Redistribution, Sovereign Debt, and Optimal Taxation

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Abstract

This paper examines the interaction between a country's concern for redistribution and its external indebtedness. I document a positive correlation between income inequality and external debt across countries and time periods. I propose a model of small open economy where the government has a redistributive motive and lacks commitment, and taxes are distortionary. Domestic and external credit markets are state-contingent, and the government faces endogenous borrowing constraints. Theoretically, when borrowing constraints bind, the government increases its ability to repay debt by lowering labor taxes and levying taxes on domestic borrowing. Default is endogenously costly because the redistribution requires high and volatile labor distortions in financial autarky. I calibrate the model using Italy's data and show that the model accounts for (i) the average level and volatility of Italy's external debt-to-output ratio, (ii) the cross-sectional positive correlation between pre-tax income inequality and external debt, and (iii) the increase in external debt-to-output following an increase in Italy's income inequality from 1985-2001 to 2002-2015. Following a negative productivity shock, the optimal austerity policies are increasing external debt and redistribution, while temporarily decreasing average tax rates. These responses are smaller in magnitude for a higher level of wage inequality.

Keywords: Redistribution; Inequality; Sovereign debt; Optimal taxation; Limited commitment

JEL Classifications: F34; F38; H21; H23; H63

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1 Introduction

In the recent European debt crises, there have been intense policy debates on the design of fiscal policies during severe economic downturns. Austerity policies such as increasing tax revenue or reducing government's expenditure help reduce debt levels, but have unequal consequences across domestic residents[1]. Such policy designs often require a thorough understanding about the interaction between a government's concern for redistribution and its commitment to debt repayment.

This paper aims to fill in the knowledge gap by answering three key questions: first, how does limited borrowing affect a government's ability to redistribute? Second, how does a government's redistributive goals affect its incentive to repay debt? Third, given the answers to the first two questions, how does one design optimal austerity policies taking into account their distributional consequences?

The answers come from the theoretical and quantitative analysis of a small open economy where the government has a redistributive motive, taxes are distortionary, and the government lacks commitment and faces endogenous borrowing constraints. I theoretically characterize optimal taxes in the presence of borrowing constraints and show that when these constraints bind, the government's redistribution relies less on labor taxes and more on claims and taxes from the domestic credit market. I calibrate the model to Italy's data and argue that the government's concern for redistribution provides a strong incentive to sustain debt. This mechanism also accounts for the positive relationship between income inequality and external debt in the data. The optimal austerity policies following a negative productivity shock are increasing external debt, decreasing average tax rates, and increasing redistribution.

The paper first documents the empirical properties of income inequality and external debt by using two multi-country panel data sets on inequality and balance of payments. I estimate that a high pre-tax income inequality is correlated with a high external debt-to-output both across countries and over time. This relationship still holds after controlling for output levels and output growth[2].

[1]The United Kingdom and Ireland implemented expenditure cuts, while Greece, Italy, Portugal, and Spain implemented both tax increases and expenditure cuts for their austerity plans. Most of these plans include cuts in public services, pension, and education programs. Monastiriotis (2011) argued that the Greek prolonged fiscal consolidation exacerbated the regional disparities and imbalances. Leventi and Matsaganis (2016) used micro-simulation model to assess distributional effects of austerity policies and found that such policies have led to higher poverty and after-tax income inequality, worsening the adverse distributional effects of the recession.

[2]In the later part, I provide an overview of other papers that document similar and related empirical patterns, including Berg and Sachs (1988), Aizenman and Jinjarak (2012),Jeon et al. (2014), and Ferriere (2015) using different measures and estimation techniques.

Given the empirical patterns, I construct a model where domestic agents are impatient and differ by labor productivity types. The aggregate shocks are in terms of aggregate productivity and government spending[3]. Both domestic and external credit markets are state-contingent. Domestic agents only have access to the domestic credit market, while the government has access to both markets[4]. Tax policies include marginal taxes on labor, marginal taxes on domestic saving, and lump-sum taxes that do not depend on individual income levels. The government lacks commitment in all fiscal policies and cares about the social welfare of domestic agents, which captures its redistributive preference.

Concerns for redistribution rationalize the need for distortionary taxation. Since all domestic agents face the same tax rates, the government redistributes resources by levying a positive labor tax and a lump-sum transfer. In this way, highly-skilled, high-income agents bear a larger tax burden[5] than lowly-skilled low-income ones. Alternatively, the government can use a tax on domestic borrowing and a lump-sum transfer, which implies that the highly-indebted agents will pay more taxes than the lowly-indebted agents. In this environment, levels of tax distortions represent the equilibrium cost of redistribution[6].

The government's lack of commitment imposes endogenous limits on the economy's external borrowing. In every period, the government cannot commit to future choices on repayments of debt and taxes. The policies are determined in a repeated game between the government, domestic agents, and international creditors. The contract is an ex-ante set of policies such that if the government deviates from any policies, it triggers punishment to financial autarky, in which there is permanent exclusions from domestic and external credit markets. For example, even if the government only defaults on external debt, it is still excluded from both domestic and external lending markets[7]. The subgame perfect equilibrium of the game is characterized by self-enforcing constraints, where the continuation value of staying in the contract has to be at least the value of financial autarky. These constraints act as borrowing constraints.

The impatience of the domestic agents means that they would want to borrow. The do-

[3]Later on when I match the model to the data, the government spending is measured as the government's final consumption on public goods, etc., excluding spending on social welfare programs.

[4]This assumption on market structure does not affect the main results because the government can control the after-tax return on domestic savings. Allowing domestic agents to access to the external credit market only changes the interpretation of external debt.

[5]In terms of levels and not percentage

[6]This is because the government can raise lump-sum taxes to finance expenditures and debt repayment without distorting the domestic agents' decisions. Werning (2007) provided a similar intuition. The presence of lump-sum taxes removes the revenue purposes of distortionary taxation.

[7]This is equivalent to the assumption of nondiscriminatory defaults on domestic or external lenders. See, for example, D'Erasmo and Mendoza (2016) for a similar assumption. For an example on discriminatory defaults, see Gonzalez-Aguado (2018).

mestic need for borrowing leads to the country run up debt and eventually hit the borrowing constraints. However, an infinitesimal domestic agent does not internalize the fact that as she borrows more, the borrowing constraints get tightened. When the borrowing constraint binds, the government can use the tax on domestic borrowing to make the domestic agent's inter-temporal substitution constraint take into account the additional cost of debt.

This framework builds on state-contingent financial markets and features no equilibrium default, in contrast to the sovereign default model with incomplete market and equilibrium default[8]. However, the threat of default still affects the optimal allocation in the similar fashion as in Thomas and Worrall (1988) and Kehoe and Levine (1993). The endogenous borrowing constraints imply that there is imperfect insurance against the aggregate risk. These constraints also endogenously determine the optimal portfolio of debt, in contrast to the incomplete framework where there is only the risk-free bond. Furthermore, defaults in reality often accompany with a non-zero net capital flow as a country often goes through a lengthy process of renegotiation and haircuts[9]. This framework embeds part of the default procedure into self-enforcing borrowing constraints, instead of assuming zero net capital flows as in the standard sovereign default model[10].

The main theoretical result establishes how optimal taxation dynamically respond to borrowing constraints[11]. Intuitively, without borrowing constraints, it is optimal to redistribute by distorting intratemporal margins (labor taxes). When the borrowing constraints do not bind, labor taxes do not change, and domestic borrowing taxes are zero. The optimal labor taxes balance the marginal benefit of redistribution and the marginal cost of distortion. Since borrowing is not costly, the government can use debt to smooth out the labor distortions[12]Nevertheless, when the borrowing constraints bind, all future labor taxes weakly decrease, and domestic borrowing taxes are positive. . When borrowing is limited, distorting intertemporal margins (borrowing taxes) become beneficial because it aligns the intertemporal interests of the government and the domestic agents. It turns out that these borrowing taxes are beneficial in redistribution. Therefore, when facing borrowing constraints, the government can maintain the same redistribution as before by using more borrowing taxes and

[8] Restrepo-Echavarria (2019) also provides an excellent discussion on these issues

[9] Standard & Poor's defines default as the failure to meet a principal or interest payment on the due date contained in the original terms of a debt issue. This definition covers both missed payments (breach of contract) and distressed debt restructurings that involve losses for creditors. This is the standard default definition used in the literature (e.g. Reinhart and Rogoff (2009)). Ams et al. (2018) provides a discussion of existing definitions of default and provides a new measure.

[10] See, for example, Eaton and Gersovitz (1981)

[11] See Tran Xuan (2019) for more results on optimal taxation in the deterministic case.

[12] See Lucas and Stokey (1983) and Werning (2007) for more details on the argument for labor tax smoothing. Both frameworks have state-contingent asset markets but are for closed economy and do not have the borrowing constraints.

less labor taxes. The declines in labor distortions improve the economy's efficiency and allow the government to sustain its external debt.

The government's concern for redistribution affects its repayment incentive via changing the cost of default. This endogenous component of the cost of default is novel to the literature. The main idea is that, without access to financial markets, redistribution becomes costly. In the contract, labor taxes decreases over time as borrowing constraints bind, which implies that the government in the long run uses claims and taxes from the domestic credit markets to redistribute and to finance interest payments on debt. Default makes this redistributive tool no longer available, and so the government needs to levy a high labor distortion to redistribute. Moreover, in financial autarky, the government cannot use debt to smooth out fluctuations in labor distortion. Therefore, the labor distortion in autarky is more volatile than the labor distortion of the contract in the long run.

I quantify the effect of the government's concern for redistribution on its sovereign debt policies in the case of Italy for the period of 1985-2015. I calibrate the model to match key macroeconomic statistics and the average cross-sectional wage inequality and show that the model accounts for the average level and volatility of external debt-to-output ratio in Italy without generating any counterfactual business cycle statistics.

In a cross-sectional simulation, I show that the model produces a positive association between pre-tax income inequality and external debt across countries. Specifically, I perform a regression analysis on a sample of simulated economies differentiated only by wage inequalities and find a statistically significant and positive effect of the pre-tax income inequality on the external debt-to-output. A counterfactual exercise for Italy in the periods of 1985-2001 and 2002-2015 shows that an increase in the underlying wage inequality that matches the increase in income inequality can account for 93% of the increase in the average debt-to-output ratio. The theory predicts that a higher inequality, or a higher redistributive motive, corresponds to a higher cost of default. It is then optimal for the government to sustain a higher debt level.

Following a negative innovation of the productivity shock, external debt increases, labor taxes remain unchanged, and borrowing taxes decrease initially and increase afterwards. In addition, I show responses of two different measures of redistribution in response to shock: average tax-to-income ratios and the variance of log utilities across individuals. A negative productivity shock leads to initial decreases in average tax rates among agents, with a higher reduction for high-income agents, and decreases in the variance of log utilities. More borrowing allows the government to reduce average taxes and provides more redistribution. In addition, for a higher level of inequality, responses to a negative shock is smaller in external debt is smaller and larger in utility inequality and average taxes. Additional unit of external borrowing allows for more redistribution when inequality is higher.

The paper then discusses the roles of aggregate uncertainty, heterogeneity, distortionary taxation, and government spending. Aggregate uncertainty implies an insurance cost of default, which is quantitatively small and so does not significantly change the optimal level of debt. Wage dispersion determines the optimal level of redistribution and so the optimal level of external debt. When the government can use fully income-dependent lump-sum transfer to redistribute, there is no need for distortionary taxation, and so financial autarky is not distributively costly. In this case, the model cannot sustain a high debt level. In addition, the government spending does not significantly affect the optimal external debt.

Related literature. This paper contributes to the sovereign debt literature. Seminal papers study sovereign debt in the limited commitment environment include Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), Aguiar and Amador (2011), and Aguiar and Amador (2014). There are a few recent works on optimal policies in the Eaton-Gersovitz framework in which the government sometimes defaults in equilibrium and the spread of the sovereign debt depends on the probability of default. Cuadra et al. (2010) show how the sovereign default model of incomplete market and endogenous fiscal policy generates optimal pro-cyclical policy. I introduce distributive effect of fiscal policies in a complete market framework and finds that the distortionary labor tax drifts down over time as the borrowing constraint is tightened. Pouzo and Presno (2015) study the optimal fiscal policies in a representative-agent closed economy with distortionary taxes and defaultable government bonds. One of their main results that is similar to this paper is that the government's lack of commitment hinders the ability to use debt to smooth taxes. Arellano and Bai (2016) quantitatively find that in the case of the government committing to the tax policy, higher tax distortion would have made the country more likely to default, and hence it is not optimal. I find that without commitment to the tax policy, the government finds it optimal to reduce the tax distortion when the borrowing constraint binds. Karantounias (2018) studies optimal time-consistent tax policy in a representative-agent, closed economy with defaultable domestic government bond and shows how the two incentives of "greed" and "fear" determine the optimal back-loading or front-loading tax distortions. In this paper, with the interest rate is exogenously given and the binding borrowing constraints, the optimal policy is to front load the tax distortion.

The framework of limited commitment has been explored by Kehoe and Perri (2002) with two-country international real business cycle and Bauducco and Caprioli (2014) with optimal fiscal policy. I add heterogeneous agents to this framework and study the implication on debt sustainability.

Endogenous cost of default beyond insurance motive has been explored by Mendoza and Yue (2012), which is the efficiency loss in production as default prevents the final good producers to finance the purchase of imports, which only have imperfect substitutes at home. Balke (2017) shows how default limits the supply of bank's loans that firms use to finance vacancies and wages. Therefore, default leads to a large increase in unemployment, which is endogenously costly. In this paper, distortionary taxation plays an important role in determining the cost of default. Default is costly because of the government cannot redistribute as much, and that they do not have access to the inter-temporal tax on labor to mitigate the labor distortion.

This paper finds optimal policy by characterizing the best allocation of any tax-distorted equilibrium, i.e. the primal approach as in the public finance literature (Aiyagari and McGrattan (1998), Aiyagari et al. (2002), Barro (1979) Chari et al. (1994), Chari and Kehoe (1999),Lucas and Stokey (1983), and many other papers). The argument for labor tax smoothing in these papers relies on the fact that the government can issue debt that is contingent to all states and is not constrained (in a sense of beyond the natural debt limit). In this paper, tax smoothing is not always optimal; the government's ability to smooth tax distortion is restricted by the willingness to lend by the international lenders.

Bhandari et al. (2016),Bhandari et al. (2017) and Werning (2007) study optimal taxation with heterogeneity and find that redistribution has significant impact on optimal policies. This paper relaxes their assumption on the government's commitment to policies. Bhandari et al. (2016) establishes the existence of the ergodic distribution of debt in the long run and finds that the average long-run on optimal public debt is not positive. In contrast, this paper shows the quantitatively high cost of default of redistribution and the high positive debt level in the long run. Bhandari et al. (2017) emphasizes the impact of the distribution of initial asset holdings on optimal allocation. In this paper, the initial distribution of after-tax net asset holdings matters in determining the equilibrium redistribution across agents, which indirectly affect the long-run repaying capacity. Werning (2007) develops the conditions for perfect tax smoothing under heterogeneity. Here, I establish that labor tax smoothing only occurs when the borrowing constraint does not bind. Long-run binding borrowing constraints then alters the dynamic of the labor tax, resulting in imperfect tax smoothing.

Several recent papers addressed the trade-off between redistribution and external debt. Ferriere (2015) shows that committing (one-period ahead) to the tax progressivity reduces the incentive for the government to default. In this paper, I extend the lack of commitment to both tax and debt policies and finds a related result that a more progressive economy finds a higher cost of default since redistribution is more distortionary in autarky. D'Erasmo and Mendoza (2016) focus on how redistribution incentives affected defaults on domestic debt.

They asserted that equilibrium with debt could be supported only when the government was politically biased towards bond holders. Dovis et al. (2016) argues how the interaction between inequality and debt endogenized the dynamic cycles of debt, taxes, and transfers over time. Balke and Ravn (2016) studies time-consistent fiscal policy in a sovereign debt model à la Eaton and Gersovitz (1981) with inequality through unemployment. They find that austerity policies are optimal during debt crises since they reduce the default premium, which was correlated with debt issuance, and increased access to international lending market. This paper instead emphasizes the endogenous borrowing constraints arising from a self-enforcing contracting problem among the government, international lenders, and domestic agents. The paper features the endogenous binding debt constraints, in which austerity policies might not be optimal if they generate more distortion. The main result is that taking into account the distributitive and distortionary effect of fiscal policies, the model can quantitatively sustain a high level of debt.

Other papers have documented the positive relationship between the income dispersion and sovereign debt. Berg and Sachs (1988) shows that income dispersion was a key predictor of a country's probability of rescheduling debt and the bond spread in secondary markets. Aizenman and Jinjarak (2012) describes a negative correlation between income dispersion and the tax base and a positive correlation with sovereign debt. Jeon et al. (2014) and Ferriere (2015) also provide evidence of rising income dispersion significantly increases sovereign default risk. This paper extends the panel analysis to a more recent data set on income inequalityprovides a theory that can account for the increase in debt and income dispersion together.

Outline. The paper is organized as follows. Section 2 documents the relationship between income inequality and external debt. Section 3 describes the environment and sets up the competitive equilibrium. Section 4 formulates the planning problem and the main theoretical results. Section 5 provides the quantitative analysis and analyzes the optimal austerity. Section 6 discusses the assumptions and robustness. Section 7 then concludes.

2 Empirical Motivation

This section presents the empirical relationship between income inequality and external debt. I document that income inequality is positively correlated with external debt in both the cross section and time series. To measure a country's external indebtedness, I use the negative of net foreign asset-to-GDP[13] from the External Wealth of Nations Database of Lane and Milesi-Ferretti (2018). The database contains data on foreign assets and foreign liabilities for a large sample of countries for the period 1970-2015. For income inequality, I use pre-tax (market) Gini indices from Standardized World Income Inequality Database (SWIID) of Solt (2019). This database contains 169 countries and covers from 1960 to 2018.

2.1 Cross-Sectional Properties

In this subsection, I show the cross-sectional properties in a selected sample of advanced and emerging market economies[14].

Figure 1 plots averages across the period of 1985-2015 of net foreign liability-to-GDP, pre-tax Gini index, and log GDP per capita in constant 2010 US Dollars. Panel (a) establishes a positive relationship between income inequality and external debt across countries. Panel (b) and (c) show the relationship of net-foreign liability-to-GDP and pre-tax Gini index with respect to GDP per capita. Countries with high GDP per capita tend to have lower net foreign liability and lower pre-tax income inequality.

The levels of GDP per capita and GDP growth can account for the observed levels in both income inequality and external debt across countries. However, the following exercises show that the positive correlation is robust such factors. Increases in the pre-tax Gini index can account for increases in net foreign liability-to-GDP, excluding from changes in GDP per capita and GDP growth rates. The exercises are as follows. For a given country in the sample, I calculate average statistics across the time period of 1985-2015. I consider two regressions of net foreign liability-to-GDP and pre-tax Gini index on log GDP per capita and GDP growth rates:

$$\text{Net foreign liability-to-GDP}_i = \beta_0 + \beta_1 \text{GDP per capita}_i + \beta_3 \text{GDP growth}_i + \epsilon_i^{nfl} \quad (1)$$

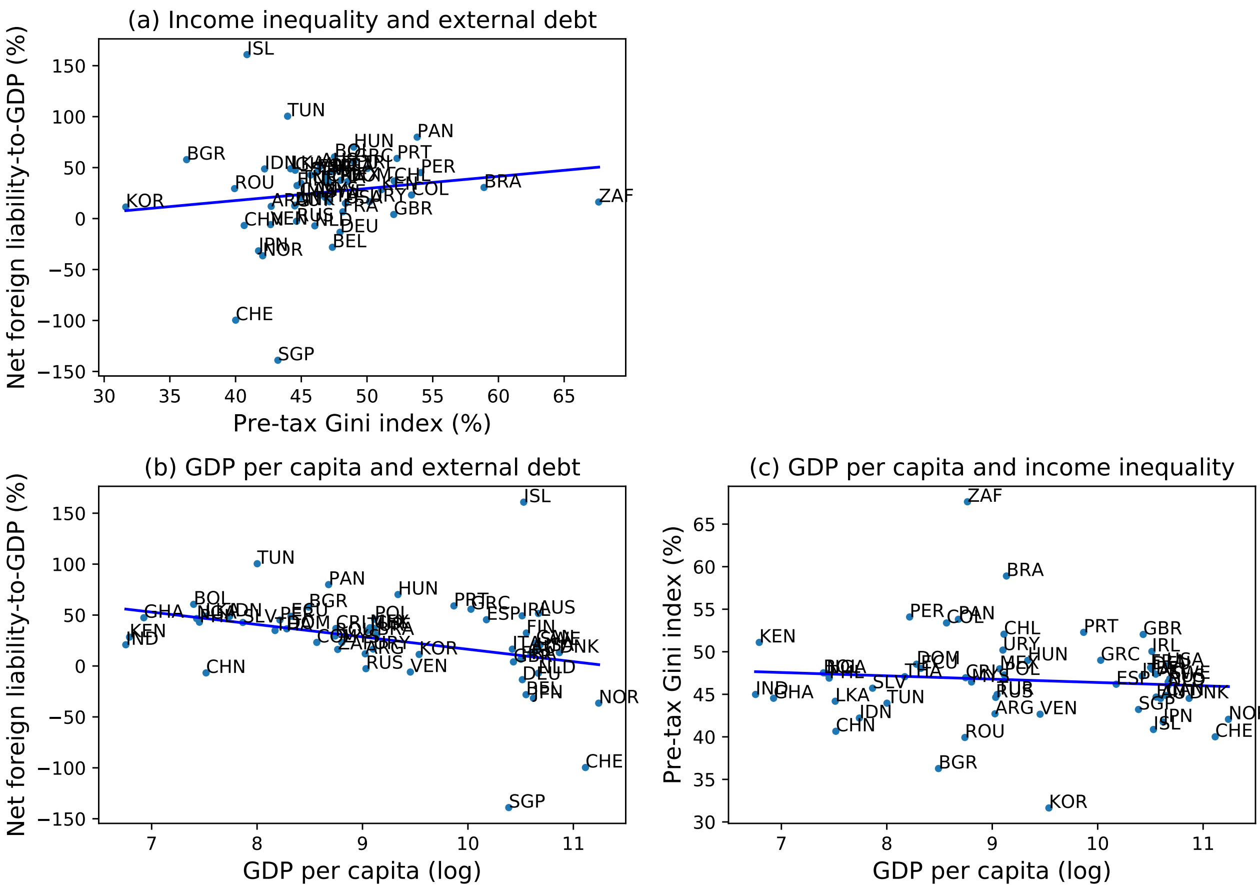
$$\text{Pre-tax Gini Index}_i = \beta_0 + \beta_1 \text{GDP per capita}_i + \beta_3 \text{GDP growth}_i + \epsilon_i^{gini} \quad (2)$$

In Figure 2, panel (a) of plots the residuals ϵ_i^{nfl} of equation (1) with respect to the pre-tax Gini index, and panel (b) plots the net foreign liability-to-GDP with respect to the residuals ϵ_i^{gini} of equation (2). The positive trend in Panel (a) implies that a higher pre-tax Gini index

[13]The net foreign asset (NFA) position of a country is the value of the assets that country owns abroad, minus the value of the domestic assets owned by foreigners, adjusted for changes in valuation and exchange rates. A different measure of a country's external position is the net international investment position (NIIP), which is the difference between a country's stock of foreign assets and foreigner's stock of that country's assets. See Appendix A.2 for the estimation using NIIP.

[14]See appendix G.2 for the list of advanced and emerging market economies. Appendix A.1 displays the graphs for the whole dataset.

Figure 1: Income inequality, external debt, and GDP per capita across countries



Note: The graph shows the 1985-2015 time averages of net financial liability-to-GDP, pre-tax Gini index, and GDP per capita in constant 2010 US Dollars for advanced and emerging market economies. Panel (a) plots averages of pre-tax Gini index (%) and net foreign liability-to-GDP (%). Panel (b) plots averages of log of GDP per capita and net foreign liability-to-GDP (%). Panel (c) plots averages of log of GDP per capita and pre-tax Gini index (%). Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and World Bank (2019).

is associated with a higher net foreign liability-to-GDP that do not come from GDP per capita or GDP growth rates. The positive trend in Panel (b) implies that a higher pre-tax Gini index that is not due to different GDP per capita or GDP growth levels is correlated with a higher net foreign liability-to-GDP level.

2.2 Time Series Properties

Income inequality has been rising over time across countries (Alvaredo et al. (2018)). At the same time, external debt is also increasing across many countries. The 2007-2009 financial crises contributed to the increase in borrowing across countries, particularly across European ones[15]. Figure 3 plots the GDP-weighted average of net financial liability-to-GDP and pre-tax Gini index for countries in the European Union. Over time, there are increasing trends in both income inequality and external debt.

2.3 Estimation

To estimate the effect of income inequality on external debt, I use the following specification

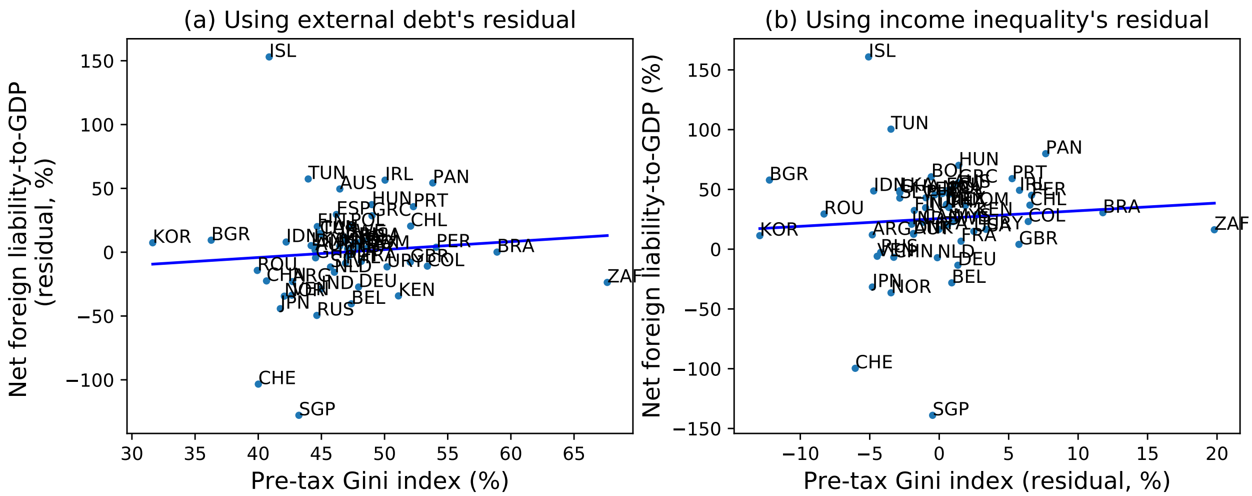
$$\begin{aligned}\text{Net foreign liability-to-GDP}_{i,t} = \alpha_0 + \alpha_1 \text{Gini}_{i,t} + \alpha_2 \text{GDP per capita}_{i,t} \\ + \alpha_3 \text{GDP growth}_{i,t} + \alpha_4 \text{Inflation}_{i,t} + u_i + z_t + \epsilon_{i,t}\end{aligned} \tag{3}$$

where GDP per capita is the log of real GDP per capita series in constant 2010 US Dollars and inflation is calculated from GDP deflators. Both GDP per capita and inflation series are from the World Development Database[16].

[15] Reinhart and Rogoff (2010) reported large increases in public debt across countries, especially in the period 2007-2009. External debt levels were particularly high among European countries.

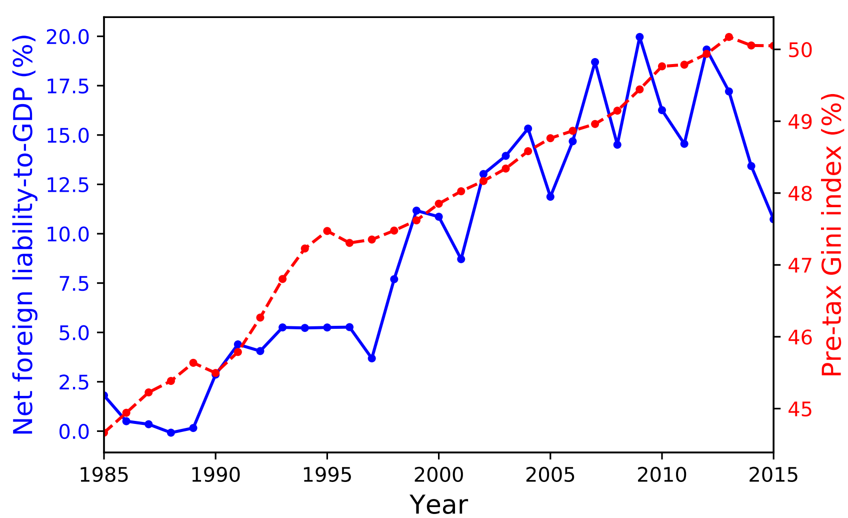
[16] Appendix G.2 lists all countries considered in the regression.

Figure 2: Cross-sectional relationship between income inequality and external debt



Note: Panel (a) plots the residuals ϵ_i^{nfl} (in percentage) of equation (1) and the pre-tax Gini index (%). Panel (b) plots the net foreign liability-to-GDP (%) and the residuals ϵ_i^{gini} (in percentage) of equation (2). The sample includes advanced and emerging market economies. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and World Bank (2019)

Figure 3: Time series of income inequality and external debt



Note: The graph shows the GDP-weighted average of net financial liability-to-GDP and pre-tax Gini index for countries in the European Union from 1985 to 2015. Sources: Lane and Milesi-Ferretti (2018) and Solt (2019).

Table 1: Regression analysis of income inequality and external debt

	Dependent Variable: Net foreign liability-to-GDP (%)	
	Time periods: 1985-2015	
	(1)	(2)
Gini index, pre tax (%)	1.4367***	1.3272***
	(0.4150)	(0.4576)
GDP per capita (log)		-21.741***
		(7.0379)
GDP growth (%)		-0.3330
		(0.2802)
Inflation (%)		-0.0083**
		(0.0043)
Country fixed effects	Yes	Yes
Time fixed effects	Yes	Yes
No. Countries	179	176
No. Observations	3932	3848

Note: The table describes the panel regression results using all countries in the data set. The first column shows the regression coefficient and standard error in parenthesis of pre-tax Gini index (%) with respect to net foreign liability-to-GDP (%). The second column shows the regression coefficients and standard errors in parentheses that include other control variables: log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). Both regressions have country and time fixed effects. All standard errors are clustered. *,**,*** represent significant levels of 10%, 5%, and 1%, respectively. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and World Bank (2019).

Table 1 shows the results for the panel regression of all countries in the data set for the time period of 1985-2015. The first column presents the estimations of equation (3) without control variables. The second column presents the estimations with control variables. The clustered standard errors are in parentheses. The correlation between inequality and external debt is positive and statistically significant, and the result is robust to country and time fixed effects, GDP per capita, GDP per capita growth, and inflation. GDP per capita, GDP growth, and inflation are negatively correlated with net foreign liability-to-GDP, but the effect of GDP growth is not statistically significant.

3 Model

This section describes the main framework and sets up the competitive equilibrium given the government's policies. The competitive equilibrium can be characterized by a set of aggregate allocation and a distribution of marginal utility shares.

3.1 Environment

A small open economy faces publicly observed aggregate shocks $s_t \in S$ in period t, where S is some finite set. Let $\Pr(s^t)$ denote the probability of any history $s^t = (s_0, s_1, ..., s^t)$, where $\Pr(s^{t+j}|s^t)$ denotes the probability conditional on history s^t, $j \geq 0$. Similarly, $\Pr(s_{t+1}|s^t)$ is the probability period $t+1$'s state is s_{t+1}, conditional on history s^t. The exogenous risk-free international interest rate for borrowing is r^*. There is a measure-one continuum of infinitely-lived agents different by labor productivity types $(\theta^i)_{i\in I}$, which are publicly observable. The fraction of agents with productivity θ^i is π^i, where $(\pi^i)_{i\in I}$ is normalized such that $\sum_{i\in I}\pi^i\theta^i = 1$. All agents have the same discount factor β and the static utility $U(c, n)$ over consumption c and hours worked n. The utility of agent with productivity θ^i over consumption $c_t^i \geq 0$ and efficiency-unit labor $l_t^i \geq 0$ is

$$\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0 U^i(c_t^i, l_t^i) \tag{4}$$

where $U^i(c, l) = U\left(c, \frac{l}{\theta^i}\right)$.

In addition, there is a representative firm that uses labor to produce a single final good. The production function in period t with history s^t is$F(L, s^t, t)$, constant return to scale, where L is the aggregate labor. The economy is subject to an exogenous sequence of government spending $\{G_t(s^t)\}_{t=0,s^t\in S^t}^{\infty}$. Both the production function and government expenditures depend on the time period t, capturing deterministic changes such as growth, and the history s^t, capturing the uncertainty impact.

An allocation specifies consumption and labor in every period after every history: $\{c^i(s^t), l^i(s^t)\}$. The aggregates are denoted by $C(s^t) \equiv \sum_{i\in I}\pi^i c^i(s^t)$ and $L(s^t) \equiv \sum_{i\in I}\pi^i l^i(s^t)$.

Both the domestic and international financial markets are competitive. The government can issue domestic debt from a full set of state-contingent bonds, which can be traded across agents. The government also have access to a full set of state-contingent external bonds. Let $R^* = 1 + r^*$ denote the gross risk-free interest rate. Define $Q(s_{t+1}|s^t) = \Pr(s_{t+1}|s^t)/R^*$ as the international price of one unit of consumption at state s_{t+1} in period $t+1$, conditional on history s^t, in units of consumption at history s^t. Similarly, $q(s^t) = \Pr(s^t)/(R^*)^t$ is the international price of one unit of consumption at history s^t in units of consumption at s^0. Let normalize $q(s^0) = 1$[17]. Note that $q(s^{t+1}) = Q(s_{t+1}|s^t)q(s^t)$. Assume only the government can borrow abroad[18].

[17]This normalization is without loss of generality since the initial level of external debt is fixed.

[18]In the data, domestic residents often hold a very small amount of foreign assets, so most models assume that they do not have access to the external credit market. In this environment, the set up is equivalent to the case where the domestic agents can save abroad with the bond price $Q^*(s^t)$, but then face a residence-based tax $\tau^d(s^t)$. External debt will be the net foreign liability of both the private and public sectors, instead of

3.2 Competitive Equilibrium

In every period and history s^t, the government can issue both domestic and foreign bonds, impose a lump-sum tax $T(s^t)$, a marginal tax on labor income $\tau^n(s^t)$ and levies a tax on the return of domestic saving $\tau^d(s^t)$. Both the firm and agents face the labor wage $w(s^t)$.

Domestic Agent. Individual agent of type $i \in I$ faces the sequential budget constraint in period t and history s^t

$$c^i(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t) b^{d,i}(s^{t+1}) \leq (1-\tau^n(s^t))w(s^t)l^i(s^t) + (1-\tau^d(s^t))b^{d,i}(s^t) - T(s^t) \tag{5}$$

where $c^i(s^t), l^i(s^t), b^{d,i}(s^t)$ denote the consumption, labor, and domestic bond holding of agent i in period t, history s^t, respectively. $Q^d(s_{t+1}|s^t)$ is the price of one unit of domestic asset for realization s_{t+1} in period $t+1$ given history s^t.

Representative Firm. The firm chooses the amount of capital and labor to maximize profit in each history node s^t

$$\max_{\{L(s^t)\}} \quad F\left(L(s^t), s^t, t\right) - w(s^t)L(s^t)$$

which gives the first-order condition

$$w(s^t) = F_L\left(L(s^t), s^t, t\right) \tag{6}$$

The firm's profit is zero in equilibrium because of the constant-return-to-scale production function.

Government. There is an exogenous government expenditure $\{G_t(s^t)\}_{t=0, s^t \in S^t}^{\infty}$. Given the one-period state-contingent domestic bond $B^d(s^t)$ and external bond $B(s^t)$, the government's budget constraint in each history node s^t is

$$\begin{aligned} &G(s^t) + (1-\tau^d(s^t))B^d(s^t) + B(s^t) \\ \leq &\tau^n(s^t)w(s^t)L(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) + \sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t)B(s^{t+1}) + T(s^t) \end{aligned}$$

only the public sector here. I choose this particular set up so that it is more straightforward to characterize the strategic game later on in Appendix B.

where $B^d(s^t) = \sum_{i \in I} \pi^i b^{d,i}(s^t)$ is the aggregate domestic bond, and $B(s^t)$ is the amount of the government's external debt. There is a no-Ponzi condition such that the present value of external debt is bounded below.

The government's present-value budget constraint is

$$\sum_{t \geq 0, s^t \in S^t} q(s^t) \Big\{ G(s^t) - \tau^n(s^t) w(s^t) L(s^t) - T(s^t) \\ + \sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t) B^d(s^{t+1}) - (1 - \tau^d(s^t)) B^d(s^t) \Big\} \leq B(s^0) \tag{7}$$

Resource constraint. Using the agent's budget constraints and government's budget constraint, one can obtain a present-value resource constraint in terms of the inter-temporal international prices and the initial external debt,

$$\sum_{t \geq 0, s^t \in S^t} q(s^t) \left[F(L(s^t), s^t, t) - G(s^t) - C(s^t) \right] \geq B(s^0) \tag{8}$$

Competitive equilibrium. Given the above equations, one can define the following competitive equilibrium with taxes.

Definition 3.1. Given initial external debt $B(s^0)$ and individual individual bond positions $\left(b^{i,d}(s^0)\right)_{i \in I}$, a competitive equilibrium with taxes for an open economy is individual agent's allocation $z^{H,i} = \left\{c^i(s^t), l^i(s^t), b^{i,d}(s^t)\right\}_{t=0, s^t \in S^t}^{\infty}$, $\forall i \in I$, the representative firm's allocation $z^F = \{L(s^t)\}_{t=0,s^t}^{\infty}$, prices $p = \left\{q(s^t), w(s^t), Q^d(s_{t+1}|s^t)\right\}_{t=0,s^t \in S^t}^{\infty}$, and government's policy $z^G = \left\{\tau^n(s^t), \tau^d(s^t),\ T(s^t), B^d(s^t), B(s^t)\right\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves individual i's problem that maximizes (4) subject to (5) and a no-Ponzi condition of agent's debt value, (ii) given p and z^G, z^F solves firm's problem, (iii) the government budget constraint (7) holds, (iv) the aggregate resource constraint (8) is satisfied, (iv) the domestic bond market clears $B^d(s^t) = \sum_{i \in I} \pi^i b^{d,i}(s^t)$, and (v) p satisfies $q(s^t) = \Pr(s^t)/(R^*)^t$ and equation (6) given z^G.

3.3 Characterizing Equilibrium

In equilibrium, the intra-temporal and inter-temporal rates of substitution are the same across agents, i.e. in each period t and each history s^t, for any individual i,

$$\begin{aligned}(1-\tau^n(s^t))w(s^t) &= -\frac{U^i_l(c^i(s^t),l^i(s^t))}{U^i_c(c^i(s^t),l^i(s^t))}\\ \frac{Q^d(s_{t+1}|s^t)}{1-\tau^d(s^{t+1})} &= \beta\Pr(s^{t+1}|s^t)\frac{U^i_c(c^i(s^{t+1}),l^i(s^{t+1}))}{U^i_c(c^i(s^t),l^i(s^t))}\end{aligned}$$

Given the aggregate allocation $(C(s^t),L(s^t))$, there is an efficient assignment of individual allocation $(c^i(s^t),l^i(s^t))_{i\in I}$ due to the equal marginal rates of substitution between consumption and labor. Moreover, the efficient assignment needs to be the same across time, because of the equal marginal rates of substitution of future to current consumption. Any inefficiencies due to tax distortions are captured by the aggregate allocation. This property allows the competitive equilibrium allocation to be characterized in terms of aggregates and a static rule for individual allocation.

For any equilibrium, there exist a set of Neghishi (market) weights $\boldsymbol{\varphi}=(\varphi^i)_{i\in I}$, with $\varphi^i\geq 0$ and $\sum_i\pi^i\varphi^i=1$, such that individual allocation solve a static problem

$$\begin{aligned}V(C,L;\boldsymbol{\varphi})\equiv &\max_{(c^i,l^i)_{i\in I}}\sum_{i\in I}\varphi^i\pi^iU^i(c^i,l^i)\\ &s.t.\qquad \sum_{i\in I}\pi^ic^i=C;\quad \sum_{i\in I}\pi^il^i=L\end{aligned}$$

This problem gives the policy functions for each individual i

$$h^i(C,L;\boldsymbol{\varphi})=\left(h^{i,c}(C,L;\boldsymbol{\varphi}),\ h^{i,l}(C,L;\boldsymbol{\varphi})\right)$$

A competitive equilibrium allocation must satisfy $(c^i(s^t),l^i(s^t))=h^i(C(s^t),L(s^t);\boldsymbol{\varphi})$ for all i and s^t. The associate competitive equilibrium prices can be computed as if the economy were populated by a fictitious representative agent with utility function $V(C,L;\varphi)$. The envelope conditions of the static problem give

$$(1-\tau^n(s^t))w(s^t) = -\frac{V_L\left[h^i(C(s^t),L(s^t);\boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t),L(s^t);\boldsymbol{\varphi})\right]} \tag{9}$$

$$\frac{Q^d(s_{t+1}|s^t)}{1-\tau^d(s^{t+1})} = \beta\Pr(s^{t+1}|s^t)\frac{V_C\left[h^i(C(s^{t+1}),L(s^{t+1});\boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t),L(s^t);\boldsymbol{\varphi})\right]} \tag{10}$$

Furthermore, the present-value budget constraint for individual i can be written as

$$\sum_{t\geq 0, s^t\in S^t} \beta^t \Pr(s^t) \Big[V_C(C(s^t), L(s^t); \boldsymbol{\varphi}) h^{i,c}(C(s^t), L(s^t); \boldsymbol{\varphi}) \\ + V_L(C(s^t), L(s^t); \boldsymbol{\varphi}) h^{i,l}(C(s^t), L(s^t); \boldsymbol{\varphi})\Big] = V_C(C(s^0), L(s^0); \boldsymbol{\varphi}) \left(b^i(s^0) - T\right) \tag{11}$$

where T is the present-value of lump-sum taxes[19]. Equation (11) is the individual implementability constraint.

One has the following characterization proposition.

Proposition 3.1. *Given the initial external debt $B(s^0)$ and individual bond holdings$\{b^i(s_0)\}_{i\in I}$,an allocation $\{C(s^t), L(s^t), K(s^t)\}_{t=0,s^t\in S^t}^{\infty}$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if the resource constraint (8) holds, and there exist market weights $\boldsymbol{\varphi} = (\varphi^i)_{i\in I}$ and lump-sum tax T such that the implementability constraint (11) holds for all $i \in I$.*

4 A Planning Problem

This section characterizes the planning problem of a benevolent government that cares about redistribution but lacks commitment in both tax and debt policies. The main theoretical result entails how limited borrowing affects optimal taxes and hence the government's redistributive ability.

4.1 Lack of Commitment

The government cares about all residents in the country, and its objective is the social welfare

$$\sum_{i\in I} \lambda^i \pi^i \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 U^i \left(c^i(s^t), l^i(s^t)\right), \tag{12}$$

given by a set of social welfare weights $\boldsymbol{\lambda} = (\lambda^i)_{i\in I}$. In every period and history node, the government cannot commit to future choices on repayments of debt and taxes. Following Chari and Kehoe (1990, 1993), the policies are determined in a repeated game between the government, a continuum of domestic agents, and a continuum of international creditors. The subgame perfect equilibrium supported by trigger strategies to autarky is characterized by the competitive equilibrium conditions described in Proposition 3.1 and the following

[19] $T \equiv \sum_{t=0}^{\infty} \beta^t \sum_{s^t\in S^t} \frac{V_C\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s_0), L(s_0); \boldsymbol{\varphi})\right]} T(s^t)$

self-enforcing constraint

$$\sum_{i\in I}\lambda^i\pi^i\sum_{k\geq t,s^t\subseteq s^{t+j}}\beta^{k-t}\Pr(s^k|s^t)U^i(c^i(s^k),l^i(s^k))\geq \underline{U}(s^t,t),\ \forall t,\forall s^t \tag{13}$$

where $\underline{U}(s^t,t)$ is the one-shot deviation value in which the government fully redistributes wealth among the domestic agents and default on external debt[20]. The government is then in financial autarky, in which it it has no access to the domestic and international financial markets. $\underline{U}(s^t,t)$ is the value associated with an allocation of a closed economy where the initial states are realized s_t at period t and history s^t, the initial wealth inequality among agents are equal, and the net supplies of domestic and international bonds are zero.

The self-enforcing constraint captures the time-inconsistency of the government's policies. If there is a positive net external debt, the government has an incentive to default externally to increase domestic consumption and leisure. In addition, in every history node, there is a non-degenerate distribution of wealth across the domestic agents. The inequality-averse government will also have an incentive to expropriate all the wealth and equally redistribute it.

The self-enforcing constraint imposes a limit on the utility, which endogenously determines a limit on external debt for every period and history. These constraints act as endogenous borrowing constraints.

4.2 Efficient Allocation

Given the above set-up, an efficient allocation is defined as follows

Definition 4.1. An efficient allocation $\{C(s^t),L(s^t)\},\boldsymbol{\varphi}$ maximizes the social welfare function (12) and satisfies the conditions in Proposition 3.1 and the self-enforcing constraint (13)

The efficient allocation is part of the solution to a planning problem

$$(P)\equiv\max_{\{C(s^t),L(s^t)\},\boldsymbol{\varphi},T}\sum_{i\in I}\lambda^i\pi^i\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0\left[h^i(s^t;\boldsymbol{\varphi})\right]$$

[20]See Appendix B for the formal set up of the sovereign game and its equilibrium characterization.

$$
\begin{aligned}
s.t. \qquad & \sum_{t\geq 0, s^t \in S^t} q(s^t)\left[F(L(s^t), s^t, t) - G(s^t) - C(s^t)\right] \geq B(s^0) \\
\forall i, \ & \sum_{t\geq 0, s^t \in S^t} \beta^t \left[V_C(s^t;\boldsymbol{\varphi})h^{i,c}(s^t;\boldsymbol{\varphi}) + V_L(s^t;\boldsymbol{\varphi})h^{i,l}(s^t;\boldsymbol{\varphi})\right] = V_C(s_0;\boldsymbol{\varphi})\left(b^i(s^0) - T\right) \\
\forall t, \forall s^t, \ & \sum_{i\in I} \lambda^i \pi^i \sum_{k\geq t, s^t \subseteq s^{t+j}} \beta^{k-t} \Pr(s^k|s^t) U^i\left[h^i(s^k;\boldsymbol{\varphi})\right] \geq \underline{U}(s^t,t) \geq \underline{U}(s^t,t)
\end{aligned}
$$

where $(s^t;\boldsymbol{\varphi}) \equiv (C(s^t), L(s^t);\boldsymbol{\varphi})$ for notation convenience.

The first constraint is the resource constraint. The second constraint is the implementability constraints that take into account the distortionary effect of the government's policies on individuals' decision. The last constraint is the borrowing constraint due to the government's lack of commitment. Domestic agents do not directly internalize the effect of their borrowing decisions on these borrowing constraints. The government, on the other hand, has to consider these constraints when choosing optimal allocation and policies. Therefore, the borrowing constraints indirectly affect domestic borrowing choices via the government's decision on saving taxes.

4.3 Optimal Taxation and Borrowing Constraints

I derive optimal policies that implements the efficient allocation. Optimal labor and saving taxes reflect the trade-off between redistribution and efficiency. I show that this trade-off is dynamically changed over time in the presence of borrowing constraints.

I make the following assumptions

Assumption 1 (Separable isoelastic utility)**.** *The individual preference follows*

$$
U^i(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \omega\frac{\left(\frac{l}{\theta^i}\right)^{1+\nu}}{1+\nu}
$$

Assumption 2. *The welfare weights, skill distribution, and initial wealth satisfy the following properties*

1. *Redistribution motive towards the low skills:* $\theta^i \leq \theta^j \iff \lambda^i \geq \lambda^j, \ \forall i,j \in I$

2. *Perfect correlation between skill and initial wealth:* $\theta^i \leq \theta^j \iff b^i(s^0) \leq b^j(s^0), \ \forall i,j \in I$

3. *High elasticity of substitution:* $\sigma \geq 1$

The first assumption implies that the individual preference is separable between consumption and labor, and the elasticities of substitution are constant across periods and histories. For the second set of assumptions, the first part is on the welfare weights, meaning that the planner has a high motive of redistribution towards the lower skill, lower income individuals. The second part makes sure that there direction of inequality in skill is the same as the direction of inequality in initial wealth, meaning that lower skill individuals start off with lower initial wealth. The last assumption implies that the intra-temporal elasticity of substitution is at least above log preference. This assumption determines the direction of change in the optimal tax rate in response to inter-temporal changes.

Using the solution to the planning problem, I establish the main theoretical result on the optimal labor tax,

Proposition 4.1. *For any period $\mathcal{T}$, history $s^{\mathcal{T}}$, and $\forall s^{\mathcal{T}-1} \subseteq s^{\mathcal{T}}$,*

1. *If the borrowing constraint does not bind at $s^{\mathcal{T}}$, the optimal labor tax does not change, i.e. $\tau^n(s^{\mathcal{T}}) = \tau^n(s^{\mathcal{T}-1})$, and the optimal saving tax is zero, i.e. $\tau^d(s^{\mathcal{T}}) = 0$*

2. *If the borrowing constraint binds at $s^{\mathcal{T}}$, the optimal labor tax is weakly decreasing in the future, i.e. $\tau^n(s^t) \leq \tau^n(s^{\mathcal{T}-1})$, $\forall t \geq \mathcal{T}, \forall s^{\mathcal{T}} \subseteq s^t$, and there is optimal saving subsidy, i.e. $\tau^d(s^{\mathcal{T}}) < 0$*

The proposition first implies that given non-binding borrowing constraints, optimal labor taxes do not change over time and across histories, and optimal saving taxes are zero. The labor tax result is related to the standard tax smoothing argument as in Lucas and Stokey (1983). Since the borrowing constraint does not bind, it is optimal to use debt to smooth out the distortionary cost across all states. Zero saving taxes, on the other hand, are related to the idea of no intertemporal distortion in Judd (1985) and Chamley (1986).

When the borrowing constraint binds, all future labor tax rates weakly decrease. The tax smoothing property implies that a one-time binding borrowing constraint does not decrease the labor tax in one current period but the reduction is spread out to all tax rates in the subsequent periods. In addition, the government finds it optimal to use a saving subsidy to discourage the impatient domestic agents from over-borrowing.

Intuitively, distortionary taxation is a mechanism for redistribution. A positive marginal labor tax and lump-sum rebate to all agents imply that the higher skilled, higher income individuals pay more taxes than the lower skilled, lower income individuals. Therefore, a planner that cares about redistribution towards low income agents will find it necessary to levy a positive labor tax. Optimal labor taxes balance the marginal benefit of redistribution and the marginal cost of distortion. The determinants of inequality and distributive motive do not vary over time and are independent of the aggregate shocks. When borrowing is not costly (non-binding borrowing constraints), both the distributive benefit and the distortionary cost do not vary, so it is optimal to keep labor taxes unchanged. When borrowing is limited (binding constraints), distortion becomes more costly because it reduces the amount of resources that the economy can use to repay debt. A decrease in distortionary labor tax is then optimal to increase the repaying capacity and relax borrowing constraints.

The forward-looking borrowing constraints induce a backward-looking effect of the optimal policies. Suppose that the labor tax τ^n decreases in node $s^{\mathcal{T}}$, then efficiency (e.g. aggregate output Y) is higher in node $s^{\mathcal{T}}$. Not only this increase in efficiency allows the economy to repay more debt in $s^{\mathcal{T}}$, but also to repay more debt in any previous histories $s^t \subseteq s^{\mathcal{T}}$ where $0 \leq t \leq \mathcal{T}$. As a result, a lower labor tax in the future relaxes all of the past borrowing constraints. Therefore, instead of a one-time large decrease in the tax rate when the borrowing constraint binds, a permanent small decrease in all future tax rates will relax more borrowing constraints.

Optimal saving taxes, on the other hand, relate to intertemporal distortions. When borrowing constraints do not bind, it is optimal not to distort the intertemporal margins[21]. When borrowing constraints bind, saving subsidies make the domestic agents internalize the additional cost of borrowing that comes from the binding constraints.

Formally, the proof relies on the property that the only component in the optimal taxes that varies across time periods and histories is the sum of all Larange multipliers on the borrowing constraints (γ) up to node $s^{\mathcal{T}}$, i.e. $\sum_{k=0.s^k \subseteq s^{\mathcal{T}}}^{\mathcal{T}} \gamma(s^k)$. If the borrowing constraint does not bind ($\gamma(s^{\mathcal{T}}) = 0$), the sum stays constant, and so labor taxes remain the same as before, and saving taxes are zero in $s^{\mathcal{T}}$. However, if borrowing constraints bind ($\gamma(s^{\mathcal{T}}) > 0$), the sum increases, which leads to a permanent decline in labor taxes. Differences in today's sum at $s^{\mathcal{T}}$ and yesterday's sum at $s^{\mathcal{T}-1}$ leads to a tax in domestic borrowing's return at $s^{\mathcal{T}}$.

Proposition 4.1 implies how limited borrowing affect the government's redistributive policies. When borrowing constraints do not bind, it is optimal to distort the intratemporal margins and not the intertemporal margins. Hence, the government optimally redistribute using labor taxes and levies no borrowing taxes. However, when borrowing constraints bind, distorting the intertemporal margins is beneficial since it aligns the domestic agents' interests with the government's interests. It turns out that the domestic borrowing taxes also redistribute resources among agents. The government optimally redistributes via more borrowing taxes and less labor taxes, which reduces the overall cost of distortion in the economy.

[21]This is an insight from the public finance literature, found in Judd (1985), Chamley (1986), Chari and Kehoe (1999), Werning (2007), and many other papers.

5 Quantitative Analysis

The previous section provides a theoretical foundation on how borrowing constraints affect optimal taxation and the government's ability to redistribute. This section shows the impact of the government's concern for redistribution on its incentive to repay debt and how that implies the positive correlation between income inequality and external debt in the cross section and over time periods. The benchmark calibration uses Italy's data. Lastly, the section presents optimal austerity policies, specifically the fiscal and redistributive implications of a negative productivity shock.

Throughout this section, there are the following assumptions on the domestic discount factor and deviation utility

Assumption 3 (Impatience)**.** *There exists* $0 < \mathcal{M} < 1$ *such that* $\beta R^* < \mathcal{M} < 1$*.*

Assumption 4. $\underline{U}$ *is bounded below, i.e. there exists a finite real* M_U *such that* $\inf_{s^t,t} \underline{U}(s^t, t) \geq M_U$*.*

5.1 Computation

Given the foward-looking borrowing constraints, I implement the recursive formulation developed by Marcet and Marimon (2019). Appendix E.2 provides more details on the computational algorithm. The key co-state variable is the discounted sum of the Larange multipliers on the borrowing constraints: $\Gamma(s^t) = (\beta R^*)^t \sum_{s^k \subseteq s^t} \gamma(s^k)$. Given that the domestic agents are impatient, the borrowing constraint will bind infinitely often in the long run. It turns out that Γ is bounded, i.e.

Proposition 5.1. *Suppose Assumptions 1, 3, and 4 hold, if an interior efficient allocation exisits, then* $\lim_{t\to\infty} \Gamma(s^t) > 0$*.*

$\Gamma(s^t)$ reflects the marginal benefit of relaxing the current and previous borrowing constraints from increasing one unit of utility in period t and history s^t (either by giving more consumption or leisure). In the long run, Γ corresponds to the amount of external debt that can be sustained in equilibrium. Impatience implies that in the long run, there exists a positive component in the price of borrowing coming from the lack of commitment. If there exists an ergodic distribution of the efficient allocation, then Γ will also follows an ergodic distribution. This property allows the computational algorithm using Γ as one of the state variables to converge.

5.2 Calibration

For the quantitative exercise, I assume the following distributional and functional forms. The economy is populated by two types of agents with labor productivity $\{\theta^H, \theta^L\}$, where $\theta^H \geq \theta^L > 0$ and $\pi^H = \pi^L = 0.5$. The planner is utilitarian, i.e. $\lambda^H = \lambda^L$. The individual preference has the form of

$$U^i(c, l) = \log c - \frac{l^{1+\nu}}{1+\nu}$$

The production function is linear in labor, i.e. $F(L, z) = zL$, where z is the aggregate productivity. The aggregate shock is z_t that follows a logged $AR(1)$ process,

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t^z,\ \epsilon_t^z \sim \mathcal{N}(0, \sigma_z),$$

where ρ_z, σ_z are the auto-correlation and the residual standard deviation, respectively. I discretize the productivity process into a Markov chain using Tauchen's method with 31 evenly-spaced nodes. Let $s_t = z_t$. The government expenditure is constant over time and across histories: $G(s^t, t) = \bar{g}$. The initial debt levels are $B(s^0) = 0$ and $b^{H,d}(s^0) = b^{L,d}(s^0) = 0$, where s^0 is the mean of the productivity distribution. The deviation utility $\underline{U}(s^t)$ is calculated as the closed-economy version of the model that starts with productivity z_t, zero external debt, and all domestic individuals start with the same initial wealth. $\underline{U}(s^t, t)$ varies with respect to the realized shock $s_t = z_t$.

With these assumptions, the model requires giving values to the parameters of (i) the aggregate productivity process, ρ_z and σ_z; (ii) the cross-sectional wage ratio, θ^H/θ^L; (iii) the individual preference, β and ν; (iv) the government expenditure $\bar{g}$; and (v) the risk-free rate r^*.

A period in the model is one year. For output, I use the logged and linear detrended real GDP series from 1985-2015. I set the auto-correlation of productivity,ρ_z, equals to the auto-correlation of output, which is 0.928. To calculate the wage ratio θ^H/θ^L, I use the data on cross-sectional inequality by Jappelli and Pistaferri (2010). For each year in the database, I calculate the ratio of the mean wage of the top 50% of the wage distribution to the mean wage of the bottom 50%. Then θ^H/θ^L is set to 1.9475, which is the time-average of these wage ratios for the period 2002-2006. The discount factor β is set to 0.967 so that the average real domestic interest rate is 3.4% for Italy from 2002 to 2015. I choose $\nu = 2$ so that the elasticity of labor supply is 0.5, a standard value in the literature. The risk-free rate is set at 0.017, which is the real rate of return on the German government bonds for the period 2002-2015 (these are secondary market returns, gross of tax, with around 10 years' residual maturity). The interest rate series start at 2002 to isolate the effect of currency and

Table 2: Calibrated Parameters and Targets

Parameter	Description	Value	Target
Externally calibrated parameters			
r^*	Risk-free rate	0.017	Avg. real return on German bond
β	Discount factor	0.967	Avg. Italian real interest rate = 3.4%
$1/\nu$	Labor elasticity	0.5	Standard literature value
θ^H/θ^L	Wage ratio	1.9475	Mean top 50% wage / mean bottom 50% wage
ρ_z	Auto-corr. of prod.	0.927	Auto-corr. of log GDP
Internally calibrated parameters			
σ_z	Std. dev. of prod. res.	0.019	Std. dev. log GDP
$\bar{g}$	Govt. spending	0.202	Avg. govt. consumption-to-GDP

Note: The table describes the parameters, their values, and the targets in the calibration exercise. Statistics are annual. The risk-free rate and discount factor cover the period of 2002-2015. Wage ratio is the author's calculation from the cross-sectional data set by Jappelli and Pistaferri (2010), covering the period of 2002-2006. Auto-correlation and standard deviation of GDP and government final consumption cover the period of 1985-2015. Data sources: Jappelli and Pistaferri (2010), Eurostat (2019), and World Bank (2019)

exchange rate risks[22].

The two remaining parameters, σ_z and $\bar{g}$, are selected to match (i) the standard deviation of logged output and (ii) the government's final consumption-to-GDP ratio for the period 1985-2015. I use the simulated method of moments (SMM). Departing from the quantitative literature on sovereign debt, I do not target the average external debt-to-output ratio but instead leave it as one of the non-targeted moments[23].

Table 2 summarizes the parameter values and targets from the calibration exercise.

5.3 Calibration Results

Table 3 shows the moment matching exercise of the model and the data. The first column reports the statistics from the data for Italy in the period of 1985-2015. The second column reports the statistics from simulating the model and taking the long-run averages[24].

[22] See Appendix G for more data descriptions and sources

[23] See Section 5.3 for the results of non-targeted moments. Alternatively, the discount factor β can be used to target the debt-to-output ratio.

[24] All model statistics are long-run averages of simulating the economy for 10500 periods and discarding the first 500 periods.

The calibration successfully matches the standard deviation of output and the government consumption-output ratio for Italy.

Table 3: Targeted Statistics: Data and Model

Statistics	Data: 1985-2015	Model
Std. dev. log GDP	0.053	0.053
Avg. govt. consumption-to-GDP	0.19	0.19

Note: The table describes the targeted statistics from the calibration exercise. The first column reports data statistics which are across the period of 1985-2015. The second column reports the model statistics which come from the model's simulation for 10500 periods and excluding the first 500 periods. Sources: World Bank (2019)

Table 4 reports the non-targeted statistics of the model comparing to the data. The first column is from the Italian data, and the second column is from the model. The key cyclical properties are the volatility and correlation with respect to output of consumption and net saving ratio. I consider net savings as the amount of output minus the total consumption. In the model, net saving is the net amount of resources used to repay external debt in every period.

Several cyclical features of the Italian data stand out. First, consumption is as volatile as output and is highly correlated with output. Net saving only has a volatility of more than a quarter of the volatility of output, and has a positive correlation with output that is around 40%[25]. The model correctly gets the qualitative patterns of the data. The volatility of consumption and net savings relative to output are slightly higher in the model than in the data. Both model consumption and net saving are pro-cyclical with similar correlation levels as in the data. The model is able to generate realistic cyclical patterns of the data, in contrast to the standard model of complete markets. The main reason is that, even with state-contingent assets, the occasionally binding borrowing constraints lead to an imperfect insurance across states and time periods.

External debt. In the data, external debt is defined as the net foreign liability position, as reported by Lane and Milesi-Ferretti (2018)'s External Wealth of Nation Database. The model explains well both the first and second moments of external debt for Italy from 2002

[25] Neumeyer and Perri (2005) reported key business cycle statistics for both advanced and emerging market economies.

Table 4: Non-targeted Statistics: Data and Model

Statistics	Data	Model
Cyclical property		
std (C) / std (Y)	1.0	1.2
std (NS/Y) / std (Y)	0.29	0.34
corr (C,Y)	0.97	0.94
corr (NS/Y,Y)	0.40	0.30
External debt property[a]		
Mean external debt/Y	0.24	0.21
Std. dev. external debt/Y	0.027	0.022

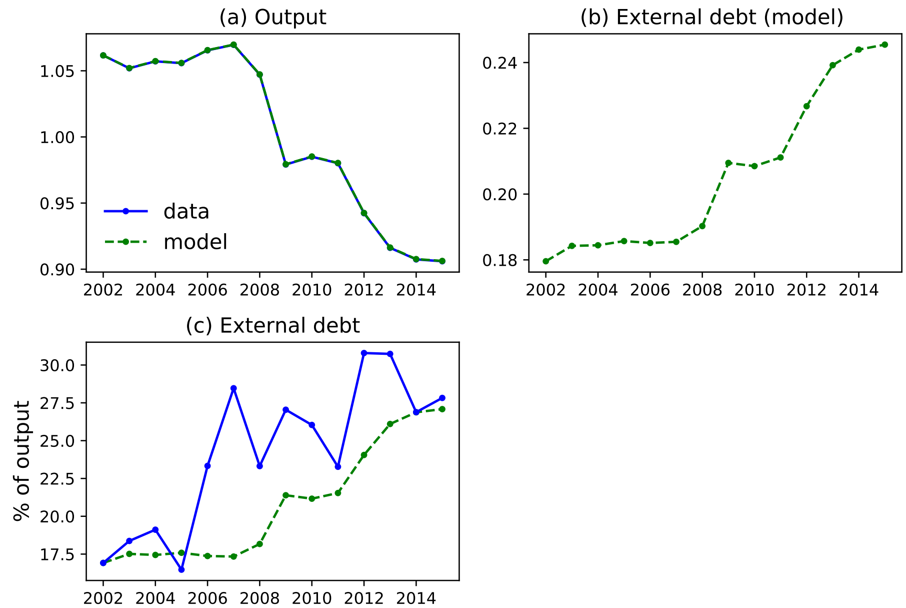
[a]Sample period: 2002-2015.

Note: This table reports the non-targeted statistics of the data and the model. The first column reports data statistics which are across the period of 1985-2015, unless specified. The second column reports the model statistics which come from the model's simulation for 10500 periods and excluding the first 500 periods. Net saving (NS) is defined as output minus total private and government consumption in the data and the model. External debt is defined as the country's net financial liability in the data. For the second moments, output and consumption series are logged and linear detrended. Net saving and external debt ratio series is linear detrended.

to 2015. The model generates on average around 21% of external debt-to-output ratio, comparing to 24% of net foreign liability-to-output ratio in the data. This model feature is with a relatively high discount factor (0.969) with respect to the literature. The model also matches the volatility of external debt-to-output ratio in the data.

Event analysis. I now conduct an event analysis for Italy in period of 2002-2015. I feed into the model a sequence of productivity shock realizations such that the model's outputs matches ones in Italy from 2002 to 2015. I simulate a time path of external debt in the model given that the initial external debt-to-output is the data value in 2002 . I then compare the evolution of external debt-to-output in the data and in the model's simulation over time. Figure 4 plots the exercise's results. Panel (a) plots the output paths of the data and the model. Panel (b) plots the time path of external debt in the model. Panel (c) plots external debt-to-output time paths for both the data and the model. From 2011 to 2015, Italy's output has dropped by 7.4% below trend, while external debt-to-output has increased by 4.6%. In the model's simulation, external debt has increased by 3.4%, which leads to a 5.5% increase in external debt-to-output.

Figure 4: Italy's Recession: Data and Model



Note: The graph depicts the time paths of output, external debt, and external debt-to-output for the data and the model's simulation. Panel (a) plots the output path. Panel (b) plots the external debt paths of the model. Panel (c) plots external debt-to-output.The simulation uses a sequence of productivity shock realization such that the model's output matches the data output for Italy in 2002-2015. The initial external debt level is such that the model's external debt-to-output matches with the starting value in 2002 from the data. Data sources: Lane and Milesi-Ferretti (2018) and World Bank (2019).

5.4 Model Mechanics

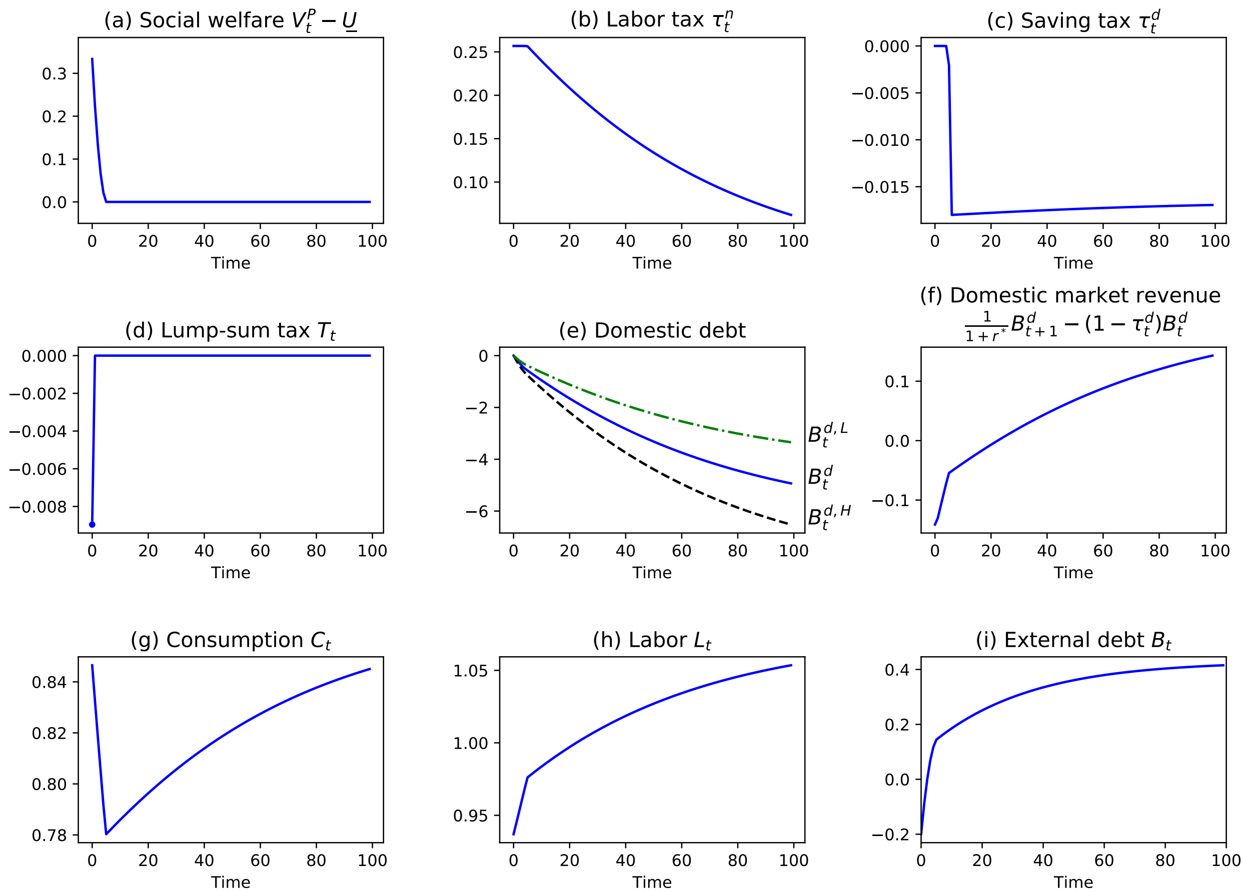
This subsection explains the mechanism on how the government's concerns for redistribution affect its incentive to repay debt. The first part analyzes dynamics of optimal policies, which provides insights on the properties of redistributive policies that allow the government to sustain debt. The second part studies the cost of default and its interaction with the government's redistributive motive. This interaction is the key explanation for the positive association between income inequality and external debt.

5.4.1 Dynamics of optimal policies

I first consider a special case of the model that faces a deterministic path of productivity: $z_t = \bar{z} = 1, \forall t$, and the economy starts with an initial external asset position: $B_0 = -0.2$. The other parameter values are the same as in Table 2. Without shocks, when the borrowing constraint binds, all future borrowing constraint will also bind. This framework can clearly show the effect of the binding borrowing constraint on optimal policies, in which there is a region where the borrowing constraint does not bind, and a region where it binds. The Ricardian equivalence implies that there is indeterminacy between lump-sum taxes and domestic debt holdings. Therefore, a particular implementation is that the government only levies the present-value of lump-sum taxes in the initial period. These lump-sum taxes determine the effective initial wealth positions of domestic agents.

Figure 5 depicts the aggregate time paths of this special case. Panel (a) plots the difference between the social welfare and deviation utility over time. Panel (b) and (c) describes the optimal labor tax and saving tax. Panel (d) depicts the lump-sum taxes. Panel (e) plots the time paths of total and individual domestic debt. Panel (f) plots the net revenue that the government collects from participating in the domestic credit market, $\frac{1}{1+r^*}B^d_{t+1} - (1-\tau^d_t)B^d_t$. Panel (g) and (h) plot aggregate consumption and labor, respectively. Panel (f) describes the time path of the external debt.

Figure 5: Time paths of aggregates in the special case: $z_t = 1$ and $B_0 = -0.2$



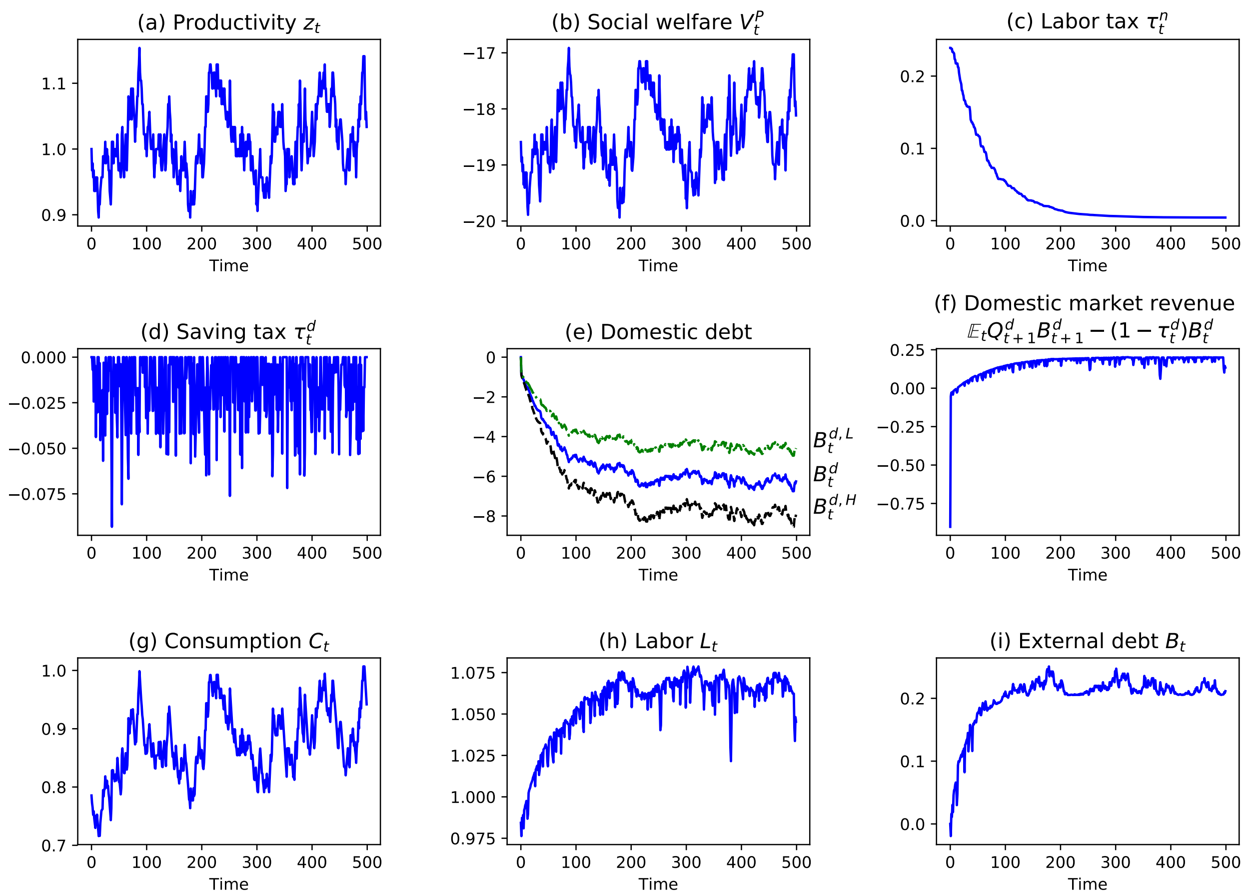
Note: The graph plots the deterministic time paths of optimal policies and aggregates from the planning problem in which $z_t = \bar{z} = 1, \forall t$ and $B_0 = -0.2$. The implementation is that lump-sum taxes only occur in period 0. Panel (a) plots the difference between the social welfare and deviation utility. Panel (b) and (c) describes the optimal labor tax and saving tax. Panel (d) depicts the lump-sum taxes. Panel (e) plots the time paths of total and individual domestic debt. Panel (f) plots the net government's revenue of domestic market. Panel (g) and (h) plot aggregate consumption and labor, respectively. Panel (f) describes the time path of the external debt.

The borrowing constraint does not bind when $V^P > \underline{U}$, and binds when $V^P = \underline{U}$. Due to impatience, the social welfare decreases over time until it reaches the deviation utility value. Optimal taxes follow the properties in Proposition 4.1. Optimal labor taxes are positive and constant, and saving taxes are zero when borrowing constraints do not bind. However, optimal labor taxes permanently decrease as the borrowing constraints start binding, and saving taxes become negative, imply taxes on domestic borrowing. In present value terms, the planner gives a small lump-sum transfer (negative lump-sum tax) to all residents. Domestic debt decreases over time, implying that agents are borrowing. Since all agents borrow at the same fraction of their income, high-income agents borrow more than low-income agents and are net debtors. The government acts as an financial intermediary between the domestic agents and international lenders. Panel (f) describes the net resources that the government receives from the domestic credit market. In the beginning of time, the government gives resources to domestic agents. The government collects taxes on labor income in return. When borrowing constraints bind, the government uses borrowing taxes ($\tau^d < 0$) and start collecting net revenue from the domestic credit market. As labor taxes decline over time, there is less labor tax revenue. Impatience implies that the planner front loads consumption and leisure when borrowing constraints do not bind. When borrowing constraints bind, declining labor taxes encourage increases in labor and output, while saving subsidies encourage back-loading consumption.

The similar dynamics occur in the baseline model with a stochastic aggregate productivity. Figure 6 depicts the time paths of the aggregates and policies of the baseline model for a simulation. In this case, the borrowing constraint is occasionally binding, generating the fluctuations in the efficient allocation and policies. Both the social welfare V^P and aggregate consumption C are highly correlated with the productivity shock z. Over time, since the borrowing constraint binds more often, both aggregate consumption and labor go up, as predicted by the deterministic case. The labor tax drifts down, while the domestic market revenue increases. The external debt increases over time and eventually reaches an ergodic distribution.

Figure 6 points out the changes in the government's redistributive policies over time. Since high-income agents borrow more than low-income ones, borrowing taxes act as an additional redistributive policy when borrowing constraints bind. The government redistributes via taxes on borrowing instead of labor taxes, which increases the efficiency gain and allows the government to sustain the existing level of debt.

Figure 6: Simulated time paths of aggregates in the baseline model



Note: The graph plots the simulated time paths of optimal policies and aggregates of the planning problem for the baseline model. The implementation is that lump-sum taxes only occur in period 0. Panel (a) plots the difference between the social welfare and deviation utility. Panel (b) and (c) describes the optimal labor tax and saving tax. Panel (d) depicts the lump-sum taxes. Panel (e) plots the time paths of total and individual domestic debt. Panel (f) plots the net government's revenue of domestic market. Panel (g) and (h) plot aggregate consumption and labor, respectively. Panel (f) describes the time path of the external debt.

5.4.2 Cost of default

The cost of default determines the equilibrium level of external debt. The more costly is to default, the more likely that the government is willing to repay debt, and so the higher level of external debt is.

The cost of default is the opportunity cost of not having access to domestic and external credit markets. In this framework, there are two main components: insurance and redistributive costs. The former comes from the fact that without credit markets, the government cannot insure itself against aggregate fluctuations. The insurance cost is present in many representative-agent models in the literature (Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), Chatterjee and Eyigungor (2012), and many other papers). This paper introduces the redistributive cost, which is novel to the literature. The main idea is that redistribution is more costly in financial autarky than in the contract.

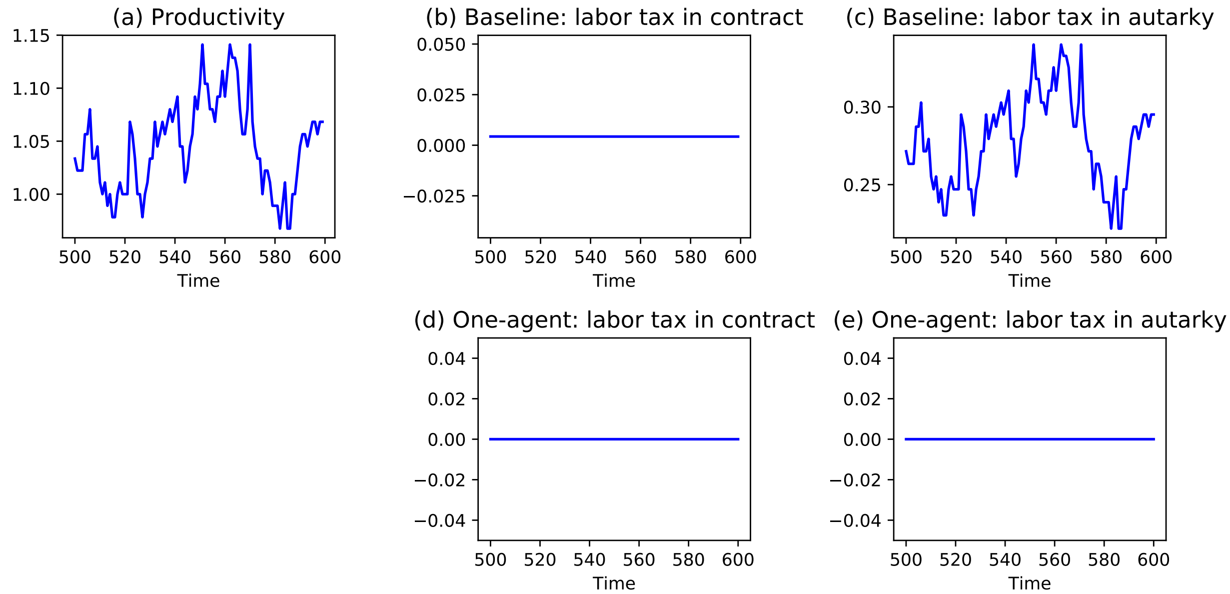
The government's redistributive motive affects its repayment incentive via changing the cost of default. This an endogenous component of the cost of default that is novel to the literature. The main idea is that redistribution is more costly in financial autarky than in the contract, so the government is more willing to repay its debt.

What entails the redistributive cost of default, or the benefit of repayment? First, note that if the government defaults, it does so both domestically and externally. It turns out that domestic default creates an adverse distributive effect. Figure 6 shows that the high-income agents are net domestic debtors in the long run. Domestic default erases the distribution of domestic wealth and so implicitly transfer more resources to the high-income agents. Furthermore, the labor distortion needed for redistribution in financial autarky is higher and more volatile than the labor distortion in the contract. Access to both domestic and external credit markets allows the government to use less labor distortion by redistributing via borrowing taxes when borrowing constraints bind, and is able to smooth out the distortions over aggregate fluctuations.

Figure 7 shows optimal labor taxes in different scenarios. Panel (a) plots the path of productivity shock in the long run, starting at period 500. Panel (b) and (c) plot the baseline simulation of optimal labor taxes in contract and in autarky, respectively. The autarky case means that the government defaults at period 500 and faces financial autarky onward. Panel (d) and (e) are the analogs of panel (b) and (c) for the one-agent's model.

In the one-agent model, there is no need for redistribution, so the labor taxes are zero across time periods and histories. If the government defaults and goes into financial autarky, labor taxes remain zero. However, in the baseline model with heterogeneous agents, there are differences in labor taxes between the contract and autarky. The labor tax in autarky is higher and more volatile than the labor tax in the contract. These properties lead to

Figure 7: Labor distortion in contract and in autarky



Note: The graph describes the time paths of productivity shock and optimal labor taxes in contract and in financial autarky for the baseline model and the one-agent model. The autarky case is when the government defaults at period 500 and is permanently excluded from all credit markets.

financial autarky, or default, be more costly than repayment.

Quantitatively, the welfare cost of insurance is trivial, so the amount of external debt that the one-agent model can sustain is quantitatively small[26]. When the government has concerns for redistribution, the redistributive cost of default arise endogenously. This cost turns out to be quantitatively large enough to account for the observed external debt levels. Given the calibrated parameters, the long-run average external debt-to-output is 2.8% in the one-agent model, while it is 21% in the heterogeneous-agent model. These results imply that the insurance cost accounts for 13% , and the redistributive cost accounts for 87% of the long-run average external debt-to-output. Appendix F.1 provides a measure of the distributive cost of default in terms of productivity loss.

Higher income inequality is correlated with higher external debt because of the higher redistributive cost of default. Given the government's redistributive preference towards low-income agents, a higher income inequality implies a higher motive for redistribution and a larger labor tax distortion in financial autarky, making it more costly to default. Therefore, the government is willing to sustain a higher amount of external debt.

[26] Lucas (1987) pointed out that the cost of eliminating business cycles is quantitatively small for the standard neoclassical growth model. In the sovereign debt framework, Aguiar and Gopinath (2006), Arellano (2008), and Chatterjee and Eyigungor (2012) have shown that in order to match the observed debt levels in the data, they need to impose additional output losses in default.

In the next subsections, I estimate the effect of income inequality on external debt in the cross section and over time.

5.5 Cross-Sectional Estimation

Table 5 shows the estimation results of the correlation between pre-tax Gini index and net foreign liability-to-GDP from the model and the data. The data values are from the second column of Table 1, robust to country, time, and other controls. The model values come from the regression on the model's simulated data. Given the calibrated parameters, I solve different versions of the model differentiated only by wage inequality and compute the pre-tax Gini indices, long-run averages of external debt-to-output ratios, and output per capita. I then estimate the regression $\text{NFL}_i = \beta_0 + \beta_1 \text{Gini}_i + \beta_2 \log \text{GDP per capita}_i + \epsilon_i$ and report $\hat{\beta}_1$ and its standard error[27].

Table 5: Cross-sectional estimation: Data and Model

	Dependent Variable: Net foreign liability-to-GDP (%)	
	Data: 1985-2015	Model
Gini index, pre tax (%)	1.3272***	2.4233***
	(0.4576)	(0.0674)
Controls	Yes	Yes
No. Observations	3848	30

Note: The table describes the cross-sectional estimation of the coefficient of pre-tax Gini index (%) on net foreign liability-to-GDP (%) in the data and in the model. Details on data estimation are from Table 1. The model estimation comes from simulated data of 30 different versions of the model that are differentiated by wage ratios.

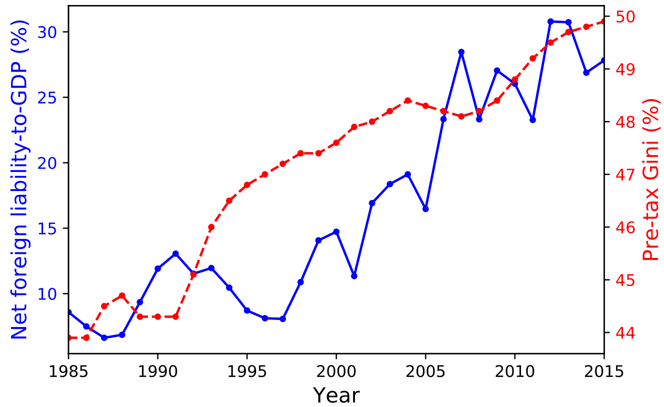
The model produces a positive and statistically significant coefficient of the pre-tax Gini index. The coefficients imply that a one percent increase in the pre-tax Gini index corresponds to a 1.3272% increase in net foreign liability-to-GDP in the data, comparing to a 2.4233% increase in the model.

5.6 Comparative Statics Exercise

This subsection estimates the effect of income inequality on external debt over time. I conduct a comparative statics exercise in the case of Italy for two time periods of 1985-2001

[27] In the model, average output growth rates and inflation are zero, so I omit them as control variables. Since the regression uses the ergodic means of the model as variables, country and time fixed effects are redundant.

Figure 8: Income inequality and external debt in Italy



Notes: The graph shows the time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. The left y-axis depicts the values in net foreign liablity-to-GDP (%), and the right y-axis depicts the values in pre-tax Gini (%) . Sources: Lane and Milesi-Ferretti (2018), and Solt (2019).

and 2002-2015. Figure 8 plots the time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. On average, in the period of 1985-2001, Italy has a lower levels of income inequality and external debt comparing to the period of 2002-2015.

The comparative statics exercise is as follows. I feed into the model a value of wage inequality for the period 1985-2001 and keep other parameter values fixed. I compute ergodic means of pre-tax Gini index and external-debt-to-output ratios. The 1985-2001 value of wage inequality is such that the change in the average pre-tax Gini income from 1985-2001 to 2002-2015 is the same as the change in the data. Table 6 reports the results of the policy experiment. Given the targeted increase in the pre-tax Gini indices in Italy from 1985-2001 to 2002-2015, the model can account for 93% of the increase in the external debt-to-output ratio.

Table 6: Comparative statics results for periods 1985-2001 and 2002-2015

Statistics	Data	Model
Targeted		
Δ Pre-tax Gini	3.0%	3.0%
Non-targeted		
Δ Extenal debt/Y	14%	13%

Notes: The table reports the results of the comparative statics exercise. The first column reports the changes in the data statistics, computed as the average statistics of period 2002-2015 minus the average statistics of period 1985-2001. The second column reports the results from the model. The change in the model statistics is computed as the average statistic of a simulation for the model with the wage ratio equal to 1.9475 minus the same statistic of the model with the wage ratio equal to 1.73.

5.7 Optimal Austerity

This section evaluates responses of optimal austerity policies to a negative productivity shock in the presence of inequality.

Figure 9 plots the impulse response functions of aggregate variables and tax policies with respect to a one standard deviation decline in productivity growth, computed via local projection[28]. Panel (a) plots the path of productivity growth given the negative innovation shock occurring in period 0. There are three groups of responses: aggregates, fiscal policies, and redistribution. Panel (b) and (c) plot the first group of responses of output and consumption, respectively. For fiscal policies, Panel (d), (e), and (f) plot the responses of external debt, labor taxes, and saving taxes. Lastly, Panel (g), (h), and (i) plot the responses in redistribution measures: variance of log utilities and average tax-to-income across individuals[29].

A decline in productivity growth leads to declines in both output and consumption with a higher drop in output. External debt-to-output increases in response to a low productivity. Labor taxes remains unchanged, while there is a sharp increase in saving taxes in the first period, accompanying with a decline. Note that the optimal saving taxes are negative in the long run. The initial increase in saving taxes comes from the non-binding borrowing constraints in the first few periods after a negative shock[30]. However, as borrowing constraints bind in the future, it is then optimal to increase borrowing taxes, or reducing saving taxes. In terms of redistribution, utility inequality decreases. Initially, average tax rates decrease for both agents, and decrease more for high-income agents. Average taxes then increase and increase more for high-income agents.

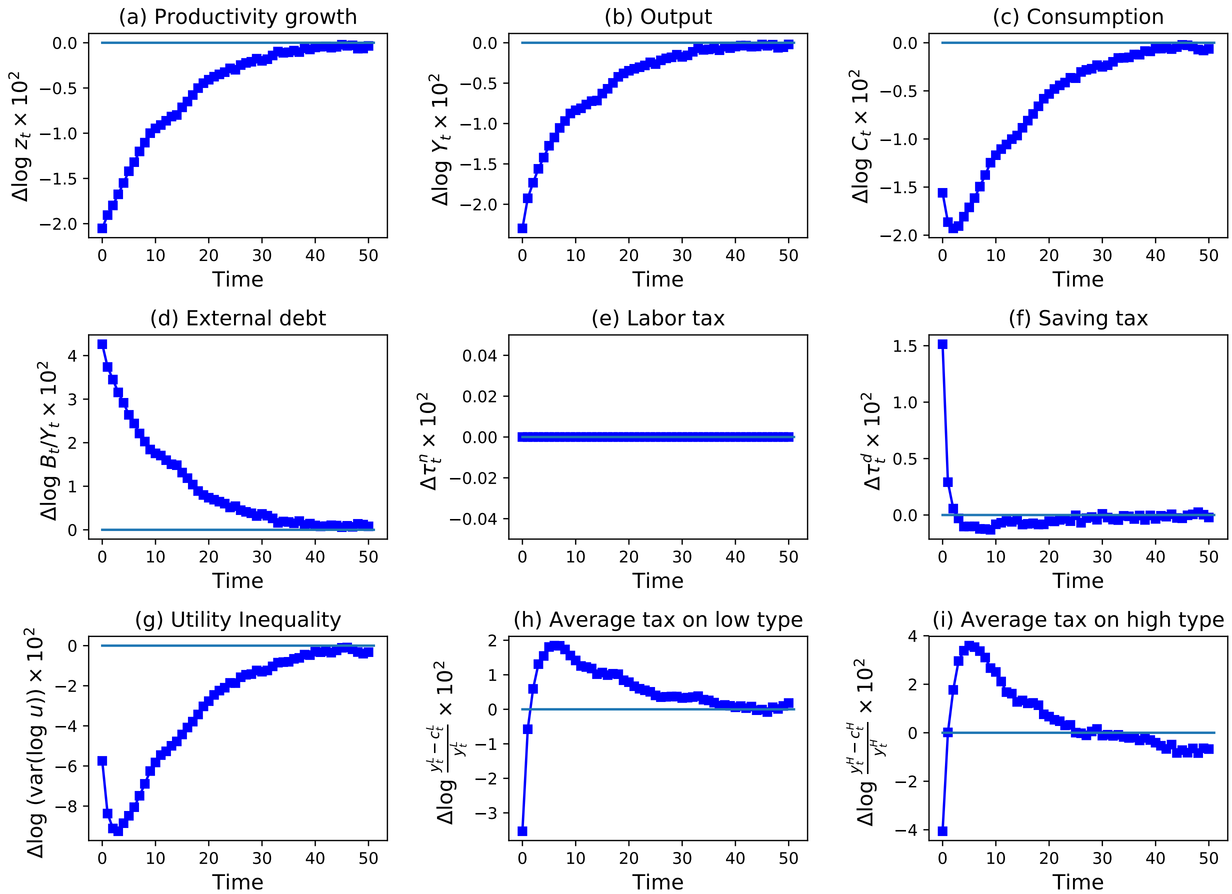
Intuitively, a negative shock leads to a reduction in the deviation utility, and so the borrowing constraint becomes non-binding. The non-binding constraint allows the govern-

[28]The approach to calculate impulse respond functions is econometrically equivalent to the approach of Jordà (2005). I simulate the economy for 10500 periods with aggregate productivity shocks and exclude the first 500 periods. I then calculate the realized time series of shocks to productivity, ϵ_t^z. To compute the response of a variable X to the shock ϵ_t^z, I perform the OLS regressions $\Delta \log X_t = \alpha + \beta_k \epsilon_{t+k}^z + \eta_t$ to get the estimated $\hat{\beta}_k$. The horizontal τ IRF is then $IRF_\tau = \sum_{k=0}^{\tau} \hat{\beta}_t$. The effect of one standard deviation shock to ϵ_t^z is the responses $\sigma_z \times IRF_\tau$. Since labor and saving taxes can be zero or negative, the dependent variable in the OLS regressions are ΔX_t instead of $\Delta \log X_t$. To my best knowledge, Mongey (2019) is the first paper that applies this computational technique in calculating impulse responses.

[29]The variance of log utilities is equal to $var_i(\log(u^i))$. The average tax-to-income ratio is defined as total amount of taxes/total income for individuals, equal to $\frac{y^i - c^i}{y^i}$ for an individual i with income y^i and consumption c^i

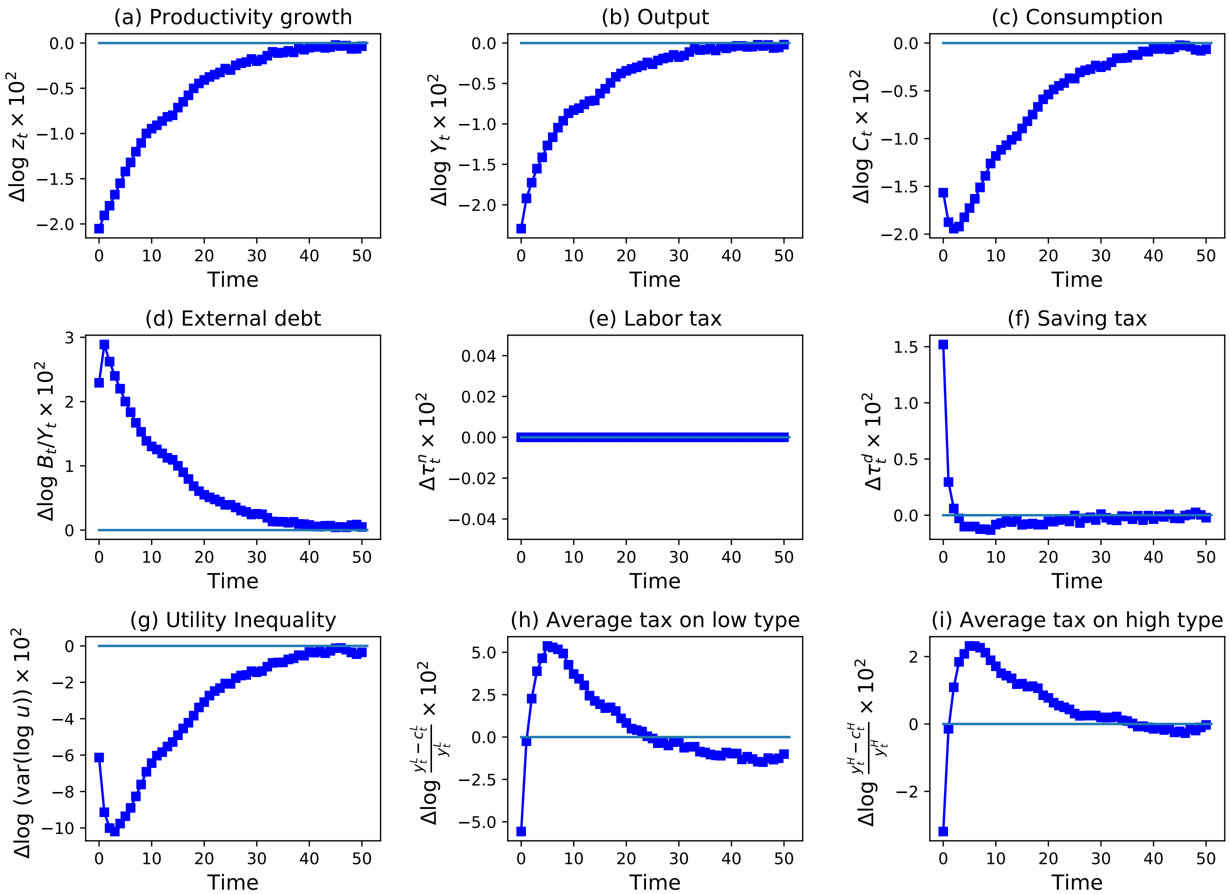
[30]Proposition 4.1 shows that $\tau^d = 0$ when borrowing constraints do not bind. So τ^d goes from a negative number to zero initially.

Figure 9: Benchmark impulse response functions to a negative productivity shock



Notes: The graph shows the impulse response functions $\sigma_z \times IRF_\tau$ computed by local projection methods as in Jordà (2005). Panel (a) plots the productivity growth response. Panel (b) and (c) plot the responses of output and consumption, respectively. Panel (d), (e), and (f) plot the responses of fiscal policies: external debt, labor, and saving taxes, respectively. Panel (g), (h), and (i) show the responses of redistribution: variance of log utilities and average tax-to-income ratios across agents.

Figure 10: Counterfactual impulse response functions to a negative productivity shock



Notes: The graph shows the impulse response functions $\sigma_z \times IRF_\tau$ computed by local projection methods as in Jordà (2005) for the model with wage ratio equal to 2.5

ment to accumulate external debt, temporarily decreases average tax rates, and increases redistribution. In future periods, the government raises taxes to repay the debt and reduces redistribution.

The responses are non-linear with respect to the underlying inequality level. Figure 10 plots the responses to a negative productivity shock given a higher wage inequality than the benchmark value. The responses to a negative shock in consumption, output, labor and saving taxes are similar to the benchmark case. However, increases in external debt is smaller, and responses of utility inequality and average taxes are larger than the benchmark case.

Higher inequality is correlated with higher external debt-to-output level. However, additional unit of external borrowing allows for more redistribution when inequality is higher.

6 Discussions

This section discusses how optimal policies respond to different model ingredients. Throughout this section, I consider the previous implementation in which the government only uses lump-sum taxes in the initial period.

6.1 Role of Aggregate Uncertainty and Heterogeneity

This subsection shows how the two main ingredients: aggregate uncertainty and heterogeneity affect the optimal policies. Table 7 describes the long-run statistics and optimal policies for different cases of the model and data. The first column reports the long run statistics in the data. The other columns specify the statistics from model simulations and the optimal tax policies that supported the efficient allocation.

Table 7: Role of Aggregate Uncertainty and Heterogeneity

	Data	Baseline	One agent	No shock	One agent & no shock
Long-run statistics					
std (C) / std (Y)	1.0	1.2	1.2	–	–
Mean B/Y	0.24	0.21	0.028	0.25	0.0
Std. dev. B/Y	0.027	0.022	0.082	–	–
Tax policies					
Initial τ^n	–	0.25	0.0	0.25	0.0
Mean LR τ^n	–	0.0042	0.0	0.0029	0.0
Lump-sum tax T_0/Y_0	–	0.87	5.8	0.6	5.7

Notes: The table reports long-run statistics and tax policies for data and different cases of the model. The first column reports data values. The second column reports the baseline values. The third column presents the results of the representative-agent case. The fourth column shows the results of the deterministic case. The last column shows the results of the deterministic and representative-agent case.

In the case of one agent and no shock, there is no external debt sustainable, and the government spending is financed via the initial lump-sum taxes. Labor taxes are zero in all periods. It is because there are no insurance nor distributive costs of default. In the case of one agent, a small amount of debt is sustainable financed partially by the initial lump-sum taxes. In this case, default is costly due to the government's inability to smooth consumption fluctuations. However, the cost is quantitatively trivial, and so the sustainable debt level is small. The deterministic case produces similar statistics as the baseline case.

External debt is high. The labor tax is high initially and low in the long run. In the long run, the borrowing constraints bind in all periods in the deterministic case, while they only bind occasionally in the baseline. This property leads to the deterministic level of external debt higher than its stationary mean.

6.2 Role of Distortionary Taxation

The previous section has shown how the redistributive tax policy comes with a cost of distortion, and by front-loading the distortion, the economy sustains a high debt. I now consider a scenario in which the planner has access to skill-specific lump-sum transfer so that the planner achieves perfect redistribution without any distortion. I show that the need to use distortionary taxation as a redistributive tool makes the government be willing to sustain highly positive debt in the long run. Table 8 reports the external debt and tax policies of the baseline model using linear taxes comparing to the alternative framework with lump-sum tax depending on income.

Table 8: Role of Distortionary Taxation

	Baseline	Skill-dependent lump-sum tax
External debt statistics		
Mean B/Y	0.21	0.029
Std. dev. B/Y	0.022	0.082
Tax policies		
Initial τ^n	0.25	0.0
Mean LR τ^n	0.0042	0.0
Lump-sum tax T_0/Y_0	0.88	5.7, high type = 20, low type $=-8.6$

Notes: The table reports long-run statistics and tax policies for the baseline and the case in which the government has access to fully skill-dependent lump-sum taxes, $T^i, \forall i \in I$. The last row and column reports the average lump-sum tax, as well as the individual lump-sum taxes.

The alternative framework quantitatively generates a much smaller amount of debt with a higher volatility than the baseline model. Across all periods, the labor tax is zero, since all of the redistribution is done via the type-dependent lump-sum taxes. In present-value terms, the government taxes the high-income agents and transfers to the low-income agents.

6.3 Role of Government Spending

This subsection shows the role of the government spending, G_t in the optimal tax and debt policies. Table 9 reports the results for the baseline case and the case of zero government spending. Without government spending, the mean external debt-to-output slightly decreases, while its volatility reduces by a half comparing to the baseline case. The optimal labor and lump-sum taxes are also lower in the case without G than the baseline case.

Table 9: Role of Government Consumption

	Baseline	$G_t = 0$
External debt statistics		
Mean B/Y	0.21	0.20
Std. B/Y	0.022	0.011
Tax Policies		
Initial τ^n	0.25	0.21
Mean LR τ^n	0.0042	-0.016
Lump-sum tax T_0/Y_0	0.88	4.4

Notes: The table reports long-run external debt statistics and tax policies for the baseline and the case in which $G_t = 0, \forall t$.

7 Conclusion

This paper studies the interaction between a country's concern for redistribution and its external indebtedness. I introduce the government's motive for redistribution and distortionary taxes into the sovereign debt framework and analyze the interaction between distortionary and distributive effect of fiscal policies and the government's lack of commitment. The endogenous borrowing constraints arise from the government's lack of commitment, and become relevant in the long run due to the domestic agents' impatience. Taxes are distortionary, and redistribution comes with the cost of tax distortions.

The paper's theoretical contribution is the effect of borrowing constraints on optimal taxation in the presence of inequality. While the magnitude of inequality determines the optimal level of taxes and redistribution, the borrowing constraints determine the optimal dynamics. The main conclusion is that labor taxes are unchanged, and borrowing taxes are zero when borrowing constraints do not bind. Binding borrowing constraints lead to permanent declines in labor taxes and positive borrowing taxes to discourage domestic borrowing.

Borrowing taxes have a redistributive benefit, which allows the government to distort labor less and increases the economy's efficiency.

The quantitative contribution is showing that the government's redistributive motive plays an important role in determining the equilibrium level of external debt. This channel comes from the additional cost of redistribution during financial autarky. The result contributes to the ongoing literature on endogenous default costs in sovereign debt models. The redistributive cost of default quantitatively accounts for 87% of the long-run average external debt-to-output, while the insurance cost of default only accounts for 13%. Another contribution of the paper is that the theory can account for the cross-sectional and time-series relationship between income inequality and external debt in the data.

The model has implications on optimal austerity policies in the presence of inequality. Estimations from the model's simulation points out that a negative productivity shock leads to an increase in external debt and a temporary decrease in borrowing taxes, while labor taxes remain unchanged. In terms of redistribution, average tax-to-income initially decreases for all agents and more for high-income agents, and utility inequality decreases. In the future, the government raises taxes to repay the debt and reduces redistribution.

The future work includes allowing for equilibrium defaults and incorporating various types of debt crises. My next project examines the interaction between inequality and default risks and their implications on austerity policies.

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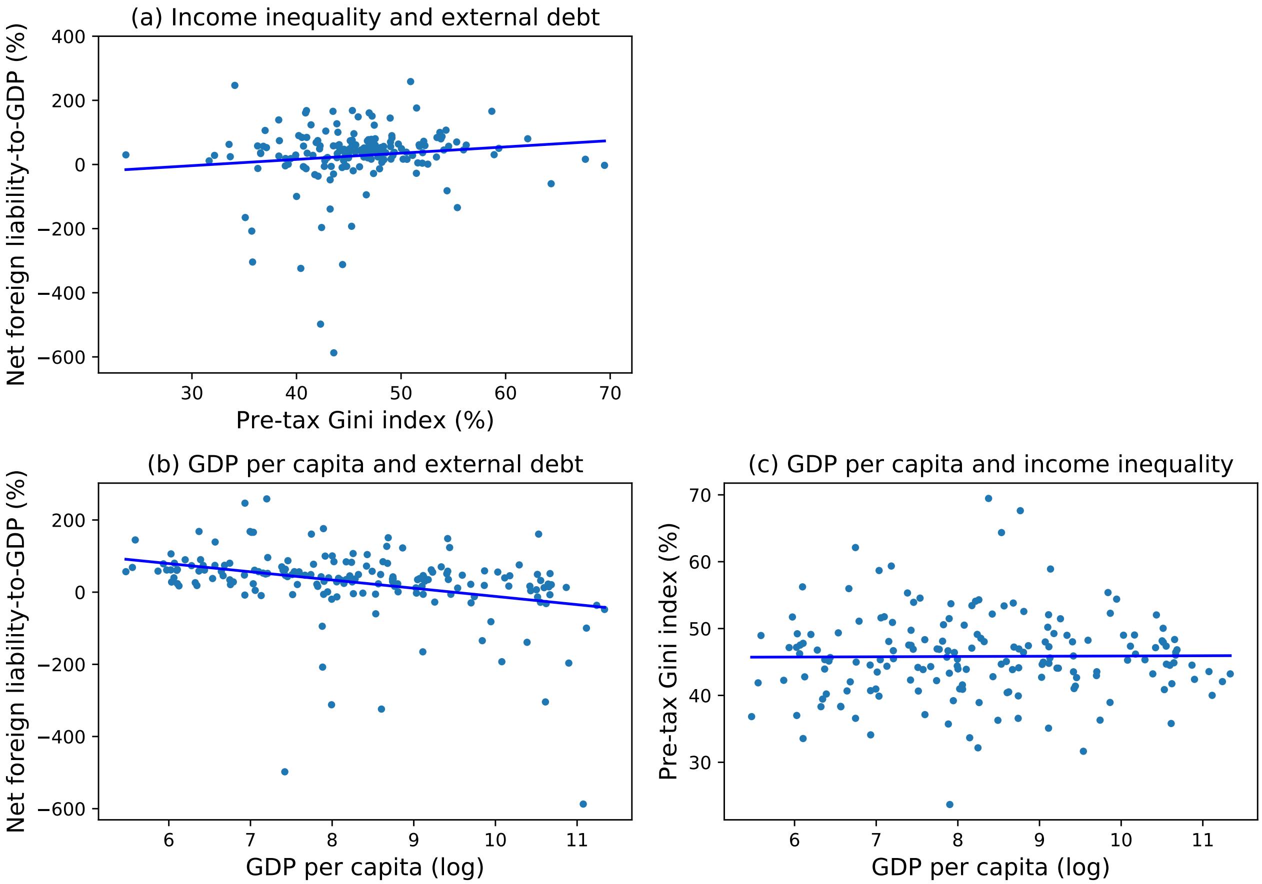
Appendix

A Additional Empirics

A.1 Figures

This subsection presents the analogs of Figure 1 and Figure 2 for all countries in the dataset.

Figure 11: Income inequality, external debt, and GDP per capita across countries

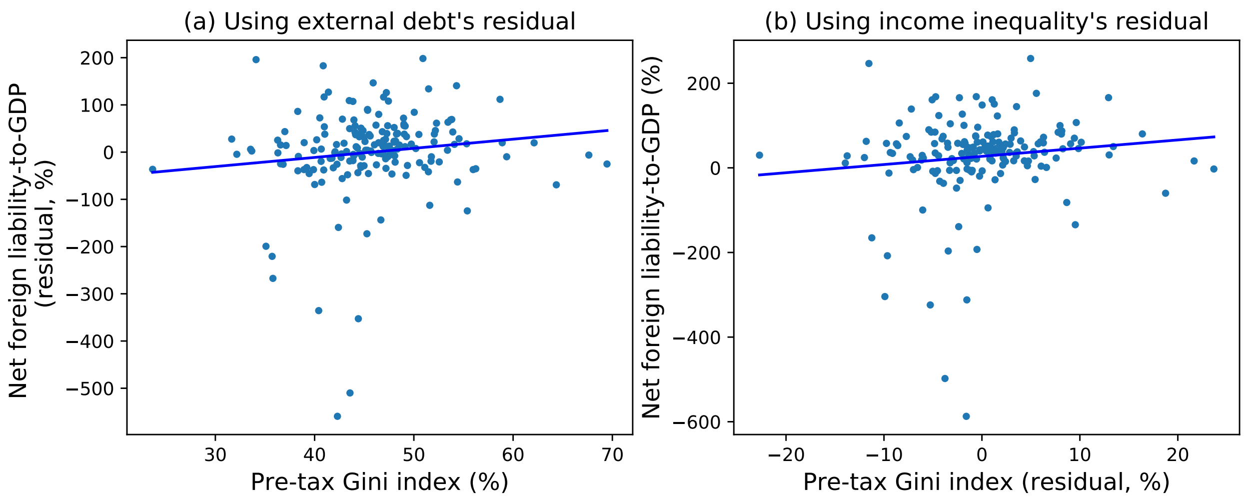


Note: The graph shows the 1985-2015 time averages of net financial liability-to-GDP, pre-tax Gini index, and GDP per capita in constant 2010 US Dollars for all economies. Panel (a) plots averages of pre-tax Gini index (%) and net foreign liability-to-GDP (%). Panel (b) plots averages of log of GDP per capita and net foreign liability-to-GDP (%). Panel (c) plots averages of log of GDP per capita and pre-tax Gini index (%). Sources: World Development Indicator Database (2019), Lane and Milesi-Ferretti (2018), and Solt (2019).

Figure 11 shows the 1985-2015 time averages of net financial liability-to-GDP, pre-tax Gini index, and GDP per capita in constant 2010 US Dollars for all countries in the data set. I omit country's labels for visual purposes. The pre-tax Gini index is positively correlated with the net foreign liability-to-GDP. The GDP per capita negatively associates with the net foreign liability-to-GDP, while it does not have a strong correlation with the pre-tax Gini index.

Figure 12 shows the cross-sectional relationship between income inequality and external debt controlling for other common factors, for all countries in the sample. Panel (a) plots the residuals ϵ_i^{nfl} (in percentage) of equation (1) and the pre-tax Gini index (%). Panel (b) plots the net foreign liability-to-GDP (%) and the residuals ϵ_i^{gini} (in percentage) of equation (2). Both panels show a positive correlation between the two main variables in the cross section.

Figure 12: Cross-sectional relationship between income inequality and external debt



Note: Panel (a) plots the residuals ϵ_i^{nfl} (in percentage) of equation (1) and the pre-tax Gini index (%). Panel (b) plots the net foreign liability-to-GDP (%) and the residuals ϵ_i^{gini} (in percentage) of equation (2). The sample includes all economies in the dataset. Sources: World Development Indicator Database (2019), Lane and Milesi-Ferretti (2018), and Solt (2019).

A.2 Net international investment position

This subsection provides the estimation using the negative of net international investment position an alternative definition of a country's external indebtedness. Table 10 reports the regression results. High income inequality levels are correlated with high external debt positions, though the coefficients are not statistically significant. Fewer countries and observations is a potential reason for differences in the results between this estimation and the one presented in the main text.

Table 10: Regression analysis of income inequality and external debt

	Dependent Variable: Net international liability position-to-GDP (%) Time periods: 1985-2015	
	(1)	(2)
Gini index, pre tax (%)	0.2681	0.1755
	(0.7528)	(0.7707)
GDP per capita (log)		20.793***
		(7.5818)
GDP growth (%)		-1.0290**
		(0.4211)
Inflation (%)		0.0787**
		(0.0386)
Country fixed effects	Yes	Yes
Time fixed effects	Yes	Yes
No. Countries	137	137
No. Observations	2028	2028

Note: The table describes the panel regression results using all countries in the data set. The first column shows the regression coefficient and standard error in parenthesis of pre-tax Gini index (%) with respect to net international liability position-to-GDP (%). The second column shows the regression coefficients and standard errors in parentheses that include other control variables: log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). Both regressions have country and time fixed effects. All standard errors are clustered. *,**,*** represent significant levels of 10%, 5%, and 1%, respectively. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and World Bank (2019).

B Sovereign Game

Before setting up the game, consider the general environment where the government's policy includes the decision to default on external bond $\{\delta(s^t)\}$, where $\delta \in \{0, 1\}$ and $\delta = 0$ implies default[31]. The government's budget constraint becomes

$$\begin{aligned} & G(s^t) + (1 - \tau^d(s^t))B^d(s^t) + \delta(s^t)B(s^t) \\ \leq & \tau^n(s^t)w(s^t)L(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) + \sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t)B(s^{t+1}) + T(s^t) \end{aligned}$$

The price of international debt takes into account the probability of default is

$$Q(s_{t+1}|s^t) = \frac{\Pr(s_{t+1}|s^t)\delta(s_{t+1}|s^t)}{1 + r^*}$$

[31]Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its foreign debt ($d_t = 1$).

As the government cannot commit to any of its policies, one can think that the government, private agents, and international lenders enter in a sovereign game where they determine their actions sequentially. In every period and every history, the state variable for the game is $\left\{B(s^t), \left(b^{i,d}(s^t)\right)_{i\in I}\right\}$. The timing of the actions is as follows.

- Aggregate shock s_t is realized
- Government chooses $z_t^G = \left(\tau^n(s^t), \tau^d(s^t), T(s^t), \delta(s^t), B(s_{t+1}, s^t), B^d(s_{t+1}, s^t)\right) \in \Pi$such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c^i(s^t), l^i(s^t), b^{d,i}(s_{t+1}, s^t)\right)$ subject to their budget constraints, the representative firm produce output by choosing $z_t^F = L(s^t)$, and the international lenders choose holdings of government's bonds $z_t^* = B(s_{t+1}, s^t)$.

Define $h^t = \left(h^{t-1}, z_{t-1}^G, \left(z_{t-1}^{H,i}\right)_{i\in I}, z_{t-1}^F, z_t^*, s_t\right) \in H^t$ as the history after shock s_t is realized. Note that the history incorporates the government's policy, allocation and prices. Define $h_p^t = \left(h^t, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period t. The government strategy is $\sigma_t^G : H^t \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i} : H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has strategy $\sigma_t^F : H_p^t \to \mathbb{R}_+^2$, and the international lenders have strategy $\sigma_t^* : H_p^t \to \mathbb{R}_+$.

Definition B.1 (Sustainable equilibrium)**.** A sustainable equilibrium is $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ such that (i) for all h^t, the policy z_t^G induced by the government strategy maximizes the socially weighted utility given λ subject to the government's budget constraint (7) (ii) for all h_p^t, the strategy induced policy $\left\{z_t^G\right\}_{t=0}^{\infty}$, allocation $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^{\infty}$, and prices $\{Q_t,\}_{t=0}^{\infty}$ constitute a competitive equilibrium with taxes.

The following focuses on characterizing a set of sustainable equilibrium in which deviation triggers autarky, where there is no domestic and foreign borrowing. In this case, the value of deviation includes the autarkic payoff.

By definitions, autarky is a sustainable equilibrium. Given that the domestic agents do not save/invest, the representative firm produces only with labor, and the international creditors do not lend, the government finds it optimal to default on its external debt, set saving and capital taxes such that the after-tax gross returns on domestic bonds and capital are zero, and set the labor tax such that it maximizes the socially weighted utility. Given the government defaulting and fully taxing all returns from domestic savings and capital, international creditors do not want to lend, agents do not save or invest in capital, and output is produced only by labor. Lastly, given that the government will be in autarky in the future, it is optimal in the current period for the government to also follow the autarkic strategies.

Reverting to autarky equilibrium is defined as a sustainable equilibrium of the above game such that following any government's deviation from the promised plans, the economy reverts to autarky. One can characterize the equilibrium as follows.

Proposition B.1 (Reverting to autarky equilibrium)**.** *An allocation and policy* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *can be supported by reverting to autarky equilibrium if and only if (i) given* z^G*, there exist prices* p *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G, p\right\}$ *is a competitive equilibrium with taxes for an open economy, and (ii) for any* t *and any* s^t*, there exists* $\underline{U}(s^t,t)$ *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *satisfies the constraint*

$$\sum_{i\in I}\lambda^i\pi^i\sum_{k\geq t, s^t\subseteq s^k}\beta^{k-t}\Pr(s^k|s^t)U^i(c^i(s^k),l^i(s^k))\geq \underline{U}(s^t,t) \tag{13}$$

Proof. Define $\underline{U}(s^t,t)$ as the maximum discounted weighted utility for the agents in period t, history s^t, when the government deviates. At period t and history s^t, the government taxes all domestic wealth ($\tau^d(s^t)=1$) and redistributes equally across agents, and the government defaults on the external debt. In subsequent period $k>t$, the economy reverts to financial autarky where agents do not save in domestic bonds, and the government is excluded from international lending. This economy ensembles a neoclassical growth closed economy that has an initial aggregate state s_t, distortionary taxation on labor, and equal initial wealth across individuals.

Suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an outcome of the reverting to autarky equilibrium. Then by the optimal problems of the government, agents, and foreign lenders, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes the weighted utility of the agents, satisfies government budget constraint and foreign lender's problem at period 0. Thus, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an open-economy tax-distorted competitive equilibrium. For any period t and history h^t, an equilibrium strategy that has the government deviates in period t triggers reverting to autarky in period $k>t$. Such strategy must deliver the weighted value at least as high as the right-hand side of (13). So $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii).

Next, suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies conditions (i) and (ii). Let h^t be any history such that there is no deviation from z^G up until period t and history s^t. Since $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t and history s^t onward. Consider a deviation plan $\hat{\sigma}^G$at period t that receives $U^d(s_t,t)$ in period t and $U^{aut}(s_t)$ for the subsequent period $k>t$. Because the plan is constructed to maximize the utility in period t, the right-hand side of (13) is the maximum attainable utility under $\hat{\sigma}^G$. Given that

$\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. $\square$

Proposition B.1 can be extended to the general characterization of sustainable equilibrium, as in Chari and Kehoe (1990).

C Separable Isoelastic Utility Case

This section provides details on the characterization of the efficient allocation and optimal policies given that the individual utility is

$$U^i(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \omega\frac{\left(\frac{l}{\theta^i}\right)^{1+\nu}}{1+\nu}$$

The analysis is an extension to Tran Xuan (2019).

C.1 Equilibrium Characterization

Individual consumption and efficient labor supply are time- and history-independently proportional to the aggregates:

$$\begin{aligned} c^i(s^t) &= h^{i,c}(C(s^t), L(s^t); \boldsymbol{\varphi}) = \psi_c^i C(s^t) \\ l^i(s^t) &= h^{i,l}(C(s^t), L(s^t); \boldsymbol{\varphi}) = \psi_l^i L(s^t) \end{aligned} \tag{B.1}$$

where

$$\psi_c^i = \frac{(\varphi^i)^{1/\sigma}}{\sum_{i\in I}\pi^i(\varphi^i)^{1/\sigma}}, \quad \psi_l^i = \frac{(\theta^i)^{\frac{1+\nu}{\nu}}(\varphi^i)^{-1/\nu}}{\sum_{i\in I}\pi^i(\theta^i)^{\frac{1+\nu}{\nu}}(\varphi^i)^{-1/\nu}} \tag{B.2}$$

C.2 Planning Problem

Let μ be the multiplier on the resource constraint, $\pi^i\eta^i$ be the multiplier on the implementability constraint for agent i, and $\beta^t \Pr(s^t)\gamma(s^t)$ be the multiplier on the aggregate debt constraint for period t. Define $\boldsymbol{\eta} = (\eta^i)_{i\in I}$ and rewrite the Larangian of the planning problem with a new pseudo-utility function that incorporates the implementability constraints:

$$\sum_{t=0}^{\infty}\beta^t W\left[s^t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] - V_C(s_0;\boldsymbol{\varphi})\sum_{i\in I}\pi^i\eta^i\left(b^i(s^0) - T\right)$$

where

$$W\left[s^t;\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\eta}\right]\equiv\sum_{i\in I}\lambda^i\pi^iU^i\left[h^i(s^t;\boldsymbol{\varphi})\right]+\sum_{i\in I}\pi^i\eta^i\left[V_C(s^t;\boldsymbol{\varphi})h^{i,c}(s^t;\boldsymbol{\varphi})+V_L(s^t;\boldsymbol{\varphi})h^{i,l}(s^t;\boldsymbol{\varphi})\right]$$

Then V and W inherit the separable and isoelastic properties from U, i.e. $\forall t,\forall s^t$,

$$\begin{aligned}V(C(s^t),L(s^t);\boldsymbol{\varphi})&=\Phi_C^V\frac{C(s^t)^{1-\sigma}}{1-\sigma}-\Phi_L^V\frac{L(s^t)^{1+\nu}}{1+\nu}\\ W\left[C(s^t),L(s^t);\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\eta}\right]&=\Phi_C^W\frac{C(s^t)^{1-\sigma}}{1-\sigma}-\Phi_L^W\frac{L(s^t)^{1+\nu}}{1+\nu}\end{aligned}$$

and the social welfare is

$$\sum_{t\geq 0,s^t\in S^t}\beta^t\Pr(s^t)\left(\Phi_C^P\frac{C(s^t)^{1-\sigma}}{1-\sigma}-\Phi_L^P\frac{L(s^t)^{1+\nu}}{1+\nu}\right)$$

where

$$\begin{aligned}\Phi_C^V&=\left[\sum_i\pi^i(\varphi^i)^{1/\sigma}\right]^\sigma; & \Phi_L^V&=\omega\left[\sum_i\pi^i\left(\varphi^i\right)^{-1/\nu}(\theta^i)^{(1+\nu)/\nu}\right]^{-\nu}\\ \Phi_C^W&=\Phi_C^V\sum_{i\in I}\pi^i\psi_c^i\left[\frac{\lambda^i}{\varphi^i}+(1-\sigma)\eta^i\right]; & \Phi_L^W&=\Phi_L^V\sum_{i\in I}\pi^i\psi_l^i\left[\frac{\lambda^i}{\varphi^i}+(1+\nu)\eta^i\right]\\ \Phi_C^P&=\Phi_C^V\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_c^i; & \Phi_L^P&=\Phi_L^V\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_l^i\end{aligned}$$

C.3 Optimal Policy

The first-order conditions of the planning problem for any period $t\geq 1$ can be summarized as

$$F_L(s^t,t)=\frac{\left\{\Phi_L^W+\Phi_L^P\sum_{k=0,s^k\subseteq s^t}^t\gamma(s^k)\right\}L(s^t)^\nu}{\left\{\Phi_C^W+\Phi_C^P\sum_{k=0,s^k\subseteq s^t}^t\gamma(s^k)\right\}C(s^t)^{-\sigma}}\tag{B.3}$$

and

$$Q(s_{t+1}|s^t)=\beta\Pr(s^{t+1}|s^t)\frac{C(s^{t+1})^{-\sigma}}{C(s^t)^{-\sigma}}\frac{\Phi_C^W+\Phi_C^P\sum_{k=0,s^k\subseteq s^{t+1}}^{t+1}\gamma(s^k)}{\Phi_C^W+\Phi_C^P\sum_{k=0,s^k\subseteq s^t}^t\gamma(s^k)}\tag{B.4}$$

The optimal policies follow

$$\tau^n(s^t)=1-\frac{1}{F_L(s^t,t)}\frac{\Phi_L^VL(s^t)^\nu}{\Phi_C^VC(s^t)^{-\sigma}}\tag{B.5}$$

$$\frac{Q^d(s_{t+1}|s^t)}{1-\tau^d(s^{t+1})}=\beta\Pr(s^{t+1}|s^t)\frac{C(s^{t+1})^{-\sigma}}{C(s^t)^{-\sigma}}\tag{B.6}$$

D Proofs

D.1 Proof of Proposition 3.1

Proof. ($\Rightarrow$) Let $\{C(s^t), L(s^t)\}_{t=0, s^t\in S^t}^{\infty}$ be an aggregate allocation of an open economy competitive equilibrium with taxes. Then by definition, $\{C(s^t), L(s^t)\}$ satisfies aggregate resource constraint for every period. Moreover, given any market weights $\boldsymbol{\varphi}$, $\{C(s^t), L(s^t)\}$ satisfies

$$
\begin{aligned}
(1-\tau^n(s^t))w(s^t) &= -\frac{V_L\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]} \\
\frac{Q^d(s_{t+1}|s^t)}{1-\tau^d(s^{t+1})} &= \beta \Pr(s^{t+1}|s^t)\frac{V_C\left[h^i(C(s^{t+1}), L(s^{t+1}); \boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s^t), L(s^t); \boldsymbol{\varphi})\right]}
\end{aligned}
$$

Substituting for $w(s^t)$ into the budget constraint (5), and using $(c^i(s^t), l^i(s^t)) = h^i(C(s^t), L(s^t); \boldsymbol{\varphi})$ gives the implementability constraint for each agent. Importantly, choose $\boldsymbol{\varphi}$ and T such that the individual implementability constraints hold with equality.

($\Leftarrow$) Given $\boldsymbol{\varphi}$, T and an allocation $\{C(s^t), L(s^t)\}$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\{w(s^t)\}$ using the firm's first-order condition (6). $\{\tau^n(s^t)\}$ can be calculated using the intra-temporal condition (9), and choosing $\left\{Q^d(s^t)\right\}$ to satisfy the inter-temporal constraint (10). Define $\{q(s^t)\}$ by $q(s^t) = \Pr(s^t)/(R^*)^t$.

Rewriting the aggregate resource constraint using $F(L) = wL$ gives

$$
\begin{aligned}
&\sum_{t\geq 0, s^t\in S^t} q(s^t)\left\{C(s^t) - (1-\tau^n(s^t))w(s^t)L(s^t) + T(s^t)\right\} \\
&+ \sum_{t\geq 0, s^t\in S^t} q(s^t)\left[G(s^t, t) - \tau^n(s^t)w(s^t)L(s^t) - T(s^t)\right] \leq -B(s^0)
\end{aligned}
\tag{C.1}
$$

Aggregating up the agent's budget constraints implies

$$
C(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) = (1-\tau^n(s^t))w(s^t)L(s^t) + (1-\tau^d(s^t))B^d(s^t) - T(s^t)
$$

or

$$
C(s^t) - (1-\tau^n(s^t))w(s^t)L(s^t) + T(s^t) = (1-\tau^d(s^t))B^d(s^t) - \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1})
$$

Substituting the last equation into (C.1) gives the government's budget constraint (7). Thus, $\{C(s^t), L(s^t)\}$ is the aggregate allocation of the constructed competitive equilibrium with taxes. $\square$

D.2 Proof of Proposition 4.1

Proof. Given equations (B.3) and (B.5), the optimal labor tax is

$$\tau^n(s^t) = 1 - \frac{\Phi_L^V \Phi_C^W + \Phi_L^V \Phi_C^P \sum_{k=0,s^k \subseteq s^t}^{t} \gamma(s^k)}{\Phi_C^V \Phi_L^W + \Phi_C^V \Phi_L^P \sum_{k=0,s^k \subseteq s^t}^{t} \gamma(s^k)} \tag{C.2}$$

Given equations (B.4) and (B.6), the optimal saving tax is[32]

$$\tau^d(s^t) = \frac{\Phi_C^W + \Phi_C^P \sum_{k=0,s^k \subseteq s^{t-1}}^{t-1} \gamma(s^k)}{\Phi_C^W + \Phi_C^P \sum_{k=0,s^k \subseteq s^t}^{t} \gamma(s^k)} - 1 \tag{C.3}$$

Suppose that the borrowing constraint does not bind at period $\mathcal{T}$ and history $s^{\mathcal{T}}$, then $\gamma(s^{\mathcal{T}}) = 0$, which implies $\tau^n(s^{\mathcal{T}}) = \tau^n(s^{\mathcal{T}-1})$, and $\tau^d(s^t) = 0$

To prove the second part of the proposition, I first show that $\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} \leq \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$. By definitions,

$$\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} = \frac{\sum_i \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]}{\sum_i \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right]} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

and

$$\frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

using the optimal conditions $\eta^i = \sum_j \pi^j \lambda^j / \varphi^j - \lambda^i / \varphi^i$, and the definitions $\mathbb{E}[x^i] \equiv \sum_i \pi^i x^i$, $\text{cov}(x^i, y^i) \equiv \mathbb{E}[x^i y^i] - \mathbb{E}[x^i]\,\mathbb{E}[y^i]$.

Lemma D.1. *cov* $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ *and cov* $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$

Proof. The first step is to show that $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j$.

Suppose $\theta^i \geq \theta^j$ and $\varphi^i < \varphi^j$, then $\psi_l^i < \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < \frac{\varphi^i}{\varphi^j} < 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq 1$, which is a contradiction.

Suppose $\varphi^i \geq \varphi^j$ and $\theta^i < \theta^j$, then $\psi_l^i \geq \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq \frac{\varphi^i}{\varphi^j} \geq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < 1$, which is a contradiction.

[32]There is an indeterminacy between Q^d and τ^d. Here I assume a particular implementation where $Q^d(s_{t+1}|s^t) = \frac{\Pr(s_{t+1}|s^t)}{1+r^*} = Q(s_{t+1}|s^t)$, that is the price of the domestic debt is the same as the price of the external debt. The government uses τ^d to manipulate the domestic stochastic discount factor.

Next, the individual implementability constraint is

$$\psi_c^i \Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma} - \psi_l^i \Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu} = \Phi_C^V C(s_0)^{-\sigma} \left(a^i(s_0) - T \right)$$

or

$$\psi_c^i = \psi_l^i \frac{\Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu}}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}} + \frac{\Phi_C^V C_0^{-\sigma} \left(a^i(s_0) - T \right)}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}}$$

By definition of ψ_c^i, $\varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j$, and by assumption, $\theta^i \geq \theta^j \iff a^i(s_0) \geq a^j(s_0)$, which implies that $\theta^i \geq \theta^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$.

Thus, $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$.

In addition, $\theta^i \geq \theta^j \iff \lambda^i \leq \lambda^j$, which implies that

$$\psi_c^i \geq \psi_c^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$
$$\psi_l^i \geq \psi_l^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$

Hence, $\operatorname{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ and $\operatorname{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$. □

Lemma D.1 and $\sigma \geq 1, \nu > 0$ imply that $\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} \leq \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$.

Suppose that the borrowing constraint binds at period $\mathcal{T}$ and history $s^{\mathcal{T}}$, then $\gamma(s^{\mathcal{T}}) > 0$, which leads to $\sum_{\tau=0,s^\tau}^{\mathcal{T}} \gamma(s^\tau) > \sum_{\tau=0,s^\tau}^{\mathcal{T}-1} \gamma(s^\tau)$. Applying equation (C.2) gives $\tau^n(s^{\mathcal{T}}) \leq \tau^n(s^{\mathcal{T}-1})$. In addition, equation (C.3) implies that $\tau^d(s^{\mathcal{T}}) < 0$. □

D.3 Proof of Proposition 5.1

Proof. Let $\{C^*(s^t), L^*(s^t), K^*(s^t)\}_{t,s^t}, \boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exists $\boldsymbol{\lambda}$ such that $\{C^*(s^t), L^*(s^t), K^*(s^t)\}_{t,s^t}, \boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). Define

$$A_C = \sum_{i \in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_c^i, \quad A_L = \sum_{i \in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_l^i \tag{C.4}$$

where ψ_c^i, ψ_l^i are defined by equations (B.2) using $\boldsymbol{\varphi}^*$. First, one can show that A_C and A_L are positive and bounded:

Lemma D.2. *Given an interior allocation,* $0 < A_C < \infty$ *and* $0 < A_L < \infty$

Proof. Interior allocation means that for any i, $c_t^i, l_t^i > 0, \ \forall t$. This implies that $\psi_c^i, \psi_l^i > 0$. By (B.2), $\boldsymbol{\varphi}^{*i} > 0$.

For all i, $\pi^i > 0, \lambda^i \geq 0$ and since $\sum_{i\in I} \pi^i \lambda^i = 1$, there exists at least an i such that $\lambda^i > 0$. Given that $\psi_c^i, \psi_l^i > 0$, $\forall i$, it must be that $A_C, A_L > 0$.

Since $\sum_{i\in I} \pi^i \boldsymbol{\varphi}^{*i} = 1 < \infty$ and $\forall i$, $\pi^i, \boldsymbol{\varphi}^{*i} > 0$, it must be that $\boldsymbol{\varphi}^{*i} < \infty$. So by definition, $\psi_c^i, \psi_l^i < \infty$. Moreover, $\boldsymbol{\varphi}^{*i} > 0$ implies that $\lambda^i / \boldsymbol{\varphi}^{*i} < \infty$. Then by definition, $A_C, A_L < \infty$. $\square$

For any M and s^M, define (P^{s^M}) the same problem as (P) with the restriction that $(C(s^t), L(s^t)) = (C^*(s^t), L^*(s^t)), \forall t > M,\ s^t \supset s^M$, $\boldsymbol{\varphi} = \boldsymbol{\varphi}^*$, $T = T^*$, and $K_t = K_t^*, \forall t$. Note that $\{C^*(s^t), L^*(s^t), K^*(s^t)\}$ is a solution to (P^{s^M}), and (P^{s^M}) has a finite number of constraints. By a Lagrangian theorem in Luenberger (1969), there exists non-negative, not-identically zero vector $\left\{r^{s^M}, \mu^{s^M}, \eta^{s^M,1}, \ldots, \eta^{s^M,I}, \gamma^{s^M}(s^0), \ldots, \gamma^{s^M}(s^M)\right\}$ such that the first-order and complementarity conditions hold for $t \in \{1, .., M\}, s^t \subseteq s^M$, i.e.

$$(\beta R^*)^t \left\{ r^{s^M} A_C + \sum_i \pi^i \eta^{s^M,i} (1-\sigma)\psi_c^i + \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) A_C \right\} \Phi_C^V C(s^t)^{-\sigma} = \mu^{s^M} \tag{C.5}$$

$$(\beta R^*)^t \left\{ r^{s^M} A_L + \sum_i \pi^i \eta^{s^M,i} (1+\nu)\psi_l^i + \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) A_L \right\} \Phi_L^V L(s^t)^{\nu} = \mu^{s^M} F_L(K(s^t), L(s^t), s^t) \tag{C.6}$$

Equation (C.5) can be rewritten as

$$(\beta R^*)^t \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) = \frac{\mu^{s^M}}{A_C \Phi_C^V C(s^t)^{-\sigma}} - (\beta R^*)^t \left[r^{s^M} + \frac{1}{A_C} \sum_i \pi^i \eta^{s^M,i} (1-\sigma)\psi_c^i \right] \tag{C.7}$$

The following lemma shows that μ^{s^M} and $C(s^t)^{-\sigma}$ are always positive for the sub-problem (P^{s^M}) for any $M \geq 1$ and any s^M.

Lemma D.3. *In the sub-problem (P^{s^M}) for any $M \geq 1$ and s^M, $\mu^{s^M} > 0$*

Proof. Suppose, by contradiction, that $\mu^{s^M} = 0$ so the resource constraint does not bind. Consider allocation $\{C(s^t), L(s^t), K(s^t)\}$ which is the solution to (P^{s^M}). Then there exists $\epsilon > 0$ such that

$$\sum_{t \geq 0, s^t} q(s^t) \left[F(L(s^t), s^t, t) - G(s^t, t) - C(s^t) \right] - B(s^0) - \epsilon \geq 0$$

Define $\left\{\hat{L}(s^t)\right\}$ such that for a fixed s^1, $\hat{L}(s^1) < L(s^1)$ such that $F(\hat{L}(s^1), s^1, 1) = F(L(s^1), s^1, 1) - \epsilon/q(s^1)$, and $\hat{L}(s^t) = L(s^t),\ \forall t > 1, \forall s^t$. The allocation $\left\{C(s^t), \hat{L}(s^t)\right\}$ satisfies the resource constraint and because of the preference's strict monotonicity, $\left\{C(s^t), \hat{L}(s^t)\right\}$ also satisfies the implementability constraints and the aggregate debt constraints. However,

$$\sum_{i\in I}\lambda^i\pi^i\sum_{t\geq 0,s^t}\beta^t\Pr(s^t)U^i\left[h^i(C(s^t),\hat{L}(s^t);\varphi)\right] > \sum_{i\in I}\lambda^i\pi^i\sum_{t\geq 0,s^t}\beta^t\Pr(s^t)U^i\left[h^i(C(s^t),L(s^t);\varphi)\right]$$

which contradicts $\{(C(s^t), L(s^t))_{s^t}\}_{t=0}^{\infty}$ being optimal solution for (P^{s^M}). □

The consumption path is bounded below by zero in the long run, i.e.

Lemma D.4 (No immiseration)**.** *Suppose Assumptions 1 and 4 hold, then for any efficient allocation* $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$, $\liminf_{t\to\infty} C_t^* > 0$.

Proof. Given an efficient allocation $\{C^*(s^t), L^*(s^t)\}$, suppose, by contradiction that for a sequence of shocks $\{s_0, ..., s_t, ...\}$, $\liminf_{t\to\infty} C^*(s^t) \leq 0$. Find $\epsilon > 0$ such that $\forall t, \forall s^t$,

$$\sum_{k=t}^{\infty}\beta^{\tau-t}\sum_{s^t\subseteq s^k}Pr(s^\tau)\left[\Phi_C^V\frac{C(s^k)^{1-\sigma}}{1-\sigma}-\Phi_L^V\frac{L^*(s^k)^{1+\nu}}{1+\nu}\right]\leq M_U$$

with $C(s^t) = \epsilon$ and $C(s^k) = C^*(s^k)$, $\forall k > t,\ s^t \subset s^k$. Such ϵ exists since the utility function is unbounded below. Because $\liminf_{t\to\infty} C^*(s^t) \leq 0$, there exists a t_0 such that $C^*(s^{t_0}) < \epsilon$. Then by monotonicity,

$$\begin{aligned}
&\sum_{k=t_0}^{\infty}\beta^{\tau-t_0}\sum_{s^{t_0}\subseteq s^k}\Pr(s^k)\left[\Phi_C^V\frac{C^*(s^k)^{1-\sigma}}{1-\sigma}-\Phi_L^V\frac{L^*(s^k)^{1+\nu}}{1+\nu}\right]\\
<&\sum_{k=t_0}^{\infty}\beta^{\tau-t_0}\sum_{s^{t_0}\subseteq k}\Pr(s^k)\left[\Phi_C^V\frac{C(s^k)^{1-\sigma}}{1-\sigma}-\Phi_L^V\frac{L^*(s^k)^{1+\nu}}{1+\nu}\right]\\
\leq& M_U\\
\leq& \underline{U}(s^{t_0}, t_0)
\end{aligned}$$

which contradicts the aggregate debt constraint at s^{t_0}. □

Taking the limit on both sides of equation (C.7) gives

$$\begin{aligned}
\lim_{t\to\infty}(\beta R^*)^t\sum_{k=0}^{t}\sum_{s^k\subseteq s^M}\gamma^{s^M}(s^k) &= \lim_{t\to\infty}\left\{\frac{\mu^{s^M}}{A_C\Phi_C^V C(s^t)^{-\sigma}}-(\beta R^*)^t\left[r^{s^M}+\frac{1}{A_C}\sum_i\pi^i\eta^{s^M,i}(1-\sigma)\psi_c^i\right]\right\}\\
&= \lim_{t\to\infty}\frac{\mu^{s^M}}{A_C\Phi_C^V C(s^t)^{-\sigma}}\\
&> 0
\end{aligned}$$

□

E Computation

This section provides additional details that is implemented in Section 5 .

E.1 Deviation Utility

The deviation utility $\underline{U}(z)$ is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes of a closed economy where the government does not issue both domestic and external debts.

$$\underline{U}(z) \equiv \max_{c^i(s^t), l^i(s^t), \tau^n(s^t), T(s^t)} \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i \mathbb{E}_z U^i(c^i(s^t), l^i(s^t))$$

$$\begin{aligned}
s.t. \qquad & C(s^t) + G = z(s^t)L(s^t) \\
& c^i(s^t) + \sum_{s_{t+1}} Q(s_{t+1}|s^t) b^{d,i}(s^{t+1}) = (1-\tau^n(s^t))z(s^t)l^i(s^t) + b^{d,i}(s^t) - T(s^t) \\
& (1-\tau^n(s^t))z(s^t) = -\frac{U_l^i(c^i(s^t), l^i(s^t)}{U_c^i(c^i(s^t), l^i(s^t)} \\
& Q(s_{t+1}|s^t) = \beta \Pr(s^{t+1}|s^t) \frac{U_c^i(c^i(s^{t+1}), l^i(s^{t+1}))}{U_c^i(c^i(s^t), l^i(s^t))} \\
& b^{d,i}(s^0) = b^{d,j}(s^0), \ \forall t \geq 1, \sum_{i \in I} b^{d,i}(s^t) = 0 \\
& z(s^0) = z
\end{aligned}$$

There exist a vector of market weights $\hat{\boldsymbol{\varphi}}$ that satisfies the conditions in Proposition 3.1 such that

$$\begin{aligned}
\underline{U}(z) \equiv \max_{C(s^t), L(s^t), \hat{\boldsymbol{\varphi}}, T} & \sum_{t=0}^{\infty} \beta^t \mathbb{E}_z \left[\hat{\Phi}_C^W \log C(s^t) - \hat{\Phi}_L^W \frac{L(s^t)^{1+\nu}}{1+\nu} \right] \\
s.t. \qquad & C(s^t) + G = z(s^t)L(s^t) \\
& z(s^0) = z
\end{aligned}$$

where $\hat{\psi}_c^i, \hat{\psi}_l^i, \hat{\Phi}_C^V, \hat{\Phi}_L^V, \hat{\Phi}_C^W, \hat{\Phi}_L^W$ are calculated using $\hat{\boldsymbol{\varphi}}$ (see Appendix C for the formulas).

E.2 Computational Algorithm

1. Guess μ and φ. Compute η.

- Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ
  Initial guess for $V(s_t, \mu_t, \Gamma_{t-1}) = \sum_{j\geq 0, s^t \subseteq s^{t+j}} \beta^j \Pr(s^{t+j}) \left[\Phi_C^P \frac{C(s^{t+j})^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^{t+j})^{1+\nu}}{1+\nu}\right]$.

- Assume the constraint does not bind in s_t: $\gamma(s_t) = 0$. Solve for the allocation $C(s_t), L(s_t)$ using the first-order conditions

$$\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1}\right] C(s_t)^{-\sigma} = \mu$$
$$\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1}\right] L(s_t)^{\nu} = \mu F_L(s_t)$$

- Since $\gamma(s_t) = 0$, compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$ (interpolating the expectation), then compute

$$\begin{aligned} A(s_t) &= \sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^\tau \subseteq s^t} Pr(s^\tau) \left[\Phi_C^P \frac{C(s^\tau)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^\tau)^{1+\nu}}{1+\nu}\right] \\ &= \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1}) \end{aligned}$$

- Check if $A(s_t) \geq \underline{U}(s_t)$. If it is, proceed to the next step. If not, solve for $C(s_t), L(s_t), \gamma(s_t)$ using these equations

$$\begin{aligned} \left[\mu_t \Phi_C^W + \Phi_C^V \left(\Gamma_{t-1} + \gamma(s_t)\right)\right] C(s_t)^{-\sigma} &= \mu \\ \left[\mu_t \Phi_L^W + \Phi_L^V \left(\Gamma_{t-1} + \gamma(s_t)\right)\right] L(s_t)^{\nu} &= \mu F_L(s_t) \\ \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} & \\ +\beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V\left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t)\right)\right) &= \underline{U}(s_t) \end{aligned}$$

- Given $C(s_t), L(s_t), \gamma(s_t)$($\gamma$ can be zero or not), compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \Gamma_{t-1} + \gamma(s_t))$. Update the value function

$$\begin{aligned} V^{n+1}(s_t, \Gamma_{t-1}) = {} & \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} \\ & + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V^n\left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t)\right)\right) \end{aligned}$$

2. Compute residuals to find μ and φ

$$r^{\mu} = \sum_{t\geq 0, s^t \in S^t} q^*(s^t)\left[F(L(s^t), s^t) - G(s^t) - C(s^t)\right] - B(s^0)$$

$$r^{\varphi} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^V \left(\psi_c^i - \psi_c^j\right) C(s^t)^{1-\sigma} - \Phi_L^V \left(\psi_l^i - \psi_l^j\right) L(s^t)^{1+\nu}\right]$$

3. Find μ and φ such that $r^{\mu} = 0$ and $r^{\varphi} = 0$.

F Additional Quantitative Exercises

F.1 Measuring the distributive cost of default

This subsection performs a counterfactual exercise that measures the equivalent productivity loss of the distributive component of the cost of default. I consider a representative-agent model in which default not only leads to exclusions from financial markets, but also a permanent loss of productivity. I model a proportional loss of productivity in default as in Aguiar and Gopinath (2006)[33], i.e. $z^{default} = (1-\kappa)z$, for $0 \leq \kappa \leq 1$. I calibrate κ such that the representative-agent model generates the same amount of external debt-to-output as the hetereogenous-agent model.

Table 11: Measuring the distributive cost of default

Description	Parameter	Value	Target	Baseline	One agent exog. prod. loss
Fraction of prod. loss in default	κ	0.3%	Mean B/Y	21%	21%

Notes: The table reports the value of κ, fraction of productivity that is lost in default such that the long-run average external debt-to-output of the baseline model is the same as the one-agent model. The statistics come from simulations of the models for 10500 periods, excluding the first 500 periods.

Table 11 reports the results of the exercise. The equivalent loss of productivity for the distributive component is 0.3%

[33] For non-linear default costs that are widely used in the literature, see Arellano (2008) and Chatterjee and Eyigungor (2012).

G Data

G.1 Data sources

Most data are annual series covering the 1985-2015 period. Some data samples cover the 2002-2015 period.

- Net foreign liability is the negative of net foreign asset (NFA) from the External Wealth of Nations Database, Lane and Milesi-Ferretti (2018)

- Pre-tax Gini Index the market Gini from the Standardized World Income Inequality Database, Solt (2019).

- GDP per capita is the constant 2010 US Dollar GDP per capita series from World Development Indicator Database (2019)

- GDP growth is the log difference of constant 2010 US Dollar GDP series from World Development Indicator Database (2019)

- Inflation is the annual inflation series measured by the GDP deflator from World Development Indicator Database (2019)

- Real GDP is GDP series in constant local currency units from World Development Indicator Database (2019)

- Real return on German bond is the interest rate on Gernman bond adjusted for inflation measured by the GDP deflator. The interest rate is the long-term interest rate for converenge purposes from the Eurostat Database (2019). These bonds have 10-year maturity and are denominated in Euro.

- Real interest rate is the lending interest rate adjusted for inflation as measured by the GDP deflator from World Development Indicator Database (2019)

- Italy's cross-sectional wage inequality is calculated from the micro-data by Jappelli and Pistaferri (2010) using Surveys of Household Income and Wealth conducted by the Bank of Italy for the period 1980-2006.

- Government consumption is the general government final consumption expenditure series from World Development Indicator Database (2019)

- Private consumption is the households and NPISHs final consumption expenditure series from World Development Indicator Database (2019)

G.2 Lists of countries

- List of all countries in the data set:

Afghanistan, Albania, Algeria, Angola, Antigua and Barbuda, Argentina, Armenia, Australia, Austria, Azerbaijan, Bahrain, Bangladesh, Barbados, Belarus, Belgium, Belize, Benin, Bhutan, Bolivia, Bosnia and Herzegovina, Botswana, Brazil, Bulgaria, Burkina Faso, Burundi, Cabo Verde, Cambodia, Cameroon, Canada, Central African Republic, Chad, Chile, China, Colombia, Comoros, Congo, Costa Rica, Côte d'Ivoire, Croatia, Cyprus, Czech Republic, Dem. Rep. Congo, Denmark, Dominica, Dominican Republic, Ecuador, Egypt, Arab Rep., El Salvador, Equatorial Guinea, Estonia, Ethiopia, Fiji, Finland, France, Gabon, Gambia, The, Georgia, Germany, Ghana, Greece, Grenada, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, Hong Kong SAR, China, Hungary, Iceland, India, Indonesia, Iran, Islamic Rep., Iraq, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kazakhstan, Kenya, Kiribati, Korea, Kosovo, Kuwait, Kyrgyz Republic, Lao PDR, Latvia, Lebanon, Lesotho, Libya, Lithuania, Luxembourg, Madagascar, Malawi, Malaysia, Maldives, Mali, Malta, Mauritania, Mauritius, Mexico, Micronesia, Fed. Sts., Moldova, Mongolia, Montenegro, Morocco, Mozambique, Myanmar, Namibia, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Oman, Pakistan, Palau, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Qatar, Romania, Russian Federation, Rwanda, Samoa, Sao Tome and Principe, Saudi Arabia, Senegal, Serbia, Seychelles, Sierra Leone, Singapore, Slovak Republic, Slovenia, Solomon Islands, South Africa, South Sudan, Spain, Sri Lanka, St. Kitts and Nevis, St. Lucia, Sudan, Suriname, Sweden, Switzerland, Tajikistan, Tanzania, Thailand, Timor-Leste, Togo, Tonga, Trinidad and Tobago, Tunisia, Turkey, Turkmenistan, Tuvalu, Uganda, Ukraine, United Arab Emirates, United Kingdom, United States, Uruguay, Uzbekistan, Vanuatu, Venezuela, RB, Vietnam, Yemen, Rep., Zambia, Zimbabwe.

- List of advanced and emerging market economies:

Argentina, Australia, Austria, Belgium, Bolivia, Brazil, Bulgaria, Canada, Chile, China, Colombia, Costa Rica, Denmark, Dominican Republic, Ecuador, El Salvador, Finland, France, Germany, Ghana, Greece, Hungary, Iceland, India, Indonesia, Ireland, Italy, Japan, Kenya, Korea, Malaysia, Mexico, Netherlands, Nigeria, Norway, Panama, Peru, Philippines, Poland, Portugal, Romania, Russian Federation, Singapore, South Africa, Spain, Sri Lanka, Sweden, Switzerland, Thailand, Tunisia, Turkey, United Kingdom, United States, Uruguay, Venezuela.