

BiCGSTAB

In numerical linear algebra, the biconjugate gradient stabilized method, often abbreviated as BiCGSTAB, is an iterative method developed by H. A. van der Vorst for the numerical solution of nonsymmetric linear systems. It is a variant of the biconjugate gradient method (BiCG) and has faster and smoother convergence than the original BiCG as well as other variants such as the conjugate gradient squared method (CGS).

Unpreconditioned BiCGSTAB

To solve a linear system $Ax = b$, BiCGSTAB starts with an initial guess x_0 and proceeds as follows:

1. $r_0 = b - Ax_0$
2. Choose an arbitrary vector \hat{r}_0 such that $(\hat{r}_0, r_0) \neq 0$, e.g., $\hat{r}_0 = r_0$
3. $\rho_0 = \alpha = \omega_0 = 1$
4. $v_0 = p_0 = 0$
5. For $i = 1, 2, 3, \dots$
 - a. $\rho_i = (\hat{r}_0, r_{i-1})$
`double roc=0;`
`for (int i=1; i<=n; ++i) {`
`double v1=0;`
`if (r.getLine(i)!=NULL) v1=r.getLine(i)->val;`

`roc+=v1*r1.getLine(i)->val;`
`}`
 - b. $\beta = (\rho_i/\rho_{i-1})(\alpha/\omega_{i-1})$
`double beta=(roc/ro)*(alpha/w);`
 - c. $p_i = r_{i-1} + \beta(p_{i-1} - \omega_{i-1}v_{i-1})$
`p=r+(p-v*w)*beta;`
 - d. $v_i = Ap_i$
`v=A*p;`
 - e. $\alpha = \rho_i/(\hat{r}_0, v_i)$
`alpha=roc;`
`double sub=0;`
`for (int i=1; i<=n; ++i) {`
`double v1=0;`
`if (v.getLine(i)!=NULL) v1=v.getLine(i)->val;`
`sub+=v1*r1.getLine(i)->val;`
`}`
`alpha/=sub;`
 - f. $h = x_{i-1} + \alpha p_i$
`h=x+p*alpha;`
 - g. If h is accurate enough, then set $x_i = h$ and quit
`double eBic=(A*h-b).norm();`
`if (eBic<=eSor) {`
`x=h;`

```

        break;
    }

```

h. $s = r_{i-1} - \alpha v_i$

```
s=r-v*alpha;
```

i. $t = As$

```
t=A*s;
```

j. $\omega_i = (t, s) / (t, t)$

```
double sus=0, jos=0;
```

```
for (int i=1; i<=n; ++i) {
```

```
    double v1=0;
```

```
    if (t.getLine(i)!=NULL)
```

```
        v1=t.getLine(i)->val;
```

```
    double v2=0;
```

```
    if (s.getLine(i)!=NULL)
```

```
        v2=s.getLine(i)->val;
```

```
    sus+=v1*v2;
```

```
    jos+=v1*v1;
```

```
}
```

```
w=sus/jos;
```

k. $x_i = h + \omega_i s$

```
x=h+s*w;
```

l. If x_i is accurate enough, then quit

```
double eBic=(A*x-b).norm();
```

```
if (eBic<=eSor) {
```

```
    ++k;
```

```
    Break;
```

```
}
```

m. $r_i = s - \omega_i t$

```
r=s-t*w;
```

```
ro=roc;
```