

# Online Appendix

## Optimal Redistributive Policy in Debt Constrained Economies

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### A Sovereign game

Before setting up the game, consider the general environment in which the government's policy includes the decision to default on debt  $\{d_t\}_{t=0}^{\infty}$ , where  $d_t \in \{0, 1\}$  and  $d_t = 0$  implies default.<sup>1</sup> The government's budget constraint becomes

$$G_t + (1 + r_t)B_t^g \leq \tau_t^n w_t L_t + \tau_t^k r_t K_t + \tau_t^d \left( r_t A_t + \left[ (1 - \tau_t^k) r_t^k - \delta \right] K_t \right) + d_t B_{t+1}^g + T_t.$$

As the government cannot commit to any of its policies, one can think that the government, domestic agents, and international lenders enter into a sovereign game in which they determine their actions sequentially. In every period, the state variable for the game is  $\left\{ B_t^g, (k_t^i, a_t^i)_{i \in I} \right\}$ . The timing of the actions is as follows:

- Government chooses  $z_t^G = (\tau_t^n, \tau_t^d, \tau_t^k, T_t, d_t, B_{t+1}^g) \in \Pi$  such that it is consistent with the government budget constraint.
- Agents choose allocation  $z_t^{H,i} = (c_t^i, l_t^i, k_{t+1}^i, a_{t+1}^i)$  subject to their budget constraints, the representative firm produces output by choosing  $z_t^F = (K_t, L_t)$ , and the international lenders choose holdings of bonds  $z_t^* = (B_{t+1}^g, A_{t+1})$  given the interest rates  $r_t^*$ .

Define  $h^t = \left( h^{t-1}, z_t^G, \left( z_t^{H,i} \right)_{i \in I}, z_t^F, z_t^*, p \right) \in H^t$  as the history at the end of period  $t$ . Note that the history incorporates the government's policy, allocation, and prices. Define  $h_p^t = (h^{t-1}, z_t^G) \in H_p^t$  as the history after the government announce its policies at period  $t$ . The government strategy

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<sup>1</sup>Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its international debt ( $d_t = 1$ ).

is  $\sigma_t^G : H^{t-1} \rightarrow \Pi$ . The individual agent's strategy is  $\sigma_t^{H,i} : H_p^t \rightarrow \mathbb{R}_+^3 \times \mathbb{R}$ . The firm has the strategy  $\sigma_t^F : H_p^t \rightarrow \mathbb{R}_+^2$ , and the international lenders have the strategy  $\sigma_t^* : H_p^t \rightarrow \mathbb{R}_+^2$ . Prices are determined by the pricing rule:  $p : H_p^t \rightarrow \mathbb{R}_+$ .

**Definition 1** (Sustainable equilibrium). A sustainable equilibrium is  $(\sigma^G, \sigma^H, \sigma^F, \sigma^*)$  such that (i) for all  $h^{t-1}$ , the policy  $z_t^G$  induced by the government strategy maximizes the weighted utility by  $\lambda$  subject to the government's budget constraint; (ii) for all  $h_p^t$ , the strategy induced policy  $\{z_t^G\}_{t=0}^\infty$ , allocation  $\{z_t^{H,i}, z_t^F, z_t^*\}_{t=0}^\infty$ , and prices  $\{r_t^*\}_{t=0}^\infty$  constitute a competitive equilibrium with taxes.

The following focuses on characterizing a set of sustainable equilibria in which deviation triggers the worst payoff. In this case, the value of deviation is the worst equilibrium payoff.

**Proposition 1** (Sustainable equilibrium). *An allocation and policy  $\{(z^{H,i})_{i \in I}, z^F, z^G\}$  can be part of sustainable equilibrium if and only if (i) given  $z^G$ , there exist prices  $p$  such that  $\{(z^{H,i})_{i \in I}, z^F, z^G, p\}$  is a competitive equilibrium with taxes for an open economy; and (ii) for any  $t$ , there exists  $\underline{U}_t(\cdot)$  such that  $\{(z^{H,i})_{i \in I}, z^F, z^G\}$  satisfies the constraint*

$$\sum_{s=t}^{\infty} \beta^{s-t} \sum_{i \in I} \lambda^i \pi^i U^i(c_s^i, l_s^i) \geq \underline{U}_t(K_t).$$

*Proof.* Define  $\underline{U}_t(K_t)$  as the maximum discounted weighted utility for the domestic agents in period  $t$  when the government deviates. In period  $t$ , the domestic agents and the government can borrow abroad. In subsequent period  $s > t$ , the economy reverts to the worst equilibrium.

Suppose  $\{(z^{H,i})_{i \in I}, z^F, z^G\}$  is an outcome of the sustainable equilibrium. Then by the optimal problems of the government, domestic agents, and international lenders,  $\{(z^{H,i})_{i \in I}, z^F, z^G\}$  maximizes the weighted utility of the agents, satisfies the government budget constraint, and satisfies the international lender's problem at period 0. Thus,  $\{(z^{H,i})_{i \in I}, z^F, z^G\}$  is a competitive equilibrium with policies. For any period  $t$  and history  $h^{t-1}$ , an equilibrium strategy that has the government deviate in period  $t$  triggers reverting to the worst equilibrium in period  $s > t$ . Such a strategy must deliver a weighted utility value that is at least as high as the right-hand side of (1). So  $\{(z^{H,i})_{i \in I}, z^F, z^G\}$  satisfies condition (ii).

Next, suppose  $\{(z^{H,i})_{i \in I}, z^F, z^G\}$  satisfies conditions (i) and (ii). Let  $h^{t-1}$  be any history such that there is no deviation from  $z^G$  up until period  $t$ . Since  $\{(z^{H,i})_{i \in I}, z^F, z^G\}$  maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period  $t$  onward. Consider a deviation plan  $\hat{\sigma}^G$  at period  $t$  that receives  $U_t^d(K_t)$  in period  $t$  and  $U^{aut}$  for period  $s > t$ . Because the plan is constructed to maximize period- $t$  utility at  $K_t$ , the right-hand side of (1) is the maximum attainable utility under  $\hat{\sigma}^G$ . Given that  $\{(z^{H,i})_{i \in I}, z^F, z^G\}$  satisfies condition (ii), the original no-deviation plan is optimal.  $\square$

## B Numerical appendix

This section explains the calculation of deviation utility and the computational algorithm solving the model.

### B.1 Deviation utility

The deviation utility  $\underline{U}$  is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes where there is no access to international financial markets. Given that output is equal to the total efficiency-unit labor supply, one has

$$\begin{aligned} \underline{U} &\equiv \max_{c_t^i, l_t^i, \tau_t^n, T_t} \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i U^i(c_t^i, l_t^i) \\ s.t. \quad &c_t^i + b_{t+1}^{i,d} = (1 - \tau_t^n) l_t^i - T_t + (1 + r_t) b_t^{i,d} \\ &G_t + (1 + r_t) B_t^d \leq \tau_t^n L_t + T_t + B_{t+1}^d \end{aligned}$$

There exist a vector of market weights  $\hat{\varphi}$  such that

$$\begin{aligned} \underline{U} &\equiv \max_{C_t, L_t, \hat{\varphi}, T} \sum_{t=0}^{\infty} \beta^t \left[ \hat{\Phi}_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \hat{\Phi}_L^W \frac{L_t^{1+\nu}}{1+\nu} \right] \\ s.t. \quad &C_t + G_t \leq L_t \end{aligned}$$

where  $\hat{\psi}_c^i, \hat{\psi}_l^i, \hat{\Phi}_C^V, \hat{\Phi}_L^V, \hat{\Phi}_C^W, \hat{\Phi}_L^W$  are calculated using  $\hat{\varphi}$ .

### B.2 Computational algorithm

State variables:  $\mu, \Gamma$

1. Guess  $\mu$  and  $\varphi$ . Compute  $\eta$ .
  - (a) Construct a grid for  $\mu_t = (\beta R^*)^t$  for  $t$  periods. Construct a grid for  $\Gamma$   
Initial guess of the expectation  $V(\mu_t, \Gamma_{t-1}) = \sum_{s=t}^{\infty} \beta^{\tau-t} \left[ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s^{1+\nu}}{1+\nu} \right]$ .
  - (b) Assume the constraint does not bind in  $t$ :  $\gamma_t = 0$ . Solve for the allocation  $C_t, L_t$  using the following first-order conditions

$$\begin{aligned} [\mu_t \Phi_C^W + \Phi_C^m \Gamma_{t-1}] C_t^{-\sigma} &= \mu \\ [\mu_t \Phi_L^W + \Phi_L^m \Gamma_{t-1}] L_t^{\nu} &= \mu \end{aligned}$$

(c) Compute  $V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$ , then compute

$$\begin{aligned} A_t &= \sum_{s=t}^{\infty} \beta^{\tau-t} \left[ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s^{1+\nu}}{1+\nu} \right] \\ &= \Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1}, \Gamma_t) \end{aligned}$$

(d) Check if  $A_t \geq \underline{U}_t$ . If it is, proceed to the next step. If not, solve for  $C_t, L_t, \gamma_t$  using these optimality equations

$$\begin{aligned} [\mu_t \Phi_C^W + \Phi_C^m \Gamma_{t-1}] C_t^{-\sigma} &= \mu \\ [\mu_t \Phi_L^W + \Phi_L^m \Gamma_{t-1}] L_t^\nu &= \mu \\ \Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* (\Gamma_{t-1} + \gamma_t)) &= \underline{U}_t \end{aligned}$$

(e) Given  $C_t, L_t, \gamma_t$  ( $\gamma_t$  can be zero or not), compute  $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* (\Gamma_{t-1} + \gamma_t))$ . Update the value function

$$V^{n+1}(s_t, \Gamma_{t-1}) = \Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} + \beta V^n(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* (\Gamma_{t-1} + \gamma_t))$$

2. Compute residuals to find  $\mu$  and  $\varphi$

$$\begin{aligned} r^\mu &= \sum_{t=0}^{\infty} q_t [L_t - G_t - C_t] - B_0 \\ r_{ij}^\varphi &= \sum_{t=0}^{\infty} \beta^t \left[ \Phi_C^m (\psi_c^i - \psi_c^j) C_t^{1-\sigma} - \Phi_L^m (\psi_l^i - \psi_l^j) L_t^{1+\nu} \right] \end{aligned}$$

$$r = (r^\mu)^2 + \sum_{i,j} (r_{ij}^\varphi)^2 \tag{B.1}$$

3. Find  $\mu$  and  $\varphi$  such that  $r$  is minimized using a Nelder-Mead algorithm.