Optimal Redistributive Policy in Debt Constrained Economies

Monica Tran-Xuan*

October 2020

Abstract

This paper studies optimal taxation when the government has a redistributive motive and faces limited borrowing. The model is a small open economy in which agents are heterogeneous in labor productivity, and the government is subject to self-enforcing debt constraints that arise endogenously from its limited commitment. Redistributive policies are proportional taxes on labor and domestic saving. The standard Ramsey results of labor tax smoothing and a zero capital tax in the limit no longer hold. Instead, optimal labor taxes decrease over time and eventually converge to a non-zero limit, and the optimal capital tax is positive in the limit. The government's redistributive motive determines both the initial and long-run optimal tax levels, while binding debt constraints change optimal tax dynamics. The numerical comparative statics demonstrates that a stronger government's redistributive motive requires greater tax distortions in initial periods and a higher external debt level in the long run.

Keywords: Optimal taxation; Redistribution; Limited commitment; Sovereign debt
JEL Classifications: F34; F38; H21; H23; H63

*Department of Economics, University at Buffalo. 415 Fronczak Hall, Buffalo NY 14260, U.S.A. Email: monicaxu@buffalo.edu. This is a revised version of the first chapter of my dissertation at the University of Minnesota. I thank V.V. Chari, Larry Jones, and Manuel Amador for their invaluable advice and guidance. I am grateful for comments from Fernando Arce, Anmol Bhandari, Javier Bianchi, Jonathan Heathcote, Emily Moschini, Juan Pablo Nicolini, Chris Phelan, David Rahman, Aldo Rustichini, and Jan Werner. I also benefit from discussions with the seminar participants at University of Minnesota, the Federal Reserve Bank of Minneapolis, the Federal Reserve Bank of Richmond, 2018 Midwest Macroeconomic Meetings, 2018 EEA Annual Congress, and 2019 North American Summer Meeting of the Econometric Society. I acknowledge the support from the 2017 AEA Summer Economics Fellowship.

Introduction

How does a government determine its tax policies while facing limited borrowing but having a redistributive motive? Taxes are the main source of the government's revenue that affects its ability to issue or repay debt. At the same time, distortionary taxes and transfers redistribute resources across agents, so the government's preferences for greater redistribution significantly affect its willingness to raise revenue and influence its borrowing capacity. When borrowing is constrained, redistributing via taxes is limited.

Empirical works on the recent European debt crisis have demonstrated how the government's redistributive motive and debt accumulation are closely related.[1] First, the rapid accumulation of external debt in the periods leading up to a crisis led to many countries facing such high costs of borrowing that they could not roll over their debt. Second, highly indebted countries such as Greece, Portugal, and Spain also experienced high levels of income dispersion. According to the European Union's statistics on Income and Living Conditions, the Gini coefficients and S80/S20 income quintile share ratio in these countries were both higher than the EU-27 country average. Countries proposed different policy strategies to tackle the problems of constrained borrowing while maintaining the desired level of redistribution.

Motivated by these observations, this paper questions how both the government's redistributive motive and its limited ability to borrow affect optimal tax policies. The paper explores insights on the trade-off between redistribution and efficiency as in Werning (2007) and studies how limited borrowing restricts the government's ability to smooth taxes over time, as in Aguiar and Amador (2016). While tax policies allow the government to achieve a desired level of redistribution, the government has an additional incentive to change tax levels to sustain more debt. Through tax policies, the government's redistributive motive influences its borrowing capacity.

I address these issues in a model of a small open economy with heterogeneous agents and the government's limited commitment. The model combines the sovereign debt framework of limited commitment (Aguiar et al. (2009) and Aguiar and Amador (2011, 2014, 2016)) with the Ramsey taxation framework (Chari et al. (1994), Chari and Kehoe (1999), and Werning (2007)). I follow the primal approach in the Ramsey taxation literature to characterize the optimal fiscal policies. I then examine how changes in the levels of income inequality and redistributive preferences affect the optimal policies.

I find that redistributive concerns change optimal tax levels via the trade-off between equality and efficiency, whereas binding debt constraints alter optimal tax dynamics by imposing an additional cost borrowing on the government. The optimal labor tax decreases

[1] See Berg and Sachs (1988), Aizenman and Jinjarak (2012), Ferriere (2015) and Jeon and Kabukcuoglu (2018) for more empirical analysis on how high income inequality is correlated with high spreads and likelihood of debt crises.

over time and eventually converges to a non-zero limit, and the optimal capital and domestic borrowing taxes are positive in the limit. These results are in contrast of the standard Ramsey ones in which optimal labor tax is smooth (Lucas and Stokey (1983)), and the optimal capital and domestic borrowing taxes are zero in the limit (Chamley (1986)). The non-zero result of optimal labor tax in the limit differs from the zero value found in Aguiar and Amador (2016) in a representative-agent setting. Here, the optimal labor tax in the limit depends on the level of skill heterogeneity and the government's redistributive motive.

Second, the government's redistributive motive, not heterogeneity, is the main source for changes in optimal policy levels. If the social welfare weights are equal to the inverse equilibrium marginal utilities, the optimal tax levels are independent of the distributional preference. In this case, even with heterogeneity, the optimal labor tax converges to zero in the limit as in the representative agent case of Aguiar and Amador (2016).

Furthermore, the economy's long-run external debt level depends on the heterogeneity level and the government's distributional preference. In the numerical analysis, as the skill dispersion and the distributional preference towards low-skilled workers increase, the government is willing to sustain higher debt in the limit. The government achieves this outcome by front-loading greater tax distortions to redistribute initially and further reducing the distortions to increase efficiency and borrowing capacity in the future.

These results rely on the following key elements of the model. Domestic agents are differentiated by their labor productivity types. In each period, a benevolent government, without commitment, chooses both a level of debt going forward and a system of labor taxes, capital income taxes, and lump-sum taxes that are independent of individual income. Each period debt constraint reflects the level of debt that the government can sustain, which depends on the current and future fiscal policies. The objective function of the government features a desire to redistribute, captured by the social weights on domestic agents' utilities. The government has the same discount factor as the domestic agents, which are less patient than the international lenders.

Debt constraints are limited commitment constraints on the government in which the value of committing to the optimal path of policies onward is weakly higher than the value of deviating from the optimal paths. In the appendix, I show that these constraints characterize the subgame perfect equilibrium of a game between domestic agents, international lenders, and the government, similar to the approach in Chari and Kehoe (1990, 1993). When the government deviates from the contract, it faces a punishment imposed by domestic agents and international lenders, which corresponds to a value of deviation. I assume this punishment to be financial autarky, where the government cannot have access to both domestic and international financial markets. These constraints imply that the debt limits are endogenous and dependent upon current and future fiscal policies.

Impatience provides an ex-ante motive for debt accumulation in the economy. Impatience means that the intertemporal discounting rate of future utilities is lower than the international intertemporal price of resources. This assumption can be rationalized by political frictions or weak domestic financial markets. The economy has an incentive to borrow abroad to front-load consumption and leisure, resulting in binding debt constraints in the future.

The government's redistributive motive provides a natural reason for its need to levy distortionary taxation. A government that wants to redistribute more toward lower-income agents will levy positive marginal labor taxes and lump-sum transfers. In this case, higher-income individuals will bear a higher tax burden. However, marginal labor taxes discourage labor supply and reduce the economy's efficiency. Therefore, the optimal tax balances the marginal benefit of redistribution and the marginal cost of distortions.

The optimal labor tax is a function of the distributional preference and heterogeneity, which are invariant over time, and of the shadow cost of borrowing that is varying over time. When debt constraints do not bind, the borrowing cost remains constant, and so does the optimal labor tax. In this case, the government finds it optimal to use debt to smooth finances over time and keep labor distortions equal to the level that maximizes the redistributive benefit. When debt constraints bind, there is an additional motive to increase social welfare and relax the debt constraints. The government then finds it optimal to adjust current and future tax rates to lower the borrowing cost. An increase in the current period's efficiency relaxes the debt constraints for all previous periods. Anticipating this effect, the government changes the tax rates to increase efficiency in all future periods.

In the long run, the efficient allocation minimizes the cost of delivering the promised utility and maximizes the net payment to the international financial markets. The optimal labor tax converges to a limit associated with the maximal sustainable debt level of the whole economy. The skill distribution and distributional preference influence both sides of the debt constraints: the value of staying in the contract and the value of deviating. Therefore, they affect the endogenous level of the maximum aggregate debt, which in turn determines the tax limit.

With the government's inequality aversion and high consumption-inequality aversion, the optimal labor tax permanently drifts downward as the debt constraints bind. This result implies that, when the willingness to lend of the international lenders is high (as the debt constraints do not bind), the government would like to redistribute via a high labor tax rate. When the willingness to lend decreases (as the debt constraints bind), the government reduces distortions (lower tax rates) to increase the economy's repaying capacity.

I illustrate how the government's redistributive motive affects the optimal labor tax in the limit using the sub-market analysis in Werning (2007). I show that the competitive equilibrium generates an efficient assignment of individual allocation, which is captured by a set of market–Negishi weights that determine individual utility shares. Without heterogeneity or redistributive motive, the model collapses to the representative setting as in Aguiar and Amador (2016), which proved that the labor tax limit is zero. When there is heterogeneity, the labor tax is zero as long as the equilibrium Negishi weights are equal to the social weights. These findings imply that the optimal labor tax in the limit depends on the government's preferred distribution only when it is different from the equilibrium distribution.

The numerical analysis validates the theoretical results by illustrating that the optimal labor tax is positive when borrowing is not limited and decreases over time until reaching its negative limit. When the borrowing is tightened, the government discourages domestic borrowing by taxing it. It turns out that the higher-income agents borrow more over time comparing to the lower-income agents, so the borrowing taxes play a role in redistribution. The government redistributes more via the borrowing taxes and less via labor taxes until it reaches the highest efficiency level where it does not face any distortionary cost. The limit represents the maximum debt capacity of the economy. The key feature is that the government dynamically substitutes between the intratemporal and intertemporal distortions.

The comparative statics demonstrates that a stronger redistributive motive requires greater tax distortions at the beginning of time, lower tax distortions and a higher external debt level in the long run. In detail, I look at the changes in the optimal tax rates and external debt with respect to skill dispersion, measured as the ratio of individual labor productivity levels. A higher skill-dispersion means a higher motive for redistribution, translating into a higher level of tax before the debt constraint binds. When reaching the debt constraints, the economy faces a higher distortion. In order to relax the debt constraints, the planner needs to reduce the higher efficiency cost. Consequently, the labor tax limit declines when the skill dispersion rises. Higher the skill dispersion means higher return on domestic savings, but eventually the saving return will converge to the same steady-state rate. The external debt levels also change with respect to skill dispersion, in which for the higher the skill dispersion, the longer the periods of unconstrained borrowing, and the higher the external debt positions in the long run. The main reason is that it is more costly for the higher dispersed economy to redistribute in financial autarky. Redistribution now leads to high debt accumulation before reaching the constraints, which leads to a higher debt later on to sustain the allocation.

Related Literature. This paper provides a theoretical model that qualitatively accounts for the positive correlation between income dispersion and debt level. Several empirical papers have documented this relationship in the data. Specifically, Berg and Sachs (1988) showed that income dispersion was a key predictor of a country's probability of rescheduling debt and the bond spread in secondary markets. Aizenman and Jinjarak (2012) described a negative correlation between income dispersion and the tax base and a positive correlation with sovereign debt. Recently, Ferriere (2015) and Jeon and Kabukcuoglu (2018) also provided empirical evidence of rising income dispersion significantly increases sovereign default risk.

This paper contributes to the optimal taxation and debt management research of the public finance literature (Barro (1979), Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1994), Aiyagari and McGrattan (1998), Chari and Kehoe (1999), Aiyagari, Marcet, Sargent, and Seppälä (2002), and many other papers). The argument for labor tax smoothing often relies on the fact that the government can issue debt that is contingent to all states and is not constrained beyond the natural debt limit. In this paper, tax smoothing is not always optimal; the government's ability to smooth tax distortion is restricted by the willingness to lend by the international lenders. The non-zero capital and domestic borrowing taxes are contrast to the zero convergence of the capital tax from Judd (1985), Chamley (1986), Chari, Christiano, and Kehoe (1994), and Straub and Werning (2020), because the endogenous debt limits depend on the capital stock, and the domestic agents do not internalize the effect of their borrowing on the aggregate debt constraints.

The paper contributes to the sovereign debt literature by studying optimal tax and sovereign debt policies with heterogeneous agents. Seminal papers that studied sovereign debt in the lack-of-commitment environment include Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) Arellano (2008), Aguiar, Amador, and Gopinath (2009), and Aguiar and Amador (2011). There are a few recent works on optimal policies in the sovereign default framework (Ferriere (2015) and Arellano and Bai (2016)). In this paper, I allow the government not to commit to both tax and debt policies. I also highlight the effect of the government's redistributive motive on sovereign debt policies.

The dynamic environment in this paper is an extension to one in Aguiar and Amador (2016), adding heterogeneity and distributional preference and allowing for richer tax systems. Aguiar and Amador (2016) found that labor tax must go to zero in the long run as a result of front-loading efficient consumption and leisure allocation. In this paper, the tax limit can be any real value. More interestingly, when turning off the effect of redistribution in the model, the limit of labor tax is zero, consistent with their findings. The paper shows that the government's redistributive consideration, not heterogeneity, is the main source for the changes in optimal policies.

This paper also contributes to the current research in optimal fiscal policies with heterogeneity and redistributive motive. Bhandari, Evans, Golosov, and Sargent (2017) and Werning (2007) both found that the government's redistributive motive had significant impact on optimal policies. Werning (2007) developed the conditions for perfect tax smoothing with redistribution, while Bhandari, Evans, Golosov, and Sargent (2017) emphasized the impact of the initial distribution of heterogeneity on optimal allocation and optimal debt in the long run. In this paper, I highlight the role of endogenous debt constraints in changing optimal tax dynamics, i.e. no perfect tax smoothing. I also argue that distributions of heterogeneity and the government's redistributive motive are important in determining the optimal debt level that is sustainable in the long run.

This paper is related to recent research on the interaction between inequality and external debt. D'Erasmo and Mendoza (2016) focused on how the government's redistributive incentives affected defaults on domestic debt. They asserted that equilibrium with debt could be supported only when the government was politically biased towards bond holders. Ferriere (2015) showed how modifying tax progressiveness could mitigate the cost of default. Dovis, Golosov, and Shourideh (2016) argued how this interaction endogenized the dynamic cycles of fiscal policies over time. Balke and Ravn (2016) studied time-consistent fiscal policy in a sovereign debt model à la Eaton and Gersovitz (1981) with inequality through unemployment. This paper instead emphasizes endogenous debt constraints that are due to limited commitment. The results feature long-run binding debt constraints in which austerity policies might not be optimal if they generate more distortion. In this paper, I also characterize the optimal policies for a general distributional preference of the government.

Outline. The paper is organized as follows. Section 1 describes the environment, the competitive equilibrium, and the government's lack-of-commitment problem. Section 2 characterizes the equilibrium. Section 3 formulates the efficiency problem, while section 4 derives the main results of the optimal policies. Section 5 analyzes the effect of redistribution on optimal taxes. Section 6 provides the numerical exercise, explaining the properties of optimal policies and efficient allocation. Section 7 demonstrates the comparative statics with respect to changes in the skill dispersion, and Section 8 concludes. In the appendix, I extend the optimal taxation results to economies with aggregate uncertainty and with general separable preferences.

1 A Model of Redistribution and Limited Commitment

1.1 Environment

The model is a small open economy facing exogenous world interest rates $\{r_t^*\}_{t=0}^{\infty}$. Time is discrete. There is a measure-one continuum of infinitely-lived agents different by labor productivity types $(\theta^i)_{i\in I}$, which are publicly observable. The fraction of agents with productivity θ^i is π^i, where $(\pi^i)_{i\in I}$ is such that $\sum_{i\in I}\pi^i = 1$ and normalize $\sum_{i\in I}\pi^i\theta^i = 1$. All agents have the same discount factor β and the static utility $U(c,n)$ over consumption c and hours worked n. The utility function of the agent with productivity θ^i over consumption $c_t^i \geq 0$ and efficiency units of labor $l_t^i \geq 0$ is

$$\sum_{t=0}^{\infty} \beta^t U^i(c_t^i, l_t^i), \tag{1}$$

where $U^i(c,l) = U\left(c, \frac{l}{\theta^i}\right)$.

In addition, there is a representative firm that uses both capital and labor to produce a single output good. The production function $F(K,L)$ is constant return to scale, where K and L are respectively the aggregate capital and labor. Capital is depreciated at the δ rate each period, where $0 < \delta \leq 1$. The economy is subject to an exogenous sequence of government spending $\{G_t\}_{t=0}^{\infty}$. In each period, the government policies are issuances of domestic and international bonds, a lump-sum tax T_t, a marginal tax on labor income τ_t^n, a marginal tax on capital income τ_t^k, and the return on domestic bond r_t. I assume that only the government can borrow abroad.[2]

1.2 Competitive Equilibrium with Government Policies

The set of prices facing by the domestic agents and firm are the labor wage w_t and the return on capital r_t^k.

Domestic Agents. Agent of type $i \in I$ faces the sequential budget constraint in period t,

$$c_t^i + k_{t+1}^i + b_{t+1}^{d,i} \leq (1-\tau_t^n) w_t l_t^i + \left[1 + (1-\tau_t^k) r_t^k - \delta\right] k_t^i + (1+r_t)\, b_t^{d,i} - T_t, \tag{2}$$

where $c_t^i, l_t^i, k_t^i, b_t^{d,i}$ denote the consumption, efficiency-unit labor, capital holding, and domestic bond holding of agent i in period t, respectively.

Moreover, no arbitrage exists such that the after-tax return is the same when investing in capital or in domestic bonds

$$1 + (1-\tau_t^k) r_t^k - \delta = 1 + r_t$$

which implies $(1-\tau_t^k) r_t^k = r_t + \delta$.

Representative Firm. The firm chooses the amount of capital and labor to maximize profit each period:

$$\max_{\{K_t, N_t\}} \quad F(K_t, L_t) - w_t L_t - r_t^k K_t,$$

[2] This assumption is based on the empirical observation that domestic households often hold a very small amount of international assets. The assumption is also without loss of generality. This economy is equivalent to an economy where domestic agents can borrow abroad and the government sets residence-based taxes on the returns to assets of the domestic agents.

which gives the first-order conditions

$$
\begin{aligned}
r_t^k &= F_K(K_t, L_t) \\
w_t &= F_L(K_t, L_t)
\end{aligned}
\tag{3}
$$

The firm's profits are zero in equilibrium because of the constant return to scale production function.

Government. The government needs to finance an exogenous sequence of expenditures$\{G_t\}_{t=0}^{\infty}$. The government sells one-period bond B_t^d to domestic agents and B_{t+1} to the international lenders at a price Q_{t+1}. The government's budget constraint in each period is

$$G_t + (1 + r_t)B_t^d + B_t \leq \tau_t^n w_t L_t + \tau_t^k r_t^k K_t + B_{t+1}^d + Q_{t+1}B_{t+1} + T_t,$$

where $L_t = \sum_{i \in I} \pi^i l_t^i$ is the aggregate labor, $K_t = \sum_{i \in I} \pi^i k_t^i$ is the aggregate capital, $B_t^d = \sum_{i \in I} \pi^i b_t^{d,i}$ is the aggregate domestic bond, and B_t is the government's external debt. The government faces a no-Ponzi condition such that the present value of external debt is bounded below.

Define q_t as international price of a unit period-t consumption in terms of period-0 consumption units,

$$q_t = \Pi_{s=0}^{t} \frac{1}{1 + r_s^*} \tag{4}$$

The optimality of the risk-neutral international lenders' problem gives $Q_t = \frac{1}{1+r_t^*}$. Using $\{q_t\}_{t=0}^{\infty}$ as the relevant intertemporal price, one can write the government's present-value budget constraint as

$$\sum_{t=0}^{\infty} q_t \left\{ G_t - \tau_t^n w_t L_t - \tau_t^k r_t^k K_t + (1 + r_t)\, B_t^d - B_{t+1}^d - T_t \right\} \leq -B_0 \tag{5}$$

with normalizing $1 + r_0^* = 1$.[3]

Aggregate resource constraint. In a small open economy, markets do not have to clear in every period. However, using the agents' budget constraints and the government's budget constraint, one can obtain an aggregate resource constraint in terms of the intertemporal prices and the initial external debt

$$\sum_{t=0}^{\infty} q_t \left[F(K_t, L_t) + (1 - \delta)K_t - K_{t+1} - C_t - G_t \right] \geq B_0 \tag{6}$$

[3]This assumption is without loss of generality to fix the initial level of external debt.

This constraint implies that the initial level of external debt is weakly less than the present-value of net resources of the economy.

Competitive equilibrium. Given the above equations, one can define the following competitive equilibrium with government policies.

Definition 1.1. Given initial external debt B_0 and individual wealth positions $\left(a_0^i\right)_{i\in I}$,[4] a competitive equilibrium with government policies for an open economy is domestic agents' allocation $z^{H,i} = \left\{c_t^i, l_t^i, k_{t+1}^i, b_{t+1}^{i,d}\right\}_{t=0}^{\infty}$, $\forall i \in I$, the firm's allocation $z^F = \{K_t, L_t\}_{t=0}^{\infty}$, prices $p = \{q_t, Q_t, w_t, r_t, r_t^k\}_{t=0}^{\infty}$, and the government policy $z^G = \{\tau_t^n, \tau_t^k, T_t, r_t, B_{t+1}^d, B_{t+1}\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves agent i's problem that maximizes (1) subject to (2) and a no-Ponzi condition for the agent's debt value, (ii) given p and z^G, z^F solves firm's problem, which implies the first-order conditions (3), (iii) government budget constraint (5) holds, (iv) aggregate resource constraint (6) is satisfied, $\sum_{i\in I} b_t^{d,i} = B_t^d$, (iv) no arbitrage condition $(1-\tau_t^k)r_t^k = r_t + \delta$, and (v) p satisfies (4) given z^G.

1.3 Lack of Commitment

The government is benevolent in that its objective is the weighted discounted utility of all agents in the economy, given by a set of social welfare weights $\boldsymbol{\lambda} = \left(\lambda^i\right)_{i\in I}$.[5] The government enters into contracts with domestic agents and international creditors that specify the allocation of consumption, capital, labor, and domestic and international bonds. Nevertheless, in every period, the government can default on its external and domestic obligations, change the tax schedules, and expropriate all capital holdings. When deviating from the contracted allocation, the government faces punishment from the domestic agents and international lenders. The government then receives a deviation utility $\underline{U}_t(K_t)$ that depends on the current aggregate capital level that it can expropriate. The limited commitment implies that there exists a lower bound on future discounted aggregate utility. Specifically, for all $t \geq 0$,

$$\sum_{s=t}^{\infty} \beta^{s-t} \sum_{i\in I} \lambda^i \pi^i U^i\left(c_s^i, l_s^i\right) \geq \underline{U}_t(K_t) \tag{7}$$

Following Chari and Kehoe (1990, 1993), this limited commitment constraint characterizes the sub-game perfect equilibrium of a dynamic sovereign game between the government, domestic agents, and international creditors. Appendix C provides the details of the game and the equilibrium characterization. In general, the set of sustainable equilibrium payoffs of this sovereign game can be supported by trigger strategies to the equilibrium that has the

[4] $a_0^i \equiv \left[1 + (1-\tau_0^k)r_0^k - \delta\right] k_0^i + (1+r_0)\, b_0^{d,i}$

[5] Section 3 describes in details the relationship between the welfare weights and the utility frontier.

worst payoff. Due to the complication in characterizing the worst equilibrium of this dynamic game, this paper uses financial autarky as the punishment, in which there are no international and domestic financial markets, as the punishment for deviation. This assumption does not change the main results of the paper. The reverting-to-autarky equilibrium is characterized by the constraint (7) where $\underline{U}_t(K_t)$ incorporates the financial autarky value.[6] In each period, this constraint imposes an endogenous aggregate debt limit which depends on the government's current and future fiscal policies. Moreover, the debt limit represents the sustainable debt level of the economy in equilibrium.

2 Characterizing Equilibrium

In equilibrium, because all agents have the same preference, facing the same tax rates, earn the same wage on their efficient labor units, and have the same return on savings, the intratemporal and intertemporal conditions are the same across agents, i.e. in each period t, for all i,

$$\begin{aligned}(1-\tau_t^n)w_t &= -\frac{U_l^i(c_t^i,l_t^i)}{U_c^i(c_t^i,l_t^i)}\\ 1+r_{t+1} &= \frac{U_c^i(c_t^i,l_t^i)}{\beta U_c^i(c_{t+1}^i,l_{t+1}^i)}\end{aligned}$$

Given the aggregate allocation (C_t, L_t) in every period, there is an efficient assignment of individual allocation $(c_t^i, l_t^i)_{i\in I}$ due to the equal marginal rates of substitution between consumption and efficient labor. Moreover, because of the equal marginal rates of substitution of future to current consumption, the efficient assignment needs to be the same across time. Any inefficiencies due to tax distortions are captured by the aggregate allocation. Therefore, the competitive equilibrium allocation can be completely characterized in terms of aggregates and a static rule for individual allocation. Following Werning (2007), I first analyze the static distortion problem and then look at the dynamics in aggregate levels.

Sub-market analysis. For any equilibrium, there exist market weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i\in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i\varphi^i = 1$, such that individual assignments solve a static problem

$$\begin{aligned}V(C,L;\boldsymbol{\varphi}) \equiv \max_{(c^i,l^i)_{i\in I}} &\sum_{i\in I}\varphi^i\pi^i U^i(c^i,l^i)\\ s.t. \qquad &\sum_{i\in I}\pi^i c^i = C; \quad \sum_{i\in I}\pi^i l^i = L\end{aligned}$$

[6]Many papers have pointed out the problem in characterizing the worst equilibrium in this type of dynamic games as it might not be the repeated worst static Nash equilibrium. Instead, they made the same assumption of using autarky as the worst equilibrium (see Chari, and Kehoe (1993); Dovis, Golosov, and Shourideh (2016)).

The market weights capture how individual allocation is determined given any aggregate allocation. This problem gives the policy functions for each agent i:

$$h^i(C, L; \boldsymbol{\varphi}) = \left(h^{i,c}(C, L; \boldsymbol{\varphi}),\ h^{i,l}(C, L; \boldsymbol{\varphi})\right)$$

A competitive equilibrium allocation then must satisfy: $(c_t^i, l_t^i) = h^i(C_t, L_t; \boldsymbol{\varphi})$ for all i and t. The associate competitive equilibrium prices can be computed as if the economy were populated by a fictitious representative agent with utility function $V(C, L; \varphi)$. The envelope conditions of the static problem give

$$(1 - \tau_t^n) w_t = -\frac{V_L\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]}{V_C\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]} \tag{8}$$

$$1 + r_{t+1} = \frac{V_C\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]}{\beta V_C\left[h^i(C_{t+1}, L_{t+1}; \boldsymbol{\varphi})\right]} \tag{9}$$

Furthermore, the present-value budget constraint for individual i can be written as

$$\sum_{t=0}^{\infty} \beta^t \left[V_C(C_t, L_t; \boldsymbol{\varphi}) h^{i,c}(C_t, L_t; \boldsymbol{\varphi}) + V_L(C_t, L_t; \boldsymbol{\varphi}) h^{i,l}(C_t, L_t; \boldsymbol{\varphi})\right] = V_C(C_0, L_0; \boldsymbol{\varphi}) \left(a_0^i - T\right) \tag{10}$$

where T is the present-value of lump-sum taxes, and a_0^i is the individual initial after-tax wealth.[7] Equation (10) is the individual implementability constraint.

Now one has the following characterization proposition.

Proposition 2.1. *Given initial individual wealth $\left\{a_0^i\right\}_{i \in I}$ and external debt B_0, an allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ can be supported as an aggregate allocation of an open economy's competitive equilibrium with taxes if and only if aggregate resource constraint (6) holds, and there exist market weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i \in I}$ and lump-sum tax T such that the implementability constraint (10) holds for all $i \in I$.*

Proof. See Appendix D.

3 Efficiency

This section formulates the planning problem in terms of a Ramsey problem with the additional aggregate debt constraints due to limited commitment. It follows the primal approach in public finance to characterize the best equilibrium allocation and derive the optimal policies.

[7] $T \equiv \sum_{t=0}^{\infty} \beta^t \frac{V_C\left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]}{V_C\left[h^i(C_0, L_0; \boldsymbol{\varphi})\right]} T_t$ and $a_0^i \equiv \left[1 + (1 - \tau_0^k) r_0^k - \delta\right] k_0^i + (1 + r_0)\, b_0^{d,i}$

3.1 Planning Problem

The set of equilibrium with limited commitment can be supported as a competitive equilibrium with taxes and the limited commitment constraint (7). Define the set $\mathcal{U}$ of attainable utilities $\{u^i\}_{i\in I}$ such that $u^i = \sum_{t=0}^{\infty} \beta^t U^i \left(c_t^i, l_t^i\right)$ for any such equilibrium allocation. Given Proposition 2.1, $\{u^i\}_{i\in I}$ is the individual lifetime utilities for any allocation $\{C_t, L_t, K_t\}_{t=0}^{\infty}$ and a vector of market weights φ such that the aggregate resource constraint and the implementability constraint all $i \in I$ are satisfied. Specifically, $u^i = \sum_{t=0}^{\infty} \beta^t U^i \left[h^i(C_t, L_t; \boldsymbol{\varphi})\right]$. An efficient allocation is defined as one that reaches the northeastern frontier of $\mathcal{U}$, i.e. maximizing lifetime utility of one agent given that the utilities of other agents are above feasible thresholds. The necessary conditions can be derived by an alternative problem of maximizing a Pareto-weighted utility, where the Pareto weights are closely related to the feasible thresholds.[8]

Therefore, given the social welfare weights $\boldsymbol{\lambda} = \{\lambda^i\}_{i\in I}$ and exogenous international interest rates $\{r_t^*\}_{t=0}^{\infty}$, an efficient allocation maximizes the weighted utility subject to the aggregate resource constraint, the individual implementability constraints, and each-period sustainability constraint. The planning problem is

$$
\begin{aligned}
(P) \equiv \max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T} & \sum_{t=0}^{\infty} \beta^t \sum_{i\in I} \lambda^i \pi^i U^i \left[h^i(t; \boldsymbol{\varphi})\right] \\
s.t. \qquad & \sum_{t=0}^{\infty} q_t \left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 \geq 0 \\
\forall i, \ & \sum_{t=0}^{\infty} \beta^t \left[V_C(t; \boldsymbol{\varphi}) h^{i,c}(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) h^{i,l}(t; \boldsymbol{\varphi})\right] \geq V_C(0; \boldsymbol{\varphi}) \left(a_0^i - T\right) \\
\forall t, \ & \sum_{s=t}^{\infty} \sum_{i\in I} \beta^{s-t} \lambda^i \pi^i U^i \left[h^i(s; \boldsymbol{\varphi})\right] \geq \underline{U}_t(K_t)
\end{aligned}
$$

3.2 Characterizing Efficient Allocation

Let μ be the multiplier on the resource constraint, $\pi^i \eta^i$ be the multiplier on the implementability constraint for agent i, and $\beta^t \gamma_t$ be the multiplier on the aggregate debt constraint for period t. Define $\boldsymbol{\eta} = (\eta^i)_{i\in I}$ and rewrite the Lagrangian of the planning problem with a new pseudo-utility function that incorporates the implementability constraints

$$
\sum_{t=0}^{\infty} \beta^t W \left[t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] - V_C(0; \boldsymbol{\varphi}) \sum_{i\in I} \pi^i \eta^i \left(a_0^i - T\right),
$$

[8] As the set of attainable utilities $\mathcal{U}$ might not be convex, an allocation that solves (P) might not attain the utilities in $\mathcal{U}$. The analysis focuses on the necessary conditions, as they are enough to develop the properties of the optimal taxes. The set of optimal taxes is a subset of the set of taxes that implement any allocation satisfying the necessary conditions for efficiency. Therefore, the optimal taxes also satisfy the attributes of taxes deriving from the necessary analysis. Park (2014); Werning (2007) made the similar argument in their work.

where

$$W\left[t;\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\eta}\right] \equiv \sum_{i\in I}\lambda^i\pi^i U^i\left[h^i(t;\boldsymbol{\varphi})\right] + \sum_{i\in I}\pi^i\eta^i\left[V_C(t;\boldsymbol{\varphi})h^{i,c}(t;\boldsymbol{\varphi}) + V_L(t;\boldsymbol{\varphi})h^{i,l}(t;\boldsymbol{\varphi})\right]$$

The necessary conditions to characterize the set of efficient allocation are the first-order conditions of the planning problem, the aggregate resource constraint, the implementability constraints, and the aggregate debt constraints.

4 Optimal Taxation

This section provides the main optimal taxation results in the case of separable isoelastic preferences. The optimal taxes are derived such that the efficient allocation can be implemented as an allocation of a competitive equilibrium with government policies. Key assumptions throughout this section are the impatience of domestic agents with respect to the international intertemporal interest rates that the country faces when borrowing abroad, i.e.

Assumption 1 (Impatience)**.** *There exists* $M > 0$ *and* T *such that* $\forall t > T,\ \beta(1 + r^*_t) < M < 1$.

and following separable isoelastic utility:

Assumption 2 (Separable isoelastic preference)**.** *The utility function* $U:\ \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ *satisfies*

$$U(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \omega\frac{n^{1+\nu}}{1+\nu}$$

for $\sigma, \omega, \nu > 0$.

Individual consumption and efficient labor supply are time-independently proportional to the aggregates

$$\begin{aligned} c^i_t &= h^{i,c}(C_t, L_t; \boldsymbol{\varphi}) = \psi^i_c C_t \\ l^i_t &= h^{i,l}(C_t, L_t; \boldsymbol{\varphi}) = \psi^i_l L_t, \end{aligned} \tag{11}$$

where

$$\psi^i_c = \frac{\left(\varphi^i\right)^{1/\sigma}}{\sum_{i\in I}\pi^i(\varphi^i)^{1/\sigma}}, \quad \psi^i_l = \frac{(\theta^i)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}}{\sum_{i\in I}\pi^i(\theta^i)^{\frac{1+\nu}{\nu}}\left(\varphi^i\right)^{-1/\nu}} \tag{12}$$

$V,\ W$ inherit the separable and isoelastic properties from U, i.e. $\forall t$,

$$V(C_t, L_t; \boldsymbol{\varphi}) = \Phi_C^V \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L_t^{1+\nu}}{1+\nu}$$
$$W\left[C_t, L_t; \boldsymbol{\varphi}, \boldsymbol{\lambda}, \boldsymbol{\eta}\right] = \Phi_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^W \frac{L_t^{1+\nu}}{1+\nu}$$

and the planning objective is

$$\sum_{t=0}^{\infty} \beta^t \left(\Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} \right),$$

where Φ_C^V, Φ_L^V depend on $\boldsymbol{\varphi}$, Φ_C^W, Φ_L^W depend on $\boldsymbol{\varphi}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\eta}$, and Φ_C^P, Φ_L^P are functions of $\boldsymbol{\lambda}$ and $\boldsymbol{\varphi}$ (see Appendix D.1).

The first-order conditions of the planning problem for any period $t \geq 1$ can be summarized as

$$F_L(K_t, L_t) = \frac{\left\{\Phi_L^W + \Phi_L^P \sum_{s=0}^{t} \gamma_s\right\} L_t^{\nu}}{\left\{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t} \gamma_s\right\} C_t^{-\sigma}} \tag{13}$$

$$F_K(K_t, L_t) = r_t^* + \delta + \frac{\beta^t}{q_t} \frac{\gamma_t}{\mu} \underline{U}_t'(K_t) \tag{14}$$

and

$$C_t^{-\sigma} = \beta C_{t+1}^{-\sigma}(1 + r_{t+1}^*) \left[\frac{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t+1} \gamma_s}{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t} \gamma_s} \right] \tag{15}$$

To implement the efficient allocation as part of a competitive equilibrium with government policies, optimal taxes on labor, capital and saving return must satisfy

$$(1 - \tau_t^n) F_L(K_t, L_t) = \frac{\Phi_L^V L_t^{\nu}}{\Phi_C^V C_t^{-\sigma}} \tag{16}$$

$$(1 - \tau_t^k) F_K(K_t, L_t) = r_t + \delta \tag{17}$$

$$C_t^{-\sigma} = \beta \left(1 + r_{t+1}\right) C_{t+1}^{-\sigma} \tag{18}$$

4.1 Optimal Labor Tax

Dividing equation (13) by equation (16) gives

$$\tau_t^n = 1 - \frac{\Phi_L^V}{\Phi_C^V} \left[\frac{\Phi_C^W + \Phi_C^P \sum_{s=0}^{t} \gamma_s}{\Phi_L^W + \Phi_L^P \sum_{s=0}^{t} \gamma_s} \right], \tag{19}$$

which is the equation for optimal labor income tax. The time-variant component of the optimal tax is the sum of the Lagrange multipliers on the debt constraints, reflecting how limited borrowing influences the optimal tax dynamics. When the aggregate debt constraints never bind, i.e. $\gamma_s = 0, \ \forall s \leq t$, the tax on labor income becomes

$$\tau_t^n = 1 - \frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} \equiv \bar{\tau}^n \tag{20}$$

which is the optimal labor tax equation in Werning (2007). The optimal labor tax is constant when the debt constraint is not relevant. Intuitively, distortionary tax is a mechanism for redistribution. The distortion reflects the trade-off between the dispersion level, which is determined by the skill distribution, and the government's redistributive motive, which is the social welfare weights. Because the skill distribution and welfare weights do not change, and borrowing is unconstrained, the government finds it optimal keep the intratemporal distortion constant and borrow as needed to finance expenditure. The unconstrained optimal level is formulated by (20), which is a function of the skill distribution and the social welfare weights (see Appendix for the formulas of Φ's).

On the other hand, binding debt constraints limit the government's ability to borrow and to maintain constant labor tax distortions over time. If the debt constraint binds in the current period, the debt-constraint multiplier, which represents the shadow cost of borrowing, shows up in equation (19) and changes the dynamics of optimal labor taxes for all future periods. Intuitively, any change in the labor tax distortion at any period t affects the economy's output in period t and in turn affects the government's ability to borrow not only in the current period but also in all of the previous periods $0 \leq s \leq t$. Therefore, to relax the debt constraint in the current period, the government finds it optimal to permanently change all future taxes. In this way, the government is back-loading the cost of limited borrowing.

Debt constraints are binding in the long run because the aggregate debt increases over time as the domestic agents are impatient, the country's aggregate debt increases over time and the debt constraint eventually binds. In the environment without debt constraints, or when debt constraints never bind, the Ramsey allocation features immiseration in the long run such that the marginal utility of consumption is growing without bound. Such scenario happens when the deviation utility is unbounded below, i.e. the value of deviating is low enough such that the government will always commit to the contract. However, if the deviation utility is bounded below, the full-commitment Ramsey allocation cannot be supported. There is no immiseration in the long run as the future utility is always bounded below, and so debt constraints eventually bind.

As the debt constraints bind, the multiplier γ_t increases over time. In the long run, the cumulative sum of multipliers will diverge.[9] Given equation (19), it must be that $\lim_{t\to\infty} \tau_t^n =$

[9]Intuitively, optimal allocation must satisfies $\beta^t/q_t \left(\Phi_C^W + \Phi_C^P \sum_{s=0}^t \gamma_s\right) C_t^{-\sigma} = \mu$. No immiseration implies that $C_t^{-\sigma}$ is bounded below. Since $\mu > 0$, as $\beta^t/q_t \to 0$, it must be that $\sum_{s=0}^t \gamma_s \to \infty$

$1-\frac{\Phi_L^V\Phi_C^P}{\Phi_C^V\Phi_L^P}$, or by substituting in the definitions,

$$\lim_{t\to\infty}\tau_t^n = 1-\frac{\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_c^i}{\sum_{i\in I}\pi^i\frac{\lambda^i}{\varphi^i}\psi_l^i}, \tag{21}$$

which is a different level from the unconstrained distortion level. The optimal labor tax in limit depends on the distributional preference, $\{\lambda^i\}_{i\in I}$, and inequality, captured by $\{\varphi^i\}_{i\in I}$ (utility shares), $\{\psi_c^i\}_{i\in I}$ (consumption shares), and $\{\psi_l^i\}_{i\in I}$ (labor shares).

Formally, consider the following assumption on deviation utility,

Assumption 3. *$\underline{U}_t(\cdot)$ is bounded below, i.e. there exists a finite real M_U such that $\inf_{K_t}\underline{U}_t(K_t)\geq M_U$.*

Given this assumption, the consumption path is bounded below by zero in the long run, i.e.

Lemma 4.1 (No immiseration)**.** *Suppose assumptions 2 and 3 hold, then for any efficient allocation $\{C_t^*,L_t^*,K_t^*\}_{t=0}^{\infty}$, $\liminf_{t\to\infty}C_t^*>0$.*

Proof. See Appendix D.

The following proposition characterizes the optimal tax on labor income in an economy facing debt constraints and distributive concern.

Proposition 4.1 (Optimal labor tax)**.** *Given assumption 2, if an efficient allocation exists and debt constraint does not bind, there is constant labor tax given by (20). Moreover, if assumptions 1 and 3 also hold, and an interior efficient allocation exists, then the optimal labor tax converges to a real constant given by (21) that depends on skill distribution and the government's redistributive preference. These results hold with or without the lump-sum transfers.*

Proof. See Appendix D.

Both the government's redistributive motive and the limited borrowing determine the optimal level of tax distortion in the economy, expressed in equation (19). When borrowing is not constrained, it is optimal to set the marginal cost of distortion equal to the marginal benefit of redistribution, which is constant over time. Therefore, optimal tax rates do not change during the periods that debt constraints do not bind. As debt constraint binds, there is an additional benefit of relaxing the constraints. The marginal cost of distortion is equal to the net marginal benefit of redistribution and relaxing debt constraints. The following Proposition shows that this marginal cost of distortion decreases over time as the debt constraint binds, in an environment of only skill heterogeneity, high consumption-inequality aversion, measured as σ, and high inequality aversion.

Proposition 4.2. *Given assumptions 1–3, and additionally suppose that there are (i) equal initial wealth distribution: $a_0^i = a_0^j$, $\forall i, j \in I$, (iii) high consumption-inequality aversion: $\sigma \geq 1$, and (iv) inequality aversion: $\theta^i \geq \theta^j \iff \lambda^i \leq \lambda^j, \forall i, j \in I$, then for any period t such that the debt constraint binds, the optimal labor tax decreases, i.e. $\tau_t^n \leq \tau_{t-1}^n$. Moreover, $\tau_s^n \leq \tau_{t-1}^n$, $\forall s \geq t$.*

Proof. See Appendix D.

This Proposition points out that it requires a lower labor tax rate not only at the period the debt constraint binds, but also at any periods afterwards. The optimal labor tax drifts downward over time as the debt constraints bind, and given Proposition 4.1, it will eventually converge to a limit. As a result, the optimal labor tax before the debt constraint binds is at least as high as the optimal limit, i.e.

Corollary 4.1. *Given the assumptions of Proposition 4.2, $\bar{\tau}^n \geq \lim_{t\to\infty} \tau_t^n$.*

Intuitively, a government that has a high inequality aversion would like to keep a high labor tax to redistribute. However, labor taxes weakly decrease whenever the debt constraints bind. A lower tax rate, in return, encourages more labor supply and output and relaxes the debt constraints by increasing the government's borrowing capacity.

In general, this efficiency motive is a driver for the dynamic in the optimal tax rates. Consider the following expenditure minimization problem for each period t

$$
\begin{aligned}
(EM_t) \equiv \min_{C_s, L_s, K_{s+1}} & \sum_{s=t}^{\infty} q_s \left[C_s + G_s + K_{s+1} - F(K_s, L_s) - (1-\delta)K_s\right] \\
s.t. \quad & \sum_{s=t}^{\infty} \beta^{s-t} \left(\Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s^{1+\nu}}{1+\nu} \right) = \underline{U}_t(K_t)
\end{aligned}
$$

which is the problem of minimizing the present value of resourced needed for the planner to deliver $\underline{U}_t(K_t)$ as the promised utility at period t. The solution to this minimization problem can be implemented with the labor tax $\tau_s^n = 1 - \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$ for $s \geq t$. Note that this equation is exactly the labor tax limit. The optimal labor tax formula (19) incorporates the solution to the Ramsey planner when debt is unconstrained (Φ^W's), and as debt constraint binds, part of the solution to this expenditure minimization problem shows up (Φ^P's) with respect to the tightness of the constraints ($\sum_{s\leq t} \gamma_s$). When there is unlimited borrowing, the planner should set the tax rate to achieve the most redistributive allocation, which is the allocation that the planner would choose if she never runs into the debt constraints. However, when debt constraint binds, the planner wants the international lenders to continue the contract by offering an allocation such that it delivers the promised utility $\underline{U}_t(K_t)$ in a less costly way. As the debt constraint binds in the long run, the optimal allocation continues to lower the delivering cost and eventually reaches the allocation with minimal cost, which is the solution to (EM_∞). The labor tax starts off with the most redistributive level for the country, which is $1-\frac{\Phi_L^V\Phi_C^W}{\Phi_C^V\Phi_L^W}$ and gradually converges to the most efficient level, i.e. $1-\frac{\Phi_L^V\Phi_C^P}{\Phi_C^V\Phi_L^P}$.

In the case without lump-sum transfers, besides the redistributive benefit, the marginal tax rate is also set to meet the budgetary needs of the government. This additional need only changes the optimal tax levels, but not the dynamics. The limited borrowing still requires the distortion to be front-loaded.

4.2 Optimal Capital and Domestic Saving Taxes

Combining equations (15) and (18), the optimal domestic return satisfies

$$1+r_t=(1+r_t^*)\frac{\Phi_C^W+\Phi_C^P\sum_{s=0}^{t}\gamma_s}{\Phi_C^W+\Phi_C^P\sum_{s=0}^{t-1}\gamma_s} \tag{22}$$

Note that when the sustainability constraint is not relevant, i.e. $\gamma_s = 0\ \forall s \leq t$, it is optimal to set the domestic interest rates equal to the exogenous international interest rates. However, as the sustainability constraints start binding, equation (22) implies that $r_t > r_t^*$, which implies a saving subsidy. As the economy reaches its debt limits, shown as the binding sustainability constraints, the government has incentive to subsidize more on saving, or tax more on borrowing, to discourage domestic agents from accumulating debt.

Since capital is a form of domestic savings, capital investment is also subject to the domestic saving tax. To disentangle the intratemporal and the intertemporal incentives in taxing capital, define $\hat{\tau}_t^k$ that satisfies

$$(1-\tau_t^k)F_K(t)+1-\delta=\frac{1+r_t}{1+r_t^*}\left[(1-\hat{\tau}_t^k)F_K(t)+1-\delta\right],\ \forall t$$

where $\frac{1+r}{1+r^*}$ represents the effective intertemporal return induced by domestic saving tax. The first-order conditions (14) and (17), without loss of generality, imply an implementation

$$\hat{\tau}_t^k=\frac{\frac{\beta^t}{q_t}\gamma_t\underline{U}_t'(K_t)}{F_K(t)} \tag{23}$$

Assume that $\underline{U}_t'$ is positive.[10] Then the above equation implies that there is a higher tax on capital income when debt constraints bind. A greater capital tax reflects that there is capital under-investment of the efficient allocation. Indeed, the first-order condition (14) shows that $F_K(K_t, L_t) > r_t^* + \delta$ when $\gamma_t > 0$, where $r_t^* + \delta$ is the first-best interest rate. Because the government can expropriate more capital and receive higher utility from reneging, the optimal

[10] The higher the amount of capital the government can expropriate, the higher the deviation utility. Proposition C.1 proves a case when it is true.

contract discourages capital accumulation, which can be implemented by imposing more tax on capital income.

Define a tax on domestic savings τ_t^d as $1 - r_t/r_t^*$. Since the debt constraints bind in the long run, the optimal domestic saving and capital taxes are negative and positive, respectively, in the limit:

Proposition 4.3. *Given assumptions 1–3, and* $\underline{U}_t' > 0$*, if the interior efficient allocation exists, then* $\lim_{t\to\infty} \tau_t^d < 0$ *and* $\lim_{t\to\infty} \hat{\tau}_t^k > 0$

Proof. The proof follows directly from the above discussion. □

5 Effect of Redistribution

This section analyzes how the government's need for redistribution affects the optimal labor tax. Redistribution influences not only the tax levels from the trade-off between equity and efficiency, but also the tax dynamics from the interaction with the debt constraints. Suppose that there is no heterogeneity: then the problem becomes the standard representative Ramsey problem in a small open economy. If the government can use lump-sum taxes, there is no need for distortion, and the optimal labor tax will be zero. If the government can only impose distortionary taxes, the model collapses to the representative setting as in Aguiar and Amador (2016), where the zero labor tax in the limit is optimal.

Proposition 5.1 (No heterogeneity)**.** *Suppose that* $\theta^i = \theta^j, a_0^i = a_0^j, \ \forall i,j \in I$*. Then there is zero labor tax in the long run. This result holds with or without lump-sum transfers.*

Proof. Follow from equation (21) with $\varphi^i = \psi_c^i = \psi_l^i = 1, \ \forall i \in I$. □

While the market weights determine how the competitive market chooses individual shares of utility, the Pareto weights regulate the social shares of utility. Any agent has an exogenous Pareto weight that is based on the government's distributional preference and a market weight that depends on her relative skill and initial wealth. An interesting case is when the vector of market weights is equal to the vector of Pareto weights ($\boldsymbol{\psi} = \boldsymbol{\lambda}$). This implies that there is no distributive effect, because the government, as a planner, distributes aggregate utility exactly the same way as the competitive market does. In this situation, (21) turns out to be $\lim_{t\to\infty} \tau_t^n = 0$, that is, the optimal labor tax converges to zero. These results are summarized in the following Proposition.

Proposition 5.2. *There exists an efficient allocation* $\left\{C_t^*, L_t^*, K_{t+1}^*\right\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ *such that for all* i*,* $\lambda^i = \varphi^{*i}$*. Such an allocation can be implemented with a zero labor tax in the long run.*

Proof. See Appendix D.

Generally, changing the government's distributional preference affects the social utility, and thus the debt constraints.[11] Proposition 5.2 shows that the distributional preference also influences the optimal tax levels, yet only when it can deviate from the distribution rising from the competitive equilibrium markets. This is the special case in which the social welfare weights are equal to the inverse of marginal utilities. Therefore, the efficient allocation is the non-distorted competitive equilibrium allocation. The government's redistributive motive, not heterogeneity, is the source of differences in optimal policies comparing to the representative agent setting.

Another way to interpret the effect of redistribution is to rewrite the tax formulas as

$$\bar{\tau}^n = 1 - \frac{\tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)} \tag{24}$$

$$\lim_{t\to\infty} \tau^n = 1 - \frac{\tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] + \tilde{\text{cov}}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\tilde{\mathbb{E}}\left[\frac{\lambda^i}{\varphi^i}\right] + \tilde{\text{cov}}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)} \tag{25}$$

using the definitions $\tilde{\mathbb{E}}\left[x^i\right] \equiv \sum_i \pi^i x^i$, $\tilde{cov}(x^i, y^i) \equiv \tilde{\mathbb{E}}\left[x^i y^i\right] - \tilde{\mathbb{E}}\left[x^i\right]\tilde{\mathbb{E}}\left[y^i\right]$.

The optimal tax levels depend on the co-variance between fractions of individual allocation to aggregates $\left(\psi_c^i, \psi_l^i\right)_{i\in I}$ and the relative distribution ratios $\left(\lambda^i/\varphi^i\right)_{i\in I}$. The relevant redistribution component is the ratio between Pareto and Negishi weights λ/φ. Changes in the redistribution preference or skill distribution affect $\boldsymbol{\varphi}$, $\boldsymbol{\psi_c}$, and $\boldsymbol{\psi}_l$, which in turn affectcov $\left(\psi_c^i, \lambda^i/\varphi^i\right)$ and cov $\left(\psi_l^i, \lambda^i/\varphi^i\right)$ and the optimal tax levels.

6 Numerical Analysis

This section illustrates the theoretical results by a numerical exercise. There is no capital. Assume that production is linear in efficiency-unit labor, i.e. $F(L_t) = L_t$. The deviation utility is a constant finite $\underline{U}$ so that it is consistent with Assumption 3.

Lemma 6.1. *If the sustainability constraint binds for some finite S , then it will bind for all $t > S$.*

Proof. See Appendix D.

It must be true that the socially weighted utility, $\sum_{i\in I} \lambda^i \pi^i U^i[h^i(C_t, L_t; \boldsymbol{\varphi})]$ is equal to $\underline{U}$ across all period $t > S$ that the debt constraint binds. Combining this feature and the planner's first-order conditions solves the efficient allocation at each period after the constraints bind: then the allocation at period $t < S$ can be derived by solving backwards.

[11]The welfare weights determine the discounted future utility and the deviation utility, derived in the Appendix, so they influence both sides of the sustainability constraints.

Consider a parametric economy consisting of two types of agents, denoted $I = \{H, L\}$, where $\theta^H \geq 1 \geq \theta^L$. Let $\pi^H = \pi^L = 0.5$, which implies that $\theta^H = 2 - \theta^L$. The two types have zero initial wealth positions, i.e. $a_0^i = 0, \ \forall i$. Let $U(c, n) = \log c - \omega \frac{n^{1+\nu}}{1+\nu}$, $\omega = 1, \ \nu = 2$ so that the Frisch elasticity of labor supply is 0.5. Assign $\beta = 0.94$ and $r_t^* = r^* = 0.05$ so that $\beta(1 + r^*) < 1$. The planner is utilitarian: $\lambda^H = \lambda^L = 1$. The government expenditure is constant across time and set to be 20 percent of the average productivity. The economy starts with an initial external asset position of 20%. $\underline{U}$ is the value of autarky, which is calculated as the maximal utility attained from a tax-distorted competitive equilibrium of the economy with no domestic and international credit markets.

6.1 Dynamics of Optimal Policies and Efficient Allocation

Figure 1 depicts the time paths of optimal policies and the efficient allocation when the relative skill dispersion is such that $\theta^H = 3\theta^L$. Figure 2 expands the time periods to show the long-run properties.

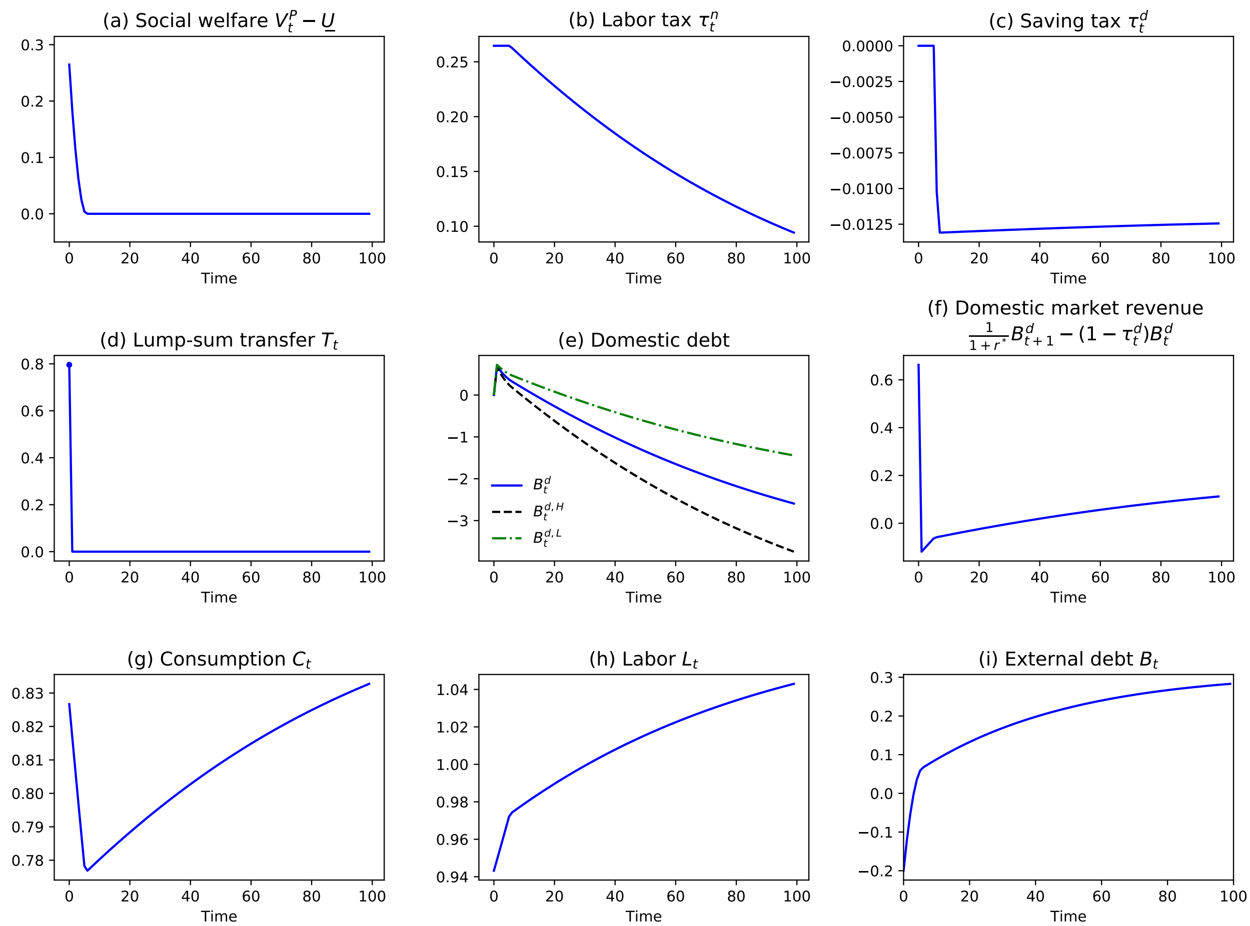
Time is the horizontal axis. Panel (a), (b), and (c) plot the planner's utility relative to the deviation utility, the labor tax, and the domestic saving tax, respectively. The planner's utility decreases over time until it reaches the deviation utility and stays constant, as the debt constraint binds. When the debt constraint does not bind, the labor tax starts at a positive level and remains constant, while the saving tax is zero. As the debt constraint starts binding, labor tax decreases while saving is subsidized, implying a positive tax on borrowing. Figure 2 shows that in the long run, there is labor subsidy and borrowing tax.

Due to Ricardian equivalence, I consider a particular implementation of the efficient allocation where the planner gives the present-value lump-sum transfer only in period 0. Panel (d) depicts the lump-sum transfer over time. Given this implementation, panel (e) plots the aggregate and individual domestic debt levels. Because of impatience, domestic agents borrow over time, and the planner acts as an intermediary that borrows abroad and lends to domestic agents. However, when debt constraints bind, the planner levies taxes on borrowing. In net, the planner collects revenue from the domestic market, as described in panel (f).

Panel (g) and (h) plot the dynamics of the aggregate consumption and labor. When debt constraints do not bind, there is front-loading consumption and leisure. When debt constraints bind, given the positive tax on borrowing and the decreasing labor tax, aggregate consumption and labor increase over time. Panel (f) shows the path of external debt B_t. The economy accumulates external debt quickly in the beginning of time. However, when the debt constraint hits, there is a slower accumulation of debt that eventually reaches its steady state, which is the maximum debt capacity of the economy.

One important point from panel (e) is that the higher-income agents borrow more over time.

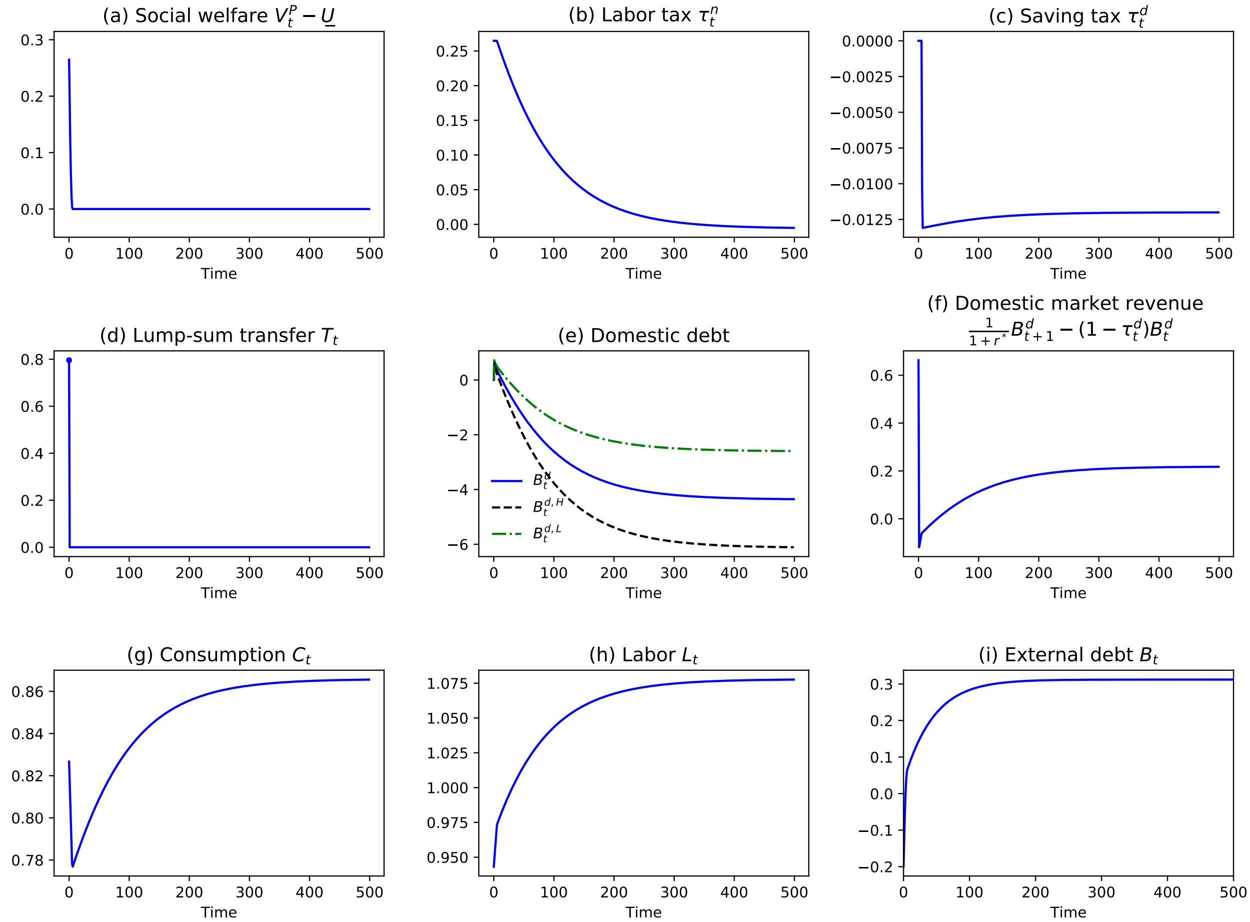
Figure 1: Time paths of economic aggregates



Note: This figure plots the time paths of the social welfare, aggregate variables, and optimal policies from the model's simulation for 100 periods in the case of no capital and $\theta^H = 3\theta^L$.

This is because all agents borrow at similar fractions of their income over time. Therefore, when the planner uses the marginal tax on borrowing, she also redistributes more resources towards the lower-income agents, as the higher-income agents pay higher taxes on borrowing. When there is no cost of borrowing, the planner uses the labor distortion to redistribute. The high initial labor tax rate reflects the government's redistributive motive, and that the marginal benefit of redistribution is high. When hitting debt constraints, the planner can use taxes on borrowing to redistribute. This allows a lower labor distortion until subsidy in the long run. In this case, the high-skilled agent is more productive than the average productivity. A lower labor tax encourages her to produce more output, which increases the economy's ability to repay.

Figure 2: Time paths of economic aggregates in the long run



Note: This figure plots the time paths of the social welfare, aggregate variables, and optimal policies from the model's simulation for 500 periods in the case of no capital and $\theta^H = 3\theta^L$.

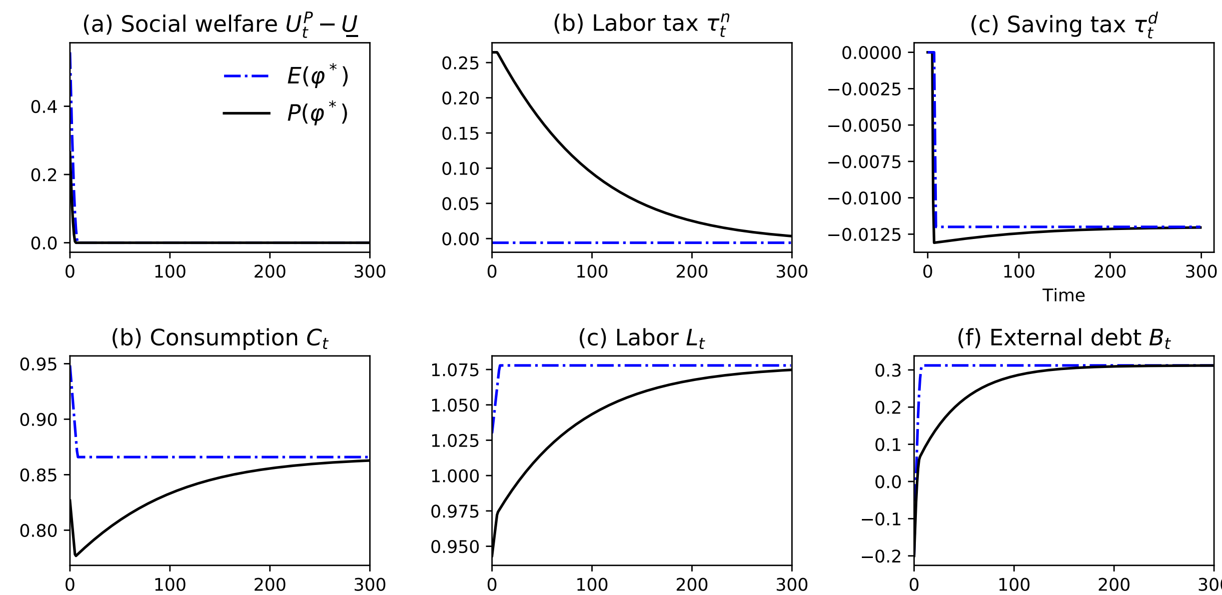
6.2 Optimal Properties and Debt Constraints

Why do allocation, taxes, and debt change even when debt constraint binds? To answer this question, consider the following relaxed planning problem that does not incur any distortionary cost of redistribution but is subject to delivering the same distribution of individual outcomes, which is fixing the optimal φ^*,

$$
\begin{aligned}
E(\varphi^*) \equiv \max_{\{C_t, L_t\}_t} & \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i U^i \left[h^i(t; \varphi^*)\right] \\
s.t. \quad & \sum_{t=0}^{\infty} q_t \left[L_t + -C_t - G_t\right] - B_0 \geq 0 \\
& \forall t, \ \sum_{s=t}^{\infty} \sum_{i \in I} \beta^{s-t} \lambda^i \pi^i U^i \left[h^i(s; \varphi^*)\right] \geq \underline{U}_t
\end{aligned}
$$

Figure 3 compares the dynamics of aggregates between the benchmark problem $P(\varphi^*)$ and the relaxed problem $E(\varphi^*)$. $E(\varphi^*)$ delivers a higher ex-ante welfare (panel (a)). It takes longer for $E(\varphi^*)$ to reach the debt constraint, but when the debt constraint binds, consumption, labor, taxes, and external debt stay constant. The implemented labor tax for the allocation of $E(\varphi^*)$ is constant at a negative level. The solution of $P(\varphi^*)$ converges to the solution of $E(\varphi^*)$ in the long run. $E(\varphi^*)$ is the most efficient way to deliver φ^*, while $P(\varphi^*)$ is the best way to deliver φ^* taking into account the distortionary cost of redistribution. The solution to $P(\varphi^*)$ increases its efficiency every time the debt constraint binds and eventually reaches the most efficient outcome.

Figure 3: Time paths economic aggregates in the benchmark and relaxed problems



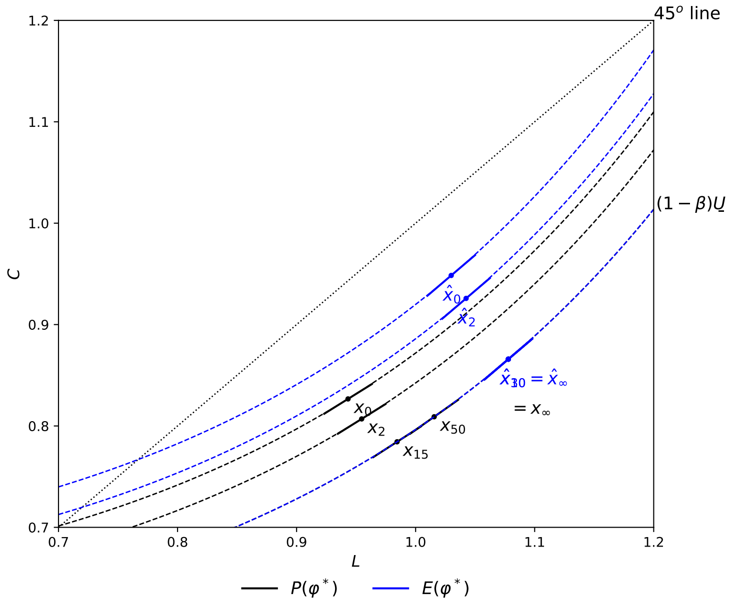
Note: This figure plots the time paths of the social welfare, aggregate variables, and optimal policies from the model simulations for 300 periods for the benchmark planning problem $P(\varphi^*)$ and the relaxed planning problem $E(\varphi^*)$.

Figure 4 provides the trade-off path between aggregate consumption and aggregate labor of the benchmark and relaxed allocation over time. The dash curves represent the intra-period indifference curve of the planning utility with respect to the aggregate consumption and labor. The benchmark and relaxed allocation for each period t are indexed by x_t and $\hat{x}_t$, respectively. The slope of each associated line is the marginal rate of substitution between consumption and labor at period t.[12] For the periods that the debt constraints do not bind, the marginal rate of substitution remains constant, as the benchmark allocation drifts down the utility indifference curve, decreasing consumption and increasing labor. The decline in consumption and leisure reflects the impatience of domestic agents, while the constant marginal rate of substitution

[12]The slope of the planning utility indifference curve is $\left.\frac{\Phi_C^P}{\Phi_L^P}\frac{C_t^{-\sigma}}{L_t^{\nu}}\right|_{u_t}$

comes from the tax smoothing argument. When the debt constraint starts binding, as illustrated before, it is not sustainable to stay at the same allocation. Therefore, the allocation moves along the autarkic utility indifference curve.

Figure 4: The consumption-labor trade-offs over time in the benchmark and relaxed planning problems



Note: This figure plots the trade-offs between aggregate consumption C and aggregate labor L over time for the benchmark planning problem $P(\varphi^*)$ and the relaxed planning problem $E(\varphi^*)$.

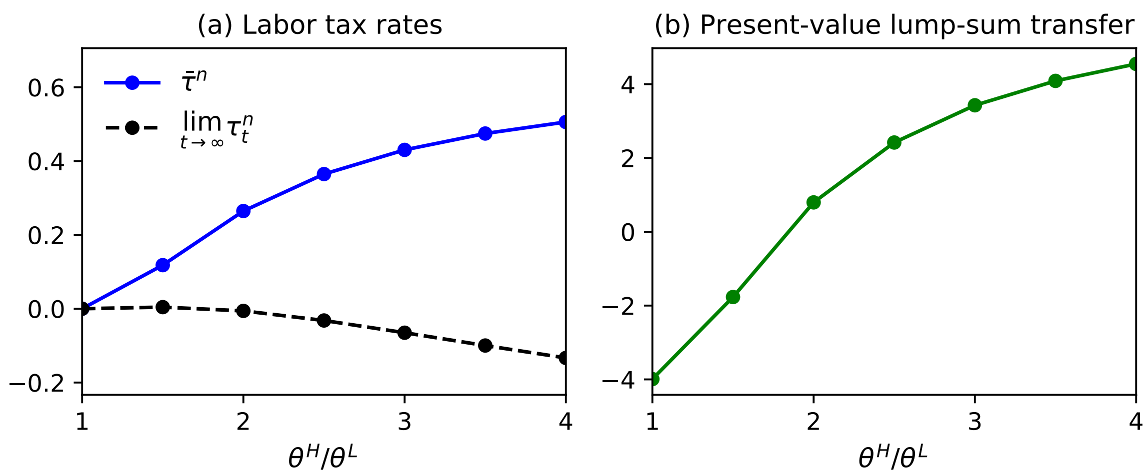
The benchmark allocation moves up along the flow autarkic utility indifference curve. In the benchmark planning problem, the marginal rate of substitution between consumption and labor starts at a low level, as the slope of the indifference curve at x_0 is less than one. The argument is that it is always better for the planner to redistribute by distorting intratemporal decisions instead of intertemporal decisions. The marginal rate of substitution is then lower than one because of the distortionary cost of redistribution. On the other hand, the relaxed allocation does not have to take into account this distortionary cost, so its allocation ($\hat{x}_t$) always has a slope of one, in which the slope of the indifference curve equals the slope of the aggregate resource constraint. Tax smoothing implies that at the end of the periods that the debt constraints do not bind, the benchmark allocation's marginal rate of substitution has not changed and is less than one. Given the same promised utility, at the moment the debt constraint binds, suppose that the planner decreases one unit of labor, then she can only decrease consumption by less than one unit, implying that the planner will need to take more debt. If the planner instead increases one unit of labor, she will only need to increase consumption by less than one unit. Then the planner can gain additional resources to pay back the existing debt. As a result, the benchmark allocation moves up along the indifference curve, until it reaches the most efficient allocation with a slope of one.[13]

7 Comparative Statics: Skill Dispersion

In this section, I demonstrate how optimal labor taxes, lump-sum transfers, and external debt change with respect to changes in the skill dispersion. I find that higher skill dispersion, which represents a stronger redistributive motive of the government, requires greater labor tax distortions and lump-sum transfers initially yet lower labor tax distortions and higher external debt level in the long run.

Figure 5 illustrates the changes with respect to the relative skill dispersion θ^H/θ^L of the optimal labor and lump-sum taxes. Panel (a) plots the labor tax rate in periods where borrowing is unconstrained ($\bar{\tau}^n$) and at the limit ($\lim_{t\to\infty}\tau_t^n$), for the utilitarian planner. Panel (b) depicts the present-value of lump-sum tax. When there is no heterogeneity ($\theta^H = \theta^L$), the problem collapses to a Ramsey's problem of a representative-agent small open economy. Due to the presence of the lump-sum tax, it is optimal to have zero labor distortion in all periods. As the skill dispersion increases, the government increases its distributional preference towards low income agents. While debt constraints do not bind, it is optimal to levy higher tax rates. Therefore, as shown in panel (a),$\bar{\tau}^n$ increases with θ^H/θ^L.

Figure 5: Optimal labor taxes and lump-sum transfers by relative skill dispersion



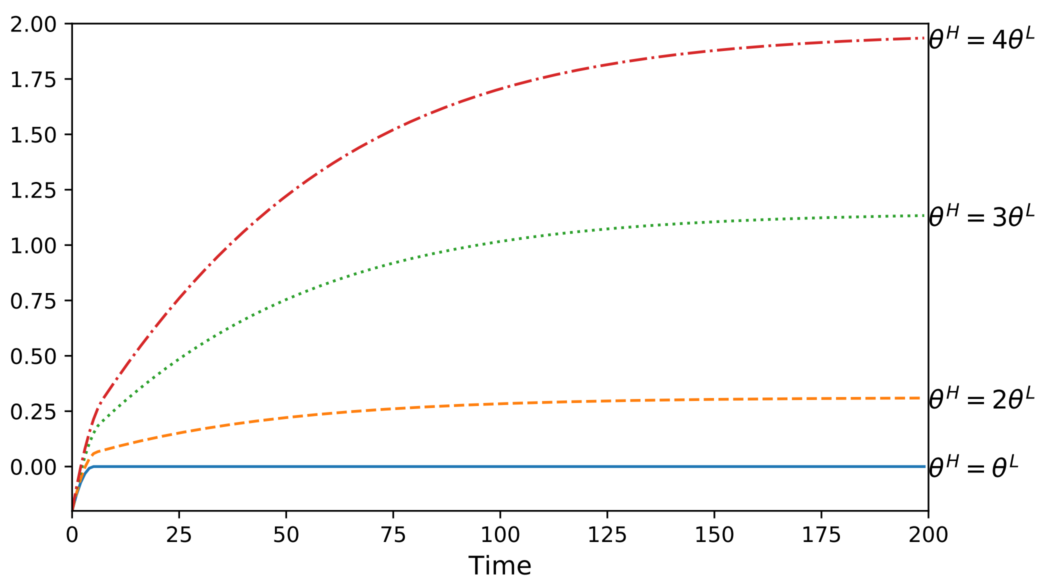
Note: This figure plots the unconstrained optimal labor tax $\bar{\tau}^n$, the optimal labor tax in the limit $\lim_{t\to\infty}\tau_t^n$, and the present-value lump-sum transfers $-T$ for different levels of relative skill dispersion θ^H/θ^L.

[13]In this example, one can show that $\frac{\Phi_C^W}{\Phi_L^W} > \frac{\Phi_C^P}{\Phi_L^P}$, which implies that $\frac{\Phi_C^P C_t^{-\sigma}}{\Phi_L^P L_t^{\nu}} < \frac{\Phi_C^W C_t^{-\sigma}}{\Phi_L^W L_t^{\nu}} = 1$ for any period t such that debt constraints have not binded before. In the long run, $\lim_{t\to\infty}\frac{\Phi_C^P C_t^{-\sigma}}{\Phi_L^P L_t^{\nu}} = 1$.

On the other hand, setting high tax rates during the not-binding-debt-constraint periods means a high cost of distortion when the economy first reaches the debt constraints. At a higher level of skill dispersion, the government wants to redistribute by increasing the marginal tax rates on labor income. Therefore, the marginal rate of substitution between consumption and labor in the planning utility starts at a lower level than before, remains constant during the no-binding periods, and gradually increases to one as the debt constraints bind (as shown in Figure 4). In the long-run, panel (a) shows that $\lim_{t\to\infty} \tau_t^n$ becomes negative and declines with respect to skill dispersion. A higher skill dispersion implies that the highly productive agent becomes more productive. Increasing the subsidy in labor in the long run can encourage a greater output to sustain the high level of debt. Although labor is subsidized in the long run, the government still achieves its redistributive purpose by combining the initial high tax rates and positive lump-sum transfer. Indeed, panel (b) shows that the lump-sum transfer increases with respect to the relative skill dispersion.

Figure 6 presents the dynamics of the government's external debt B_t with respect to skill dispersion. While all economies start with the same initial external debt position, an economy with higher skill dispersion accumulates higher debt over time. A highly-dispersed economy wants to redistribute more by levying a higher labor tax rate during the periods that the debt constraints have not bound. The higher tax rate means that there is lower output, which is compensated by more borrowing.

Figure 6: Time paths of external debt by relative skill dispersion



Note: This figure plots the time paths of external debt B_t from the model simulations for 300 periods for different levels of relative skill dispersion θ^H/θ^L.

The higher debt capacity of the economy corresponds to the need to stabilize the higher debt level that the economy accumulates beforehand because of a higher redistributive motive. In addition, a higher skill dispersion is associated with a longer time of unconstrained borrowing. Since a highly-dispersed economy has more redistributive motive, it is more costly to redistribute during financial autarky. Therefore, it is optimal to prolong the periods that the debt constraint does not bind, in which the government can redistribute the most.

8 Conclusion

This paper analyzes optimal taxation for a small open economy with the government's redistributive motive and endogenous debt constraints. Impatient agents borrow over time, which makes the debt constraints relevant in the long run. Optimal labor taxes feature constant rates when borrowing is unconstrained, yet later a gradual convergence to non-zero values in the limit that are associated with the economy's aggregate debt limit. As the debt constraints bind, it is optimal to increase the taxes on capital and domestic borrowing.

The government's redistributive motive significantly changes the implication for the optimal fiscal policies. Specifically, it alters the long-run limit of taxes through interacting with the heterogeneity. It also changes both the inside and outside values of the contract, which indirectly determines the debt limits. Any country's optimal levels of taxes and debt issuance crucially depends on its labor productivity distribution as well as its social distribution preference. On the other hand, the debt constraints limit a government's ability to redistribute, in which the optimal policies decrease the redistribution and increase the efficiency whenever the debt constraints bind.

The paper also provides a mechanism explaining the relationship between sovereign debt accumulation and redistribution. A government with high redistributive motive wants to set a high tax rates and accumulate a high level of debt. When the debt constraints bind, by lowering the tax rates and possibly subsidizing labor in the long run, the government can sustain this high debt position.

In the model, the main source of income heterogeneity comes from the skill dispersion. Other sources of heterogeneity, such as capital income and wealth, are worth exploring in future research. In addition, enriching the tax system to non-linear taxes can help study the optimal tax progressiveness in the presence of sovereign debt.

References

Aguiar, Mark and Gita Gopinath, "Defaultable debt, interest rates and the current account," *Journal of International Economics*, 2006, *69* (1), 64–83.

__ **and Manuel Amador**, "Growth in the Shadow of Expropriation," *Quarterly Journal of Economics*, 2011, *126* (2), 651–697.

__ **and** __ , "Sovereign Debt," *Handbook of International Economics*, 2014, *4*, 647–87.

__ **and** __ , "Fiscal policy in debt constrained economies," *Journal of Economic Theory*, 2016, *161*, 37–75.

__ **,** __ **, and Gita Gopinath**, "Investment cycles and sovereign debt overhang," *Review of Economic Studies*, 2009, *76* (1), 1–31.

Aiyagari, S. Rao, Albert Marcet, Thomas J. Sargent, and Juha Seppälä, "Optimal taxation without state-contingent debt," *Journal of Political Economy*, 2002, *110* (6), 1220–1254.

__ **and Ellen R. McGrattan**, "The optimum quantity of debt," *Journal of Monetary Economics*, 1998, *42* (3), 447–469.

Aizenman, Joshua and Yothin Jinjarak, "Income inequality, tax base and sovereign spreads," *FinanzArchiv: Public Finance Analysis*, 2012, *68* (4), 431–444.

Arellano, Cristina, "Default risk and income fluctuations in emerging economies," *American Economic Review*, 2008, *98* (3), 690–712.

__ **and Yan Bai**, "Fiscal austerity during debt crises," *Economic Theory*, 2016, pp. 1–17.

Balke, Neele and Morten O. Ravn, "Time-Consistent Fiscal Policy in a Debt Crisis," 2016.

Barro, Robert J., "On the determination of the public debt," *Journal of Political Economy*, 1979, *87* (5), 940–71.

Berg, Andrew and Jeffrey Sachs, "The debt crisis structural explanations of country performance," *Journal of Development Economics*, 1988, *29* (3), 271–306.

Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J Sargent, "Public debt in economies with heterogeneous agents," *Journal of Monetary Economics*, 2017, *91*, 39–51.

Chamley, Christophe, "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 1986, *54* (3), 607–622.

Chari, Varadarajan V. and Patrick J. Kehoe, “Sustainable plans,” *Journal of Political Economy*, 1990, pp. 783–802.

— **and** — , “Sustainable plans and debt,” *Journal of Economic Theory*, 1993, *61* (2), 230–261.

— **and** — , “Optimal fiscal and monetary policy,” *Handbook of Macroeconomics*, 1999, *1*, 1671–1745.

— **, Lawrence J. Christiano, and Patrick J. Kehoe**, “Optimal fiscal policy in a business cycle model,” *Journal of Political Economy*, 1994, *102* (4), 617–652.

D’Erasmo, Pablo and Enrique G. Mendoza, “Distributional incentives in an equilibrium model of domestic sovereign default,” *Journal of the European Economic Association*, 2016, *14* (1), 7–44.

Dovis, Alessandro, Mikhail Golosov, and Ali Shourideh, “Political Economy of Sovereign Debt: A Theory of Cycles of Populism and Austerity,” 2016.

Eaton, Jonathan and Mark Gersovitz, “Debt with potential repudiation: Theoretical and empirical analysis,” *Review of Economic Studies*, 1981, *48* (2), 289–309.

Ferriere, Axelle, “Sovereign Default, Inequality, and Progressive Taxation,” 2015.

Jeon, Kiyoung and Zeynep Kabukcuoglu, “Income Inequality and Sovereign Default,” *Journal of Economic Dynamics and Control*, 2018, *95* (C), 211–232.

Judd, Kenneth L., “Redistributive taxation in a simple perfect foresight model,” *Journal of Public Economics*, 1985, *28* (1), 59–83.

Lucas, Robert E. and Nancy L. Stokey, “Optimal fiscal and monetary policy in an economy without capital,” *Journal of Monetary Economics*, 1983, *12* (1), 55–93.

Luenberger, David G., *Optimization by vector space methods*, John Wiley & Sons, 1969.

Park, Yena, “Optimal Taxation in a Limited Commitment Economy,” *Review of Economic Studies*, 2014, *81* (2), 884–918.

Straub, Ludwig and Iván Werning, “Positive Long-Run Capital Taxation: Chamley-Judd Revisited,” *American Economic Review*, January 2020, *110* (1), 86–119.

Werning, Iván, “Optimal Fiscal Policy with Redistribution,” *Quarterly Journal of Economics*, 08 2007, *122* (3), 925–967.

A Economy with Aggregate Uncertainty

This section extends optimal taxation results in a model with aggregate shocks in terms of the aggregate productivity and government expenditures. I find that optimal taxes on labor, capital, and domestic saving have the same properties as the benchmark model.

Denote the aggregate shock to be $s_t \in S$ in period t, where S is some finite set. Let $\Pr(s^t)$ denote the probability of any history $s^t = (s_0, s_1, ..., s^t)$. The preference of the domestic agents is

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 U^i(c_t^i, l_t^i) \tag{A.1}$$

The production function in period t with history s^t is $F(K, L, s^t, t)$, constant return to scale, where K is the aggregate capital, and L is the aggregate labor. The economy is subject to an exogenous sequence of government spending $\{G_t(s^t)\}_{t=0,s^t\in S^t}^{\infty}$. Both the production function and government expenditures depend on the time period t (to capture deterministic changes such as growth) and the history s^t (to capture the uncertainty impact).

A.1 Competitive Equilibrium with Government Policies

Markets are assumed to be competitive and complete. An interpretation is that the government can issue debt from a rich set of Arrow-Debreu state-contingent bonds. A more general interpretation is that, even only non-contingent debt is available, assets may span the necessary payoffs to complete the market.

In every period and history s^t, the government can issue both domestic and international bonds, impose a lump-sum tax $T(s^t)$, a marginal tax on labor income $\tau^n(s^t)$ and set the price domestic bond $Q(s^t)$.

Domestic Agent. Individual agent of type $i \in I$ faces the sequential budget constraint in period t, history s^t

$$c^i(s^t) + k^i(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}) b^{d,i}(s^{t+1}) \leq (1-\tau^n(s^t)) w(s^t) l^i(s^t) + \left[1 + (1-\tau^k(s^t)) r^k(s^t) - \delta\right] k^i(s^{t-1}) \tag{A.2}$$

$$+ b^{d,i}(s^t) - T(s^t)$$

where $c^i(s^t), l^i(s^t), k^i(s^t), b^{d,i}(s^t)$ denote the consumption, efficiency-unit labor, capital holding, and domestic bond holding of agent i in period t, history s^t, respectively.

Representative Firm. The firm chooses the amount of capital and labor to maximize profit each period and each history:

$$\max_{\{K(s^{t-1}),L(s^t)\}} \quad F\left(K(s^{t-1}), L(s^t), s^t, t\right) - w(s^t)L(s^t) - r^k(s^t)K(s^{t-1})$$

which gives the first-order conditions:

$$\begin{aligned} w(s^t) &= F_L\left(K(s^{t-1}), L(s^t), s^t, t\right) \\ r^k(s^t) &= F_K\left(K(s^{t-1}), L(s^t), s^t, t\right) \end{aligned} \tag{A.3}$$

The firm profits are zero in equilibrium because of the constant-return-to-scale production function.

Government. The government's budget constraint in each period is

$$\begin{aligned} G(s^t) + B^d(s^t) + B(s^t) \leq \tau^n(s^t)w(s^t)L(s^t) + \tau^k(s^t)r(s^t)K(s^{t-1}) \\ + \sum_{s^{t+1}|s^t} Q(s^{t+1})B^d(s^{t+1}) + Q^*(s^{t+1})B(s^{t+1}) + T(s^t) \end{aligned}$$

where $B^d(s^t) = \sum_{i \in I} \pi^i b^{d,i}(s^t)$ is the aggregate domestic bond, and $B(s^t)$ is the amount of the government's external debt. The government faces a no-Ponzi condition such that the present value of external debt is bounded below.

Define $q(s^t)$ as an Arrow-Debrew price of a unit of period-t, history-s^t consumption:

$$q(s^t) = \Pi^t_{\tau=0, s^\tau \subseteq s^t} \frac{\Pr(s_\tau)}{1 + r^*_t}, \tag{A.4}$$

normalized so that $q(s_0) = 1$.

The government's present-value budget constraint is

$$\begin{aligned} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} q(s^t) \Big[G(s^t) - \tau^n(s^t)w(s^t)L(s^t) - \tau^k(s^t)r(s^t)K(s^{t-1}) \\ + \sum_{s^{t+1}|s^t} Q(s^{t+1})B^d(s^{t+1}) - B^d(s^t) - T(s^t) \Big] \leq B(s_0) \end{aligned} \tag{A.5}$$

Aggregate resource constraint.

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} q(s^t) \left[F(K(s^{t-1}), L(s^t), s^t) - G(s^t) - C(s^t) \right] \geq B(s_0) \tag{A.6}$$

Competitive equilibrium.

Definition A.1. Given initial external debt $B(s_0)$, individual wealth positions $\left(a^i(s_0)\right)_{i\in I}$[14], a competitive equilibrium is domestic agents' allocation $z^{H,i} = \left\{c^i(s^t), l^i(s^t), k^i(s^t), b^{i,d}(s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$, $\forall i \in I$, the firm's allocation $z^F = \left\{L(s^t)\right\}_{t=0,s^t}^{\infty}$, prices $p = \left\{q(s^t), w(s^t), r^k(s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$, and government's policy $z^G = \left\{\tau^n(s^t), \tau^k(s^t),\ T(s^t), Q(s^t), B^d(s^t), B(s^t)\right\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves agent i's problem that maximizes (1) subject to (A.2) and a no-Ponzi condition of agent's debt value, (ii) given p and z^G, z^F solves firm's problem, which implies the first-order conditions (A.3), (iii) the government budget constraint (A.5) holds, (iv) the aggregate resource constraint (A.6) is satisfied, and (v) p satisfies (A.4) given z^G.

A.2 Characterizing Equilibrium

For any equilibrium, there exist a set of Neghishi (market) weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i\in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i\varphi^i = 1$, such that individual allocation solve a static problem

$$
\begin{aligned}
V(C, L; \boldsymbol{\varphi}) \equiv \max_{(c^i,l^i)_{i\in I}} & \sum_{i\in I} \varphi^i\pi^i U^i(c^i, l^i) \\
s.t. \qquad & \sum_{i\in I} \pi^i c^i = C; \quad \sum_{i\in I} \pi^i l^i = L
\end{aligned}
$$

The equilibrium allocation is $\left(c^i(s^t), l^i(s^t)\right) = h^i(C(s^t), L(s^t); \boldsymbol{\varphi})$ for all i and s^t. Furthermore, the individual implementability constraint is

$$
\begin{aligned}
\sum_{t\geq 0, s^t\in S^t} \beta^t \Pr(s^t) & \left[V_C(C(s^t), L(s^t); \boldsymbol{\varphi}) h^{i,c}(C(s^t), L(s^t); \boldsymbol{\varphi})\right. \\
& \left. + V_L(C(s^t), L(s^t); \boldsymbol{\varphi}) h^{i,l}(C(s^t), L(s^t); \boldsymbol{\varphi})\right] = V_C(C(s^0), L(s^0); \boldsymbol{\varphi}) \left(a^i(s_0) - T\right)
\end{aligned} \tag{A.7}
$$

where $\alpha^i(s^0)$ is the initial wealth position, and T is the present-value of lump-sum taxes.[15]

One has the following characterization proposition.

Proposition A.1. *Given the initial external debt $B(s_0)$ and individual wealth positions $\left(a^i(s_0)\right)_{i\in I}$, an allocation $\left\{C(s^t), L(s^t), K(s^t)\right\}_{t=0,s^t\in S^t}^{\infty}$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if the resource constraint (A.6) holds, and there exist market weights $\boldsymbol{\varphi} = \left(\varphi^i\right)_{i\in I}$ and lump-sum tax T such that the implementability constraint (A.7) holds for all $i \in I$.*

Proof. See Appendix D.

[14] $a^i(s_0) \equiv \left[1 + (1 - \tau^k(s_0))r^k(s_0) - \delta\right] k^i(s_0) + b_0^{d,i}$

[15] $T \equiv \sum_{t=0}^{\infty} \beta^t \sum_{s^t\in S^t} \frac{V_C\left[h^i(C(s^t),L(s^t);\boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s_0),L(s_0);\boldsymbol{\varphi})\right]} T(s^t)$

A.3 Planning Problem

The benevolent government objective is

$$\sum_{i\in I}\lambda^i\pi^i\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0U^i\left(c^i(s^t),l^i(s^t)\right), \tag{A.8}$$

and the government's limited commitment constraint is

$$\sum_{i\in I}\lambda^i\pi^i\sum_{k\geq t,s^t\subseteq s^{t+j}}\beta^{k-t}\Pr(s^k|s^t)U^i(c^i(s^k),l^i(s^k))\geq \underline{U}(s^t,t),\ \forall t,\forall s^t \tag{A.9}$$

where $\underline{U}(s^t,t)$ is the one-shot deviation value in which the government defaults on both domestic and external debt and fully redistributes wealth among domestic agents. The government is then in financial autarky, in which it it has no access to external financial markets. $\underline{U}(s^t,t)$ is the value associated with an allocation of a closed economy where the initial states are realized s_t at period t and history s^t, the initial wealth inequality among agents are equal, and the net supplies of domestic and international bonds are zero.

The efficient allocation is part of the solution to a planning problem

$$(P)\equiv\max_{\{C(s^t),L(s^t)\},\varphi,T}\sum_{i\in I}\lambda^i\pi^i\sum_{t=0}^{\infty}\beta^t\mathbb{E}_0\left[h^i(s^t;\varphi)\right]$$

$$\begin{aligned}
s.t.\quad &\sum_{t\geq 0,s^t\in S^t}q(s^t)\left[F(K(s^{t-1}),L(s^t),s^t,t)-G(s^t)-C(s^t)\right]\geq B(s_0)\\
\forall i,\quad &\sum_{t\geq 0,s^t\in S^t}\beta^t\left[V_C(s^t;\varphi)h^{i,c}(s^t;\varphi)+V_L(s^t;\varphi)h^{i,l}(s^t;\varphi)\right]=V_C(s_0;\varphi)\left(a^i(s_0)-T\right)\\
\forall t,\forall s^t,\quad &\sum_{i\in I}\lambda^i\pi^i\sum_{k\geq t,s^t\subseteq s^{t+j}}\beta^{k-t}\Pr(s^k|s^t)U^i\left[h^i(s^k;\varphi)\right]\geq\underline{U}(K(s^t),s^t,t)
\end{aligned}$$

where $(s^t;\varphi)\equiv(C(s^t),L(s^t);\varphi)$ for notation convenience.

A.4 Optimal Taxation under Separable Isoelastic Preferences

First, the equilibrium allocation is

$$\begin{aligned}c^i(s^t)&=\psi_c^iC(s^t)\\ l^i(s^t)&=\psi_l^iL(s^t)\end{aligned} \tag{A.10}$$

where ψ_c^i,ψ_l^i follow 12

The fictious representative-agent utility is

$$V(C(s^t), L(s^t); \varphi) = \Phi_C^V \frac{C(s^t)^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L(s^t)^{1+\nu}}{1+\nu},$$

and the social welfare is

$$\sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \Pr(s^t) \left(\Phi_C^P \frac{C(s^t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^t)^{1+\nu}}{1+\nu} \right)$$

Optimal labor tax. The optimal labor tax is

$$\tau^n(s^t) = 1 - \frac{\Phi_L^V}{\Phi_C^V} \left[\frac{\Phi_C^W + \Phi_C^P \sum_{\tau=0,s^\tau}^{t} \gamma(s^\tau)}{\Phi_L^W + \Phi_L^P \sum_{\tau=0,s^\tau}^{t} \gamma(s^\tau)} \right]$$

where $\gamma(s^t)$ is the Lagrange multiplier on the debt constraint of at period t and history s^t.

We have the followings:

Assumption 4. *$\underline{U}$ is bounded below, i.e. there exists a finite real M_U such that* $\inf_{K,s^t,t} \underline{U}(K, s^t, t) \geq M_U$.

Given this assumption, the consumption path is bounded below by zero in the long run, i.e.

Lemma A.1 (No immiseration)**.** *Suppose assumptions 2 and 4 hold, then for any efficient allocation* $\{C^*(s^t), L^*(s^t), K^*(s^t)\}_{t,s^t}$, $\liminf_{t\to\infty} C^*(s^t) > 0$.

Proof. See Appendix D.

The following proposition characterizes the optimal labor tax that is similar to the main text,

Proposition A.2 (Optimal labor tax)**.** *Given assumption 2, if an efficient allocation exists and debt constraint does not bind, there is constant labor tax given by (20). Moreover, if assumptions 1 and 4 also hold, and an interior efficient allocation exists, then the optimal labor tax converges to a real constant given by (21) that depends on skill distribution and redistribution preference. These results hold with or without the lump-sum transfers.*

Proof. See Appendix D.

Optimal capital and domestic saving taxes. The optimal domestic return satisfies

$$1 + r(s^t) = (1 + r_t^*) \frac{\Phi_C^W + \Phi_C^P \sum_{\tau=0,s^\tau}^{t} \gamma(s^\tau)}{\Phi_C^W + \Phi_C^P \sum_{\tau=0,s^\tau}^{t-1} \gamma(s^\tau)} \tag{A.11}$$

Define $\hat{\tau}^k(s^t)$ that satisfies

$$(1 - \tau^k(s^t)) F_K(s^t, t) + 1 - \delta = \frac{1 + r(s^t)}{1 + r^*(s^t)} \left[(1 - \hat{\tau}^k(s^t)) F_K(s^t, t) + 1 - \delta \right], \ \forall t$$

where $\frac{1+r}{1+r^*}$ represents the effective intertemporal return induced by domestic saving tax. The first-order conditions (14) and (17), without loss of generality, imply an implementation

$$\hat{\tau}^k(s^t) = \frac{\frac{\beta^t}{q_t}\gamma_t \frac{d\underline{U}(K(s^t),s^t,t)}{dK}}{F_K(s^t,t)} \tag{A.12}$$

Define a tax on domestic savings $\tau^d(s^t) \equiv 1 - r(s^t)/r^*_t$,then we have the following Proposition

Proposition A.3. *Given assumptions 1, 2, 4, and* $\frac{d\underline{U}(\cdot,s^t,t)}{dK} > 0$, *as* $t \to \infty$, $\tau^d(s^t) < 0$ *and* $\hat{\tau}^k(s^t) > 0$

Proof. Follows directly from equations (A.11) and (A.12). □

B Economy With Separable Preferences

This section extends the results of optimal labor taxation with separable preferences. As the elasticity of consumption intertemporal substitution and the elasticity of labor supply vary across time, the optimal labor tax fluctuates. In general, the labor tax is bounded in the long run. If the steady states exist, in the long run, the optimal tax goes to a real limit as in the case with separable isoelastic preferences. In either case, the distributive preference alters the level of optimal taxes in the long run. The results rely on the following assumptions of separability and boundedness.

Assumption 5 (Separable preference)**.** $U^i(c,l) = u(c) - v\left(l/\theta^i\right)$, *where* $u_c(\cdot) > 0$, $u_{cc}(\cdot) < 0$, $\lim_{c\to 0} u_c(c) = \infty$, $\lim_{c\to\infty} u_c(c) = 0$, $v_l(\cdot) > 0$, $v_{ll}(\cdot) > 0$, *and* $\lim_{l\to 0} v_l(l) = \infty$

Assumption 6 (Bounded elasticities)**.** u *and* v *are such that* $\forall c, n \in \mathbb{R}_+$, $0 < -\frac{u"(c)}{u'(c)}c < \infty$ *and* $0 < \frac{v"(n)}{v'(n)}n < \infty$

Since the preference is separable between consumption and leisure, individual consumption (labor) only depends on the aggregate consumption (labor). In addition, each agent's allocation is increasing with respect to the aggregates.

Lemma B.1. *Given assumption 5, for any competitive equilibrium, there exist time-invariant functions* $h^{i,c}(\cdot;\boldsymbol{\varphi}), h^{i,l}(\cdot;\boldsymbol{\varphi}),\ \forall i$ *such that* $\forall i, \forall t$,

$$c^i_t = h^{i,c}(C_t;\boldsymbol{\varphi})$$
$$l^i_t = h^{i,l}(L_t;\boldsymbol{\varphi})$$

where $h^{i,c}(\cdot;\boldsymbol{\varphi}), h^{i,l}(\cdot;\boldsymbol{\varphi})$ *are strictly increasing.*

The characterization of individual allocation from Lemma B.1 and the bounded elasticities from assumption 6 give the long-run property of efficient allocation and optimal labor tax. Specifically, the efficient allocation also features no immiseration in the long run,

Lemma B.2 (No immiseration)**.** *Suppose assumptions 3 and 5 hold, then for any efficient allocation* $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$, $\liminf_{t\to\infty} C_t^* > 0$.

and the long-run optimal labor tax is bounded.

Proposition B.1 (Optimal labor tax in the long run)**.** *If assumptions 1,3,5, and 6 hold, and an interior efficient allocation* $\{C_t, L_t, K_t\}_{t=0}^{\infty}, \varphi, T$ *exists, then there exist* $-\infty < \underline{\tau}, \bar{\tau} < \infty$ *such that* $\liminf_{t\to\infty} \tau_t^n = \underline{\tau}$ *and* $\limsup_{t\to\infty} \tau_t^n = \bar{\tau}$*. Moreover, if the steady states exist, then* $\lim_{t\to\infty} \tau_t^n$ *exists. These results hold with or without the lump-sum transfers.*

The optimal tax's long-run value relies on the marginal changes in individual allocation with respect to the aggregates in the long run. With constant elasticities, the individual allocation is linear in the aggregate allocation, as in equations (11), so the marginal change is constant over time, which means that the limit exists. However, when preferences are not isoelastic, the marginal change fluctuates over time. Therefore, the optimal labor tax does not necessarily converge to a constant. Given the bounded elasticities, the marginal changes are bounded and so is the optimal labor tax. In case the steady state allocation exists, the marginal changes will converge to the steady state values, which implies the convergence to limit of the labor tax.

C Sovereign Game

Before setting up the game, consider the general environment where the government's policy includes the decision to default on external bond $\{d_t\}_{t=0}^{\infty}$, where $d_t \in \{0,1\}$ and $d_t = 0$ implies default[16]. The government's budget constraint becomes

$$G_t + (1+r_t)B_t^d + d_t B_t \leq \tau_t^n w_t L_t + \tau_t^k r_t K_t + B_{t+1}^d + Q_{t+1}B_{t+1} + T_t$$

As the government cannot commit to any of its policies, one can think that the government, domestic agents, and international lenders enter in a sovereign game where they determine their actions sequentially. In every period, the state variable for the game is $\left\{B_t, \left(k_t^i, b_t^{i,d}\right)_{i\in I}\right\}$. The timing of the actions is as follows.

- Government chooses $z_t^G = \left(\tau_t^n, \tau_t^k, T_t, d_t, r_t, B_{t+1}, B_{t+1}^d\right) \in \Pi$such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c_t^i, l_t^i, k_{t+1}^i, b_{t+1}^{d,i}\right)$ subject to their budget constraints, the representative firm produce output by choosing $z_t^F = (K_t, L_t)$, and the international lenders choose holdings of government's bonds $z_t^* = (B_{t+1})$ given the price Q_{t+1}.

[16]Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its international debt ($d_t = 1$).

Define $h^t = \left(h^{t-1}, z_t^G, \left(z_t^{H,i}\right)_{i\in I}, z_t^F, z_t^*, p\right) \in H^t$ as the history at the end of period t. Note that the history incorporates the government's policy, allocation and prices. Define $h_p^t = \left(h^{t-1}, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period t. The government strategy is $\sigma_t^G: H^{t-1} \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i}: H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has strategy $\sigma_t^F: H_p^t \to \mathbb{R}_+^2$, and the international lenders have strategy $\sigma_t^*: H_p^t \to \mathbb{R}_+$. The prices are determined by the pricing rule: $p: H_p^t \to \mathbb{R}_+$

Definition C.1 (Sustainable equilibrium)**.** A sustainable equilibrium is $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ such that (i) for all h^{t-1}, the policy z_t^G induced by the government strategy maximizes the socially weighted utility given λ subject to the government's budget constraint (5) (ii) for all h_p^t, the strategy induced policy $\{z_t^G\}_{t=0}^{\infty}$, allocation $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^{\infty}$, and prices $\{Q_t\}_{t=0}^{\infty}$ constitute a competitive equilibrium with taxes.

The following focuses on characterizing a set of sustainable equilibrium in which deviation triggers autarky, where there is no domestic and international borrowing. In this case, the value of deviation includes the autarkic payoff.

By definitions, autarky is a sustainable equilibrium. Given that the domestic agents do not save/invest, the representative firm produces only with labor, and the international creditors do not lend, the government finds it optimal to default on its external debt, set saving and capital taxes such that the after-tax gross returns on domestic bonds and capital are zero, and set the labor tax such that it maximizes the socially weighted utility. Given the government defaulting and fully taxing all returns from domestic savings and capital, international creditors do not want to lend, agents do not save or invest in capital, and output is produced only by labor. Lastly, given that the government will be in autarky in the future, it is optimal in the current period for the government to also follow the autarkic strategies.

Reverting to autarky equilibrium is defined as a sustainable equilibrium of the above game such that following any government's deviation from the promised plans, the economy reverts to autarky. One can characterize the equilibrium as follows.

Proposition C.1 (Reverting to autarky equilibrium)**.** *An allocation and policy* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *can be supported by reverting to autarky equilibrium if and only if (i) given* z^G*, there exist prices* p *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G, p\right\}$ *is a competitive equilibrium with taxes for an open economy, and (ii) for any* t*, there exists* $\underline{U}_t(\cdot)$ *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *satisfies the constraint*

$$\sum_{s=t}^{\infty} \beta^{s-t} \sum_{i\in I} \lambda^i \pi^i U^i\left(c_s^i, l_s^i\right) \geq \underline{U}_t(K_t) \tag{7}$$

Furthermore, $\underline{U}_t(\cdot)$ *is increasing.*

Proof. Define $\underline{U}_t(K_t)$ as the maximum discounted weighted utility for the agents in period t when the government deviates. In period t, the agents save and the government can borrow abroad. In subsequent period $s > t$, the economy reverts to financial autarky where the agents do not save and the government is excluded from international lending. This economy ensembles a neoclassical growth closed economy that starts at period t and in which distortionary taxes and savings are only in the initial period. Then it is true that the higher the initial capital stock (in this case K_t), the higher utility that the agents receive. Hence, $\underline{U}_t(\cdot)$ is increasing.

Suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an outcome of the reverting to autarky equilibrium. Then by the optimal problems of the government, domestic agents, and international lenders, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes the weighted utility of the agents, satisfies government budget constraint and international lender's problem at period 0. Thus, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an open-economy tax-distorted competitive equilibrium. For any period t and history h^{t-1}, an equilibrium strategy that has the government deviates in period t triggers reverting to autarky in period $s > t$. Such strategy must deliver the weighted value at least as high as the right-hand side of (7). So $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii).

Next, suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies conditions (i) and (ii). Let h^{t-1} be any history such that there is no deviation from z^G up until period t. Since $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t onward. Consider a deviation plan $\hat{\sigma}^G$at period t that receives $U_t^d(K_t)$ in period t and U^{aut} for period $s > t$. Because the plan is constructed to maximize period-t utility at K_t, the right-hand side of (7) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. $\square$

D Formulas and Proofs

D.1 Formulas for separable isoelastic preference

Given the formulas for ψ_c^i and ψ_l^i in (12),

$$\Phi_C^V = \left[\sum_i \pi^i (\varphi^i)^{1/\sigma}\right]^\sigma; \qquad \Phi_L^V = \omega \left[\sum_i \pi^i \left(\varphi^i\right)^{-1/\nu} (\theta^i)^{(1+\nu)/\nu}\right]^{-\nu}$$

$$\Phi_C^W = \Phi_C^V \sum_{i\in I} \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]; \qquad \Phi_L^W = \Phi_L^V \sum_{i\in I} \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right]$$

$$\Phi_C^P = \Phi_C^V \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i}\psi_c^i; \qquad \Phi_L^P = \Phi_L^V \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^i}\psi_l^i$$

D.2 Proof of Proposition 2.1

Proof. $(\Rightarrow)$ Let $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ be an aggregate allocation of an open economy competitive equilibrium with taxes. Then by definition, $\{C_t, L_t, K_t\}_{t=0}^{\infty}$ satisfies aggregate resource constraint for every period. Moreover, given any market weights φ, $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ satisfies

$$
\begin{aligned}
(1-\tau_t^n)w_t &= -\frac{V_L(C_t, L_t; \varphi)}{V_C(C_t, L_t; \varphi)} \\
1 + r_{t+1} &= \frac{V_C\left[h^i(C_t, L_t; \varphi)\right]}{\beta V_C\left[h^i(C_{t+1}, L_{t+1}; \varphi)\right]}
\end{aligned}
$$

Substituting for w_t and r_t into the budget constraint (2), and using $(c_t^i, l_t^i) = h^i(C_t, L_t; \varphi)$ gives the implementability constraint for each agent. Importantly, choose φ and T such that the individual implementability constraints hold with equality.

$(\Leftarrow)$ Given φ, T and an allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\{w_t, r_t^k\}_{t=0}^{\infty}$ using firm's first-order conditions (3). $\{\tau_t^n\}_{t=0}^{\infty}$ can be calculated using the intratemporal condition (8), while one can choose $\{r_t\}_{t=0}^{\infty}$ that satisfy the intertemporal constraint (9). The tax on capital $\{\tau_t^k\}_{t=0}^{\infty}$ can be derived from $(1-\tau_t^k)r_t^k = r_t + \delta$. Define $\{q_t\}_{t=0}^{\infty}$ by (4).

Rewriting the aggregate resource constraint using $F(K, L) = wL + rK$ gives

$$
\begin{aligned}
\sum_{t=0}^{\infty} q_t \left\{C_t + K_{t+1} - (1-\tau_t^n)w_t L_t - \left[1 + (1-\tau_t^k)r_t^k - \delta\right] K_t + T_t\right\} \\
+ \sum_{t=0}^{\infty} q_t \left[G_t - \tau_t^k r_t K_t - \tau_t^n w_t L_t - T_t\right] \leq -\delta_0 B_0
\end{aligned} \tag{D.1}
$$

Aggregating up the agent's budget constraints implies

$$
C_t + K_{t+1} + B_{t+1}^d = (1-\tau_t^n)w_t L_t + \left[1 + (1-\tau_t^k)r_t^k - \delta\right] K_t + (1+r_t)\, B_t^d - T_t
$$

or

$$
C_t + K_{t+1} - (1-\tau_t^n)w_t L_t - \left[1 + (1-\tau_t^k)r_t^k - \delta\right] K_t + T_t = (1+r_t)\, B_t^d - B_{t+1}^d
$$

Substituting the last equation into (D.1) gives the government's budget constraint (5). Thus, $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ is the aggregate allocation of the constructed competitive equilibrium with taxes. $\square$

D.3 Proof of Lemma 4.1

Proof. Given an efficient allocation $\{C_t^*, L_t^*, K_{t+1}^*\}_{t=0}^{\infty}$, suppose, by contradiction that $\liminf_{t\to\infty} C_t^* \leq 0$. Find $\epsilon > 0$ such that $\forall t$,

$$\sum_{s=t}^{\infty} \beta^{s-t} \left\{ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu} \right\} \leq M_U$$

with $C_t = \epsilon$ and $C_s = C_s^*,\ \forall s > t$. Such ϵ exists since the utility function is unbounded below. Because $\liminf_{t\to\infty} C_t^* \leq 0$, there exists t_0 such that $C_{t_0}^* < \epsilon$. Then by monotonicity,

$$\begin{aligned}
&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \Phi_C^P \frac{(C_s^*)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu} \right\} \\
<&\sum_{s=t_0}^{\infty} \beta^{s-t_0} \left\{ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{(L_s^*)^{1+\nu}}{1+\nu} \right\} \\
\leq& M_U \\
\leq& \underline{U}_{t_0}(K_{t_0}^*)
\end{aligned}$$

which contradicts the aggregate debt constraint at t_0. $\square$

D.4 Proof of Proposition 4.1

Proof. The first statement directly follows from equations (19) and (20).

Let $\{C_t^*, L_t^*, K_{t+1}^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exists $\boldsymbol{\lambda}$ such that $\{C_t^*, L_t^*, K_{t+1}^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). Define

$$A_C = \sum_{i\in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_c^i, \quad A_L = \sum_{i\in I} \pi^i \frac{\lambda^i}{\boldsymbol{\varphi}^{*i}} \psi_l^i \tag{D.2}$$

where ψ_c^i, ψ_l^i are defined by equations (12) using $\boldsymbol{\varphi}^*$. First, one can show that A_C and A_L are positive and bounded:

Lemma D.1. *Given an interior allocation,* $0 < A_C < \infty$ *and* $0 < A_L < \infty$

Proof. Interior allocation means that for any i, $c_t^i, l_t^i > 0,\ \forall t$. This implies that $\psi_c^i, \psi_l^i > 0$. By (12), $\boldsymbol{\varphi}^{*i} > 0$.

For all i, $\pi^i > 0, \lambda^i \geq 0$ and since $\sum_{i\in I} \pi^i \lambda^i = 1$, there exists at least an i such that $\lambda^i > 0$. Given that $\psi_c^i, \psi_l^i > 0,\ \forall i$, it must be that $A_C, A_L > 0$.

Since $\sum_{i\in I} \pi^i \boldsymbol{\varphi}^{*i} = 1 < \infty$ and $\forall i,\ \pi^i, \boldsymbol{\varphi}^{*i} > 0$, it must be that $\boldsymbol{\varphi}^{*i} < \infty$. So by definition, $\psi_c^i, \psi_l^i < \infty$. Moreover, $\boldsymbol{\varphi}^{*i} > 0$ implies that $\lambda^i/\boldsymbol{\varphi}^{*i} < \infty$. Then by definition, $A_C, A_L < \infty$. $\square$

Define (P^M) the same problem as (P) with the restriction that $(C_t, L_t) = (C_t^*, L_t^*), \forall t > M$, $\varphi = \varphi^*$, $T = T^*$, and $K_t = K_t^*, \forall t$. Note that $\{C_t^*, L_t^*, K_{t+1}^*\}_{t=0}^{\infty}$ is a solution to (P^M), and (P^M) has a finite number of constraints. By a Lagrangian theorem in Luenberger (1969), there exists non-negative, not identically zero vector $\{r^M, \mu^M, \eta^{M,1}, \ldots, \eta^{M,I}, \gamma_0^M, \ldots, \gamma_M^M\}$ such that the first-order and complementarity conditions hold, i.e. $\forall t \geq 1$

$$\frac{\beta^t}{q_t}\left\{r^M A_C + \sum_i \pi^i \eta^{M,i}(1-\sigma)\psi_c^i + \sum_{s=0}^{t} \gamma_s^M A_C\right\} \Phi_C^V C_t^{-\sigma} = \mu^M \tag{D.3}$$

$$\frac{\beta^t}{q_t}\left\{r^M A_L + \sum_i \pi^i \eta^{M,i}(1+\nu)\psi_l^i + \sum_{s=0}^{t} \gamma_s^M A_L\right\} \Phi_L^V L_t^{\nu} = \mu^M F_L(K_t, L_t) \tag{D.4}$$

Since the allocation is interior and $A_C, A_L > 0$, one can rewrite the first-order conditions as

$$\frac{\beta^t}{q_t}\left\{r^M A_C + \sum_i \pi^i \eta^{M,i}(1-\sigma)\psi_c^i + \sum_{s=0}^{t} \gamma_s^M A_C\right\} \Phi_C^V C_t^{-\sigma} = \mu^M$$

$$\frac{\beta^t}{q_t}\left\{r^M A_C + \sum_i \pi^i \eta^{M,i}(1+\nu)\psi_l^i \frac{A_C}{A_L} + \sum_{s=0}^{t} \gamma_s^M A_C\right\} \Phi_C^V C_t^{-\sigma} = \mu^M \frac{A_C}{A_L} F_L(K_t, L_t) \frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}}$$

Subtracting the first from the second line gives

$$\frac{\beta^t}{q_t}\left\{\Phi_C^V \sum_i \pi^i \eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right]\right\} C_t^{-\sigma} = \mu^M \left[\frac{A_C}{A_L} F_L(K_t, L_t) \frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}} - 1\right] \tag{D.5}$$

The following lemma shows that the resource constraint binds for any sub-problem (P^M) and $M \geq 1$.

Lemma D.2. *In any sub-problem (P^M) with $M \geq 1$, $\mu^M > 0$*

Proof. Suppose, by contradiction, that $\mu^M = 0$ so the resource constraint does not bind. Consider allocation $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ which is the solution to (P^M). Then there exists $\epsilon > 0$ such that

$$\sum_{t=0}^{\infty} q_t \left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 - \epsilon \geq 0$$

Define $\left\{\hat{L}_t\right\}_{t=0}^{\infty}$ where $\hat{L}_1 < L_1$ such that $F(K_1, \hat{L}_1) = F(K_1, L_1) - \epsilon/q_1$, and $\hat{L}_t = L_t, \ \forall t > 1$. The allocation $\left\{C_t, \hat{L}_t, K_{t+1}\right\}_{t=0}^{\infty}$ satisfies the resource constraint and because of the preference's strict monotonicity, $\left\{C_t, \hat{L}_t, K_{t+1}\right\}_{t=0}^{\infty}$ also satisfies the implementability constraints and the aggregate debt constraints. However,

$$\sum_{t=0}^{\infty}\sum_{i\in I}\beta^t\lambda^i\pi^iU^i\left[h^i(C_t,\hat{L}_t;\varphi)\right] > \sum_{t=0}^{\infty}\sum_{i\in I}\beta^t\lambda^i\pi^iU^i\left[h^i(C_t,L_t;\varphi)\right]$$

which contradicts $\{C_t, L_t, K_t\}_{t=0}^{\infty}$ being optimal solution for (P^M). □

By Lemma D.2 and interior allocation, we can rewrite equation (D.5) as

$$\frac{\Phi_C^V}{\mu^M}\sum_i \pi^i\eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i-(1-\sigma)\psi_c^i\right] = \frac{q_t}{\beta^t}C_t^{\sigma}\left[\frac{A_C}{A_L}F_L(K_t,L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}}-1\right]$$

Specifically, for any $M \geq 1$,

$$\frac{\Phi_C^V}{\mu^M}\sum_i \pi^i\eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i-(1-\sigma)\psi_c^i\right] = \frac{q_1}{\beta}\left(C_1^*\right)^{\sigma}\left[\frac{A_C}{A_L}F_L(1)\frac{\Phi_C^V\left(C_1^*\right)^{-\sigma}}{\Phi_L^V\left(L_1^*\right)^{\nu}}-1\right]$$

Note that the left-hand side is a function of (C_1^*, L_1^*, K_1^*) , which implies that there exists a constant κ such that $\forall M \geq 1$,

$$\frac{\Phi_C^V}{\mu^M}\sum_i \pi^i\eta^{M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i-(1-\sigma)\psi_c^i\right] = \kappa$$

Hence, (D.5) can be rewritten as

$$\frac{\beta^t}{q_t}C_t^{-\sigma}\kappa = \frac{A_C}{A_L}F_L(K_t,L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}}-1$$

Note that $\lim_{t\to\infty}\beta^t/q_t = 0$ and $C_t^{-\sigma}$ is bounded by Lemma 4.1, so taking the limit on both sides gives

$$\lim_{t\to\infty}\frac{A_C}{A_L}F_L(K_t,L_t)\frac{\Phi_C^V C_t^{-\sigma}}{\Phi_L^V L_t^{\nu}} = 1$$

Hence, given the definition of τ_t^n and the fact that A_C,A_L are bounded,

$$\lim_{t\to\infty}\tau_t^n = \lim_{t\to\infty}\left[1-\frac{\Phi_L^V L_t^{\nu}}{\Phi_C^V C_t^{-\sigma}}\frac{1}{F_L(K_t,L_t)}\right] = 1-\frac{A_C}{A_L}$$

In addition, the above argument does not rely on the existence of lump-sum transfers. □

D.5 Proof of Proposition 4.2

Proof. Rewrite the optimal labor tax formulas as

$$\tau_t^n = 1 - \frac{\Phi_L^V \Phi_C^W + \Phi_L^V \Phi_C^P \sum_{s=0}^t \gamma_s}{\Phi_C^V \Phi_L^W + \Phi_C^V \Phi_L^P \sum_{s=0}^t \gamma_s} \tag{D.6}$$

By definitions,

$$\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} = \frac{\sum_i \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma)\eta^i\right]}{\sum_i \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu)\eta^i\right]} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

and

$$\frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P} = \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

using the optimal conditions $\eta^i = \sum_j \pi^j \lambda^j/\varphi^j - \lambda^i/\varphi^i$, and the definitions $\mathbb{E}\left[x^i\right] \equiv \sum_i \pi^i x^i$, $\text{cov}(x^i, y^i) \equiv \mathbb{E}\left[x^i y^i\right] - \mathbb{E}\left[x^i\right]\mathbb{E}\left[y^i\right]$.

Lemma D.3. *cov* $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ *and cov* $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$

Proof. Given that $a_0^i = A_0, \forall i \in I$, the individual implementability constraints can be rewritten as

$$\psi_c^i \Phi_C^V \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma} - \psi_l^i \Phi_L^V \sum_{t=0}^{\infty} \beta^t L_t^{1+\nu} = \Phi_C^V C_0^{-\sigma}(A_0 - T)$$

or

$$\psi_c^i = \psi_l^i \frac{\Phi_L^V \sum_{t=0}^{\infty} \beta^t L_t^{1+\nu}}{\Phi_C^V \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma}} + \frac{\Phi_C^V C_0^{-\sigma}(A_0 - T)}{\Phi_C^V \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma}}$$

which implies that $\psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$. By definition of ψ_c^i, $\varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$.

The next step is to show that $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j$.

Suppose $\theta^i \geq \theta^j$ and $\varphi^i < \varphi^j$, then $\psi_l^i < \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < \frac{\varphi^i}{\varphi^j} < 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq 1$, which is a contradiction.

Suppose $\varphi^i \geq \varphi^j$ and $\theta^i < \theta^j$, then $\psi_l^i \geq \psi_l^j$. By definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \geq \frac{\varphi^i}{\varphi^j} \geq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} < 1$, which is a contradiction.

Thus, $\theta^i \geq \theta^j \iff \varphi^i \geq \varphi^j \iff \psi_c^i \geq \psi_c^j \iff \psi_l^i \geq \psi_l^j$. In addition, $\theta^i \geq \theta^j \iff \lambda^i \leq \lambda^j$, which implies that

$$\psi_c^i \geq \psi_c^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$
$$\psi_l^i \geq \psi_l^j \iff \frac{\lambda^i}{\varphi^i} \leq \frac{\lambda^j}{\varphi^j}$$

Hence, $\text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$ and $\text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) \leq 0$. □

Lemma D.3 and $\sigma \geq 1, \nu > 0$ imply that $\frac{\Phi_L^V \Phi_C^W}{\Phi_C^V \Phi_L^W} \leq \frac{\Phi_L^V \Phi_C^P}{\Phi_C^V \Phi_L^P}$. Suppose that the debt constraint binds at period t, then $\gamma_t > 0$, which leads to $\sum_{s=0}^{t} \gamma_s > \sum_{s=0}^{t-1} \gamma_s$. Applying equation (D.6) gives $\tau_t^n \leq \tau_{t-1}^n$. □

D.6 Proof of Proposition 5.2

Proof. $\lambda^i = \boldsymbol{\varphi}^{*i}$, $\forall i \in I$ implies that $A_C = 1$ and $A_L = 1$. Therefore, $\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ solves

$$\begin{aligned}
\max_{\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T} \quad & \sum_{t=0}^{\infty} \beta^t V(C_t, L_t; \boldsymbol{\varphi}) \\
s.t. \quad & \sum_{t=0}^{\infty} q_t \left[F(K_t, L_t) + (1-\delta)K_t - K_{t+1} - C_t - G_t\right] - B_0 \geq 0 \\
& \sum_{t=0}^{\infty} \beta^t \left[V_C(t; \boldsymbol{\varphi}) h^{i,c}(t; \boldsymbol{\varphi}) + V_L(t; \boldsymbol{\varphi}) h^{i,l}(t; \boldsymbol{\varphi})\right] \geq V_C(0; \boldsymbol{\varphi}) \left(a_0^i - T\right) \\
& \sum_{s=t}^{\infty} \beta^{s-t} V(C_t, L_t; \boldsymbol{\varphi}) \geq \underline{U}_t(K_t)
\end{aligned}$$

To implement $\{C_t^*, L_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ given the specified tax system, by (21), it must be that

$$\lim_{t \to \infty} \tau_t^n = 0$$

□

D.7 Proof of Proposition A.1

Proof. $(\Rightarrow)$ Let $\left\{C(s^t), L(s^t), K(s^t)\right\}_{t=0, s^t \in S^t}^{\infty}$ be an aggregate allocation of an open economy competitive equilibrium with taxes. Then by definition, $\left\{C(s^t), L(s^t), K(s^t)\right\}$ satisfies aggregate resource constraint for every period. Moreover, given any market weights φ,

$\{C(s^t), L(s^t), K(s^t)\}$ satisfies

$$(1-\tau^n(s^t))w(s^t) = -\frac{V_L\left[h^i(C(s^t), L(s^t); \varphi)\right]}{V_C\left[h^i(C(s^t), L(s^t); \varphi)\right]}$$
$$1 + r(s^{t+1}) = \frac{V_C\left[h^i(C(s^t), L(s^t); \varphi)\right]}{\beta \Pr(s^{t+1}|s^t) V_C\left[h^i(C(s^{t+1}), L(s^{t+1}); \varphi)\right]}$$

Substituting for $w(s^t)$ and $r(s^t)$ into the budget constraint (2), and using $(c^i(s^t), l^i(s^t)) = h^i(C(s^t), L(s^t); \varphi)$ gives the implementability constraint for each agent. Importantly, choose φ and T such that the individual implementability constraints hold with equality.

($\Leftarrow$) Given φ, T and an allocation $\{C(s^t), L(s^t), K(s^t)\}$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\{w(s^t), r^k(s^t)\}$ using the firm's first-order conditions (3). $\{\tau^n(s^t)\}$ can be calculated using the intratemporal condition (8), while one can choose $\{r(s^t)\}$ that satisfy the intertemporal constraint (9). The tax on capital $\{\tau^k(s^t)\}$ can be derived from $(1-\tau^k(s^t))r^k(s^t) = r(s^t) + \delta$. Define $\{q(s^t)\}$ by (4).

Rewriting the aggregate resource constraint using $F(K, L) = wL + rK$ gives

$$\sum_{t,s^t} q(s^t)\left\{C(s^t) + K(s^t) - (1-\tau^n(s^t))w(s^t)L(s^t) - \left[1 + (1-\tau^k(s^t))r^k(s^t) - \delta\right] K(s^{t-1}) + T(s^t)\right\}$$
$$+ \sum_{t,s^t} q(s^t)\left[G(s^t, t) - \tau^k(s^t)r^k(s^t)K(s^{t-1}) - \tau^n(s^t)w(s^t)L(s^t) - T(s^t)\right] \leq -B(s_0) \tag{D.7}$$

Aggregating up the agent's budget constraints implies

$$C(s^t) + K(s^t) + \sum_{s^{t+1}|s^t} B^d(s^{t+1}) = (1-\tau^n(s^t))w(s^t)L(s^t) + \left[1 + (1-\tau^k(s^t))r^k(s^t) - \delta\right] K(s^{t-1})$$
$$+ \left(1 + r(s^t)\right) B^d(s^t) - T(s^t)$$

or

$$C(s^t) + K(s^t) - (1-\tau^n(s^t))w(s^t)L(s^t) - \left[1 + (1-\tau^k(s^t))r^k(s^t) - \delta\right] K(s^{t-1}) + T(s^t)$$
$$= \left(1 + r(s^t)\right) B^d(s^t) - \sum_{s^{t+1}|s^t} B^d(s^{t+1})$$

Substituting the last equation into (D.7) gives the government's budget constraint (5). Thus, $\{C(s^t), L(s^t), K(s^t)\}$ is the aggregate allocation of the constructed competitive equilibrium with taxes. $\square$

D.8 Proof of Lemma A.1

Proof. Given an efficient allocation $\{C^*(s^t), L^*(s^t), K^*(s^t)\}$, suppose, by contradiction that for a sequence of shocks $\{s_0, ..., s_t, ...\}$, $\liminf_{t\to\infty} C^*(s^t) \leq 0$. Find $\epsilon > 0$ such that $\forall t$,$\forall s^t$,

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^t \subseteq s^\tau} Pr(s^\tau) \left[\Phi_C^V \frac{C(s^\tau)^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L^*(s^\tau)^{1+\nu}}{1+\nu}\right] \leq M_U$$

with $C(s^t) = \epsilon$ and $C(s^\tau) = C^*(s^\tau),\ \forall \tau > t,\ s^t \subset s^\tau$. Such ϵ exists since the utility function is unbounded below. Because $\liminf_{t\to\infty} C^*(s^t) \leq 0$, there exists a t_0 such that $C^*(s^{t_0}) < \epsilon$. Then by monotonicity,

$$\begin{aligned}
&\sum_{\tau=t_0}^{\infty} \beta^{\tau-t_0} \sum_{s^{t_0} \subseteq s^\tau} Pr(s^\tau) \left[\Phi_C^V \frac{C^*(s^\tau)^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L^*(s^\tau)^{1+\nu}}{1+\nu}\right] \\
<& \sum_{\tau=t_0}^{\infty} \beta^{\tau-t_0} \sum_{s^{t_0} \subseteq s^\tau} Pr(s^\tau) \left[\Phi_C^V \frac{C(s^\tau)^{1-\sigma}}{1-\sigma} - \Phi_L^V \frac{L^*(s^\tau)^{1+\nu}}{1+\nu}\right] \\
\leq& M_U \\
\leq& \underline{U}(K^*(s^{t_0}), s^{t_0})
\end{aligned}$$

which contradicts the aggregate debt constraint at s^{t_0}. □

D.9 Proof of Proposition A.2

Proof. The first statement directly follows from equations (19) and (20).

Let $\{C^*(s^t), L^*(s^t), K^*(s^t)\}_{t,s^t}, \boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exists $\boldsymbol{\lambda}$ such that $\{C^*(s^t), L^*(s^t), K^*(s^t)\}_{t,s^t}, \boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). Redefine

$$A_C = \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^{*i}} \psi_c^i, \quad A_L = \sum_{i\in I} \pi^i \frac{\lambda^i}{\varphi^{*i}} \psi_l^i \tag{D.8}$$

where ψ_c^i, ψ_l^i are defined by equations (12) using $\boldsymbol{\varphi}^*$. First, one can show that A_C and A_L are positive and bounded:

Lemma D.4. *Given an interior allocation,* $0 < A_C < \infty$ *and* $0 < A_L < \infty$

Proof. Interior allocation means that for any i, $c_t^i, l_t^i > 0,\ \forall t$. This implies that $\psi_c^i, \psi_l^i > 0$. By (12), $\varphi^{*i} > 0$.

For all i, $\pi^i > 0, \lambda^i \geq 0$ and since $\sum_{i\in I} \pi^i \lambda^i = 1$, there exists at least an i such that $\lambda^i > 0$. Given that $\psi_c^i, \psi_l^i > 0,\ \forall i$, it must be that $A_C, A_L > 0$.

Since $\sum_{i\in I}\pi^i\boldsymbol{\varphi}^{*i} = 1 < \infty$ and $\forall i,\ \pi^i, \boldsymbol{\varphi}^{*i} > 0$, it must be that $\boldsymbol{\varphi}^{*i} < \infty$. So by definition, $\psi_c^i, \psi_l^i < \infty$. Moreover, $\boldsymbol{\varphi}^{*i} > 0$ implies that $\lambda^i/\boldsymbol{\varphi}^{*i} < \infty$. Then by definition, $A_C, A_L < \infty$. $\square$

For any M and s^M, define (P^{s^M}) the same problem as (P) with the restriction that $\big(C(s^t), L(s^t)\big) = \big(C^*(s^t), L^*(s^t)\big), \forall t > M,\ s^t \supset s^M$, $\boldsymbol{\varphi} = \boldsymbol{\varphi}^*$, $T = T^*$, and $K_t = K_t^*, \forall t$. Note that $\{C^*(s^t), L^*(s^t), K^*(s^t)\}$ is a solution to (P^{s^M}), and (P^{s^M}) has a finite number of constraints. By a Lagrangian theorem in Luenberger (1969), there exists non-negative, not-identically zero vector $\left\{r^{s^M}, \mu^{s^M}, \eta^{s^M,1}, \dots, \eta^{s^M,I}, \gamma^{s^M}(s^0), \dots, \gamma^{s^M}(s^M)\right\}$ such that the first-order and complementarity conditions hold for $t \in \{1, .., M\}$, $s^t \subseteq s^M$, i.e.

$$\frac{\beta^t}{\tilde{q}(s^t)}\left\{r^{s^M}A_C + \sum_i \pi^i\eta^{s^M,i}(1-\sigma)\psi_c^i + \sum_{\tau=0}^{t}\sum_{s^\tau\subseteq s^M}\gamma^{s^M}(s^\tau)A_C\right\}\Phi_C^V C(s^t)^{-\sigma} = \mu^{s^M} \tag{D.9}$$

$$\frac{\beta^t}{\tilde{q}(s^t)}\left\{r^{s^M}A_L + \sum_i \pi^i\eta^{s^M,i}(1+\nu)\psi_l^i + \sum_{\tau=0}^{t}\sum_{s^\tau\subseteq s^M}\gamma^{s^M}(s^\tau)A_L\right\}\Phi_L^V L(s^t)^{\nu} = \mu^{s^M}F_L(K(s^t), L(s^t), s^t) \tag{D.10}$$

where $\tilde{q}(s^t) = q(s^t)/Pr(s^t)$.

Since the allocation is interior and $A_C, A_L > 0$, one can rewrite the first-order conditions as

$$\frac{\beta^t}{\tilde{q}(s^t)}\left\{r^{s^M}A_C + \sum_i \pi^i\eta^{s^M,i}(1-\sigma)\psi_c^i + \sum_{\tau=0}^{t}\sum_{s^\tau\subseteq s^M}\gamma^{s^M}(s^\tau)A_C\right\}\Phi_C^V C(s^t)^{-\sigma} = \mu^{s^M}$$

$$\begin{aligned}\frac{\beta^t}{\tilde{q}(s^t)}\left\{r^{s^M}A_C + \sum_i \pi^i\eta^{s^M,i}(1+\nu)\psi_l^i\frac{A_C}{A_L} + \sum_{\tau=0}^{t}\sum_{s^\tau\subseteq s^M}\gamma^{s^M}(s^\tau)A_C\right\}\Phi_C^V C(s^t)^{-\sigma}\\ = \mu^{s^M}\frac{A_C}{A_L}F_L(K(s^t), L(s^t), s^t)\frac{\Phi_C^V C(s^t)^{-\sigma}}{\Phi_L^V L(s^t)^{\nu}}\end{aligned}$$

Subtracting the first from the second line gives

$$\begin{aligned}&\frac{\beta^t}{\tilde{q}(s^t)}\left\{\Phi_C^V\sum_i \pi^i\eta^{s^M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right]\right\}C(s^t)^{-\sigma}\\ =&\mu^{s^M}\left[\frac{A_C}{A_L}F_L(K(s^t), L(s^t), s^t)\frac{\Phi_C^V C(s^t)^{-\sigma}}{\Phi_L^V L(s^t)^{\nu}} - 1\right]\end{aligned} \tag{D.11}$$

The following lemma shows that the resource constraint binds for the sub-problem (P^{s^M}) for any $M \geq 1$ and any s^M

Lemma D.5. *In the sub-problem (P^{s^M}) for any $M \geq 1$ and s^M, $\mu^{s^M} > 0$*

Proof. Suppose, by contradiction, that $\mu^{s^M} = 0$ so the resource constraint does not bind. Consider allocation $\{C(s^t), L(s^t), K(s^t)\}$ which is the solution to (P^{s^M}). Then there exists $\epsilon > 0$ such that

$$\sum_{t=0}^{\infty}\sum_{s^t} q(s^t)\left[F(K(s^{t-1}), L(s^t), s^t) + (1-\delta)K(s^{t-1}) - K(s^t) - G(s^t) - C(s^t)\right] - B(s^0) - \epsilon \geq 0$$

Define $\left\{\hat{L}(s^t)\right\}$ such that for a fixed s^1, $\hat{L}(s^1) < L(s^1)$ such that $F(K(s^1), \hat{L}(s^1), s^1) = F(K(s^1), L(s^1)) - \epsilon/q(s^1)$, and $\hat{L}(s^t) = L(s^t),\ \forall t > 1, \forall s^t$. The allocation $\left\{C(s^t), \hat{L}(s^t), K(s^t)\right\}$ satisfies the resource constraint and because of the preference's strict monotonicity, $\left\{C(s^t), \hat{L}(s^t), K(s^t)\right\}$ also satisfies the implementability constraints and the aggregate debt constraints. However,

$$\sum_{i\in I}\lambda^i\pi^i\sum_{t=0}^{\infty}\beta^t\sum_{s^t} Pr(s^t)U^i\left[h^i(C(s^t), \hat{L}(s^t);\varphi)\right] > \sum_{i\in I}\lambda^i\pi^i\sum_{t=0}^{\infty}\beta^t\sum_{s^t} Pr(s^t)U^i\left[h^i(C(s^t), L(s^t);\varphi)\right]$$

which contradicts $\left\{\left(C(s^t), L(s^t), K(s^t)\right)_{s^t}\right\}_{t=0}^{\infty}$ being optimal solution for (P^{s^M}). □

By Lemma D.5 and interior allocation, we can rewrite equation (**??**) as

$$\begin{aligned}&\frac{\Phi_C^V}{\mu^{s^M}}\sum_i \pi^i\eta^{s^M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right]\\ =&\frac{\tilde{q}(s^t)}{\beta^t}C(s^t)^{\sigma}\left[\frac{A_C}{A_L}F_L(K(s^t), L(s^t), s^t)\frac{\Phi_C^V C(s^t)^{-\sigma}}{\Phi_L^V L(s^t)^{\nu}} - 1\right]\end{aligned}$$

Specifically, for any $M \geq 1$ and s^M

$$\begin{aligned}&\frac{\Phi_C^V}{\mu^{s^M}}\sum_i \pi^i\eta^{s^M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right]\\ =&\frac{\tilde{q}(s^1)}{\beta}\left(C^*(s^1)\right)^{\sigma}\left[\frac{A_C}{A_L}F_L(K^*(s^1), L^*(s^1), s^1)\frac{\Phi_C^V C^*(s^1)^{-\sigma}}{\Phi_L^V L^*(s^1)^{\nu}} - 1\right]\end{aligned} \tag{D.12}$$

for a given s^1. Note that the left-hand side is a function of $\left(C^*(s^1), L^*(s^1), K^*(s^1)\right)$, which implies that there exists a constant κ such that $\forall M \geq 1, \forall s^M$,

$$\frac{\Phi_C^V}{\mu^{s^M}}\sum_i \pi^i\eta^{s^M,i}\left[\frac{A_C}{A_L}(1+\nu)\psi_l^i - (1-\sigma)\psi_c^i\right] = \kappa$$

Hence, (D.12) can be rewritten as

$$\frac{\beta^t}{\tilde{q}(s^t)}C(s^t)^{-\sigma}\kappa = \mu^{s^M}\left[\frac{A_C}{A_L}F_L(K(s^t), L(s^t), s^t)\frac{\Phi_C^V C(s^t)^{-\sigma}}{\Phi_L^V L(s^t)^{\nu}} - 1\right]$$

Note that $\lim_{t\to\infty} \beta^t/\tilde{q}(s^t) = 0$ and $C(s^t)^{-\sigma}$ is bounded by Lemma 4.1, so taking the limit on both sides gives

$$\lim_{t\to\infty} \frac{A_C}{A_L} F_L(K(s^t), L(s^t), s^t) \frac{\Phi_C^V C(s^t)^{-\sigma}}{\Phi_L^V L(s^t)^{\nu}} = 1$$

Hence, given the definition of $\tau^n(s^t)$ and the fact that A_C,A_L are bounded,

$$\lim_{t\to\infty} \tau^n(s^t) = \lim_{t\to\infty} \left[1 - \frac{\Phi_L^V L(s^t)^{\nu}}{\Phi_C^V C(s^t)^{-\sigma}} \frac{1}{F_L(K(s^t), L(s^t), s^t)}\right] = 1 - \frac{A_C}{A_L}$$

In addition, the above argument does not rely on the existence of lump-sum transfers. □

D.10 Proof of Lemma B.1

Proof. For any competitive equilibrium, there exists market weight $\boldsymbol{\varphi} = \{\varphi^i\}_{i\in I}$ such that $\forall t$, given C_t, L_t, individual assignment$\{c_t^i, l_t^i\}_{i\in I}$ solves

$$\begin{aligned} V(C_t, L_t; \boldsymbol{\varphi}) = \max_{(c^i,l^i)_{i\in I}} & \sum_i \varphi^i \pi^i \left[u(c^i) - v\left(l^i/\theta^i\right)\right] \\ s.t. \quad & \sum_i \pi^i c^i = C_t; \ \sum_i \pi^i l^i = L_t \end{aligned}$$

Let μ^m and η^m be the Lagrange multipliers on the consumption and labor constraints. The first-order conditions for interior solutions are

$$c_t^i = u_c^{-1}\left(\mu^m/\varphi^i\right) \tag{D.13}$$
$$l_t^i = \theta^i v_l^{-1}\left(\theta^i \eta^m/\varphi^i\right) \tag{D.14}$$

Substituting for c_t^i and l_t^i in the constraints gives

$$\sum_{i\in I} \pi^i u_c^{-1}\left(\mu^m/\varphi^i\right) = C_t$$
$$\sum_{i\in I} \pi^i \theta^i v_l^{-1}\left(\theta^i \eta^m/\varphi^i\right) = L_t$$

These equations imply functions $\mu^m(C_t)$ and $\eta^m(L_t)$. Substituting in (D.13) and (D.14), for all i implies that

$$c_t^i = u_c^{-1}\left(\mu^m(C_t)/\varphi^i\right)$$
$$l_t^i = \theta^i v_l^{-1}\left(\theta^i \eta^m(L_t)/\varphi^i\right)$$

Thus, the time-invariant functions $h^{i,c}(\cdot;\boldsymbol{\varphi}), h^{i,l}(\cdot;\boldsymbol{\varphi})$ are

$$\begin{aligned} h^{i,c}(C_t;\boldsymbol{\varphi}) &= u_c^{-1}\left(\mu^m(C_t)/\varphi^i\right) \\ h^{i,l}(L_t;\boldsymbol{\varphi}) &= \theta^i v_l^{-1}\left(\theta^i\eta^m(L_t)/\varphi^i\right) \end{aligned}$$

Note that $u_c(\cdot)$ is strictly decreasing, so $u_c^{-1}(\cdot)$ is strictly decreasing. This implies that $\mu^m(\cdot)$ is strictly decreasing. Then $h^{i,c}(\cdot;\boldsymbol{\varphi})$ must be strictly increasing. Similarly, one can argue that $h^{i,l}(\cdot;\boldsymbol{\varphi})$ is also strictly increasing. □

D.11 Proof of Lemma B.2

Proof. Given an efficient allocation $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$, suppose that $\liminf_{t\to\infty} C_t^* \leq 0$. Find $\epsilon > 0$ such that $\forall t$,

$$\sum_{s=t}^{\infty}\beta^{s-t}\left\{\sum_{i\in I}\lambda^i\pi^i\left[u\left(h^{i,c}(C_s;\boldsymbol{\varphi})\right) - v\left(h^{i,l}(L_s^*;\boldsymbol{\varphi})\right)\right]\right\} \leq M_U$$

with $C_t = \epsilon$ and $C_s = C_s^*$, $\forall s \geq t$. Such ϵ exists since the utility function is unbounded. Furthermore, there exists t_0 such that $C_{t_0}^* < \epsilon$. Then since $u(\cdot)$ and $h^{i,c}(\cdot;\boldsymbol{\varphi})$ are strictly increasing,

$$\begin{aligned} &\sum_{s=t_0}^{\infty}\beta^{s-t_0}\left\{\sum_{i\in I}\lambda^i\pi^i\left[u\left(h^{i,c}(C_s^*;\boldsymbol{\varphi})\right) - v\left(h^{i,l}(L_s^*;\boldsymbol{\varphi})\right)\right]\right\} \\ <&\sum_{s=t_0}^{\infty}\beta^{s-t_0}\left\{\sum_{i\in I}\lambda^i\pi^i\left[u\left(h^{i,c}(C_s;\boldsymbol{\varphi})\right) - v\left(h^{i,l}(L_s^*;\boldsymbol{\varphi})\right)\right]\right\} \\ \leq& M_U \\ \leq& \underline{U}_t(K_t^*) \end{aligned}$$

which is a contradiction. □

D.12 Proof of Proposition B.1

Proof. The proof follows a similar structure of the proof of Proposition 4.1. Let $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ be an interior efficient allocation. Then there exist $\boldsymbol{\lambda}$ such that $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}, \boldsymbol{\varphi}^*, T^*$ solves the planning problem (P). For any interior allocation $\{C_t, L_t, K_t\}_{t=0}^{\infty}, \boldsymbol{\varphi}, T$ from problem

(P), define the followings

$$A_C(t) = \sum_{i \in I} \pi^i \frac{\lambda^i}{\varphi^i} \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} \tag{D.15}$$

$$A_L(t) = \sum_{i \in I} \pi^i \frac{\lambda^i}{\varphi^i} \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} \tag{D.16}$$

Then the following lemma holds.

Lemma D.6. *Given an interior allocation, for all* t, $0 < \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}, \ \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} < \infty$, *and so* $0 < A_C(t), \ A_L(t) < \infty$

Proof. First, it must be that $\varphi^i > 0, \ \forall i$. Suppose there exists an i such that $\varphi^i = 0$. Then from the static sub-problem, it is optimal to set $c_t^i = 0$ for all t, which contradicts the assumption of interior allocation.

Note that from the proof of Lemma B.1, using implicit function derivatives, one has

$$\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} = \frac{\frac{1}{\varphi^i u_{cc}(h^{i,c}(t;\boldsymbol{\varphi}))}}{\sum_i \pi^i \frac{1}{\varphi^i u_{cc}(h^{i,c}(t;\boldsymbol{\varphi}))}}$$

$$\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} = \frac{\frac{\theta^i}{\varphi^i v_{ll}(h^{i,l}(t;\boldsymbol{\varphi})/\theta^i)}}{\sum_i \pi^i \frac{\theta^i}{\varphi^i v_{ll}(h^{i,l}(t;\boldsymbol{\varphi})/\theta^i)}}$$

Given $u_{cc}(\cdot) < 0$, $v_{ll}(\cdot) > 0$ by assumption 5, and $\varphi^i > 0, \ \forall i$, it must be that $\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}, \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} > 0$. Moreover, $\sum_{i \in I} \pi^i \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} = \sum_{i \in I} \pi^i \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} = 1 < \infty$ implies that $\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}, \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} < \infty$.

Since all the terms are positive and bounded, by definition, $A_C(t)$ and $A_L(t)$ are positive and bounded. □

Define (P^T) the same problem as (P) with the restriction that $(C_t, L_t) = (C_t^*, L_t^*), \forall t > T$, $\boldsymbol{\varphi} = \boldsymbol{\varphi}^*$, $T = T^*$, and $K_t = K_t^*, \forall t$. Note that $\{C_t^*, L_t^*, K_t^*\}_{t=0}^{\infty}$ is a solution to (P^T), and (P^T) has a finite number of constraints. By a Lagrangian theorem in Luenberger (1969), there exists non-negative, not identically zero vector $\{r^T, \mu^T, \eta^{T,1}, \ldots, \eta^{T,I}, \gamma_0^T, \ldots, \gamma_T^T\}$ such that the first-order and complementarity conditions hold, i.e. $\forall t \geq 1$

$$\frac{\beta^t}{q_t}\left\{r^T A_C(t) + \sum_i \pi^i \eta^{T,i}\left[\frac{V_{CC}(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})} h^{i,c}(t;\boldsymbol{\varphi}) + \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}\right] + \sum_{s=0}^{t} \gamma_s^T A_C(t)\right\} \\ * V_C(t;\boldsymbol{\varphi}) = \mu^T \tag{D.17}$$

$$\frac{\beta^t}{q_t}\left\{r^T A_L(t) + \sum_i \pi^i \eta^{T,i}\left[\frac{V_{LL}(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})} h^{i,l}(t;\boldsymbol{\varphi}) + \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}\right] + \sum_{s=0}^{t} \gamma_s^T A_L(t)\right\} \\ * V_L(t;\boldsymbol{\varphi}) = -\mu^T F_L(K_t, L_t) \tag{D.18}$$

Using the Envelope conditions of the static sub-problem, one can show that

$$\frac{V_{CC}(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})}h^{i,c}(t;\boldsymbol{\varphi}) = \frac{u_{cc}\left[h^{i,c}(t;\boldsymbol{\varphi})\right]}{u_c\left[h^{i,c}(t;\boldsymbol{\varphi})\right]}h^{i,c}(t;\boldsymbol{\varphi})\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t}$$
$$\frac{V_{LL}(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})}h^{i,l}(t;\boldsymbol{\varphi}) = \frac{v_{ll}\left[h^{i,l}(t;\boldsymbol{\varphi})\right]}{v_l\left[h^{i,l}(t;\boldsymbol{\varphi})\right]}h^{i,l}(t;\boldsymbol{\varphi})\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}$$

Define $\sigma_t^i = -\frac{u_{cc}\left[h^{i,c}(t;\boldsymbol{\varphi})\right]}{u_c\left[h^{i,c}(t;\boldsymbol{\varphi})\right]}h^{i,c}(t;\boldsymbol{\varphi})$ and $\nu_t^i = \frac{v_{ll}\left[h^{i,l}(t;\boldsymbol{\varphi})\right]}{v_l\left[h^{i,l}(t;\boldsymbol{\varphi})\right]}h^{i,l}(t;\boldsymbol{\varphi})$, then equations (D.17) and (D.18) become

$$\frac{\beta^t}{q_t}\left\{r^T A_C(t) + \sum_i \pi^i\eta^{T,i}(1-\sigma_t^i)\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} + \sum_{s=0}^{t}\gamma_s^T A_C(t)\right\}V_C(t;\boldsymbol{\varphi}) = \mu^T \tag{D.19}$$
$$\frac{\beta^t}{q_t}\left\{r^T A_L(t) + \sum_i \pi^i\eta^{T,i}(1+\nu_t^i)\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} + \sum_{s=0}^{t}\gamma_s^T A_L(t)\right\}V_L(t;\boldsymbol{\varphi}) = -\mu^T F_L(K_t, L_t) \tag{D.20}$$

Since the allocation is interior and $A_C(t),\ \ A_L(t) > 0$ by Lemma D.6, one can combine equations (D.19) and (D.20) to get

$$\begin{aligned}\frac{\beta^t}{q_t}\left\{\sum_i \pi^i\eta^{T,i}(1-\sigma_t^i)\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} - \frac{A_C(t)}{A_L(t)}\sum_i \pi^i\eta^{T,i}(1+\nu_t^i)\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}\right\}V_C(t;\boldsymbol{\varphi})\\ = \mu^T\left[1 + F_L(K_t, L_t)\frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})}\frac{A_C(t)}{A_L(t)}\right]\end{aligned} \tag{D.21}$$

Lemma D.7. *In any subproblem* (P^T) *with* $T \geq 1$, $\mu^T > 0$, *i.e. the resource constraint binds.*

Proof. Follows directly from the proof of Lemma D.2. □

Given Lemma D.7 and interior allocation, (D.21) becomes

$$\begin{aligned}\frac{1}{\mu^T}\left\{\sum_i \pi^i\eta^{T,i}\left[(1-\sigma_t^i)\frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} - \frac{A_C(t)}{A_L(t)}(1+\nu_t^i)\frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t}\right]\right\}V_C(t;\boldsymbol{\varphi})\\ = \frac{q_t}{\beta^t}\frac{1}{V_C(t;\boldsymbol{\varphi})}\left[1 + F_L(K_t, L_t)\frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})}\frac{A_C(t)}{A_L(t)}\right]\end{aligned}$$

Define the left-hand side of the above equation as $\kappa(t)$, then the following lemma gives an important property of $\kappa(t)$.

Lemma D.8. *For any sub-problem* (P^T) *with* $T \geq 1$,$\kappa(t)$ *is bounded* $\forall t \geq 1$.

Proof. Note that $\forall t, \forall i$, by assumption 6, σ_t^i and ν_t^i are bounded.

Any sub-problem (P^T) with $T \geq 1$ has

$$\sum_i \frac{\eta^{T,i}}{\mu^T} \pi^i \left[(1-\sigma_s^i) \frac{\partial h^{*i,c}(s;\boldsymbol{\varphi})}{\partial C_s^*} - \frac{A_C^*(s)}{A_L^*(s)} (1+\nu_1^i) \frac{\partial h^{*i,l}(s;\boldsymbol{\varphi})}{\partial L_s^*} \right] \\ = \frac{q_s}{\beta^s} \frac{1}{V_C^*(s;\boldsymbol{\varphi})} \left[1 + F_L^*(s) \frac{V_C^*(s;\boldsymbol{\varphi})}{V_L^*(s;\boldsymbol{\varphi})} \frac{A_C^*(s)}{A_L^*(s)} \right]$$

for $s = 1, ..., \|I\|$.

The above equations formulate a linear system with respect to $\|I\|$ variables $\left\{ \frac{\eta^{T,i}}{\mu^T} \right\}_{i \in I}$. By Lemma D.6 and interior allocation, the right-hand sides and the coefficients are bounded. Therefore, for any T, $\left\{ \frac{\eta^{T,i}}{\mu^T} \right\}_{i \in I}$ are functions of $\{C_s^*, L_s^*, K_s^*\}_{s=0}^{\|I\|}, \boldsymbol{\varphi}^*$and bounded.

So $\forall t \geq 1$,

$$\kappa(t) = \sum_i \frac{\eta^{T,i}}{\mu^T} \pi^i \left[(1-\sigma_t^i) \frac{\partial h^{i,c}(t;\boldsymbol{\varphi})}{\partial C_t} - \frac{A_C(t)}{A_L(t)} (1+\nu_1^i) \frac{\partial h^{i,l}(t;\boldsymbol{\varphi})}{\partial L_t} \right]$$

is bounded. □

Substituting for $\kappa(t)$ into equation (D.21) provides

$$\frac{\beta^t}{q_t} \kappa(t) V_C(t;\boldsymbol{\varphi}) = \left[1 + F_L(K_t, L_t) \frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})} \frac{A_C(t)}{A_L(t)} \right]$$

Assumption 1 implies that $\lim_{t\to\infty} \beta^t / q_t = 0$. By Lemma D.8, $\kappa(t)$ is bounded. Since $\liminf_{t\to\infty} C_t > 0$ from Lemma B.2, $V_C(t;\boldsymbol{\varphi}) = \varphi^i u_c(h^{i,c}(t;\boldsymbol{\varphi}))$ is bounded. Then taking the limit as $t \to \infty$ on both sides of the above equation gives

$$\lim_{t\to\infty} \left[1 + F_L(K_t, L_t) \frac{V_C(t;\boldsymbol{\varphi})}{V_L(t;\boldsymbol{\varphi})} \frac{A_C(t)}{A_L(t)} \right] = 0$$

From Lemma D.6, it must be true that as t approaches infinity, we have that $0 < A_C(t), A_L(t) < \infty$, so $-\infty < \liminf_{t\to\infty} \frac{A_C(t)}{A_L(t)}$, $\limsup_{t\to\infty} \frac{A_C(t)}{A_L(t)} < \infty$. Define $\underline{\tau} = 1 - \limsup_{t\to\infty} \frac{A_C(t)}{A_L(t)}$ and $\bar{\tau} = 1 - \liminf_{t\to\infty} \frac{A_C(t)}{A_L(t)}$. Then using the definition of τ_t^n gives

$$\liminf_{t\to\infty} \tau_t^n = \liminf_{t\to\infty} \left[1 + \frac{1}{F_L(K_t, L_t)} \frac{V_L(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})} \right] = 1 - \limsup_{t\to\infty} \frac{A_C(t)}{A_L(t)} = \underline{\tau}$$
$$\limsup_{t\to\infty} \tau_t^n = \limsup_{t\to\infty} \left[1 + \frac{1}{F_L(K_t, L_t)} \frac{V_L(t;\boldsymbol{\varphi})}{V_C(t;\boldsymbol{\varphi})} \right] = 1 - \liminf_{t\to\infty} \frac{A_C(t)}{A_L(t)} = \bar{\tau}$$

In the case of steady states, it must be true that $A_C(\infty), A_L(\infty)$ exist and that $0 < A_C(\infty), A_L(\infty) < \infty$. Hence, $\lim_{t\to\infty} \tau_t^n = 1 - \frac{A_C(\infty)}{A_L(\infty)}$.

Similarly, the argument of the proof does not rely on lump-sum transfers. □

D.13 Proof of Lemma 6.1

Proof. Note that the sustainability constraint is rewritten as $\forall t$,

$$\sum_{s=t}^{\infty} \beta^{s-t} \left\{ \Phi_C^P \frac{C_s^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s^{1+\nu}}{1+\nu} \right\} \geq \underline{U}$$

Define $u_t = \Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu}$. Then the proof is similar to Lemma 2 in Aguiar and Amador (2016). □

E Numerical Appendix

This section explains the numerical algorithm that is implemented in Section 6 for a simple environment with no capital, and additional plots.

E.1 Deviation Utility

The deviation utility $\underline{U}$ is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes where the government does not issue external debt. Given that output is equal to the total efficiency-unit labor supply, one has

$$\begin{aligned}
\underline{U} \equiv \max_{c_t^i, l_t^i, \tau_t^n, T_t} & \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i U^i(c_t^i, l_t^i) \\
s.t. \quad & c_t^i + b_{t+1}^{i,d} = (1-\tau_t^n) l_t^i - T_t + (1+r_t) b_t^{i,d} \\
& G_t + (1+r_t) B_t^d \leq \tau_t^n L_t + T_t + B_{t+1}^d
\end{aligned}$$

There exist a vector of market weights $\hat{\varphi}$ such that

$$\begin{aligned}
\underline{U} \equiv \max_{C_t, L_t, \hat{\varphi}, T} & \sum_{t=0}^{\infty} \beta^t \left[\hat{\Phi}_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \hat{\Phi}_L^W \frac{L_t^{1+\nu}}{1+\nu} \right] \\
s.t. \quad & C_t + G_t \leq L_t
\end{aligned}$$

where $\hat{\psi}_c^i, \hat{\psi}_l^i, \hat{\Phi}_C^V, \hat{\Phi}_L^V, \hat{\Phi}_C^W, \hat{\Phi}_L^W$ are calculated using $\hat{\varphi}$.

E.2 Algorithm

State variables: μ, Γ

1. Guess μ and φ. Compute η.

(a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ

Initial guess of the expectation $V(\mu_t, \Gamma_{t-1}) = \sum_{s=t}^{\infty} \beta^{\tau - t} \left[\Phi_C^P \frac{C_s{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s{}^{1+\nu}}{1+\nu} \right]$.

(b) Assume the constraint does not bind in t: $\gamma_t = 0$. Solve for the allocation C_t, L_t using FOCs

$$\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1} \right] C_t{}^{-\sigma} = \mu$$
$$\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1} \right] L_t{}^{\nu} = \mu$$

(c) Compute $V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$, then compute

$$\begin{aligned} A_t &= \sum_{s=t}^{\infty} \beta^{\tau - t} \left[\Phi_C^P \frac{C_s{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s{}^{1+\nu}}{1+\nu} \right] \\ &= \Phi_C^P \frac{C_t{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t{}^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1}, \Gamma_t) \end{aligned}$$

(d) Check if $A_t \geq \underline{U}_t$. If it is, proceed to the next step. If not, solve for C_t, L_t, γ_t using these optimality equations

$$\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1} \right] C_t{}^{-\sigma} = \mu$$
$$\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1} \right] L_t{}^{\nu} = \mu$$
$$\Phi_C^P \frac{C_t{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t{}^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* (\Gamma_{t-1} + \gamma_t)) = \underline{U}_t$$

(e) Given C_t, L_t, γ_t (γ_t can be zero or not), compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* (\Gamma_{t-1} + \gamma_t))$. Update the value function

$$V^{n+1}(s_t, \Gamma_{t-1}) = \Phi_C^P \frac{C_t{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t{}^{1+\nu}}{1+\nu} + \beta V^n(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* (\Gamma_{t-1} + \gamma_t))$$

2. Compute residuals to find μ and φ

$$r^{\mu} = \sum_{t=0}^{\infty} q_t \left[L_t - G_t - C_t \right] - B_0$$
$$r_{ij}^{\varphi} = \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^V \left(\psi_c^i - \psi_c^j \right) C_t{}^{1-\sigma} - \Phi_L^V \left(\psi_l^i - \psi_l^j \right) L_t{}^{1+\nu} \right]$$

$$r = (r^{\mu})^2 + \sum_{i,j} (r_{ij}^{\varphi})^2 \tag{E.1}$$

3. Find μ and φ such that (E.1) is minimized using a Nelder-Mead algorithm.