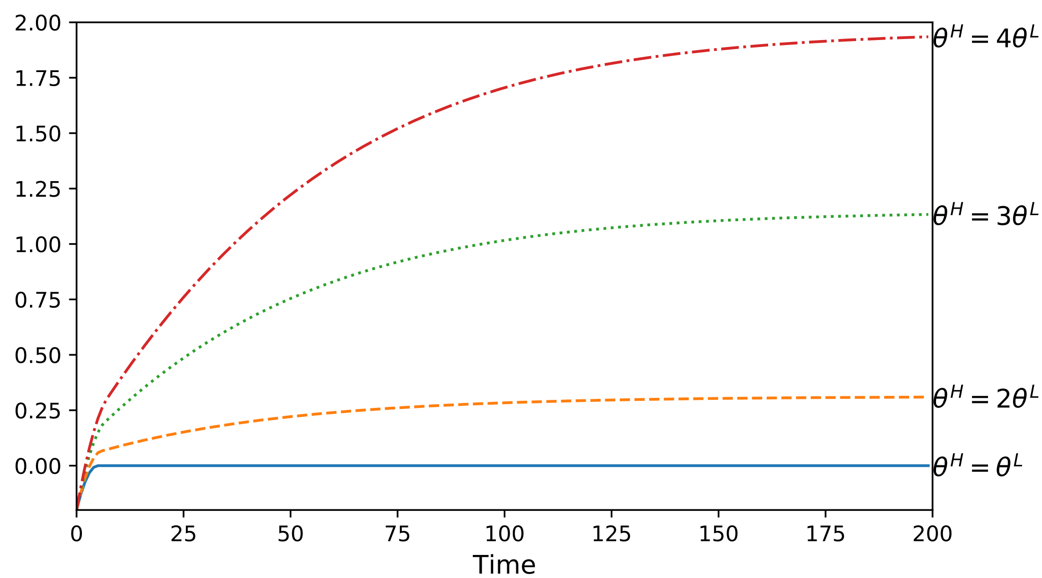


On the other hand, setting high tax rates during the not-binding-debt-constraint periods means a high cost of distortion when the economy first reaches the debt constraints. At a higher level of skill dispersion, the government wants to redistribute by increasing the marginal tax rates on labor income. Therefore, the marginal rate of substitution between consumption and labor in the planning utility starts at a lower level than before, remains constant during the no-binding periods, and gradually increases to one as the debt constraints bind (as shown in Figure 3). In the long-run, panel (a) shows that $\lim_{t\to\infty}\tau_t^n$ becomes negative and declines with respect to the skill dispersion. A higher skill dispersion implies that the highly productive agent becomes more productive. Increasing the subsidy in labor in the long run can encourage a greater output to sustain the high level of debt. Although labor is subsidized in the long run, the government still achieves its redistributive purpose by combining the initial high tax rates and positive lump-sum transfer. Indeed, panel (b) shows that the lump-sum transfer increases with respect to the relative skill dispersion.

Figure 5 presents the dynamics of government's external debt B_t with respect to the skill dispersion. While all economies start with the same initial external debt position, an economy with higher skill dispersion accumulates higher debt over time. A highly-dispersed economy wants to redistribute more by levying a higher labor tax rate during the periods that the debt constraints have not bound. The higher tax rate means that there is lower output, which is compensated by more borrowing.

The higher debt capacity of the economy corresponds to the need of stabilizing the higher debt level that the economy accumulates beforehand because of a higher redistributive motive. In addition, a higher skill dispersion is associated with a longer time of unconstrained borrowing. Since a highly-dispersed economy has more redistributive motive, it is more costly to redistribute during financial autarky. Therefore, it is optimal to prolong the periods that the debt constraint does not bind, in which the government can redistribute the most.

Figure 5: Time paths of external debt by relative skill dispersion



8 General Case: Separable Preference

This section extends the results of optimal labor taxation with separable preferences. As the elasticity of consumption intertemporal substitution and the elasticity of labor supply vary across time, the optimal labor tax fluctuates. In general, the labor tax is bounded in the long run. If the steady states exist, in the long run, the optimal tax goes to a real limit as in the case with separable isoelastic preferences. In either case, the distributive preference alters the level of optimal taxes in the long run. The results rely on the following assumptions of separability and boundedness.

Assumption 4 (Separable preference). *$U^i(c,l) = u(c) - v\left(l/\theta^i\right)$, where $u_c(\cdot) > 0$, $u_{cc}(\cdot) < 0$, $\lim_{c\to 0} u_c(c) = \infty$, $\lim_{c\to\infty} u_c(c) = 0$, $v_l(\cdot) > 0$, $v_{ll}(\cdot) > 0$, and $\lim_{l\to 0} v_l(l) = \infty$*

Assumption 5 (Bounded elasticities). *u and v are such that $\forall c, n \in \mathbb{R}_+$, $0 < -\frac{u"(c)}{u'(c)}c < \infty$ and $0 < \frac{v"(n)}{v'(n)}n < \infty$*

Since the preference is separable between consumption and leisure, individual consumption (labor) only depends on the aggregate consumption (labor). In addition, each agent's allocation is increasing with respect to the aggregates.

Lemma 8.1. *Given assumption 4, for any competitive equilibrium, there exist time-invariant functions $h^{i,c}(\cdot;\boldsymbol{\varphi}), h^{i,l}(\cdot;\boldsymbol{\varphi})$, $\forall i$ such that $\forall i, \forall t$,*

$$c_t^i = h^{i,c}(C_t;\boldsymbol{\varphi})$$
$$l_t^i = h^{i,l}(L_t;\boldsymbol{\varphi})$$

where $h^{i,c}(\cdot;\boldsymbol{\varphi}), h^{i,l}(\cdot;\boldsymbol{\varphi})$ are strictly increasing.

The characterization of individual allocation from Lemma 8.1 and the bounded elasticities from assumption 5 give the long-run property of efficient allocation and optimal labor tax. Specifically, the efficient allocation also features no immiseration in the long run,

Lemma 8.2 (No immiseration). *Suppose assumptions 3 and 4 hold, then for any efficient allocation $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$, $\liminf_{t\to\infty} C_t^* > 0$.*

and the long-run optimal labor tax is bounded.

Proposition 8.1 (Optimal labor tax in the long run). *If assumptions 1,3,4, and 5 hold, and an interior efficient allocation $\{C_t, L_t, K_t\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T$ exists, then there exist $-\infty < \underline{\tau}, \bar{\tau} < \infty$ such that $\liminf_{t\to\infty} \tau_t^n = \underline{\tau}$ and $\limsup_{t\to\infty} \tau_t^n = \bar{\tau}$. Moreover, if the steady states exist, then $\lim_{t\to\infty} \tau_t^n$ exists. These results hold with or without the lump-sum transfers.*

The results rely on the fact that the tax's long-run value relies on the marginal changes in individual allocation with respect to the aggregates in the long run. With constant elasticities, the individual allocation is linear in the aggregate allocation, as in equations (11), so the marginal change is constant over time, which means that the limit exists. However, when preferences are not isoelastic, the marginal change fluctuates over time. Therefore, the optimal labor tax does not necessary converge to a constant. Given the bounded elasticities, the marginal changes are bounded and so is the optimal labor tax. In case the steady state allocation exists, the marginal changes will converge to the steady state values, which implies the convergence to limit of the labor tax.

9 Conclusion

This paper analyzes optimal taxation and debt management for a small open economy with impatient agents, endogenous debt constraints, and redistributive motive. Impatient agents borrow over time, which makes the debt constraints relevant in the long run. Optimal labor taxed feature constant rates when borrowing is unconstrained, yet later a gradual convergence to non-zero values in the limit that are associated with the economy's aggregate debt limit. As the debt constraints bind, it is optimal to increase the taxes on capital and domestic borrowing.

The redistributive motive significantly changes the implication for the optimal fiscal policies. Specifically, it alters the long-run limit of taxes through interacting with the heterogeneity. It also changes both the inside and outside values of the contract, which indirectly determines the debt limits. Any country's optimal levels of taxes and debt issuance crucially depends on its labor productivity distribution as well as its social distribution preference. On the other hand, the debt constraints limit a government's ability to redistribute, in which the optimal policies decrease the redistribution and increase the efficiency whenever the debt constraints bind.

The paper also provides a mechanism explaining the relationship between sovereign debt accumulation and redistribution. A government with high redistributive motive wants to set a high tax rates and accumulate a high level of debt. When the debt constraints bind, by lowering the tax rates and possibly subsidizing labor in the long run, the government can sustain this high debt position.

The main source of income heterogeneity in the model comes from the skill dispersion, which highlights the redistributive effect on labor taxes. Other sources of heterogeneity, such as capital income and wealth are worth being explored in future research. It is also useful to understand how optimal policies response in this model with aggregate shock that leads to periods of fiscal and debt crises. The government will have an additional incentive to insure the economy against aggregate shocks. The trade-off between insurance and redistributive motive can result in interesting tax dynamics, as illustrated in Arellano and Bai (2016) and Balke and Ravn (2016). In addition, enriching the tax system to non-linear taxes can help study the optimal tax progressiveness in the presence of sovereign debt.

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A Sovereign Game

Before setting up the game, consider the general environment where the government's policy includes the decision to default on external bond $\{d_t\}_{t=0}^{\infty}$, where $d_t \in \{0,1\}$ and $d_t = 0$ implies default[13]. The government's budget constraint becomes

$$G_t + (1+r_t)B_t^d + d_t B_t \leq \tau_t^n w_t L_t + \tau_t^k r_t K_t + B_{t+1}^d + Q_{t+1}B_{t+1} + T_t$$

As the government cannot commit to any of its policies, one can think that the government, private agents, and international lenders enter in a sovereign game where they determine their actions sequentially. In every period, the state variable for the game is $\left\{B_t, \left(k_t^i, b_t^{i,d}\right)_{i\in I}\right\}$. The timing of the actions is as follows.

- Government chooses $z_t^G = \left(\tau_t^n, \tau_t^k, T_t, d_t, r_t, B_{t+1}, B_{t+1}^d\right) \in \Pi$such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c_t^i, l_t^i, k_{t+1}^i, b_{t+1}^{d,i}\right)$ subject to their budget constraints, the representative firm produce output by choosing $z_t^F = (K_t, L_t)$, and the international lenders choose holdings of government's bonds $z_t^* = (B_{t+1})$ given the price Q_{t+1}.

Define $h^t = \left(h^{t-1}, z_t^G, \left(z_t^{H,i}\right)_{i\in I}, z_t^F, z_t^*, p\right) \in H^t$ as the history at the end of period t. Note that the history incorporates the government's policy, allocation and prices. Define $h_p^t = \left(h^{t-1}, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period t. The government strategy is $\sigma_t^G: \ H^{t-1} \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i}: \ H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has strategy $\sigma_t^F: \ H_p^t \to \mathbb{R}_+^2$, and the international lenders have strategy $\sigma_t^*: H_p^t \to \mathbb{R}_+$. The prices are determined by the pricing rule: $p: \ H_p^t \to \mathbb{R}_+$

Definition A.1 (Sustainable equilibrium)**.** A sustainable equilibrium is $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ such that (i) for all h^{t-1}, the policy z_t^G induced by the government strategy maximizes the socially weighted utility given λ subject to the government's budget constraint (5) (ii) for all h_p^t, the strategy induced policy $\left\{z_t^G\right\}_{t=0}^{\infty}$, allocation $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^{\infty}$, and prices $\{Q_t\}_{t=0}^{\infty}$ constitute a competitive equilibrium with taxes.

[13] Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its foreign debt ($d_t = 1$).

The following focuses on characterizing a set of sustainable equilibrium in which deviation triggers autarky, where there is no domestic and foreign borrowing. In this case, the value of deviation includes the autarkic payoff.

By definitions, autarky is a sustainable equilibrium. Given that the domestic agents do not save/invest, the representative firm produces only with labor, and the international creditors do not lend, the government finds it optimal to default on its external debt, set saving and capital taxes such that the after-tax gross returns on domestic bonds and capital are zero, and set the labor tax such that it maximizes the socially weighted utility. Given the government defaulting and fully taxing all returns from domestic savings and capital, international creditors do not want to lend, agents do not save or invest in capital, and output is produced only by labor. Lastly, given that the government will be in autarky in the future, it is optimal in the current period for the government to also follow the autarkic strategies.

Reverting to autarky equilibrium is defined as a sustainable equilibrium of the above game such that following any government's deviation from the promised plans, the economy reverts to autarky. One can characterize the equilibrium as follows.

Proposition A.1 (Reverting to autarky equilibrium)**.** *An allocation and policy* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *can be supported by reverting to autarky equilibrium if and only if (i) given* z^G*, there exist prices* p *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G, p\right\}$ *is a competitive equilibrium with taxes for an open economy, and (ii) for any* t*, there exists* $\underline{U}_t(\cdot)$ *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *satisfies the constraint*

$$\sum_{s=t}^{\infty} \beta^{s-t} \sum_{i\in I} \lambda^i \pi^i U^i\left(c_s^i, l_s^i\right) \geq \underline{U}_t(K_t) \tag{7}$$

Furthermore, $\underline{U}_t(\cdot)$ *is increasing.*

Proof. Define $\underline{U}_t(K_t)$ as the maximum discounted weighted utility for the agents in period t when the government deviates. In period t, the agents save and the government can borrow abroad. In subsequent period $s > t$, the economy reverts to financial autarky where the agents do not save and the government is excluded from international lending. This economy ensembles a neoclassical growth closed economy that starts at period t and in which distortionary taxes and savings are only in the initial period. Then it is true that the higher the initial capital stock (in this case K_t), the higher utility that the agents receive. Hence, $\underline{U}_t(\cdot)$ is increasing.

Suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an outcome of the reverting to autarky equilibrium. Then by the optimal problems of the government, agents, and foreign lenders, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes the weighted utility of the agents, satisfies government budget constraint and foreign lender's problem at period 0. Thus, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an open-economy tax-distorted competitive equilibrium. For any period t and history h^{t-1}, an equilibrium strategy that has the government deviates in period t triggers reverting to autarky in period $s > t$. Such strategy must deliver the weighted value at least as high as the right-hand side of (7). So $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii).

Next, suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies conditions (i) and (ii). Let h^{t-1} be any history such that there is no deviation from z^G up until period t. Since $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t onward. Consider a deviation plan $\hat{\sigma}^G$at period t that receives $U_t^d(K_t)$ in period t and U^{aut} for period $s > t$. Because the plan is constructed to maximize period-t utility at K_t, the right-hand side of (7) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. $\square$

B Formulas and Proofs

B.1 Formulas for separable isoelastic preference

Given the formulas for ψ_c^i and ψ_l^i in (12),

$$\begin{aligned}
\Phi_C^V &= \left[\sum_i \pi^i (\varphi^i)^{1/\sigma}\right]^\sigma; & \Phi_L^V &= \omega\left[\sum_i \pi^i \left(\varphi^i\right)^{-1/\nu}\left(\theta^i\right)^{(1+\nu)/\nu}\right]^{-\nu} \\
\Phi_C^W &= \Phi_C^V \sum_{i\in I}\pi^i\psi_c^i\left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]; & \Phi_L^W &= \Phi_L^V \sum_{i\in I}\pi^i\psi_l^i\left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right] \\
\Phi_C^P &= \Phi_C^V \sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_c^i; & \Phi_L^P &= \Phi_L^V \sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_l^i
\end{aligned}$$

B.2 Proof of Proposition 3.1

Proof. $(\Rightarrow)$ Let $\{C_t, L_t, K_{t+1}\}_{t=0}^\infty$ be an aggregate allocation of an open economy competitive equilibrium with taxes. Then by definition, $\{C_t, L_t, K_t\}_{t=0}^\infty$ satisfies aggregate resource constraint for every period. Moreover, given any market weights $\boldsymbol{\varphi}$, $\{C_t, L_t, K_{t+1}\}_{t=0}^\infty$ satisfies

$$\begin{aligned}
(1-\tau_t^n)w_t &= -\frac{V_L(C_t, L_t; \boldsymbol{\varphi})}{V_C(C_t, L_t; \boldsymbol{\varphi})} \\
1 + r_{t+1} &= \frac{V_C\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]}{\beta V_C\left[h^i(C_{t+1}, L_{t+1}; \boldsymbol{\varphi})\right]}
\end{aligned}$$

Substituting for w_t and r_t into the budget constraint (2), and using $(c_t^i, l_t^i) = h^i(C_t, L_t; \boldsymbol{\varphi})$ gives the implementability constraint for each agent. Importantly, choose $\boldsymbol{\varphi}$ and T such that the individual implementability constraints hold with equality.

($\Leftarrow$) Given $\boldsymbol{\varphi}$, T and an allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\left\{w_t, r_t^k\right\}_{t=0}^{\infty}$ using firm's first-order conditions (3). $\{\tau_t^n\}_{t=0}^{\infty}$ can be calculated using the intratemporal condition (8), while one can choose $\{r_t\}_{t=0}^{\infty}$ that satisfy the intertemporal constraint (9). The tax on capital $\left\{\tau_t^k\right\}_{t=0}^{\infty}$ can be derived from $(1-\tau_t^k)r_t^k = r_t + \delta$. Define $\{q_t\}_{t=0}^{\infty}$ by (4).

Rewriting the aggregate resource constraint using $F(K, L) = wL + rK$ gives

$$\sum_{t=0}^{\infty} q_t \left\{C_t + K_{t+1} - (1-\tau_t^n)w_t L_t - \left[1 + (1-\tau_t^k)r_t^k - \delta\right] K_t + T_t\right\} + \sum_{t=0}^{\infty} q_t \left[G_t - \tau_t^k r_t K_t - \tau_t^n w_t L_t - T_t\right] \leq -\delta_0 B_0 \tag{B.1}$$

Aggregating up the agent's budget constraints implies

$$C_t + K_{t+1} + B_{t+1}^d = (1-\tau_t^n)w_t L_t + \left[1 + (1-\tau_t^k)r_t^k - \delta\right] K_t + (1+r_t)\, B_t^d - T_t$$

or

$$C_t + K_{t+1} - (1-\tau_t^n)w_t L_t - \left[1 + (1-\tau_t^k)r_t^k - \delta\right] K_t + T_t = (1+r_t)\, B_t^d - B_{t+1}^d$$

Substituting the last equation into (B.1) gives the government's budget constraint (5). Thus, $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ is the aggregate allocation of the constructed competitive equilibrium with taxes. □

B.3 Proof of Lemma 5.1

Proof. Given an efficient allocation $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}$, suppose, by contradiction that $\liminf_{t\to\infty} C_t^* \leq 0$. Find $\epsilon > 0$ such that $\forall t$,

$$\sum_{s=t}^{\infty} \beta^{s-t} \left\{\Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu}\right\} \leq M_U$$

with $C_t = \epsilon$ and $C_s = C_s^*$, $\forall s > t$. Such ϵ exists since the utility function is unbounded below. Because $\liminf_{t\to\infty} C_t^* \leq 0$, there exists t_0 such that $C_{t_0}^* < \epsilon$. Then by monotonicity,

$$
\begin{aligned}
&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \Phi_C^P \frac{(C_s^*)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu} \right\} \\
<& \sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu} \right\} \\
\leq& M_U \\
\leq& \underline{U}_{t_0}(K_{t_0}^*)
\end{aligned}
$$

which contradicts the aggregate debt constraint at t_0. □

B.4 Proof of Proposition 5.1

Proof. The first statement directly follows from equations (19) and (20). Let $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exists $\boldsymbol{\lambda}$ such that $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). Define

$$
A_C = \sum_{i\in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_c^i, \quad A_L = \sum_{i\in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_l^i \tag{B.2}
$$

where ψ_c^i, ψ_l^i are defined by equations (12) using $\boldsymbol{\varphi}^*$. First, one can show that A_C and A_L are positive and bounded:

Lemma B.1. *Given an interior allocation,* $0 < A_C < \infty$ *and* $0 < A_L < \infty$

Proof. Interior allocation means that for any i, $c_t^i, l_t^i > 0$, $\forall t$. This implies that $\psi_c^i, \psi_l^i > 0$. By (12), $\boldsymbol{\varphi}^{*i} > 0$.

For all i, $\pi^i > 0, \lambda^i \geq 0$ and since $\sum_{i\in I} \pi^i \lambda^i = 1$, there exists at least an i such that $\lambda^i > 0$. Given that $\psi_c^i, \psi_l^i > 0$, $\forall i$, it must be that $A_C, A_L > 0$.

Since $\sum_{i\in I} \pi^i \boldsymbol{\varphi}^{*i} = 1 < \infty$ and $\forall i$, $\pi^i, \boldsymbol{\varphi}^{*i} > 0$, it must be that $\boldsymbol{\varphi}^{*i} < \infty$. So by definition, $\psi_c^i, \psi_l^i < \infty$. Moreover, $\boldsymbol{\varphi}^{*i} > 0$ implies that $\lambda^i / \boldsymbol{\varphi}^{*i} < \infty$. Then by definition, $A_C, A_L < \infty$. □

Define (P^M) the same problem as (P) with the restriction that $(C_t, L_t) = (C_t^*, L_t^*)$, $\forall t > M$, $\boldsymbol{\varphi} = \boldsymbol{\varphi}^*$, $T = T^*$, and $K_t = K_t^*, \forall t$. Note that $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}$ is a solution to (P^M), and (P^M) has a finite number of constraints. By a Lagrangian theorem in Luenberger (1969), there exists non-negative, not identically zero vector $\left\{r^M, \mu^M, \eta^{M,1}, \ldots, \eta^{M,I}, \gamma_0^M, \ldots, \gamma_M^M\right\}$ such that the first-order and complementarity conditions hold, i.e. $\forall t \geq 1$

$$\frac{\beta^t}{q_t}\left\{r^M A_C + \sum_i \pi^i \eta^{M,i}(1-\sigma)\psi_c^i + \sum_{s=0}^{t}\gamma_s^M A_C\right\}\Phi_C^V C_t^{-\sigma} = \mu^M \tag{B.3}$$

$$\frac{\beta^t}{q_t}\left\{r^M A_L + \sum_i \pi^i \eta^{M,i}(1+\nu)\psi_l^i + \sum_{s=0}^{t}\gamma_s^M A_L\right\}\Phi_L^V L_t^{\nu} = \mu^M F_L(K_t, L_t) \tag{B.4}$$

Since the allocation is interior and $A_C, A_L > 0$, one can rewrite the first-order conditions as

$$\frac{\beta^t}{q_t}\left\{r^M A_C + \sum_i \pi^i \eta^{M,i}(1-\sigma)\psi_c^i + \sum_{s=0}^{t}\gamma_s^M A_C\right\}\Phi_C^V C_t^{-\sigma} = \mu^M$$

$$\frac{\beta^t}{q_t}\left\{r^M A_C + \sum_i \pi^i \eta^{M,i}(1+\nu)\psi_l^i \frac{A_C}{A_L} + \sum_{s=0}^{t}\gamma_s^M A_C\right\}\Phi_C^V C_t^{-\sigma} = \mu^M \frac{A_C}{A_L} F_L(K_t, L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}}$$

Subtracting the first from the second line gives

$$\frac{\beta^t}{q_t}\left\{\Phi_C^V \sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right]\right\} C_t^{-\sigma} = \mu^M\left[\frac{A_C}{A_L}F_L(K_t, L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}} - 1\right] \tag{B.5}$$

The following lemma shows that the resource constraint binds for any sub-problem (P^M) and $M \geq 1$.

Lemma B.2. *In any sub-problem (P^M) with $M \geq 1$, $\mu^M > 0$*

Proof. Suppose, by contradiction, that $\mu^M = 0$ so the resource constraint does not bind. Consider allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ which is the solution to (P^M). Then there exists $\epsilon > 0$ such that

$$\sum_{t=0}^{\infty} q_t\left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 - \epsilon \geq 0$$

Define $\left\{\hat{L}_t\right\}_{t=0}^{\infty}$ where $\hat{L}_1 < L_1$ such that $F(K_1, \hat{L}_1) = F(K_1, L_1) - \epsilon/q_1$, and $\hat{L}_t = L_t,\ \forall t > 1$. The allocation $\left\{C_t, \hat{L}_t, K_{t+1}\right\}_{t=0}^{\infty}$ satisfies the resource constraint and because of the preference's strict monotonicity, $\left\{C_t, \hat{L}_t, K_{t+1}\right\}_{t=0}^{\infty}$ also satisfies the implementability constraints and the aggregate debt constraints. However,

$$\sum_{t=0}^{\infty}\sum_{i\in I}\beta^t \lambda^i \pi^i U^i\left[h^i(C_t, \hat{L}_t; \boldsymbol{\varphi})\right] > \sum_{t=0}^{\infty}\sum_{i\in I}\beta^t \lambda^i \pi^i U^i\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]$$

which contradicts $\{C_t, L_t, K_t\}_{t=0}^{\infty}$ being optimal solution for (P^M). $\square$

By Lemma B.2 and interior allocation, we can rewrite equation (B.5) as

$$\frac{\Phi_C^V}{\mu^M}\sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i-(1-\sigma)\psi_c^i\right]=\frac{q_t}{\beta^t}C_t^{\sigma}\left[\frac{A_C}{A_L}F_L(K_t,L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}}-1\right]$$

Specifically, for any $M \geq 1$,

$$\frac{\Phi_C^V}{\mu^M}\sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i-(1-\sigma)\psi_c^i\right]=\frac{q_1}{\beta}\left(C_1^*\right)^{\sigma}\left[\frac{A_C}{A_L}F_L(1)\frac{\Phi_C^V\left(C_1^*\right)^{-\sigma}}{\Phi_L^V\left(L_1^*\right)^{\nu}}-1\right]$$

Note that the left-hand side is a function of (C_1^*, L_1^*, K_1^*) , which implies that there exists a constant κ such that $\forall M \geq 1$,

$$\frac{\Phi_C^V}{\mu^M}\sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i-(1-\sigma)\psi_c^i\right]=\kappa$$

Hence, (B.5) can be rewritten as

$$\frac{\beta^t}{q_t}C_t^{-\sigma}\kappa=\frac{A_C}{A_L}F_L(K_t,L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}}-1$$

Note that $\lim_{t\to\infty}\beta^t/q_t=0$ and $C_t^{-\sigma}$ is bounded by Lemma 5.1, so taking the limit on both sides gives

$$\lim_{t\to\infty}\frac{A_C}{A_L}F_L(K_t,L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}}=1$$

Hence, given the definition of τ_t^n and the fact that A_C,A_L are bounded,

$$\lim_{t\to\infty}\tau_t^n=\lim_{t\to\infty}\left[1-\frac{\Phi_L^V L_t^{\nu}}{\Phi_C^V C_t^{-\sigma}}\frac{1}{F_L(K_t,L_t)}\right]=1-\frac{A_C}{A_L}$$

In addition, the above argument does not rely on the existence of lump-sum transfers. $\square$

B.5 Proof of Proposition 5.2

Proof. Rewrite the optimal labor tax formulas as

$$\tau_t^n=1-\frac{\Phi_L^V\Phi_C^W+\Phi_L^V\Phi_C^P\sum_{s=0}^t\gamma_s}{\Phi_C^V\Phi_L^W+\Phi_C^V\Phi_L^P\sum_{s=0}^t\gamma_s} \tag{B.6}$$

By definitions,

$$\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} = \frac{\sum_i \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]}{\sum_i \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right]} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \mathrm{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \mathrm{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

and

$$\frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \mathrm{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \mathrm{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

using the optimal conditions $\eta^i = \sum_j \pi^j \lambda^j / \varphi^j - \lambda^i / \varphi^i$, and the definitions $\mathbb{E}[x^i] \equiv \sum_i \pi^i x^i$, $\mathrm{cov}(x^i, y^i) \equiv \mathbb{E}[x^i y^i] - \mathbb{E}[x^i]\mathbb{E}[y^i]$.

Lemma B.3. *cov* $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ *and cov* $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$

Proof. Given that $a_0^i = A_0, \forall i \in I$, the individual implementability constraints can be rewritten as

$$\psi_c^i \Phi_C^V \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma} - \psi_l^i \Phi_L^V \sum_{t=0}^{\infty} \beta^t L_t^{1+\nu} = \Phi_C^V C_0^{-\sigma}(A_0 - T)$$

or

$$\psi_c^i = \psi_l^i \frac{\Phi_L^V \sum_{t=0}^{\infty} \beta^t L_t^{1+\nu}}{\Phi_C^V \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma}} + \frac{\Phi_C^V C_0^{-\sigma}(A_0 - T)}{\Phi_C^V \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma}}$$

which implies that $\psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$. By definition of ψ_c^i, $\varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$.

The next step is to show that $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j$.

Suppose $\theta^i \geq \theta^j$ and $\varphi^i < \varphi^j$, then $\psi_l^i < \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < \frac{\varphi^i}{\varphi^j} < 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq 1$, which is a contradiction.

Suppose $\varphi^i \geq \varphi^j$ and $\theta^i < \theta^j$, then $\psi_l^i \geq \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq \frac{\varphi^i}{\varphi^j} \geq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < 1$, which is a contradiction.

Thus, $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$. In addition, $\theta^i \geq \theta^j \iff \lambda^i \leq \lambda^j$, which implies that

$$\psi_c^i \geq \psi_c^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$
$$\psi_l^i \geq \psi_l^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$

Hence, $\mathrm{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ and $\mathrm{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$. $\square$

Lemma B.3 and $\sigma \geq 1, \nu > 0$ imply that $\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} \leq \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$. Suppose that the debt constraint binds at period t, then $\gamma_t > 0$, which leads to $\sum_{s=0}^{t} \gamma_s > \sum_{s=0}^{t-1} \gamma_s$. Applying equation (B.6) gives $\tau_t^n \leq \tau_{t-1}^n$. □

B.6 Proof of Proposition 6.2

Proof. $\lambda^i = \boldsymbol{\varphi}^{*i}$, $\forall i \in I$ implies that $A_C = 1$ and $A_L = 1$. Therefore, $\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ solves

$$
\begin{aligned}
\max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T} \quad & \sum_{t=0}^{\infty} \beta^t V(C_t, L_t; \boldsymbol{\varphi}) \\
s.t. \quad & \sum_{t=0}^{\infty} q_t \left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 \geq 0 \\
& \sum_{t=0}^{\infty} \beta^t \left[V_C(t; \boldsymbol{\varphi}) h^{i,c}(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) h^{i,l}(t; \boldsymbol{\varphi})\right] \geq V_C(0; \boldsymbol{\varphi}) \left(a_0^i - T\right) \\
& \sum_{s=t}^{\infty} \beta^{s-t} V(C_t, L_t; \boldsymbol{\varphi}) \geq \underline{U}_t(K_t)
\end{aligned}
$$

To implement $\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ given the specified tax system, by (21), it must be that

$$
\lim_{t \to \infty} \tau_t^n = 0
$$

□

B.7 Proof of Lemma 8.1

Proof. For any competitive equilibrium, there exists market weight $\boldsymbol{\varphi} = \{\varphi^i\}_{i \in I}$ such that $\forall t$, given C_t, L_t, individual assignment$\{c_t^i, l_t^i\}_{i \in I}$ solves

$$
\begin{aligned}
V(C_t, L_t; \boldsymbol{\varphi}) = \max_{(c^i, l^i)_{i \in I}} & \sum_i \varphi^i \pi^i \left[u(c^i) - v\left(l^i / \theta^i\right)\right] \\
s.t. \quad & \sum_i \pi^i c^i = C_t; \ \sum_i \pi^i l^i = L_t
\end{aligned}
$$

Let μ^m and η^m be the Lagrange multipliers on the consumption and labor constraints.

The first-order conditions for interior solutions are

$$c_t^i = u_c^{-1}\left(\mu^m/\varphi^i\right) \tag{B.7}$$
$$l_t^i = \theta^i v_l^{-1}\left(\theta^i\eta^m/\varphi^i\right) \tag{B.8}$$

Substituting for c_t^i and l_t^i in the constraints gives

$$\sum_{i\in I}\pi^i u_c^{-1}\left(\mu^m/\varphi^i\right) = C_t$$
$$\sum_{i\in I}\pi^i\theta^i v_l^{-1}\left(\theta^i\eta^m/\varphi^i\right) = L_t$$

These equations imply functions $\mu^m(C_t)$ and $\eta^m(L_t)$. Substituting in (B.7) and (B.8), for all i implies that

$$c_t^i = u_c^{-1}\left(\mu^m(C_t)/\varphi^i\right)$$
$$l_t^i = \theta^i v_l^{-1}\left(\theta^i\eta^m(L_t)/\varphi^i\right)$$

Thus, the time-invariant functions $h^{i,c}(\cdot;\boldsymbol{\varphi}), h^{i,l}(\cdot;\boldsymbol{\varphi})$ are

$$h^{i,c}(C_t;\boldsymbol{\varphi}) = u_c^{-1}\left(\mu^m(C_t)/\varphi^i\right)$$
$$h^{i,l}(L_t;\boldsymbol{\varphi}) = \theta^i v_l^{-1}\left(\theta^i\eta^m(L_t)/\varphi^i\right)$$

Note that $u_c(\cdot)$ is strictly decreasing, so $u_c^{-1}(\cdot)$ is strictly decreasing. This implies that $\mu^m(\cdot)$ is strictly decreasing. Then $h^{i,c}(\cdot;\boldsymbol{\varphi})$ must be strictly increasing. Similarly, one can argue that $h^{i,l}(\cdot;\boldsymbol{\varphi})$ is also strictly increasing. □

B.8 Proof of Lemma 8.2

Proof. Given an efficient allocation $\{C_t^*, L_t^*, K_t^*\}_{t=0}^\infty$, suppose that $\liminf_{t\to\infty} C_t^* \leq 0$. Find $\epsilon > 0$ such that $\forall t$,

$$\sum_{s=t}^\infty \beta^{s-t}\left\{\sum_{i\in I}\lambda^i\pi^i\left[u\left(h^{i,c}(C_s;\boldsymbol{\varphi})\right) - v\left(h^{i,l}(L_s^*;\boldsymbol{\varphi})\right)\right]\right\} \leq M_U$$

with $C_t = \epsilon$ and $C_s = C_s^*$, $\forall s \geq t$. Such ϵ exists since the utility function is unbounded. Furthermore, there exists t_0 such that $C_{t_0}^* < \epsilon$. Then since $u(\cdot)$ and $h^{i,c}(\cdot;\boldsymbol{\varphi})$ are strictly increasing,

$$
\begin{aligned}
&\sum_{s=t_0}^{\infty}\beta^{s-t_0}\left\{\sum_{i\in I}\lambda^i\pi^i\left[u\left(h^{i,c}(C_s^*;\boldsymbol{\varphi})\right)-v\left(h^{i,l}(L_s^*;\boldsymbol{\varphi})\right)\right]\right\}\\
<&\sum_{s=t_0}^{\infty}\beta^{s-t_0}\left\{\sum_{i\in I}\lambda^i\pi^i\left[u\left(h^{i,c}(C_s;\boldsymbol{\varphi})\right)-v\left(h^{i,l}(L_s^*;\boldsymbol{\varphi})\right)\right]\right\}\\
\leq& M_U\\
\leq& \underline{U}_t(K_t^*)
\end{aligned}
$$

which is a contradiction. $\square$

B.9 Proof of Proposition 8.1

Proof. The proof follows a similar structure of the proof of Proposition 5.1. Let $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$, $\boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exist $\boldsymbol{\lambda}$ such that $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$, $\boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). For any interior allocation $\{C_t, L_t, K_t\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T$ from problem (P), define the followings

$$
A_C(t) = \sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} \tag{B.9}
$$

$$
A_L(t) = \sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} \tag{B.10}
$$

Then the following lemma holds.

Lemma B.4. *Given an interior allocation, for all t, $0 < \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}, \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} < \infty$, and so $0 < A_C(t),\ A_L(t) < \infty$*

Proof. First, it must be that $\varphi^i > 0$, $\forall i$. Suppose there exists an i such that $\varphi^i = 0$. Then from the static sub-problem, it is optimal to set $c_t^i = 0$ for all t, which contradicts the assumption of interior allocation.

Note that from the proof of Lemma 8.1, using implicit function derivatives, one has

$$
\begin{aligned}
\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} &= \frac{\frac{1}{\varphi^i u_{cc}(h^{i,c}(t;\boldsymbol{\varphi}))}}{\sum_i \pi^i \frac{1}{\varphi^i u_{cc}(h^{i,c}(t;\boldsymbol{\varphi}))}}\\
\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} &= \frac{\frac{\theta^i}{\varphi^i v_{ll}(h^{i,l}(t;\boldsymbol{\varphi})/\theta^i)}}{\sum_i \pi^i \frac{\theta^i}{\varphi^i v_{ll}(h^{i,l}(t;\boldsymbol{\varphi})/\theta^i)}}
\end{aligned}
$$

Given $u_{cc}(\cdot) < 0$, $v_{ll}(\cdot) > 0$ by assumption 4, and $\varphi^i > 0$, $\forall i$, it must be that $\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}, \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} > 0$. Moreover, $\sum_{i\in I} \pi^i \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} = \sum_{i\in I} \pi^i \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} = 1 < \infty$ implies that $\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}, \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} < \infty$.

Since all the terms are positive and bounded, by definition, $A_C(t)$ and $A_L(t)$ are positive and bounded. □

Define (P^T) the same problem as (P) with the restriction that $(C_t, L_t) = (C_t^*, L_t^*)\,, \forall t > T$, $\boldsymbol{\varphi} = \boldsymbol{\varphi}^*$, $T = T^*$, and $K_t = K_t^*, \forall t$. Note that $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$ is a solution to (P^T), and (P^T) has a finite number of constraints. By a Lagrangian theorem in Luenberger (1969), there exists non-negative, not identically zero vector $\left\{r^T, \mu^T, \eta^{T,1}, \ldots, \eta^{T,I}, \gamma_0^T, \ldots, \gamma_T^T\right\}$ such that the first-order and complementarity conditions hold, i.e. $\forall t \geq 1$

$$\frac{\beta^t}{q_t}\left\{r^T A_C(t) + \sum_i \pi^i \eta^{T,i}\left[\frac{V_{CC}(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})} h^{i,c}(t;\boldsymbol{\varphi}) + \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}\right] + \sum_{s=0}^{t} \gamma_s^T A_C(t)\right\} * V_C(t;\boldsymbol{\varphi}) = \mu^T \tag{B.11}$$

$$\frac{\beta^t}{q_t}\left\{r^T A_L(t) + \sum_i \pi^i \eta^{T,i}\left[\frac{V_{LL}(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})} h^{i,l}(t;\boldsymbol{\varphi}) + \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}\right] + \sum_{s=0}^{t} \gamma_s^T A_L(t)\right\} * V_L(t;\boldsymbol{\varphi}) = -\mu^T F_L(K_t, L_t) \tag{B.12}$$

Using the Envelope conditions of the static sub-problem, one can show that

$$\begin{aligned}
\frac{V_{CC}(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})} h^{i,c}(t;\boldsymbol{\varphi}) &= \frac{u_{cc}\left[h^{i,c}(t;\boldsymbol{\varphi})\right]}{u_c\left[h^{i,c}(t;\boldsymbol{\varphi})\right]} h^{i,c}(t;\boldsymbol{\varphi}) \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} \\
\frac{V_{LL}(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})} h^{i,l}(t;\boldsymbol{\varphi}) &= \frac{v_{ll}\left[h^{i,l}(t;\boldsymbol{\varphi})\right]}{v_l\left[h^{i,l}(t;\boldsymbol{\varphi})\right]} h^{i,l}(t;\boldsymbol{\varphi}) \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}
\end{aligned}$$

Define $\sigma_t^i = -\frac{u_{cc}\left[h^{i,c}(t;\boldsymbol{\varphi})\right]}{u_c\left[h^{i,c}(t;\boldsymbol{\varphi})\right]} h^{i,c}(t;\boldsymbol{\varphi})$ and $\nu_t^i = \frac{v_{ll}\left[h^{i,l}(t;\boldsymbol{\varphi})\right]}{v_l\left[h^{i,l}(t;\boldsymbol{\varphi})\right]} h^{i,l}(t;\boldsymbol{\varphi})$, then equations (B.11) and (B.12) become

$$\frac{\beta^t}{q_t}\left\{r^T A_C(t) + \sum_i \pi^i \eta^{T,i} (1-\sigma_t^i) \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} + \sum_{s=0}^{t} \gamma_s^T A_C(t)\right\} V_C(t;\boldsymbol{\varphi}) = \mu^T \tag{B.13}$$

$$\frac{\beta^t}{q_t}\left\{r^T A_L(t) + \sum_i \pi^i \eta^{T,i} (1+\nu_t^i) \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} + \sum_{s=0}^{t} \gamma_s^T A_L(t)\right\} V_L(t;\boldsymbol{\varphi}) = -\mu^T F_L(K_t, L_t) \tag{B.14}$$

Since the allocation is interior and $A_C(t),\ A_L(t) > 0$ by Lemma B.4, one can combine equations (B.13) and (B.14) to get

$$\frac{\beta^t}{q_t}\left\{\sum_i \pi^i\eta^{T,i}(1-\sigma_t^i)\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}-\frac{A_C(t)}{A_L(t)}\sum_i \pi^i\eta^{T,i}(1+\nu_t^i)\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}\right\}V_C(t;\boldsymbol{\varphi}) \\ =\mu^T\left[1+F_L(K_t,L_t)\frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})}\frac{A_C(t)}{A_L(t)}\right] \tag{B.15}$$

Lemma B.5. *In any subproblem (P^T) with $T\geq 1$, $\mu^T>0$, i.e. the resource constraint binds.*

Proof. Follows directly from the proof of Lemma B.2. □

Given Lemma B.5 and interior allocation, (B.15) becomes

$$\frac{1}{\mu^T}\left\{\sum_i \pi^i\eta^{T,i}\left[(1-\sigma_t^i)\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}-\frac{A_C(t)}{A_L(t)}(1+\nu_t^i)\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}\right]\right\}V_C(t;\boldsymbol{\varphi}) \\ =\frac{q_t}{\beta^t}\frac{1}{V_C(t;\boldsymbol{\varphi})}\left[1+F_L(K_t,L_t)\frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})}\frac{A_C(t)}{A_L(t)}\right]$$

Define the left-hand side of the above equation as $\kappa(t)$, then the following lemma gives an important property of $\kappa(t)$.

Lemma B.6. *For any sub-problem (P^T) with $T\geq 1$,$\kappa(t)$ is bounded $\forall t\geq 1$.*

Proof. Note that $\forall t,\forall i$, by assumption 5, σ_t^i and ν_t^i are bounded.

Any sub-problem (P^T) with $T\geq 1$ has

$$\sum_i \frac{\eta^{T,i}}{\mu^T}\pi^i\left[(1-\sigma_s^i)\frac{\partial h^{*i,c}(s;\boldsymbol{\varphi})}{\partial C_s^*}-\frac{A_C^*(s)}{A_L^*(s)}(1+\nu_1^i)\frac{\partial h^{*i,l}(s;\boldsymbol{\varphi})}{\partial L_s^*}\right] \\ =\frac{q_s}{\beta^s}\frac{1}{V_C^*(s;\boldsymbol{\varphi})}\left[1+F_L^*(s)\frac{V_C^*(s;\boldsymbol{\varphi})}{V_L^*(s;\boldsymbol{\varphi})}\frac{A_C^*(s)}{A_L^*(s)}\right]$$

for $s=1,...,\|I\|$.

The above equations formulate a linear system with respect to $\|I\|$ variables $\left\{\frac{\eta^{T,i}}{\mu^T}\right\}_{i\in I}$. By Lemma B.4 and interior allocation, the right-hand sides and the coefficients are bounded. Therefore, for any T, $\left\{\frac{\eta^{T,i}}{\mu^T}\right\}_{i\in I}$ are functions of $\{C_s^*,L_s^*,K_s^*\}_{s=0}^{\|I\|}$, $\boldsymbol{\varphi}^*$and bounded.

So $\forall t\geq 1$,

$$\kappa(t)=\sum_i \frac{\eta^{T,i}}{\mu^T}\pi^i\left[(1-\sigma_t^i)\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}-\frac{A_C(t)}{A_L(t)}(1+\nu_1^i)\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}\right]$$

is bounded. □

Substituting for $\kappa(t)$ into equation (B.15) provides

$$\frac{\beta^t}{q_t}\kappa(t)V_C(t;\boldsymbol{\varphi}) = \left[1 + F_L(K_t, L_t)\frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})}\frac{A_C(t)}{A_L(t)}\right]$$

Assumption 1 implies that $\lim_{t\to\infty}\beta^t/q_t = 0$. By Lemma B.6, $\kappa(t)$ is bounded. Since $\liminf_{t\to\infty} C_t > 0$ from Lemma 8.2, $V_C(t;\boldsymbol{\varphi}) = \varphi^i u_c(h^{i,c}(t;\boldsymbol{\varphi}))$ is bounded. Then taking the limit as $t \to \infty$ on both sides of the above equation gives

$$\lim_{t\to\infty}\left[1 + F_L(K_t, L_t)\frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})}\frac{A_C(t)}{A_L(t)}\right] = 0$$

From Lemma B.4, it must be true that as t approaches infinity, we have that $0 < A_C(t), A_L(t) < \infty$, so $-\infty < \liminf_{t\to\infty}\frac{A_C(t)}{A_L(t)}$, $\limsup_{t\to\infty}\frac{A_C(t)}{A_L(t)} < \infty$. Define $\underline{\tau} = 1 - \limsup_{t\to\infty}\frac{A_C(t)}{A_L(t)}$ and $\bar{\tau} = 1 - \liminf_{t\to\infty}\frac{A_C(t)}{A_L(t)}$. Then using the definition of τ_t^n gives

$$\liminf_{t\to\infty}\tau_t^n = \liminf_{t\to\infty}\left[1 + \frac{1}{F_L(K_t, L_t)}\frac{V_L(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})}\right] = 1 - \limsup_{t\to\infty}\frac{A_C(t)}{A_L(t)} = \underline{\tau}$$
$$\limsup_{t\to\infty}\tau_t^n = \limsup_{t\to\infty}\left[1 + \frac{1}{F_L(K_t, L_t)}\frac{V_L(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})}\right] = 1 - \liminf_{t\to\infty}\frac{A_C(t)}{A_L(t)} = \bar{\tau}$$

In the case of steady states, it must be true that $A_C(\infty), A_L(\infty)$ exist and that $0 < A_C(\infty), A_L(\infty) < \infty$. Hence, $\lim_{t\to\infty}\tau_t^n = 1 - \frac{A_C(\infty)}{A_L(\infty)}$.

Similarly, the argument of the proof does not rely on lump-sum transfers. □

B.10 Proof of Lemma 7.1

Proof. Note that the sustainability constraint is rewritten as $\forall t$,

$$\sum_{s=t}^{\infty}\beta^{s-t}\left\{\Phi_C^P\frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P\frac{L_s^{1+\nu}}{1+\nu}\right\} \geq \underline{U}$$

Define $u_t = \Phi_C^P\frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P\frac{L_t^{1+\nu}}{1+\nu}$. Then the proof is similar to Lemma 2 in Aguiar and Amador (2016). □

C Numerical Appendix

This section explains the numerical algorithm that is implemented in Section 7 for a simple environment with no capital, and additional plots.

C.1 Deviation Utility

The deviation utility $\underline{U}$ is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes where the government does not issue external debt. Given that output is equal to the total effective labor supply, one has

$$\begin{aligned}
\underline{U} \equiv \max_{c_t^i, l_t^i, \tau_t^n, T_t} & \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i U^i(c_t^i, l_t^i) \\
s.t. \qquad & c_t^i + b_{t+1}^{i,d} = (1-\tau_t^n) l_t^i - T_t + (1+r_t) b_t^{i,d} \\
& G_t + (1+r_t) B_t^d \leq \tau_t^n L_t + T_t + B_{t+1}^d
\end{aligned}$$

There exist a vector of market weights $\hat{\boldsymbol{\varphi}}$ such that

$$\begin{aligned}
\underline{U} \equiv \max_{C_t, L_t, \hat{\boldsymbol{\varphi}}, T} & \sum_{t=0}^{\infty} \beta^t \left[\hat{\Phi}_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \hat{\Phi}_L^W \frac{L_t^{1+\nu}}{1+\nu} \right] \\
s.t. \qquad & C_t + G_t \leq L_t
\end{aligned}$$

where $\hat{\psi}_c^i, \hat{\psi}_l^i, \hat{\Phi}_C^V, \hat{\Phi}_L^V, \hat{\Phi}_C^W, \hat{\Phi}_L^W$ are calculated using $\hat{\boldsymbol{\varphi}}$.

C.2 Algorithm

State variables: μ, Γ

1. Guess μ and φ. Compute η.

   (a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ
   Initial guess of the expectation $V(\mu_t, \Gamma_{t-1}) = \sum_{s=t}^{\infty} \beta^{\tau-t} \left[\Phi_C^P \frac{{C_s}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{{L_s}^{1+\nu}}{1+\nu} \right]$.

   (b) Assume the constraint does not bind in t: $\gamma_t = 0$. Solve for the allocation C_t, L_t using FOCs

$$\begin{aligned}
\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1} \right] {C_t}^{-\sigma} &= \mu \\
\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1} \right] {L_t}^{\nu} &= \mu
\end{aligned}$$

   (c) Compute $V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$, then compute

$$\begin{aligned}
A_t &= \sum_{s=t}^{\infty} \beta^{\tau-t} \left[\Phi_C^P \frac{{C_s}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{{L_s}^{1+\nu}}{1+\nu} \right] \\
&= \Phi_C^P \frac{{C_t}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{{L_t}^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1}, \Gamma_t)
\end{aligned}$$

(d) Check if $A_t \geq \underline{U}_t$. If it is, proceed to the next step. If not, solve for C_t, L_t, γ_t using these optimality equations

$$\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1}\right] C_t{}^{-\sigma} = \mu$$
$$\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1}\right] L_t{}^{\nu} = \mu$$
$$\Phi_C^P \frac{C_t{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t{}^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^*(\Gamma_{t-1} + \gamma_t)) = \underline{U}_t$$

(e) Given C_t, L_t, γ_t (γ_t can be zero or not), compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^*(\Gamma_{t-1} + \gamma_t))$. Update the value function

$$V^{n+1}(s_t, \Gamma_{t-1}) = \Phi_C^P \frac{C_t{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t{}^{1+\nu}}{1+\nu} + \beta V^n(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^*(\Gamma_{t-1} + \gamma_t))$$

2. Compute residuals to find μ and φ

$$r^\mu = \sum_{t=0}^{\infty} q_t \left[L_t - G_t - C_t\right] - B_0$$
$$r_{ij}^{\varphi} = \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^V \left(\psi_c^i - \psi_c^j\right) C_t{}^{1-\sigma} - \Phi_L^V \left(\psi_l^i - \psi_l^j\right) L_t{}^{1+\nu}\right]$$

$$r = (r^\mu)^2 + \sum_{i,j} (r_{ij}^{\varphi})^2 \tag{B.16}$$

3. Find μ and $\boldsymbol{\varphi}$ such that (B.16) is minimized using a Nelder-Mead algorithm.

C.3 Additional figures

Figure 6: Time paths of economic aggregates when $\theta^H = 2\theta^L$