Sovereign Debt Sustainability and Redistribution*

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Abstract

This paper develops a theory of sovereign debt sustainability driven by the government's redistributive motive. I study a heterogeneous-agent small open economy where redistribution relies on distortionary labor taxation and the government lacks commitment. Access to international credit markets lowers the cost of redistribution, while default into financial autarky raises it, generating an endogenous cost of default. Quantitatively, the model accounts for the buildup of Italy's external debt and the positive cross-country correlation between pre-tax income inequality and external debt. Optimal austerity is more gradual when distributional concerns are present.

Keywords: Inequality; Limited commitment; Optimal taxation; Redistribution; Sovereign debt

JEL Classifications: E62; F38; F41; H21; H23; H63

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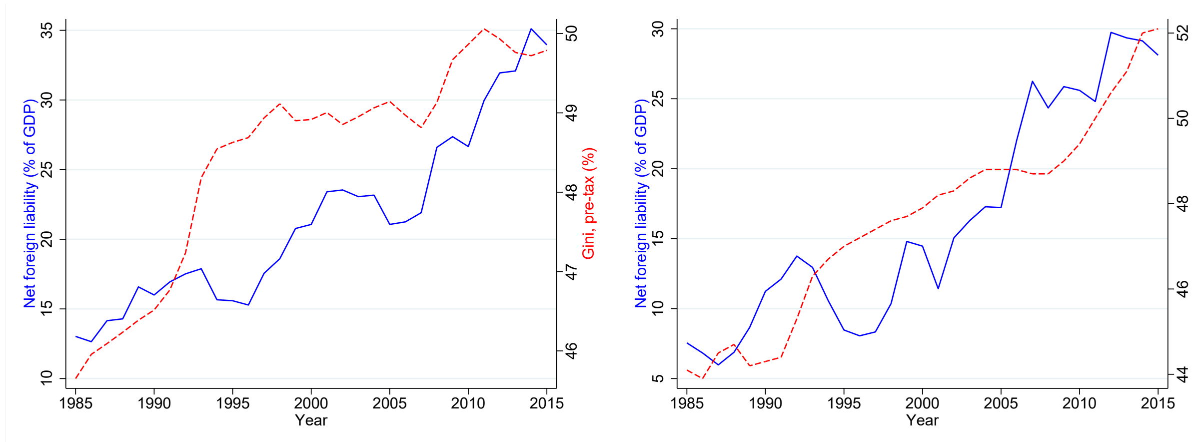
Introduction

The recent European debt crises have prompted policy debates on the design of fiscal policies during severe downturns, when output contracts and external debt rises until constrained by repayment capacity. Austerity measures—higher taxes or lower government spending—expand repayment capacity but impose unequal burdens across residents. [1]At the same time, inequality and external debt have risen together across many economies. Figure 1 documents this joint evolution for a broad set of countries and for Italy, where both debt accumulation and inequality have been particularly pronounced.



(a) Countries with positive external debt in 1985-2015

(b) Italy

Figure 1: Income Inequality and External Debt

Note: Figure 1 plots the time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) for the average indebted country (Panel (a)) and Italy (Panel (b)) from 1985 to 2015. The left y-axis depicts the values in net foreign liablity-to-GDP (%), and the right y-axis depicts the values in pre-tax Gini (%) . Sources: Lane and Milesi-Ferretti (2018), and Solt (2019).

These observations raise two central questions: First, how do redistributive motives affect a government's incentive to sustain debt? Second, how does inequality shape the design of optimal austerity policies? This paper addresses these questions by introducing redistributive concerns into a sovereign debt model with limited commitment, highlighting the tradeoff between redistribution and debt sustainability.

The paper makes three main contributions. First, it develop a theory of external debt sustainability driven by redistributive motives. Costly redistribution generates an endogenous cost of default: defaulting into financial autarky raises reliance on distortionary taxation, making re-

[1]The United Kingdom and Ireland pursued austerity mainly through spending cuts, while Greece, Italy, Portugal, and Spain combined tax increases with expenditure reductions, often targeting public services, pensions, and education. Evidence suggests that these policies exacerbated inequality: Monastiriotis (2011) links Greece's fiscal consolidation to deeper regional imbalances, Leventi and Matsaganis (2016) find higher poverty and after-tax inequality, and Brinca et al. (2019) show that consolidations are more contractionary when inequality is high.

distribution less efficient. Second, the paper quantifies the redistributive channel by calibrating the model to Italy. Redistribution accounts for roughly 60 percent of Italy's long-run external debt, compared to only 12 percent explained by the standard aggregate insurance channel. The framework is also consistent with the upward trend of Italian debt and the positive cross-country correlation between inequality and external debt. Third, the paper examines austerity design in the presence of inequality. The optimal adjustment to a negative productivity shock is gradual: the government initially expands borrowing and redistribution, then repays debt over time by raising taxes and reducing redistribution.

The mechanism operates through the cost of redistribution, which arises from the need to use distortionary labor taxes to reallocate resources across agents. The government therefore faces a tradeoff between redistributive gains and efficiency costs, and the costs increase with inequality or stronger redistributive motives. Sustaining external debt mitigates these costs by allowing the government to substitute away from distortionary taxation. Impatient private agents borrow abroad without internalizing that their borrowing tightens the limited commitment constraint, creating an overborrowing externality. An optimal borrowing tax corrects this externality while also provides redistributive benefits, since high-skilled agents hold more foreign debt and thus bear a larger tax burden. In contrast, default into financial autarky removes this instrument and forces the government to rely solely on distortionary taxes, making positive debt sustainability preferable.

The findings highlight the importance of inequality for sovereign borrowing and fiscal policy. The theoretical contribution is to show that costly redistribution generates an endogenous cost of default, linking inequality directly to sustainable debt levels. The quantitative contribution is to demonstrate that this channel is quantitatively relevant in both Italy and cross-country data. The policy contribution is to provide a rationale for more gradual austerity adjustments in unequal economies.

Related literature. This paper contributes to the sovereign debt literature with limited commitment. Seminal work emphasizes self-enforcing borrowing constraints and the role of default risk in international borrowing (Eaton and Gersovitz, 1981; Bulow and Rogoff, 1989; Aguiar and Amador, 2011, 2014, 2016). Later work examines the quantitative implications of these models, highlighting volatile consumption, taxation, and fiscal policies (e.g., Kehoe and Perri, 2002; Cuadra et al., 2010; Pouzo and Presno, 2015; Arellano and Bai, 2016; Karantounias, 2018). I extend this literature by introducing heterogeneity and redistributive concerns into a limited commitment framework, showing how costly redistribution shapes sustainable debt.

The paper also relates to a growing literature linking inequality and sovereign default risk. Recent work shows that inequality shapes sovereign risk premia, debt dynamics, and fiscal policy responses (Ferriere, 2015; Balke and Ravn, 2016; D'Erasmo and Mendoza, 2016, 2020; Dovis et al., 2016; Jeon and Kabukcuoglu, 2018; Bianchi et al., 2023; Tran-Xuan, 2021). My contribution is to identify costly redistribution as a novel source of endogenous default costs, complementing mechanisms such as efficiency losses in production (Mendoza and Yue, 2012) or unemployment dynamics (Balke, 2017).

Finally, the paper connects to the public finance literature on redistribution and debt management. While Werning (2007)shows that debt is used to smooth redistribution costs, and Bhandari et al. (2016) find that optimal long-run debt is not necessarily positive, I show that sustaining positive external debt is optimal when redistribution is costly. The paper further contributes to the policy perspective by demonstrating that redistribution concerns justify more gradual fiscal adjustment during episodes of austerity.

Outline. The paper is organized as follows. Section 1 sets up the environment and competitive equilibrium. Section 2 defines the sustainable equilibrium and establishes the main theoretical result. Section 3 illustrates costly redistribution in an example economy. Section 4presents the quantitative results and implications for austerity. Section 5 conducts sensitivity analysis, while Section 6 provides empirical evidence. Section 7 concludes.

1 A Model of Sovereign Debt and Inequality

In this section, I set up a small open economy model with aggregate uncertainty, heterogeneous agents, and a benevolent government, where the competitive equilibrium is characterized by aggregate allocations and a time-invariant distribution of marginal utility shares.

1.1 Environment

A small open economy consists of a measure-one continuum of infinitely-lived agents different by labor productivity types $(\theta^i)_{i\in I}$, which are publicly observable. The fraction of agents with productivity θ^i is π^i, where $(\pi^i)_{i\in I}$ and $(\theta^i)_{i\in I}$ are normalized such that $\sum_{i\in I}\pi^i = 1$ and $\sum_{i\in I}\pi^i\theta^i = 1$. All agents have the same discount factor β and the static utility $U(c,n)$ over consumption c and hours worked n. The utility of agent with productivity θ^i over consumption $c^i_t \geq 0$ and efficiency-unit labor $l^i_t \geq 0$ is

$$\sum_{t=0}^{\infty}\beta^t U^i(c^i_t, l^i_t) \tag{1}$$

where $U^i(c,l) = U\left(c, \frac{l}{\theta^i}\right)$. An individual allocation specifies consumption and labor in every period: (c^i, l^i). The aggregate allocation are defined as $C \equiv \sum_{i\in I}\pi^i c^i$ and $L \equiv \sum_{i\in I}\pi^i l^i$.

There is a representative firm that uses labor to produce a single final good with the constant-returns-to-scale production function $F(L,t)$, where L is the aggregate labor.

Government spending $\{G_t\}_{t=0}^{\infty}$ is exogenous. In every period t, the government can issue both domestic and foreign bonds and impose a lump-sum tax T_t, a marginal tax on labor income τ^n_t, and a tax on the return of private saving τ_t^a.

Domestic and international financial markets are competitive. The exogenous risk-free international interest rate for borrowing is $\{r_t^*\}_{t=0}^{\infty}$, and $R_t^* = 1 + r_t^*$ denote the gross risk-free interest rate in period t. Let $Q_t^* = \Pi_{\tau=1}^t \frac{1}{1+r_\tau^*}$ be the international price of one unit of period-t consumption in units of period-0 consumption, and normalize $Q_0^* = 1$.

1.2 Competitive Equilibrium

Private agent. Private agents have access to both domestic and international financial markets. Individual agent of type $i \in I$ faces the sequential budget constraint

$$c_t^i + q_{t+1}^d a_{t+1}^{d,i} + q_{t+1}^f a_{t+1}^{f,i} \leq (1-\tau_t^n) w_t l_t^i + (1-\tau_t^a) a_t^{d,i} + (1-\tau_t^a) a_t^{f,i} - T_t, \tag{2}$$

where $c_t^i, l_t^i, a_t^{d,i}, a_t^{f,i}$ denote the consumption, labor, and domestic bond and foreign bond holdings of agent i in period t, respectively. q_{t+1}^d and q_{t+1}^f are the prices of one unit of domestic and foreign bonds in period $t+1$. Define $a_t^i = a_t^{d,i} + a_t^{f,i}$ as the individual i's private bond holding. No-Ponzi conditions for both bonds apply. No arbitrage implies that

$$q_t^d = q_t^f = \frac{1}{1+r_t^*} \tag{3}$$

Representative firm. The firm takes as given the labor wage w_t and chooses aggregate labor L_t to maximize profit

$$\max_{L_t} \quad F(L_t, t) - w_t L_t,$$

which gives the following first-order condition

$$w_t = F_L(L_t, t). \tag{4}$$

The firm's profit is zero in equilibrium because of the constant-returns-to-scale production function.

Government. The government's budget constraint is

$$G_t + B_t^d + B_t^f \leq \tau_t^n w_t L_t + \tau_t^a \left(A_t^d + A_t^f\right) + T_t + q_{t+1}^d B_{t+1}^d + q_{t+1}^f B_{t+1}^f$$

where B_t^d and B_t^f are government's domestic and foreign bonds. Define $B_t^g = B_t^d + B_t^f$ as the total government debt, and $A_t = A_t^d + A_t^f$ as the total private debt. There is a no-Ponzi condition such that the present value of government debt is bounded below.

The government's present-value budget constraint is

$$\sum_{t=0}^{\infty} Q_t^* \left[\tau_t^n w_t L_t + \tau_t^a A_t + T_t - G_t\right] \geq B_0^g, \tag{5}$$

that is, the present-value of the government's primary balance is at least the initial value of total government debt.

Resource constraint. Define $B_t = B_t^f - A_t^f$ as the external debt issued by the economy. The present-value resource constraint of the economy is

$$\sum_{t=0}^{\infty} Q_t^* \left[F(L_t) - C_t - G_t\right] \geq B_0, \tag{6}$$

that is, the present-value of the economy's net resources is at least the initial value of external debt.

Competitive equilibrium. Given the above equations, one can define the following competitive equilibrium with government policies.

Definition 1.1. Given initial external debt B_0 and individual private bond holdings $(a_0^i)_{i\in I}$, a competitive equilibrium with government policies for an open economy is individual agent's allocation $z^{H,i} = \left\{\left(c_t^i, l_t^i, a_{t+1}^{d,i}, a_{t+1}^{f,i}\right)\right\}_{t=0}^{\infty}$, $\forall i \in I$, the representative firm's allocation $z^F = \{L_t\}_{t=0}^{\infty}$, prices $p = \left\{q_t^d, q_t^f, w_t\right\}_{t=0}^{\infty}$, and government's policy $z^G = \left\{\tau_t^n, \tau_t^a, T_t, B_t^d, B_t^f\right\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves individual i's problem that maximizes (1) subject to (2) and no-Ponzi conditions of agent's debts, (ii) given p and z^G, z^F solves firm's problem, (iii) the government budget constraint (5) holds, (iv) the aggregate resource constraint (6) is satisfied, (iv) the domestic bond market clears $B_t^d = \sum_{i\in I} \pi^i a_t^{d,i}$, and (v) p satisfies equations (3) and (4) given z^G.

1.3 Characterizing Competitive Equilibrium

Following Werning (2007), we show that the competitive equilibrium allocation can be characterized by a set of aggregate allocation and a time-invariant distribution of marginal utility. In equilibrium, the intra-temporal and inter-temporal rates of substitution are the same across agents, i.e. in each period t and for any individual i,

$$\begin{aligned}(1-\tau_t^n)w_t &= -\frac{U_l^i(c_t^i, l_t^i)}{U_c^i(c_t^i, l_t^i)}\\ 1-\tau_{t+1}^a &= \frac{U_c^i(c_t^i, l_t^i)}{\beta R_{t+1}^* U_c^i(c_{t+1}^i, l_{t+1}^i)}.\end{aligned}$$

Therefore, there exist a set of Neghishi weights $\boldsymbol{\varphi} = (\varphi^i)_{i\in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i \varphi^i = 1$, such that individual allocation solve a static problem

$$V(C, L; \boldsymbol{\varphi}) \equiv \max_{(c^i,l^i)_{i\in I}} \sum_{i\in I} \varphi^i \pi^i U^i(c^i, l^i)$$
$$s.t. \qquad \sum_{i\in I} \pi^i c^i = C; \quad \sum_{i\in I} \pi^i l^i = L.$$

This problem gives the allocation rule for individual i that is time-invariant prorportional to the aggregate allocation

$$h^i(C, L; \boldsymbol{\varphi}) = \left(c^i(C, L; \boldsymbol{\varphi}),\ l^i(C, L; \boldsymbol{\varphi})\right).$$

The intra-temporal and inter-temporal conditions become

$$(1 - \tau_t^n)w_t = -\frac{V_L(C_t, L_t; \boldsymbol{\varphi})}{V_C(C_t, L_t; \boldsymbol{\varphi})} \tag{7}$$

$$1 - \tau_{t+1}^a = \frac{V_C(C_t, L_t; \boldsymbol{\varphi})}{\beta R_{t+1}^* V_C(C_{t+1}, L_{t+1}; \boldsymbol{\varphi})}, \tag{8}$$

and the implementability constraint of individual i is

$$\sum_{t=0}^{\infty} \beta^t \left[V_C(C_t, L_t; \boldsymbol{\varphi})c^i(C_t, L_t; \boldsymbol{\varphi}) + V_L(C_t, L_t; \boldsymbol{\varphi})l^i(C_t, L_t; \boldsymbol{\varphi})\right] = V_C(C_0, L_0; \boldsymbol{\varphi}) \left(a_0^i - T\right), \tag{9}$$

where $T \equiv \sum_{t=0}^{\infty} \beta^t \frac{V_C(C_t,L_t;\boldsymbol{\varphi})}{V_C(C_0,L_0;\boldsymbol{\varphi})} T_t$ is the present-value of lump-sum taxes.
The following proposition summarizes the competitive equilibrium characterization

Proposition 1.1. *Given the initial external debt B_0 and individual bond holdings$(a_0^i)_{i\in I}$,an allocation $\{C_t, L_t\}_{t=0}^{\infty}$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if the resource constraint (6) holds, and there exist market weights $\boldsymbol{\varphi} = (\varphi^i)_{i\in I}$ and lump-sum tax T such that the implementability constraint (9) holds for all $i \in I$.*

Proof. See Appendix. □

2 Optimal Debt Sustainability

In this section, I define the notion of debt sustainability in the context of a government that is benevolent and cares about redistribution, but lacks commmitment in its fiscal policies. I then show that under certain assumptions, it is optimal to sustain positive external debt in the long run.

2.1 Sustainable Equilibrium

Two sources of time inconsistency arise in this environment. First, the government has an incentive to default on positive external debt to increase private consumption and leisure. Second, persistent wealth heterogeneity generates incentives for an inequality-averse government to expropriate and redistribute wealth. A sustainable debt policy is therefore defined as the ex-ante optimal, time-consistent policy under which the government has no incentive to default or renege on its tax commitments.

I begin by specifying the government's objective and defining the sustainable equilibrium in the absence of commitment. The resulting sustainable allocation then characterizes the sustainable debt policy.

Given a set of social welfare weights $\boldsymbol{\lambda} = (\lambda^i)_{i \in I}$, the government's objective is the weighted utility of all domestic agents

$$\sum_{i \in I} \lambda^i \pi^i \sum_{t=0}^{\infty} \beta^t U^i(c_t^i, l_t^i). \tag{10}$$

Following Chari, and Kehoe (1990, 1993), I define the sustainable equilibrium is a subgame perfect equilibrium of a repeated game between the government, a continuum of domestic agents, and a continuum of external creditors. The sustainable equilibrium is characterized by the competitive equilibrium conditions described in Proposition 1.1 and the following sustainability constraint

$$\sum_{i \in I} \lambda^i \pi^i \sum_{k \geq t} \beta^{k-t} U^i(c_k^i, l_k^i) \geq \underline{U}_t, \ \forall t, \tag{11}$$

where $\underline{U}_t$ is the one-shot deviation value in which the government defaults on its debt and fully redistributes wealth among private agents.[2] Then the government faces punishment imposed by private agents and external lenders for deviating from the contracted policies. As argued in Chari, and Kehoe (1990, 1993), $\underline{U}_t$ is a value of a sustainable equilibrium. If it is the worst sustainable equilibrium value, then $\underline{U}_t$ represents the notion of the worst punishment, which will support the best sustainable equilibrium. In the next subsection, we will focus on the case of financial autarky as the punishment. Constraint (11) imposes a limit on the utility, which endogenously determines a limit on external debt for every period.

Given the above set-up, a sustainable allocation and a sustainable external debt policy are defined as follows

Definition 2.1. A sustainable allocation $(\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*)$ maximizes the social welfare function (10) and satisfies the conditions in Proposition 1.1 and the sustainability constraint (11) for a given function $\underline{U}_t$. A sustainable external debt policy is $\{B_t^*\}_{t=0}^{\infty}$ that is consistent with the sustainable allocation in equilibrium.

[2] See Appendix A for the formal set up of the sovereign game and its equilibrium characterization.

The sustainable allocation solves the following contracting problem

$$
\begin{aligned}
(P) \equiv \max_{\{C_t, L_t\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T} & \sum_{t=0}^{\infty} \beta^t U^P(C_t, L_t; \boldsymbol{\varphi}, \boldsymbol{\lambda}) \\
s.t. \quad & \sum_{t=0}^{\infty} Q_t^* \left[F(L_t) - C_t - G_t\right] - B_0 \geq 0 \\
& \forall i, \ \sum_{t=0}^{\infty} \beta^t \left[V_C(t; \boldsymbol{\varphi}) c^i(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) l^{i,}(t; \boldsymbol{\varphi})\right] \geq V_C(0; \boldsymbol{\varphi}) \left(a_0^i - T\right) \\
& \forall t, \ \sum_{k \geq t} \beta^{k-t} U^P(C_k, L_k; \boldsymbol{\varphi} \boldsymbol{\lambda}) \geq \underline{U}_t.
\end{aligned}
$$

where $U^P(C, L; \boldsymbol{\varphi}, \boldsymbol{\lambda}) = \sum_{i \in I} \lambda^i \pi^i U^i \left[c^i(C, L; \boldsymbol{\varphi}), l^i(C, L; \boldsymbol{\varphi})\right]$.

The first constraint is the resource constraint. The second constraint is the implementability constraints that take into account the distortionary effect of the government's policies on individual decisions. The last constraint is the sustainability constraint due to the government's lack of commitment. Appendix B provides the characterization of this optimal contracting problem, sustainable allocation, and optimal tax policies.

2.2 Optimal Debt Policy

In this subsection, I show that the optimal policy is to sustain positive external debt in the long run. To do so, I first impose the following assumptions on preferences

Assumption 1 (Separable Utility Function)**.** *The utility function satisfies (i) additive separability:* $U(c, n) = u(c) - v(n)$*, where (ii)* u, v *are twice differentiable with* $u^{'}, v^{'} > 0, u^{"} < 0$ *and* $v^{"} < 0$ *for all* $c > 0$ *and* $0 < n < \bar{n}$ *for some* $\bar{n}$*, and (iv) bounded marginal utilities and elasticities* $u^{'}, v^{'}, -u^{''}/u^{'}, v^{''}/v^{'}, -u^{''}c/u^{'}, v^{''}n/v^{'}$ *are bounded functions in* $(c, n) \in (\epsilon_c, \infty) \times (0, \epsilon_n)$ *for some* $\epsilon_c > 0$ *and* $\epsilon_n \in (0, \bar{n})$

The first assumption of eliminates the cross-effects in marginal utilities between consumption and labor and allow the analysis to be separate in terms of the effects on consumption and labor supply. The second assumption is standard in that the utility functions satisfy monotonicity and concavity. The last assumption insures that the marginal utilities and elasticities are well behaved as consumption becomes large or labor approaches zero. These assumptions hold for several preferences commonly used in macroeconomics, including the separable isoelastic preferences $U(c, n) = c^{1-\sigma}/(1-\sigma) - \omega n^{1+\nu}/(1+\nu)$ with $\sigma, \omega, \nu > 0$.

In addition, I assume that the private agents are more impatient than the international financial markets in the limit.

Assumption 2 (Impatience)**.** *There exists* $0 < \mathcal{M} < 1$ *and* $\mathcal{T}$ *such that for all* $t > \mathcal{T}, \beta R_t^* < \mathcal{M} < 1$.

Impatience implies a need for the country to accumulate external debt in the long run as it is cheaper to borrow abroad.

For what follows, I assume that the deviation utility in sustainable equilibrium is the value that the government receives in financial aukarky

Assumption 3. *$\underline{U}_t$ is the value of the economy under financial autarky, that is,*

$$
\begin{aligned}
\underline{U}_t \equiv \max_{\tau^n_t, \tau^a_t, T_t, B^d_{t+1}} & \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i U^i(c^i_t, l^i_t) \\
s.t. \quad & c^i_t + q^d_{t+1} a^{i,d}_{t+1} = (1-\tau^n_t) l^i_t - T_t + (1-\tau^a_t) a^{i,d}_t \\
& (1-\tau^n_t) w_t = -\frac{U^i_l(c^i_t, l^i_t)}{U^i_c(c^i_t, l^i_t)} \\
& 1-\tau^a_{t+1} = \frac{U^i_c(c^i_t, l^i_t)}{\beta / q^d_{t+1} U^i_c(c^i_{t+1}, l^i_{t+1})} \\
& G_t + B^d_t \leq \tau^n_t L_t + \tau^a_t A^d_t + T_t + q^d_{t+1} B^d_{t+1} \\
& B^d_t = A^d_t \\
& C_t + G_t \leq F(L_t, t) \\
& a^{i,d}_0 = 0
\end{aligned}
$$

In details, I assume that punishment imposed by private agents and external lenders for government's deviation is financial autarky, in which the government has no access to international financial markets. The government can issue domestic debt and imposes taxes. However, given the above setup, one can show that there is no role for domestic government debt and the optimal savings tax is zero.[3] The value of financial autarky can be rewritten as

$$
\begin{aligned}
\underline{U}_t \equiv \max_{\{C_t, L_t\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T} & \sum_{t=0}^{\infty} \beta^t U^P(C_t, L_t, \boldsymbol{\varphi}; \boldsymbol{\lambda}) \\
s.t. \quad & C_t + G_t \leq F(L_t, z_t) \\
& \sum_{t=0}^{\infty} \beta^t \left[V_C(t; \boldsymbol{\varphi}) c^i(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) l^{i,}(t; \boldsymbol{\varphi}) \right] \geq -V_C(0; \boldsymbol{\varphi}) T
\end{aligned}
$$

The assumption of financial autarky as deviation value is consistent with the notion of worst sustainable punishment to support the best sustainable equilibrium, as discussed in Chari, and Kehoe (1990, 1993) and subsection 2.1. First, financial autarky is a sustainable equilibrium. Second, it delivers the repeated worst static outcome of the subgame perfect equilibrium, making it a natural candidate for the punishment/deviation value.[4] Alternatively, one can relax the assumption by considering temporary exclusion or debt restructuring as part of the deviation.

[3] Domestic government debt has no role for two reasons. First, there is Ricardian equivalence given the presence of lump-sum transfers. Second, as private agents face permanent heterogeneous differences in labor productivity, there is no role for domestic government debt to insure them like in the case of idiosyncratic risks. The zero optimal savings tax follows the no optimal intertemporal distortion result as in Lucas and Stokey (1983) .

[4] Under certain cases, the repeated worst static outcome might not be the worst sustainable equilibrium given the dynamic time inconsistencies. See Chari and Kehoe (1993) for more details.

However, these extensions would improve the deviation value and depart from the worst punishment benchmark. Accordingly, I interpret the analysis in this paper as characterizing the maximal debt sustainable in a renegotiation-proof, never-default equilibrium.

Define the aggregate labor distortion as

$$\Omega = 1 + \frac{1}{F_L(L)} \frac{U_L^P(C, L, \boldsymbol{\varphi}; \boldsymbol{\lambda})}{U_C^P(C, L, \boldsymbol{\varphi}; \boldsymbol{\lambda})}$$

I now show that the government is willing to sustain positive external debt in the long run if there is positive aggregate labor distortion in financial autarky. That is,

Proposition 2.1 (Debt Sustainability)**.** *Suppose Assumptions 1–3 hold. If the aggregate labor distortion is positive in financial autarky and the steady state allocation exists, then the optimal external debt is positive in the long run.*

Proof. See Appendix. □

The proof relies on two properties of the optimal contract. First, given impatience, the government front-loads consumption and leisure by accumulating external debt. In the long run, the government reaches the maximal external debt. Second, deviating to financial autarky is never optimal. If autarky were optimal, the positive labor distortion under autarky would permit a profitable deviation: lower future labor distortions without changing the continuation value allows the economy to produce more than its consumption in those periods. This deviation is possible by borrowing today and repaying the debt in the future. The extra unit of borrowing allows for more consumption today, and so welfare would be higher than under financial autarky. Combining the two properties implies that, in the long run, the government optimally finances positive external debt with taxes rather than defaulting into financial autarky.

2.3 Cost of Redistribution and Debt Sustainability

In this subsection, I highlight the mechanism that links debt sustainability to the cost of redistribution. Proposition 2.1 establishes that the government sustains positive external debt because financial autarky entails aggregate labor distortions. In this framework, the distortion arises from redistributive motives, as in Werning (2007). Since taxes are uniform across agents, redistribution requires higher labor taxation to transfer resources from high- to low-skilled households. Such taxation distorts labor supply and reduces output. Thus, the government sustains positive external debt because redistribution is costly, and access to international financial markets alleviates this cost.

Why does access to international financial markets reduce the cost of redistribution? Impatient private agents borrow abroad without internalizing that their borrowing tightens the sustainability constraint. When the constraint binds, a tax on borrowing is optimal, as it corrects the borrowing externality. The tax on borrowing also generates redistributive gains because high-skilled agents hold more debt abroad than lowly-skilled agents and thus bear a larger burden. By mitigating the limited-commitment friction, the borrowing tax serves as an additional redistributive instrument and allows a lower labor tax under the optimal contract.

Figure 2 illustrates the mechanism for an example economy. The optimal contract features private agents accumulating debt externally over time, as shown in Panel (a), with high-skilled agents accumulating more than low-skilled agents. The economy's external debt position grows over time (Panel (b)). The government imposes a tax on borrowing to relax the binding sustainability constraint (Panel (c)), and labor tax declines over time (Panel (d)). In contrast, in financial autarky, there is no external debt nor private debt (Panels (a) and (b)), and the optimal tax on borrowing is zero (Panel (c)). The optimal labor tax remains high as the only tool of redistribution (Panel (d)). In the long run, sustaining positive external debt is optimal because defaulting into autarky entails a higher distortionary cost of redistribution.

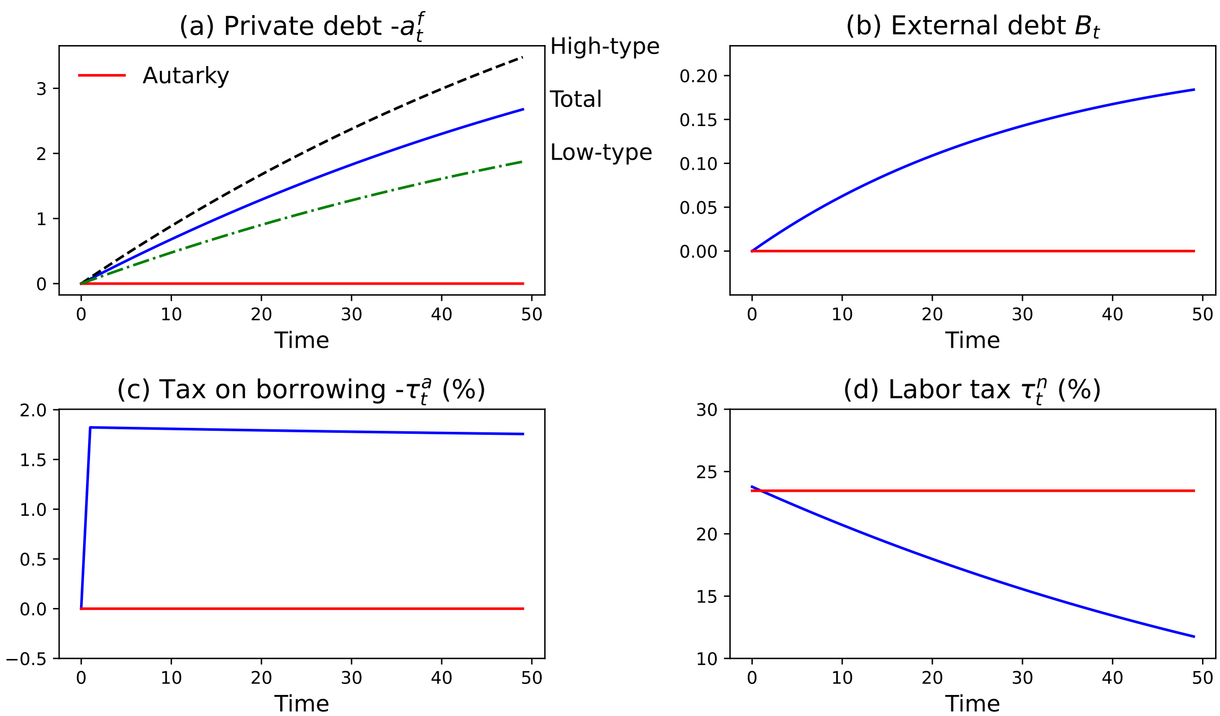


Figure 2: Time paths of aggregates in equilibrium and autarky

Note: Figure 2 plots the simulated time paths of aggregates in equilibrium and autarky. The implementation is that lump-sum taxes only occur in period 0. Panel (a) plots the private debt, which is the external debt held by private sectors. Panel (b) shows the economy's external debt position. Panels (c) and (d) plot the optimal labor tax and borrowing tax, respectively.

In summary, I show theoretically that when redistribution comes with efficiency cost, the government has an incentive to sustain external debt, since access to international financial markets lowers the cost of redistribution. The cost of redistribution generates an endogenous mechanism for the cost of default. When inequality, or the government's redistributive motive, is higher, the efficiency cost also increases as tax rates are higher. This makes financial autarky more costly and increasing the incentive to sustain external debt. Thus, the model predicts a positive link between inequality and sustainable debt levels.

3 An Economy with Separable Isoelastic Preferences

In section 2, I show theoretically that debt sustainability emerges as the optimal policy in the presence of aggregate labor distortions under financial autarky. I attribute this distortion to the government's redistributive motives without formal proof. In this section, I study a specific open-economy environment with separable isoelastic preferences and demonstrate that the aggregate labor distortion is strictly positive in financial autarky when the government has redistribution concerns.

Preferences. The utility of agent with productivity θ^i over consumption $c^i \geq 0$ and efficiency-unit labor $l^i \geq 0$ is

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c^i\right)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right] \tag{12}$$

with $\sigma, \omega, \nu > 0$.

In this case, the allocation rule for individual i is time-invariant prorportional to the aggregate allocation

$$c_t^i = \psi_c^i C_t, \quad l_t^i = \psi_l^i L_t, \tag{13}$$

In addition, V and U^P inherit the separable and isoelastic properties,

$$\begin{aligned} V(C_t, L_t; \boldsymbol{\varphi}) &= \Phi_C^V \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L_t^{1+\nu}}{1+\nu}, \\ U^P(C, L; \boldsymbol{\varphi}, \boldsymbol{\lambda}) &= \Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} \end{aligned}$$

where Φ_C^V, Φ_L^V depend on $\boldsymbol{\varphi}$, and Φ_C^P, Φ_L^P depend on $\boldsymbol{\varphi}$, $\boldsymbol{\lambda}$ (see Appendix B.1). The implementability constraint for individual i becomes

$$\sum_{t\geq 0} \beta^t \left(\Phi_C^V \psi_c^i C_t^{1-\sigma} - \Phi_L^V \psi_l^i L_t^{1+\nu} \right) = \Phi_C^V C_0^{-\sigma} \left(a_0^i - T \right), \tag{14}$$

I impose the following assumptions:

Assumption 4. *The welfare weights, skill distribution, and initial wealth distribution satisfy the following properties*

1. *Redistributive motive towards the low skills:* $\theta^i < \theta^j \iff \lambda^i > \lambda^j,\ \forall i, j \in I$
2. *Perfect correlation between skill and initial wealth:* $\theta^i < \theta^j \iff b_0^i < b_0^j,\ \forall i, j \in I$
3. *Elasticity of substitution is such that* $\sigma \geq 1$

The first assumption concerns the welfare weights: the government assigns relatively high weight to redistribution toward lower-skill, lower-income individuals, reflecting inequality aversion. The second assumption requires that the ordering of skill inequality coincides with that of initial wealth, so that lower-skill individuals are also endowed with lower initial wealth. Finally, the third assumption. Finally, the third assumption implies that the intratemporal elasticity of substitution is at least above the $\log$-preference benchmark. This assumption governs the direction of adjustment in optimal tax and debt policies in response to both intratemporal and intertemporal changes. Parameter values commonly used in quantitative macroeconomic analysis satisfy this assumption.

Given that the government cares about redistribution towards low-skilled agents, I argue that it is costly for the government to redistribute in financial autarky. That is,

Proposition 3.1. *Suppose Assumption 4 holds, then the aggregate labor distortion is positive in financial autarky.*

Proof. See Appendix. □

The key component that governs the aggregate labor distortion is the ratio between Pareto and Negishi weights λ^i/φ^i, which captures the extent to which the government's distributional preferences diverge from the market's equilibrium allocation of individual utilities. If the distributional preference agrees with the market distribution (i.e., $\lambda^i = \varphi^i, \forall i \in I$), then there is no aggregate labor distortion. This result happens under two cases: the representative-agent case and the heterogeneous-agent case in which $\lambda^i = \varphi^{*i}, \forall i \in I$.

Proposition 3.2 (Zero aggregate labor distortion)**.** *The aggregate labor distortion in financial autarky is zero if either of the following cases holds:*

1. *There is no heterogeneity:* $\theta^i = \theta^j, a_0^i = a_0^j,\ \forall i, j \in I$.
2. *There exists* $\boldsymbol{\varphi}^*$ *such that* $\lambda^i = \varphi^{*i}, \forall i \in I$.

Proof. See Appendix.

In the above cases, redistribution comes with no efficiency costs. Consequently, access to international financial markets yields no additional benefits. By contrast, default enables the government to raise consumption not financing the external debt. Ex ante, this implies that the sustainable level of debt is zero. □

4 Quantitative Analysis

In this section, I extend the framework to a stochastic setting with state-contingent domestic and foreign bonds. Beyond the redistributive motive for sustaining debt, the government also values external debt for insurance against aggregate shocks. The insurance role is well established in the literature (Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), Chatterjee and Eyigungor (2012), and many other papers). I quantify the relative importance of these two channels for debt sustainability using Italian data. In addition, I characterize the optimal austerity policies in the presence of redistribution concerns.

4.1 Stochastic Economy with State-Contingent Debt

Environment. A small open economy faces publicly observed aggregate shocks $s_t \in S$ in period t, where S is some finite set. Let $\Pr(s^t)$ denote the probability of any history $s^t = (s_0, s_1, ..., s_t)$, where $\Pr(s^{t+j}|s^t)$ denotes the probability conditional on history s^t, $j \geq 0$. Similarly, $\Pr(s_{t+1}|s^t)$ is the probability period $t+1$'s state is s_{t+1}, conditional on history s^t. The individual utility is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i(c_t^i, l_t^i)$$

The production function in period t with history s^t is $F(L, s^t, t)$.

State-contingent bonds and budget constraints. Domestic and foreign bonds are state-contingent. The individual agent's budget constraint is

$$c_t^i + \sum_{s_{t+1}} q_{t+1}^d(s_{t+1}) a_{t+1}^{d,i}(s_{t+1}) + \sum_{s_{t+1}} q^f(s_{t+1}) a_{t+1}^{f,i}(s_{t+1}) \leq (1-\tau_t^n) w_t l_t^i + (1-\tau_t^a) a_t^{d,i} + (1-\tau_t^a) a_t^{f,i} - T_t,$$

and the government's budget constraint is

$$G_t + B_t^d + B_t^f \leq \tau_t^n w_t L_t + \tau_t^a \left(A_t^d + A_t^f\right) + T_t + \sum_{s_{t+1}} q_{t+1}^d(s_{t+1}) B_{t+1}^d(s_{t+1}) + \sum_{s_{t+1}} q_{t+1}^f(s_{t+1}) B_{t+1}^f(s_{t+1})$$

Define $Q_t^* = \Pr(s^t)/(\Pi_{\tau=0}^t R_\tau^*)$ as the international price of one unit of consumption at history s^t in units of period-0 consumption. The optimal contracting problem becomes

$$(P) \equiv \max_{\{C_t, L_t\}, \boldsymbol{\varphi}, T} \sum_{i \in I} \lambda^i \pi^i \mathbb{E}_0 \sum_{t \geq 0, s^t} \beta^t U^P(C_t, L_t; \boldsymbol{\varphi}, \boldsymbol{\lambda})$$

$$
\begin{aligned}
s.t. \qquad & \sum_{t=0}^{\infty} Q_t^* \left[F(L_t, s^t, t) - G_t - C_t \right] \geq B_0 \\
& \forall i,\ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[V_C(t; \boldsymbol{\varphi}) c^i(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) l^{i,}(t; \boldsymbol{\varphi}) \right] \geq V_C(0; \boldsymbol{\varphi}) \left(a_0^i - T \right) \\
\forall t, \forall s^t,\ & \sum_{i \in I} \lambda^i \pi^i \mathbb{E}_t \sum_{k \geq t} \beta^{k-t} U^P(C_k, L_k; \boldsymbol{\varphi}, \boldsymbol{\lambda}) \geq \underline{U}_t(s^t, t)
\end{aligned}
$$

4.2 Parametrization

For the quantitative exercise, I assume the following distributional and functional forms. The economy is populated by two types of agents with labor productivity $\{\theta^H, \theta^L\}$, where $\theta^H \geq \theta^L > 0$ and $\pi^H = \pi^L = 0.5$. The planner is utilitarian, i.e. $\lambda^H = \lambda^L$. The individual preference has the form of $U(c, n) = c^{1-\sigma}/(1-\sigma) - \omega n^{1+\nu}/(1+\nu)$ with $\sigma, \omega, \nu > 0$. The production function is linear in labor, i.e. $F(L, z) = zL$, where z is the aggregate productivity. The aggregate shock is z_t that follows a logged $AR(1)$ process,

$$
\log z_t = \rho_z \log z_{t-1} + \epsilon_t^z,\ \epsilon_t^z \sim \mathcal{N}(0, \sigma_z),
$$

where ρ_z, σ_z are the auto-correlation and the residual standard deviation, respectively. I discretize the productivity process into a Markov chain using Tauchen method with 31 evenly-spaced nodes. From now on, I will use z in place of s as the source of aggregate uncertainty. The government expenditure is constant over time and across histories: $G_t = \bar{g}$. The initial debt levels are $B_0 = 0$ and $a_0^H = a_0^L = 0$. The economy starts at the mean of the productivity distribution. The deviation utility $\underline{U}(z^t, t)$ is calculated as the closed-economy version of the model that starts with productivity z_t, zero external debt, and all domestic individuals start with the same initial wealth. $\underline{U}(z^t, t)$ varies with respect to the realized shock z_t.

With these assumptions, the model requires giving values to the parameters of (i) the aggregate productivity process, ρ_z and σ_z; (ii) the cross-sectional wage ratio, θ^H/θ^L; (iii) the individual preferences, β, σ,ω, and ν; (iv) the government expenditure $\bar{g}$; and (v) the risk-free rate r^*. Table 1 summarizes the parameter values and targets from the calibration exercise.

Table 1: Parameters and Targets

Parameter	Description	Value	Target
Externally calibrated parameters			
r^*	Risk-free rate	0.017	Avg. real return on German bond
β	Discount factor	0.967	Avg. Italian real interest rate = 3.4%
σ	Intertemporal elasticity	1	Standard literature value
$1/\nu$	Labor elasticity	0.5	Standard literature value
ω	Labor utility weight	1	Standard literature value
θ^H/θ^L	Wage ratio	1.89	Mean top 50% wage / mean bottom 50% wage
ρ_z	Auto-corr. of prod.	0.927	Auto-corr. of log GDP
Internally calibrated parameters			
σ_z	Std. dev. of prod. res.	0.0205	Std. dev. log GDP
$\bar{g}$	Govt. spending	0.202	Avg. govt. consumption-to-GDP

Note: The table describes the parameters, their values, and the targets in the calibration exercise. Statistics are annual. The risk-free rate and discount factor cover the period of 2002-2015. Wage ratio is the author's calculation from the household-level data set by *Survey on Household Income and Wealth* covering the period of 2002-2014. Auto-correlation and standard deviation of GDP and government final consumption cover the period of 1985-2015. Data sources: *Survey on Household Income and Wealth* (2014), Eurostat (2019), and The World Bank (2019)

A period in the model is one year. For output, I use the logged and linear detrended real GDP series from 1985-2015. I set the auto-correlation of productivity,ρ_z, equals to the auto-correlation of output, which is 0.928. To measure the wage ratio θ^H/θ^L, I use the household-level data from the *Survey on Household Income and Wealth* (SHIW) conducted by the Bank of Italy.[5] Hourly wage is defined as total real compensation of employees, including fringe benefits, divided by total hours worked in a year.[6] For each year in the database, I calculate the ratio of the mean wage of the top 50% of the wage distribution to the mean wage of the bottom 50%. Then θ^H/θ^L is set to 1.89, which is the time-average of these wage ratios for the period of 2002 to 2014. The discount factor β is set to 0.967 so that the average real domestic interest rate is 3.4% for Italy from 2002 to 2015. I set $\sigma = 1$ to have log-consumption utility. I choose $\omega = 1$ and $\nu = 2$ so that the elasticity of labor supply is 0.5, a standard value in the literature. The risk-free rate is set at 0.017, which is the real rate of return on the German government bonds for the period 2002-2015 (these are secondary market returns, gross of tax, with around 10 years' residual maturity). The interest rate series start at 2002 to isolate the effect of currency and exchange rate risks.[7]

The two remaining parameters, σ_z and $\bar{g}$, are selected to match (i) the standard deviation of

[5]See Appendix E.2 for details on the microeconomic data and sample selection.
[6]Real data calculation uses CPI indices provided by the OECD.
[7]See Appendix E for more data descriptions and sources

logged output and (ii) the government's final consumption-to-GDP ratio for the period 1985-2015. I use the simulated method of moments (SMM). Departing from the quantitative literature on sovereign debt, I do not target the average external debt-to-output ratio but instead leave it as one of the non-targeted moments.[8]

4.3 Quantitative Results

Table 2 shows the quantitative results with moments for the data, baseline, and alternative models. The first column reports the statistics from the data for Italy in the period of 1985-2015, except for external debt moments covering the period of 2002-2015. The second column reports the statistics from simulating the model and taking the long-run averages.[9] The calibration successfully matches the standard deviation of output and the government consumption-output ratio for Italy.

External debt. Most importantly, the model is able to produce a quantitatively large amount of external debt-to-output, in consistence with the data. The model generates around 17% of external debt-to-output ratio, average at the ergodic distribution, comparing to 24% of external debt-to-output ratio in the data. The model also matches the volatility of external debt-to-output ratio in the data. These features are with the presence of a relatively high discount factor (0.969) and no additional exogenous cost of default in terms of output/productivity loss.

Cyclical properties. Several cyclical features of the Italian data stand out. First, consumption is as volatile as output and is highly correlated with output. Net saving, defined as output minus the total private and public consumption, only has a volatility of more than a quarter of the volatility of output and a positive correlation with output that is around 40%.[10] The model matches closely the quantitative patterns of the data. The volatility of consumption and net savings relative to output are slightly higher in the model than in the data. Both model consumption and net saving are pro-cyclical with similar correlation levels as in the data. The model is able to generate realistic cyclical patterns of the data, in contrast to the standard model of complete markets. The main reason is that, even with state-contingent assets, the occasionally binding borrowing constraints lead to an imperfect insurance across states and time periods.

The redistributive motive for sustaining external debt is quantitatively significant and is driven by the cost of redistribution. To illustrate this point, I compare the baseline model to two alternative specifications: a no-inequality model and a skill-dependent lump-sum tax model.

[8] The following subsection shows the the results of non-targeted moments. Alternatively, the discount factor β can be used to target the debt-to-output ratio.

[9] All model's moments are long-run averages of simulating the economy for 10500 periods and discarding the first 500 periods.

[10] Neumeyer and Perri (2005) reported key business cycle statistics for both advanced and emerging market economies.

Table 2: Moments: Data, Baseline, and Alternative Models

	Data	Baseline	No inequality	Skill-dependent lump sum tax
Targeted moments				
Std. output (%)	5.3	5.3	5.4	5.4
Avg. govt. expenditure/output (%)	19	19	19	19
Non-targeted moments				
External debt property				
Avg. external debt/output (%)	24	17	2.9	2.9
Std. external debt/output (%)	2.7	2.1	0.45	0.45
Cyclical property				
Std. consumption / Std. output	1.0	1.2	1.2	1.1
Std. net savings/output (%)	1.5	1.8	1.8	1.7
Correlation with output (%)				
Consumption	97	95	95	95
Net savings/output	40	31	36	35

Note: This table reports the non-targeted statistics of the data and the model. The first column reports data statistics which are across the period of 1985 to 2015, except for external debt moments covering the period of 2002 to 2015. The other columns report statistics coming from models' simulations for 10500 periods and excluding the first 500 periods. The no-inequality model corresponds to the case in which $\theta^H = \theta^L$. The skill-dependent-lump-sum-tax model corresponds to the case in which the government has access to skill-dependent lump-sum taxes $\left(T^i\right)_{i\in I}$. In the data, government expenditure is final government consumption excluding social transfers. Net saving is defined as output minus total private and government consumption in the data and the model. External debt is defined as the country's net financial liability in the data. For the second moments, output and consumption series are logged and linear detrended, and net saving and external debt ratio series are linear detrended.

The no-inequality model corresponds to the case in which $\theta^H = \theta^L$, with results reported in the third column of Table 2. The no-inequality model generates a lower level of external debt, 2.9% of GDP compared to 17% of GDP in the baseline model. The finding implies that the insurance channel for debt sustainability, in which international financial markets provide insurance against aggregate shocks, can account for only a small fraction of external debt. On the other hand, the redistributive channel, in which international borrowing reduces the distortionary costs of redistribution, accounts for 60% of observed external debt. Moreover, the no-inequality model generates lower external debt volatility relative to the baseline.

To highlight the role of distortionary taxation, I also consider a skill-dependent lump-sum tax model in which the government can levy lump-sum taxes and transfers conditional on household skill type. In this setting, perfect redistribution is feasible without reliance on distortionary labor taxation, since the government can directly tax high-skilled households and transfer resources to low-skilled households. The fourth column of Table 2 reports the moments of this model. reports the results. Relative to the baseline, this model generates both a lower external debt-to-output ratio and lower debt volatility.. The government redistributes by lump-sum taxing the highly skilled households and lump-sum transferring resources to lowly skilled households. In contrast, in the baseline model redistribution can only be done through distortionary taxes.

4.4 Effect of Inequality on External Debt Over Time

I next examine the model's implications for the relationship between income inequality and external debt over time. To this end, I conduct a comparative statics exercise for Italy across two subperiods: 1985–2001 and 2002–2015. On average, the earlier period is characterized by lower levels of both income inequality and external debt relative to the later period.

Table 3: Comparative statics

Statistics	Data	Model
Targeted moment		
Δ Pre-tax Gini	3.0%	3.0%
Non-targeted moment		
Δ Extenal debt/output	14%	10%

Notes: The table reports the results of the comparative statics exercise. The first column reports the changes in the data statistics, computed as the average statistics of period 2002-2015 minus the average statistics of period 1985-2001. The second column reports the results from the model. The change in the model statistics is computed as the average statistic of a simulation for the model with the wage ratio equal to 1.89 minus the same statistic of the model with the wage ratio equal to 1.83.

The comparative statics exercise proceeds as follows. I feed into the model the value of wage inequality corresponding to the 1985–2001 period, holding all other parameters fixed, and compute the ergodic means of the pre-tax Gini index and the external-debt-to-output ratio. The 1985–2001 wage inequality level is calibrated such that the change in the average pre-tax Gini from 1985–2001 to 2002–2015 matches the change observed in the data. Table 3 reports the results. Given the targeted increase in the pre-tax Gini in Italy between the two periods, the model accounts for roughly 71 percent of the observed rise in the external-debt-to-output ratio.

4.5 Optimal Austerity Policies

I study optimal austerity policies in the presence of government's redistribution concerns. To do so, I calculate the responses of optimal policies to a negative productivity shock. [11] Figure

[11] I simulate 30000 paths for the model for 2050 periods. From periods 1 to 2050, the aggregate productivity shock follows its underlying Markov chain so that the cross-sectional distribution of debt converges to the ergodic distri-

3 plots the impulse responses of output, consumption, external debt-to-output, and utility inequality $u^H - u^L$, for the baseline model and the model of no inequality ($\theta^H = \theta^L$). A decline in productivity growth reduces both output and consumption, with the contraction being larger in output. In response, the external debt increases. Utility inequality initially falls before increasing in subsequent periods, reflecting higher redistribution in the short run and lower redistribution in the long run. Intuitively, a negative shock reduces deviation utility, resulting in the borrowing constraint non-binding. This relaxation allows the government to accumulate external debt, temporarily lower average tax rates, and expand redistribution. In later periods, however, taxes must rise to finance the debt, and redistribution correspondingly declines.

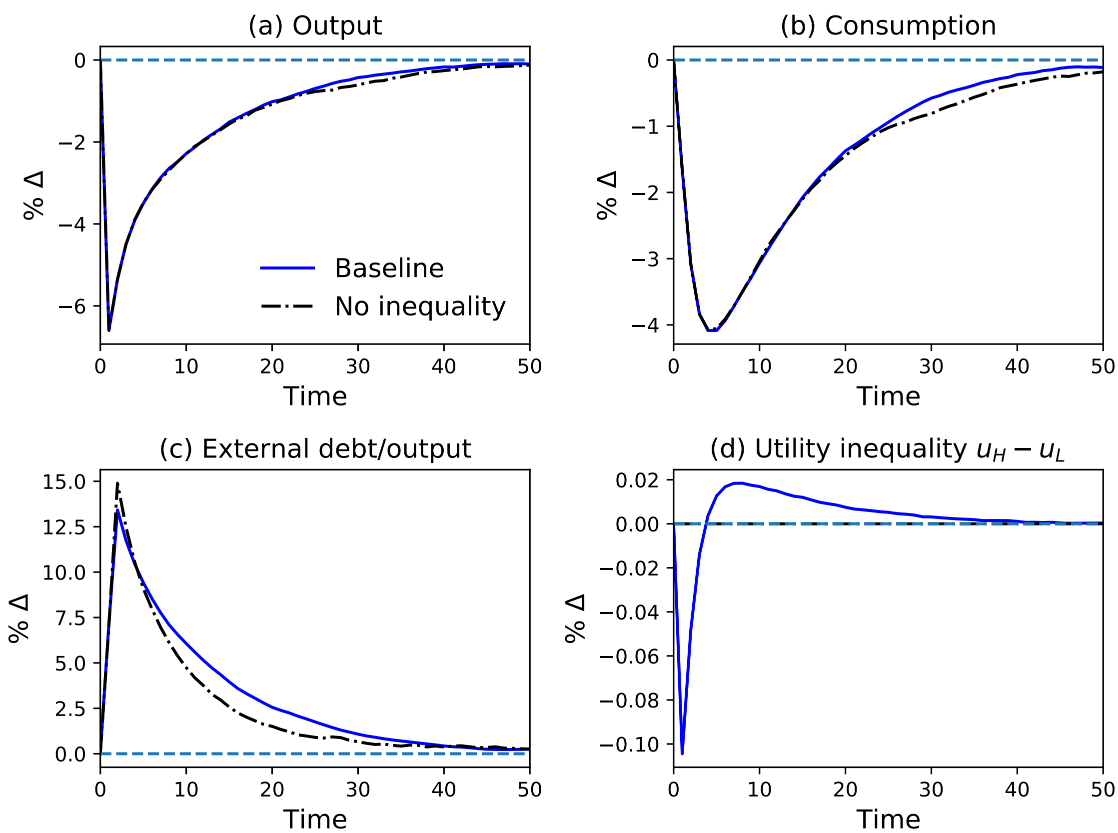


Figure 3: Impulse responses to a negative productivity shock

Notes: The graph shows the impulse responses across 30000 paths of the variables for the baseline and no-inequality models. Panel (a), (b), (c), (d) plot the responses of output, consumption, external debt-to-output, and utility inequality to a negative aggregate productivity drop in period 1, respectively.

The no-inequality model produces similar dynamics for output and consumption over time. However, the external-debt-to-output ratio rises more sharply initially and declines more rapidly

bution. In period 2051, normalized to 1 in the plots, the aggregate productivity drops by one standard deviation of the error term. From period 2051 on, the aggregate productivity shock follows the conditional Markov chain. The impulse responses plot the average, across 30000 paths, of the variables from period 2050 to 2055.

relative to the baseline model. This pattern implies that, in the baseline economy, the government—despite borrowing less initially—can pursue a more gradual fiscal consolidation by sustaining higher debt levels in later periods. The slower adjustment is implemented through reduced redistribution. Thus, the government's motive for redistribution leads to the optimal fiscal consolidation more gradual.

5 Sensitivity Analysis

In this section, I conduct a sensitivity analysis to examine how optimal policies respond quantitatively to different model ingredients: government expenditure, aggregate uncertainty, and the discount factor. Table 4 compares the moments from the baseline model with those from alternative specifications. The second column reports results from the no-government-expenditure case. Average external debt remains unchanged relative to the baseline, indicating that the level of exogenous government expenditure does not affect the optimal sustainable debt level. Higher government expenditure, however, is associated with larger second moments, with the exception of the consumption–output correlation.

Table 4: Role of government expenditure, aggregate uncertainty, and discount factor

	Baseline	No govt. exp. ($g = 0$)	Deterministic ($\sigma_z = 0$)	Lower discount ($\beta = 0.95$)
Avg. external debt/output (%)	17	17	22	12
Std. external debt/output (%)	2.1	0.99	-	1.8
Std. consumption / Std. output	1.2	0.96	-	1.2
Std. net savings/output (%)	1.8	1.5	-	0.62
Correlation with output (%)				
Consumption	95	97	-	99
Net savings/output	31	29	-	-1.1

Notes: The moments are calculated from the model's simulation for 10500 periods and then excluding the first 500 periods. For the second moments, output and consumption series are logged and linear detrended, and net saving and external debt ratio series are linear detrended.

The third column of Table 4 reports results for the deterministic case with no aggregate uncertainty. In this specification, the model sustains a higher level of debt than in the baseline. The intuition is that uncertainty in the baseline model generates a stronger precautionary motive, which limits overall external debt accumulation relative to the deterministic case.

The fourth column presents results for a lower discount factor compared to the baseline. In this case, the average external-debt-to-output ratio is lower. With a higher discount factor, the value of autarky is relatively smaller and the economy takes longer to reach the borrowing constraint, allowing the government to accumulate and sustain more debt. Finally, the lower-discount-factor model provides a poor match for the second moments of net savings in the data.

6 Empirical Evidence

I next assess whether the model's prediction linking inequality and external debt is supported empirically using a cross-country panel dataset. The theoretical and quantitative results imply that higher pre-tax income inequality—interpreted as a proxy for stronger redistributive motives—is associated with higher sustainable external debt levels. To measure external indebtedness, I use the negative of the net foreign asset–to-GDP ratio from the External Wealth of Nations database of Lane and Milesi-Ferretti (2018).[12] For inequality, I use pre-tax (market) Gini indices from the Standardized World Income Inequality Database (SWIID) Solt (2019). The analysis focuses on countries that are frequently subject to debt crises (see Appendix E.3 for the list of countries). Table 5 reports regressions of the net foreign liability–to-GDP ratio on the pre-tax Gini index for the period 1985–2015. Across specifications, the estimated coefficient is positive and statistically significant, consistent with the model's prediction that higher inequality is associated with higher external debt.

Table 5: Regression analysis of income inequality and external debt

	Net foreign liability-to-GDP (%)			
	(1)	(2)	(3)	(4)
Gini index, pre tax (%)	0.773***	0.472**	4.947***	5.456***
	(0.222)	(0.236)	(0.664)	(0.749)
GDP per capita (log)		-4.972**	-5.781	-1.608
		(1.952)	(4.093)	(7.211)
Real GDP per capita growth (%)		-1.311**	-1.371**	-1.427**
		(0.638)	(0.543)	(0.608)
Current account-to-GDP (%)		-2.061***	-1.090***	-1.100***
		(0.297)	(0.344)	(0.376)
Inflation (%)		0.013***	0.010***	0.010**
		(0.003)	(0.004)	(0.004)
Country fixed effect	No	No	Yes	Yes
Time fixed effect	No	No	No	Yes
No. Countries	30	30	30	30

Note: The table reports the regression coefficients and standard errors in parenthesis of pre-tax Gini index (%) with respect to net foreign liability-to-GDP (%). Control variables are log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). * p<0.1, ** p< 0.05, *** p<0.001. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and The World Bank (2019).

[12] The net foreign asset (NFA) position of a country is the value of the assets that country owns abroad, minus the value of the domestic assets owned by foreigners, adjusted for changes in valuation and exchange rates.

7 Conclusion

This paper develops a theory of external debt sustainability in which redistribution entails distortionary taxation, and access to international financial markets alleviates these costs. Embedding redistributive motives into a sovereign debt framework with limited commitment, I show that costly redistribution generates a novel, endogenous cost of default. Greater inequality amplifies the distortionary cost of taxation, raises the cost of financial autarky, and supports higher sustainable debt levels.

Quantitatively, redistribution channel emerges as a central role for debt sustainability. In the case of Italy, the model attributes only 12 percent of long-run debt to the standard insurance channel, while redistribution channel accounts for 60 percent, and it is consistent with the positive cross-country and time-series correlation between inequality and external debt.

The analysis also highlights the implications for fiscal austerity. Optimal austerity is shaped by distributional concerns: following a negative productivity shock, the government initially expands borrowing and redistribution, and subsequently consolidates through higher taxes and reduced transfers. This result provides a rationale for more gradual fiscal adjustment in unequal economies.

The framework departs from the sovereign default models in that equilibrium default does not occur. Future work could extend the framework to include equilibrium default risks and alternative debt crises, further linking redistribution and debt sustainability to central themes in the sovereign debt literature.

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Online Appendix

A Sovereign Game

In this section, I set up the strategic sovereign game and define the sustainable equilibrium that is characterized by the self-enforcing constraints. Consider the general environment where the government's policy includes the decision to default on its debt δ, where $\delta \in \{0,1\}$ and $\delta = 0$ implies default.[13] The government's budget constraint becomes

$$G_t + \delta_t B_t^d + \delta_t B_t^f \leq \tau_t^n w_t L_t + \tau_t^a \left(A_t^d + A_t^f\right) + T_t + q_{t+1}^d B_{t+1}^d + q_{t+1}^f B_{t+1}^f.$$

As the government cannot commit to any of its policies, one can think that the government, domestic agents, and international lenders enter into a sovereign game in which they determine their actions sequentially. In every period, the state variable for the game is $\left\{B_t^d, B_t^f, \left(a_t^{i,d}, a_t^{i,f}\right)_{i\in I}\right\}$. The timing of the actions is as follows:

- Government chooses $z_t^G = \left(\tau_t^n, \tau_t^a, T_t, \delta_t, B_{t+1}^d, B_{t+1}^f\right) \in \Pi$ such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c_t^i, l_t^i, a_{t+1}^{i,d}, a_{t+1}^{i,f}\right)$ subject to their budget constraints, the representative firm produces output by choosing $z_t^F = (L_t)$, and the international lenders choose holdings of bonds $z_t^* = \left(B_{t+1}^f, A_{t+1}^f\right)$ given the interest rates r_t^*.

Define $h^t = \left(h^{t-1}, z_t^G, \left(z_t^{H,i}\right)_{i\in I}, z_t^F, z_t^*, p\right) \in H^t$ as the history at the end of period t. Note that the history incorporates the government's policy, allocation, and prices. Define $h_p^t = \left(h^{t-1}, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period t. The government strategy is $\sigma_t^G : H^{t-1} \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i} : H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has the strategy $\sigma_t^F : H_p^t \to \mathbb{R}_+^2$, and the international lenders have the strategy $\sigma_t^* : H_p^t \to \mathbb{R}_+^2$. Prices are determined by the pricing rule: $p : H_p^t \to \mathbb{R}_+$.

Definition A.1 (Sustainable equilibrium)**.** A sustainable equilibrium is $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ such that (i) for all h^{t-1}, the policy z_t^G induced by the government strategy maximizes the weighted utility by $\boldsymbol{\lambda}$ subject to the government's budget constraint (5); (ii) for all h_p^t, the strategy induced policy $\left\{z_t^G\right\}_{t=0}^{\infty}$, allocation $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^{\infty}$, and prices $\{r_t^*\}_{t=0}^{\infty}$ constitute a competitive equilibrium with taxes.

The following focuses on characterizing a set of sustainable equilibria in which deviation triggers the worst payoff. In this case, the value of deviation is the worst equilibrium payoff.

[13] Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its debt ($\delta_t = 1$).

Proposition A.1 (Sustainable equilibrium)**.** *An allocation and policy* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *can be part of sustainable equilibrium if and only if (i) given* z^G*, there exist prices* p *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G, p\right\}$ *is a competitive equilibrium with taxes for an open economy; and (ii) for any* t*, there exists* $\underline{U}_t$ *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *satisfies the constraint*

$$\sum_{k=t}^{\infty}\beta^{k-t}\sum_{i\in I}\lambda^i\pi^i U^i\left(c_k^i, l_k^i\right) \geq \underline{U}_t. \tag{11}$$

Proof. Define $\underline{U}_t$ as the maximum discounted weighted utility for the private agents in period t when the government deviates. In period t, the private agents and the government can borrow abroad. In subsequent period $s > t$, the economy reverts to the worst equilibrium.

Suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an outcome of the sustainable equilibrium. Then by the optimal problems of the government, private agents, and international lenders, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes the weighted utility of the agents, satisfies the government budget constraint, and satisfies the international lender's problem at period 0. Thus, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is a competitive equilibrium with policies. For any period t and history h^{t-1}, an equilibrium strategy that has the government deviate in period t triggers reverting to the worst equilibrium in period $s > t$. Such a strategy must deliver a weighted utility value that is at least as high as the right-hand side of (11). So $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii).

Next, suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies conditions (i) and (ii). Let h^{t-1} be any history such that there is no deviation from z^G up until period t. Since $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t onward. Consider a deviation plan $\hat{\sigma}^G$at period t that receives U_t^d in period t and U^{aut} for period $s > t$. Because the plan is constructed to maximize period-t utility, the right-hand side of (11) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. □

B Characterizing Sustainable Allocation and Optimal Tax Policies

This section provides details on the characterization of the sustainable allocation and optimal tax policies. Section C will use this analysis to prove the propositions in the main text.

Let μ be the multiplier on the resource constraint, $\pi^i\eta^i$ be the multiplier on the implementability constraint for agent i, and $\beta^t\gamma_t$ be the multiplier on the aggregate debt constraint for period t. Define $\boldsymbol{\eta} = \left(\eta^i\right)_{i\in I}$ and rewrite the Lagrangian of the optimal contracting problem with a new pseudo-utility function that incorporates the implementability constraints,

$$\sum_{t=0}^{\infty} \beta^t U^W \left[t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] - V_C(0; \boldsymbol{\varphi}) \sum_{i \in I} \pi^i \eta^i \left(a_0^i - T\right),$$

where

$$U^W \left[t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] \equiv \sum_{i \in I} \lambda^i \pi^i U^i(t; \boldsymbol{\varphi}) + \sum_{i \in I} \pi^i \eta^i \left[V_C(t; \boldsymbol{\varphi}) h^{i,c}(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) h^{i,l}(t; \boldsymbol{\varphi})\right]$$

The first-order conditions are

$$U_C^W(t; \boldsymbol{\varphi}) + U_C^P(t; \boldsymbol{\varphi}) \sum_{k=0}^{t} \gamma_k = \mu \frac{Q_t^*}{\beta^t} \tag{B.1}$$

$$U_L^W(t; \boldsymbol{\varphi}) + U_L^P(t; \boldsymbol{\varphi}) \sum_{k=0}^{t} \gamma_k = \mu \frac{Q_t^*}{\beta^t} F_L(L_t) \tag{B.2}$$

$$\sum_i \pi^i \eta^i = 0 \tag{B.3}$$

In addition, there is the first-order condition with respect to the market weight φ^i

The necessary conditions to characterize the sustainable allocation are the first-order conditions, the aggregate resource constraint, the implementability constraints, and the aggregate debt constraints.

The optimal tax policies follow

$$\tau_t^n = 1 - \frac{U_C^W(t; \boldsymbol{\varphi}) + U_C^P(t; \boldsymbol{\varphi}) \sum_{k=0}^{t} \gamma_k}{U_L^W(t; \boldsymbol{\varphi}) + U_L^P(t; \boldsymbol{\varphi}) \sum_{k=0}^{t} \gamma_k} \frac{V_L(t; \boldsymbol{\varphi})}{V_C(t; \boldsymbol{\varphi})} \tag{B.4}$$

$$\tau_{t+1}^a = 1 - \frac{U_C^W(t+1; \boldsymbol{\varphi}) + U_C^P(t+1; \boldsymbol{\varphi}) \sum_{k=0}^{t+1} \gamma_k}{U_C^W(t; \boldsymbol{\varphi}) + U_C^P(t; \boldsymbol{\varphi}) \sum_{k=0}^{t} \gamma_k} \frac{V_C(t; \boldsymbol{\varphi})}{V_C(t+1; \boldsymbol{\varphi})} \tag{B.5}$$

B.1 Formulas for Separable Isoelastic Preferences

Consumption and labor shares ψ_c^i,ψ_l^i follow

$$\psi_c^i = \frac{\left(\varphi^i\right)^{1/\sigma}}{\sum_{i \in I} \pi^i (\varphi^i)^{1/\sigma}}, \quad \psi_l^i = \frac{\left(\theta^i\right)^{\frac{1+\nu}{\nu}} \left(\varphi^i\right)^{-1/\nu}}{\sum_{i \in I} \pi^i (\theta^i)^{\frac{1+\nu}{\nu}} \left(\varphi^i\right)^{-1/\nu}}. \tag{B.6}$$

Given equations (B.6), we have the followings:

$$\Phi_C^V = \left[\sum_i \pi^i (\varphi^i)^{1/\sigma} \right]^\sigma; \qquad \Phi_L^V = \omega \left[\sum_i \pi^i \left(\varphi^i\right)^{-1/\nu} (\theta^i)^{(1+\nu)/\nu} \right]^{-\nu}$$

$$\Phi_C^P = \Phi_C^V \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i} \psi_c^i; \qquad \Phi_L^P = \Phi_L^V \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i} \psi_l^i$$

$$\Phi_C^W = \Phi_C^V \sum_{i\in I} \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i \right]; \qquad \Phi_L^W = \Phi_L^V \sum_{i\in I} \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i \right]$$

C Proofs

C.1 Proof of Proposition 1.1

Proof. ($\Rightarrow$) Let $\{C_t, L_t\}_{t=0}^\infty$ be an aggregate allocation of an open economy competitive equilibrium with government policies. Then by definition, $\{C_t, L_t\}_{t=0}^\infty$ satisfies aggregate resource constraint for every period. Moreover, given any market weights $\boldsymbol{\varphi}$, $\{C_t, L_t\}_{t=0}^\infty$ satisfies

$$(1-\tau_t^n) w_t = -\frac{V_L(C_t, L_t; \boldsymbol{\varphi})}{V_C(C_t, L_t; \boldsymbol{\varphi})}$$

$$1 - \tau_{t+1}^a = \frac{V_C(C_t, L_t; \boldsymbol{\varphi})}{\beta R_{t+1}^* V_C(C_{t+1}, L_{t+1}; \boldsymbol{\varphi})}$$

Substituting for w_t into the budget constraint (2) and using $(c_t^i, l_t^i) = h^i(C_t, L_t; \boldsymbol{\varphi})$ gives the implementability constraint for each agent. Importantly, one can choose $\boldsymbol{\varphi}$ and T such that the individual implementability constraints hold with equality.

($\Leftarrow$) Given $\boldsymbol{\varphi}$, T and an allocation $\{C_t, L_t\}_{t=0}^\infty$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\{w_t\}_{t=0}^\infty$ using firm's first-order conditions (4). $\{\tau_t^n\}_{t=0}^\infty$ can be calculated using the intratemporal condition (7), while one can choose $\{\tau_{t+1}^a\}_{t=0}^\infty$ that satisfy the intertemporal constraint (8). Define $\{Q_t^*\}_{t=0}^\infty$ as $Q_t^* = \Pi_{\tau=1}^t \frac{1}{1+r_\tau^*}$.

Rewriting the aggregate resource constraint using $F(L) = wL$ gives

$$\sum_{t=0}^\infty Q_t^* \left\{ C_t - (1-\tau_t^n) w_t L_t + T_t \right\} + \sum_{t=0}^\infty Q_t^* \left[G_t - \tau_t^n w_t L_t - T_t \right] \le -B_0 \tag{C.1}$$

Aggregating up the agent's budget constraints implies

$$C_t + \frac{1}{1+r_t^*} \left(A_{t+1}^d + A_{t+1}^f \right) = (1-\tau_t^n) w_t L_t + (1-\tau_t^a) \left(A_t^d + A_t^f \right) - T_t$$

or

$$C_t - (1-\tau_t^n)w_t L_t + T_t = (1-\tau_t^a)\left(A_t^d + A_t^f\right) - \frac{1}{1+r_t^*}\left(A_{t+1}^d + A_{t+1}^f\right)$$

Substituting the last equation into (C.1) gives the government's budget constraint (5). Thus, $\{C_t, L_t\}_{t=0}^{\infty}$ is the aggregate allocation of the constructed competitive equilibrium with government policies. □

C.2 Proof of Proposition 2.1

The proof proceeds in two steps. First, the sustainable allocation is characterized by the steady-state level of debt that maximizes repayment capacity. Second, I demonstrate that the financial autarky allocation is never optimal at any point in time. Together, the results imply that the long-run sustainable debt corresponds to the maximum debt level and is strictly positive.

Before proceeding to the main proof, I establish a few lemmas that are useful. Consider the static problem

$$\begin{aligned} V(C, L; \boldsymbol{\varphi}) \equiv & \max_{(c^i, l^i)_{i\in I}} \sum_{i\in I} \varphi^i \pi^i U^i(c^i, l^i) \\ & s.t. \quad \sum_{i\in I} \pi^i c^i = C; \quad \sum_{i\in I} \pi^i l^i = L. \end{aligned}$$

The individual allocation is strictly increasing in the aggregate allocation.

Lemma C.1. *In equilibrium, $\forall i \in I$, $c^i(C, L; \boldsymbol{\varphi}) = c^i(C; \boldsymbol{\varphi})$ and is strictly increasing in C. Similarly, $l^i(C, L; \boldsymbol{\varphi}) = l^i(L; \boldsymbol{\varphi})$ and is strictly increasing in L .*

Proof. Let $\mu_c$and μ_l be the multipliers on the constraints. Then given assumption 1, the first-order condition with respect to $c^i$is $\varphi^i u'(c^i) = \mu_c$, which implies that $c^i = u'^{-1}(\mu_c/\varphi^i)$ and $C = \sum_i \pi^i c^i = \sum_i \pi^i u'^{-1}(\mu_c/\varphi^i)$. Since all of the functions are continuous, μ_c only depends on $C, \boldsymbol{\varphi}$. So c^i is only a function of $C, \boldsymbol{\varphi}$.

To show strict monotonicity, suppose $C_1 < C_2$ then $\sum_i \pi^i u'^{-1}\left(\frac{\mu_{c,1}}{\varphi^i}\right) < \sum_i \pi^i u'^{-1}\left(\frac{\mu_{c,2}}{\varphi^i}\right)$, which implies that $\mu_{c,1} > \mu_{c,2}$. u'^{-1} is strictly decreasing, so $c_1^i < c_2^i$.

Similarly, we can show that l^i only depends on $L, \boldsymbol{\varphi}$ and is strictly increasing in L. □

This implies that U^P is strictly increasing in C and strictly decreasing in L.

I next show that the sustainable allocation features no immiseration.

Lemma C.2. *For any sustainable allocation $\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*$, $\liminf_{t\to\infty} C_t^* > 0$ and $\limsup_{t\to\infty} L_t^* < \bar{L}$ for some $\bar{L}$.*

Proof. Suppose, by contradiction that $\liminf_{t\to\infty} C_t^* \leq 0$. Find $\epsilon > 0$ such that $\forall t$,

$$\sum_{s=t}^{\infty} \beta^{s-t} \left\{U^P\left(C_s, L_s^*; \boldsymbol{\varphi}, \boldsymbol{\lambda}\right)\right\} \leq M_U$$

with $C_t = \epsilon$ and $C_s = C_s^*,\ \forall s > t$. Such ϵ exists since the utility function is unbounded below. Because $\liminf_{t\to\infty} C_t^* \le 0$, there exists t_0 such that $C_{t_0}^* < \epsilon$. Then by monotonicity ,

$$\begin{aligned}
&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ U^P\left(C_s^*, L_s^*; \boldsymbol{\varphi}, \boldsymbol{\lambda}\right) \right\} \\
<&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{\left(L_s^*\right)^{1+\nu}}{1+\nu} \right\} \\
\le& M_U \\
\le& \underline{U}_{t_0}
\end{aligned}$$

which contradicts the aggregate debt constraint at t_0.
The similar argument applies to give $\limsup_{t\to\infty} L_t^* < \bar{L}$. □

The marginal utilities from the optimal contracting problem are bounded.

Lemma C.3. *For any sustainable allocation $\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*$, $U_C^W(t;\boldsymbol{\varphi})$ and $U_L^W(t;\boldsymbol{\varphi})$ are bounded.*

Proof. The expressions can be written as

$$\begin{aligned}
U_C^W(t;\boldsymbol{\varphi}) &= \sum_i \pi^i \frac{\partial c_t^i}{\partial C_t} u'(c_t^i) \left\{ \lambda^i + \eta^i \varphi^i \left[\frac{u''(c_t^i) c_t^i}{u'(c_t^i)} + 1 \right] \right\} \\
U_L^W(t;\boldsymbol{\varphi}) &= -\sum_i \pi^i \frac{1}{\theta^i} v'(l_t^i/\theta^i) \frac{\partial l_t^i}{\partial L_t} \left\{ \lambda^i + \eta^i \varphi^i \left[\frac{1}{\theta^i} \frac{v''(l_t^i/\theta^i) l_t^i}{v'(l_t^i/\theta^i)} + 1 \right] \right\}
\end{aligned}$$

One can show that $\frac{\partial c_t^i}{\partial C_t} = -\frac{u'(c_t^i)}{\varphi^i u''(c_t^i)}$ and $\frac{\partial l_t^i}{\partial L_t} = -\frac{v'(l_t^i/\theta^i)}{\varphi^i \frac{1}{\theta^i} v''(l_t^i/\theta^i)}$ which are bounded given Assumption 1. The first-order condition of the optimal contracting problem with respect to φ^i gives that $\eta^i = \sum_{j\in I} \frac{\lambda^j}{\varphi^j} - \frac{\lambda^i}{\varphi^i}$ which is finite. Then $U_C^W(t;\boldsymbol{\varphi})$ and $U_L^W(t;\boldsymbol{\varphi})$ are bounded. □

Now we proceed to the main proof.

Proof. **Step 1:** The sustainable allocation $\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*$ converges to the maximal steady-state debt allocation $\{C_{ss}^*, L_{ss}^*\}$that is the solution to

$$\begin{aligned}
&\max_{C,L} \left\{ \frac{1+r^*}{r^*} \left(F(L) - C - G\right) \right\} \\
&s.t. \quad U^P(C, L; \boldsymbol{\varphi}^*) \ge U^P(C_\infty^*, L_\infty^*; \boldsymbol{\varphi}^*)
\end{aligned}$$

Let ψ_u be the multiplier on the utility constraint. Then $\{C_{ss}^*, L_{ss}^*\}$ satisfies first-order conditions

$$\begin{aligned}
\psi_u U_C^P(C_{ss}^*, L_{ss}^*; \boldsymbol{\varphi}) &= 1 \\
-\psi_u U_L^P(C_{ss}^*, L_{ss}^*; \boldsymbol{\varphi}) &= F_L(C_{ss}^*, L_{ss}^*) \\
\psi_u \left(U^P(C_{ss}^*, L_{ss}^*; \boldsymbol{\varphi}) - U^P(C_\infty^*, L_\infty^*, \boldsymbol{\varphi}^*) \right) &= 0
\end{aligned}$$

U^P is strictly increasing in C, so $\psi_u > 0$.

Define $\psi_u^t = \frac{\beta^t}{\mu Q_t^*} \sum_{k=0}^{t} \gamma_k$, then from equations (B.1) and (B.2), we have that the sustainable allocation follows

$$
\begin{aligned}
1 - \psi_u^t U_C^P(C_t^*, L_t^*; \boldsymbol{\varphi}^*) &= \frac{\beta^t}{\mu Q_t^*} U_C^W(C_t^*, L_t^*; \boldsymbol{\varphi}^*) \\
F_L(L_t^*) + \psi_u^t U_L^P(C_t^*, L_t^*; \boldsymbol{\varphi}^*) &= -\frac{\beta^t}{\mu Q_t^*} U_L^W(C_t^*, L_t^*; \boldsymbol{\varphi}^*)
\end{aligned}
$$

Taking the limit as $t \to \infty$ gives

$$
\begin{aligned}
\lim_{t\to\infty} \left[1 - \psi_u^t U_C^P(C_t^*, L_t^*; \boldsymbol{\varphi}^*)\right] &= 0 \\
\lim_{t\to\infty} \left[F_L(t) + \psi_u^t U_L^P(t)\right] &= 0 \\
\lim_{t\to\infty} \left(U^P(C_{ss}^*, L_{ss}^*; \boldsymbol{\varphi}) - U^P(C_\infty^*, L_\infty^*, \boldsymbol{\varphi}^*)\right) &= 0
\end{aligned}
$$

Therefore, the long-run sustainable allocation coincides with the maximal steady-state debt allocation.

Step 2: Financial autarky allocation is never optimal at any point in time.

Suppose, by contradiction that the autarkic allocation $\{C_t^a, L_t^a, \varphi^a\}_{t=0}^{\infty}$ solves the planning problem from $t = \mathcal{T}$ onwards. Fix $\epsilon > 0$. Consider the allocation $\left\{\hat{C}_t, \hat{L}_t, \hat{\varphi}_t\right\}_{t=0}^{\infty}$ such that for $t < \mathcal{T}$,we have , $\hat{C}_t = C_t^*, \hat{L}_t = L_t^*, \hat{\varphi}_t = \varphi^*$ and $\hat{\varphi}_t = \varphi^a$ for $t \geq \mathcal{T}$

For period $\mathcal{T}$:$\hat{C}_\mathcal{T} = C_\mathcal{T}^a + \frac{\epsilon}{R^*}\left[F_L(L_{\mathcal{T}+1}^a) + \frac{U_L^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)}{U_C^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)}\right]$, $\hat{L}_\mathcal{T} = L_\mathcal{T}^a$

For period 1: $\hat{C}_{\mathcal{T}+1} = C_{\mathcal{T}+1}^a - \frac{U_L^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)}{U_C^P(C_{\mathcal{T}+1}^a, L_{\mathcal{T}+1}^a, \varphi^a)}\epsilon$, $\hat{L}_{\mathcal{T}+1} = L_{\mathcal{T}+1}^a + \epsilon$

For period $t \geq \mathcal{T}+2$:$\hat{C}_t = C_t^a, \hat{L}_t = L_t^a$

First, $\left\{\hat{C}_t, \hat{L}_t, \hat{\varphi}_t\right\}_{t=0}^{\infty}$ is feasible because

$$
\begin{aligned}
&\sum_{t\geq 0}\left(\frac{1}{R^*}\right)^t \left[F(\hat{L}_t) - \hat{C}_t - G_t\right] \\
=&F(\hat{L}_\mathcal{T}) - \hat{C}_\mathcal{T} - G_\mathcal{T} + \frac{1}{R^*}\left[F(\hat{L}_{\mathcal{T}+1}) - \hat{C}_{\mathcal{T}+1} - G_{\mathcal{T}+1}\right] + \sum_{t<\mathcal{T}}\left(\frac{1}{R^*}\right)^t \left[F(\hat{L}_t) - \hat{C}_t - G_t\right] + \sum_{t\geq\mathcal{T}+2}^{\infty}\left(\frac{1}{R^*}\right)^t \left[F(\hat{L}_t) - \hat{C}
\\
=&\sum_{t<\mathcal{T}}\left(\frac{1}{R^*}\right)^t \left[F(\hat{L}_t) - \hat{C}_t - G_t\right] + \sum_{t\geq\mathcal{T}+2}\left(\frac{1}{R^*}\right)^t \left[F(L_t^a) - C_t^a - G_t\right] - \frac{\epsilon}{R^*}\left[F_L(L_1^a) + \frac{U_L^P(C_1^a, L_1^a)}{U_C^P(C_1^a, L_1^a)}\right] + \frac{1}{R^*}\Bigg(F_L(L_1^a)\epsilon
\\
=&\sum_{t<\mathcal{T}}\left(\frac{1}{R^*}\right)^t \left[F(\hat{L}_t) - \hat{C}_t - G_t\right] + \sum_{t\geq\mathcal{T}+2}\left(\frac{1}{R^*}\right)^t \left[F(L_t^a) - C_t^a - G_t\right] \\
\geq& B_0
\end{aligned}
$$

The last inequality is guaranteed by assumption that the autarkic allocation solves the planning problem from $t = \mathcal{T}$ onwards

$\left\{\hat{C}_t, \hat{L}_t, \hat{\varphi}_t\right\}_{t=0}^{\infty}$ is implementable in equilibrium since there exists a $\hat{T}$ such that $\forall i \in I$

$$\sum_{t\geq 0} \beta^t \left[V_C(t;\boldsymbol{\varphi})c^i(t;\boldsymbol{\varphi}) + V_L(t;\boldsymbol{\varphi})l^i(t;\boldsymbol{\varphi})\right] = V_C(0;\boldsymbol{\varphi})\left(a_0^i - \hat{T}\right)$$

The flow utilities for all period $t \geq \mathcal{T}+1$ do not change. That is,
$U^P_{\mathcal{T}+1}(\hat{C}_{\mathcal{T}+1}, \hat{L}_{\mathcal{T}+1}, \hat{\varphi}_{\mathcal{T}+1}) = U^p_{\mathcal{T}+1}(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a) + U^P_L(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)\epsilon - U^P_C(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1})\frac{U^P_L(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)}{U^P_C(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)}\epsilon =$
$U^p_{\mathcal{T}+1}(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)$ (For sufficient small ϵ)
$U^P_t(\hat{C}_t, \hat{L}_t, \hat{\varphi}_t) = U^p_t(C^a_t, L^a_t, \varphi^a), \forall t \geq \mathcal{T}+2$
and the flow utility in period $\mathcal{T}$ increases. That is,

$$\begin{aligned} U^P_{\mathcal{T}}(\hat{C}_{\mathcal{T}}, \hat{L}_{\mathcal{T}}, \hat{\varphi}_{\mathcal{T}}) &= U^p_{\mathcal{T}}(C^a_{\mathcal{T}}, L^a_{\mathcal{T}}, \varphi^a) + U^p_{C,\mathcal{T}}(C^a_{\mathcal{T}}, L^a_{\mathcal{T}}, \varphi^a)\frac{\epsilon}{R^*}\left[F_L(L^a_{\mathcal{T}+1}) + \frac{U^P_L(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)}{U^P_C(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)}\right] \\ &> U^p_{\mathcal{T}}(C^a_{\mathcal{T}}, L^a_{\mathcal{T}}, \varphi^a) \end{aligned}$$

because $F_L(L^a_{\mathcal{T}+1}) + \frac{U^P_L(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)}{U^P_C(C^a_{\mathcal{T}+1}, L^a_{\mathcal{T}+1}, \varphi^a)} > 0$ as the aggregate labor distortion in autarky is positive. Thus, $V^P_0\left(\left\{\hat{C}_t, \hat{L}_t; \varphi^a\right\}_{t=0}^{\infty}\right) > V^P_0\left(\{C^a_t, L^a_t; \varphi^a\}_{t=0}^{\infty}\right)$, which contradicts $\{C^a_t, L^a_t, \varphi^a\}_{t=0}^{\infty}$ being the optimal allocation. □

C.3 Proof of Proposition 3.1

First, I show that the following lemma must hold.

Lemma C.4. *cov* $\left(\psi^i_c, \frac{\lambda^i}{\varphi^i}\right) < 0$ *and cov* $\left(\psi^i_l, \frac{\lambda^i}{\varphi^i}\right) < 0$

Proof. The first step is to show that for i and j such that $i \neq j$, $\theta^i > \theta^j \iff \varphi^i > \varphi^j$.
Suppose $\theta^i > \theta^j$ and $\varphi^i \leq \varphi^j$, then $\psi^i_l \leq \psi^j_l$. By the definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \leq \frac{\varphi^i}{\varphi^j} \leq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} > 1$, which is a contradiction.
Suppose $\varphi^i > \varphi^j$ and $\theta^i \leq \theta^j$, then $\psi^i_l > \psi^j_l$. By the definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} > \frac{\varphi^i}{\varphi^j} > 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \leq 1$, which is a contradiction.
Next, the individual implementability constraint is

$$\psi^i_c \Phi^V_C \sum_t \beta^t C_t^{1-\sigma} - \psi^i_l \Phi^V_L \sum_t \beta^t L_t^{1+\nu} = \Phi^V_C C_0^{-\sigma}\left(a^i_0 - T\right)$$

or

$$\psi^i_c = \psi^i_l \frac{\Phi^V_L \sum_t \beta^t L_t^{1+\nu}}{\Phi^V_C \sum_t \beta^t C_t^{1-\sigma}} + \frac{\Phi^V_C C_0^{-\sigma}\left(a^i_0 - T\right)}{\Phi^V_C \sum_t \beta^t C_t^{1-\sigma}}$$

By the definition of ψ^i_c, $\varphi^i > \varphi^j \iff \psi^i_c > \psi^j_c$, and by the assumption, $\theta^i > \theta^j \iff a^i_0 > a^j_0$, which implies that $\theta^i > \theta^j \iff \psi^i_c > \psi^j_c \iff \psi^i_l > \psi^j_l$.

Thus, $\theta^i > \theta^j \iff \varphi^i > \varphi^j \iff \psi_c^i > \psi_c^j \iff \psi_l^i > \psi_l^j$.
In addition, $\theta^i > \theta^j \iff \lambda^i < \lambda^j$, which implies that

$$\psi_c^i > \psi_c^j \iff \frac{\lambda^i}{\varphi^i} < \frac{\lambda^j}{\varphi^j}$$
$$\psi_l^i > \psi_l^j \iff \frac{\lambda^i}{\varphi^i} < \frac{\lambda^j}{\varphi^j}$$

Hence, $\text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < 0$ and $\text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$. □

Now I proceed to the main proof of the proposition.

Proof. In financial autarky, there exist a vector of market weights $\boldsymbol{\varphi}^{\boldsymbol{a}}$, transfer T^a, and multiplier $\boldsymbol{\eta}^{\boldsymbol{a}}$that satisfies the conditions in Proposition 1.1 such that

$$\underline{U}_t \equiv \max_{C_t, L_t, \boldsymbol{\varphi}^{\boldsymbol{a}}, T^a} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^{W,a} \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^{W,a} \frac{L_t^{1+\nu}}{1+\nu} \right] - \Phi_C^{V,a} C_0^{-\sigma} \sum_i \pi^i \eta^{i,a} T^a$$
$$s.t. \qquad C_t + G = F(L_t, t)$$

where $\beta^t \pi^i \eta^{i,a}$ is the Lagrange multiplier on the individual implementability constraint and $\Phi_C^{W,a}, \Phi_L^{W,a}$ follows the formulas in Appendix B.1.
The aggregate labor distortion is constant and equal to

$$\Omega^a = 1 - \frac{\Phi_C^W \Phi_L^P}{\Phi_L^W \Phi_C^P}$$

We have that

$$\frac{\Phi_C^W}{\Phi_C^P} = 1 + (1-\sigma)\left[\frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right]}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)} - 1 \right]; \quad \frac{\Phi_L^W}{\Phi_L^P} = 1 + (1+\nu)\left[\frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right]}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)} - 1 \right]$$

Lemma C.4 shows that $\text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right), \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$, which implies that

$$\frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right]}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)} - 1 > 0; \quad \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right]}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)} - 1 > 0$$

and given that $\sigma \geq 1$ and $\nu > 0$, we have $\frac{\Phi_C^W}{\Phi_C^P} < \frac{\Phi_L^W}{\Phi_L^P}$. Thus, $\Omega^a > 0$. □

C.4 Proof of Proposition 3.2

Proof. For both of the two cases, $\lambda^i = \varphi^i$ and $\eta^i = 0$. This implies that $\Phi_C^W = \Phi_C^P$ and $\Phi_L^W = \Phi_L^P$. The aggregate labor distortion in financial autarky is

$$\Omega^a = 1 - \frac{\Phi_C^W \Phi_L^P}{\Phi_L^W \Phi_C^P} = 0$$

□

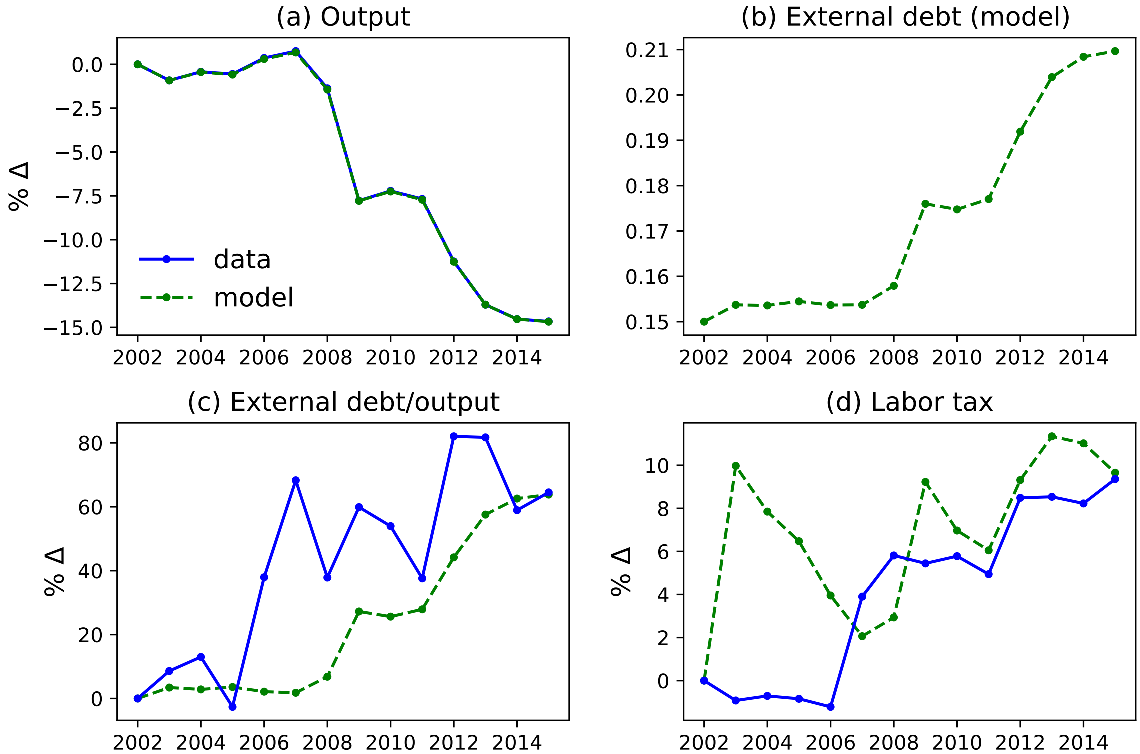
D Quantitative Appendix

This section provides additional details that is implemented in Section 4 .

D.1 Event Analysis

I conduct an event analysis for Italy in period of 2002 to 2015. I feed into the model a sequence of productivity shock realizations and the initial external debt-to-output ratio such that the model's sequence of outputs matches the sequence of Italian output and the change in external debt-to-output ratio is the same both in the model and the data from 2002 to 2015. I then compare the relative change of external debt-to-output in the data and in the model's simulation over time, given that 2002 is the benchmark year. Figure 4 plots the exercise's results. Panel (a) plots the output paths of the data and the model. Panel (b) plots the time path of external debt in the model. Panel (c) plots external debt-to-output time paths for both the data and the model. Panel (d) plots estimated labor taxes from McDaniel (2007) and the optimal labor taxes from the model. From 2011 to 2015, Italy's output has dropped by 7.6% below trend, accompanying with a 20% increase in external debt-to-output and a 4.2% increase in labor tax. In the model's simulation, the similar drop in output is associated with a 26% increase in external debt-to-output and a 3.5% increase in labor tax.

Figure 4: Italy's Recession: Data and Model



Note: The graph depicts the time paths of output, external debt, and external debt-to-output for the data and the model's simulation. Panel (a) plots the output path. Panel (b) plots the external debt paths of the model. Panel (c) plots external debt-to-output, and panel (d) plots the labor tax. The simulation uses a sequence of productivity shock realization such that the model's output matches the data output for Italy in 2002-2015. The initial external debt level is such that the model's external debt-to-output matches with the starting value in 2002 from the data. The benchmark period is 2002. Data sources: McDaniel (2007), Lane and Milesi-Ferretti (2018), and The World Bank (2019).

D.2 Computational Algorithm

Given the forward-looking borrowing constraints, I implement the recursive formulation developed by Marcet and Marimon (2019).

1. Guess μ and φ. Compute η.

   (a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ
   Initial guess for $V(s_t, \mu_t, \Gamma_{t-1}) = \sum_{j\geq 0, s^t\subseteq s^{t+j}} \beta^j \Pr(s^{t+j}) \left[\Phi_C^P \frac{C(s^{t+j})^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^{t+j})^{1+\nu}}{1+\nu}\right]$.

   (b) Assume the constraint does not bind in s_t: $\gamma(s_t) = 0$. Solve for the allocation $C(s_t), L(s_t)$ using the first-order conditions

$$\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1}\right] C(s_t)^{-\sigma} = \mu$$
$$\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1}\right] L(s_t)^{\nu} = \mu F_L(s_t)$$

(c) Since $\gamma(s_t) = 0$, compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$ (interpolating the expectation), then compute

$$\begin{aligned} A(s_t) &= \sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^\tau \subseteq s^t} Pr(s^\tau) \left[\Phi_C^P \frac{C(s^\tau)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^\tau)^{1+\nu}}{1+\nu} \right] \\ &= \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1}) \end{aligned}$$

(d) Check if $A(s_t) \geq \underline{U}(s_t)$. If it is, proceed to the next step. If not, solve for $C(s_t), L(s_t), \gamma(s_t)$ using these equations

$$\begin{aligned} \left[\mu_t \Phi_C^W + \Phi_C^V \left(\Gamma_{t-1} + \gamma(s_t) \right) \right] C(s_t)^{-\sigma} &= \mu \\ \left[\mu_t \Phi_L^W + \Phi_L^V \left(\Gamma_{t-1} + \gamma(s_t) \right) \right] L(s_t)^{\nu} &= \mu F_L(s_t) \\ \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} & \\ + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V\left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t) \right)\right) &= \underline{U}(s_t) \end{aligned}$$

(e) Given $C(s_t), L(s_t), \gamma(s_t)$($\gamma$ can be zero or not), compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \Gamma_{t-1} + \gamma(s_t))$. Update the value function

$$\begin{aligned} V^{n+1}(s_t, \Gamma_{t-1}) = {} & \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} \\ & + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V^n \left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t) \right)\right) \end{aligned}$$

2. Compute residuals to find μ and φ

$$\begin{aligned} r^\mu &= \sum_{t \geq 0} q_t \left[F(L_t) - G_t - C_t \right] - B_0 \\ r^\varphi &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^V \left(\psi_c^i - \psi_c^j \right) C_t^{1-\sigma} - \Phi_L^V \left(\psi_l^i - \psi_l^j \right) L_t^{1+\nu} \right] \end{aligned}$$

3. Find μ and φ such that $r^\mu = 0$ and $r^\varphi = 0$.

E Data

E.1 Macroeconomic Data Descriptions and Sources

Most data are annual series covering the 1985-2015 period. Some data samples cover the 2002-2015 period.

- Net foreign liability is the negative of net foreign asset (NFA) from the External Wealth of Nations Database, Lane and Milesi-Ferretti (2018)
- Net international investment position is the official international investment position (IIP) from the External Wealth of Nations Database, Lane and Milesi-Ferretti (2018)
- Pre-tax Gini Index the market Gini from the Standardized World Income Inequality Database, Solt (2019).
- GDP per capita is the constant 2010 US Dollar GDP per capita series from World Development Indicator Database, The World Bank (2019)
- GDP growth is the log difference of constant 2010 US Dollar GDP series from World Development Indicator Database, The World Bank (2019)
- Inflation is the annual inflation series measured by the GDP deflator from World Development Indicator Database, The World Bank (2019)
- Real GDP is GDP series in constant local currency units from World Development Indicator Database, The World Bank (2019)
- Real return on German bond is the interest rate on Gernman bond adjusted for inflation measured by the GDP deflator. The interest rate is the long-term interest rate for converenge purposes from the Eurostat Database (2019). These bonds have 10-year maturity and are denominated in Euro.
- Real interest rate is the lending interest rate adjusted for inflation as measured by the GDP deflator from World Development Indicator Database, The World Bank (2019)
- Italy's cross-sectional wage inequality is calculated from the micro-data by Jappelli and Pistaferri (2010) using Surveys of Household Income and Wealth conducted by the Bank of Italy for the period 1980-2006.
- Government consumption is the general government final consumption expenditure series from World Development Indicator Database, The World Bank (2019)
- Private consumption is the households and NPISHs final consumption expenditure series from World Development Indicator Database, The World Bank (2019)

E.2 Italian Household Survey

For the analysis of wage inequality, I use household-level data from the *Survey on Household Income and Wealth* (SHIW), conducted by the Bank of Italy. The SHIW includes cross-sectional and panel data on household's demographics, income, labor supply, consumption, and wealth. Jappelli and Pistaferri (2010) provide details on the survey design descriptions and data quality analysis. I use data for the period 1987 to 2014. In this period, the survey was conducted biennially, except for the period 1995 to 1998 with a 3-year interval.

The sample selection is followed. The original individual sample includes 329,446 units from 1987 to 2014. Given the focus on wage inequality and labor supply across households, I focus on heads of households between the age of 25 and 60. This selection criteria reduces the sample size to 71,621 units. To reduce the impact of outliers, I exclude observations with no hours worked, with negative income, or with hourly wage in the bottom 0.5% of the distribution. The final sample size is 67,176.

E.3 Lists of Countries

Argentina, Bolivia, Brazil, Cameroon, Colombia, Costa Rica, Ecuador, Egypt, El Salvador, Greece, Guatemala, Honduras, India, Indonesia, Italy, Ireland, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Peru, Philippines, Portugal, Spain, Sri Lanka, Thailand, Tunisia, Turkey, Venezuela.