Grafica pe calculator

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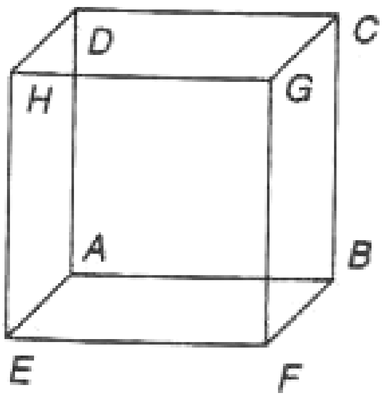
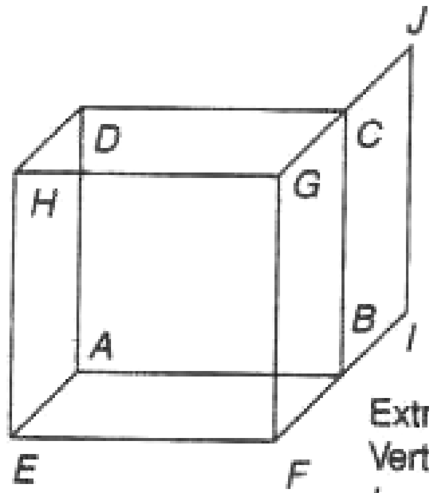
Modelarea solidelor

Reprezentarea wireframe a unui cub



Vertices		Lines
A	(0, 0, 0)	*AB*
B	(1, 0, 0)	*BC*
C	(1, 1, 0)	*CD*
D	(0, 1, 0)	*DA*
E	(0, 0, 1)	*EF*
F	(1, 0, 1)	*FG*
G	(1, 1, 1)	*GH*
H	(0, 1, 1)	*HE*
		AE
		BF
		CG
		DH

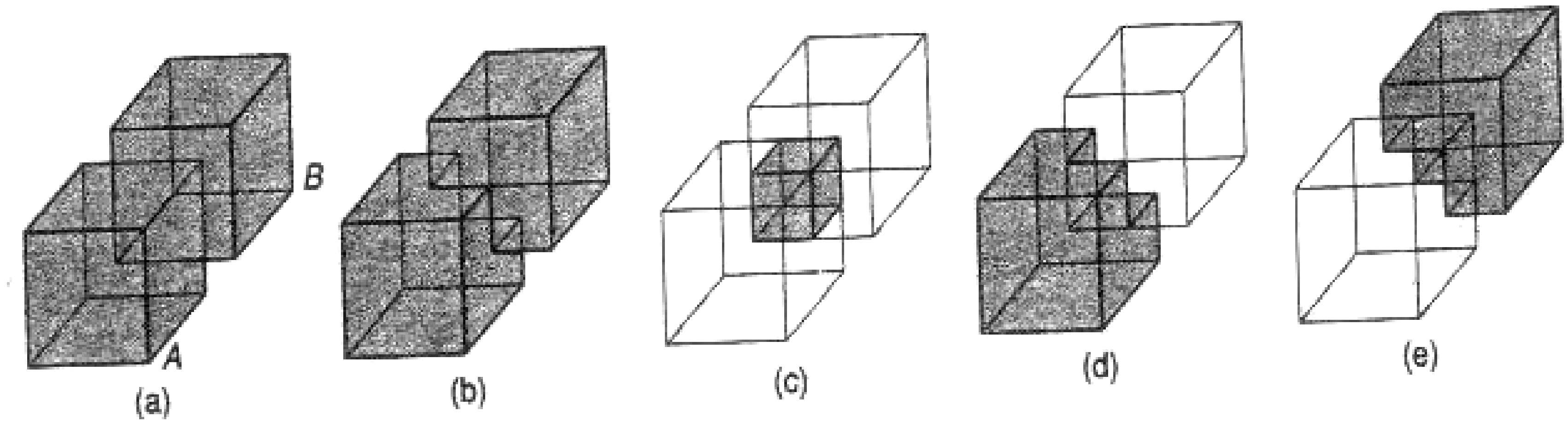


Extra Vertices		Extra Lines
I	(1, 0, −1)	*BI*
J	(1, 1, −1)	*IJ*
		JC

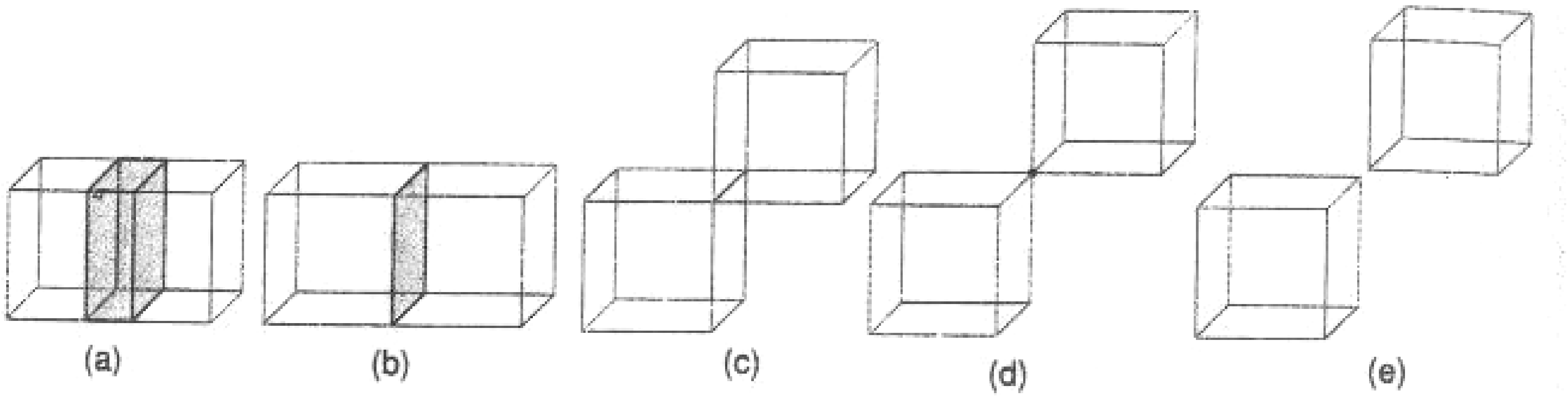
Schema de reprezentare a solidelor

- Domeniu
- Neambiguitate
- Unicitate
- Precizie
- Imposibilitatea crearii unei reprezentari nevalide
- Usurinta crearii unei reprezentari valide
- Inchiderea la translatii, rotatii, etc.
- Compactitate
- Algoritmi eficienti

Operatii booleene: A, B, A reunit B, A intersectat B, A-B si B-A



Intersectia booleana a 2 cuburi poate produce : un solid, un plan, un segment de dreapta, un punct sau multimea vida.



Elemente de topologie

- Multimi deschise
- Multimi inchise
- Punct interior
- Punct exterior
- Punct aderent
- Frontiera

Elemente de topologie

- Multimi deschise
  - Spatiu metric (X,d)
  - $$\begin{cases} S(x_0,\varepsilon) = \{x \in X \mid d(x_0,x) < \varepsilon\} \\ T(x_0,\varepsilon) = \{x \in X \mid d(x_0,x) \le \varepsilon\} \end{cases}$$
  - Definitie

$$D \subset (X,d)$$
$$D = \varnothing$$
$$\vee$$
$$(\forall x \in D)(\exists S(x_0,\varepsilon) \subset D)(x \in S(x_0,\varepsilon))$$

Elemente de topologie

- Multimi inchise
  - Definitie

$$F \subset (X, d)$$
$$X \setminus F \quad \textbf{este deschisa}$$

Elemente de topologie

- Punct interior
  - Definitie

$$A \subseteq (X, d),\ x_0 \in A$$

$$(\exists r > 0)\left(S(x_0, r) \subset A\right)$$

$$\overset{o}{A} \quad \textbf{punctele interioare ale lui A}$$

Elemente de topologie

- Punct interior
  - $A \quad \textbf{deschisa} \iff A = \overset{o}{A}$
  - $\overset{o}{A} = \bigcup_{i \in I} D_i,\ D_i \subseteq A \quad \textbf{deschise}$
  - $\overset{o}{A} = \overset{\overset{o}{o}}{A}$
  - $\overset{o}{A} \subseteq A$

Elemente de topologie

- Punct exterior

$$\boldsymbol{Ext}(\boldsymbol{A}) = \left\{ \boldsymbol{x} \in \boldsymbol{X} \middle| \boldsymbol{x} \in \overset{o}{\overline{\boldsymbol{X} \setminus \boldsymbol{A}}} \right\}$$

Elemente de topologie

- Punct aderent
  - Definitie

$$A \subseteq (X, d),\ x \in X$$

$$(\forall \varepsilon > 0)(S(x, \varepsilon) \cap A \neq \varnothing)$$

$\overline{A}$ **punctele aderente ale lui A**

Elemente de topologie

- Punct aderent
  - A **inchisa** $\Leftrightarrow A = \overline{A}$
  - $\overline{A} = \bigcap_{i \in I} F_i,\ A \subseteq F_i$ **inchise**
  - $\overline{\overline{A}} = \overline{A}$
  - $A \subseteq \overline{A}$

Elemente de topologie

- Puncte aderente/interioare

$$A \subset (X, d)$$

$$X \setminus \overset{o}{A} = \overline{X \setminus A}$$

$$X \setminus \overline{A} = \overset{o}{\overline{X \setminus A}}$$

Elemente de topologie

- Frontiera
  - Definitie

$$A \subseteq (X, d)$$

$$\mathbf{Fr}\ A = \overline{A} \cap \overline{X \setminus A} = \overline{A} \cap \left(X \setminus \overset{o}{A} \right) = \overline{A} \setminus \overset{o}{A}$$

Elemente de topologie

- Frontiera

Daca $A \subseteq (X, d)$ **atunci** $\text{Fr } A$ **este multime inchisa**

Daca $A \subseteq (X, d)$ **atunci** $\text{Fr } A = \text{Fr } X \setminus A$

Daca $A \subseteq (X, d)$ **atunci** $\text{Fr } \overset{o}{A} \subseteq \text{Fr } A, \ \text{Fr } \overline{A} \subseteq \text{Fr } A$

Daca $A \subseteq (X, d)$ **atunci** $\overline{A} = A \bigcup \text{Fr } A, \ \overset{o}{A} = A \setminus \text{Fr } A$

Daca $A \subseteq (X, d)$ **atunci** A **deschisa** $\Leftrightarrow A \bigcap \text{Fr } A = \emptyset$

Daca $A \subseteq (X, d)$ **atunci** A **inchisa** $\Leftrightarrow \text{Fr } A \subseteq A$

Elemente de topologie

Daca $A \subseteq (X, d)$ **este inchisa atunci** $A = \overset{o}{A} \cup \mathrm{Fr}\, A$

Fie $A \subseteq (X, d)$. **Atunci** $\mathrm{reg}(A) \stackrel{\bullet}{\underset{\bullet}{=}} \overline{\overset{o}{A}}$.

Regularizarea unui obiect.

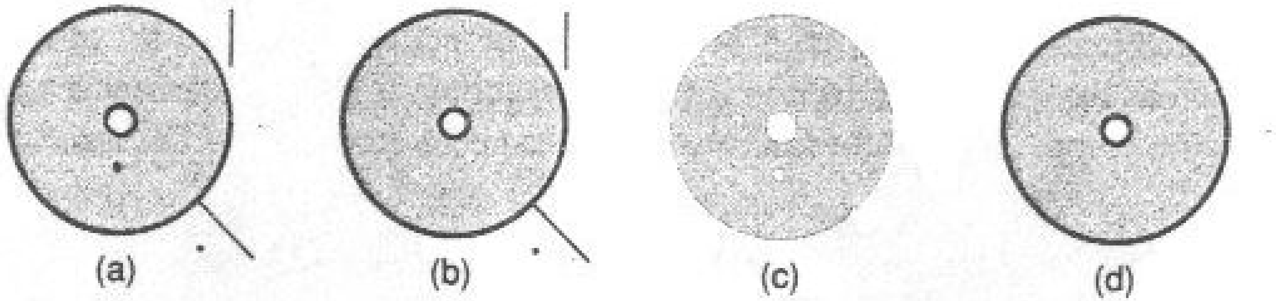


Fig. 12.4 Regularizing an object. (a) The object is defined by interior points, shown in light gray, and boundary points. Boundary points that are part of the object are shown in black; the rest of the boundary points are shown in dark gray. The object has dangling and unattached points and lines, and there is a boundary point in the interior that is not part of the object. (b) Closure of the object. All boundary points are part of the object. The boundary point embedded in the interior of (a) is now part of the interior. (c) Interior of the object. Dangling and unattached points and lines have been eliminated. (d) Regularization of the object is the closure of its interior.

Intersectia booleana

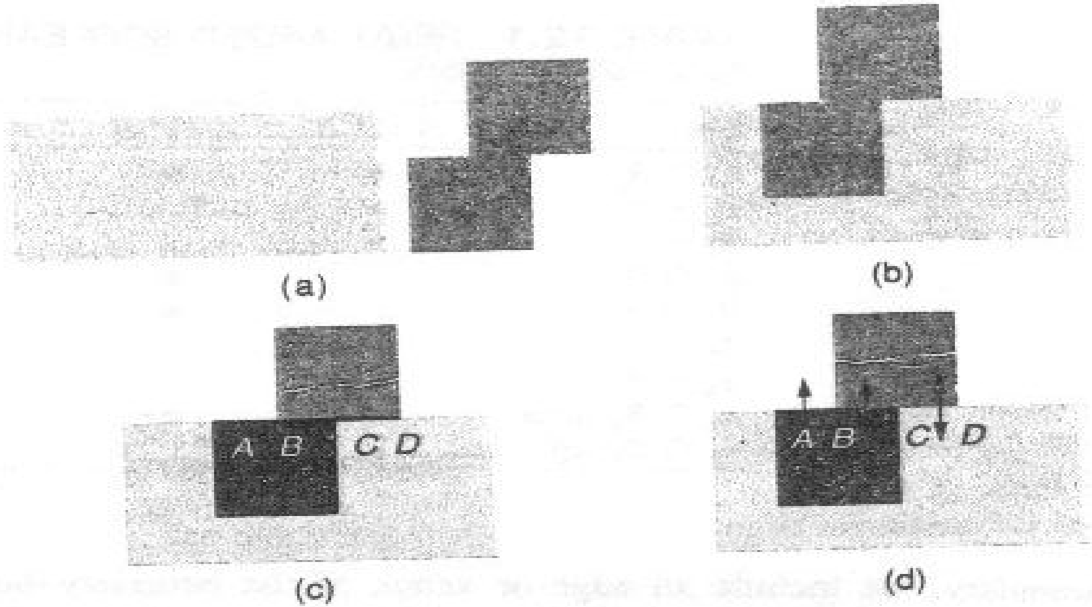
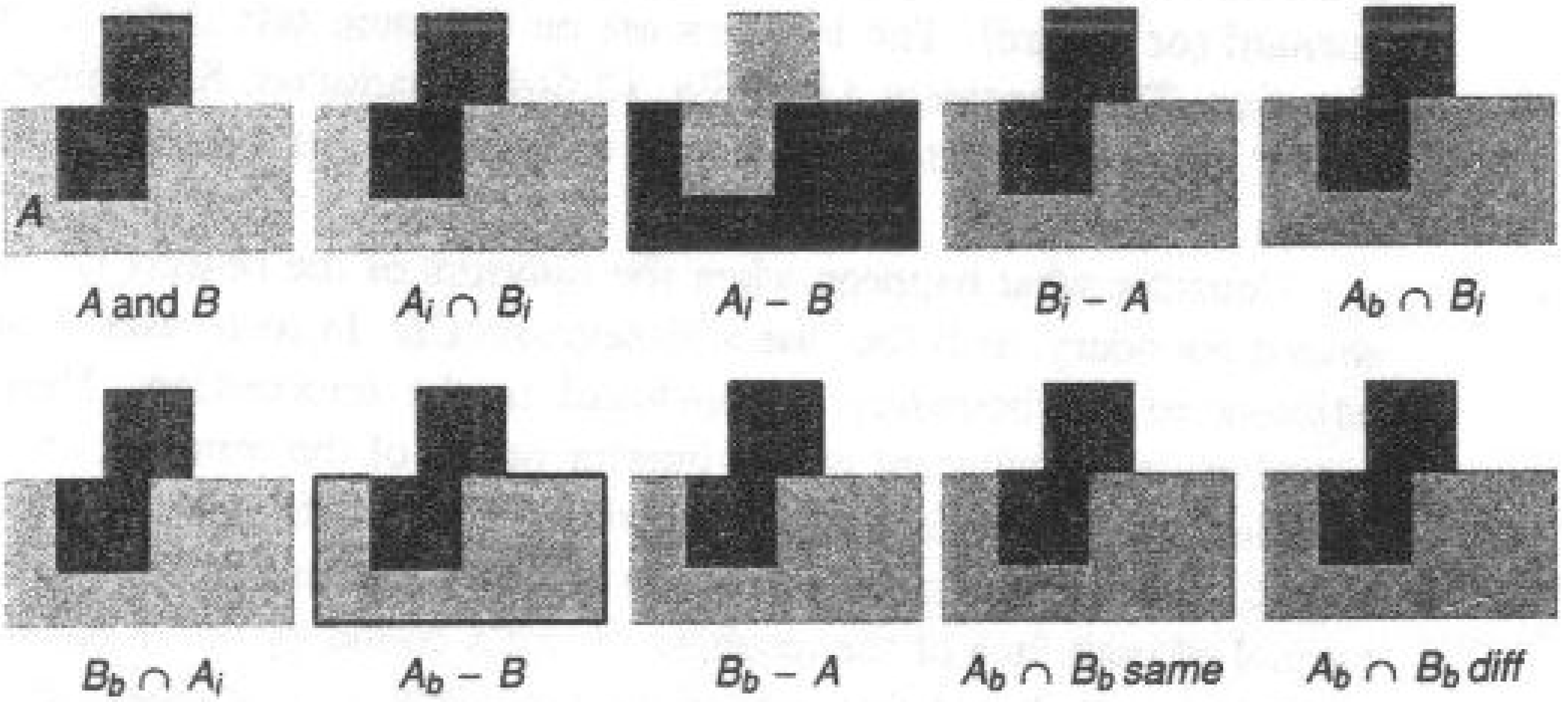
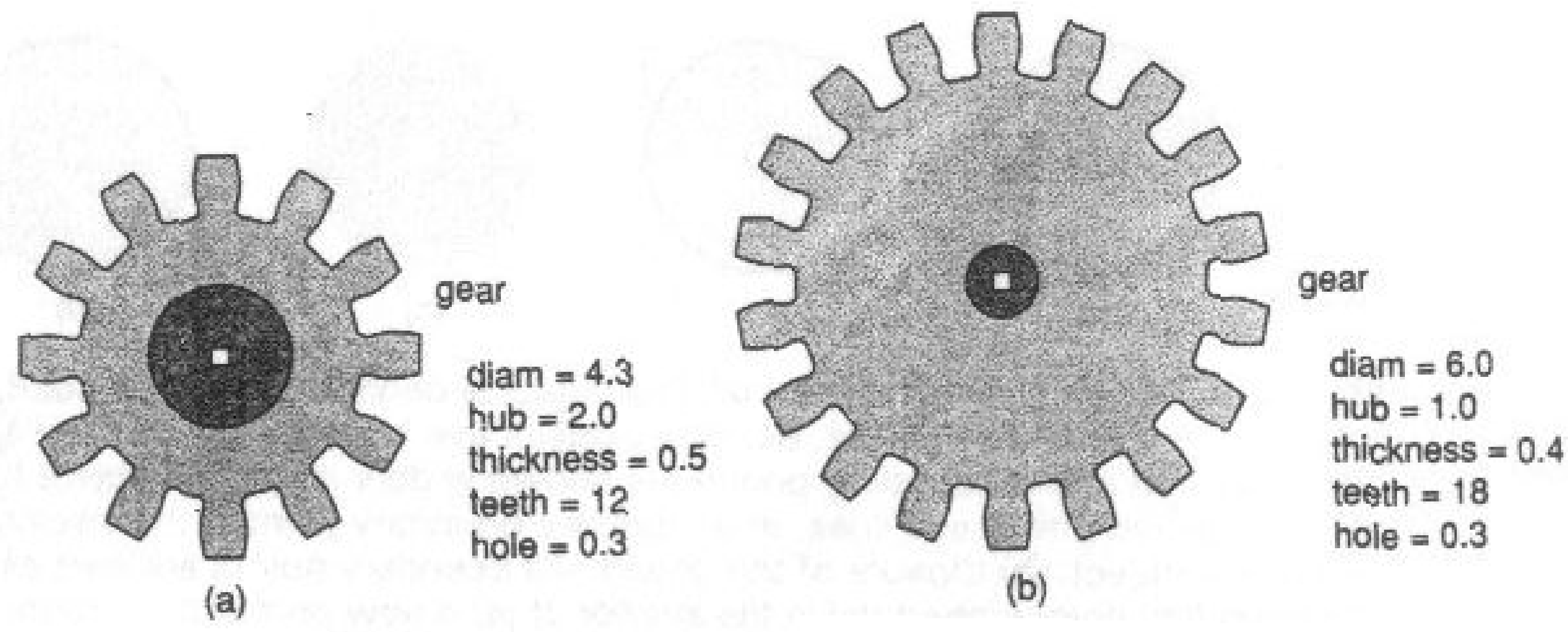


Fig. 12.5 Boolean intersection. (a) Two objects, shown in cross-section. (b) Positions of object prior to intersection. (c) Ordinary Boolean intersection results in a dangling face, shown as line *CD* in cross-section. (d) Regularized Boolean intersection includes a piece of shared boundary in the resulting boundary if both objects lie on the same side of it (*AB*), and excludes it if the objects lie on opposite sides (*CD*). Boundary–interior intersections are always included (*BC*).

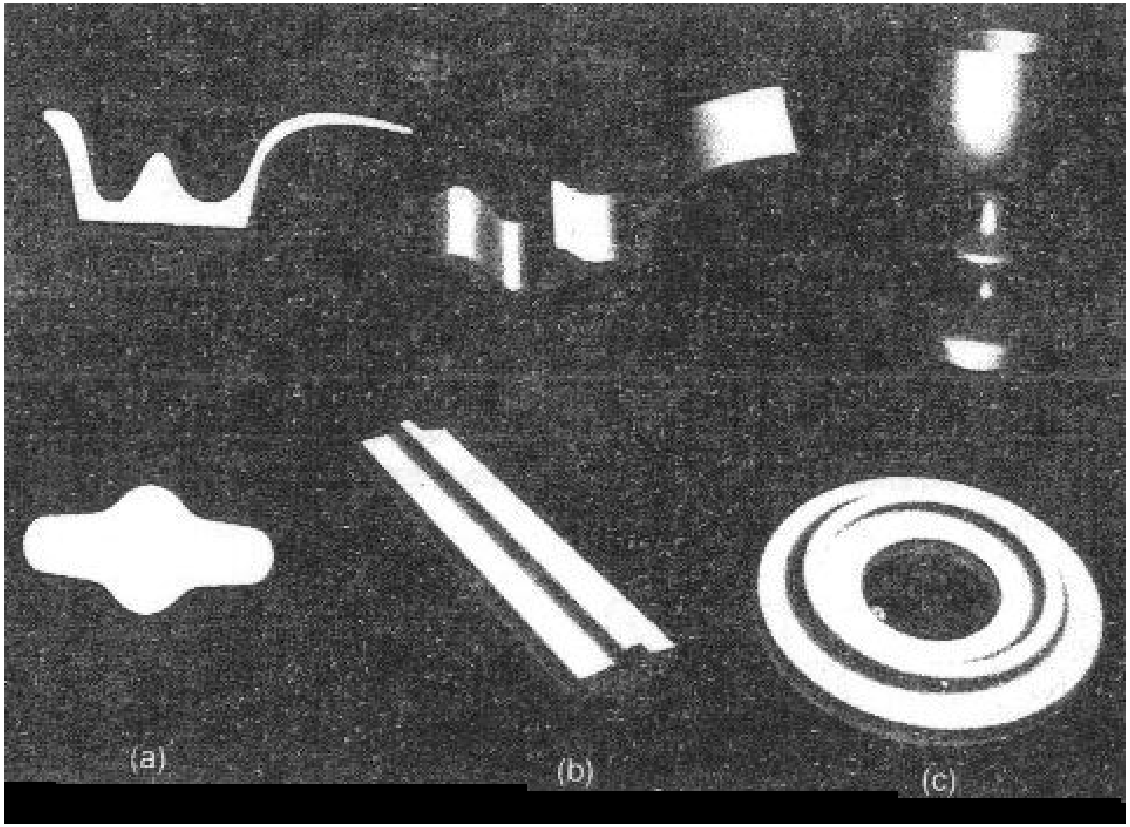
Operatiile booleene care intervin in operatiile booleene regularizate



2 roti dintate definite prin instantierea unei primitive

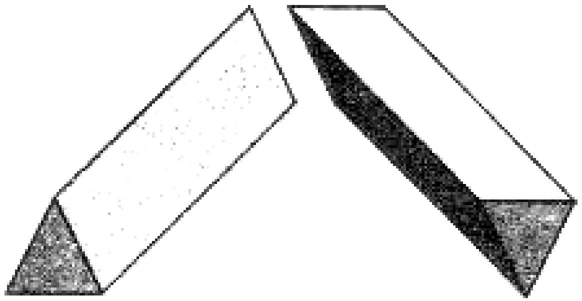


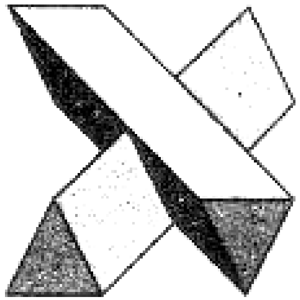
Tehnica baleierii (sweep)



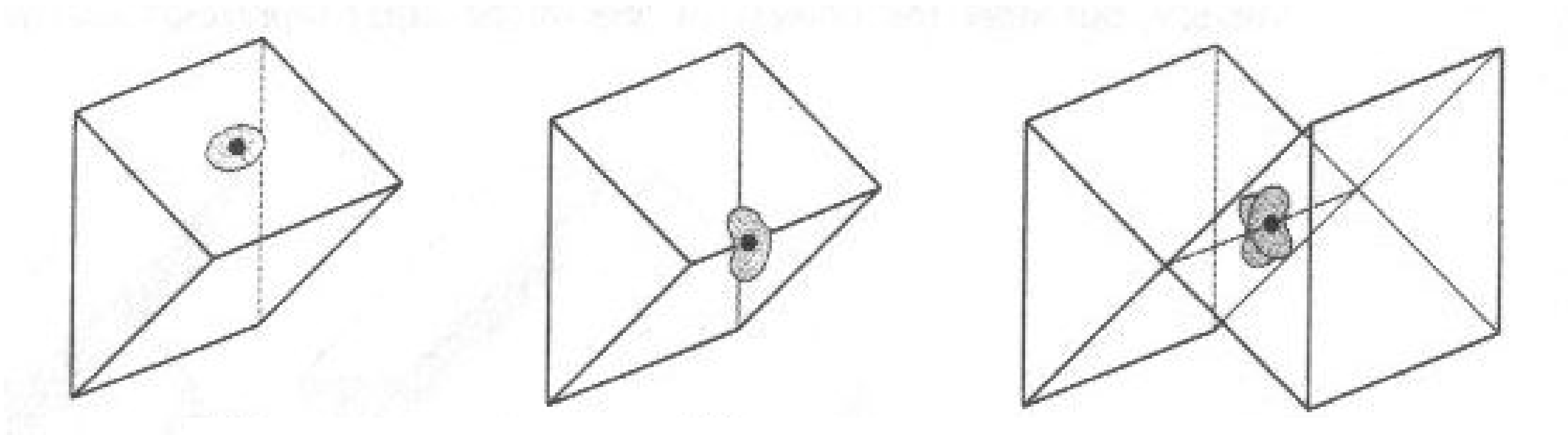
Obiecte 3D obtinute prin baleiere translationala (b) sau baleiere rotationala (c) a unor obiecte 2D

Reuniunea (b) a 2 solide obtinute prin baleiere (a) nu este, in general, un solid cu aceeasi proprietate (i.e., obtinut prin baleiere)

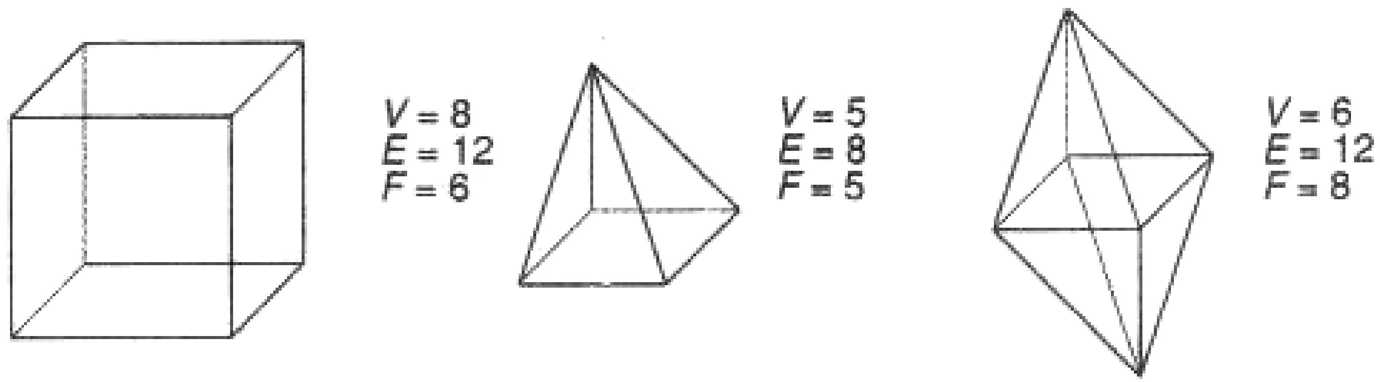

(a)


(b)

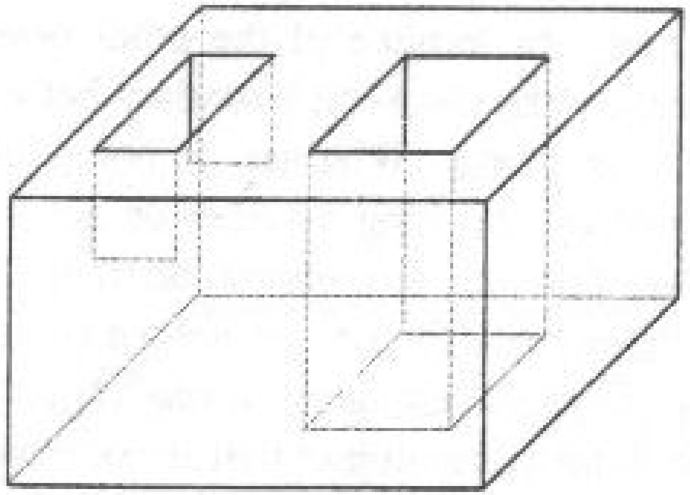
Varietati de ordin 2
(2-manifold)



Formula lui Euler : V-E+F=2

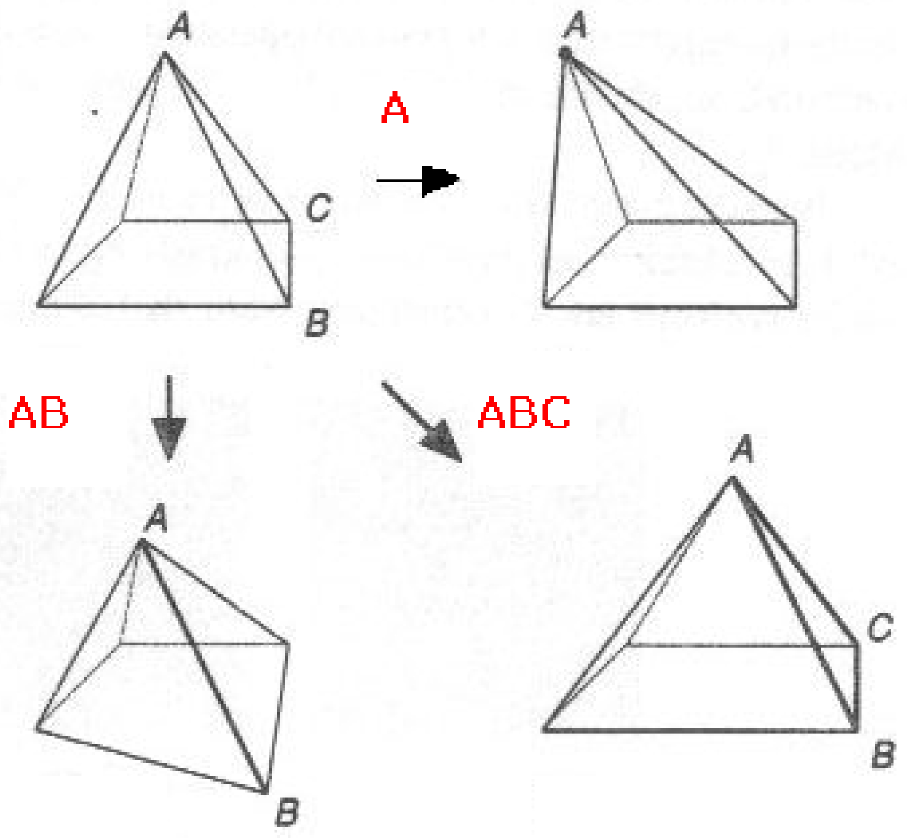


Formula lui Euler generalizata

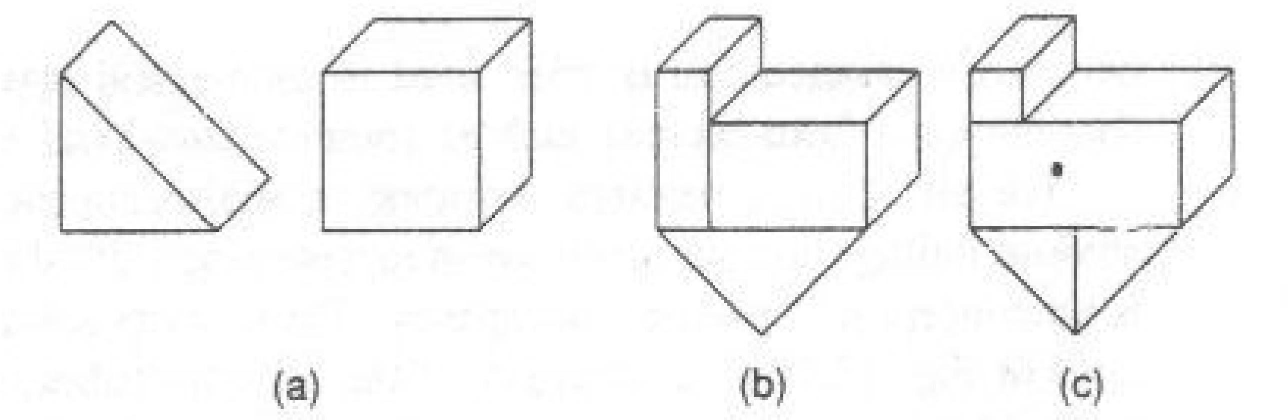


$$\begin{array}{ccccccccc} V & - & E & + & F & - & H & = & 2(C - G) \\ 24 & & 36 & & 15 & & 3 & & 1 \quad 1 \end{array}$$

Operatori “tweaking”

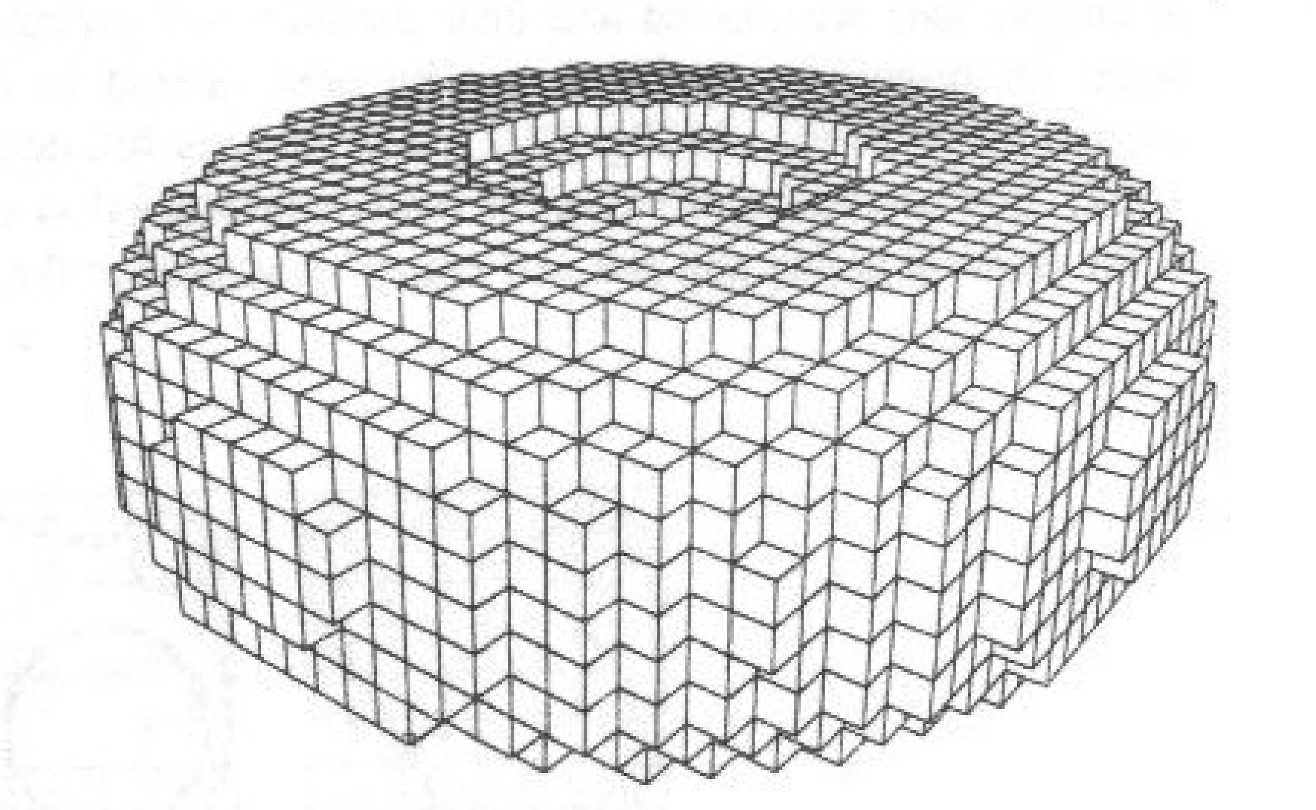


Descompunerea in celule a unui obiect este neambigua dar nu este unica

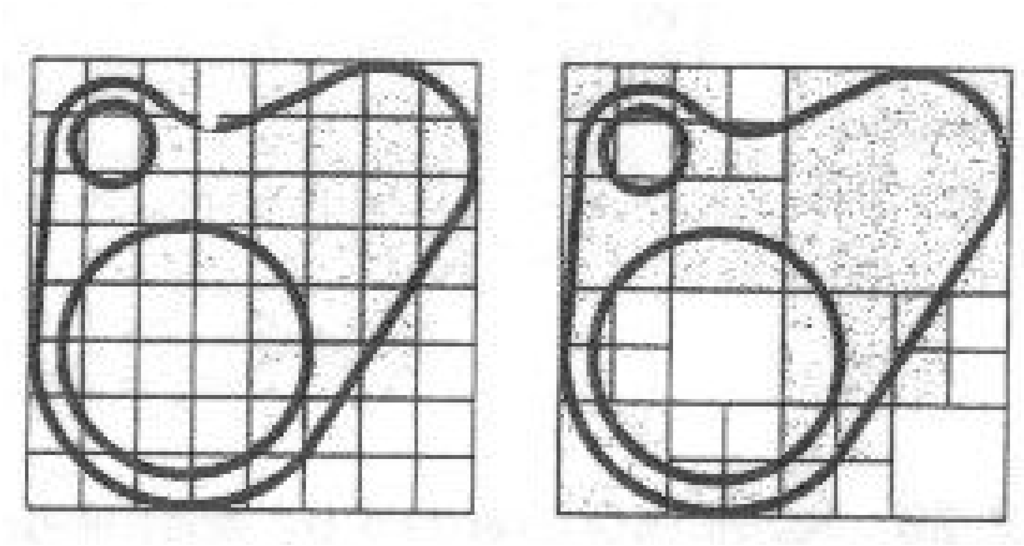


Celulele din (a) se pot transforma astfel incat sa construim obiectul (b,c) in 2 moduri diferite.

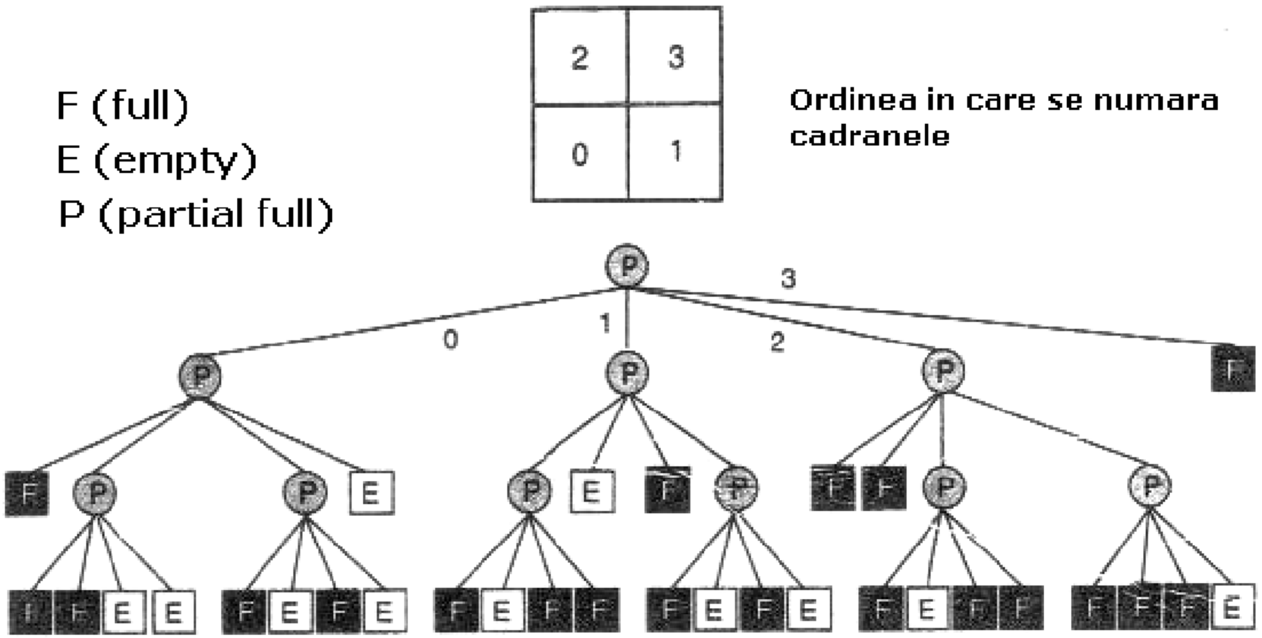
Reprezentarea unui tor prin enumerarea ocuparii spatiale



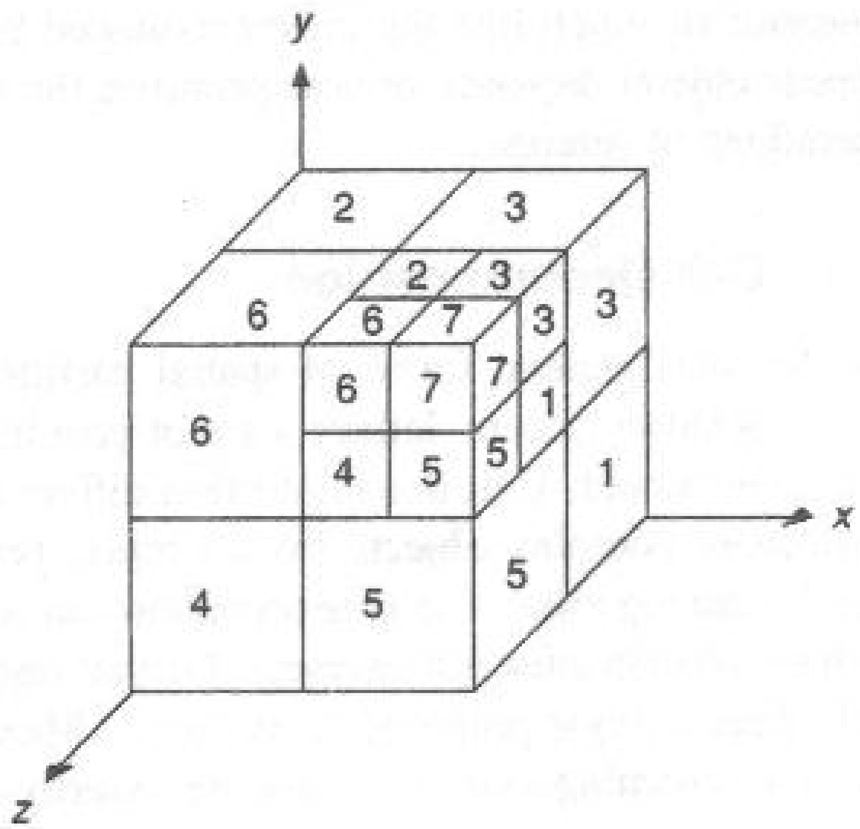
Obiect reprezentat prin enumerarea ocuparii spatiale si prin $ACOS_4$



Structura de date $ACOS_4$



Enumerarea octantilor $ACOS_8$



Structura de date ACOS_n

$$\text{ACOS}_n = \left(Q, \Sigma_n, \left\{ \xrightarrow{a} \,\middle|\, \xrightarrow{a} \subseteq Q^2, a \in \Sigma_n \right\}, q_0, \ell \right),$$

Q multimea de noduri

q_0 nodul radacina

$\ell : Q \to \{F\text{ull}, E\text{mpty}, P\text{artial Full}\}$ functie de etichetare a nodurilor

$\left\{ \xrightarrow{a} \,\middle|\, \xrightarrow{a} \subseteq Q^2, a \in \Sigma_n \right\}$ multimea arcelor etichetate cu etichete din Σ_n

$n = 4,8$

$\Sigma_4 = \{\text{sw, se, nw, ne}\}$

$\Sigma_8 = \{\text{ldb, rdb, lub, rub, ldf, rdf, luf, ruf}\}$

Structura de date ACOS$_n$

$$\text{Pre, Post } : Q \times \Sigma_n \to 2^Q$$

$$\text{Pre}(q, a) = \left\{ q' \middle| q' \in Q \wedge q' \xrightarrow{\quad a \quad} q \right\}$$

$$\text{Post}(q, a) = \left\{ q' \middle| q' \in Q \wedge q \xrightarrow{\quad a \quad} q' \right\}$$

$$\text{Pre}(q) = \bigcup_{a \in \Sigma_n} \text{Pre}(q, a)$$

$$\text{Post}(q) = \bigcup_{a \in \Sigma_n} \text{Post}(q, a)$$

Structura de date ACOS$_n$

$$(\forall q \in Q)(\forall a \in \Sigma_n)\left(\left|\mathrm{Pre}(q,a)\right| \le 1 \wedge \left|\mathrm{Post}(q,a)\right| \le 1\right)$$

$$(\forall q \in Q)\begin{pmatrix} \left|\mathrm{Post}(q)\right| \in \{0, n\} \wedge \\ \left|\mathrm{Post}(q)\right| = 0 \Leftrightarrow \ell(q) \in \{F, E\} \wedge \\ \left|\mathrm{Post}(q)\right| = n \Leftrightarrow \ell(q) = P \end{pmatrix}$$

$$\mathrm{Pre}(q_0) = \varnothing$$

Structura de date ACOS$_n$ reuniunea

Date doua obiecte ACOS

$$\mathrm{ACOS}_{\mathrm{n}}^{\mathrm{i}} = \begin{pmatrix} Q_i, \Sigma_n, \\ \left\{ \xrightarrow{a}_i \middle| \xrightarrow{a}_i \subseteq Q_i \times Q_i, a \in \Sigma_n \right\}, \\ q_0^i, \ell_i \end{pmatrix}, i = 1,2$$

sa se obtina obiectul reuniune

$$\mathrm{ACOS}_{\mathrm{n}}^{3} = \begin{pmatrix} Q_3, \Sigma_n, \\ \left\{ \xrightarrow{a}_3 \middle| \xrightarrow{a}_3 \subseteq Q_3 \times Q_3, a \in \Sigma_n \right\}, \\ q_0^3, \ell_3 \end{pmatrix}$$

Structura de date $ACOS_n$ reuniunea

$$Q_3 \subseteq Q_1 \times Q_2$$

$$q_0^3 = \left(q_0^1, q_0^2\right)$$

Definim operatia $\bigcup$ astfel :

$$\bigcup : \{P, E, F\} \times \{P, E, F\} \to \{P, E, F\}$$

$$\left(\forall x \in \{P, E, F\}\right)\begin{pmatrix} x \bigcup F = F \wedge \\ x \bigcup E = x \end{pmatrix}$$

$$P \bigcup P = P$$

Structura de date ACOS$_n$
reuniunea
reguli de inferenta

Regula 1

$$\frac{}{q_0^3 = \left(q_0^1, q_0^2\right) \in Q_3 \quad \ell_3(q_0^3) = \ell_1(q_0^1) \cup \ell_2(q_0^2)}$$

Structura de date ACOS$_n$ reuniunea reguli de inferenta

Regula 2

$$\frac{(q_1, q_2) \in Q_3 \quad \ell_1(q_1) = P \quad \ell_2(q_2) = E \quad \ell_3(q_1, q_2) = P \quad a \in \Sigma_n}{\begin{array}{c}(\text{Post}(q_1, a), q_2) \in Q_3 \\ (q_1, q_2) \xrightarrow{a}_3 (\text{Post}(q_1, a), q_2) \\ \ell_3((\text{Post}(q_1, a), q_2)) = \ell_1(\text{Post}(q_1, a))\end{array}}$$

Structura de date ACOS$_n$ reuniunea reguli de inferenta

Regula 3

$$\frac{(q_1, q_2) \in Q_3 \quad \ell_1(q_1) = E \quad \ell_2(q_2) = P \qquad \ell_3(q_1, q_2) = P \quad a \in \Sigma_n}{(q_1, \mathrm{Post}(q_2, a)) \in Q_3}$$

$$(q_1, q_2) \xrightarrow{a}_3 (q_1, \mathrm{Post}(q_2, a))$$

$$\ell_3((q_1, \mathrm{Post}(q_2, a))) = \ell_2(\mathrm{Post}(q_2, a))$$

Structura de date ACOS_n reuniunea reguli de inferenta

Regula 4

$$\frac{\left(q_1,q_2\right)\in Q_3 \quad \ell_1(q_1)=P \quad \ell_2(q_2)=P \quad \ell_3(q_1,q_2)=P \quad a\in\Sigma_n}{\left(\mathrm{Post}(q_1,a),\mathrm{Post}(q_2,a)\right)\in Q_3 \quad \left(q_1,q_2\right)\xrightarrow{a}{}_3\left(\mathrm{Post}(q_1,a),\mathrm{Post}(q_2,a)\right) \quad \ell_3(\left(\mathrm{Post}(q_1,a),\mathrm{Post}(q_2,a)\right))= \ell_1(\mathrm{Post}(q_1,a))\bigcup\ell_2(\mathrm{Post}(q_2,a))}$$

Structura de date $ACOS_n$ reuniunea reguli de inferenta

- Regulile 1,2,3,4 se aplica pana cand nici una din ele nu mai poate fi aplicata
- Din acest moment se aplica urmatoarele reguli pentru reetichetarea cu F(ull) a nodurilor cu n descendenti etichetati F si eliminarea acestor descendenti

Structura de date ACOS$_n$ reuniunea reguli de inferenta

Regula 5

$$\frac{q_3 \in Q_3 \qquad \ell_3(q_3) = P \qquad \left(\forall q \in \mathrm{Post}(q_3)\right)\left(\ell_3(q) = F\right)}{\ell_3(q_3) = F}$$

$$\left(\forall a \in \Sigma_n\right)\left(\forall q \in \mathrm{Post}(q_3, a)\right)$$

$$\left(\xrightarrow{\;a\;}_3 \; = \; \xrightarrow{\;a\;}_3 \setminus \{(q_3, q)\}\right)$$

Structura de date $ACOS_n$ reuniunea reguli de inferenta

Regula 6

$$\frac{q_3 \in Q_3 \setminus \left\{q_0^3\right\} \quad \operatorname{Pre}(q_3) = \varnothing \qquad (\forall q \in \operatorname{Post}(q_3))(\ell_3(q) = F)}{(\forall a \in \Sigma_n)(\forall q \in \operatorname{Post}(q_3, a))\left(\xrightarrow{\;a\;}_3 = \xrightarrow{\;a\;}_3 \setminus \{(q_3, q)\}\right) \qquad Q_3 = Q_3 \setminus \{q_3\}}$$

Structura de date ACOS_n intersectia

Date doua obiecte ACOS

$$\text{ACOS}_{\text{n}}^{\text{i}} = \begin{pmatrix} Q_i, \Sigma_n, \\ \left\{ \xrightarrow{a}_i \middle| \xrightarrow{a}_i \subseteq Q_i \times Q_i, a \in \Sigma_n \right\}, \\ q_0^i, \ell_i \end{pmatrix}, i = 1,2$$

sa se obtina obiectul intersecti e

$$\text{ACOS}_{\text{n}}^{3} = \begin{pmatrix} Q_3, \Sigma_n, \\ \left\{ \xrightarrow{a}_3 \middle| \xrightarrow{a}_3 \subseteq Q_3 \times Q_3, a \in \Sigma_n \right\}, \\ q_0^3, \ell_3 \end{pmatrix}$$

Structura de date $ACOS_n$ intersectia

$$Q_3 \subseteq Q_1 \times Q_2$$

$$q_0^3 = \left(q_0^1, q_0^2\right)$$

Definim operatia $\bigcap$ astfel :

$$\bigcap : \{P, E, F\} \times \{P, E, F\} \to \{P, E, F\}$$

$$\left(\forall x \in \{P, E, F\}\right)\begin{pmatrix} x \bigcap F = x \wedge \\ x \bigcap E = E \end{pmatrix}$$

$$P \bigcap P = P$$

Structura de date ACOS$_n$ intersectia reguli de inferenta

Regula 1

$$\frac{}{q_0^3 = \left(q_0^1, q_0^2\right) \in Q_3 \quad \ell_3(q_0^3) = \ell_1(q_0^1) \bigcap \ell_2(q_0^2)}$$

Structura de date $ACOS_n$ intersectia reguli de inferenta

Regula 2

$$\frac{(q_1, q_2) \in Q_3 \quad \ell_1(q_1) = P \quad \ell_2(q_2) = F \quad \ell_3(q_1, q_2) = P \quad a \in \Sigma_n}{\begin{array}{c}(\mathrm{Post}(q_1, a), q_2) \in Q_3 \\ (q_1, q_2) \xrightarrow{a}_3 (\mathrm{Post}(q_1, a), q_2) \\ \ell_3((\mathrm{Post}(q_1, a), q_2)) = \ell_1(\mathrm{Post}(q_1, a))\end{array}}$$

Structura de date ACOS$_n$ intersectia reguli de inferenta

Regula 3

$$\frac{(q_1, q_2) \in Q_3 \quad \ell_1(q_1) = F \quad \ell_2(q_2) = P \quad \ell_3(q_1, q_2) = P \quad a \in \Sigma_n}{\begin{array}{c}(q_1, \mathrm{Post}(q_2, a)) \in Q_3 \\ (q_1, q_2) \xrightarrow{a}_3 (q_1, \mathrm{Post}(q_2, a)) \\ \ell_3((q_1, \mathrm{Post}(q_2, a))) = \ell_2(\mathrm{Post}(q_2, a))\end{array}}$$

Structura de date $ACOS_n$ intersectia reguli de inferenta

Regula 4

$$\frac{(q_1, q_2) \in Q_3 \quad \ell_1(q_1) = P \quad \ell_2(q_2) = P \quad \ell_3(q_1, q_2) = P \quad a \in \Sigma_n}{\left(\mathrm{Post}(q_1, a), \mathrm{Post}(q_2, a)\right) \in Q_3}$$

$$(q_1, q_2) \xrightarrow{a}_3 \left(\mathrm{Post}(q_1, a), \mathrm{Post}(q_2, a)\right)$$

$$\ell_3\left(\left(\mathrm{Post}(q_1, a), \mathrm{Post}(q_2, a)\right)\right) =$$

$$\ell_1(\mathrm{Post}(q_1, a)) \cap \ell_2(\mathrm{Post}(q_2, a))$$

Structura de date $ACOS_n$ intersectia reguli de inferenta

- Regulile 1,2,3,4 se aplica pana cand nici una din ele nu mai poate fi aplicata
- Din acest moment se aplica urmatoarele reguli pentru reetichetarea cu E(mpty) a nodurilor cu n descendenti etichetati E si eliminarea acestor descendenti

Structura de date ACOS$_n$ intersectia reguli de inferenta

Regula 5

$$\frac{q_3 \in Q_3 \quad \ell_3(q_3) = P \quad \left(\forall q \in \mathrm{Post}(q_3)\right)\left(\ell_3(q) = E\right)}{\ell_3(q_3) = F \quad \left(\forall a \in \Sigma_n\right)\left(\forall q \in \mathrm{Post}(q_3, a)\right) \left(\xrightarrow{a}_3 = \xrightarrow{a}_3 \setminus \{(q_3, q)\}\right)}$$

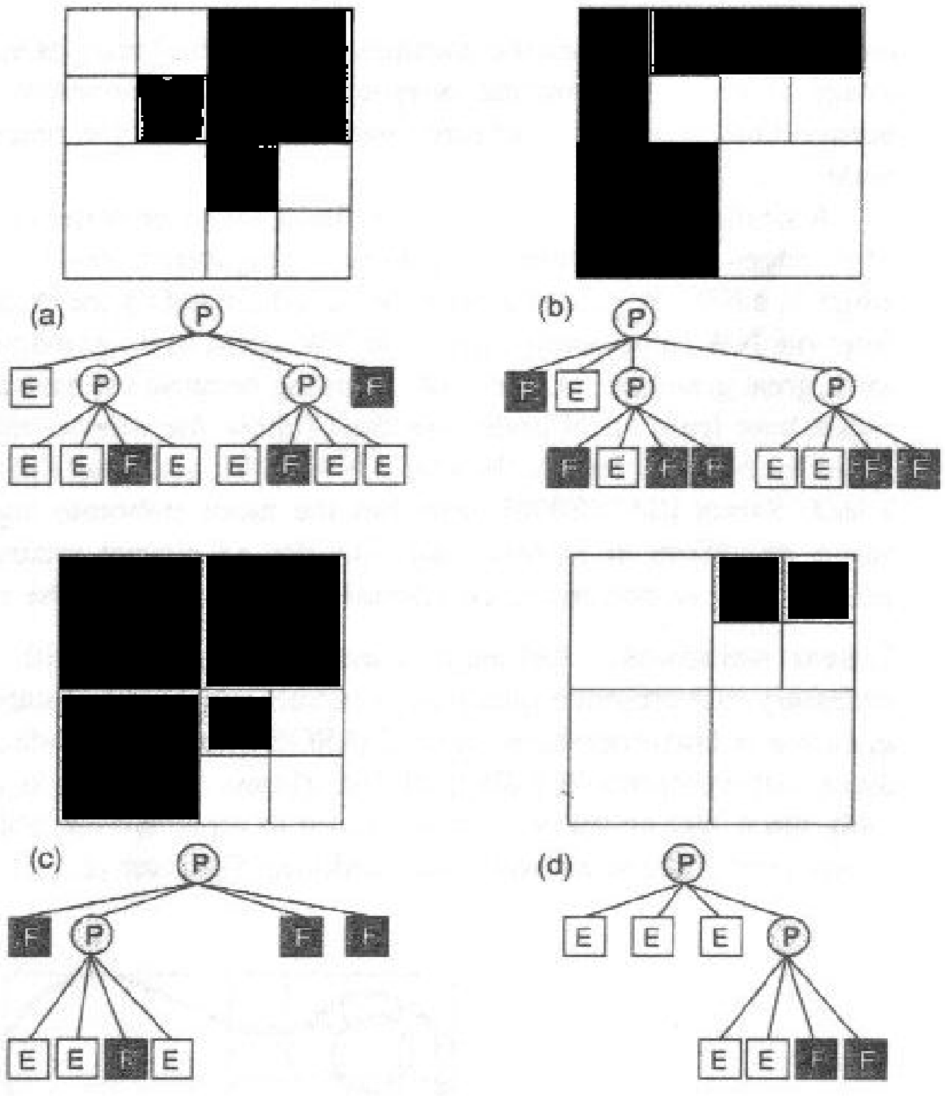
Structura de date ACOS$_n$ intersectia reguli de inferenta

Regula 6

$$\frac{q_3 \in Q_3 \setminus \{q_0^3\} \quad \mathrm{Pre}(q_3) = \emptyset \quad (\forall q \in \mathrm{Post}(q_3))(\ell_3(q) = F)}{(\forall a \in \Sigma_n)(\forall q \in \mathrm{Post}(q_3, a)) \left(\xrightarrow{a}_3 = \xrightarrow{a}_3 \setminus \{(q_3, q)\} \right)}$$

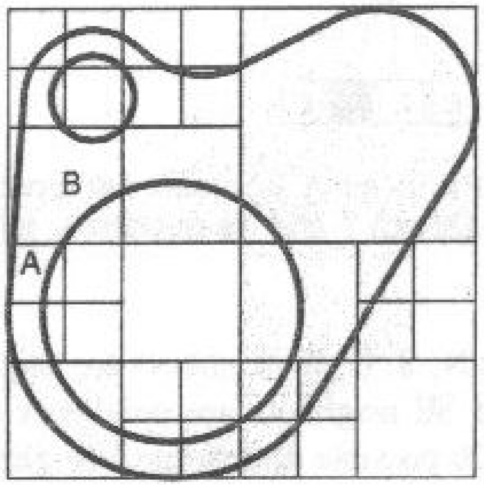
$$Q_3 = Q_3 \setminus \{q_3\}$$

Operatii booleene pe $ACOS_4$

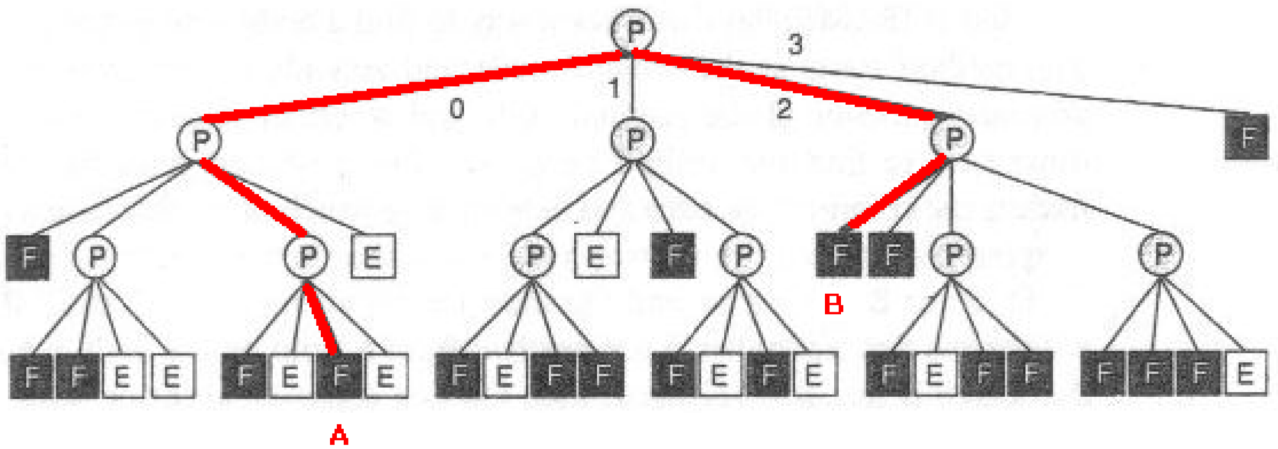


Operatii booleene pe ACOS4 : obiectele S si T (imaginile (a) si (b)) si S ∪ T (c), S ∩ T (d)

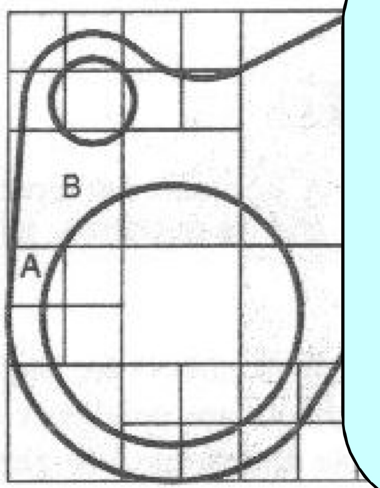
Determinarea vecinului



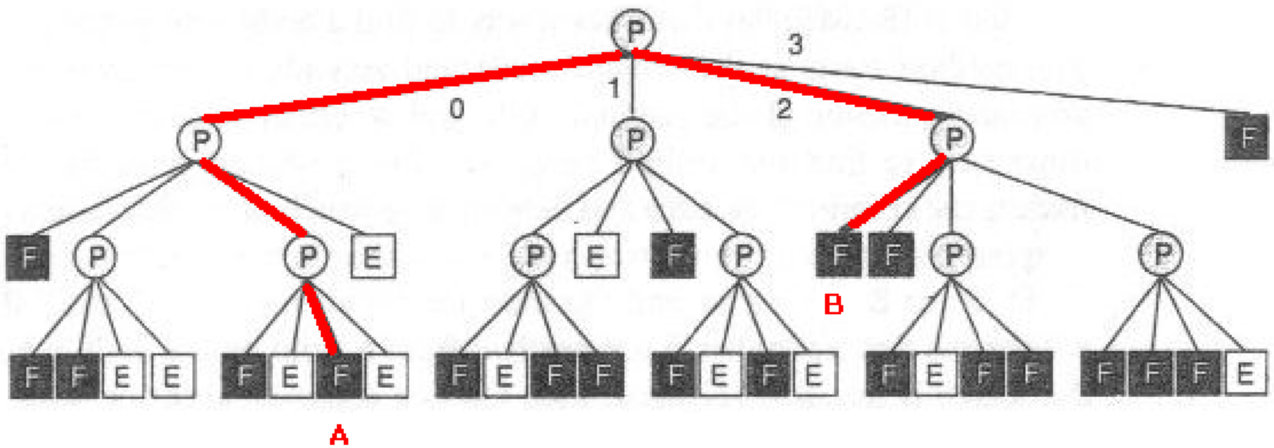
Dorim determinarea vecinului nodului A cu care acesta se invecineaza la nord (i.e., nodul B).



Determinarea vecinului



Plecam dinspre A catre radacina arborelui pana cand intalnim o eticheta care nu mai contine “nord”:
(A,2-nw,P),(P,2-nw,P), (P,0-sw,P), STOP.
De la ultimul nod astfel obtinut, in cazul ns. radacina, mergem pe drumul inversat simetric:
(P,2-nw,P),(P,0-sw,F), STOP deoarece F nu are succesori si deci am obtinut B.



Determinarea vecinului

vecin1.cpp
FindNeighbourCommonFace8

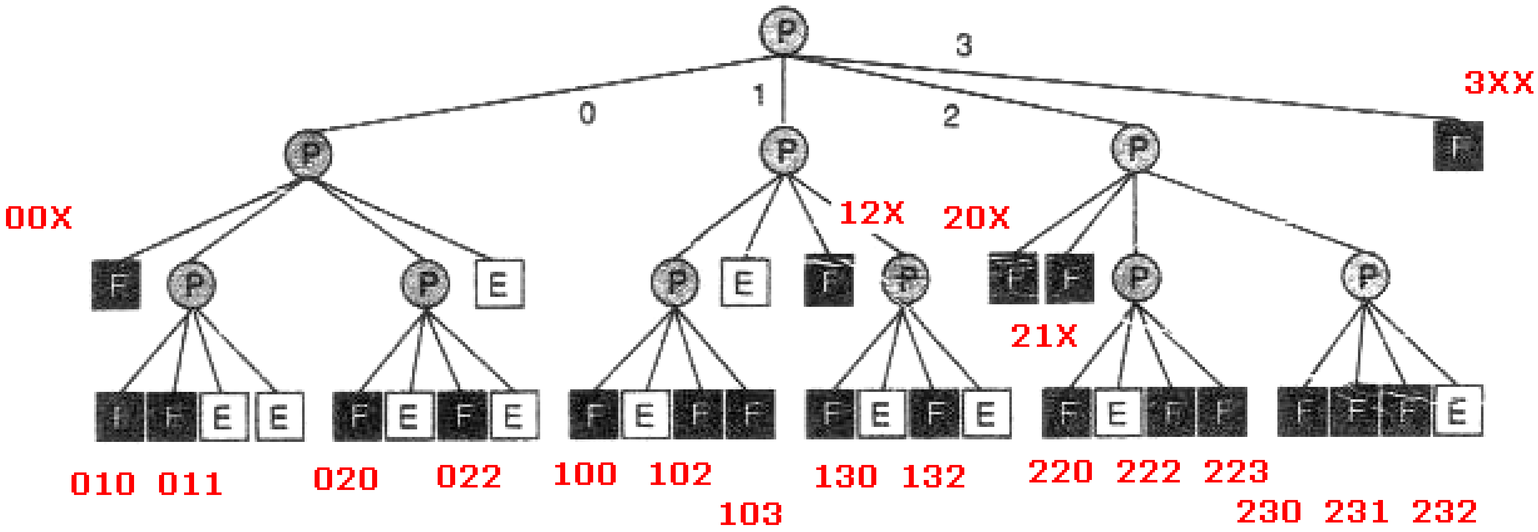
vecin2.cpp
FindNeighbourCommonEdge4

$ACOS_n$ liniare

Notatia liniara : sunt reprezentate doar nodurile F.

Nodurile F sunt reprezentate printr-o secventa de cifre care reprezinta “adresa” sa din arborele ACOS. # de cifre coincide cu # de niveluri din arborele ACOS. Nodurile F care nu se gasesc pe ultimul nivel au adaugate in secventa ‘X’ pana la completarea # de niveluri (ex. 3XX).

ACOS$_n$ liniare



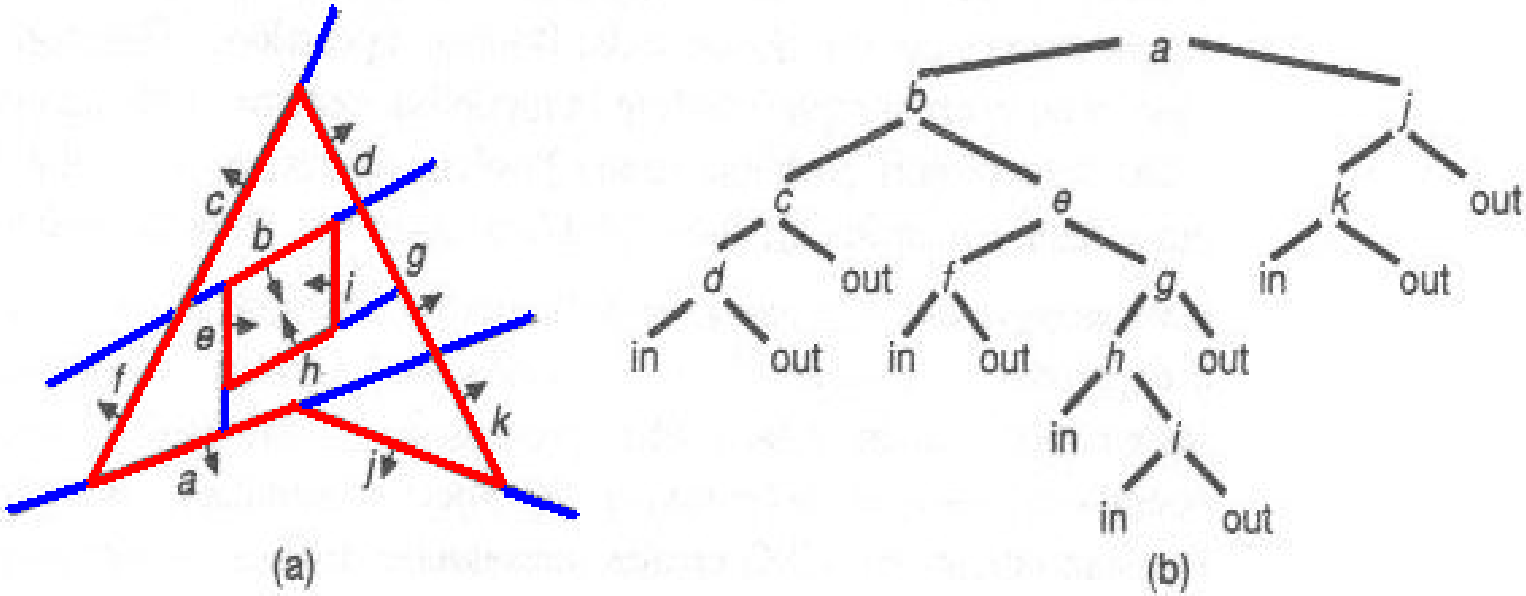
00X,010,011,020,022,100,102,103,12X,130,132,

20X,21X,220,222,223,230,231,232,3XX

$ACOS_n$ liniare

frontiera.cpp
determinareFrontieraACOS8

Reprezentarea BSP



Reprezentarea BSP a unui obiect 2D (poligonul concav din imaginea (a) delimitat de conturul de culoare rosie)

Arborele BSP corespunzator (b)

Reprezentarea BSP

- Arborii BSP (Binary Space Partitioning Trees) subdivid, in mod recursiv, spatiul in 2 subspatii delimitate printr-un plan oarecare
- Un nod intern al unui arbore BSP este asociat unui plan P si are 2 pointeri la nodurile fii (de o parte si de cealalta a planului P)

Reprezentarea BSP

- orientarea normalei la P (spre exteriorul lui P) distinge fiul stang de cel drept
  - fiul stang este in interiorul (spatele) planului
  - fiul drept este in exteriorul (fata) planului

Reprezentarea BSP

- divizarea spatiului continua in mod recursiv cu fiii nodului corespunzator planului P
- divizarea spatiului se opreste cand subspatiul obtinut este, in intregime, in interiorul sau in exteriorul obiectului de reprezentat
- astfel, nodurile frunza vor fi etichetate in sau out

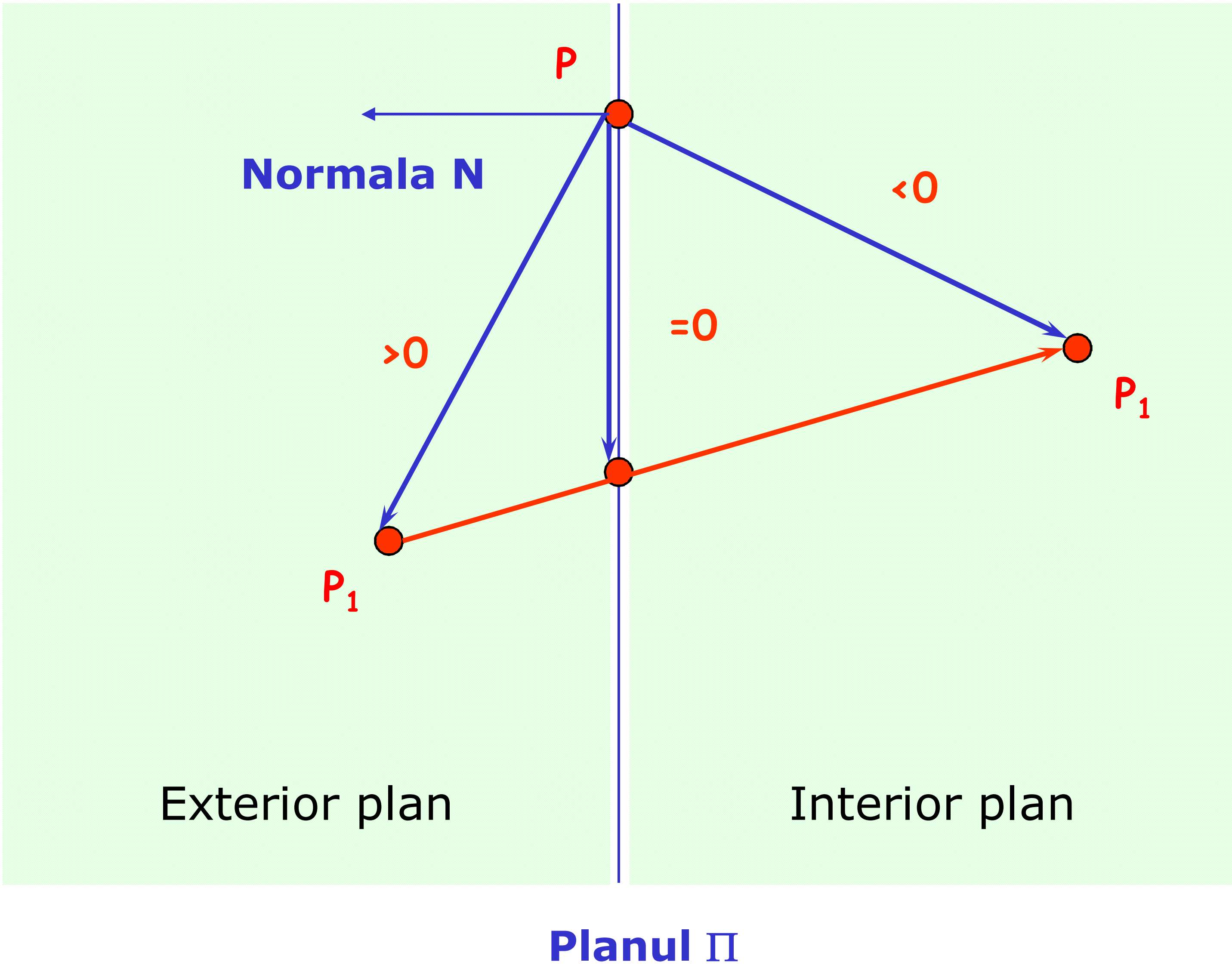
Reprezentarea BSP

- problema clasificarii unui punct: dat un punct si un obiect solid sa se determine daca punctul se gaseste in interiorul/exteriorul obiectului sau pe obiect
  - in ecuatia planului din nodul radacina $Ax + By + Cz + D = 0$ se inlocuiesc x, y si z cu coordonatele punctului

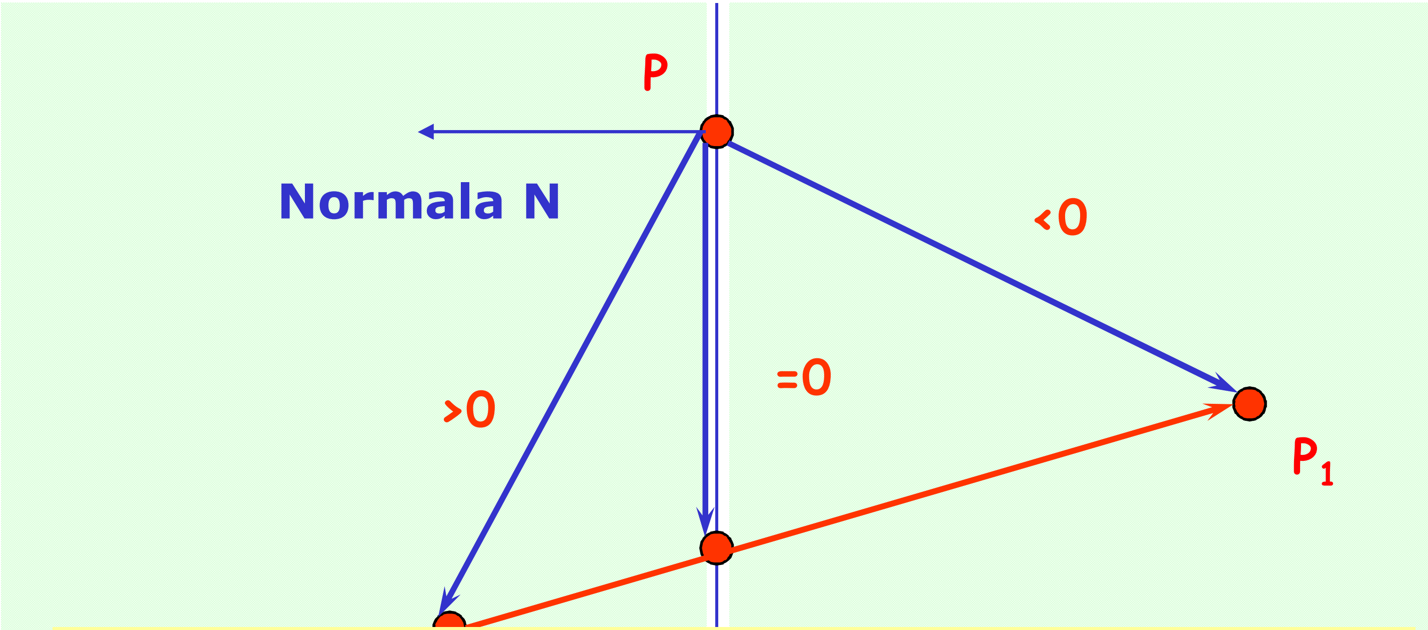
Reprezentarea BSP

- problema clasificarii unui punct
  - in ecuatia planului din nodul radacina $Ax + By + Cz + D = 0$ se inlocuiesc x, y si z cu coordonatele punctului
  - se determina, in functie de semn, daca punctul este in interiorul/exteriorul planului sau in plan

Problema clasificarii unui punct



Problema clasificarii unui punct



$$\vec{N} \cdot \overrightarrow{PP_1} < 0 \Rightarrow P_1 \text{ este interior } \Pi$$

$$\vec{N} \cdot \overrightarrow{PP_1} > 0 \Rightarrow P_1 \text{ este exterior } \Pi$$

$$\vec{N} \cdot \overrightarrow{PP_1} = 0 \Rightarrow P_1 \in \Pi$$

Problema clasificarii unui punct

$$\text{Ecuatia planului } \prod : Ax + By + Cz + D = 0$$

$$\overrightarrow{N} = A\vec{i} + B\vec{j} + C\vec{k}$$

$$P(x, y, z) \in \prod \quad \wedge \quad P_1(x_1, y_1, z_1) \Rightarrow$$

$$\overrightarrow{PP_1} = (x_1 - x)\vec{i} + (y_1 - y)\vec{j} + (z_1 - z)\vec{k}$$

$$\overrightarrow{N} \cdot \overrightarrow{PP_1} = A(x_1 - x) + B(y_1 - y) + (z_1 - z) =$$

$$= Ax_1 + By_1 + Cz_1 - Ax - By - Cz =$$

$$= Ax_1 + By_1 + Cz_1 + D$$

Reprezentarea BSP

- problema clasificarii unui punct
  - daca punctul este in interiorul sau in exteriorul planului atunci este trimis pentru clasificare fiului stang sau drept
  - daca este pe plan atunci este trimis ambilor fii si clasificarile obtinute se compara

Reprezentarea BSP

- problema clasificarii unui punct
  - daca este pe plan atunci este trimis ambilor fii si clasificarile obtinute se compara
    - daca sunt aceleasi, evident
    - daca nu, atunci punctul este clasificat ca fiind pe obiect

Reprezentarea BSP

- problema clasificarii unui punct
  - in final ajungem cu punctul intr-un nod frunza si clasificam punctul ca fiind in sau out conform etichetei nodului frunza

Obiect definit prin CSG