Sovereign Debt Sustainability and Redistribution*

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February 2021

Abstract

This paper proposes a theory of sovereign debt sustainability based on the government's motive for redistribution. I study a small open economy model in which taxes are distortionary and the government has a redistributive concern and faces endogenous borrowing constraints due to its lack of commitment. Given these borrowing constraints, the value of financial autarky (default) determines the sustainable level of debt. When the government has access to external financing, it redistributes the external funds to domestic households via domestic financial markets and levy low distortionary labor taxes. Default resulting in financial autarky is endogenously costly because, without external borrowing, redistribution requires high distortionary taxes, which reduce the economy's efficiency. Quantitatively, the theory can account for the external debt's recent buildup in Italy and is consistent with the positive correlation between pre-tax income inequality and external debt across countries. In response to a negative productivity shock, the optimal austerity policies are increasing external borrowing and redistribution while reducing redistribution to repay debt in the future.

Keywords: Inequality; Limited commitment; Optimal taxation; Redistribution; Sovereign debt

JEL Classifications: E32; F34; F38; H21; H23; H63

*This is a revised version of the second chapter of my dissertation at the University of Minnesota. I thank V.V. Chari, Larry Jones, and Manuel Amador for their invaluable advice and guidance. I am grateful for comments from Fernando Arce, Cristina Arellano, David Argente, Anmol Bhandari, Jarda Borovicka, Javier Bianchi, Amy Handlan, Jonathan Heathcote, Ben Malin, Chris Phelan, David Rahman, Paulina Restrepo-Echavarria, Aldo Rustichini, Jan Werner, the seminar participants at University of Minnesota, the Federal Reserve Bank of Minneapolis, and the Federal Reserve Bank of St. Louis.

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Introduction

The recent European debt crises have prompted intense policy debates on the design of fiscal policies during severe economic downturns, in which output is low and external debt rapidly increases until it is constrained by the country's inability to repay. Austerity policies such as increasing tax revenue or reducing government expenditure provide more resources to repay debt but have unequal effects on domestic residents.[1] Therefore, appropriate policy design requires understanding the interaction between a government's commitment to debt repayment and its commitment to maintain a level of redistribution. Two key questions arise: How does a government's redistributive goals affect its incentive to repay debt? Given the answer to the first question, how does one design optimal austerity policies taking their distributional consequences into account?

To address these questions, this paper analyzes a small open economy model in which redistribution comes with an efficiency cost in terms of labor tax distortion and the government faces endogenous borrowing constraints due to its lack of commitment. Given these constraints, the cost of financial autarky endogenously determines the sustainable level of external debt. I examine the trade off between the government's desire for redistribution and its ability to sustain external debt and evaluate optimal responses of fiscal policies to aggregate shocks in the presence of inequality.

I develop a theory of external debt sustainability based on the government's motive for redistribution. When the government can borrow externally, it redistributes the external funds to domestic households via domestic financial markets and levy low distortionary labor taxes. Default resulting in financial autarky is endogenously costly because, without external borrowing, redistribution requires high distortionary taxes, which reduce the economy's efficiency. Therefore, the government is willing to repay its external obligations in order to continue having access to external financial markets. I then show that this theory can quantitatively account for the recent buildup of external debt in Italy and is consistent with the positive correlation between pre-tax income inequality and external debt across countries and over time. The optimal austerity policies responding to a negative productivity shock are increasing external borrowing, decreasing average taxes, and increasing redistribution while raising average taxes and reducing redistribution to repay debt in the future.

As an empirical motivation, the paper first documents the cross-country and time series properties of pre-tax income inequality and external debt using two multi-country panel data sets on inequality and balance of payments. First, highly indebted countries have also experi-

[1]The United Kingdom and Ireland implemented expenditure cuts, while Greece, Italy, Portugal, and Spain implemented both tax increases and expenditure cuts for their austerity plans. Most of these plans include cuts in public services, pension programs, and education programs. Monastiriotis (2011) argues that in Greece, the prolonged fiscal consolidation has exacerbated regional disparities and imbalances. Leventi and Matsaganis (2016) use a micro-simulation model to assess the distributional effects of austerity policies and find that such policies have led to higher poverty and after-tax income inequality, worsening the adverse distributional effects of the recession. Brinca, Homem Ferreira, Franco, Holter, and Malafry (2019) show that fiscal consolidations are more recessive when income inequality is higher.

enced high levels of pre-tax income inequality. Second, the increase in the country's net financial outflows has coincided with an increase in aggregate pre-tax income inequality. The regression estimation shows that this positive relationship is robust to changes in output levels and output growth.[2] These facts point to a connection between a government's redistributive goals and its external debt management.

The model features a continuum of domestic agents that are impatient and differ by labor productivity types. The aggregate shocks are in aggregate productivity and government spending.[3] Domestic and external credit markets consist of state-contingent assets. The tax system has lump-sum taxes as well as marginal taxes that are distortionary to individual labor supply and saving decisions. The government cares about all domestic agents, assigning individual welfare weights that represent its distributional preference. The government lacks commitment in all tax and debt policies.

Concerns for redistribution rationalize the need for distortionary taxation. Since all domestic agents face the same tax rates, the government redistributes resources by levying a positive labor tax alongside a lump-sum transfer. In this way, highly-skilled, high-income agents bear a larger tax burden than low-skilled, low-income ones. Alternatively, the government can use a tax on domestic borrowing and a lump-sum transfer, which implies that the highly indebted agents will pay more taxes than the less-indebted agents. In this environment, levels of tax distortions represent the cost of redistribution.[4]

The government's lack of commitment imposes endogenous limits on the economy's external borrowing. The government chooses its policies sequentially in a repeated game between the government, domestic agents, and international creditors. The contract is an ex ante set of policies such that if the government deviates from any of its policies, it triggers a punishment to financial autarky, in which permanent exclusions from domestic and external credit markets take place. For example, even if the government only defaults on external debt, it is still excluded from both domestic and external financing.[5] Domestic agents are still able to participate in the domestic credit market. One can characterize the subgame perfect equilibrium with self-enforcing constraints, in which the continuation value of staying in the contract has to be at least the value of financial autarky. These constraints act as endogenous borrowing constraints.

The impatience of the domestic agents means that they want to borrow. The domestic need for borrowing leads to the country running up debt and eventually hitting the borrowing con-

[2]The end of this section provides an overview of other papers that document similar and related empirical patterns. These papers include Berg and Sachs (1988), Aizenman and Jinjarak (2012), Jeon, and Kabukcuoglu (2018), and Ferriere (2015) which use different measures and estimation techniques.

[3]Later on when I match the model to the data, government spending is measured as the government's final consumption of public goods, excluding spending on social welfare programs.

[4]This is because the government can raise lump-sum taxes to finance expenditures and debt repayment without distorting the domestic agents' decisions. Werning (2007) provides a similar intuition. The presence of lump-sum taxes removes the revenue purposes of distortionary taxation.

[5]This is equivalent to the assumption of nondiscriminatory defaults on domestic or external lenders. See, for example, D'Erasmo and Mendoza (2016) for a similar assumption. For an example on discriminatory defaults, see Gonzalez-Aguado (2018).

straints. However, an infinitesimal domestic agent does not internalize the fact that as she borrows more, the borrowing constraints become tighter. There is a shadow price of borrowing that does not enter into the individual problem. When the borrowing constraint binds, the government can set borrowing taxes such that domestic agents face the correct borrowing cost.

The main theoretical result establishes that the government sustains external debt in equilibrium, given that the government is inequality averse, initial wealth inequality is positively correlated with skill inequality, and the intratemporal elasticity of substitution is at least above the $\log$ preference, and the elasticity of labor supply is at most one. Debt sustainability is optimal because it is costly for the government to default and faces financial autarky.[6] Default is costly not only because the government cannot use debt to smooth consumption over the business cycles, but also because redistribution is more distortionary and less efficient in financial autarky than in the contract. The distributive cost of default is endogenous and novel to the literature, in contrast to the standard exogenous cost of default in terms of output or productivity losses.

Having access to external financing allows the government to redistribute more than in financial autarky. In the contract, future declines in labor taxes improve the economy's efficiency by encouraging higher output and allowing the government to borrow more in the present. This additional unit of resource implies higher present consumption and so higher welfare than financial autarky. Furthermore, the contract exhibits a lower level of redistribution than in financial autarky at the same amount tax distortion in the long run.

The theory is quantitatively consistent with salient features of the data. Using Italy's data, I calibrate the model to match key macroeconomic statistics and average cross-sectional wage inequality and show that the model accounts for the average level and upward trend of the external debt-to-output ratio in Italy for the period 2002-2015, while also being consistent with key business cycle statistics. The simulation points out that the model can account for the positive association between pre-tax income inequality and external debt across countries. Specifically, I perform a regression analysis on a sample of simulated economies differentiated only by wage inequalities and find a statistically significant and positive correlation of pre-tax income Gini index and external debt-to-output. Moreover, a counterfactual exercise for Italy during the periods 1985-2001 and 2002-2015 exhibits that an increase in the underlying wage inequality that matches the increase in income inequality can account for 93% of the increase in the average debt-to-output ratio. These findings are consistent with the theory's prediction that a higher level of inequality, or a higher redistributive motive, corresponds to a higher cost of default, which results in a higher sustainable debt level.

Furthermore, I study optimal austerity through the lens of optimal policy response to shocks. Following a negative innovation of the productivity shock, external debt increases while utility differences among agents decline initially and increase in subsequent periods. More external borrowing allows higher transfers to individuals and more redistribution, while higher taxes and

[6] In most of the literature, default means not repaying debt. In this model, I use the term default as the case in which the government does not honor any terms of the contract, either in debt repayment or taxes.

lower redistribution are needed in the future to repay debt. In contrast, the representative agent model only borrows a small amount in response to the negative productivity shock.

Endogenous borrowing constraints are essential to the theory because they link the government's external borrowing decisions to its concern for redistribution and tax distortions through the value of financial autarky. If the borrowing constraint is exogenous, given impatience, the only optimal policy is to borrow up to the exogenous debt limit, regardless of the desired level of redistribution or inequality.

The model builds on state-contingent financial markets and features no equilibrium default, in contrast to the sovereign default model.[7] However, the threat of default still affects the optimal allocation, as in Thomas and Worrall (1988) and Kehoe and Levine (1993). The endogenous borrowing constraints imply that there is imperfect insurance against aggregate risk. These constraints also endogenously determine the optimal debt portfolio, in contrast to the incomplete framework in which the the risk-free bond is the only financial asset. Furthermore, defaults in reality often accompany a non-zero net capital flow as a country often goes through a lengthy process of renegotiation and haircuts.[8] This framework embeds part of the default procedure into self-enforcing borrowing constraints, instead of assuming zero net capital flows, as in the standard sovereign default model.[9]

I last show that heterogeneity and distortionary taxation significantly affect the equilibrium sustainable level of external debt, whereas the level of government expenditure have only a minor impact. Intuitively, the value of financial autarky decreases in the presence of distortionary taxes and inequality, which in turn increases the government's incentive to repay external debt.

Related literature. This paper builds on the sovereign debt literature of limited commitment and state-contingent asset market that follows Aguiar and Amador (2011, 2014). I present the quantitative predictions of this type of model, including highly volatile consumption and fiscal policies, which are similar with the findings in Kehoe and Perri (2002) for a two-country international real business cycle and Bauducco and Caprioli (2014) with two-sided limited commitment.

I introduce heterogeneity and redistributive effect of fiscal policies in the literature that studies the government's lack of commitment in both tax and debt policies. The volatile tax and government expenditures are similar to Cuadra, Sanchez, and Sapriza (2010). The theoretical finding of declining labor taxes when borrowing is tightened relate to the absence of tax smoothing in Pouzo and Presno (2015) and the quantitative result of Arellano and Bai (2016), in which higher tax distortion would make the country more likely to default. The government's incentive

[7] See, for example, Eaton and Gersovitz (1981).

[8] Standard & Poor's defines default as the failure to meet a principal or interest payment on the due date contained in the original terms of a debt issue. This definition covers both missed payments (breach of contract) and distressed debt restructurings that involve losses for creditors. This is the standard default definition used in the literature (e.g., Reinhart and Rogoff (2009)). See Ams, Baqir, Gelpern, and Trebesch (2018) for a discussion on the pros and cons of different definitions of default.

[9] See Restrepo-Echavarria (2019) for a discussion on these issues.

to front-loading tax distortion is also found in Karantounias (2018).

This paper also contributes to the literature of endogenous cost of default beyond insurance motive. In Mendoza and Yue (2012), it is the efficiency loss in production as default prevents the final good producers to finance the purchase of imports, which only have imperfect substitutes at home. Balke (2017) shows how default limits the supply of bank's loans that firms use to finance vacancies and wages. Therefore, default leads to a large increase in unemployment, which is endogenously costly. In this paper, distortionary taxation plays an important role in determining the cost of default. Default is costly because of the high and volatile labor distortions need to redistribute in financial autarky.

This paper finds optimal policy by characterizing the best allocation of any tax-distorted equilibrium, i.e. the primal approach as in the public finance literature (Barro (1979),Lucas and Stokey (1983) Chari, Christiano, and Kehoe (1994), Aiyagari and McGrattan (1998), Chari and Kehoe (1999), Aiyagari, Marcet, Sargent, and Seppälä (2002), and many other papers). The argument for labor tax smoothing in these papers relies on the fact that the government can issue debt that is contingent to all states and is not constrained (in a sense of beyond the natural debt limit). In this paper, tax smoothing is not always optimal; the government's ability to smooth tax distortion is restricted by the willingness to lend by the international lenders.

This paper relaxes the assumption on the government's commitment to policies in many papers that study trade-off between redistribution and debt management. I build upon the framework of optimal taxation with redistribution in Werning (2007), in which perfect tax smoothing occurs under the same assumptions. I establish that binding borrowing constraint due to lack of commitment alter the tax dynamics, resulting in imperfect tax smoothing. In addition, I show how the quantitatively high cost of default of redistribution lead to the high positive debt level in the long run, in contrast to Bhandari, Evans, Golosov, and Sargent (2016)'s finding that the average long-run on optimal public debt is not positive. Similar to Bhandari, Evans, Golosov, and Sargent (2017) which emphasize the importance of the distribution of initial asset holdings, I find that it affects the long-run debt repayment capacity via the equilibrium level of redistribution.

Several recent papers addressed the trade-off between redistribution and external debt. This paper extends the lack of commitment to both tax and debt policies, in contrast to Ferriere (2015) that assumes one-period commitment to tax progressivity, and finds a related result that a more progressive economy finds a higher cost of default because redistribution is more distortionary in autarky. I focus on the long-run trade-off between inequality and external debt, in contrast to Dovis, Golosov, and Shourideh (2016) that argued how this trade-off endogenized the dynamic cycles of fiscal policies over time. My model features redistributive consequences of domestic defaults, as emphasized by D'Erasmo and Mendoza (2016, 2020). I incorporate this insight into linking redistributive incentives and the cost of default, which in turn affects the sustainable level of external debt. Balke and Ravn (2016) studies tax and debt policies from a Markov perfect equilibrium with inequality through unemployment. This paper allows for a more general framework of inequality and redistributive motive and focuses on the ex-ante welfare maximizing policies. My theoretical finding is consistent with their quantitative one, in which it's optimal to minimize tax distortions during crises.

Other papers have documented a positive relationship between income inequality and sovereign debt. Berg and Sachs (1988) show that income inequality is a key predictor of a country's probability of rescheduling debt and the bond spread in secondary markets. Aizenman and Jinjarak (2012) describe a negative correlation between income dispersion and the tax base and a positive correlation with sovereign debt. Jeon and Kabukcuoglu (2018) and Ferriere (2015) also provide evidence that rising income dispersion significantly increases sovereign default risk. This paper extends the panel analysis to a more recent data set on income inequality and provides a theory that can collectively account for the increase in debt and income dispersion.

Outline. The paper is organized as follows. Section 1 documents the relationship between income inequality and external debt. Section 2 describes the environment and sets up the competitive equilibrium. Section 3 formulates the planning problem and the main theoretical results. Section 5 provides a quantitative analysis and analyzes optimal austerity. Section 7 discusses assumptions and robustness, and Section 8 concludes.

1 Empirical Motivation

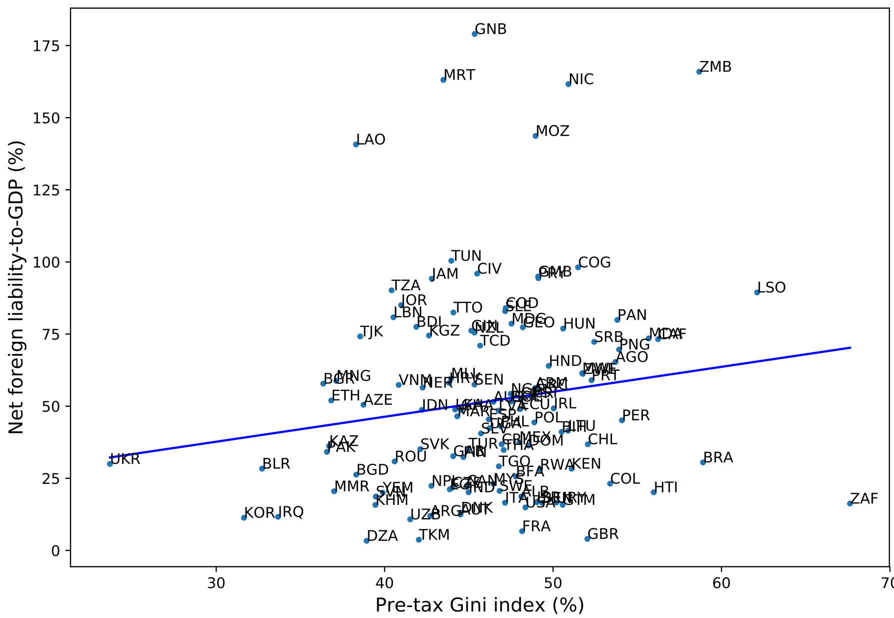
This section presents the empirical relationship between income inequality and external debt. I document that income inequality is positively correlated with external debt in both the cross section and time series. To measure a country's external indebtedness, I use the negative of the net foreign asset-to-GDP ratio from the External Wealth of Nations Database of Lane and Milesi-Ferretti (2018).[10] The database contains data on foreign assets and foreign liabilities for a large sample of countries for the period 1970-2015. For income inequality, I use pre-tax (market) Gini indices from Standardized World Income Inequality Database (SWIID) of Solt (2019) that covers from 1960 to 2018.

Fact 1: Countries with high pre-tax income inequality also experience high external debt levels. Figure 1 plots averages across the period of 1985-2015 of net foreign liability-to-GDP and pre-tax Gini index for 120 countries.[11] The figure establishes a positive correlation between income inequality and external debt across countries.

[10] The net foreign asset (NFA) position of a country is the value of the assets that country owns abroad, minus the value of the domestic assets owned by foreigners, adjusted for changes in valuation and exchange rates. A different measure of a country's external position is the net international investment position (NIIP), which is the difference between a country's stock of foreign assets and foreigner's stock of that country's assets. See Appendix A.3 for the estimation using NIIP.

[11] See Appendix A.2 for the list of all economies in the data set. I focus on the countries that have the population in 1985 over one million people and the net foreign liability above zero. See Appendix **??** for a similar analysis on a sub-sample of advanced and emerging market economies.

Figure 1: Income inequality and external debt across countries



Note: The graph shows the 1985-2015 time averages of net financial liability-to-GDP and pre-tax Gini indefor all countries in the data set. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and The World Bank (2019).

Table 1 reports the regression results of net foreign liability-to-GDP on pre-tax Gini index, reported as averages across 1985-2015. The results show a statistically significant positive correlation between income inequality and external debt.

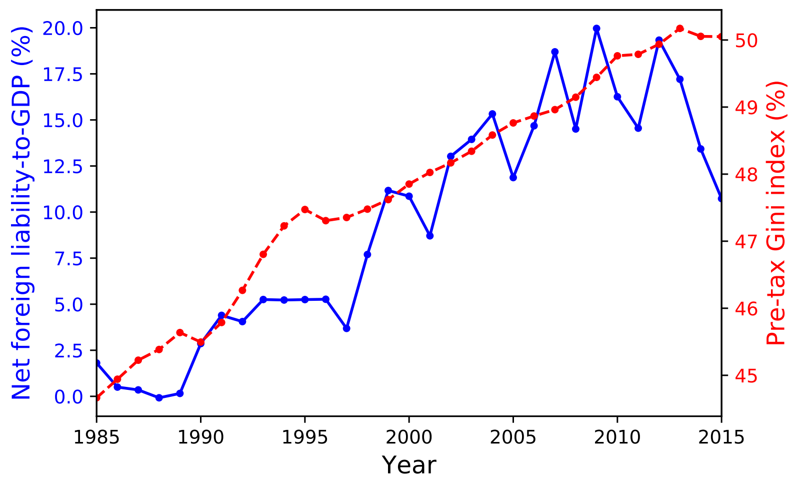
Table 1: Regression analysis of income inequality and external debt

	Dependent Variable: Net foreign liability-to-GDP (%) Averages across 1985-2015	
	(1)	(2)
Gini index, pre tax (%)	0.865*	0.968**
	(0.521)	(0.487)
Controls	No	Yes
No. Countries	120	120

Note: The table shows the regression coefficient and standard error in parenthesis of pre-tax Gini index (%) with respect to net foreign liability-to-GDP (%). Control variables are log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). *, ** represent significant levels of 10% and 5%, respectively. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and The World Bank (2019).

Fact 2: Increases in pre-tax income inequality coincides with increases in external debt in the long run for most countries. Figure 2 plots the GDP-weighted average of net financial liability-

Figure 2: Time series of income inequality and external debt



Note: The graph shows the GDP-weighted average of net financial liability-to-GDP and pre-tax Gini index for countries in the European Union from 1985 to 2015. Sources: Lane and Milesi-Ferretti (2018) and Solt (2019).

to-GDP and pre-tax Gini index for countries in the European Union. Over time, there are increasing trends in both income inequality and external debt. Income inequality has been rising over time across countries (Alvaredo et al. (2018)). At the same time, external debt is also increasing across many countries. The 2007-2009 financial crises contributed to the increase in borrowing across countries, particularly across European ones.[12]

2 A Model of Sovereign Debt and Inequality

In this section, I setup the model of a small open economy with aggregate uncertainty, heterogeneous agents, and a benevolent government. I define the competitive equilibrium given government policies and show that the competitive equilibrium can be characterized by a set of aggregate allocation and a time-invariant distribution of marginal utility shares.

2.1 Environment

A small open economy faces publicly observed aggregate shocks $s_t \in S$ in period t, where S is some finite set. Let $\Pr(s^t)$ denote the probability of any history $s^t = (s_0, s_1, ..., s_t)$, where $\Pr(s^{t+j}|s^t)$ denotes the probability conditional on history s^t, $j \geq 0$. Similarly, $\Pr(s_{t+1}|s^t)$ is the probability period $t+1$'s state is s_{t+1}, conditional on history s^t. When it does not cause confusion, I use x_t to denote ta random variable with a time t for all s^t.

There is a measure-one continuum of infinitely-lived agents different by labor productivity types $(\theta^i)_{i \in I}$, which are publicly observable. The fraction of agents with productivity θ^i is π^i,

[12] Reinhart and Rogoff (2010) reported large increases in public debt across countries, especially in the period 2007-2009. External debt levels were particularly high among European countries.

where $\left(\pi^i\right)_{i\in I}$ and $\left(\theta^i\right)_{i\in I}$ are normalized such that $\sum_{i\in I}\pi^i = 1$ and $\sum_{i\in I}\pi^i\theta^i = 1$. All agents have the same discount factor β and the static utility $U(c,n)$ over consumption c and hours worked n. The utility of agent with productivity θ^i over consumption $c_t^i \geq 0$ and efficiency-unit labor $l_t^i \geq 0$ is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^i\right)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right] \tag{1}$$

where $\sigma, \nu > 0$.

In addition, there is a representative firm that uses labor to produce a single final good. The production function in period t with history s^t is $F(L, s^t, t)$, constant returns to scale, where L is aggregate labor (in efficiency units). The economy is subject to an exogenous sequence of government spending $\{G_t\}_{t=0,s^t\in S^t}^{\infty}$. Both the production function and government expenditures depend on the time period t, capturing deterministic changes such as growth, and the history s^t, capturing the uncertainty impact.

An individual allocation specifies consumption and labor in every period after every history for each agent i: $\left\{\left(c_t^i, l_t^i\right)\right\}_t$. The aggregates are denoted by $C_t \equiv \sum_{i\in I}\pi^i c_t^i$ and $L_t \equiv \sum_{i\in I}\pi^i l_t^i$.

The exogenous risk-free international interest rate for borrowing is r^*. Both the domestic and international financial markets are competitive. The government can issue domestic debt from a full set of state-contingent bonds, which can be traded across agents. The government also have access to a full set of state-contingent external bonds. Let $R^* = 1 + r^*$ denote the gross risk-free interest rate. Define $Q_t(s_{t+1}) = \Pr(s_{t+1}|s^t)/R^*$ as the international price of one unit of consumption at state s_{t+1} in period $t+1$, conditional on history s^t, in units of consumption at history s^t. Similarly, $q_t = \Pr(s^t)/(R^*)^t$ is the international price of one unit of consumption at history s^t in units of consumption at s^0. Let normalize $q_0 = 1$.[13] Note that $q_{t+1} = Q_t(s_{t+1})q_t$. I assume that only the government can borrow abroad.[14]

2.2 Competitive Equilibrium

In every period t and history s^t, the government can issue both domestic and foreign bonds and impose a lump-sum tax T_t, a marginal tax on labor income τ_t^n, and a tax on the return of domestic saving τ_t^d. The firm and agents face the labor wage $w(s^t)$.

[13] This normalization is without loss of generality since the initial level of external debt is fixed.

[14] In the data, domestic residents often hold a very small amount of foreign assets, so most models assume that they do not have access to the external credit market. In this environment, the set up is equivalent to the case where the domestic agents can save abroad with the bond price $Q^*(s^t)$, but then face a residence-based tax $\tau^d(s^t)$. External debt will be the net foreign liability of both the private and public sectors, instead of only the public sector here. I choose this particular set up so that it is more straightforward to characterize the strategic game later on in Appendix B.

Domestic agent. Individual agent of type $i \in I$ faces the sequential budget constraint

$$c_t^i + \sum_{s_{t+1}} Q_t^d(s_{t+1}) b_{t+1}^{d,i} \leq (1-\tau_t^n) w_t l_t^i + (1-\tau_t^d) b_t^{d,i} - T_t, \tag{2}$$

where $c_t^i, l_t^i, b_t^{d,i}$ denote the consumption, labor, and domestic bond holding of agent i in period t and history s^t, respectively. $Q_t^d(s_{t+1})$ is the price of one unit of domestic asset for realization s_{t+1} in period $t+1$ given history s^t.

Representative firm. The firm chooses aggregate labor to maximize profit

$$\max_{\{L_t\}_t} \quad F\left(L_t, s^t, t\right) - w_t L_t,$$

which gives the following first-order condition

$$w_t = F_L\left(L_t, s^t, t\right). \tag{3}$$

The firm's profit is zero in equilibrium because of the constant-return-to-scale production function.

Government. There is an exogenous government expenditure $\{G_t\}_{t=0, s^t \in S^t}^{\infty}$. Given the one-period state-contingent domestic bond B_t^d and external bond B_t, the government's budget constraint is

$$G_t + (1-\tau_t^d) B_t^d + B_t \leq \tau_t^n w_t L_t + \sum_{s_{t+1}} Q_t^d(s_{t+1}) B_{t+1}^d + \sum_{s_{t+1}} Q_t(s_{t+1}) B_{t+1} + T_t,$$

where $B_t^d = \sum_{i \in I} \pi^i b_t^{d,i}$ is aggregate domestic bond, and B_t is the amount of the government's external debt. There is a no-Ponzi condition such that the present value of external debt is bounded below.

The government's present-value budget constraint is

$$\sum_{t \geq 0, s^t} q_t \left\{ G_t - \tau_t^n w_t L_t - T_t + \sum_{s_{t+1}} Q_t(s_{t+1}) B_{t+1}^d - (1-\tau_t^d) B_t^d \right\} \leq B_0. \tag{4}$$

Resource constraint. Using the agent's budget constraints and government's budget constraint, one can obtain a present-value resource constraint in terms of the inter-temporal international prices and the initial external debt,

$$\sum_{t \geq 0, s^t} q_t \left[F(L_t, s^t, t) - G_t - C_t \right] \geq B_0. \tag{5}$$

Competitive equilibrium. Given the above equations, one can define the following competitive equilibrium with government policies.

Definition 2.1. Given initial external debt B_0 and individual individual bond positions $\left(b_0^{i,d}\right)_{i\in I}$, a competitive equilibrium with government policies for an open economy is individual agent's allocation $z^{H,i} = \left\{\left(c_t^i, l_t^i, b_t^{i,d}\right)\right\}_{t=0}^{\infty}$, $\forall i \in I$, the representative firm's allocation $z^F = \{L_t\}_{t=0}^{\infty}$, prices $p = \{q_t, w_t, Q^d(s_{t+1}|s^t)\}_{t=0}^{\infty}$, and government's policy $z^G = \{\tau_t^n, \tau_t^d, T_t, B_t^d, B_t\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves individual i's problem that maximizes (1) subject to (2) and a no-Ponzi condition of agent's debt value, (ii) given p and z^G, z^F solves firm's problem, (iii) the government budget constraint (4) holds, (iv) the aggregate resource constraint (5) is satisfied, (iv) the domestic bond market clears $B_t^d = \sum_{i\in I} \pi^i b_t^{d,i}$, and (v) p satisfies $q_t = \Pr(s^t)/(R^*)^t$ and equation (3) given z^G.

2.3 Characterizing Competitive Equilibrium

In equilibrium, the intra-temporal and inter-temporal rates of substitution are the same across agents, i.e. in each period t and each history s^t, for any individual i,

$$(1-\tau_t^n)w_t = -\frac{1}{\theta^i}\frac{\left(l_t^i/\theta^i\right)^{\nu}}{\left(c_t^i\right)^{-\sigma}},$$

$$\frac{Q_t^d(s_{t+1})}{1-\tau_t^d} = \beta\Pr(s_{t+1}|s^t)\frac{\left(c_{t+1}^i\right)^{-\sigma}}{\left(c_t^i\right)^{-\sigma}}.$$

Given the aggregate allocation (C_t, L_t), there is an time-invariant efficient assignment of individual allocation $\left(c_t^i, l_t^i\right)_{i\in I}$ due to the equal marginal rates of substitution between consumption and labor. Any inefficiencies due to tax distortions are captured by the aggregate allocation. This property allows the competitive equilibrium allocation to be characterized in terms of aggregates and a static rule for individual allocation.

For any equilibrium, there exist a set of Neghishi weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i\in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i\varphi^i = 1$, such that individual allocation solve a static problem

$$V(C, L; \boldsymbol{\varphi}) \equiv \max_{(c^i, l^i)_{i\in I}} \sum_{i\in I}\varphi^i\pi^i\left[\frac{\left(c_t^i\right)^{1-\sigma}}{1-\sigma} - \omega\frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu}\right]$$

$$s.t. \qquad \sum_{i\in I}\pi^i c^i = C; \quad \sum_{i\in I}\pi^i l^i = L$$

This problem gives the allocation rule for individual i that is time-invariant prorportional to the aggregate allocation

$$c_t^i = \psi_c^i C_t, \quad l_t^i = \psi_l^i L_t, \tag{6}$$

where

$$\psi_c^i = \frac{\left(\varphi^i\right)^{1/\sigma}}{\sum_{i\in I}\pi^i(\varphi^i)^{1/\sigma}}, \quad \psi_l^i = \frac{\left(\theta^i\right)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}}{\sum_{i\in I}\pi^i(\theta^i)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}}. \tag{7}$$

In addition, V inherits the separable and isoelastic properties,

$$V(C_t, L_t; \boldsymbol{\varphi}) = \Phi_C^V \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L_t^{1+\nu}}{1+\nu},$$

where Φ_C^V, Φ_L^V depend on $\boldsymbol{\varphi}$. (see Appendix D.1). The envelope conditions of the static problem give

$$(1-\tau_t^n)w_t = \frac{\Phi_L^V L_t^\nu}{\Phi_C^V C_t^{-\sigma}}, \tag{8}$$

$$\frac{Q_t^d(s_{t+1})}{1-\tau_t^d} = \beta\Pr(s_{t+1}|s^t)\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}. \tag{9}$$

Furthermore, the present-value budget constraint for individual i can be written as

$$\mathbb{E}_0\sum_{t\geq 0}\beta^t\left(\Phi_C^V\psi_c^i C_t^{1-\sigma} - \Phi_L^V\psi_l^i L_t^{1+\nu}\right) = \Phi_C^V C_0^{-\sigma}\left(b_0^i - T\right), \tag{10}$$

where T is the present-value of lump-sum taxes.[15] Equation (10) is the individual implementability constraint.

Then I have the following proposition in which the competitive equilibrium can be characterized by a set of aggregate allocation and a time-invariant distribution of marginal utility shares.

Proposition 2.1. *Given the initial external debt B_0 and individual bond holdings$\{b_0^i\}_{i\in I}$,an allocation $\{C_t, L_t\}_{t=0}^\infty$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if the resource constraint (5) holds, and there exist market weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i\in I}$ and lump-sum tax T such that the implementability constraint (10) holds for all $i \in I$.*

Proof. See Appendix. □

3 Sustainable Equilibrium

In this section, I define the problem of a benevolent government that cares about redistribution but lacks commitment in both tax and debt policies. The solution to this problem is the sustainable allocation in which the government does not default on its debt.

[15] $T \equiv \sum_{t=0}^\infty \beta^t \sum_{s^t\in S^t} \frac{V_C\left[h^i(C(s^t),L(s^t);\boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s_0),L(s_0);\boldsymbol{\varphi})\right]} T(s^t)$

Given a set of social welfare weights $\boldsymbol{\lambda} = \left(\lambda^i\right)_{i\in I}$, the government's objective is the weighted utility of all domestic agents

$$\sum_{i\in I} \lambda^i \pi^i \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^i\right)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right]. \tag{11}$$

However, in every period and history node, the government cannot commit to future choices on repayments of debt and taxes. Following Chari, and Kehoe (1990, 1993), the policies are determined in a repeated game between the government, a continuum of domestic agents, and a continuum of international creditors. The sub-game perfect equilibrium supported by trigger strategies to autarky is characterized by the competitive equilibrium conditions described in Proposition 2.1 and the following self-enforcing constraint

$$\sum_{i\in I} \lambda^i \pi^i \mathbb{E}_t \sum_{k\geq t} \beta^{k-t} \left[\frac{\left(c_k^i\right)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_k^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right] \geq \underline{U}_t(s^t, t), \ \forall t, \forall s^t, \tag{12}$$

where $\underline{U}(s^t, t)$ is the one-shot deviation value in which the government defaults on both domestic and external debt and fully redistributes wealth among domestic agents.[16] The government is then in financial autarky, in which it it has no access to external financial markets. $\underline{U}(s^t, t)$ is the value associated with an allocation of a closed economy where the initial states are realized s_t at period t and history s^t, the initial wealth inequality among agents are equal, and the net supplies of domestic and international bonds are zero.

The self-enforcing constraint captures the time-inconsistency of the government policies. If there is a positive net external debt, the government has an incentive to default externally to increase domestic consumption and leisure. In addition, in every history node, there is a non-degenerate distribution of wealth across the domestic agents. The inequality-averse government will also have an incentive to expropriate all the wealth and equally redistribute it. The self-enforcing constraint imposes a limit on the utility, which endogenously determines a limit on external debt for every period and history. These constraints act as endogenous borrowing constraints.

Given the above set-up, a susainable allocation is defined as follows

Definition 3.1. A sustainable allocation $\left(\{C_t, L_t\}_{t=0}^{\infty}, \boldsymbol{\varphi}\right)$ maximizes the social welfare function (11) and satisfies the conditions in Proposition 2.1 and the self-enforcing borrowing constraint (12)

The objective of the government can be rewritten in terms of aggregate allocation and the Negishi weights

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} \right),$$

[16] See Appendix B for the formal set up of the sovereign game and its equilibrium characterization.

where Φ_C^P, Φ_L^P are functions of $\boldsymbol{\lambda}$ and $\boldsymbol{\varphi}$ (see Appendix D.1).

The sustainable allocation is part of the solution to the following planning problem

$$(P) \equiv \max_{\{C_t, L_t\}_t, \boldsymbol{\varphi}, T} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} \right)$$

$$\begin{aligned} s.t. \qquad & \sum_{t \geq 0, s^t} q_t \left[F(L_t, s^t, t) - G_t - C_t \right] \geq B_0 \\ & \forall i,\, \mathbb{E}_0 \sum_{t \geq 0} \beta^t \left(\Phi_C^V \psi_c^i C_t^{1-\sigma} - \Phi_L^V \psi_l^i L_t^{1+\nu} \right) = \Phi_C^V C_0^{-\sigma} \left(b_0^i - T \right) \\ & \forall t, \forall s^t,\, \sum_{i \in I} \lambda^i \pi^i \mathbb{E}_t \sum_{k \geq t} \beta^{k-t} \left[\Phi_C^P \frac{C_k^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_k^{1+\nu}}{1+\nu} \right] \geq \underline{U}_t(s^t, t). \end{aligned}$$

The first constraint is the resource constraint. The second constraint is the implementability constraints that take into account the distortionary effect of the government's policies on individual decisions. The last constraint is the borrowing constraint due to the government's lack of commitment. Domestic agents do not directly internalize the effect of their borrowing decisions on these borrowing constraints. The government, on the other hand, has to consider these constraints when choosing optimal allocation and policies. Therefore, the borrowing constraints indirectly affect domestic borrowing choices via the government's decision on domestic saving taxes.

Appendix C provides the characterization of the sustainable allocation and optimal tax policies that implement the sustainable allocation in competitive equilibrium.

4 Optimal Sustainable Debt

In this section, I argue that it is optimal for the government to sustain external debt in equilibrium. To do so, I make the following assumptions

Assumption 1 (Impatience)**.** *There exists* $0 < \mathcal{M} < 1$ *such that* $\beta R^* < \mathcal{M} < 1$.

Assumption 2. *The welfare weights, skill distribution, and initial wealth satisfy the following properties*

1. *Redistributive motive towards the low skills:* $\theta^i < \theta^j \iff \lambda^i > \lambda^j,\ \forall i, j \in I$
2. *Perfect correlation between skill and initial wealth:* $\theta^i < \theta^j \iff b_0^i < b_0^j,\ \forall i, j \in I$
3. *Elasticities of substitution and labor supply are such that* $\sigma \geq 1$ *and* $1/\nu \leq 1$

The first assumption implies that the domestic agents are impatient, and therefore, there is a need for debt accumulation. The country will accumulate debt over time, as the international interest rate is lower than the domestic intertemporal rate. For the second set of assumptions, the first part is on the welfare weights which states that the government has a high motive of redistribution towards the lower skill, lower income individuals. The government is inequality averse. The second part makes sure that there direction of inequality in skill is the same as the direction of inequality in initial wealth, meaning that lower skill individuals start off with lower initial wealth endowment. The last part implies that the intratemporal elasticity of substitution is at least above the $\log$ preference, and the elasticity of labor supply is at most 1. This assumption determines the direction of change in optimal tax and debt policies in response to intratemporal and intertemporal changes.

Given that the government cares about redistribution towards low skilled workers, I argue that it is costly for the government to redistribute in financial autarky. That is,

Proposition 4.1. *Suppose Assumptions 1–2 hold, then the optimal labor tax is positive in financial autarky.*

Proof. See Appendix. □

Because of this cost of efficiency in autarky, the government is willing to sustain external debt rather than defaulting. That is,

Proposition 4.2 (Sustainable Debt)**.** *Suppose Assumptions 1–2 hold. Then it is optimal for the government to sustain external debt in any period* $t \geq 0$.

Proof. See Appendix. □

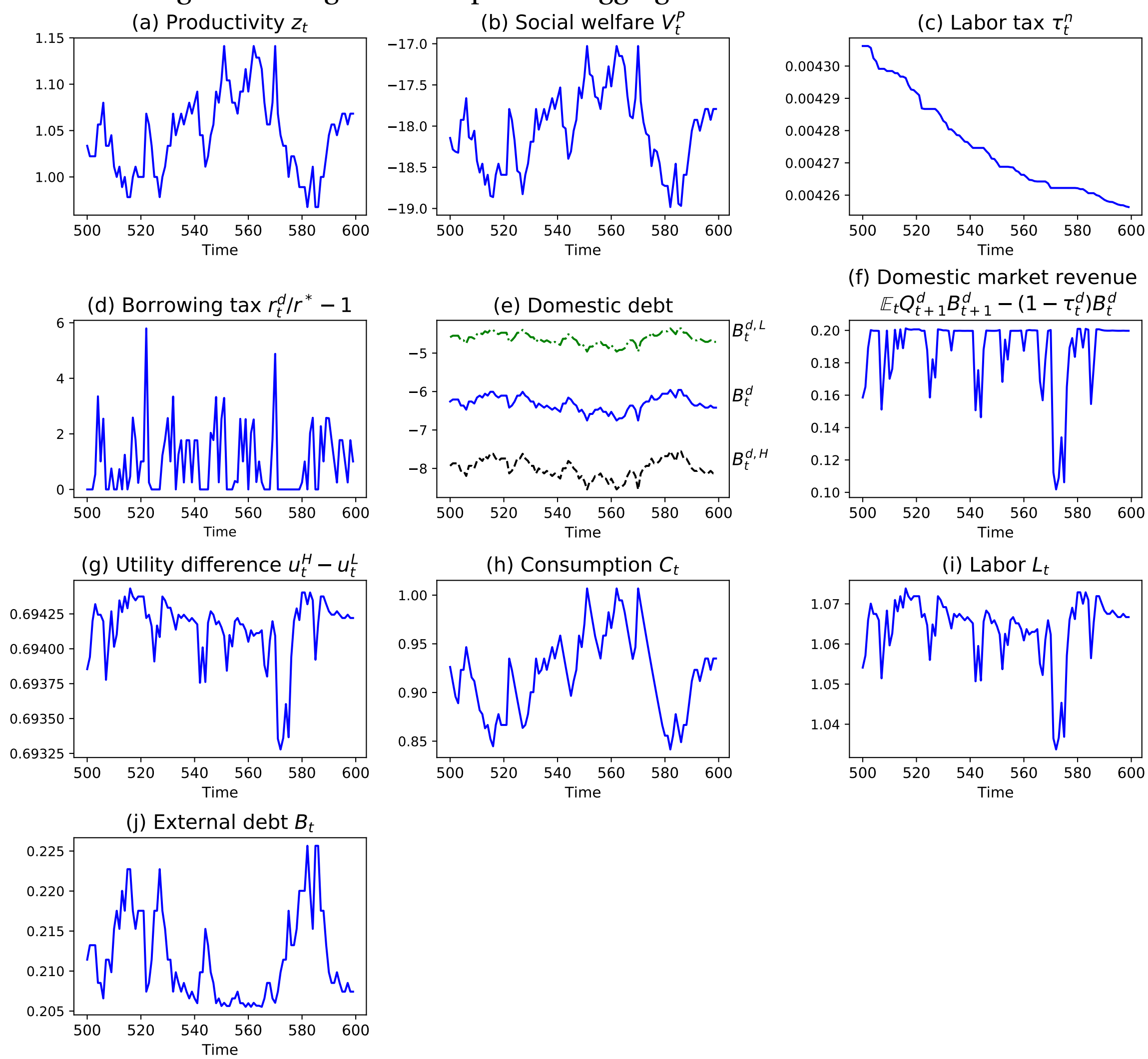
The proof relies on the argument that it is not optimal for the government to default and follows financial autarky in any period and history. Given that there is positive labor distortion in financial autarky, one can construct a deviation from the financial autarkic allocation in which lowering the labor distortion in future periods without changing the continuation value allows the economy to produce more than its consumption in those periods. This deviation implies that the government can borrow today and repay the debt in the future. The extra unit of borrowing allows for more consumption, and so the deviation gives higher welfare than financial autarky.

4.1 Mechanism

In this framework, the government is willing to sustain external debt in equilibrium instead of defaulting because of two reasons: insurance and redistribution. The former comes from the fact that without external credit markets, the government cannot insure itself against aggregate fluctuations. The insurance motive is standard in the literature (Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), Chatterjee and Eyigungor (2012), and many other papers). In this paper, I argue that the government also wants to sustain external debt because it is less costly to redistribute when the government has access to external credit markets.

What entails the redistributive benefit of debt repayment? First, note that if the government defaults and goes to financial autarky, it redistributes the domestic wealth equally across domestic agents. This feature creates an adverse distributive effect. Figure 3 plots the time paths of aggregates in the long run. Panel (e) shows that the high-income agents are net domestic debtors in the long run. Domestic wealth equalization after default erases the distribution of domestic wealth and so implicitly transfer more resources to the high-income agents.

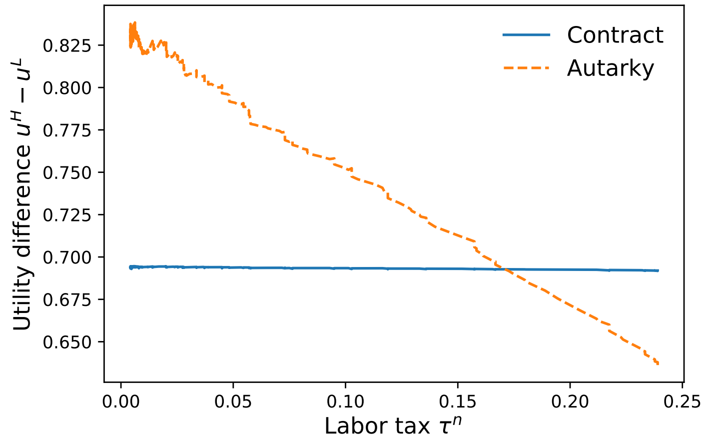
Figure 3: Long-run time paths of aggregates in the baseline model



Note: The graph plots the long-run time paths of optimal policies and aggregates of the planning problem for the baseline model. The implementation is that lump-sum taxes only occur in period 0. Panel (a) and (b) plot the realized productivity path and the social welfare, respectively. Panel (c) and (d) depict the optimal labor and saving taxes, respectively. Panel (e) plots the time paths of total and individual domestic debt. Panel (f) plots the net government's revenue of domestic market. Panel (g) depicts the utility difference between high and low income agents. Panel (h), (i), (j) plot aggregate consumption, labor, and external debt, respectively.

Second, external financing also allows for more redistribution at a lower distortionary cost. When the country can borrow externally, the government uses taxes on domestic borrowing to

Figure 4: Labor tax and utility difference



Note: The graph depicts the level of utility difference $u^H - u^L$ for a given level of labor taxes τ^n in the contract and in financial autarky after the government defaults in period 500. The contract line comes from the simulated data for 1500 periods. The autarky line comes from solving the autarkic allocation given a fixed level of labor tax.

redistribute and collect revenue to repay external debt, which means it relies less on labor taxes. Figure 4 plots the individual utility difference with respect to a given level of labor tax in the contract and in financial autarky after the government defaults in period 500. Initially, when labor tax is high, the utility difference in the contract is higher than in autarky. However, in the long run, when the optimal labor tax is low, the utility inequality in the contract is lower than in autarky. It means that in the long run, to achieve the same level of redistribution, financial autarky requires a higher cost of efficiency.

Differences in the levels of labor taxes between the contract and financial autarky can explain why the government is willing to sustain external debt in the long run. Figure 5 shows long-run optimal labor taxes in different scenarios. Panel (a) plots the path of productivity shock in the long run, starting at period 500. Panel (b) and (c) plot the baseline simulation of optimal labor taxes in contract and in autarky, respectively. Panel (d) and (e) are the analogs of panel (b) and (c) for the one-agent's model.

In the one-agent model, there is no need for redistribution, so the labor taxes are zero across time periods and histories. If the government defaults and goes into financial autarky, labor taxes remain zero. However, in the baseline model with heterogeneous agents, there are differences in labor taxes between the contract and autarky. The labor tax in autarky is higher than the labor tax in the contract. These properties lead to financial autarky, or default, be more costly than repayment in the baseline model, whereas the cost of default in the one-agent model only comes from the insurance motive.

Figure 5: Long-run labor tax in contract and autarky

Note: The graph describes the time paths of productivity shock and optimal labor taxes in contract and in financial autarky for the baseline model and the one-agent model. The autarky case is when the government defaults at period 500 and is permanently excluded from all credit markets.

5 Quantitative Analysis

The previous section provides a theoretical proof on the sovereign debt sustainability and shows that the government's incentive to sustain debt depends not only on the insurance benefit but also the redistributive benefit of having access to external credit markets. In this section, I quantify these two channels that affect debt sustainability using calibrated parameters that match Italian data. I also show that the model is quantitatively consistent with the positive correlation between income inequality and external debt in the cross section and over time as described in the empirical motivation.

5.1 Computation

Given the forward-looking borrowing constraints, I implement the recursive formulation developed by Marcet and Marimon (2019). Appendix E.2 provides more details on the computational algorithm. The key co-state variable is the discounted sum of the Lagrange multipliers on the borrowing constraints: $\Gamma_t = (\beta R^*)^t \sum_{k \le t} \gamma_k$. Given that the domestic agents are impatient, the borrowing constraint will bind infinitely often in the long run. This implies that Γ converges to a positive number in the long run. Formally, I consider the following assumption that guarantees that the value of default is finite,

Assumption 3. *$\underline{U}$ is bounded below, i.e. there exists a finite real M_U such that* $\inf_{s^t,t} \underline{U}(s^t, t) \ge M_U$.

Then the following proposition holds.

Proposition 5.1. *Suppose Assumptions 1 and 3 hold, if an interior efficient allocation exists, then* $\lim_{t\to\infty}\Gamma_t > 0$.

Γ_t reflects the marginal benefit of relaxing the current and previous borrowing constraints from increasing one unit of utility in period t and history s^t (either by giving more consumption or leisure). In the long run, Γ corresponds to the amount of external debt that can be sustained in equilibrium. Impatience implies that in the long run, there exists a positive component in the price of borrowing coming from the lack of commitment. If there exists an ergodic distribution of the efficient allocation, then Γ will also follows an ergodic distribution. This property allows the computational algorithm using Γ as one of the state variables to converge.

5.2 Calibration

For the quantitative exercise, I assume the following distributional and functional forms. The economy is populated by two types of agents with labor productivity $\{\theta^H, \theta^L\}$, where $\theta^H \geq \theta^L > 0$ and $\pi^H = \pi^L = 0.5$. The planner is utilitarian, i.e. $\lambda^H = \lambda^L$. The individual preference has the form of

$$U^i(c, l) = \log c - \frac{l^{1+\nu}}{1+\nu}$$

The production function is linear in labor, i.e. $F(L, z) = zL$, where z is the aggregate productivity. The aggregate shock is z_t that follows a logged $AR(1)$ process,

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t^z,\ \epsilon_t^z \sim \mathcal{N}(0, \sigma_z),$$

where ρ_z, σ_z are the auto-correlation and the residual standard deviation, respectively. I discretize the productivity process into a Markov chain using Tauchen method with 31 evenly-spaced nodes. From now on, I will use z in place of s as the source of aggregate uncertainty. The government expenditure is constant over time and across histories: $G_t = \bar{g}$. The initial debt levels are $B_0 = 0$ and $b_0^{H,d} = b_0^{L,d} = 0$. The economy starts at the mean of the productivity distribution. The deviation utility $\underline{U}(z^t, t)$ is calculated as the closed-economy version of the model that starts with productivity z_t, zero external debt, and all domestic individuals start with the same initial wealth. $\underline{U}(z^t, t)$ varies with respect to the realized shock z_t.

With these assumptions, the model requires giving values to the parameters of (i) the aggregate productivity process, ρ_z and σ_z; (ii) the cross-sectional wage ratio, θ^H/θ^L; (iii) the individual preference, β and ν; (iv) the government expenditure $\bar{g}$; and (v) the risk-free rate r^*.

A period in the model is one year. For output, I use the logged and linear detrended real GDP series from 1985-2015. I set the auto-correlation of productivity,ρ_z, equals to the auto-correlation of output, which is 0.928. To calculate the wage ratio θ^H/θ^L, I use the data on cross-sectional inequality by Jappelli and Pistaferri (2010). For each year in the database, I calculate the ratio of the mean wage of the top 50% of the wage distribution to the mean wage of the bottom 50%. Then θ^H/θ^L is set to 1.9475, which is the time-average of these wage ratios for the period 2002-2006. The discount factor β is set to 0.967 so that the average real domestic interest rate is 3.4% for Italy from 2002 to 2015. I choose $\nu = 2$ so that the elasticity of labor supply is 0.5, a standard value in the literature. The risk-free rate is set at 0.017, which is the real rate of return on the German government bonds for the period 2002-2015 (these are secondary market returns, gross of tax, with around 10 years' residual maturity). The interest rate series start at 2002 to isolate the effect of currency and exchange rate risks.[17]

The two remaining parameters, σ_z and $\bar{g}$, are selected to match (i) the standard deviation of logged output and (ii) the government's final consumption-to-GDP ratio for the period 1985-2015. I use the simulated method of moments (SMM). Departing from the quantitative literature on sovereign debt, I do not target the average external debt-to-output ratio but instead leave it as one of the non-targeted moments.[18]

Table 2 summarizes the parameter values and targets from the calibration exercise.

Table 2: Calibrated Parameters and Targets

Parameter	Description	Value	Target
Externally calibrated parameters			
r^*	Risk-free rate	0.017	Avg. real return on German bond
β	Discount factor	0.967	Avg. Italian real interest rate = 3.4%
$1/\nu$	Labor elasticity	0.5	Standard literature value
θ^H/θ^L	Wage ratio	1.9475	Mean top 50% wage / mean bottom 50% wage
ρ_z	Auto-corr. of prod.	0.927	Auto-corr. of log GDP
Internally calibrated parameters			
σ_z	Std. dev. of prod. res.	0.0205	Std. dev. log GDP
$\bar{g}$	Govt. spending	0.202	Avg. govt. consumption-to-GDP

Note: The table describes the parameters, their values, and the targets in the calibration exercise. Statistics are annual. The risk-free rate and discount factor cover the period of 2002-2015. Wage ratio is the author's calculation from the cross-sectional data set by Jappelli and Pistaferri (2010), covering the period of 2002-2006. Auto-correlation and standard deviation of GDP and government final consumption cover the period of 1985-2015. Data sources: Jappelli and Pistaferri (2010), Eurostat (2019), and The World Bank (2019)

5.3 Calibration Results

Table 3 shows the moment matching exercise of the model and the data. The first column reports the statistics from the data for Italy in the period of 1985-2015. The second column reports the

[17] See Appendix A for more data descriptions and sources

[18] See Section 5.3 for the results of non-targeted moments. Alternatively, the discount factor β can be used to target the debt-to-output ratio.

statistics from simulating the model and taking the long-run averages.[19] The calibration successfully matches the standard deviation of output and the government consumption-output ratio for Italy.

Table 3: Targeted Statistics: Data and Model

Statistics	Data: 1985-2015	Model
Std. dev. log GDP	0.053	0.053
Avg. govt. consumption-to-GDP	0.19	0.19

Note: The table describes the targeted statistics from the calibration exercise. The first column reports data statistics which are across the period of 1985-2015. The second column reports the model statistics which come from the model's simulation for 10500 periods and excluding the first 500 periods. Sources: The World Bank (2019)

Table 4 reports the non-targeted statistics of the model comparing to the data. The first column is from the Italian data, and the second column is from the model. The key cyclical properties are the volatility and correlation with respect to output of consumption and net saving ratio. I consider net savings as the amount of output minus the total consumption. In the model, net saving is the net amount of resources used to repay external debt in every period.

Table 4: Non-targeted Statistics: Data and Model

Statistics	Data	Model
Cyclical property		
std (C) / std (Y)	1.0	1.2
std (NS/Y) / std (Y)	0.29	0.34
corr (C,Y)	0.97	0.94
corr (NS/Y,Y)	0.40	0.30
External debt property[a]		
Mean external debt/Y	0.24	0.21
Std. (external debt/Y)	0.027	0.022

[a]Sample period: 2002-2015.

Note: This table reports the non-targeted statistics of the data and the model. The first column reports data statistics which are across the period of 1985-2015, unless specified. The second column reports the model statistics which come from the model's simulation for 10500 periods and excluding the first 500 periods. Net saving (NS) is defined as output minus total private and government consumption in the data and the model. External debt is defined as the country's net financial liability in the data. For the second moments, output and consumption series are logged and linear detrended. Net saving and external debt ratio series is linear detrended.

[19]All model statistics are long-run averages of simulating the economy for 10500 periods and discarding the first 500 periods.

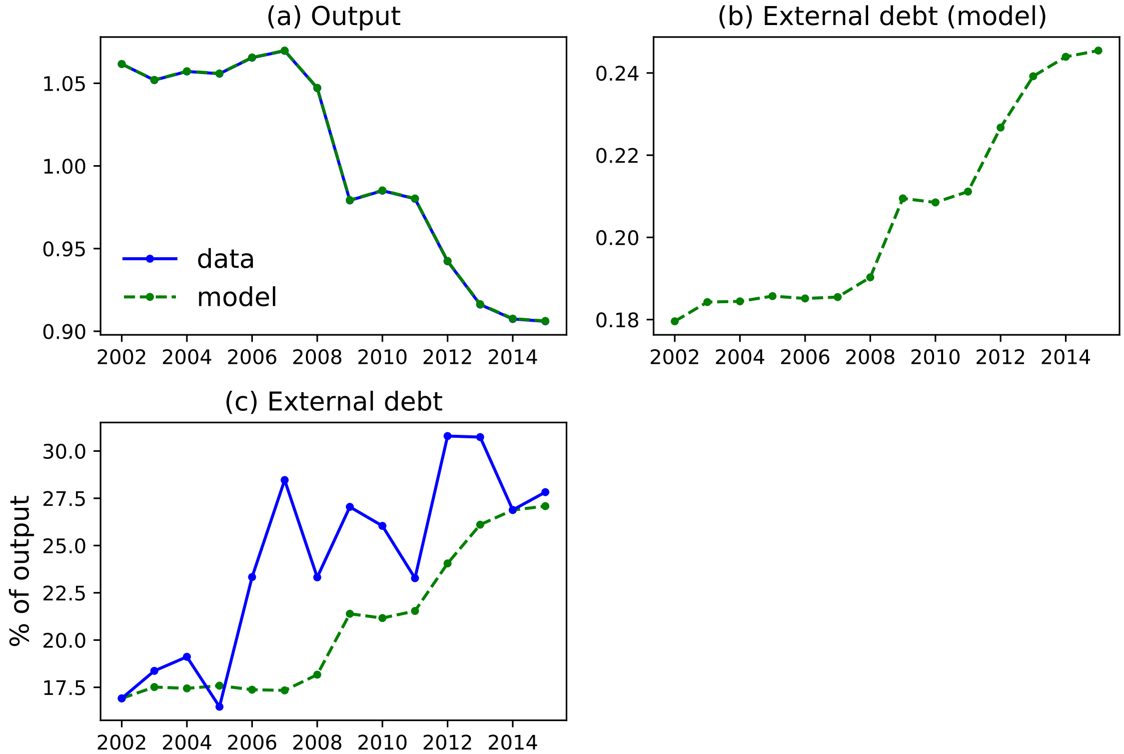
Several cyclical features of the Italian data stand out. First, consumption is as volatile as output and is highly correlated with output. Net saving only has a volatility of more than a quarter of the volatility of output, and has a positive correlation with output that is around 40%.[20] The model correctly gets the qualitative patterns of the data. The volatility of consumption and net savings relative to output are slightly higher in the model than in the data. Both model consumption and net saving are pro-cyclical with similar correlation levels as in the data. The model is able to generate realistic cyclical patterns of the data, in contrast to the standard model of complete markets. The main reason is that, even with state-contingent assets, the occasionally binding borrowing constraints lead to an imperfect insurance across states and time periods.

External debt. In the data, external debt is defined as the net foreign liability position, as reported by Lane and Milesi-Ferretti (2018)'s External Wealth of Nation Database. The model explains well both the first and second moments of external debt for Italy from 2002 to 2015. The model generates on average around 21% of external debt-to-output ratio, comparing to 24% of net foreign liability-to-output ratio in the data. This model feature is with a relatively high discount factor (0.969) with respect to the literature. The model also matches the volatility of external debt-to-output ratio in the data.

Event analysis. I now conduct an event analysis for Italy in period of 2002-2015. I feed into the model a sequence of productivity shock realizations such that the model's outputs matches ones in Italy from 2002 to 2015. I simulate a time path of external debt in the model given that the initial external debt-to-output is the data value in 2002 . I then compare the evolution of external debt-to-output in the data and in the model's simulation over time. Figure 6 plots the exercise's results. Panel (a) plots the output paths of the data and the model. Panel (b) plots the time path of external debt in the model. Panel (c) plots external debt-to-output time paths for both the data and the model. From 2011 to 2015, Italy's output has dropped by 7.4% below trend, while external debt-to-output has increased by 4.6%. In the model's simulation, external debt has increased by 3.4%, which leads to a 5.5% increase in external debt-to-output.

[20] Neumeyer and Perri (2005) reported key business cycle statistics for both advanced and emerging market economies.

Figure 6: Italy's Recession: Data and Model



Note: The graph depicts the time paths of output, external debt, and external debt-to-output for the data and the model's simulation. Panel (a) plots the output path. Panel (b) plots the external debt paths of the model. Panel (c) plots external debt-to-output.The simulation uses a sequence of productivity shock realization such that the model's output matches the data output for Italy in 2002-2015. The initial external debt level is such that the model's external debt-to-output matches with the starting value in 2002 from the data. Data sources: Lane and Milesi-Ferretti (2018) and The World Bank (2019).

5.4 Cross-Country Estimation

Table 5 shows the estimation results of the correlation between pre-tax Gini index and net foreign liability-to-GDP from the model and the data. The data values are from the second column of Table 1, robust to other controls. The model values come from the regression on the model's simulated data. Given the calibrated parameters, I solve different versions of the model differentiated only by wage inequality and compute the pre-tax Gini indices, long-run averages of external debt-to-output ratios, and output per capita. I then estimate the regression $\text{NFL}_i = \beta_0 + \beta_1 \text{Gini}_i + \beta_2 \log \text{GDP per capita}_i + \epsilon_i$ and report $\hat{\beta}_1$ and its standard error.[21]

[21] In the model, average output growth rates and inflation are zero, so I omit them as control variables. Since the regression uses the ergodic means of the model as variables, country and time fixed effects are redundant.

Table 5: Cross-country estimation: Data and Model

	Dependent Variable: Net foreign liability-to-GDP (%)	
	Data: 1985-2015	Model
Gini index, pre tax (%)	0.968**	2.42***
	(0.487)	(0.0674)
Controls	Yes	Yes
No. Observations	120	30

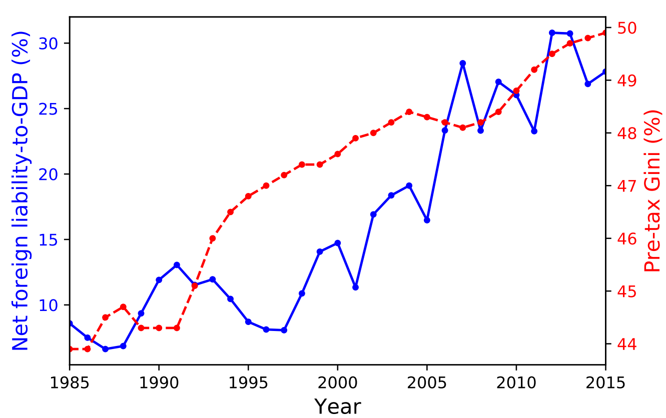
Note: The table describes the cross-country estimation of the coefficient of pre-tax Gini index (%) on net foreign liability-to-GDP (%) in the data and in the model. Details on data estimation are from Table **??**. The model estimation comes from simulated data of 30 different versions of the model that are differentiated by wage ratios.

The model produces a positive and statistically significant coefficient of the pre-tax Gini index. The coefficients imply that a one percent increase in the pre-tax Gini index corresponds to a 0.968% increase in net foreign liability-to-GDP in the data, comparing to a 2.42% increase in the model.

5.5 Time Estimation

This subsection estimates the effect of income inequality on external debt over time. I conduct a comparative statics exercise in the case of Italy for two time periods of 1985-2001 and 2002-2015. Figure 7 plots the time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. On average, in the period of 1985-2001, Italy has a lower levels of income inequality and external debt comparing to the period of 2002-2015.

Figure 7: Income inequality and external debt in Italy



Notes: The graph shows the time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. The left y-axis depicts the values in net foreign liablity-to-GDP (%), and the right y-axis depicts the values in pre-tax Gini (%) . Sources: Lane and Milesi-Ferretti (2018), and Solt (2019).

The comparative statics exercise is as follows. I feed into the model a value of wage inequality for the period 1985-2001 and keep other parameter values fixed. I compute ergodic means of pre-tax Gini index and external-debt-to-output ratios. The 1985-2001 value of wage inequality is such that the change in the average pre-tax Gini income from 1985-2001 to 2002-2015 is the same as the change in the data. Table 6 reports the results of the policy experiment. Given the targeted increase in the pre-tax Gini indices in Italy from 1985-2001 to 2002-2015, the model can account for 93% of the increase in the external debt-to-output ratio.

Table 6: Comparative statics results for periods 1985-2001 and 2002-2015

Statistics	Data	Model
Targeted		
Δ Pre-tax Gini	3.0%	3.0%
Non-targeted		
Δ Extenal debt/Y	14%	13%

Notes: The table reports the results of the comparative statics exercise. The first column reports the changes in the data statistics, computed as the average statistics of period 2002-2015 minus the average statistics of period 1985-2001. The second column reports the results from the model. The change in the model statistics is computed as the average statistic of a simulation for the model with the wage ratio equal to 1.9475 minus the same statistic of the model with the wage ratio equal to 1.73.

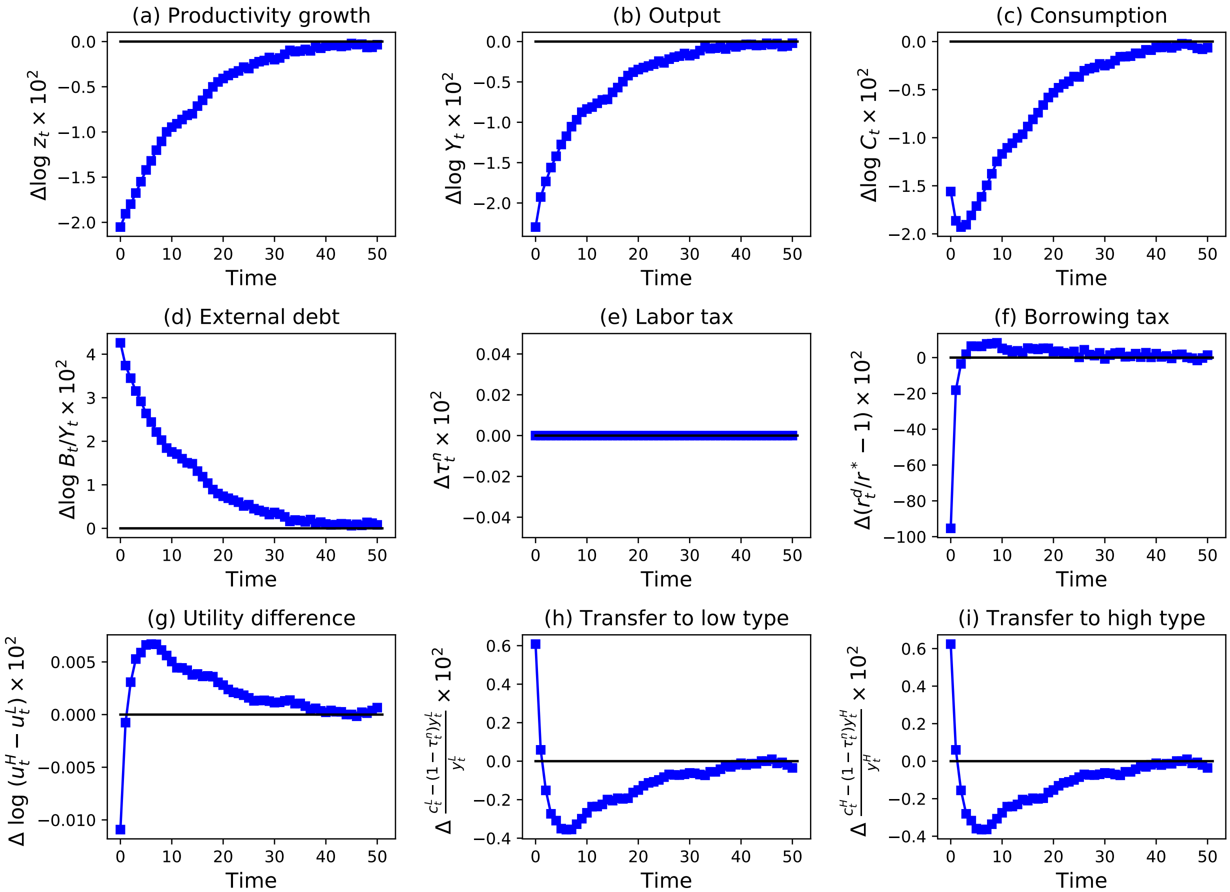
6 Optimal Austerity Policies

This section evaluates responses of optimal austerity policies to a negative productivity shock in the presence of inequality.

Figure 8 plots the impulse response functions of aggregate variables and tax policies with respect to a one standard deviation decline in productivity growth, computed via local projection.[22] Panel (a) plots the path of productivity growth given the negative innovation shock occurring in period 0. There are three groups of responses: aggregates, fiscal policies, and redistribution. Panel (b) and (c) plot the first group of responses of output and consumption, respectively. For fiscal policies, Panel (d), (e), and (f) plot the responses of external debt, labor taxes, and borrowing taxes. Lastly, Panel (g), (h), and (i) plot the responses related to redistribution: utility

[22]The approach to calculate impulse respond functions is econometrically equivalent to the approach of Jordà (2005). I simulate the economy for 10500 periods with aggregate productivity shocks and exclude the first 500 periods. I then calculate the realized time series of shocks to productivity, ϵ_t^z. To compute the response of a variable X to the shock ϵ_t^z, I perform the OLS regressions $\Delta \log X_t = \alpha + \beta_k \epsilon_{t+k}^z + \eta_t$ to get the estimated $\hat{\beta}_k$. The horizontal τ IRF is then $IRF_\tau = \sum_{k=0}^{\tau} \hat{\beta}_t$. The effect of one standard deviation shock to ϵ_t^z is the responses $\sigma_z \times IRF_\tau$. Since labor and saving taxes can be zero or negative, the dependent variable in the OLS regressions are ΔX_t instead of $\Delta \log X_t$. To my best knowledge, Mongey (2019) is the first paper that applies this computational technique in calculating impulse responses.

Figure 8: Benchmark impulse response functions to a negative productivity shock



Notes: The graph shows the impulse response functions $\sigma_z \times IRF_\tau$ computed by local projection methods as in Jordà (2005). Panel (a) plots the productivity growth response. Panel (b) and (c) plot the responses of output and consumption, respectively. Panel (d), (e), and (f) plot the responses of fiscal policies: external debt, labor, and saving taxes, respectively. Panel (g), (h), and (i) show the responses of redistribution: variance of log utilities and average tax-to-income ratios across agents.

differences and transfer rates across individuals.[23]

A decline in productivity growth leads to declines in both output and consumption with a higher drop in output. External debt-to-output increases in response to a low productivity. Labor taxes remains unchanged, while there is a sharp decrease in borrowing taxes in the first period, accompanying with a decline. Note that the optimal borrowing taxes are positive in the long run. The initial decrease in borrowing taxes comes from the non-binding borrowing constraints in the first few periods after a negative shock.[24] However, as borrowing constraints bind in the future, it is then optimal to increase borrowing taxes. In terms of redistribution, the utility difference initial decreases then increases subsequently. Similarly, transfer rates increase for

[23] The transfer rate is defined as amount of resources an individual receives excluding labor taxes over the individual income, equal to $\frac{(1-\tau^n)y^i - c^i}{y^i}$ for an individual i with income y^i and consumption c^i and faces labor tax τ^n.

[24] Proposition **??** shows that $\tau^d = 0$ when borrowing constraints do not bind. So τ^d goes from a negative number to zero initially.

both agents, following by declines which are larger for higher-skilled agents.

Intuitively, a negative shock leads to a reduction in the deviation utility, and so the borrowing constraint becomes non-binding. The non-binding constraint allows the government to accumulate external debt, temporarily decreases average tax rates, and increases redistribution. In future periods, the government raises taxes to repay the debt and reduces redistribution.

7 Discussions

This section discusses how the optimal policies respond to different model ingredients: heterogeneity/redistributive motive, distortionary taxation, and government expenditure. Throughout this section, I consider a particular implementation of the sustainable allocation in which the government only uses lump-sum taxes in the initial period.[25]

7.1 Role of Heterogeneity and Redistributive Motive

The previous section has shown that the presence of heterogeneity or concern for redistribution plays a role in debt sustainability. Here, I provide more details on their impact on the optimal allocation, debt, and tax policies. Table 7 describes the long-run statistics and optimal policies for different levels of wage inequality, measured as the ratio of high-to-low productivity levels.[26] The all columns report the statistics and the optimal tax policies from the model simulation.

Table 7: Role of heterogeneity

θ^H/θ^L	1	1.7	1.9475 Baseline	2.5
Long-run statistics				
Mean C/Y	0.82	0.82	0.82	0.81
Mean B/Y	0.028	0.073	0.21	0.63
Tax policies				
Initial τ_0^n	0.0	0.15	0.25	0.36
$\lim_{t\to\infty} \tau_t^n$	0.0	0.0077	0.0042	-0.016
Lump-sum tax T_0/Y_0	5.8	2.5	0.87	-1.7

Notes: The table reports long-run statistics and policies for different levels of wage inequality, measured as θ^H/θ^L. The model's simulation is for 10500 periods. The statistics are calculated from the sample excluding the first 500 periods. Tax policies are calculated from the whole sample.

[25] This tax is the present value of all lump-sum taxes

[26] In this environment, the average productivity is normalized to one, so increases in θ^H/θ^L imply the mean preserving spreads

When the wage inequality increases, the average consumption-to-output ratio remains around 82% (average government spending-to-output is roughly 19%), while the average external debt-to-output increases, as shown in the previous sections. The tax policies to finance external debt are such that, rising wage inequality or motive for redistribution implies a higher initial labor tax, a lower labor tax in the limit, accompanying with a lower lump-sum tax. Intuitively, the government with a higher redistributive motive redistributes via a higher labor tax distortion and a higher lump-sum transfer. The government accumulates more external debt to finance such policies. In the long run, in order to repay debt, the government reduces the labor distortion and uses claims and taxes in the domestic credit market to redistribute. The lower tax distortion in the limit encourages more output to repay.

7.2 Role of Distortionary Taxation

The previous sections have argued how the redistributive tax policy comes with a cost of distortion, and by front-loading the distortion, the economy sustains a high debt. This subsection relaxes the assumption of distortionary taxation by considering the case in which the planner has access to skill-specific lump-sum transfer, so the planner achieves perfect redistribution without generating any distortionary cost. I show that the need to use distortionary taxation as a redistributive tool makes the government be willing to sustain highly positive debt in the long run. Table 8 reports the external debt and tax policies of the baseline model using linear taxes comparing to the alternative framework with lump-sum tax depending on income.

Table 8: Role of Distortionary Taxation

	Baseline	Skill-dependent lump-sum tax
Long-run statistics		
Mean C/Y	0.82	0.82
Mean B/Y	0.21	0.029
Tax policies		
Initial τ_0^n	0.25	0.0
$\lim_{t\to\infty} \tau_t^n$	0.0042	0.0
Lump-sum tax T_0/Y_0	0.88	5.7, high type = 20, low type $=-8.6$

Notes: The table reports long-run statistics and tax policies for the baseline and the case in which the government has access to fully skill-dependent lump-sum taxes, $T^i, \forall i \in I$. The last row and column reports the average lump-sum tax, as well as the individual lump-sum taxes.

While both cases give the same average consumption-to-output ratio in the long run, the alternative case quantitatively generates a much smaller amount of debt with a higher volatility than the baseline case. Across all periods, the labor tax is zero, since all of the redistribution is done via the type-dependent lump-sum taxes. In present-value terms, the government taxes the high-income agents and transfers to the low-income agents.

7.3 Role of Government Spending

This subsection shows the role of the government expenditure, $\bar{g}$ on the efficient allocation and optimal policies. Table 9 reports the results for different values of $\bar{g}$.

Table 9: Role of government spending

$\bar{g}$	0	0.1	0.205 Baseline	0.3
Long-run statistics				
Mean C/Y	1.0	0.9	0.82	0.73
Mean B/Y	0.20	0.21	0.21	0.21
Tax Policies				
Initial τ_0^n	0.21	0.23	0.25	0.25
$\lim_{t\to\infty} \tau_t^n$	-0.016	-0.0069	0.0042	0.017
Lump-sum tax T_0/Y_0	-4.4	-1.7	0.87	3.1

Notes: The table reports long-run statistics and policies for different levels of government expenditure $\bar{g}$. The model's simulation is for 10500 periods. The statistics are calculated from the sample excluding the first 500 periods. Tax policies are calculated from the whole sample.

An increase in government expenditure implies a lower fraction of aggregate consumption-to-output in the long run, while external debt-to-output ratio remains around the same value.[27] Both labor taxes and lump-sum taxes increase in response to a higher government expenditure. In this environment, lump-sum taxes allow the government to finance its expenditure without incurring any distortionary cost. The distortionary cost only comes from the need for redistribution. Therefore, the level of government expenditure does not significantly affect the sustainable level of external debt.

8 Conclusion

This paper proposes a theory of external debt sustainability that comes from the motive for redistribution of the government. I introduce the government's redistributive concern and distortionary taxation into a sovereign debt framework. I analyze the interaction between distortionary and distributive effect of fiscal policies and the government's lack of commitment. The endogenous borrowing constraints arise from the government's lack of commitment, and become relevant in the long run due to the domestic agents' impatience. Redistribution comes

[27] In details, the external debt level slightly increases with respect to the increasing government expenditure, yet output can be increasing or decreasing, resulting in the non-linear response of external debt-to-output to government expenditure.

with the cost of tax distortions.

The paper's theoretical contribution is the effect of redistribution on optimal debt sustainability. Given that the government wants to redistribute towards lower skilled lower income households, initial wealth inequality is positively correlated with skill inequality, and conditions on the intratemporal and intertemporal elasticities, it is optimal for the government to sustain positive external debt. Default leading to financial autarky is costly not only because the government cannot use debt to smooth consumption over the business cycles, but also because redistribution is more distortionary and less efficient in financial autarky than in the contract. The distributive cost of default is endogenous and novel to the literature.

The quantitative contribution is showing that the government's redistributive motive plays an important role in determining the equilibrium level of external debt. This channel comes from the additional cost of redistribution during financial autarky. The result contributes to the ongoing literature on endogenous default costs in sovereign debt models. The redistributive cost of default quantitatively accounts for 87% of the long-run average external debt-to-output, while the insurance cost of default only accounts for 13%. Furthermore, the theory can account for the cross-country and time-series relationship between income inequality and external debt in the data.

The model has implications on optimal austerity policies in the presence of inequality. Estimations from the model's simulation points out that a negative productivity shock leads to an increase in external debt and a temporary decrease in borrowing taxes, while labor taxes remain unchanged. The average tax-to-income ratio initially decreases for all agents and more for high-income agents, and utility difference decreases. In the future, the government raises average taxes and reduces redistribution to repay debt.

Future research includes allowing for equilibrium defaults and incorporating various types of debt crises. In future projects, I examine the interaction between inequality and default risks and their implications for austerity policies.

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Appendices

A Empirical Appendix

A.1 Data Sources

Most data are annual series covering the 1985-2015 period. Some data samples cover the 2002-2015 period.

- Net foreign liability is the negative of net foreign asset (NFA) from the External Wealth of Nations Database, Lane and Milesi-Ferretti (2018)
- Net international investment position is the official international investment position (IIP) from the External Wealth of Nations Database, Lane and Milesi-Ferretti (2018)
- Pre-tax Gini Index the market Gini from the Standardized World Income Inequality Database, Solt (2019).
- GDP per capita is the constant 2010 US Dollar GDP per capita series from World Development Indicator Database, The World Bank (2019)
- GDP growth is the log difference of constant 2010 US Dollar GDP series from World Development Indicator Database, The World Bank (2019)
- Inflation is the annual inflation series measured by the GDP deflator from World Development Indicator Database, The World Bank (2019)
- Real GDP is GDP series in constant local currency units from World Development Indicator Database, The World Bank (2019)
- Real return on German bond is the interest rate on Gernman bond adjusted for inflation measured by the GDP deflator. The interest rate is the long-term interest rate for converenge purposes from the Eurostat Database (2019). These bonds have 10-year maturity and are denominated in Euro.
- Real interest rate is the lending interest rate adjusted for inflation as measured by the GDP deflator from World Development Indicator Database, The World Bank (2019)
- Italy's cross-sectional wage inequality is calculated from the micro-data by Jappelli and Pistaferri (2010) using Surveys of Household Income and Wealth conducted by the Bank of Italy for the period 1980-2006.
- Government consumption is the general government final consumption expenditure series from World Development Indicator Database, The World Bank (2019)
- Private consumption is the households and NPISHs final consumption expenditure series from World Development Indicator Database, The World Bank (2019)

Table 10: Regression analysis of income inequality and external debt

	Dependent Variable: Net international liability position-to-GDP (%) Time periods: 1985-2015	
	(1)	(2)
Gini index, pre tax (%)	0.2681	0.1755
	(0.7528)	(0.7707)
Country fixed effects	Yes	Yes
Time fixed effects	Yes	Yes
Controls	No	Yes
No. Countries	137	137
No. Observations	2028	2028

Note: The table describes the panel regression results using all countries in the data set. Both columns show the regression coefficient and standard error in parenthesis of pre-tax Gini index (%) with respect to net international liability position-to-GDP (%). Control variables are log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). Both regressions have country and time fixed effects. All standard errors are clustered. *,**,*** represent significant levels of 10%, 5%, and 1%, respectively. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and The World Bank (2019).

A.2 Lists of Countries

Albania, Algeria, Angola, Argentina, Armenia, Australia, Austria, Azerbaijan, Bangladesh, Belarus, Benin, Bolivia, Bosnia and Herzegovina, Brazil, Bulgaria, Burkina Faso, Burundi, Cambodia, Cameroon, Canada, Central African Republic, Chad, Chile, Colombia, Congo, Costa Rica, Côte d'Ivoire, Croatia, Czech Republic, Dem. Rep. Congo, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Estonia, Ethiopia, Finland, France, Gabon, Gambia, Georgia, Ghana, Greece, Guatemala, Guinea, Guinea-Bissau, Haiti, Honduras, Hungary, India, Indonesia, Iraq, Ireland, Israel, Italy, Jamaica, Jordan, Kazakhstan, Kenya, Korea, Kyrgyz Republic, Lao, Latvia, Lebanon, Lesotho, Lithuania, Madagascar, Malawi, Malaysia, Mali, Mauritania, Mexico, Moldova, Mongolia, Morocco, Mozambique, Myanmar, New Zealand, Nicaragua, Niger, Nigeria, Norway, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Rwanda, Senegal, Serbia, Sierra Leone, Slovakia, Slovenia, South Africa, Spain, Sri Lanka, Sweden, Tajikistan, Tanzania, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, Turkmenistan, Uganda, Ukraine, United Kingdom, United States, Uruguay, Uzbekistan, Vietnam, Yemen, Zambia, Zimbabwe.

A.3 Net International Investment Position

This subsection provides the estimation using the negative of net international investment position an alternative definition of a country's external indebtedness.

Table 10 reports the regression results. High income inequality levels are correlated with high external debt positions, though the coefficients are not statistically significant.

B Sovereign Game

Before setting up the game, consider the general environment where the government's policy includes the decision to default on external bond $\{\delta(s^t)\}$, where $\delta \in \{0, 1\}$ and $\delta = 0$ implies default.[28] The government's budget constraint becomes

$$
\begin{aligned}
&G(s^t) + (1 - \tau^d(s^t))B^d(s^t) + \delta(s^t)B(s^t) \\
\leq &\tau^n(s^t)w(s^t)L(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) + \sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t)B(s^{t+1}) + T(s^t)
\end{aligned}
$$

The price of international debt takes into account the probability of default is

$$
Q(s_{t+1}|s^t) = \frac{\Pr(s_{t+1}|s^t)\delta(s_{t+1}|s^t)}{1 + r^*}
$$

As the government cannot commit to any of its policies, one can think that the government, private agents, and international lenders enter in a sovereign game where they determine their actions sequentially. In every period and every history, the state variable for the game is $\left\{B(s^t), \left(b^{i,d}(s^t)\right)_{i \in I}\right\}$. The timing of the actions is as follows.

- Aggregate shock s_t is realized
- Government chooses $z_t^G = \left(\tau^n(s^t), \tau^d(s^t), T(s^t), \delta(s^t), B(s_{t+1}, s^t), B^d(s_{t+1}, s^t)\right) \in \Pi$such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c^i(s^t), l^i(s^t), b^{d,i}(s_{t+1}, s^t)\right)$ subject to their budget constraints, the representative firm produce output by choosing $z_t^F = L(s^t)$, and the international lenders choose holdings of government's bonds $z_t^* = B(s_{t+1}, s^t)$.

Define $h^t = \left(h^{t-1}, z_{t-1}^G, \left(z_{t-1}^{H,i}\right)_{i \in I}, z_{t-1}^F, z_t^*, s_t\right) \in H^t$ as the history after shock s_t is realized. Note that the history incorporates the government's policy, allocation and prices. Define $h_p^t = \left(h^t, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period t. The government strategy is $\sigma_t^G : H^t \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i} : H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has strategy $\sigma_t^F : H_p^t \to \mathbb{R}_+^2$, and the international lenders have strategy $\sigma_t^* : H_p^t \to \mathbb{R}_+$.

Definition B.1 (Sustainable equilibrium)**.** A sustainable equilibrium is $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ such that (i) for all h^t, the policy z_t^G induced by the government strategy maximizes the socially weighted utility given λ subject to the government's budget constraint (4) (ii) for all h_p^t, the strategy induced policy $\{z_t^G\}_{t=0}^{\infty}$, allocation $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^{\infty}$, and prices $\{Q_t,\}_{t=0}^{\infty}$ constitute a competitive equilibrium with taxes.

[28] Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its foreign debt ($d_t = 1$).

The following focuses on characterizing a set of sustainable equilibrium in which deviation triggers autarky, where there is no domestic and foreign borrowing. In this case, the value of deviation includes the autarkic payoff.

By definitions, autarky is a sustainable equilibrium. Given that the domestic agents do not save/invest, the representative firm produces only with labor, and the international creditors do not lend, the government finds it optimal to default on its external debt, set saving and capital taxes such that the after-tax gross returns on domestic bonds and capital are zero, and set the labor tax such that it maximizes the socially weighted utility. Given the government defaulting and fully taxing all returns from domestic savings and capital, international creditors do not want to lend, agents do not save or invest in capital, and output is produced only by labor. Lastly, given that the government will be in autarky in the future, it is optimal in the current period for the government to also follow the autarkic strategies.

Reverting to autarky equilibrium is defined as a sustainable equilibrium of the above game such that following any government's deviation from the promised plans, the economy reverts to autarky. One can characterize the equilibrium as follows.

Proposition B.1 (Reverting to autarky equilibrium). *An allocation and policy $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ can be supported by reverting to autarky equilibrium if and only if (i) given z^G, there exist prices p such that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G, p\right\}$ is a competitive equilibrium with taxes for an open economy, and (ii) for any t and any s^t, there exists $\underline{U}(s^t,t)$ such that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies the constraint*

$$\sum_{i\in I}\lambda^i\pi^i\sum_{k\geq t, s^t\subseteq s^k}\beta^{k-t}\Pr(s^k|s^t)U^i(c^i(s^k),l^i(s^k))\geq\underline{U}(s^t,t) \tag{12}$$

Proof. Define $\underline{U}(s^t,t)$ as the maximum discounted weighted utility for the agents in period t, history s^t, when the government deviates. At period t and history s^t, the government taxes all domestic wealth ($\tau^d(s^t)=1$) and redistributes equally across agents, and the government defaults on the external debt. In subsequent period $k>t$, the economy reverts to financial autarky where agents do not save in domestic bonds, and the government is excluded from international lending. This economy ensembles a neoclassical growth closed economy that has an initial aggregate state s_t, distortionary taxation on labor, and equal initial wealth across individuals.

Suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an outcome of the reverting to autarky equilibrium. Then by the optimal problems of the government, agents, and foreign lenders, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes the weighted utility of the agents, satisfies government budget constraint and foreign lender's problem at period 0. Thus, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an open-economy tax-distorted competitive equilibrium. For any period t and history h^t, an equilibrium strategy that has the government deviates in period t triggers reverting to autarky in period $k>t$. Such strategy must deliver the weighted value at least as high as the right-hand side of (12). So $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii).

Next, suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies conditions (i) and (ii). Let h^t be any history such that there is no deviation from z^G up until period t and history s^t. Since $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t and history s^t onward. Consider a deviation plan $\hat{\sigma}^G$at period t that receives $U^d(s_t,t)$ in period t and $U^{aut}(s_t)$ for the subsequent period $k>t$. Because the plan is constructed to maximize the utility in period t, the right-hand side of (12) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. □

Proposition B.1 can be extended to the general characterization of sustainable equilibrium, as in Chari and Kehoe (1990).

C Characterizing Sustainable Allocation and Optimal Tax Policies

This section provides details on the characterization of the sustainable allocation and optimal tax policies. Section D will use this analysis to prove the propositions in the main text.[29]

Let μ be the multiplier on the resource constraint, $\pi^i\eta^i$ be the multiplier on the implementability constraint for agent i, and $\beta^t\Pr(s^t)\gamma(s^t)$ be the multiplier on the aggregate debt constraint for period t. Define $\boldsymbol{\eta}=\left(\eta^i\right)_{i\in I}$ and rewrite the Larangian of the planning problem with a new pseudo-utility function that incorporates the implementability constraints:

$$\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\Phi_C^W\frac{C_t^{1-\sigma}}{1-\sigma}-\Phi_L^W\frac{L(s^t)^{1+\nu}}{1+\nu}\right]-\Phi_C^V C_0^{-\sigma}\sum_{i\in I}\pi^i\eta^i\left(b_0^i-T\right)$$

where Φ_C^W,Φ_L^W depend on $\boldsymbol{\varphi}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\eta}$ (see Appendix D.1).

The first-order conditions of the planning problem for any period $t\geq 1$ can be summarized as

$$F_L(L_t,s^t,t)=\frac{\left\{\Phi_L^W+\Phi_L^P\sum_{k=0,s^k\subseteq s^t}^{t}\gamma_k\right\}L_t^{\nu}}{\left\{\Phi_C^W+\Phi_C^P\sum_{k=0,s^k\subseteq s^t}^{t}\gamma_k\right\}C_t^{-\sigma}} \tag{C.1}$$

and

$$Q(s_{t+1}|s^t)=\beta\Pr(s_{t+1}|s^t)\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{\Phi_C^W+\Phi_C^P\sum_{k=0,s^k\subseteq s^t}^{t+1}\gamma_k}{\Phi_C^W+\Phi_C^P\sum_{k=0,s^k\subseteq s^t}^{t}\gamma_k} \tag{C.2}$$

The optimal tax policies follow

$$\tau_t^n=1-\frac{1}{F_L(L_t,s^t,t)}\frac{\Phi_L^V L_t^{\nu}}{\Phi_C^V C_t^{-\sigma}} \tag{C.3}$$

$$\frac{Q_t^d(s_{t+1})}{1-\tau_{t+1}^d}=\beta\Pr(s_{t+1}|s^t)\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \tag{C.4}$$

[29] This analysis is similar to Tran-Xuan (2020) in an economy with aggregate uncertainty.

D Formulas and Proofs

D.1 Formulas

Given the formulas for ψ_c^i and ψ_l^i in (**??**), we have the followings:

$$\Phi_C^V = \left[\sum_i \pi^i (\varphi^i)^{1/\sigma}\right]^\sigma; \qquad \Phi_L^V = \omega \left[\sum_i \pi^i \left(\varphi^i\right)^{-1/\nu} (\theta^i)^{(1+\nu)/\nu}\right]^{-\nu}$$

$$\Phi_C^P = \Phi_C^V \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i}\psi_c^i; \qquad \Phi_L^P = \Phi_L^V \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i}\psi_l^i$$

$$\Phi_C^W = \Phi_C^V \sum_{i\in I} \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]; \qquad \Phi_L^W = \Phi_L^V \sum_{i\in I} \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right]$$

D.2 Proof of Proposition 2.1

Proof. ($\Rightarrow$) Let $\{(C_t, L_t)\}_{t=0}^\infty$ be an aggregate allocation of an open economy competitive equilibrium with taxes. Then by definition,$\{(C_t, L_t)\}_{t=0}^\infty$ satisfies the aggregate resource constraint. Moreover, given any market weights $\boldsymbol{\varphi}$, (C_t, L_t) satisfies

$$(1-\tau_t^n)w_t = \frac{\Phi_L^V L_t^\nu}{\Phi_C^V C_t^{-\sigma}}$$

$$\frac{Q_t^d(s_{t+1})}{1-\tau_t^d} = \beta \Pr(s_{t+1}|s^t)\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$$

Substituting for w_t into the budget constraint (2), and using $c_t^i = \psi_c^i C_t, l_t^i = \psi_l^i L_t$, gives the implementability constraint for each agent. Importantly, choose $\boldsymbol{\varphi}$ and T such that the individual implementability constraints hold.

($\Leftarrow$) Given $\boldsymbol{\varphi}$, T and an allocation $\{(C_t, L_t)\}_{t=0}^\infty$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\{w_t\}_{t=0}^\infty$ using the firm's first-order condition (3). $\{\tau_t^n\}_{t=0}^\infty$ can be calculated using the intra-temporal condition (8), and choosing $\{Q_t^d(s_{t+1}), \tau_t^d\}_{t=0}^\infty$ to satisfy the inter-temporal constraint (9).[30] Define $\{q_t\}_{t=0}^\infty$ by $q_t = \Pr(s^t)/(R^*)^t$.

Rewriting the aggregate resource constraint using $F(L) = wL$ gives

[30]There is indeterminacy in setting the price on domestic debt and the domestic savings tax. Any combination of $\{Q_t^d(s_{t+1}), \tau_t^d\}$ that satisfies the intertemporal constraint is part of the competitive equilibrium. Later on, I consider a particular implementation in which $Q^d = Q$, the price of external debt. τ^d then captures the difference between the domestic and external prices.

$$
\begin{aligned}
&\sum_{t\geq 0,s^t} q_t \left\{C_t - (1-\tau_t^n)w_t L_t + T_t\right\} \\
&+ \sum_{t\geq 0,s^t} q_t \left[G_t - \tau_t^n w_t L_t - T_t\right] \leq -B_0
\end{aligned}
\tag{D.1}
$$

Aggregating up the agent's budget constraints implies

$$
C_t + \sum_{s_{t+1}} Q_t^d(s_{t+1})B_{t+1}^d = (1-\tau_t^n)w_t L_t + (1-\tau_t^d)B_t^d - T_t
$$

or

$$
C_t - (1-\tau_t^n)w_t L_t + T_t = (1-\tau_t^d)B_t^d - \sum_{s_{t+1}} Q_t^d(s_{t+1})B_{t+1}^d
$$

Substituting the last equation into (D.1) gives the government's budget constraint (4). Thus, $\{(C_t, L_t)\}_{t=0}^{\infty}$ is the aggregate allocation of the constructed competitive equilibrium with government policies. $\square$

D.3 Proof of Proposition 4.1

First, I show that the following lemma must hold.

Lemma D.1. *cov* $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < 0$ *and cov* $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$

Proof. The first step is to show that for i and j such that $i \neq j$, $\theta^i > \theta^j \iff \varphi^i > \varphi^j$.

Suppose $\theta^i > \theta^j$ and $\varphi^i \leq \varphi^j$, then $\psi_l^i \leq \psi_l^j$. By the definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \leq \frac{\varphi^i}{\varphi^j} \leq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} > 1$, which is a contradiction.

Suppose $\varphi^i > \varphi^j$ and $\theta^i \leq \theta^j$, then $\psi_l^i > \psi_l^j$. By the definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} > \frac{\varphi^i}{\varphi^j} > 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \leq 1$, which is a contradiction.

Next, the individual implementability constraint is

$$
\psi_c^i \Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma} - \psi_l^i \Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu} = \Phi_C^V C(s_0)^{-\sigma} \left(a^i(s_0) - T\right)
$$

or

$$
\psi_c^i = \psi_l^i \frac{\Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu}}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}} + \frac{\Phi_C^V C_0^{-\sigma}\left(b^i(s_0) - T\right)}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}}
$$

By the definition of ψ_c^i, $\varphi^i > \varphi^j \iff \psi_c^i > \psi_c^j$, and by the assumption, $\boldsymbol{\theta^i > \theta^j \iff b^i(s_0) > b^j(s_0)}$, which implies that $\theta^i > \theta^j \iff \psi_c^i > \psi_c^j \iff \psi_l^i > \psi_l^j$.

Thus, $\theta^i > \theta^j \iff \varphi^i > \varphi^j \iff \psi_c^i > \psi_c^j \iff \psi_l^i > \psi_l^j$.

In addition, $\boldsymbol{\theta}^i > \boldsymbol{\theta}^j \iff \boldsymbol{\lambda}^i < \boldsymbol{\lambda}^j$, which implies that

$$\psi_c^i > \psi_c^j \iff \frac{\lambda^i}{\varphi^i} < \frac{\lambda^j}{\varphi^j}$$
$$\psi_l^i > \psi_l^j \iff \frac{\lambda^i}{\varphi^i} < \frac{\lambda^j}{\varphi^j}$$

Hence, $\text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < 0$ and $\text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$. □

Now I proceed to the main proof of the proposition.

Proof. In financial autarky, there exist a vector of market weights $\boldsymbol{\varphi}^{\boldsymbol{a}}$, transfer T^a, and multiplier $\boldsymbol{\eta}^{\boldsymbol{a}}$that satisfies the conditions in Proposition 2.1 such that

$$\underline{U}(z) \equiv \max_{C_t, L_t, \boldsymbol{\varphi}^{\boldsymbol{a}}, T^a} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^{W,a} \log C_t - \Phi_L^{W,a} \frac{L_t^{1+\nu}}{1+\nu} \right] - \Phi_C^{V,a} C_0^{-\sigma} \sum_i \pi^i \eta^{i,a} T^a$$
$$s.t. \qquad C_t + G = z_t L_t$$
$$z_0 = z$$

where $\beta^t \pi^i \eta^{i,a}$ is the Lagrange multiplier on the individual implementability constraint and $\Phi_C^{W,a}, \Phi_L^{W,a}$ follows the formulas in Appendix D.1.

The optimal labor tax in autarky is constant over time and is equal to

$$\tau^{n,a} = 1 - \frac{\hat{\Phi}_L^V \hat{\Phi}_C^W}{\hat{\Phi}_C^V \hat{\Phi}_L^W} == 1 - \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

Lemma D.1 shows that $\text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right), \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$, which implies that

$$\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < \mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)$$

or

$$\frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)} < 1.$$

Thus, $\tau^{n,a} > 0$ □

D.4 Proof of Proposition 4.2

Without loss of generality, I consider the following proof showing that autarky is not optimal in period 0. The proof is similar for any period $t \geq 0$.

Proof. Suppose, by contradiction that the autarkic allocation $\{C_t^a, L_t^a, \varphi^a\}_{t=0}^{\infty}$ solves the planning problem given s_0.

Then the planning value at period 0 is $V_0^P\left(\{C_t^a, L_t^a, \varphi^a\}_{t=0}^{\infty}\right) = \underline{U}(z_0)$.

Fix $\epsilon > 0$. Consider the allocation $\left\{\hat{C}_t, \hat{L}_t, \varphi^a\right\}_{t=0}^{\infty}$ starting at s_0 and s_1 such that

- For period 0:$\hat{C}_0 = C_0^a + \frac{\epsilon}{R^*}\mathbb{E}_{s_1|s_0}\left[F_L(L_1^a) - \frac{\Phi_L^{P,a}\left(L_1^a\right)^{\nu}}{\Phi_C^{P,a}\left(C_1^a\right)^{-\sigma}}\right]$, $\hat{L}_0 = L_0^a$
- For period 1: $\hat{C}_1 = C_1^a + \frac{\Phi_L^{P,a}\left(L_1^a\right)^{\nu}}{\Phi_C^{P,a}\left(C_1^a\right)^{-\sigma}}\epsilon$, $\hat{L}_1 = L_1^a + \epsilon$
- For period $t \geq 2$:$\hat{C}_t = C_t^a$, $\hat{L}_t = L_t^a$

First, $\left\{\hat{C}_t, \hat{L}_t, \varphi^a\right\}_{t=0}^{\infty}$ is feasible because

$$
\begin{aligned}
&\sum_{t\geq 0}\mathbb{E}_0\left(\frac{1}{R^*}\right)^t\left[F(\hat{L}_t) - \hat{C}_t - G_t\right] \\
=&F(\hat{L}_0) - \hat{C}_0 - G_0 + \frac{1}{R^*}\mathbb{E}_{s_1|s_0}\left[F(\hat{L}_1) - \hat{C}_1 - G_1\right] + \sum_{s_1}\mathbb{E}_{s|s_1}\sum_{t\geq 2}^{\infty}\left(\frac{1}{R^*}\right)^t\left[F(\hat{L}_t) - \hat{C}_t - G_t\right] \\
=&\sum_{t\geq 0}\mathbb{E}_0\left(\frac{1}{R^*}\right)^t\left[F(L_t^a) - C_t^a - G_t\right] - \frac{\epsilon}{R^*}\mathbb{E}_{s_1|s_0}\left[F_L(L_1^a) - \frac{\Phi_L^{P,a}\left(L_1^a\right)^{\nu}}{\Phi_C^{P,a}\left(C_1^a\right)^{-\sigma}}\right] + \frac{1}{R^*}\mathbb{E}_{s_1|s_0}\left(F_L(L_1^a)\epsilon - \frac{\Phi_L^{P,a}\left(L_1^a\right)^{\nu}}{\Phi_C^{P,a}\left(C_1^a\right)^{-\sigma}}\epsilon\right) \\
=&\sum_{t\geq 0}\mathbb{E}_0\left(\frac{1}{R^*}\right)^t\left[F(L_t^a) - C_t^a - G_t\right] \\
\geq& B_0
\end{aligned}
$$

Second,$\left\{\hat{C}_t, \hat{L}_t, \varphi^a\right\}_{t=0}^{\infty}$ is implementable in equilibrium since there exists a $\hat{T}$ such that $\forall i \in I$

$$
\sum_{t\geq 0, s^t\in S^t}\beta^t\left[\Phi_C^{V,a}\psi_c^{i,a}\hat{C}_t^{1-\sigma} - \Phi_L^{V,a}\psi_l^{i,a}\hat{L}_t^{1+\nu}\right] \geq \Phi_C^{V,a}\hat{C}_0^{-\sigma}\left(b^i(s^0) - \hat{T}\right)
$$

The flow utilities for all period $t \geq 1$ do not change. That is,

$u_1^P(\hat{C}_1, \hat{L}_1, \varphi^a) = \Phi_C^{P,a}\frac{\left(\hat{C}_1\right)^{1-\sigma}}{1-\sigma} - \Phi_L^{P,a}\frac{\left(\hat{L}_1\right)^{1+\nu}}{1+\nu} = \Phi_C^{P,a}\frac{\left(C_1^a\right)^{1-\sigma}}{1-\sigma} + \Phi_C^{P,a}\left(C_1^a\right)^{-\sigma}\frac{\Phi_L^{P,a}\left(L_1^a\right)^{\nu}}{\Phi_C^{P,a}\left(C_1^a\right)^{-\sigma}}\epsilon - \Phi_L^{P,a}\frac{\left(L_1^a\right)^{1+\nu}}{1+\nu} - \Phi_L^{P,a}\left(L_1^a\right)^{\nu}\epsilon = u_1^p(C_1^a, L_1^a, \varphi^a)$

$u_t^P(\hat{C}_t, \hat{L}_t, \varphi^a) = u_t^p(C_t^a, L_t^a, \varphi^a), \forall t \geq 2$

And the flow utility in period 0 increases. That is,

$$
\begin{aligned}
u_0^P(\hat{C}_0, \hat{L}_0, \varphi^a) &= \Phi_C^{P,a} \frac{\left(\hat{C}_0\right)^{1-\sigma}}{1-\sigma} - \Phi_L^{P,a} \frac{\left(\hat{L}_0\right)^{1+\nu}}{1+\nu} \\
&= \Phi_C^{P,a} \frac{(C_0^a)^{1-\sigma}}{1-\sigma} + \Phi_C^{P,a} (C_0^a)^{-\sigma} \frac{\epsilon}{R^*} \mathbb{E}_{s_1|s_0} \left[F_L(L_1^a) - \frac{\Phi_L^{P,a} (L_1^a)^\nu}{\Phi_C^{P,a} (C_1^a)^{-\sigma}} \right] - \Phi_L^{P,a} \frac{(L_0^a)^{1+\nu}}{1+\nu} \\
&= u_0^p(C_0^a, L_0^a, \varphi^a) + \Phi_C^{P,a} (C_0^a)^{-\sigma} \frac{\epsilon}{R^*} \mathbb{E}_{s_1|s_0} \left[F_L(L_1^a) - \frac{\Phi_L^{P,a} (L_1^a)^\nu}{\Phi_C^{P,a} (C_1^a)^{-\sigma}} \right]
\end{aligned}
$$

Lemma D.1 implies that $\frac{\Phi_L^{P,a}(L_1^a)^\nu}{\Phi_C^{P,a}(C_1^a)^{-\sigma}} < \frac{\Phi_L^{W,a}(L_1^a)^\nu}{\Phi_C^{W,a}(C_1^a)^{-\sigma}} = F_L(L_1^a), \forall s_1$, so $u_0^P(\hat{C}_0, \hat{L}_0, \varphi^a) > u_0^p(C_0^a, L_0^a, \varphi^a)$. Thus, $V_0^P\left(\left\{\hat{C}_t, \hat{L}_t, \varphi^a\right\}_{t=0}^\infty\right) > V_0^P\left(\{C_t^a, L_t^a, \varphi^a\}_{t=0}^\infty\right)$, which contradicts $\{C_t^a, L_t^a, \varphi^a\}_{t=0}^\infty$ being the optimal allocation. □

D.5 Proof of Proposition 5.1

Proof. Let $\{C_t^*, L_t^*\}_t, \boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exists $\boldsymbol{\lambda}$ such that $\{C_t^*, L_t^*\}_t, \boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). Define

$$
A_C = \sum_{i \in I} \pi^i \frac{\lambda^i}{\varphi^{*i}} \psi_c^i, \quad A_L = \sum_{i \in I} \pi^i \frac{\lambda^i}{\varphi^{*i}} \psi_l^i \tag{D.2}
$$

where ψ_c^i, ψ_l^i are defined by equations (7) using $\boldsymbol{\varphi}^*$. First, one can show that A_C and A_L are positive and bounded:

Lemma D.2. *Given an interior allocation,* $0 < A_C < \infty$ *and* $0 < A_L < \infty$

Proof. Interior allocation means that for any i, $c_t^i, l_t^i > 0, \ \forall t$. This implies that $\psi_c^i, \psi_l^i > 0$. By (7), $\varphi^{*i} > 0$.

For all i, $\pi^i > 0, \lambda^i \geq 0$ and since $\sum_{i \in I} \pi^i \lambda^i = 1$, there exists at least an i such that $\lambda^i > 0$. Given that $\psi_c^i, \psi_l^i > 0, \ \forall i$, it must be that $A_C, A_L > 0$.

Since $\sum_{i \in I} \pi^i \varphi^{*i} = 1 < \infty$ and $\forall i, \ \pi^i, \varphi^{*i} > 0$, it must be that $\varphi^{*i} < \infty$. So by definition, $\psi_c^i, \psi_l^i < \infty$. Moreover, $\varphi^{*i} > 0$ implies that $\lambda^i / \varphi^{*i} < \infty$. Then by definition, $A_C, A_L < \infty$. □

For any M and s^M, define (P^{s^M}) the same problem as (P) with the restriction that $\left(C(s^t), L(s^t)\right) = \left(C^*(s^t), L^*(s^t)\right), \forall t > M, \ s^t \supset s^M, \ \varphi = \varphi^*$, and $T = T^*$. Note that $\{C_t^*, L_t^*\}$ is a solution to (P^{s^M}), and (P^{s^M}) has a finite number of constraints. By a Lagrangian theorem in Luenberger (1969), there exists non-negative, not-identically zero vector $\left\{r^{s^M}, \mu^{s^M}, \eta^{s^M,1}, \ldots, \eta^{s^M,I}, \gamma^{s^M}(s^0), \ldots, \gamma^{s^M}(s^M)\right\}$ such that the first-order and complementarity conditions hold for $t \in \{1, .., M\}, s^t \subseteq s^M$, i.e.,

$$(\beta R^*)^t \left\{ r^{s^M} A_C + \sum_i \pi^i \eta^{s^M,i}(1-\sigma)\psi_c^i + \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) A_C \right\} \Phi_C^V C_t^{-\sigma} = \mu^{s^M} \tag{D.3}$$

$$(\beta R^*)^t \left\{ r^{s^M} A_L + \sum_i \pi^i \eta^{s^M,i}(1+\nu)\psi_l^i + \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) A_L \right\} \Phi_L^V L_t^{\nu} = \mu^{s^M} F_L(L_t, s^t, t). \tag{D.4}$$

Equation (D.3) can be rewritten as

$$(\beta R^*)^t \sum_{k \le t, s^k \subseteq s^M} \gamma^{s^M}(s^k) = \frac{\mu^{s^M}}{A_C \Phi_C^V C_t^{-\sigma}} - (\beta R^*)^t \left[r^{s^M} + \frac{1}{A_C} \sum_i \pi^i \eta^{s^M,i}(1-\sigma)\psi_c^i \right]. \tag{D.5}$$

The following lemma shows that μ^{s^M} and $C_t^{-\sigma}$ are always positive for the sub-problem (P^{s^M}) for any $M \ge 1$ and any s^M.

Lemma D.3. *In the sub-problem (P^{s^M}) for any $M \ge 1$ and s^M, $\mu^{s^M} > 0$*

Proof. Suppose, by contradiction, that $\mu^{s^M} = 0$ so the resource constraint does not bind. Consider allocation $\{C_t, L_t\}_{t=0}^{\infty}$ the solution to (P^{s^M}). Then there exists $\epsilon > 0$ such that

$$\sum_{t \ge 0} q_t \left[F(L_t, s^t, t) - G_t - C_t \right] - B_0 - \epsilon \ge 0$$

Define $\left\{\hat{L}_t\right\}_{t=0}^{\infty}$ such that at period 1 for a fixed s^1, $\hat{L}_1(s^1) < L_1(s^1)$ such that $F(\hat{L}_1(s^1), s^1, 1) = F(L_1(s^1), s^1, 1) - \epsilon/q_1(s^1)$, and $\hat{L}_t = L_t$ for all other periods and histories. The allocation $\left\{C_t, \hat{L}_t\right\}_{t=0}^{\infty}$ satisfies the resource constraint and because of the preference's strict monotonicity, $\left\{C_t, \hat{L}_t\right\}_{t=0}^{\infty}$ also satisfies the implementability constraints and the aggregate debt constraints. However,

$$\mathbb{E}_0 \sum_{t \ge 0} \beta^t \left(\Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{\hat{L}_t^{1+\nu}}{1+\nu} \right) > \mathbb{E}_0 \sum_{t \ge 0} \beta^t \left(\Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{\hat{L}_t^{1+\nu}}{1+\nu} \right)$$

which contradicts $\{C_t, L_t\}_{t=0}^{\infty}$being optimal solution for (P^{s^M}). □

The consumption path is bounded below by zero in the long run, i.e.

Lemma D.4 (No immiseration)**.** *Suppose Assumptions 3 hold, then for any efficient allocation $\{C_t^*, L_t^*\}_{t=0}^{\infty}$, $\liminf_{t\to\infty} C_t^* > 0$.*

Proof. Given an efficient allocation $\{C_t^*, L_t^*\}_{t=0}^{\infty}$, suppose, by contradiction that for a sequence of shocks $\{s_0, ..., s_t, ...\}$, $\liminf_{t\to\infty} C^*(s^t) \le 0$. Find $\epsilon > 0$ such that $\forall t$,$\forall s^t$,

$$\sum_{k=t}^{\infty} \beta^{\tau-t} \sum_{s^t \subseteq s^k} Pr(s^\tau) \left[\Phi_C^V \frac{C(s^k)^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L^*(s^k)^{1+\nu}}{1+\nu}\right] \le M_U$$

with $C(s^t) = \epsilon$ and $C(s^k) = C^*(s^k),\ \forall k > t,\ s^t \subset s^k$. Such ϵ exists since the utility function is unbounded below. Because $\liminf_{t\to\infty} C^*(s^t) \le 0$, there exists a t_0 such that $C^*(s^{t_0}) < \epsilon$. Then by monotonicity,

$$\begin{aligned}
&\sum_{k=t_0}^{\infty} \beta^{\tau-t_0} \sum_{s^{t_0} \subseteq s^k} \Pr(s^k) \left[\Phi_C^V \frac{C^*(s^k)^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L^*(s^k)^{1+\nu}}{1+\nu}\right] \\
<&\sum_{k=t_0}^{\infty} \beta^{\tau-t_0} \sum_{s^{t_0} \subseteq k} \Pr(s^k) \left[\Phi_C^V \frac{C(s^k)^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L^*(s^k)^{1+\nu}}{1+\nu}\right] \\
\le& M_U \\
\le& \underline{U}(s^{t_0}, t_0)
\end{aligned}$$

which contradicts the aggregate debt constraint at t_0. □

Taking the limit on both sides of equation (D.5) gives

$$\begin{aligned}
\lim_{t\to\infty} (\beta R^*)^t \sum_{k=0}^{t} \sum_{s^k \subseteq s^M} \gamma^{s^M}(s^k) &= \lim_{t\to\infty} \left\{ \frac{\mu^{s^M}}{A_C \Phi_C^V C_t^{-\sigma}} - (\beta R^*)^t \left[r^{s^M} + \frac{1}{A_C} \sum_i \pi^i \eta^{s^M,i} (1-\sigma) \psi_c^i \right] \right\} \\
&= \lim_{t\to\infty} \frac{\mu^{s^M}}{A_C \Phi_C^V C_t^{-\sigma}} > 0
\end{aligned}$$

□

E Quantitative Appendix

This section provides additional details that is implemented in Section 5 .

E.1 Deviation Utility

The deviation utility $\underline{U}(z)$ is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes of a closed economy where the government does not issue both domestic and external debts.

$$\underline{U}(z) \equiv \max_{c_t^i, l_t^i, \tau_t^n, T} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right]$$

$$
\begin{aligned}
s.t. \quad & C_t + G = z_t L_t \\
& c_t^i + \sum_{s_{t+1}} Q_t(s_{t+1}) b_{t+1}^{d,i} = (1-\tau_t^n) z_t l_t^i + b_t^{d,i} - T_t \\
& (1-\tau_t^n) z_t = -\frac{1}{\theta^i} \frac{\omega \left(\frac{l_t^i}{\theta^i}\right)^{\nu}}{\left(c_t^i\right)^{-\sigma}} \\
& Q_t(s_{t+1}) = \beta \Pr(s^{t+1}|s^t) \frac{\left(c_{t+1}^i\right)^{-\sigma}}{\left(c_t^i\right)^{-\sigma}} \\
& b_0^{d,i} = b_0^{d,j}, \ \forall i,j \in I \\
& z_0 = z
\end{aligned}
$$

There exist a vector of market weights $\boldsymbol{\varphi^a}$, transfer T^a, and multiplier $\boldsymbol{\eta^a}$that satisfies the conditions in Proposition 2.1 such that

$$
\begin{aligned}
\underline{U}(z) \equiv \max_{C_t, L_t, \boldsymbol{\varphi^a}, T^a} \ & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^{W,a} \log C_t - \Phi_L^{W,a} \frac{L_t^{1+\nu}}{1+\nu} \right] - \Phi_C^{V,a} C_0^{-\sigma} \sum_i \pi^i \eta^{i,a} T^a \\
s.t. \quad & C_t + G = z_t L_t \\
& z_0 = z
\end{aligned}
$$

where $\beta^t \pi^i \eta^{i,a}$ is the Lagrange multiplier on the individual implementability constraint and $\Phi_C^{W,a}, \Phi_L^{W,a}$ follows the formulas in Appendix D.1.

E.2 Computational Algorithm

1. Guess μ and φ. Compute η.

   (a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ
   Initial guess for $V(s_t, \mu_t, \Gamma_{t-1}) = \sum_{j \geq 0, s^t \subseteq s^{t+j}} \beta^j \Pr(s^{t+j}) \left[\Phi_C^P \frac{C(s^{t+j})^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^{t+j})^{1+\nu}}{1+\nu} \right]$.

   (b) Assume the constraint does not bind in s_t: $\gamma(s_t) = 0$. Solve for the allocation $C(s_t), L(s_t)$ using the first-order conditions

   $$
   \begin{aligned}
   \left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1} \right] C(s_t)^{-\sigma} &= \mu \\
   \left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1} \right] L(s_t)^{\nu} &= \mu F_L(s_t)
   \end{aligned}
   $$

   (c) Since $\gamma(s_t) = 0$, compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$ (interpolating the expectation), then compute

$$A(s_t) = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^\tau \subseteq s^t} Pr(s^\tau) \left[\Phi_C^P \frac{C(s^\tau)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^\tau)^{1+\nu}}{1+\nu} \right]$$
$$= \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$$

(d) Check if $A(s_t) \geq \underline{U}(s_t)$. If it is, proceed to the next step. If not, solve for $C(s_t), L(s_t), \gamma(s_t)$ using these equations

$$\left[\mu_t \Phi_C^W + \Phi_C^V (\Gamma_{t-1} + \gamma(s_t))\right] C(s_t)^{-\sigma} = \mu$$
$$\left[\mu_t \Phi_L^W + \Phi_L^V (\Gamma_{t-1} + \gamma(s_t))\right] L(s_t)^{\nu} = \mu F_L(s_t)$$
$$\Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu}$$
$$+\beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V\left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t)\right)\right) = \underline{U}(s_t)$$

(e) Given $C(s_t), L(s_t), \gamma(s_t)$($\gamma$ can be zero or not), compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \Gamma_{t-1} + \gamma(s_t))$. Update the value function

$$V^{n+1}(s_t, \Gamma_{t-1}) = \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu}$$
$$+ \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V^n \left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t)\right)\right)$$

2. Compute residuals to find μ and φ

$$r^\mu = \sum_{t \geq 0} q_t \left[F(L_t) - G_t - C_t\right] - B_0$$
$$r^\varphi = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^V \left(\psi_c^i - \psi_c^j\right) C_t^{1-\sigma} - \Phi_L^V \left(\psi_l^i - \psi_l^j\right) L_t^{1+\nu}\right]$$

3. Find μ and φ such that $r^\mu = 0$ and $r^\varphi = 0$.

E.3 Measuring Redistributive Cost of Default

This subsection performs a counterfactual exercise that measures the equivalent productivity loss of the distributive component of the cost of default. I consider a representative-agent model in which default not only leads to exclusions from financial markets, but also a permanent loss of productivity. I model a proportional loss of productivity in default as in Aguiar and Gopinath

(2006), i.e. $z^{default} = (1-\kappa)z$, for $0 \leq \kappa \leq 1$.[31] I calibrate κ such that the representative-agent model generates the same amount of external debt-to-output as the heterogeneous-agent model.

Table E.1: Measuring the distributive cost of default

Description	Parameter	Value	Target	Baseline	One agent & exog. prod. loss
Fraction of prod. loss in default	κ	0.3%	Mean B/Y	21%	21%

Notes: The table reports the value of κ, fraction of productivity that is lost in default such that the long-run average external debt-to-output of the baseline model is the same as the one-agent model. The statistics come from simulations of the models for 10500 periods, excluding the first 500 periods.

Table E.1 reports the results of the exercise. The equivalent loss of productivity for the distributive component is 0.3%

E.4 Strategy for Cross-Country Estimation

I estimate the correlation between income inequality and external debt in the model in the following steps:

1. Construct a 31-point evenly spaced grid of θ^H/θ^L, which measures wage inequality. The grid values are between 1.5 and 3[32]
2. For each value of wage inequality, indexed by j,
   (a) Solve and simulate the model for 10500 periods
   (b) Calculate the following averages from the simulation sample, excluding the first 500 periods
      - Pre-tax income Gini Index ($y^H/y^L - 1/2$)
      - External debt-to-output (B/Y)
      - Log GDP per capita ($\log Y$)
3. Run the following regression

$$\text{External debt-to-output}_j = \alpha_0 + \alpha_1 \text{Pre-tax Gini Index}_j + \alpha_2 \text{GDP per capita (log)}_j + \epsilon_j$$

The estimation for α_1 is reported in the second column of Table 5.

[31] For non-linear default costs that are widely used in the literature, see Arellano (2008) and Chatterjee and Eyigungor (2012).

[32] The result is robust to different ranges of wage inequality.

F Sensitivity Analysis

F.1 Role of Discount Factor

This subsection consider the optimal allocation and policies under different values of the discount factor of the domestic agents. Table F.2 reports the simulation results. When the domestic agents become more impatient (their discount factor is declining), the average consumption-to-output ratio roughly remains the same (with a slight decrease), while the average external debt-to-output ratio declines. Lower external debt levels also coincide with lower labor tax and lump-sum tax rates. A higher discount factor implies a lower value of autarky and a longer time period it takes for the economy to reach the borrowing constraints. Therefore, it allows the government to accumulate and sustain a higher external debt level.

Table F.2: Role of discount factor

β	0.9	0.95	0.967 Baseline
Long-run statistics			
Mean C/Y	0.81	0.81	0.82
Mean B/Y	0.085	0.17	0.21
Tax policies			
Initial τ_0^n	0.246	0.250	0.252
$\lim_{t\to\infty} \tau_t^n$	0.0066	0.005	0.0042
Lump-sum tax T_0/Y_0	0.46	0.70	0.87

Notes: The table reports long-run statistics and policies for different levels of discount factor β. The model's simulation is for 10500 periods. The statistics are calculated from the sample excluding the first 500 periods. Tax policies are calculated from the whole sample.

F.2 Role of Aggregate Uncertainty

Table F.3 reports the statistics and policies for different levels of the shock variance. The first column reports the deterministic case, where $\sigma_z = 0$. The other columns report the other stochastic cases. Higher variance of the productivity shock implies a lower external debt-to-output, a higher consumption-to-output, a lower initial labor tax, and higher labor taxes in the limit and lump-sum taxes. In this environment, a higher uncertainty leads to more fluctuations of shocks, resulting in more time periods that the borrowing constraints do not bind in the long run. In fact, in the deterministic case, one can show that if the borrowing constraint binds, it will find forever.[33] This property leads to lower external debt levels in the long run. In addition, a higher uncertainty implies a higher precautionary motive, which helps explain the lower external debt accumulation.

[33] See Tran-Xuan (2020)

Table F.3: Role of aggregate uncertainty

σ_z	0	0.01	0.0205 Baseline	0.022
Long-run statistics				
Mean C/Y	0.81	0.81	0.82	0.82
Mean B/Y	0.25	0.245	0.21	0.21
Tax Policies				
Initial τ_0^n	0.25	0.25	0.25	0.24
Mean LR $\lim_{t\to\infty} \tau_\infty^n$	0.0029	0.0031	0.0042	0.0043
Lump-sum tax T_0/Y_0	0.6	0.65	0.87	0.9

Notes: The table reports long-run statistics and policies for different levels of the shock variance σ_z. The model's simulation is for 10500 periods. The statistics are calculated from the sample excluding the first 500 periods. Tax policies are calculated from the whole sample.