

# Grafica pe calculator

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# Vizualizari 3D

- Cum reprezentam obiecte 3D pe ecrane 2D ?
  - utilizand conceptul de **proiectie** (geometrica planara) a unui obiect pe un plan de proiectie

# Vizualizari 3D

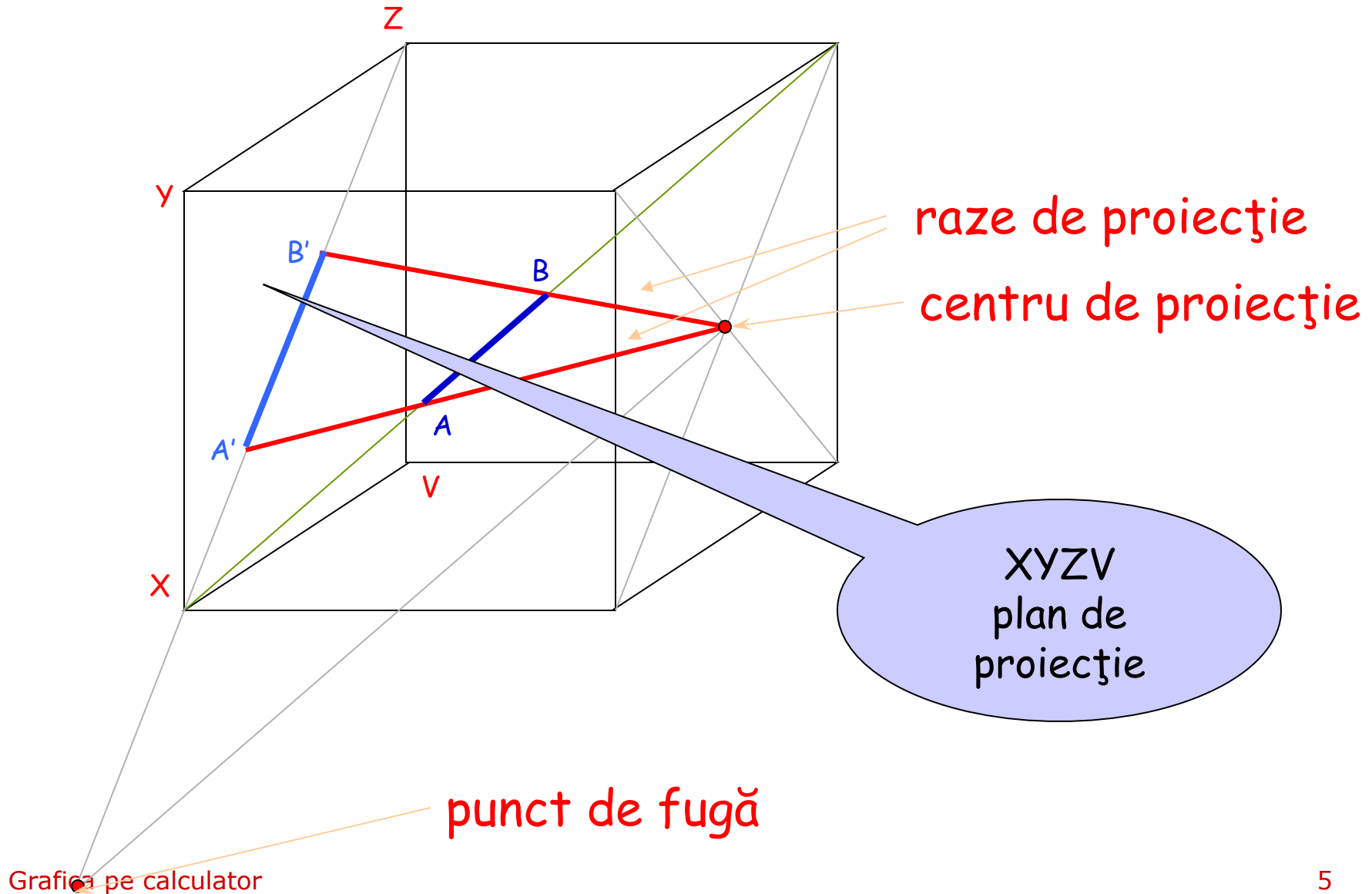
- Cum reprezentam obiecte 3D pe ecrane 2D ?
  - utilizand conceptul de **proiectie** (geometrica planara) a unui obiect pe un plan de proiectie

dintr-un centru de proiectie COP pleaca raze de proiectie (drepte) care trec prin fiecare punct al obiectului si intersecteaza planul de proiectie pentru a forma proiectia

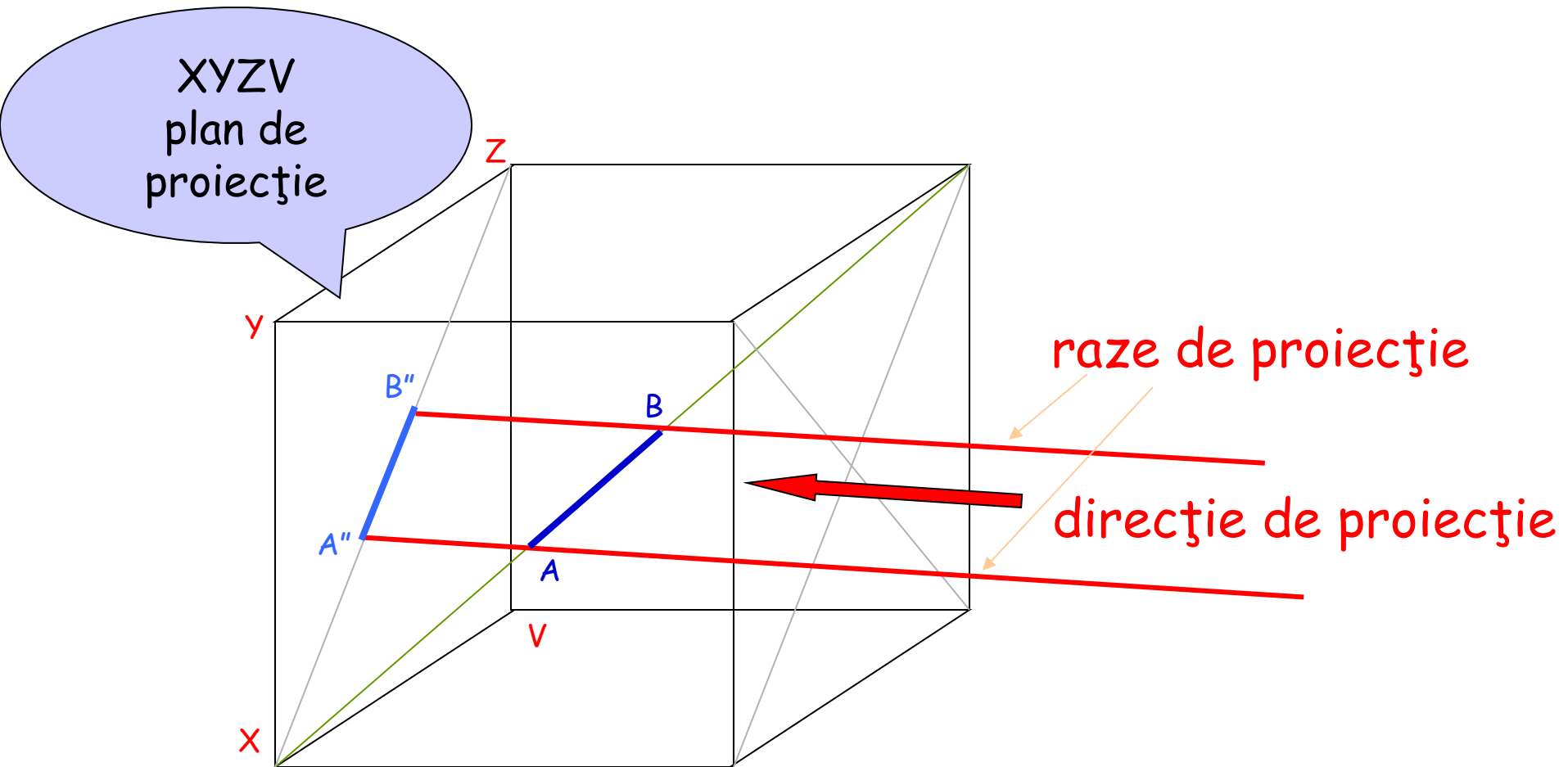
# Proiectii geometrice planare

- Clasificarea proiectiilor geometrice planare (in fct. de distanta dintre COP si planul de proiectie)
  - distanta finita -> perspectiva
  - distanta infinita -> paralele (dreptele de proiectie sunt *paralele*)

# Proiectii geometrice perspectiva



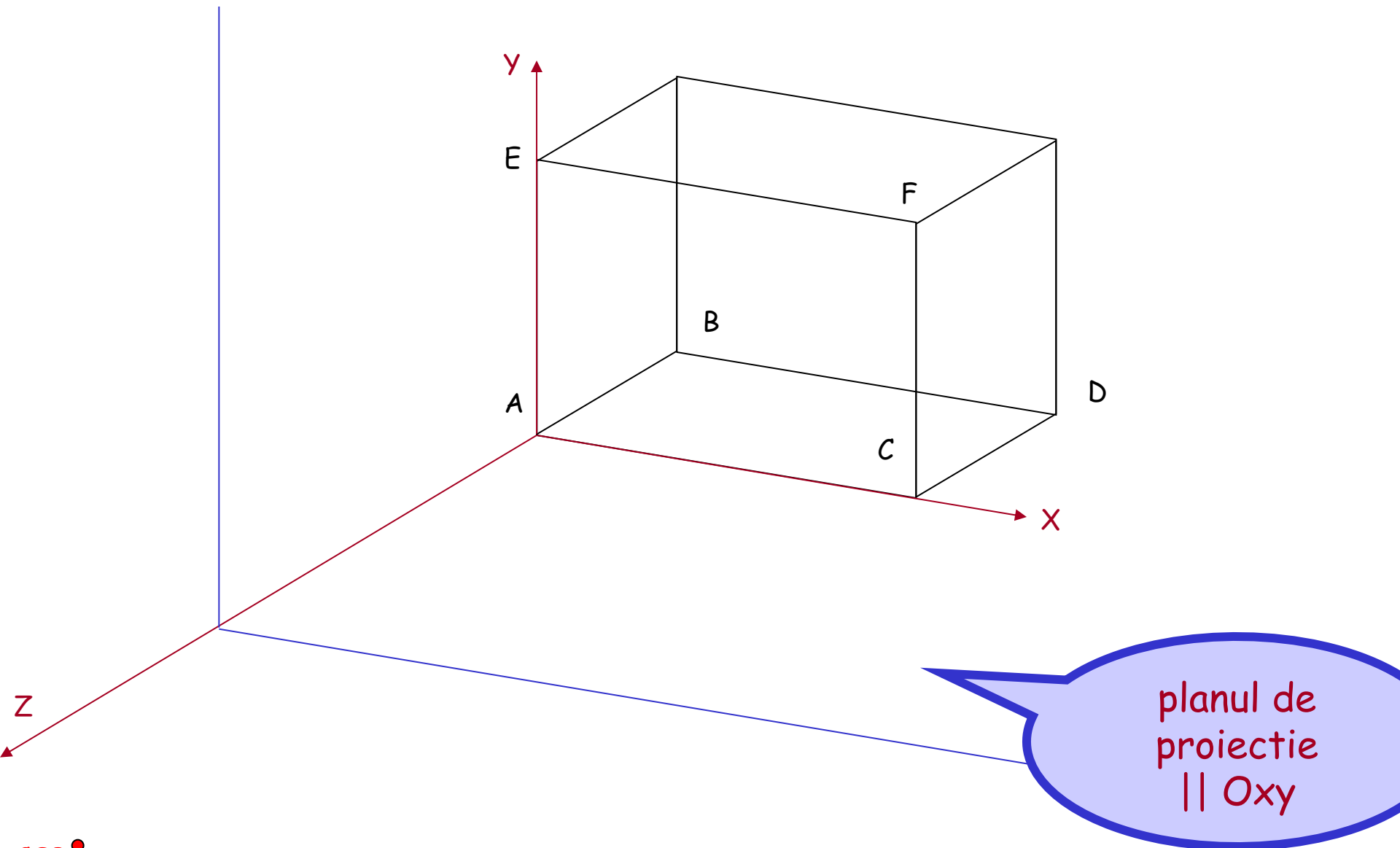
# Proiectii geometrice paralele



# Proiectii perspectiva

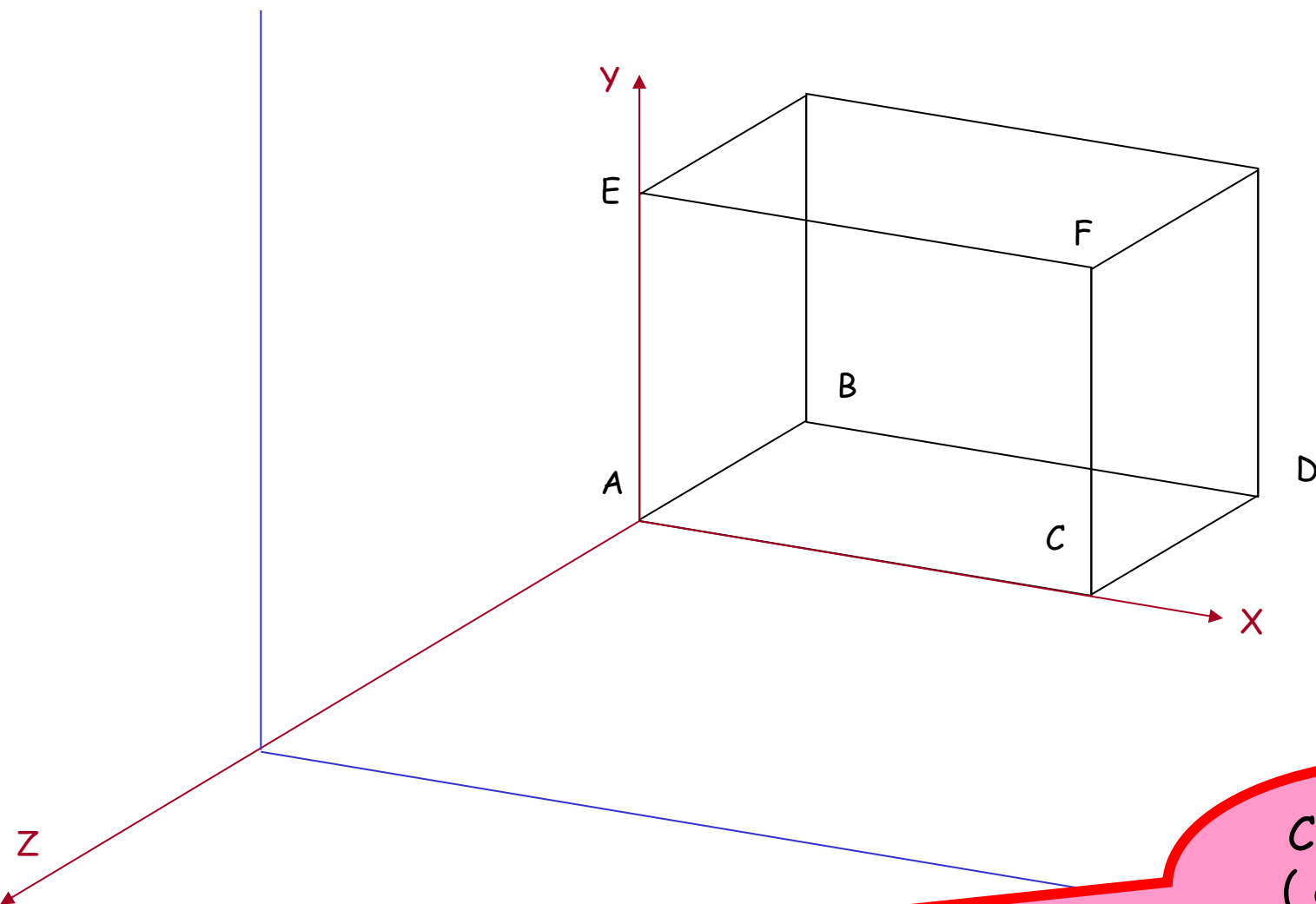
- clasificarea acestor proiectii in functie de numarul punctelor de fuga axiale
  - 1,2,3 punct(e) de fuga axial(e)
  - exemple 1, 2 puncte de fuga axiale

# Proiectii perspectiva 1 pfa



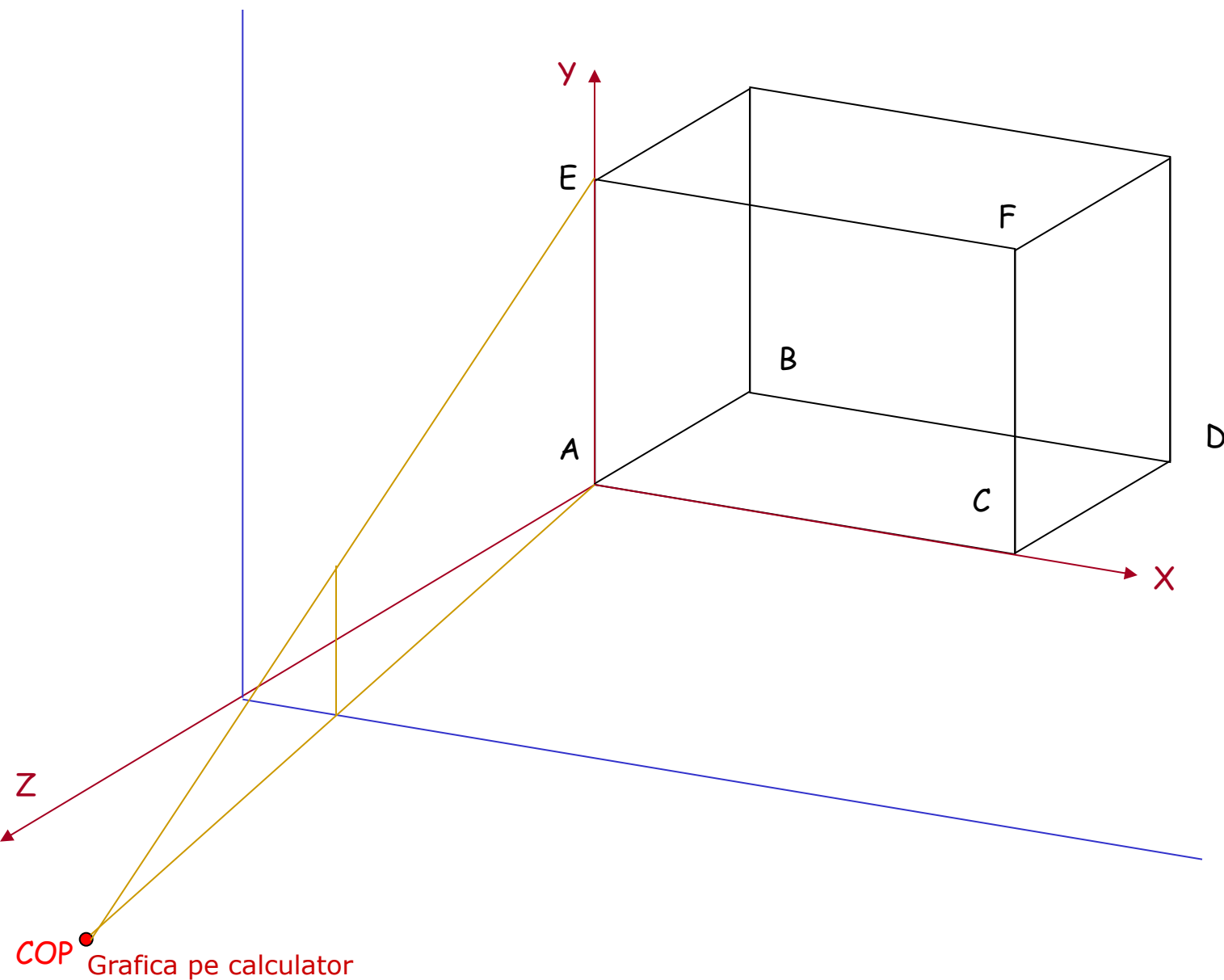


# Proiectii perspectiva 1 pfa

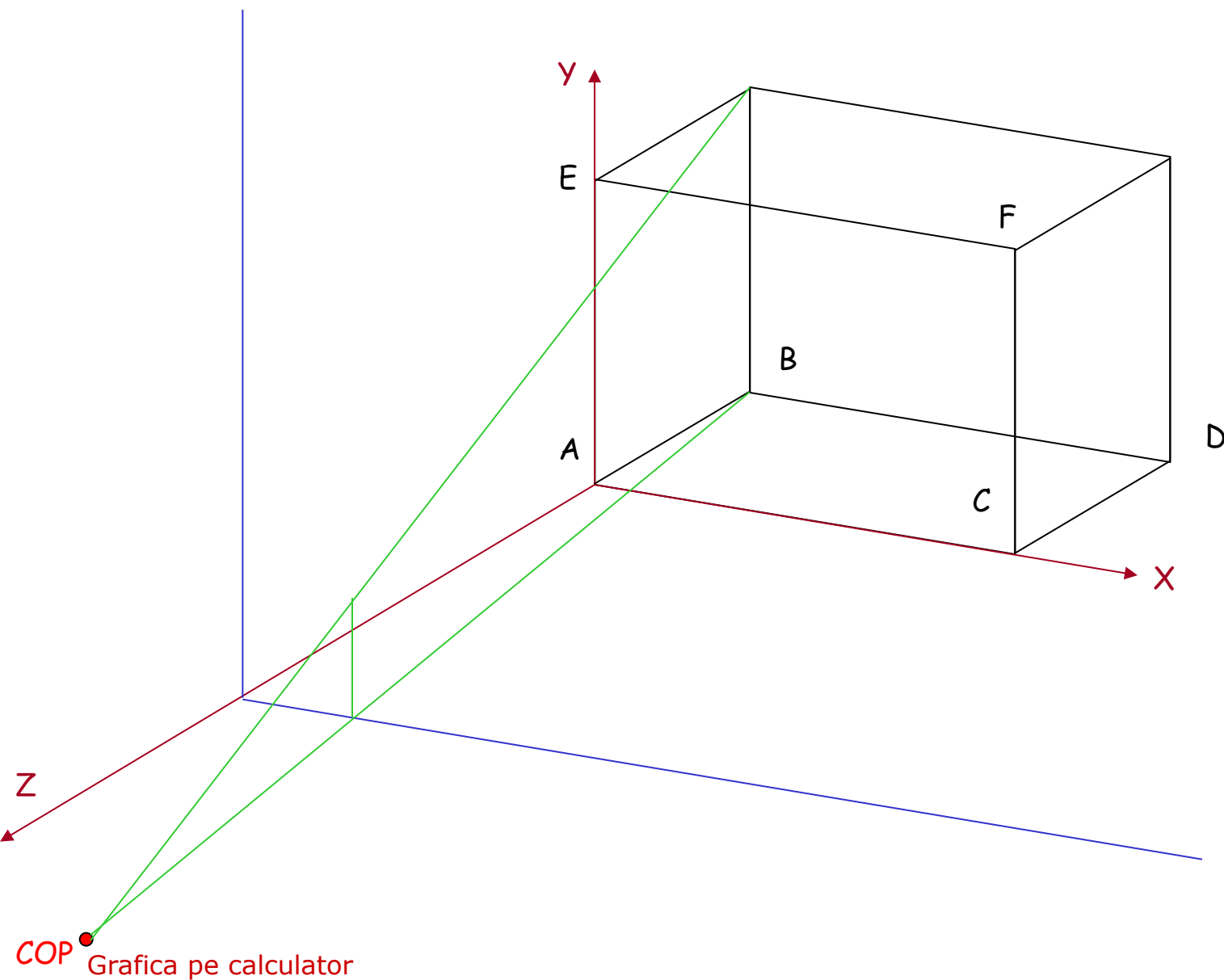


$COP \in \text{pl. } Oxyz$   
(coplanar cu  
 $A, B, C, D$ )

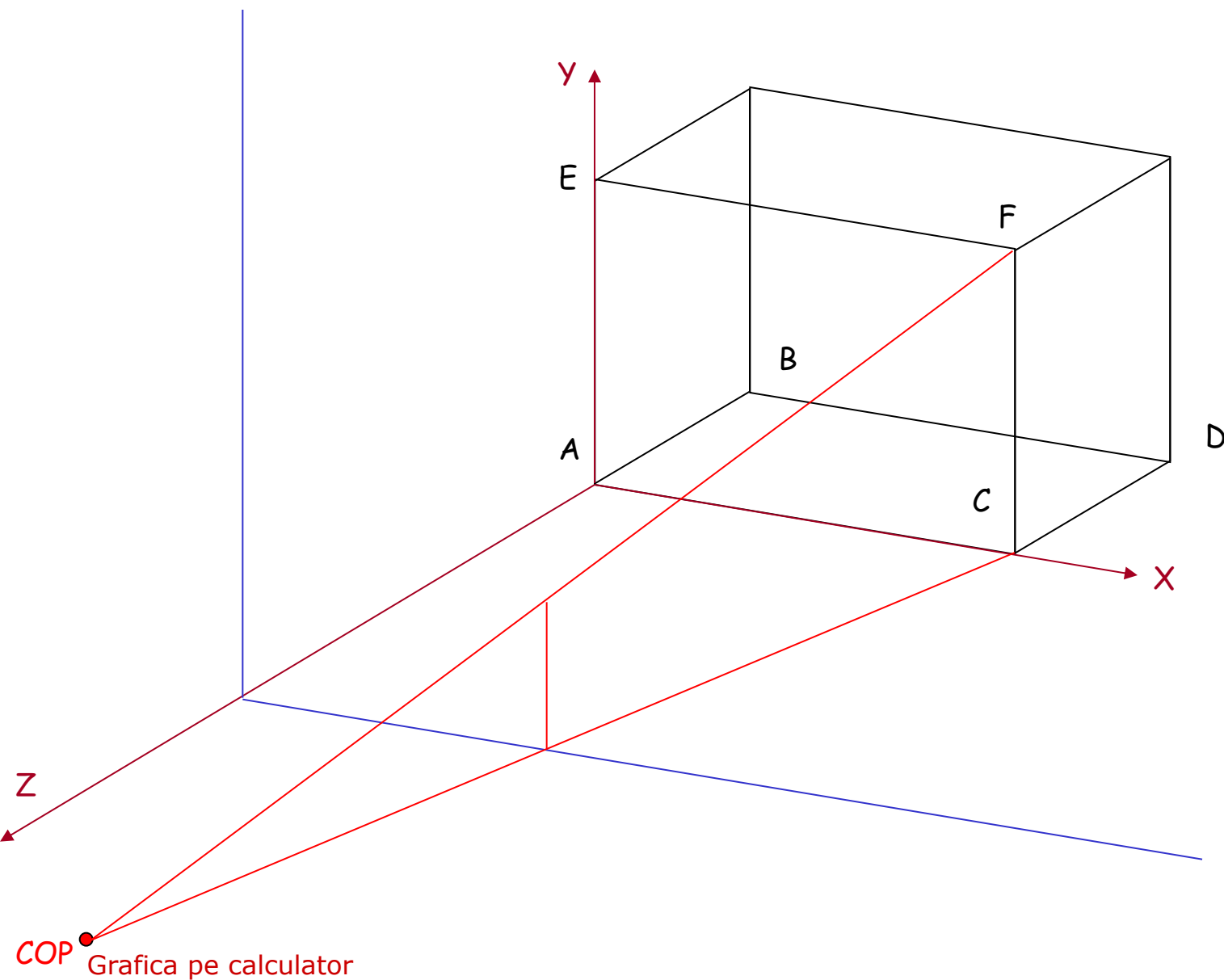
# Proiectii perspectiva 1 pfa



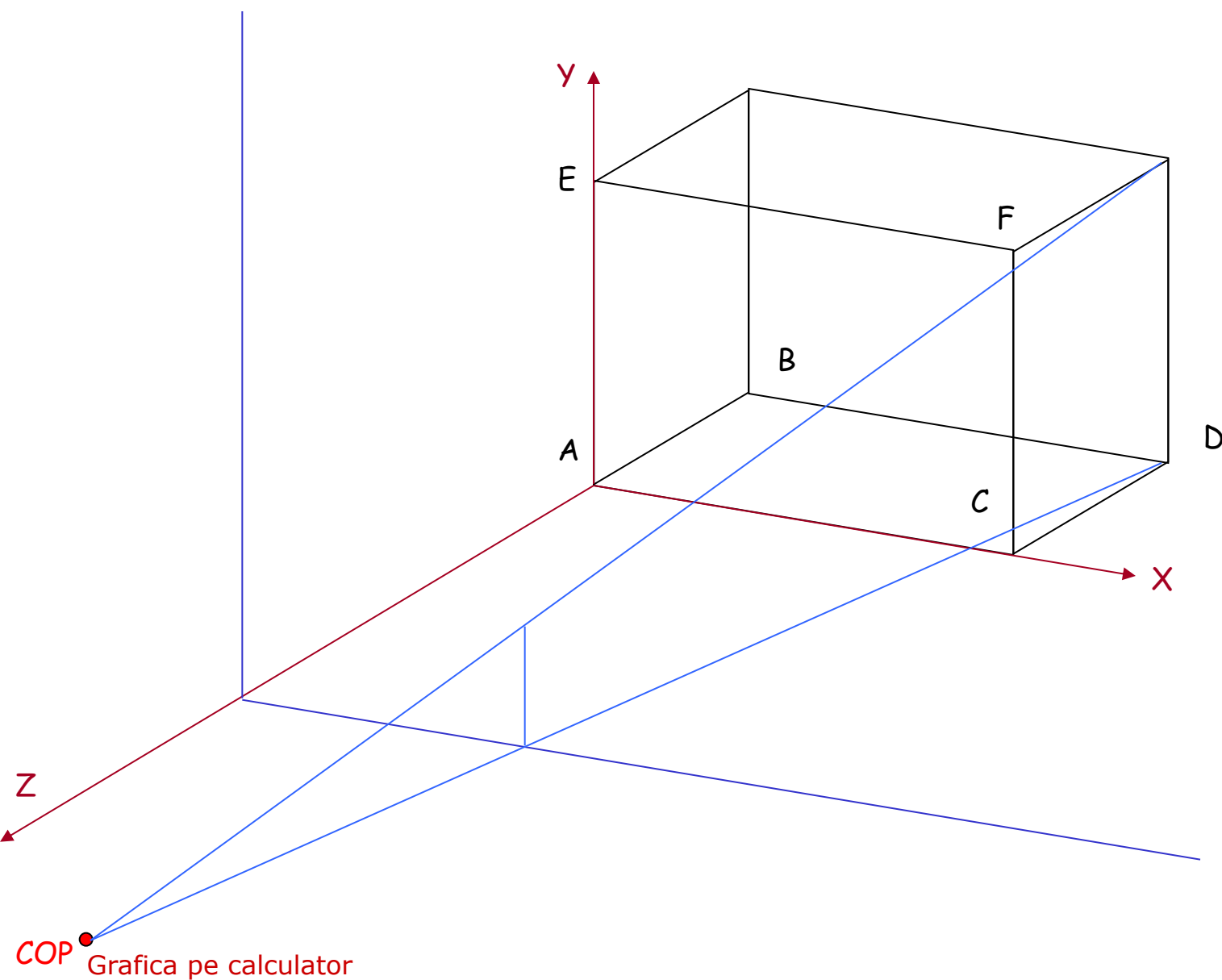
# Proiectii perspectiva 1 pfa



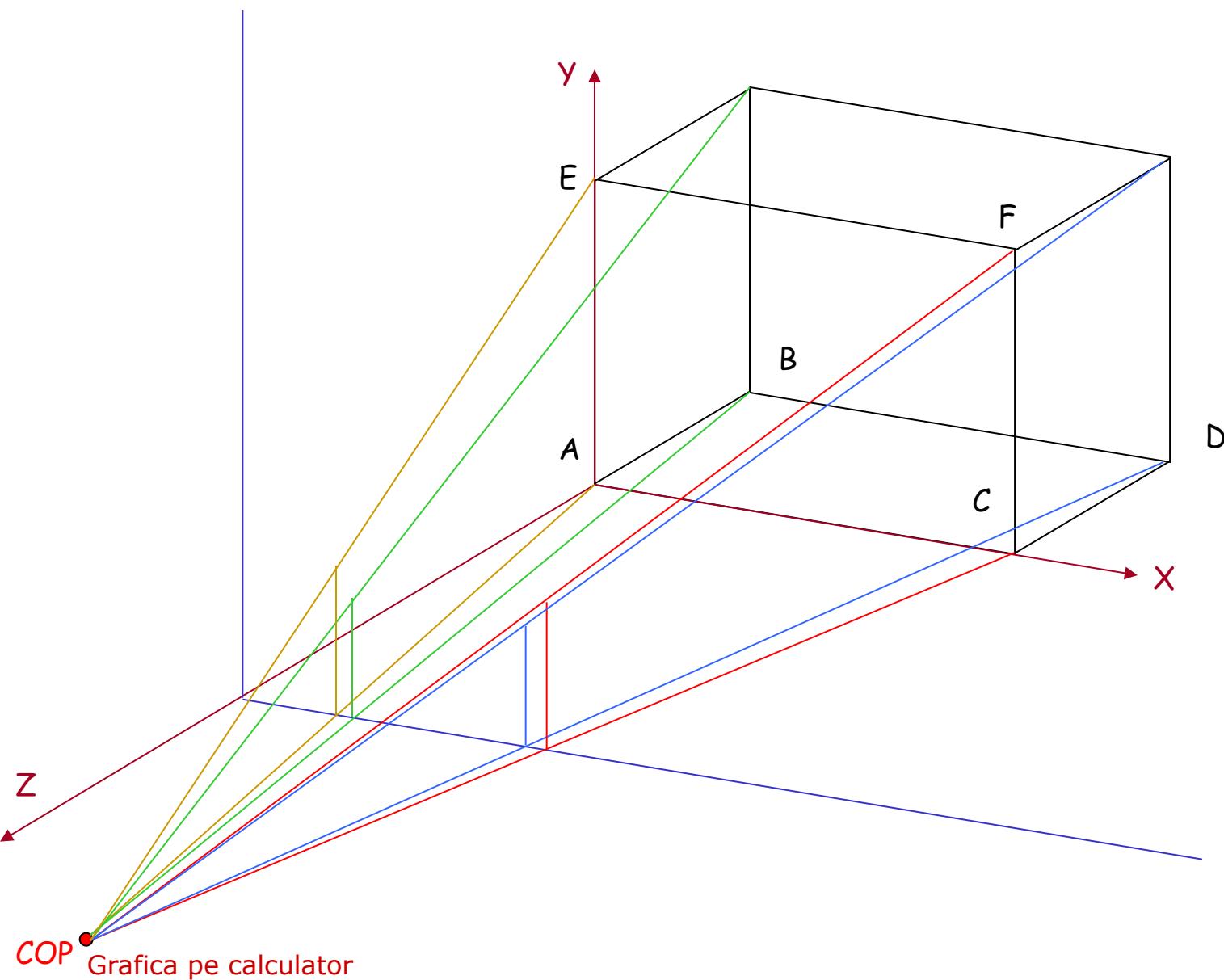
# Proiectii perspectiva 1 pfa



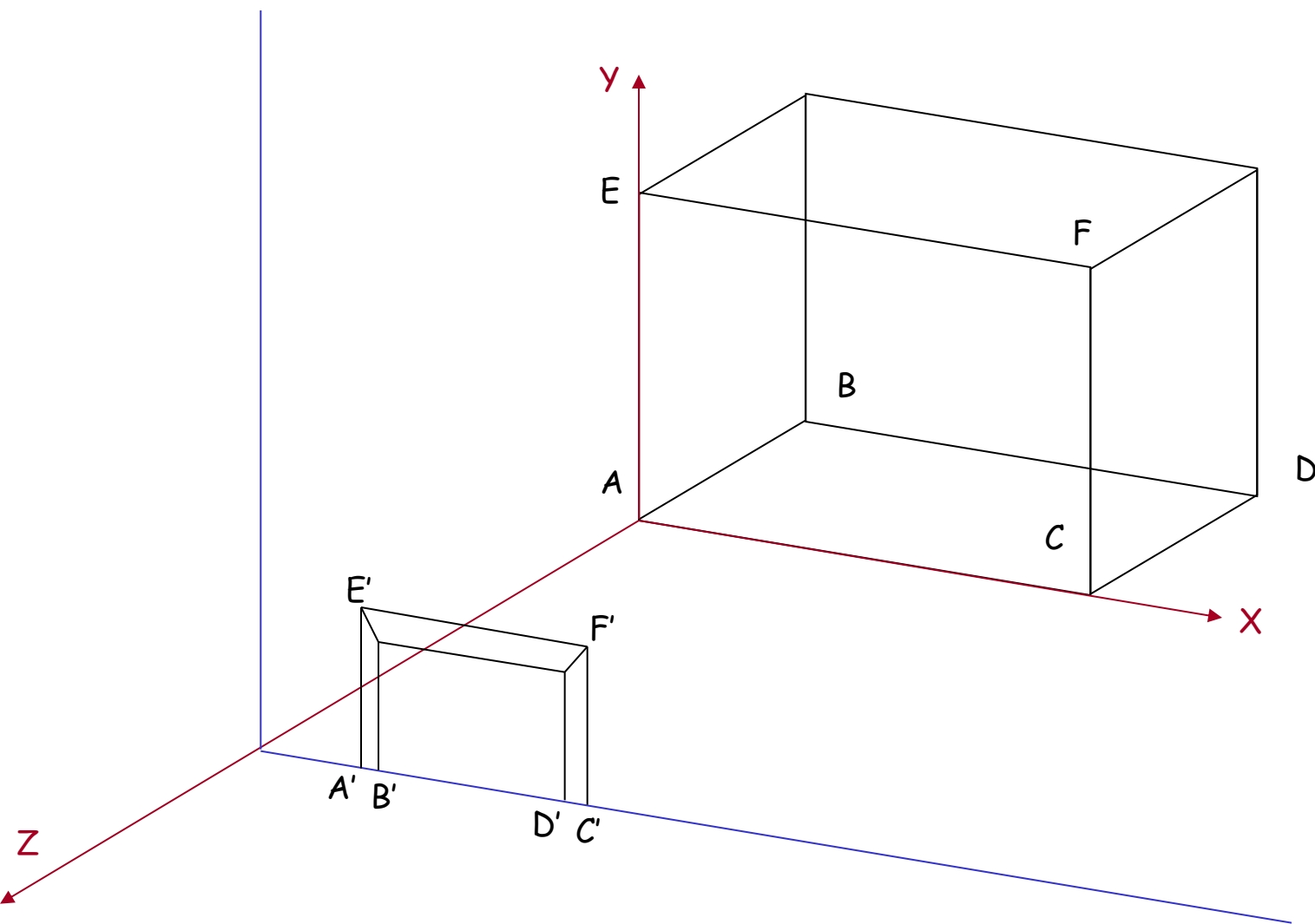
# Proiectii perspectiva 1 pfa



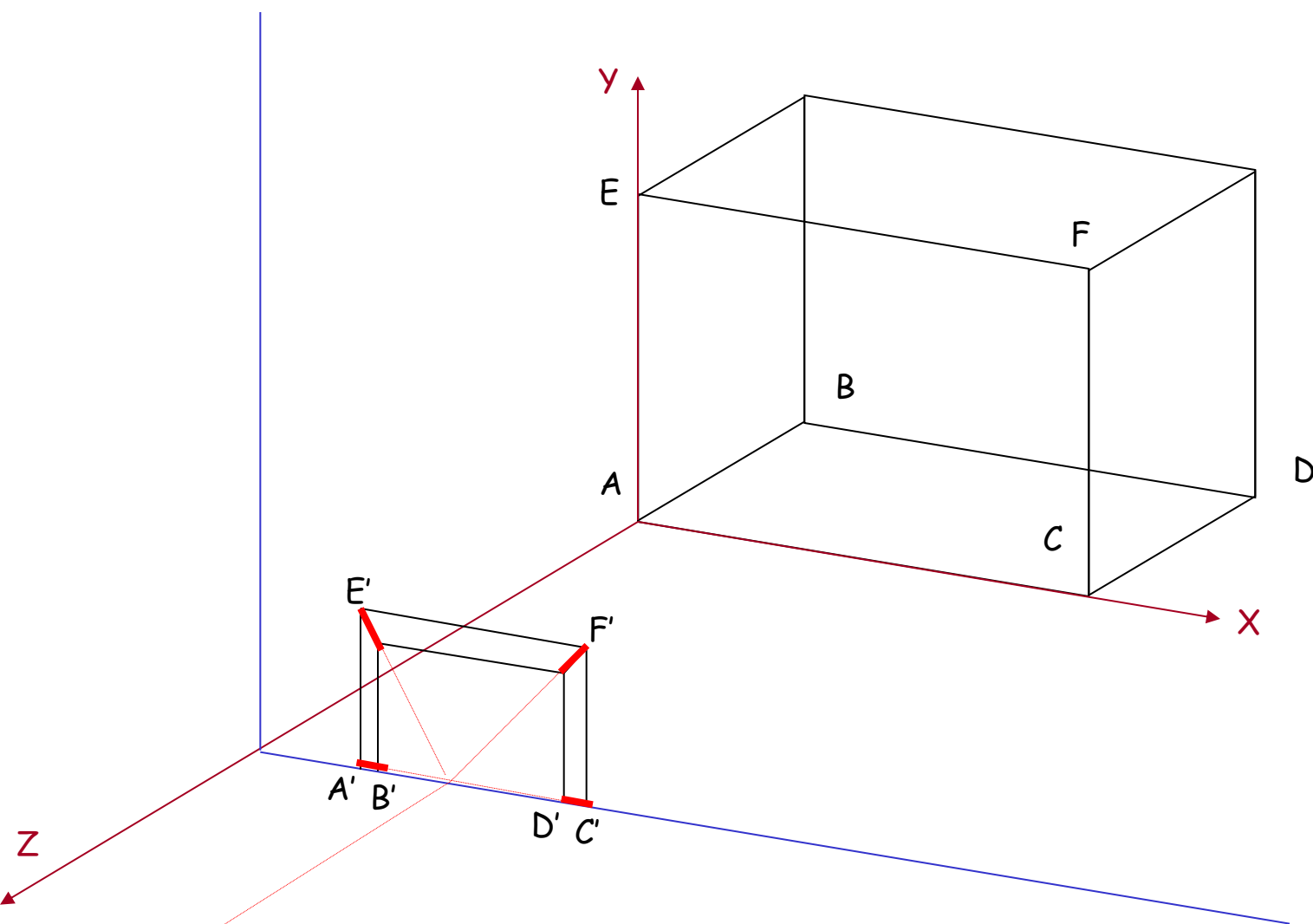
# Proiectii perspectiva 1 pfa



# Proiectii perspectiva 1 pfa

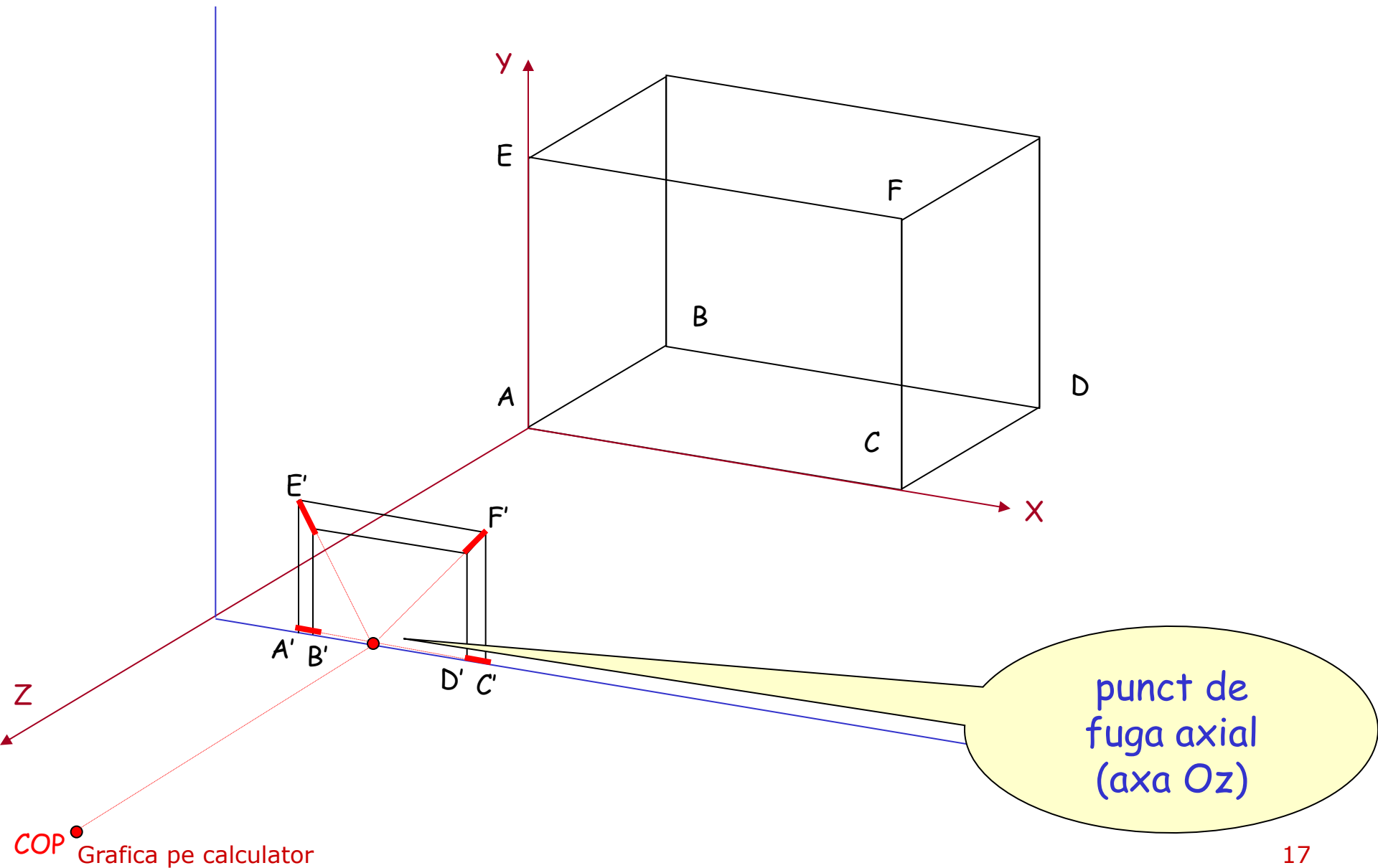


# Proiectii perspectiva 1 pfa

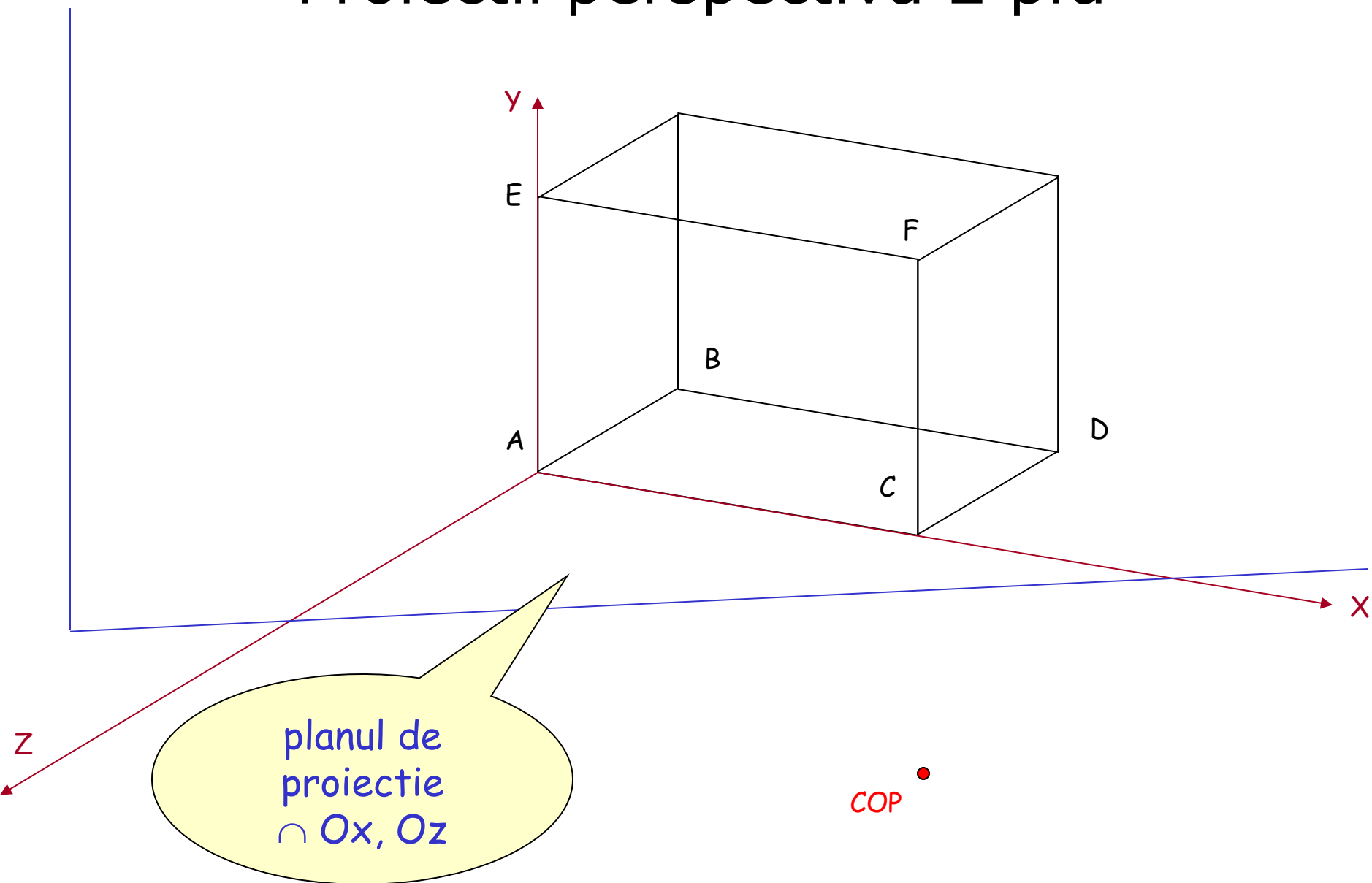




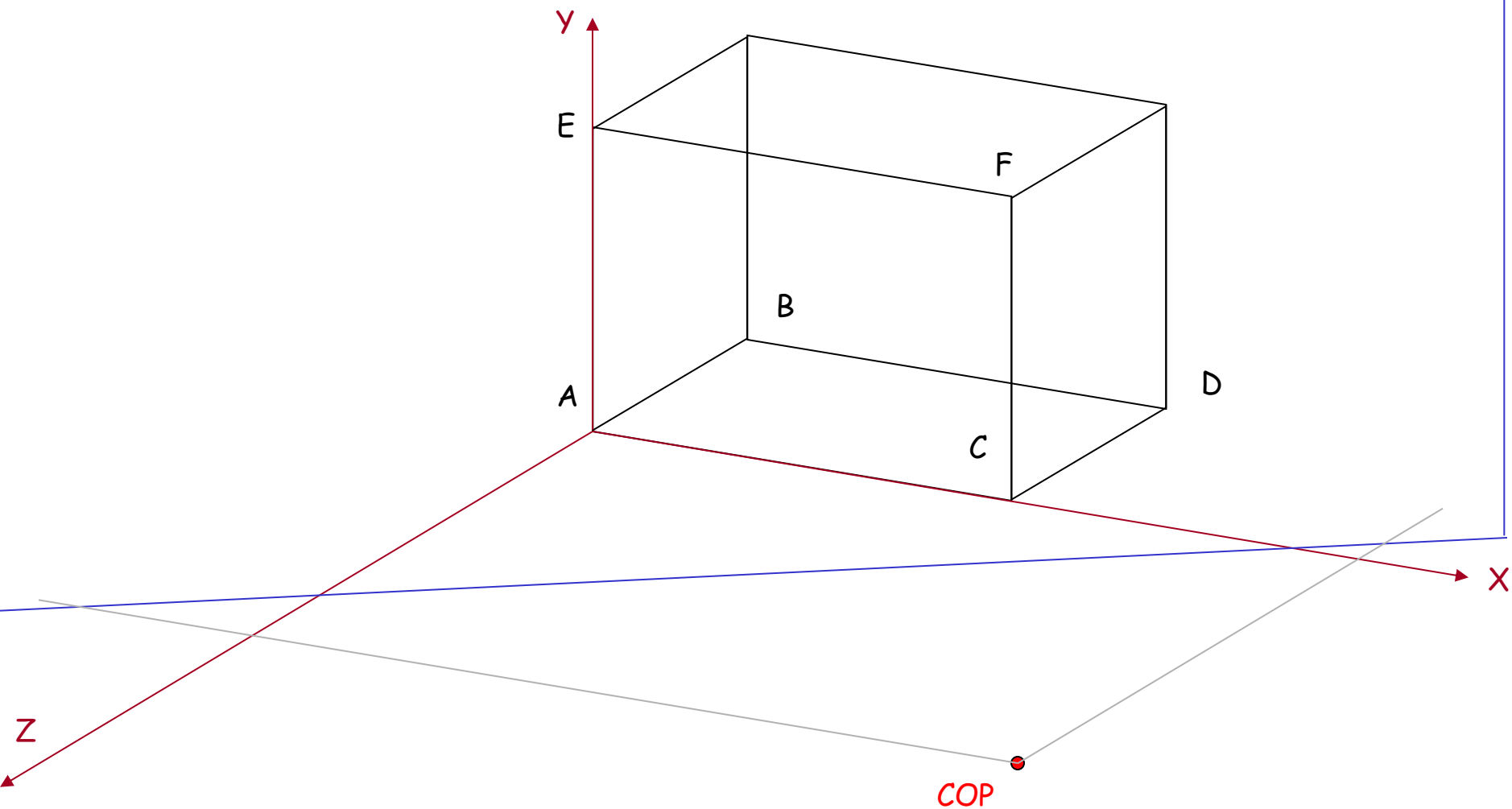
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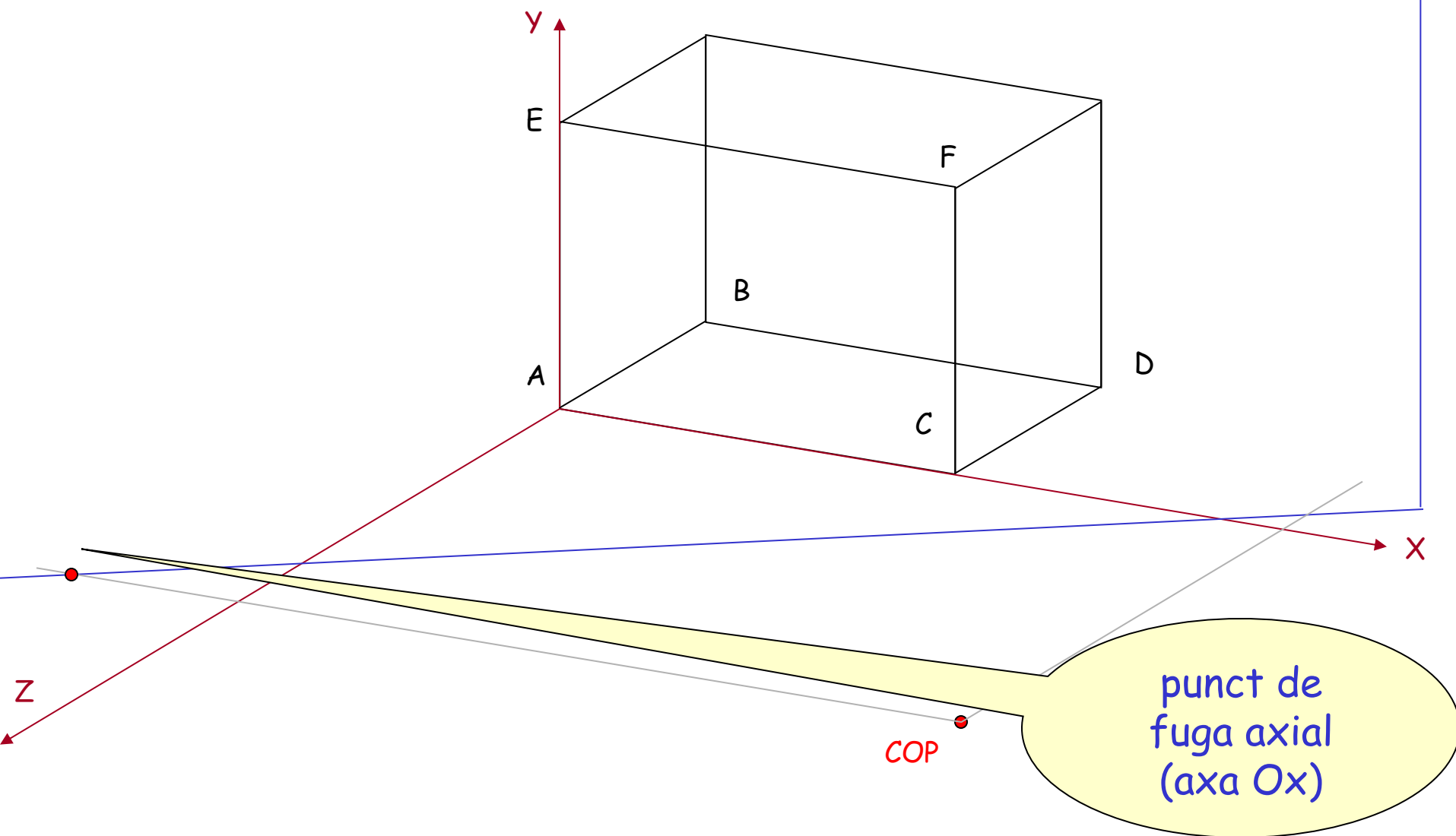
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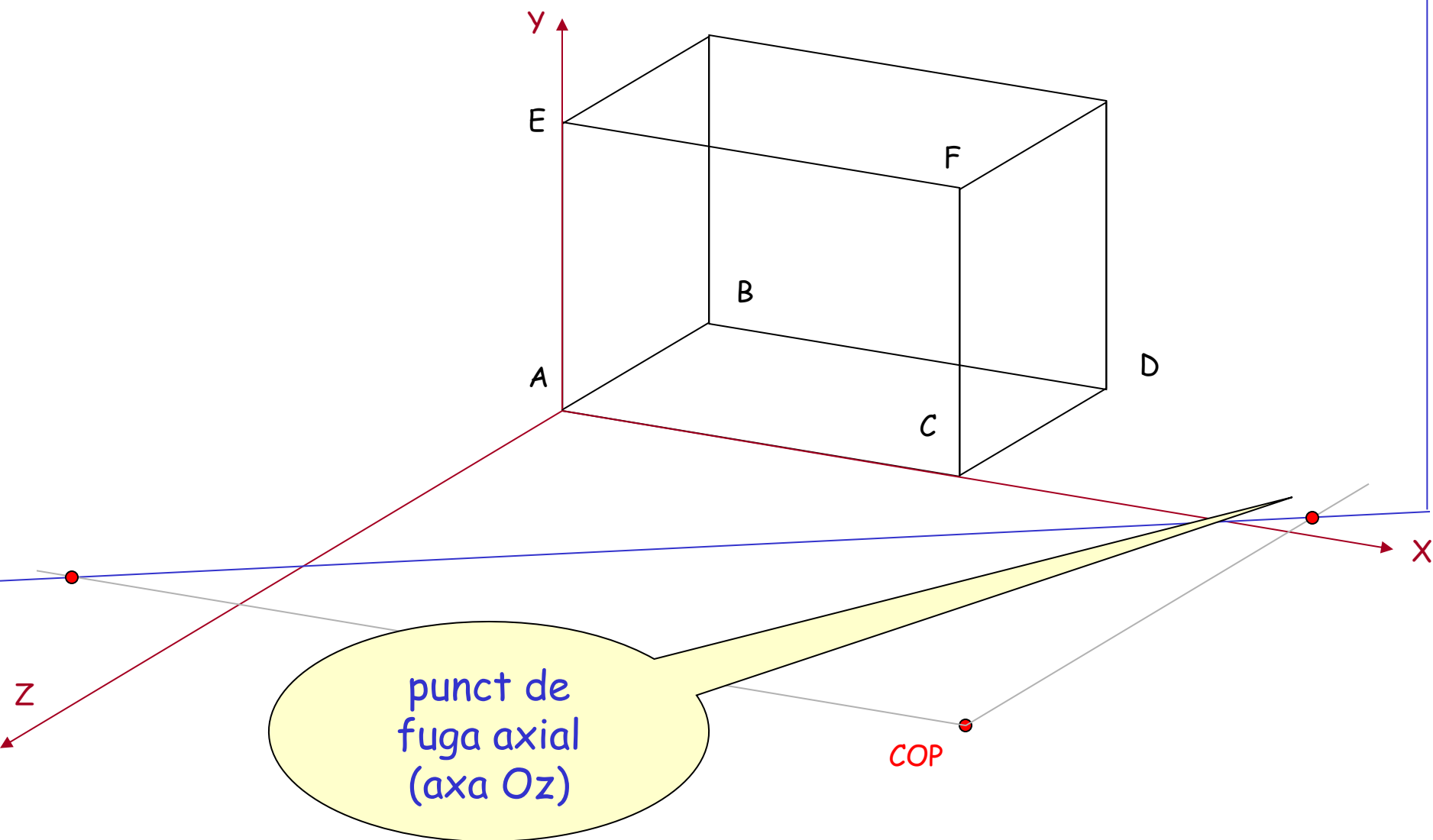
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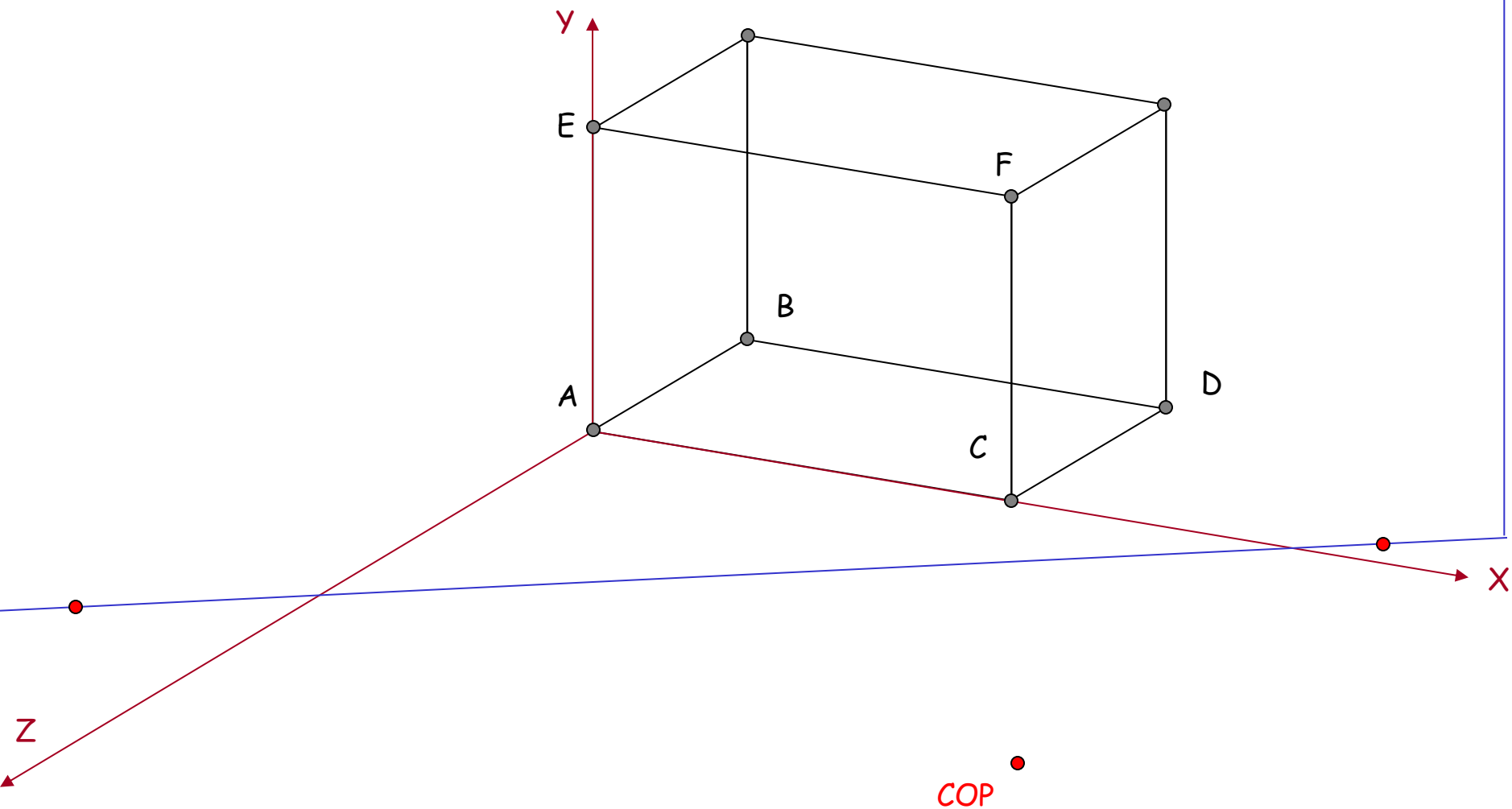
# Proiectii perspectiva 2 pfa



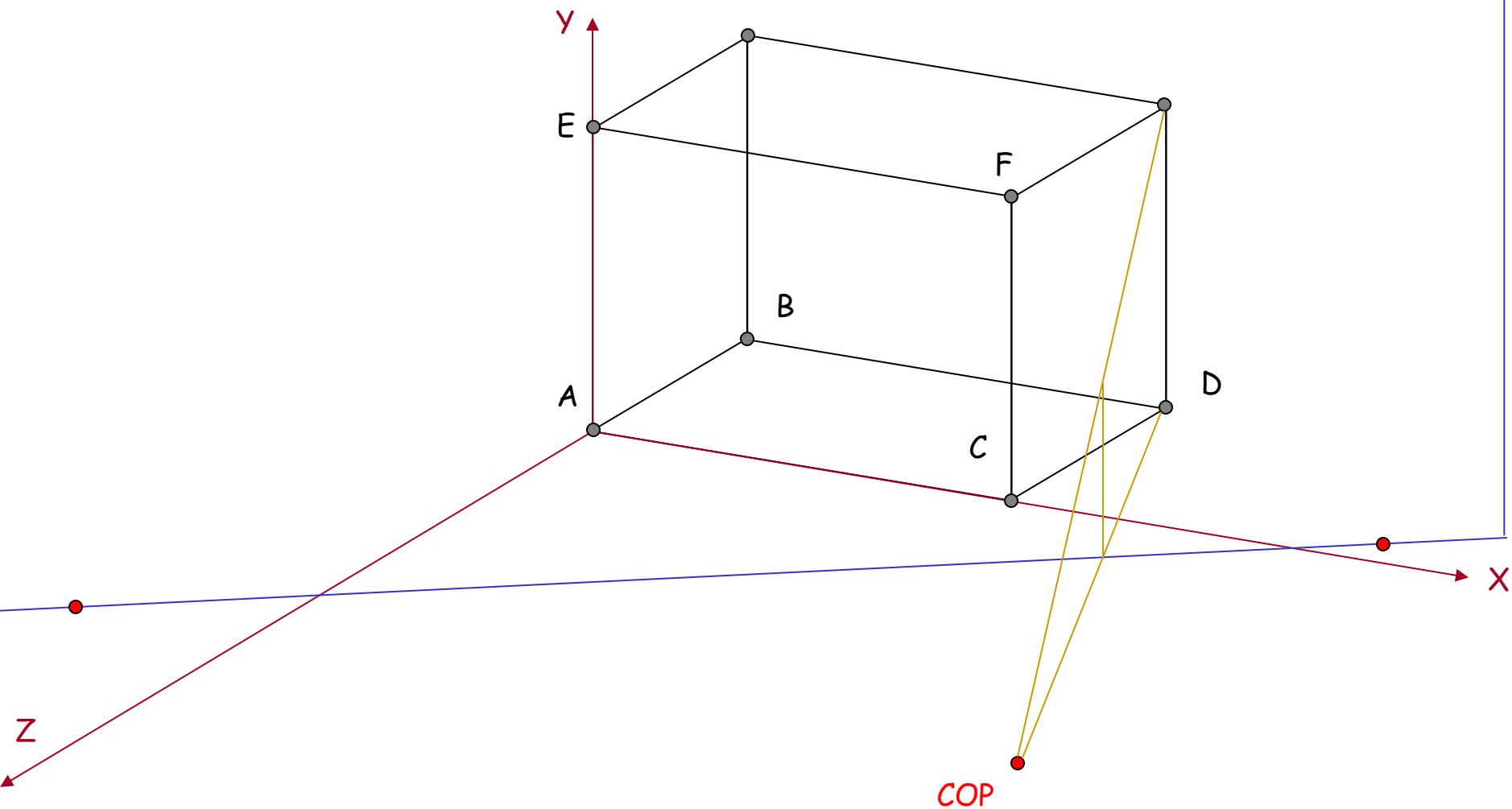
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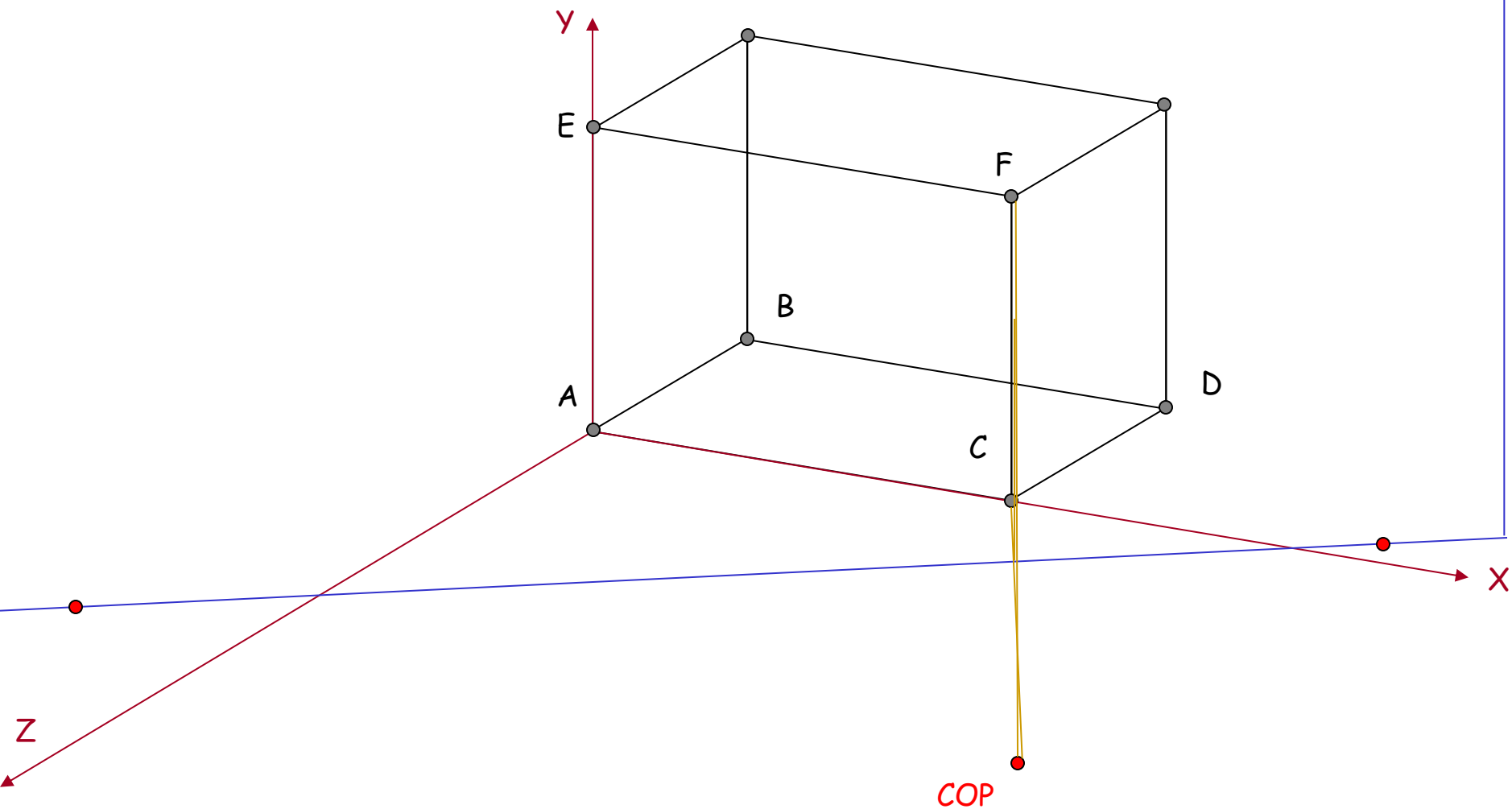
# Proiectii perspectiva 2 pfa



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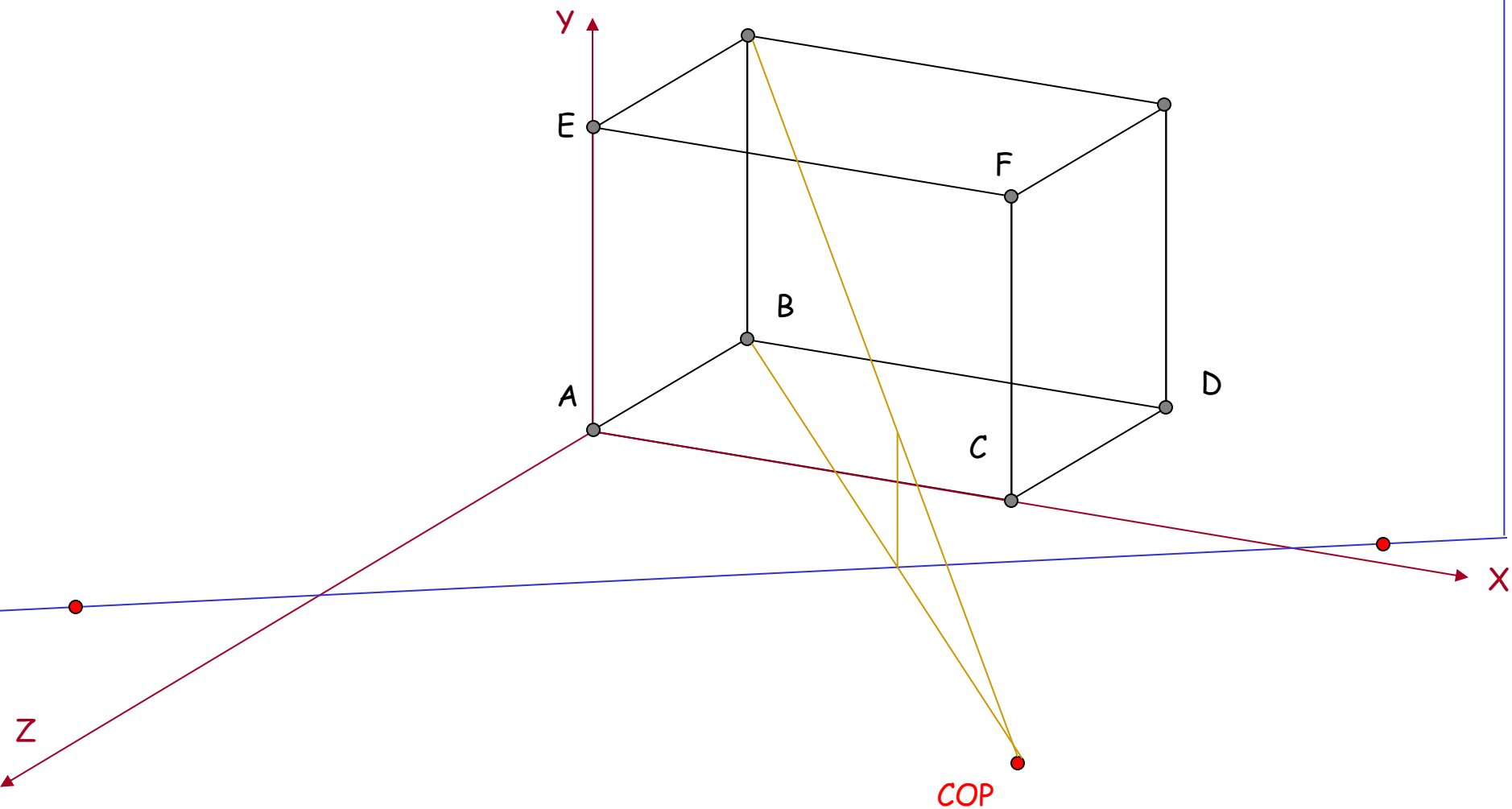


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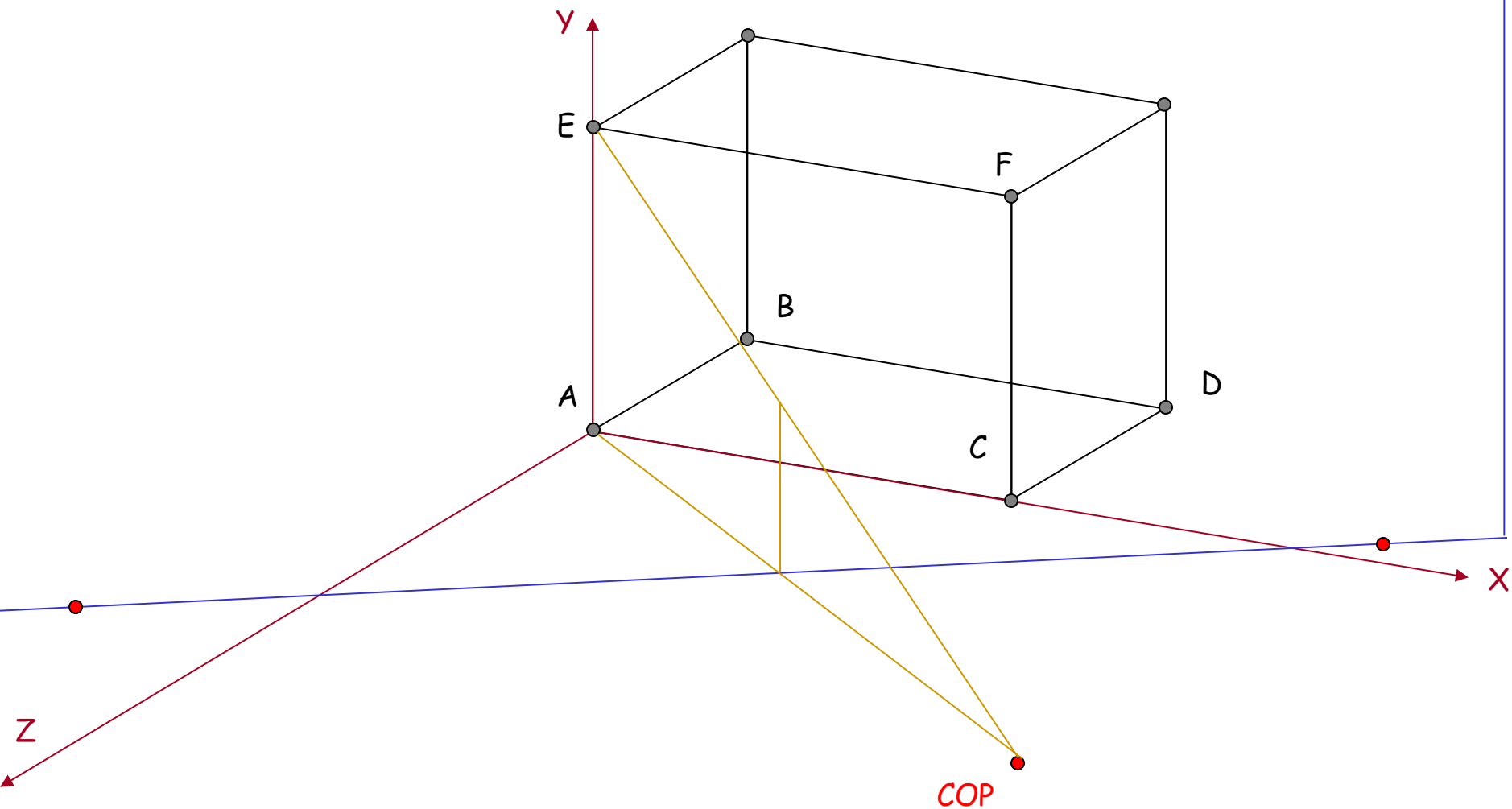




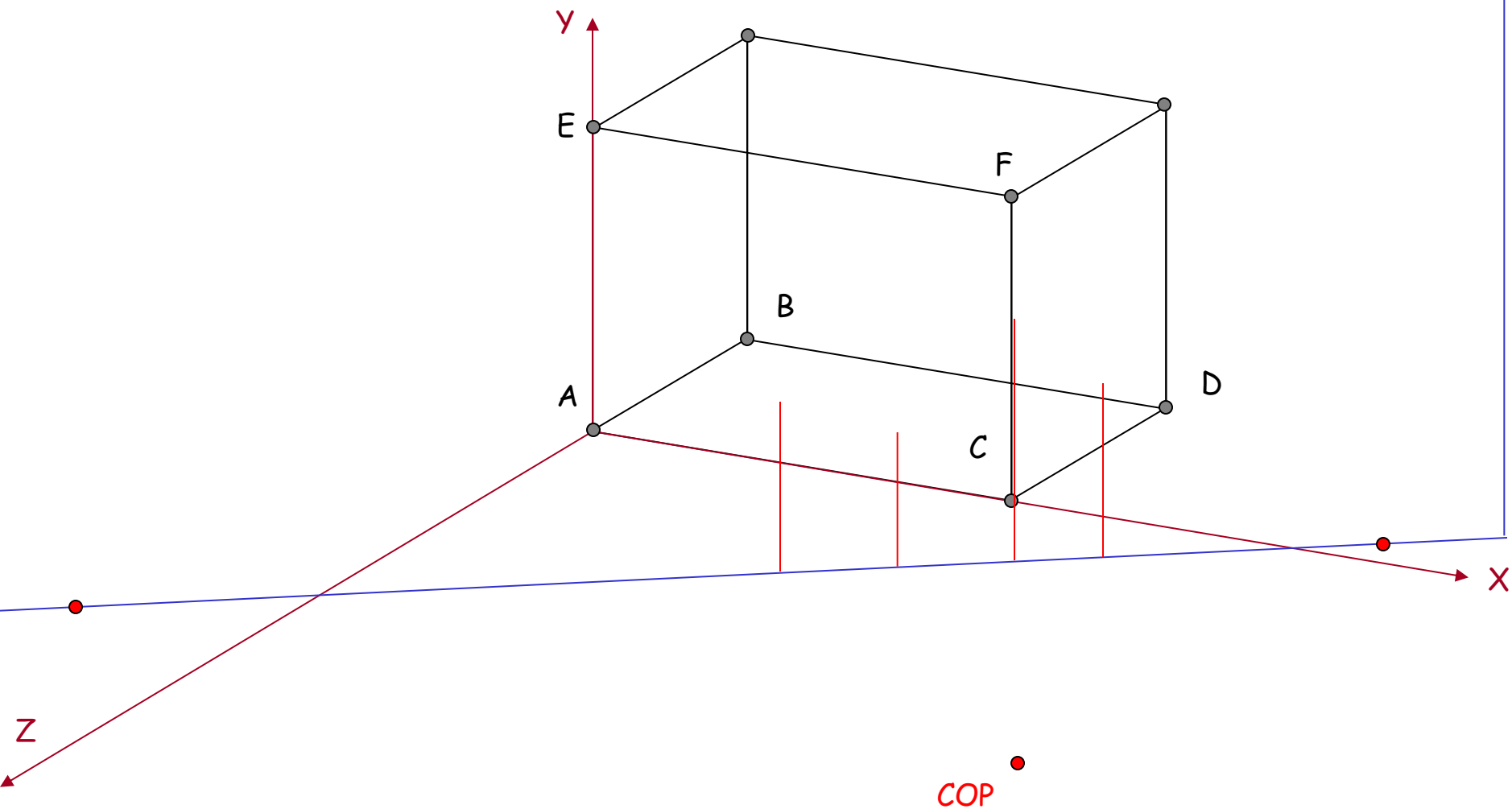
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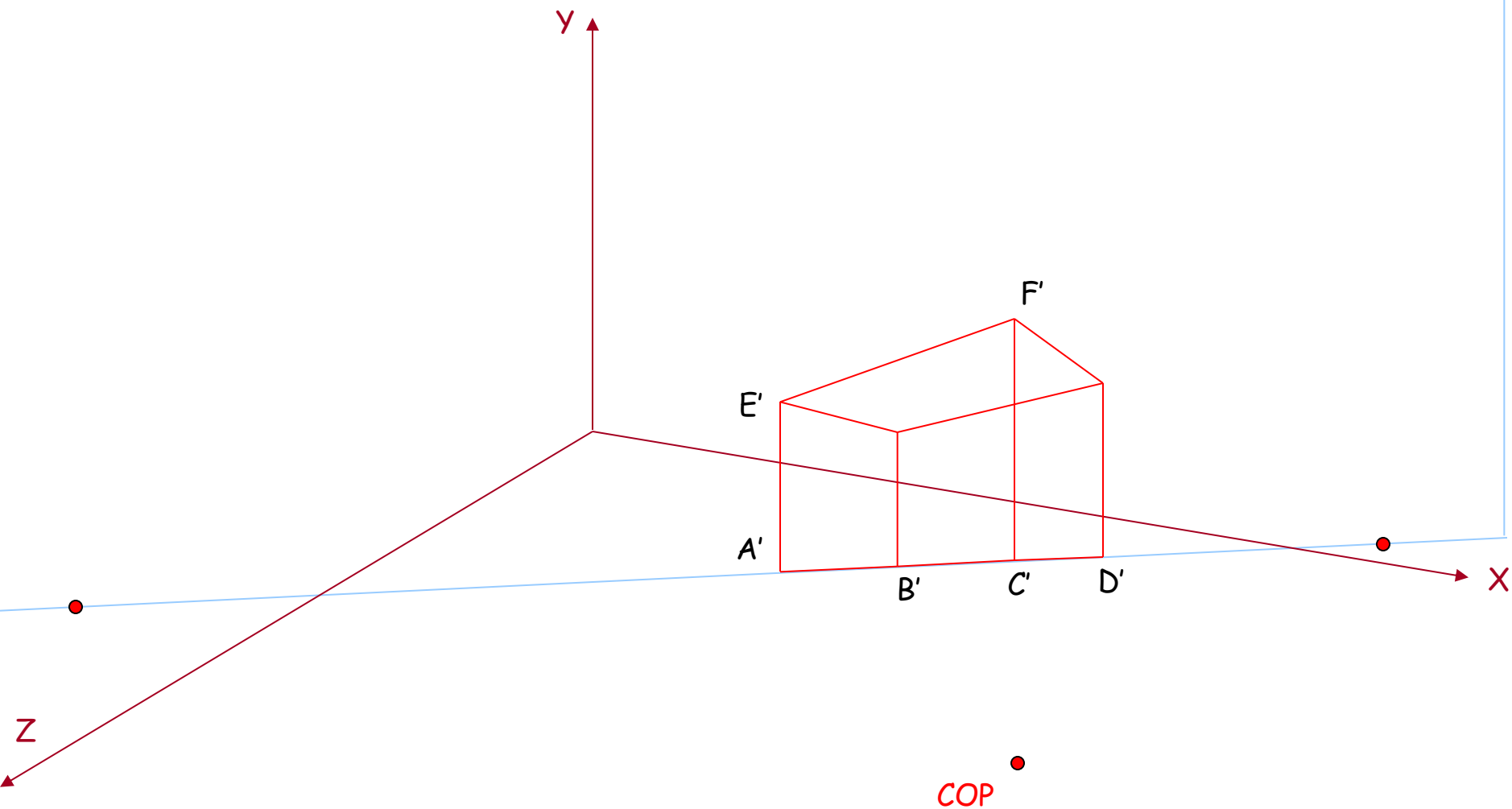
# Proiectii perspectiva 2 pfa



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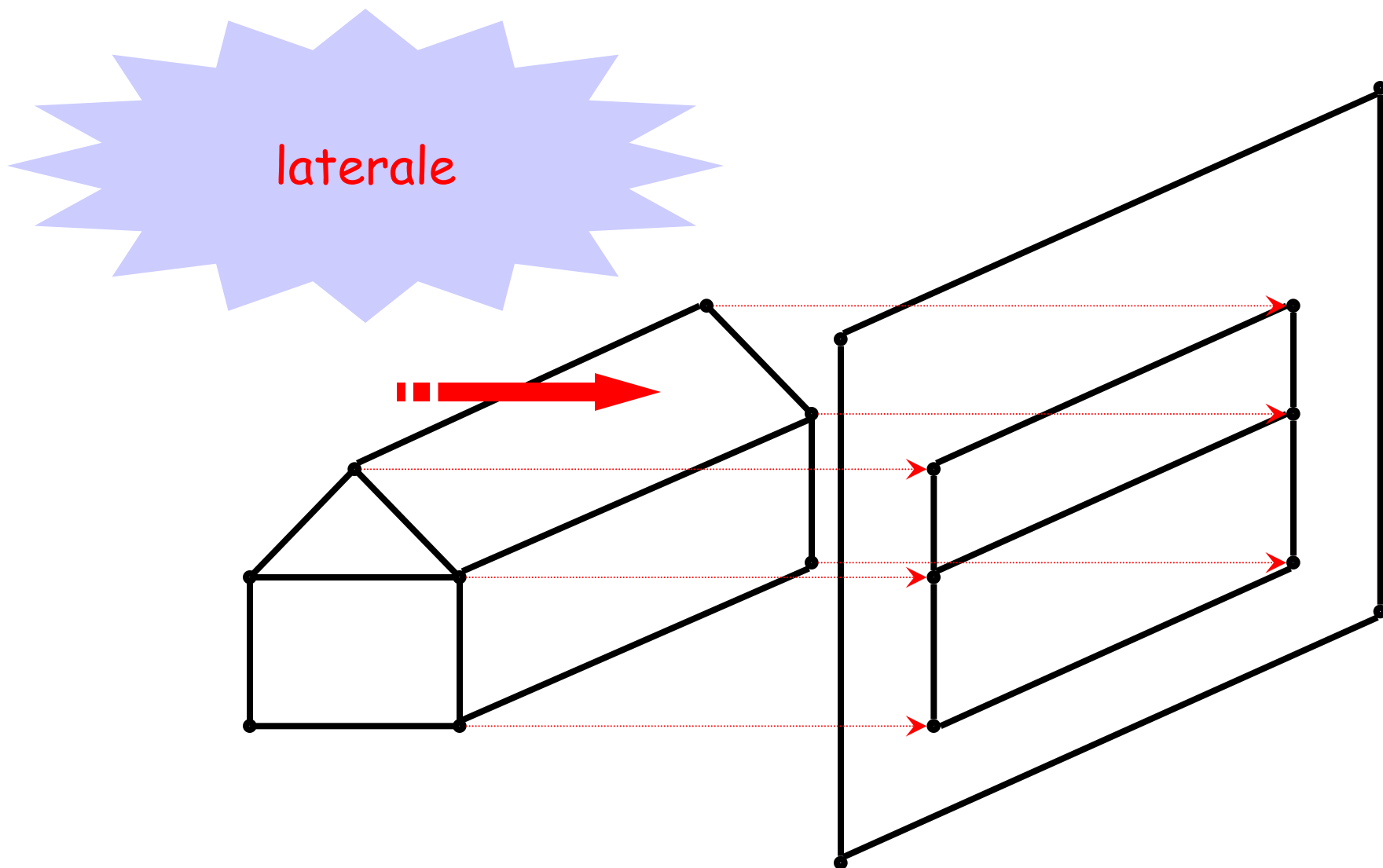
# Proiectii paralele

- clasificarea acestor proiectii in functie de pozitia DOP in raport cu planul de proiectie
  - daca DOP este perpendiculara pe planul de proiectie -> ortografice
  - altfel, -> oblice

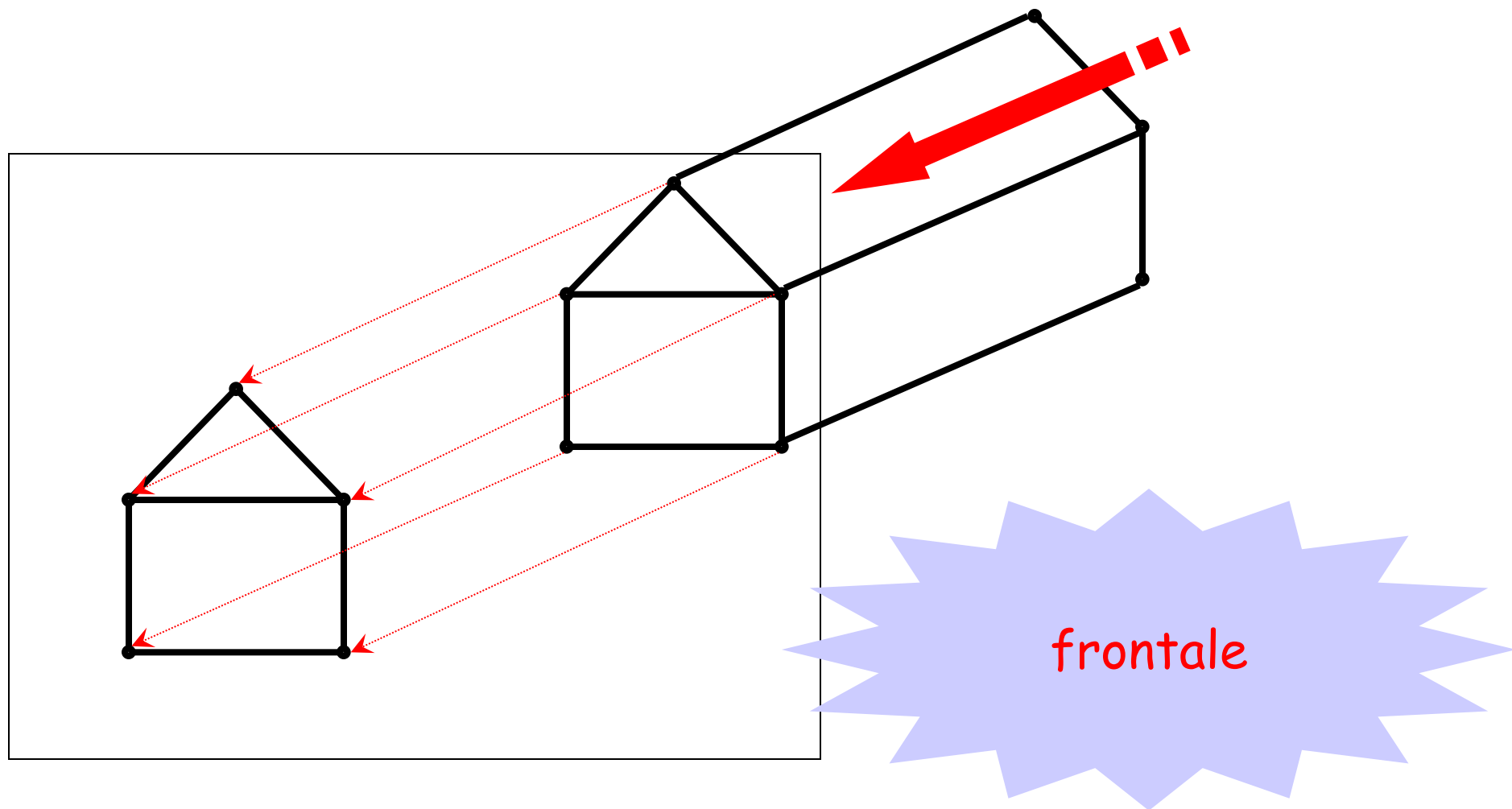
# Proiectii paralele ortografice

- Daca planul de proiectie este perpendicular pe
  - axa  $Ox$  -> laterala
  - axa  $Oy$  -> verticala
  - axa  $Oz$  -> frontala
- altfel, -> axonometrica
  - daca unghiurile (DOP,  $\{Ox, Oy, Oz\}$ ) sunt egale -> izometrica
  - altele

# Proiectii paralele ortographe

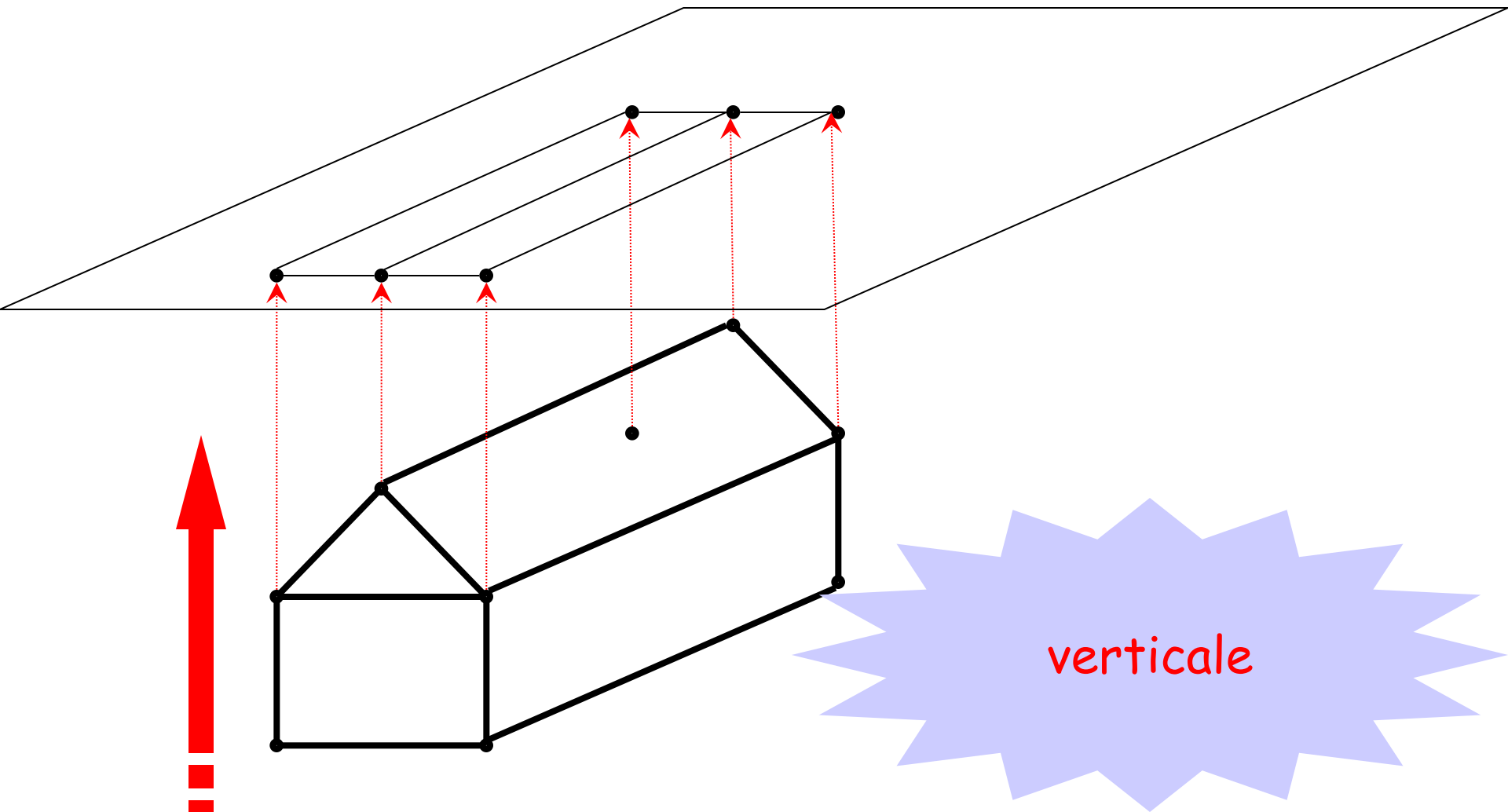


# Proiectii paralele ortografice

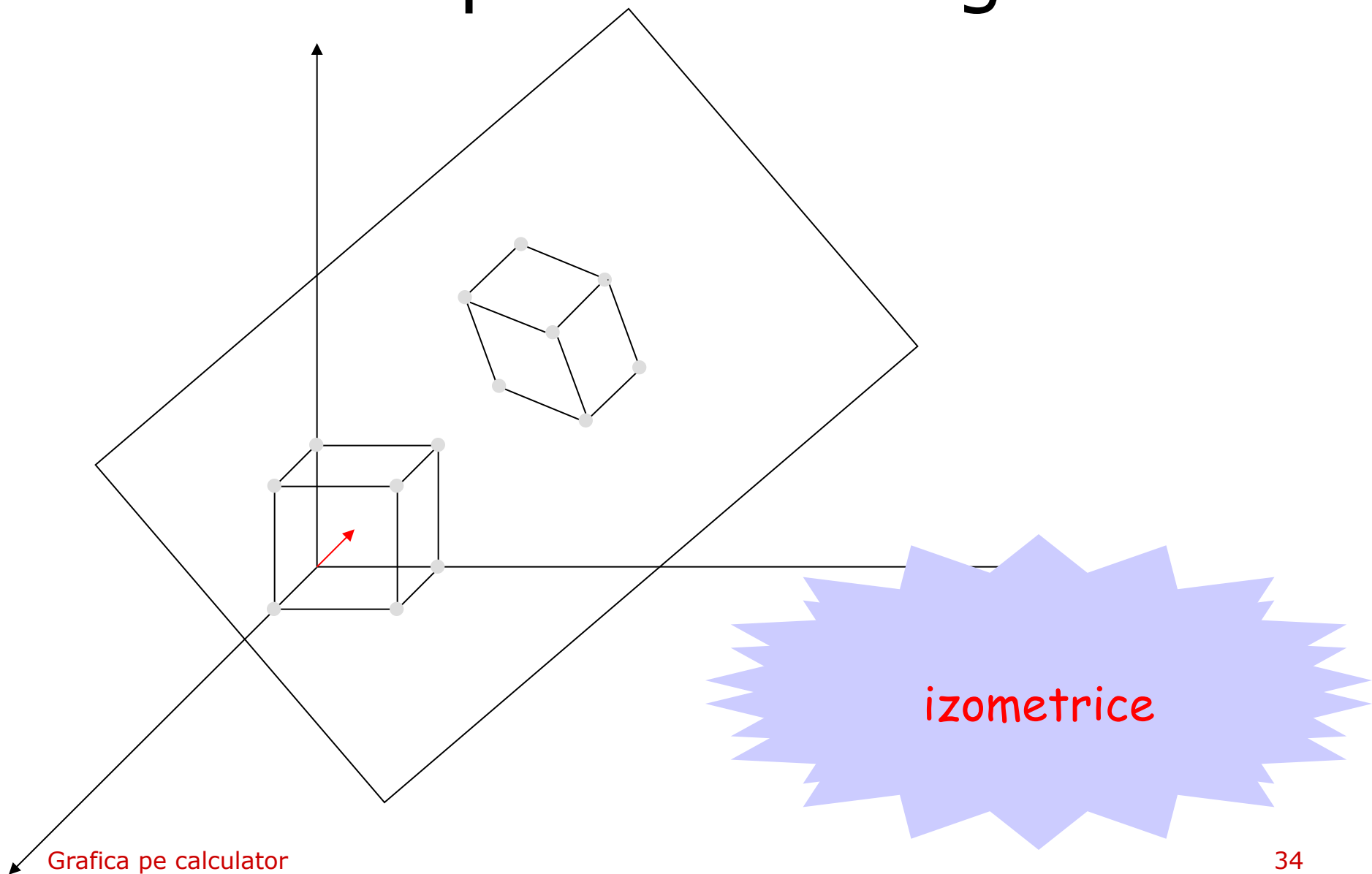




# Proiectii paralele ortografice



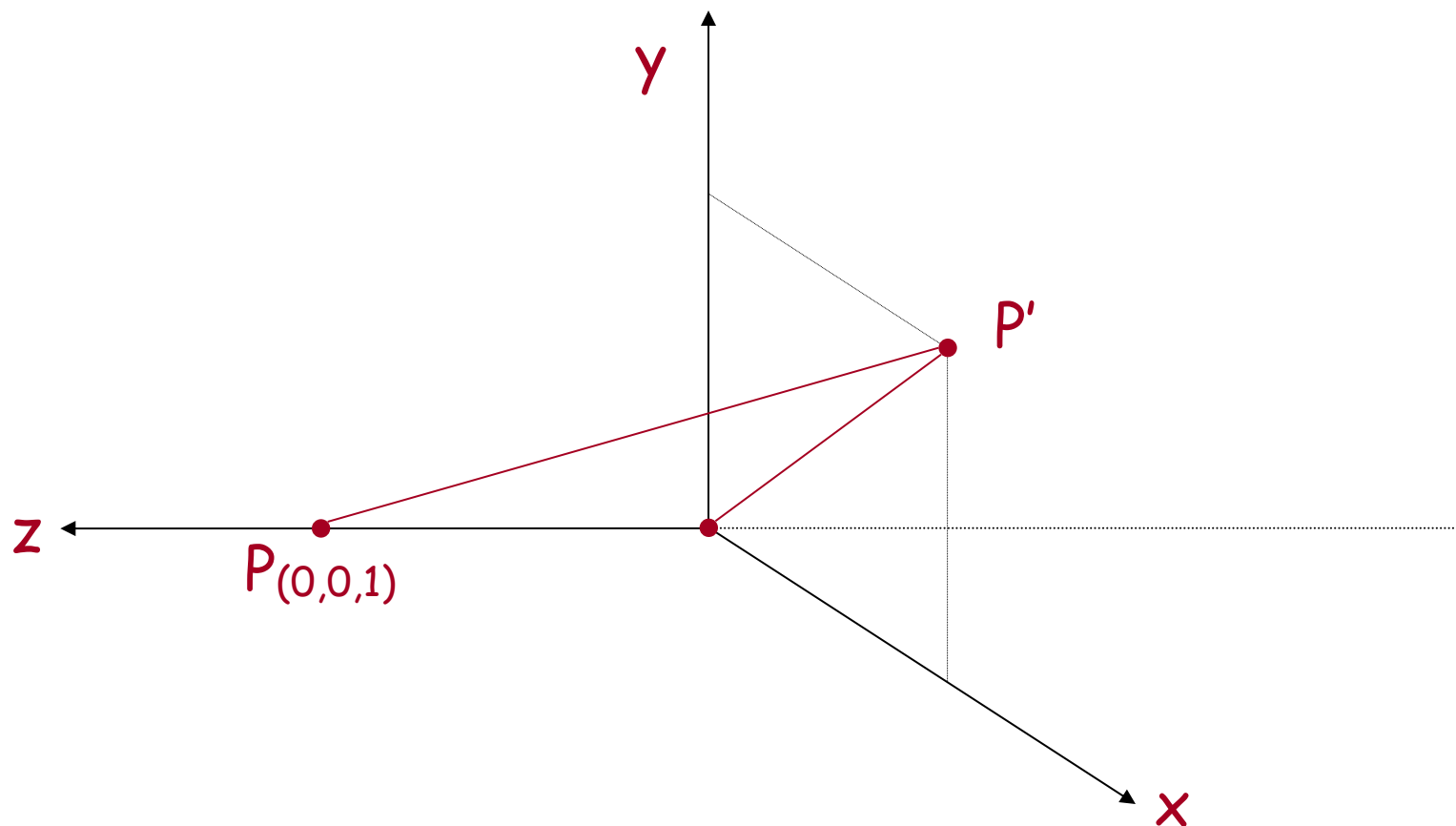
# Proiectii paralele ortografice



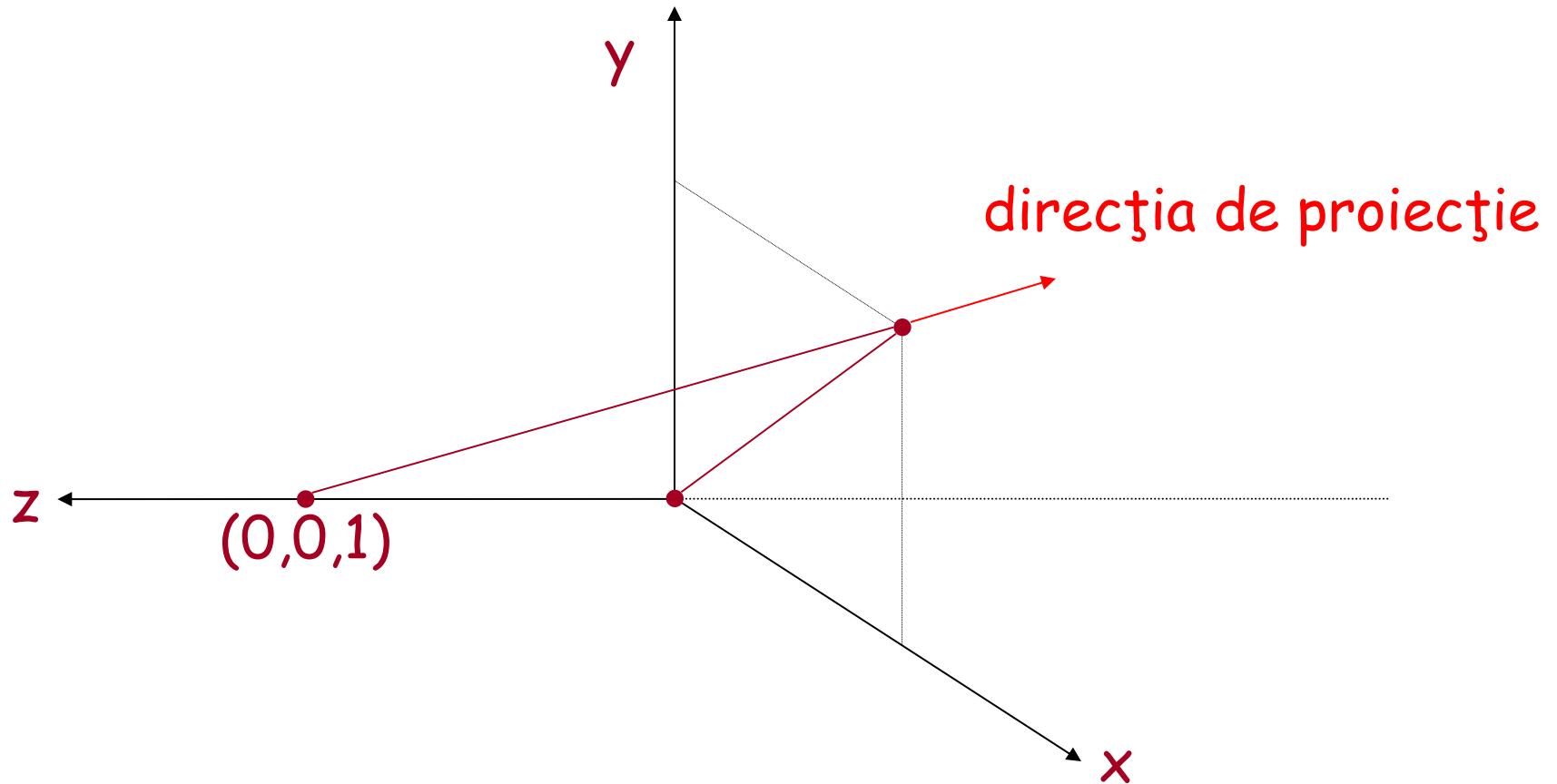
# Proiectii paralele oblice

- clasificarea acestor proiectii in functie de unghiul dintre DOP si planul de proiectie
  - cavalierea :  $45^\circ \Rightarrow$  lung.proiectiei unui seg.perp.pe pl.de pr.este egala cu lung(seg.
  - cabinet :  $\arctan(2) \Rightarrow$  lung.proiectiei unui seg.perp.pe pl.de pr.este  $\frac{1}{2}$  din lung(seg.
  - altele

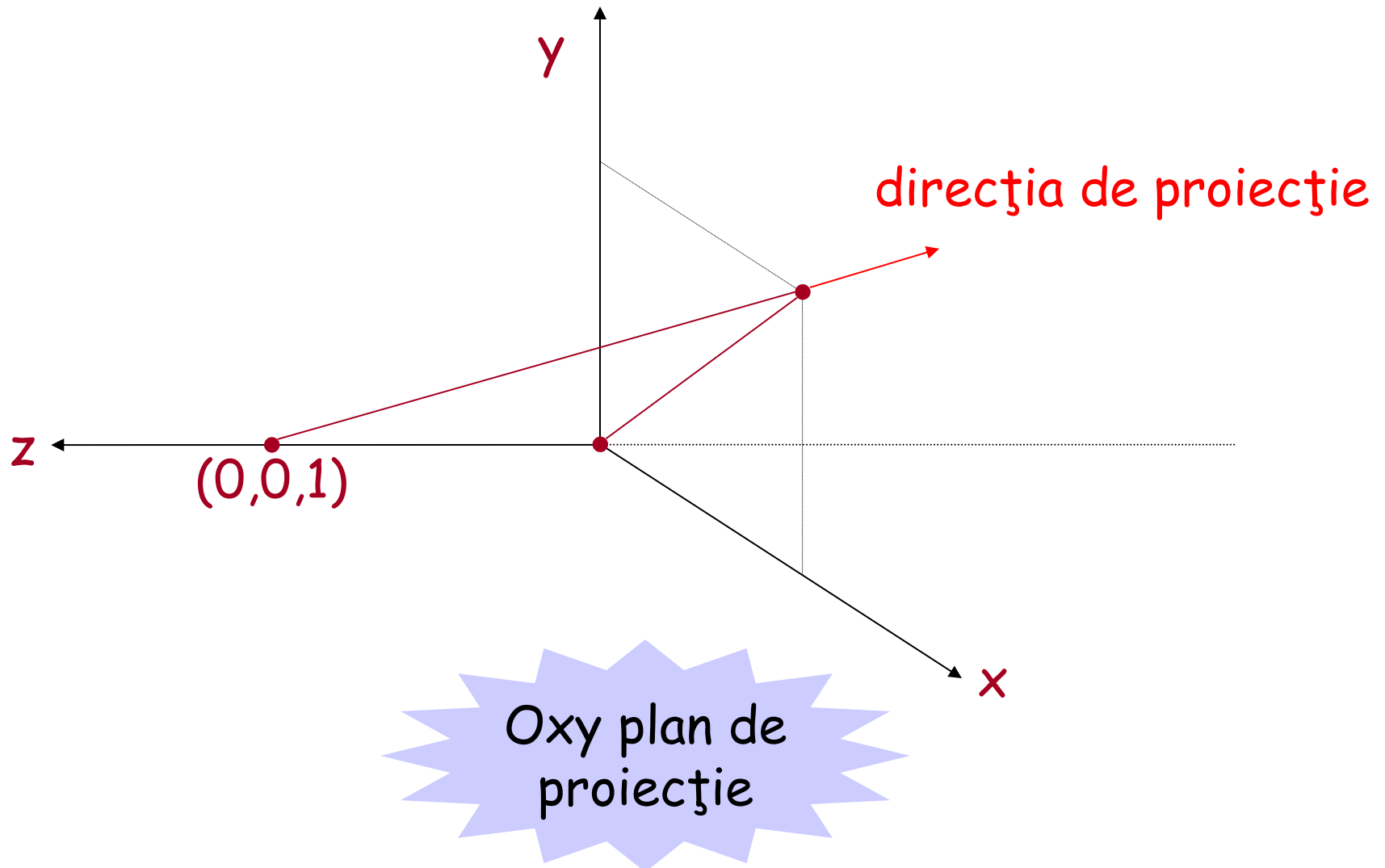
# Proiectii paralele oblice



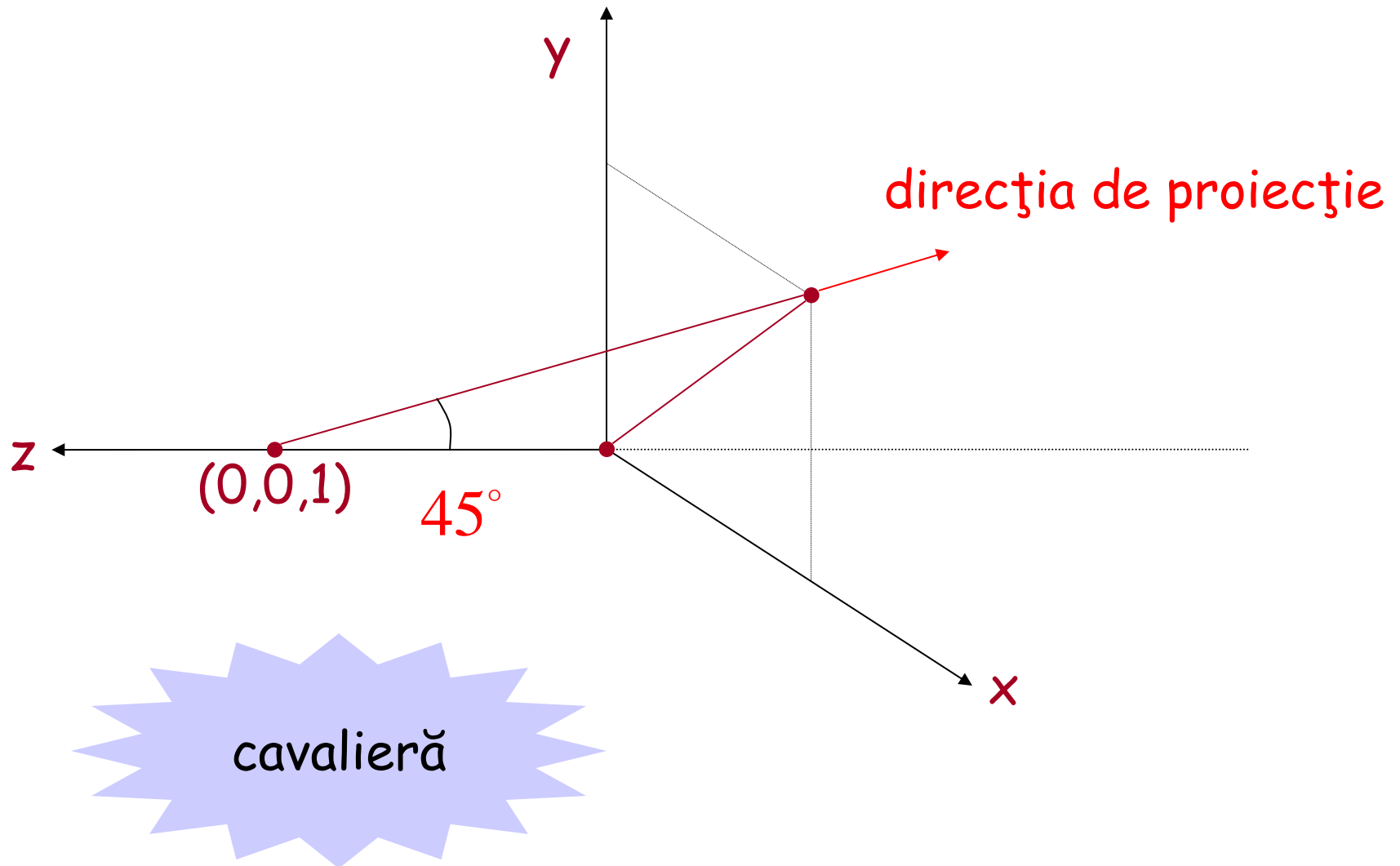
# Proiectii paralele oblice



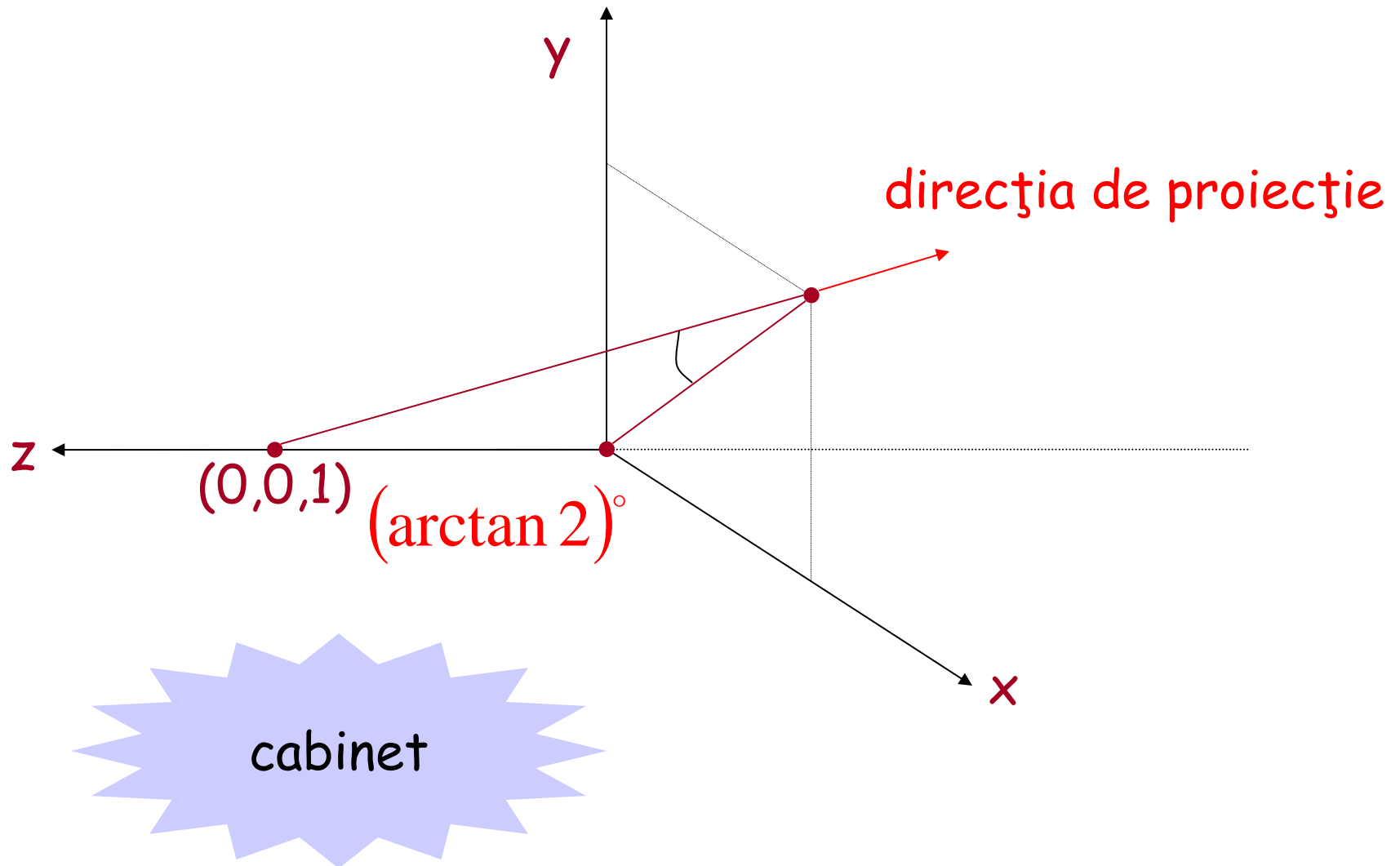
# Proiectii paralele oblice



# Proiectii paralele oblice

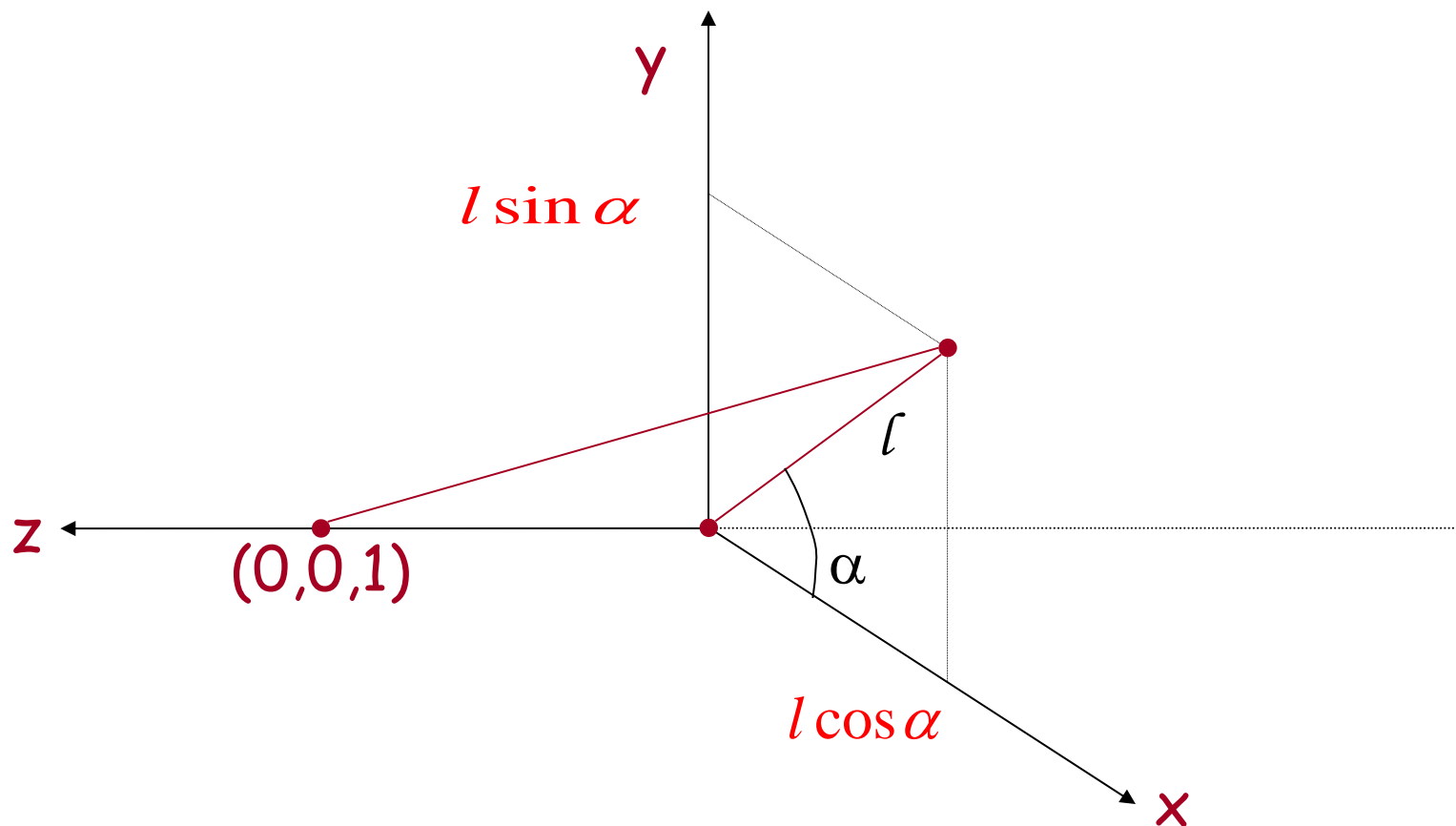


# Proiectii paralele oblice

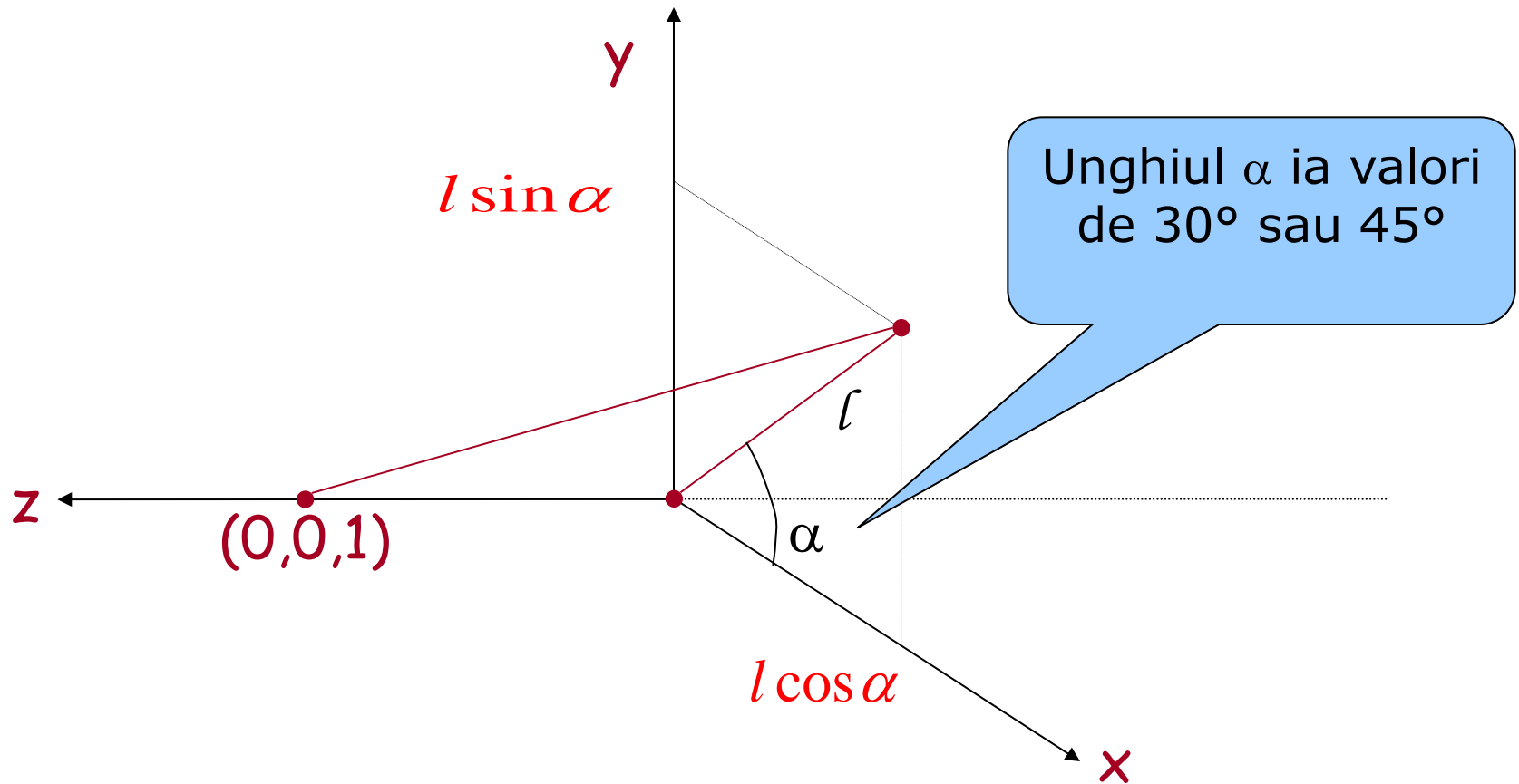




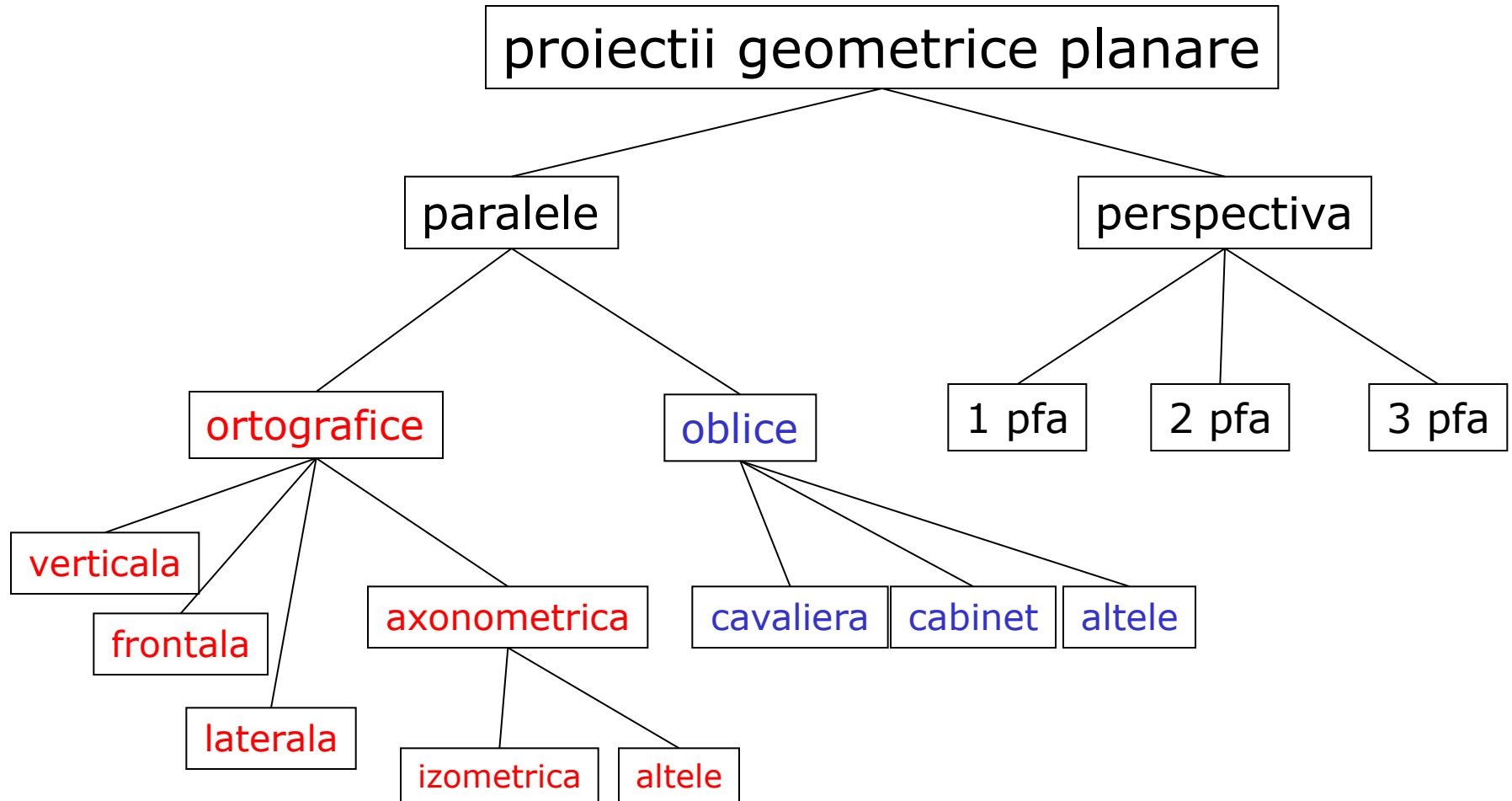
# Proiectii paralele oblice



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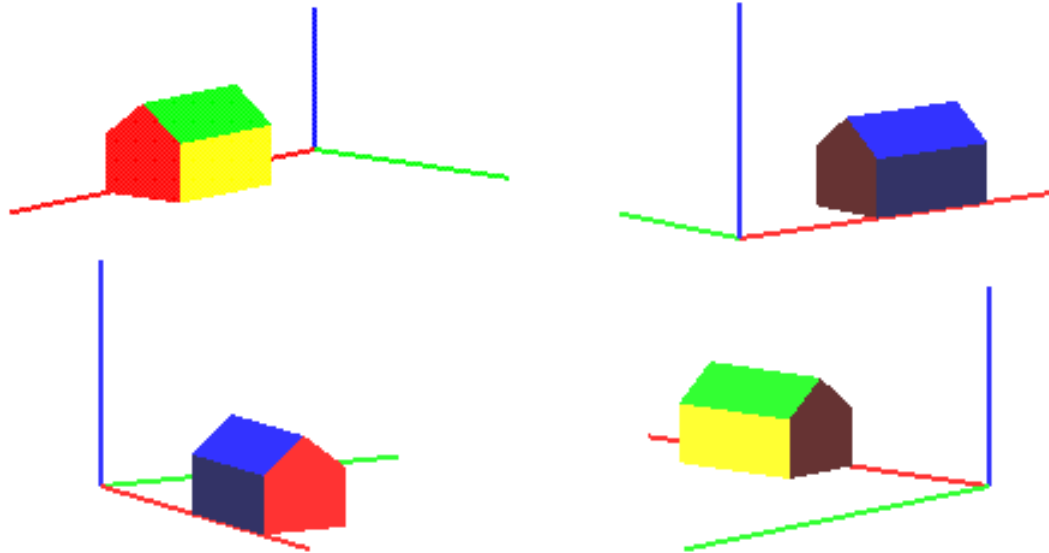


# Subclasele proiectiilor geometrice planare

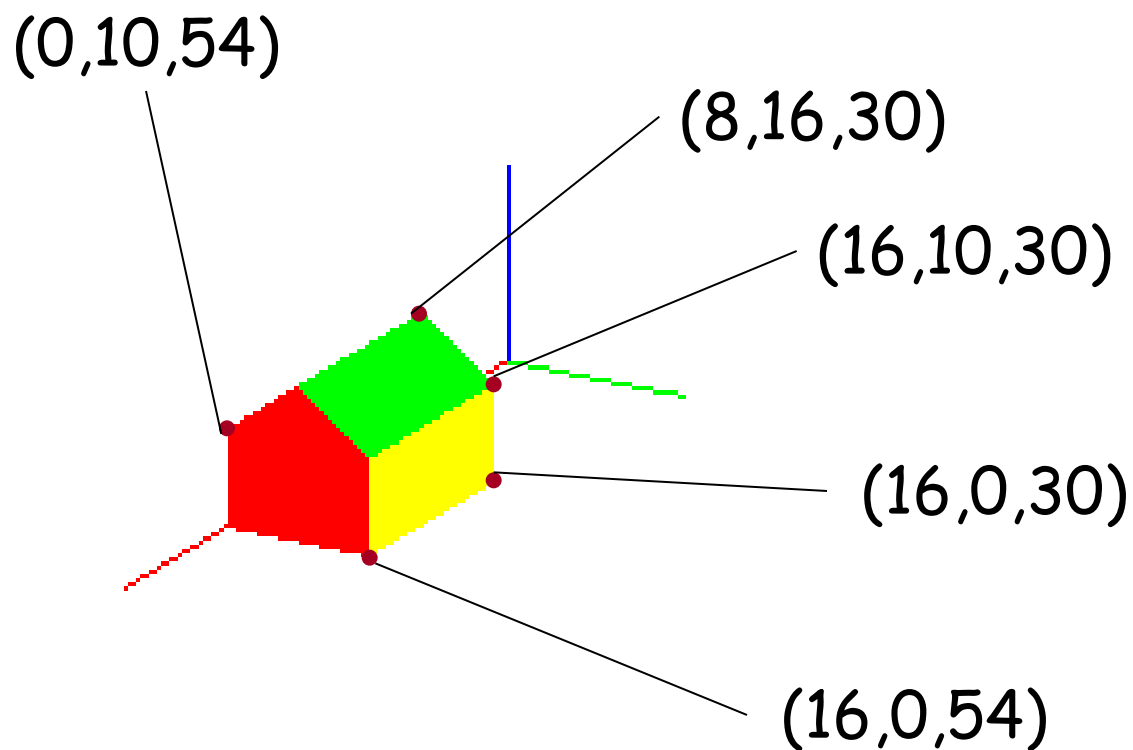


# Proiectii geometrice planare

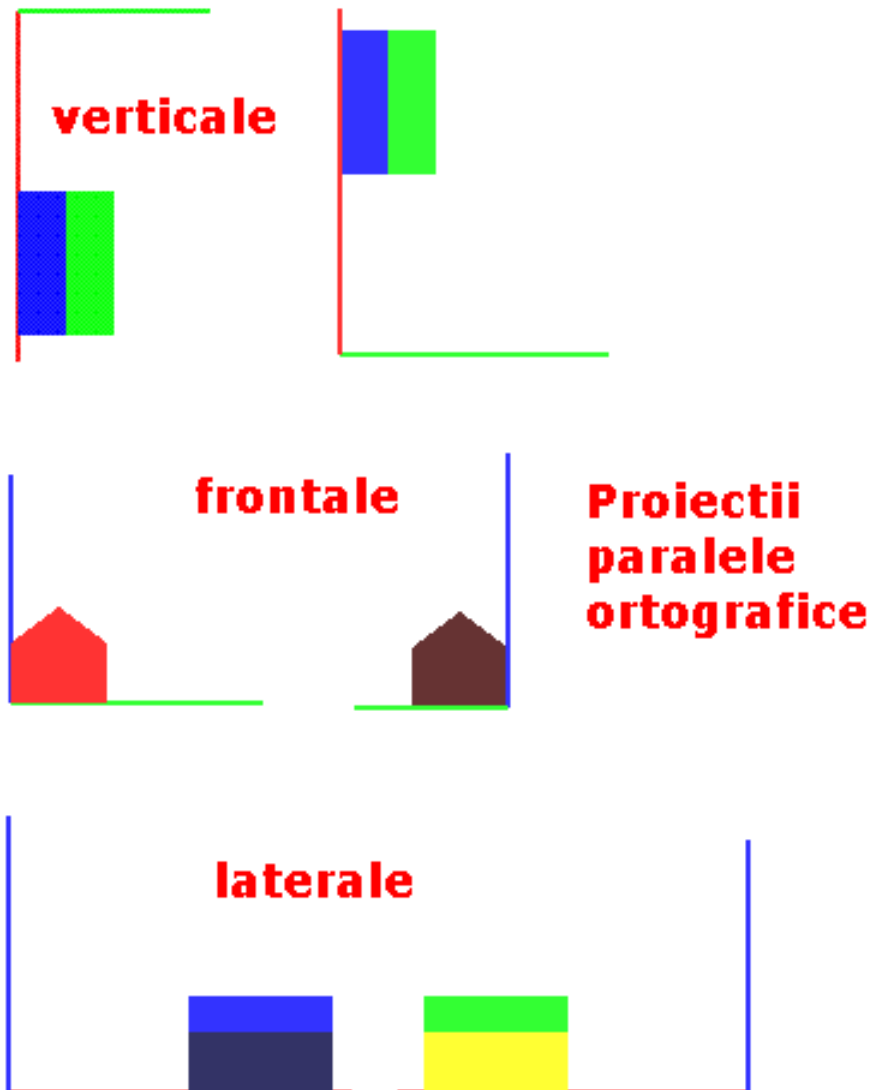
- Exemplu



# Proiectii geometrice planare



# Proiectii geometrice planare



# Proiectii geometrice planare



proiectie cavaliere  
unghi 30 grade



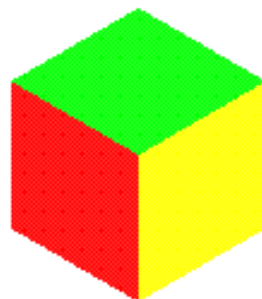
proiectie cabinet  
unghi 30 grade



proiectie cavaliere  
unghi 45 grade

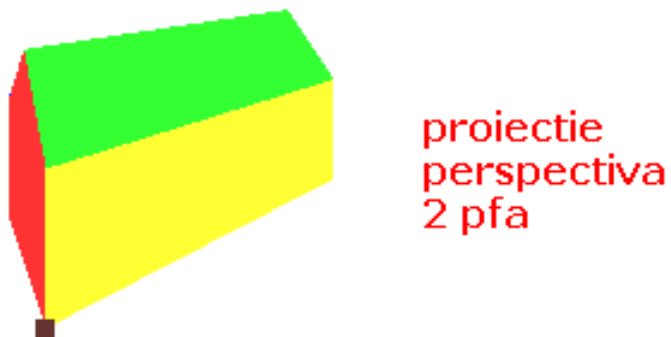
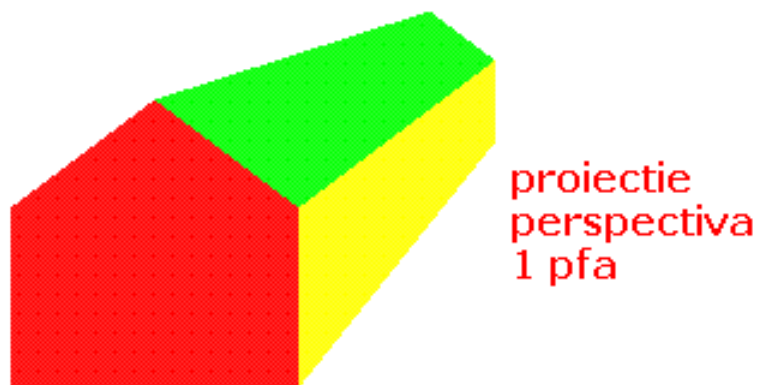


proiectie cabinet  
unghi 45 grade



Proiectie  
paralela  
axonometrica  
izometrica

# Proiectii geometrice planare





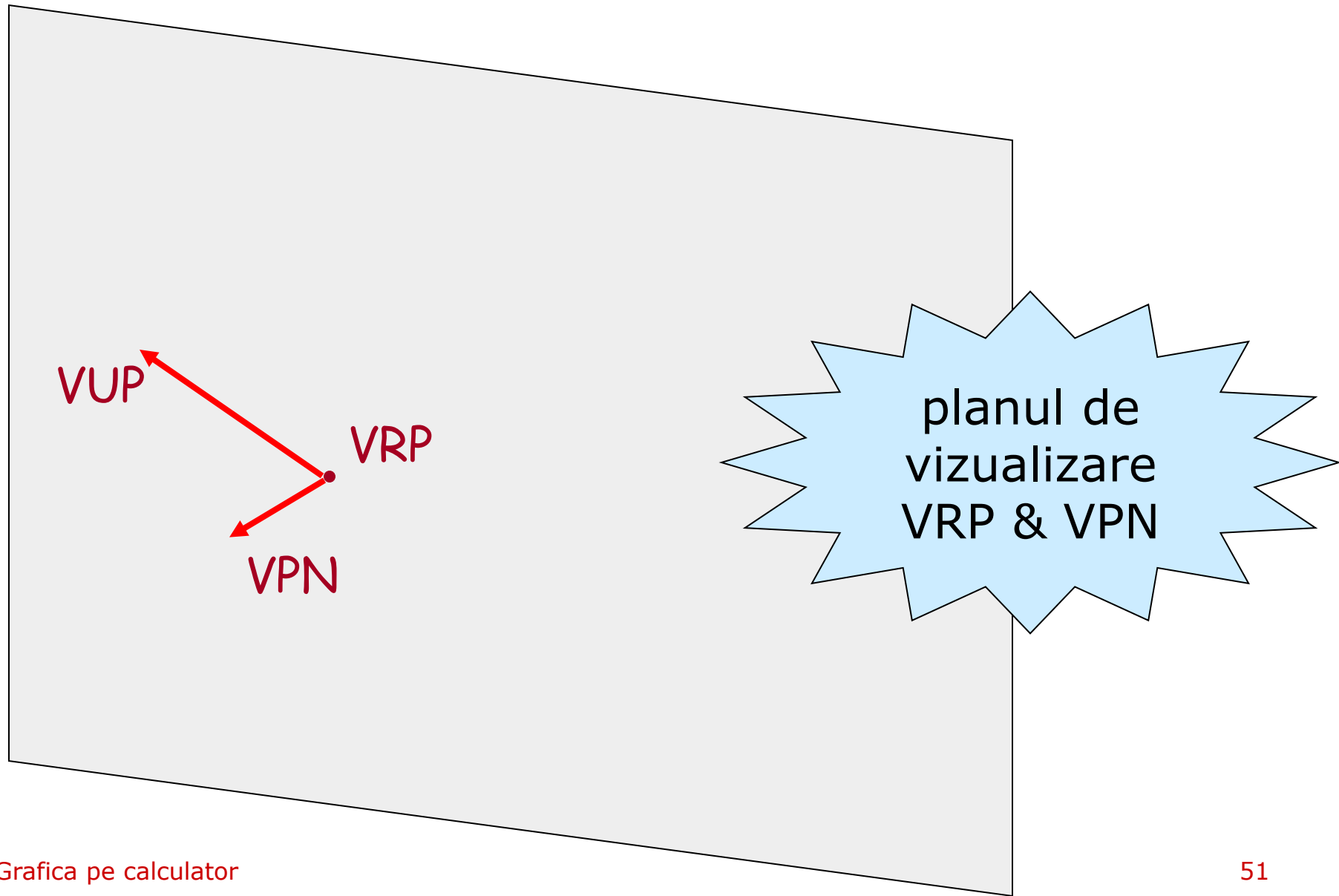
# Proiectii geometrice planare

- Rezumat
  - definitie
  - clasificare : paralele/perspectiva, perspectiva 1/2/3 pfa, paralele ortografice/oblice, etc.
  - exemple

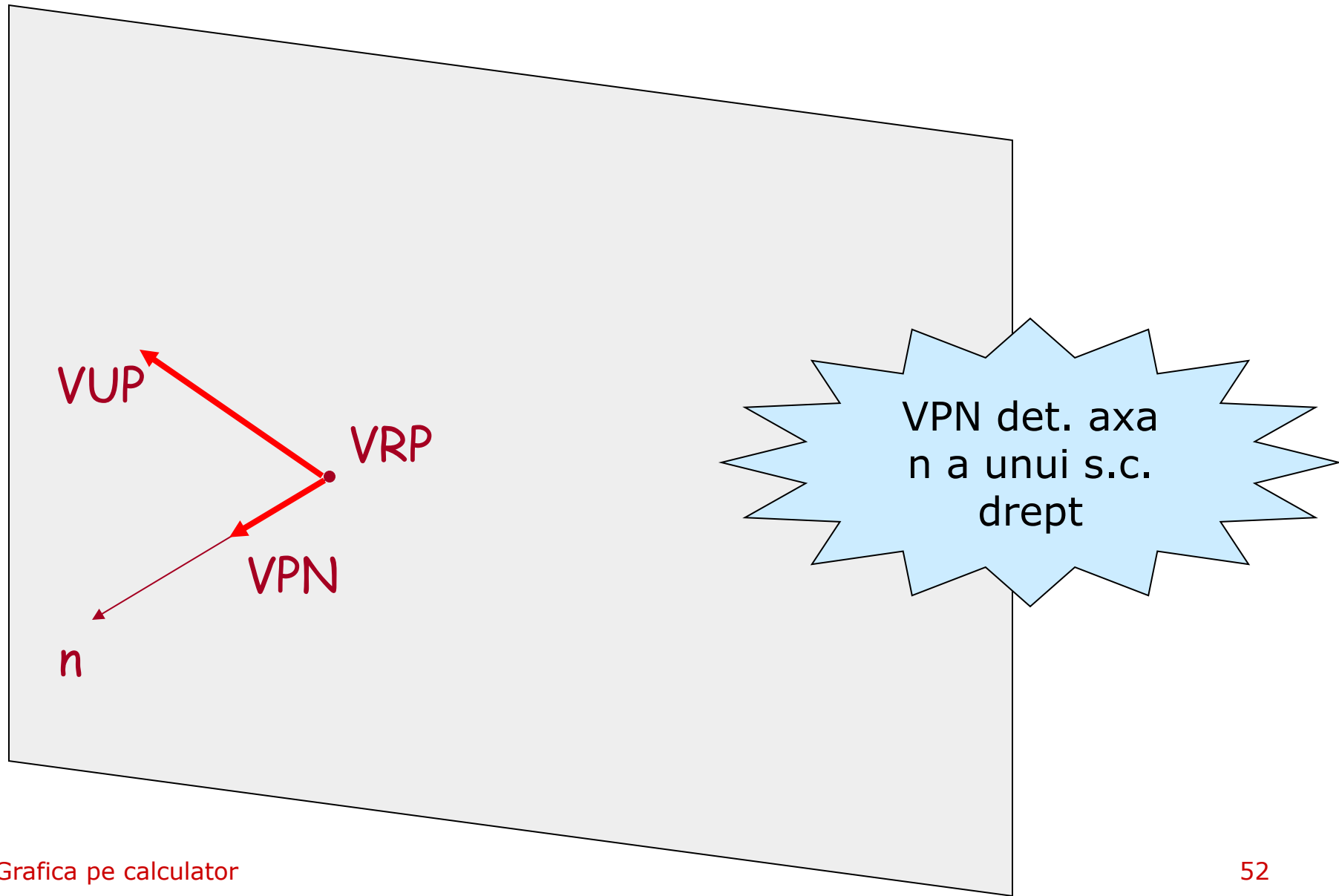
# Proiectii geometrice planare

- Specificarea unei p.g.p. (terminologie PHIGS)
  - planul de vizualizare
    - VRP : view reference point
    - VPN : view plane normal
    - VUP : view up plane
    - aceste elemente determina un sistem de coordonate orientat drept  $VRC = (VRP, u, v, n)$  : view reference coordinate system

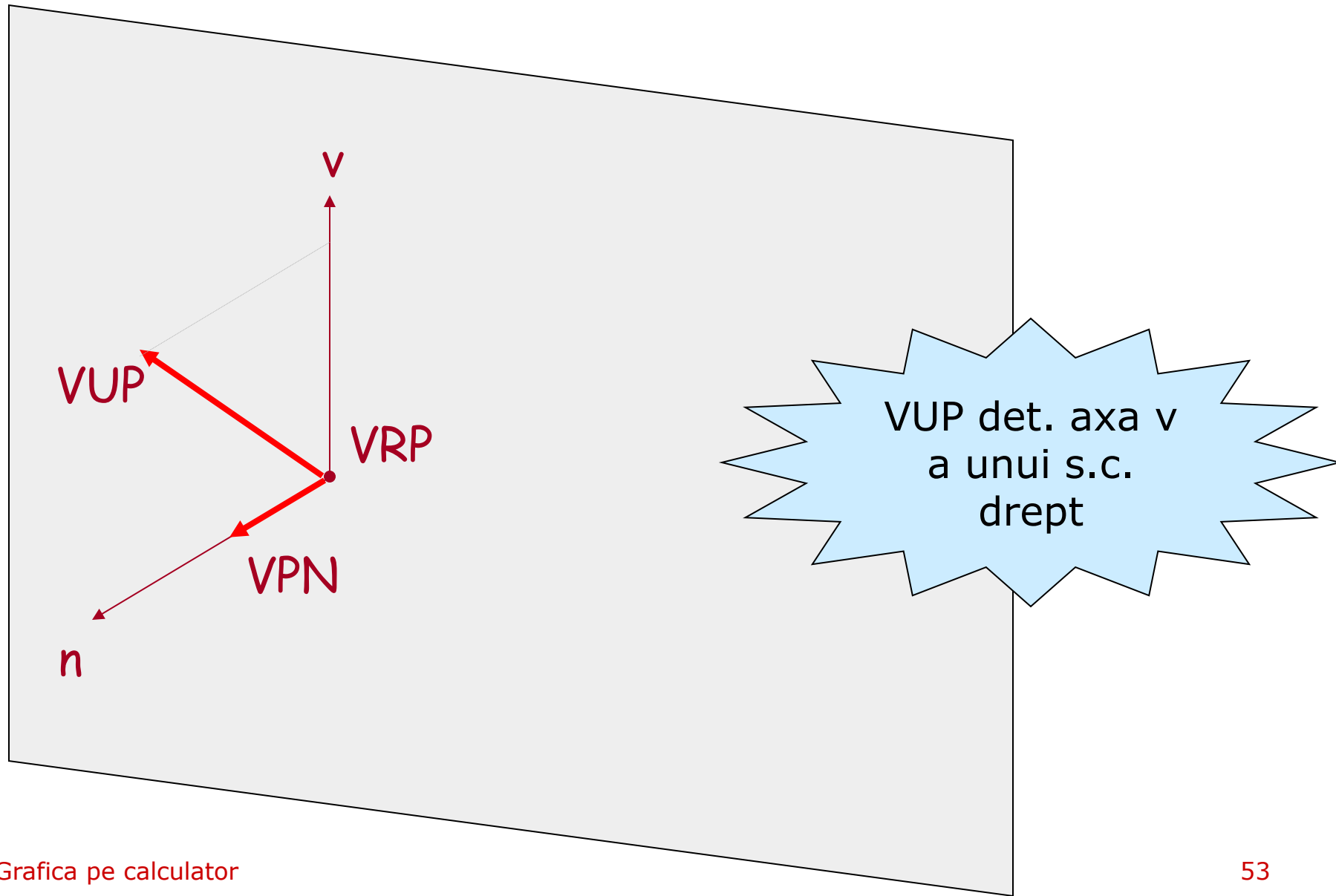
# Specificarea planului de vizualizare



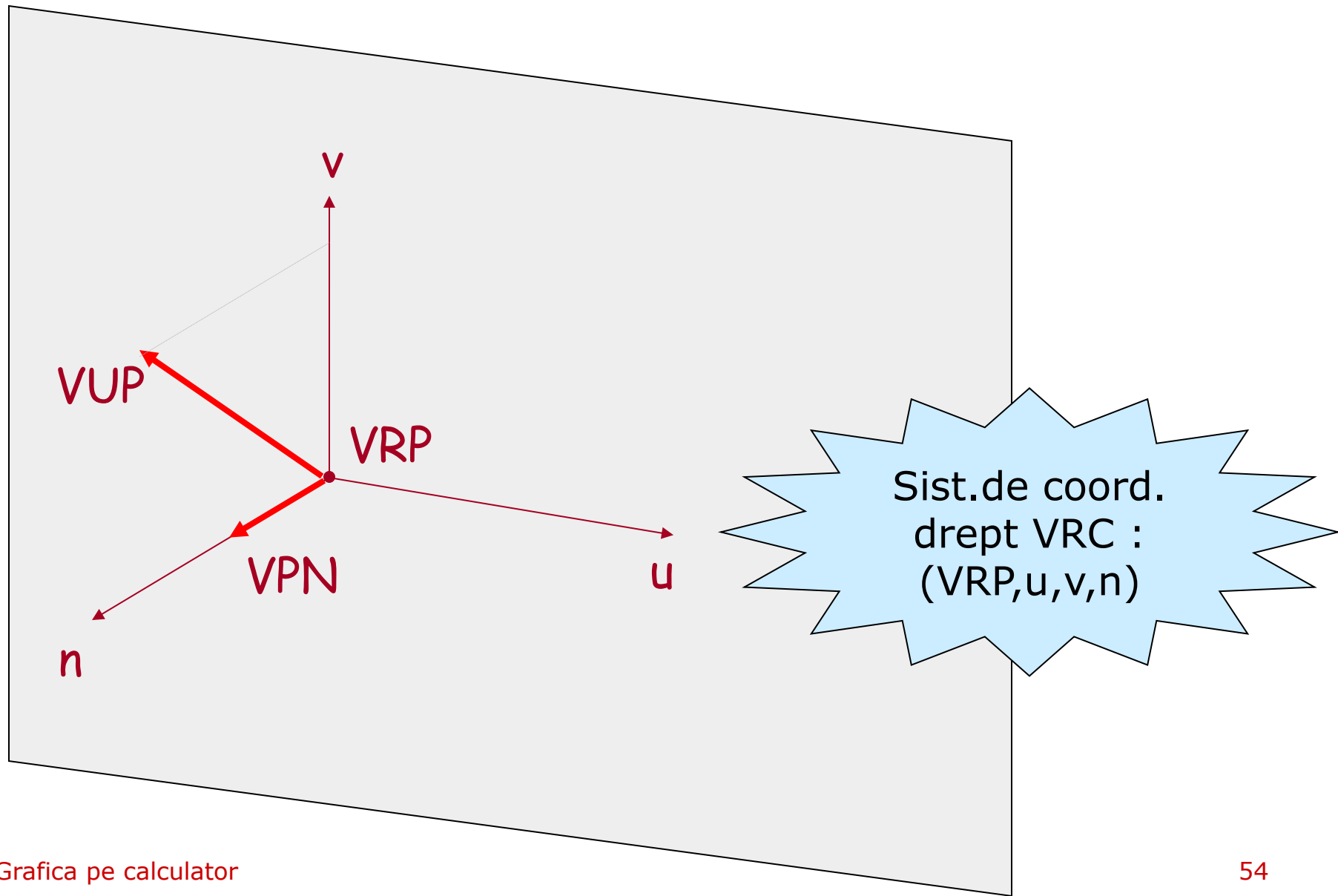
# Specificarea planului de vizualizare



# Specificarea planului de vizualizare



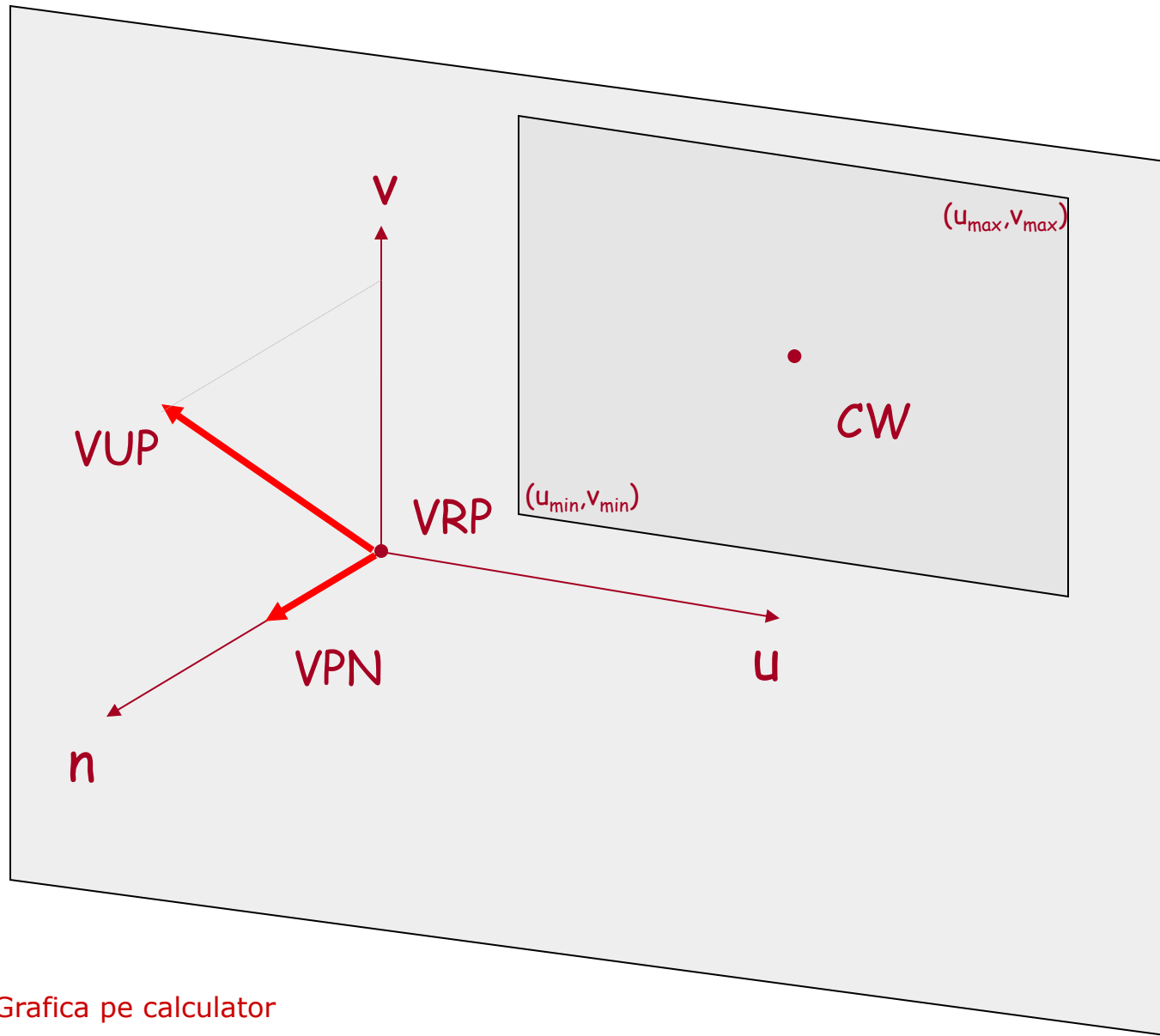
# Specificarea planului de vizualizare



# Proiectii geometrice planare

- Specificarea unei p.g.p.  
(terminologie PHIGS)
  - fereastra in planul de vizualizare
    - CW : centrul ferestrei
    - (umax, vmax) : coord.coltului din drp.sus
    - (umin, vmin) : coord.coltului din stg.jos

# Specificarea ferestrei din planul de vizualizare

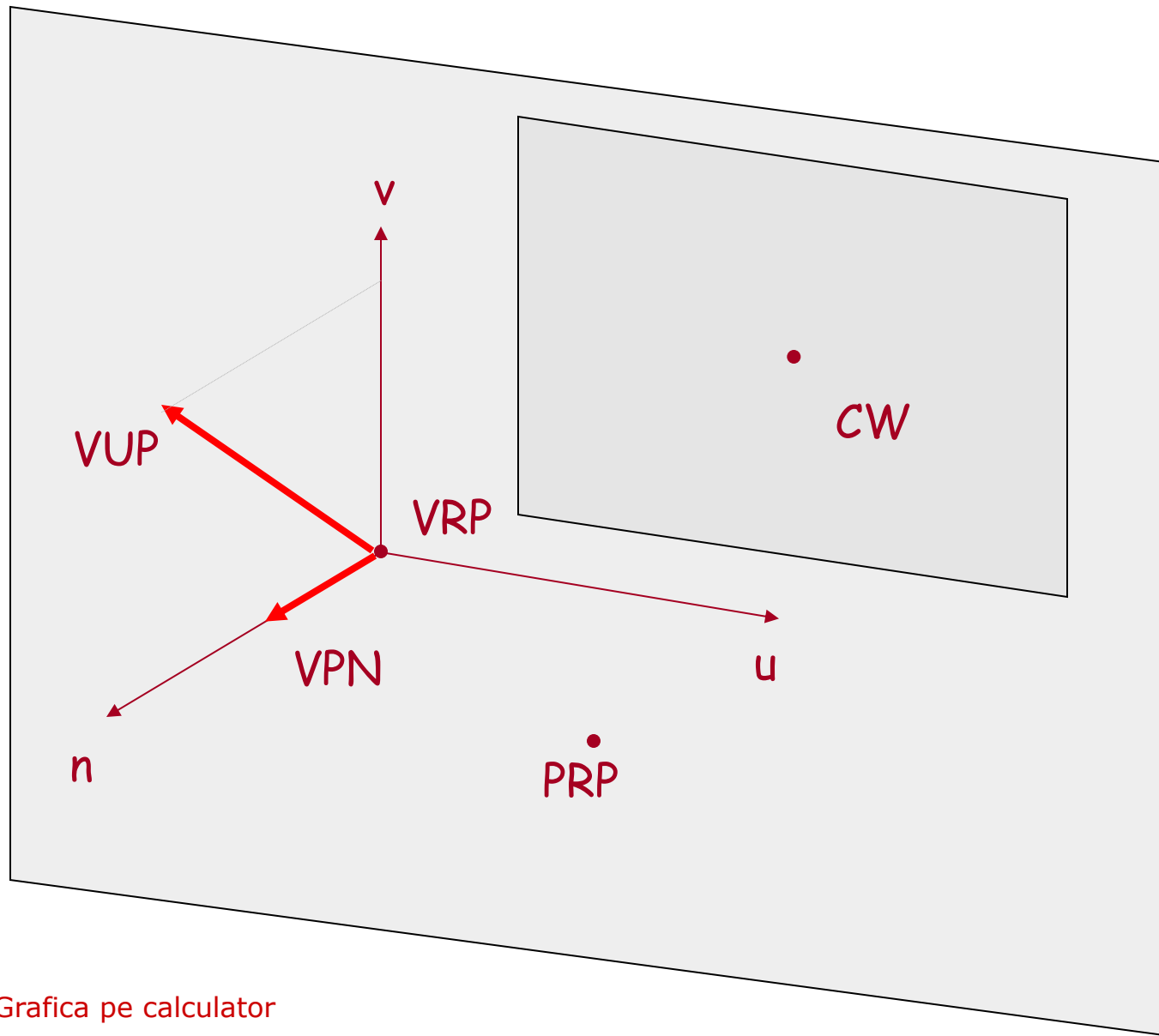




# Proiectii geometrice planare

- Specificarea unei p.g.p.  
(terminologie PHIGS)
  - punctul de referinta al proiectiei
    - PRP : projection reference point
    - proiectie perspectiva  $\Rightarrow$  PRP = COP
    - proiectie paralela  $\Rightarrow$  (PRP,CW) = DOP

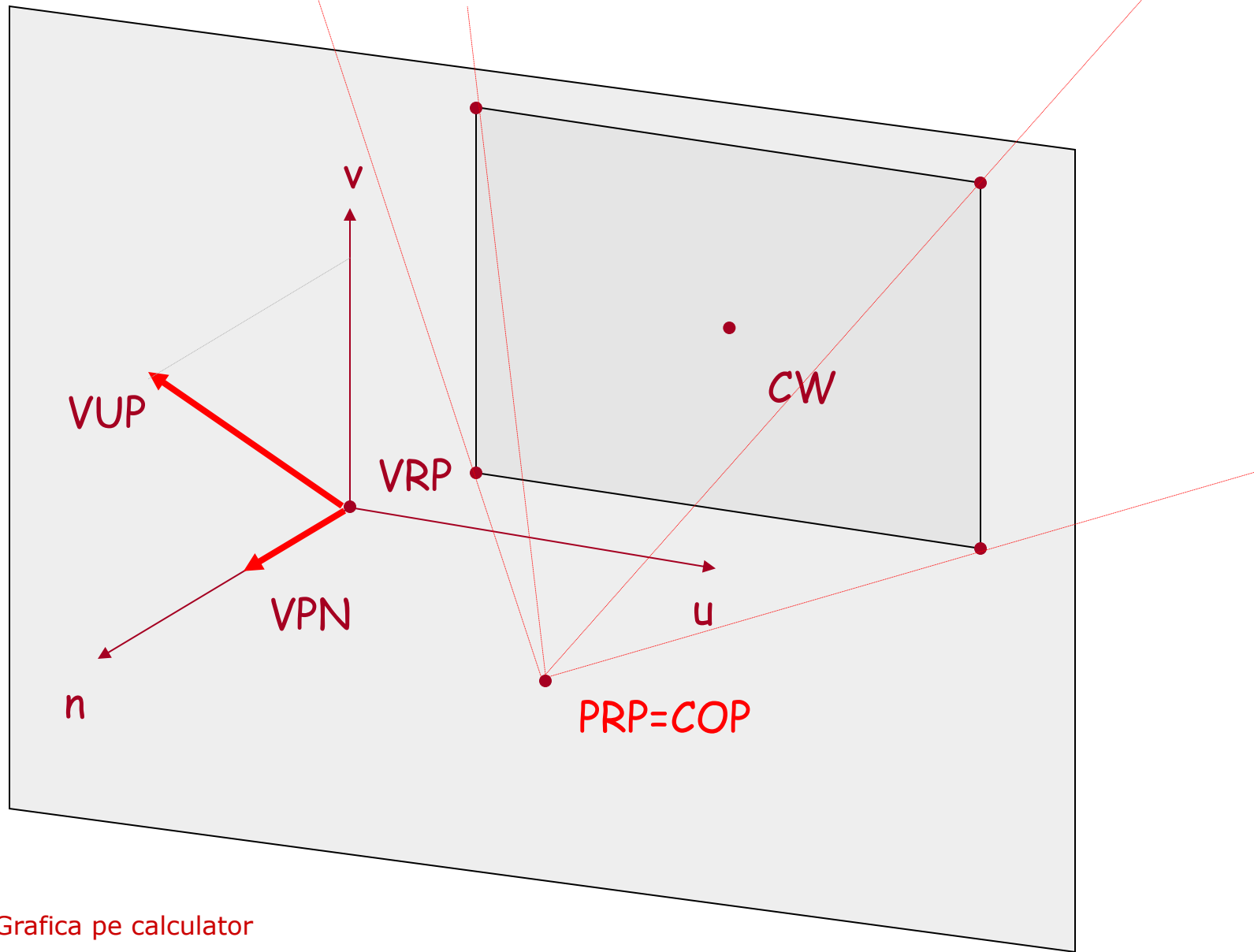
# Specificarea PRP



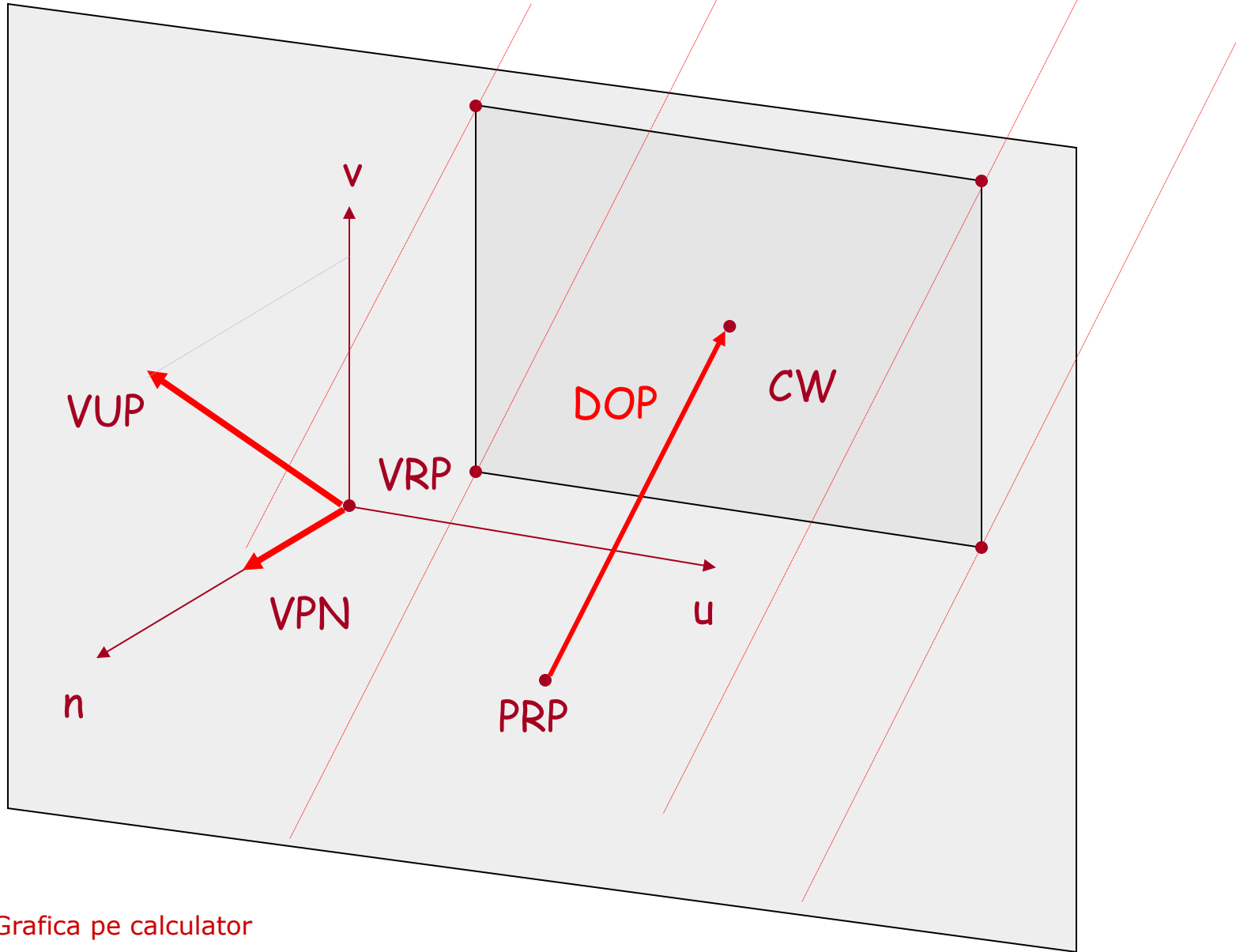
# Proiectii geometrice planare

- Specificarea unei p.g.p. (terminologie PHIGS)
  - volumului de vizualizare
    - doar (parti din) obiectele care (intersecteaza) sunt incluse in volumul de vizualizare vor fi afisate pe ecran
    - restul -> decupate ...

# Specificarea volumului de vizualizare pentru proiectia perspectiva



# Specificarea volumului de vizualizare pentru proiectia paralela

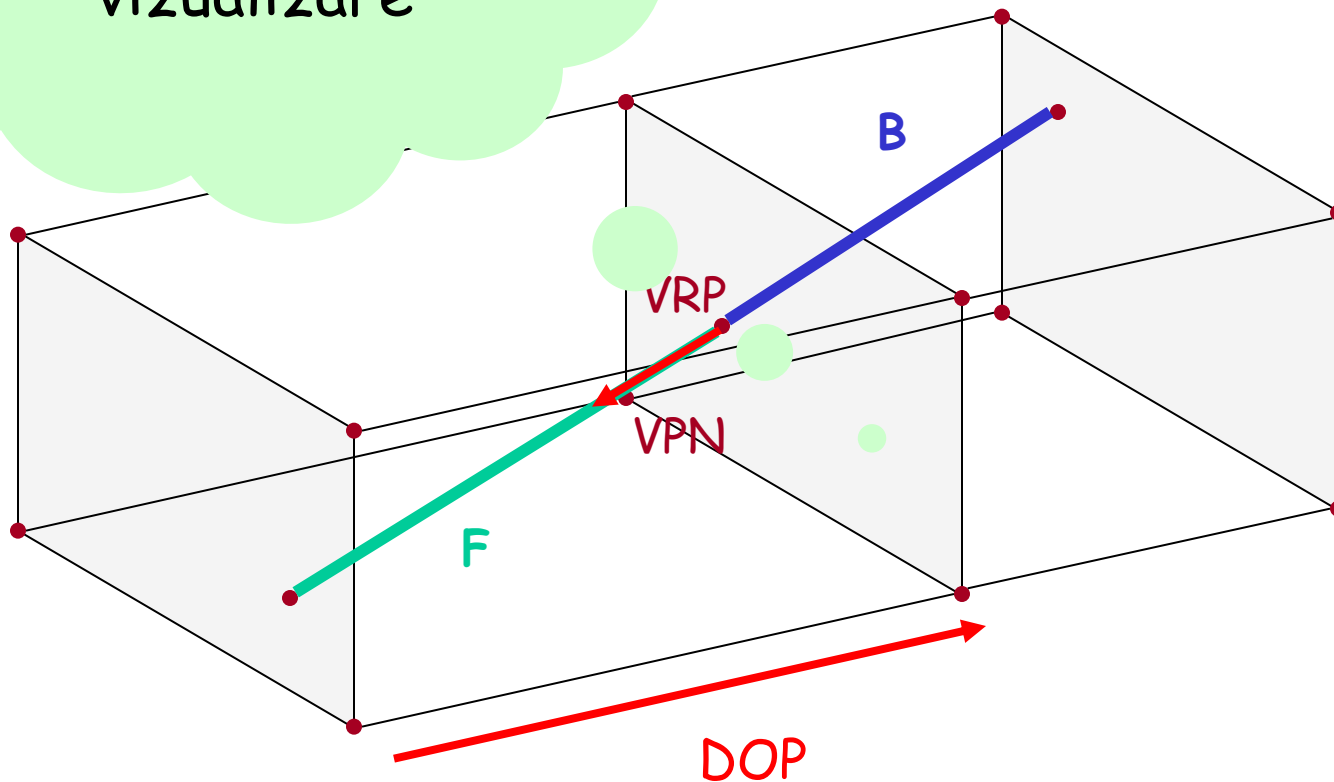


# Proiectii geometrice planare

- Specificarea unei p.g.p.  
(terminologie PHIGS)
  - planelor de decupare
    - anterior si posterior => volum de vizualizare finit

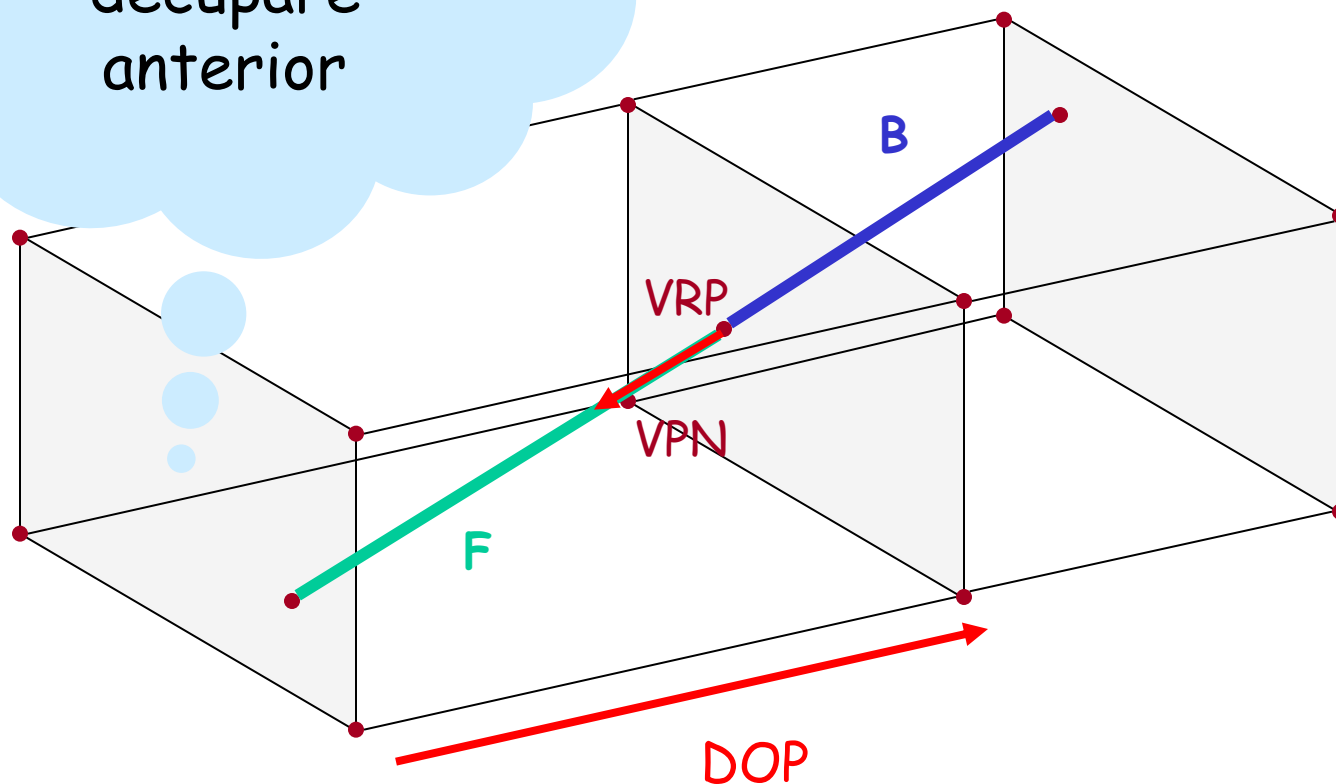
# Specificarea planelor de decupare

Plan de  
vizualizare



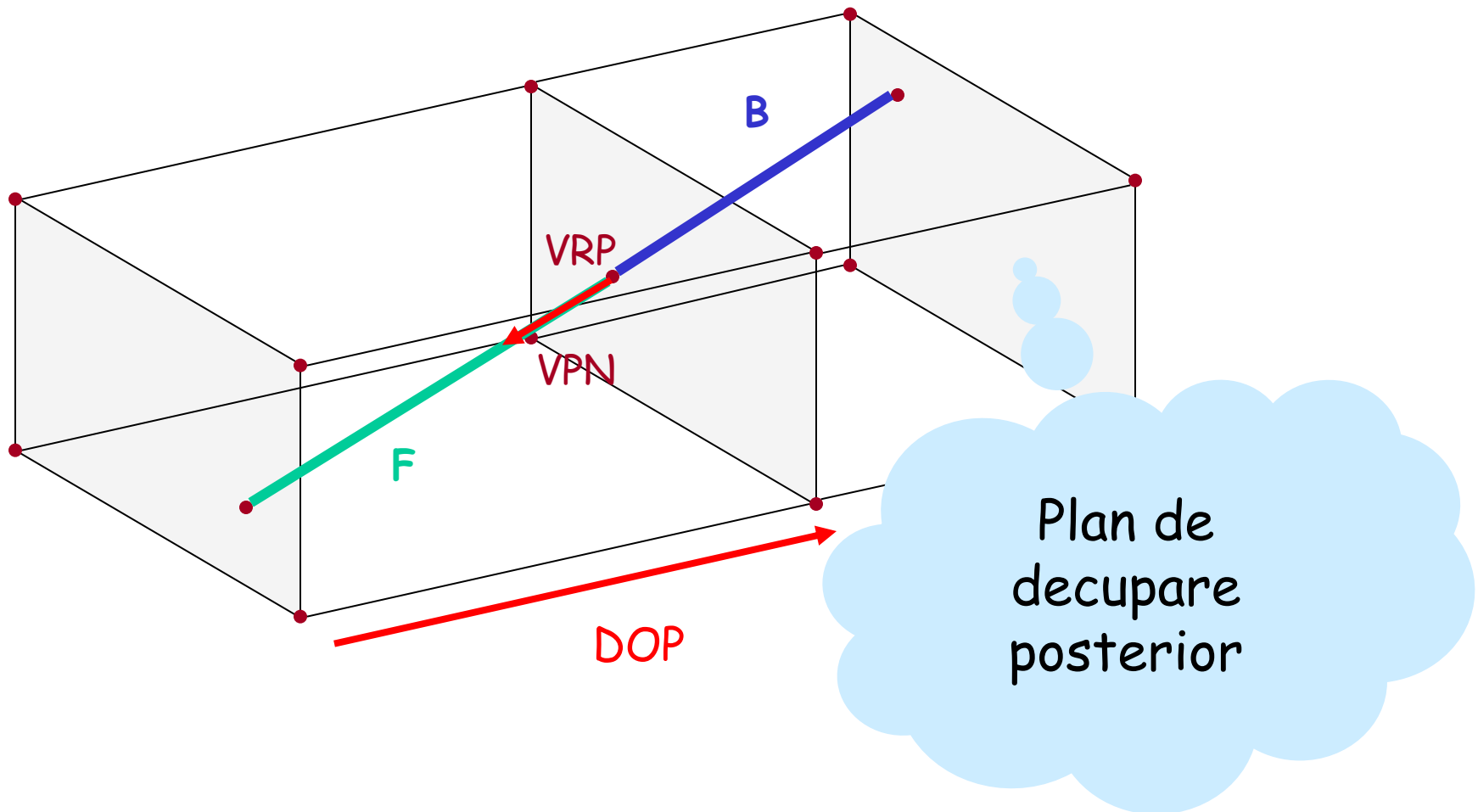
# Specificarea planelor de decupare

Plan de  
decupare  
anterior

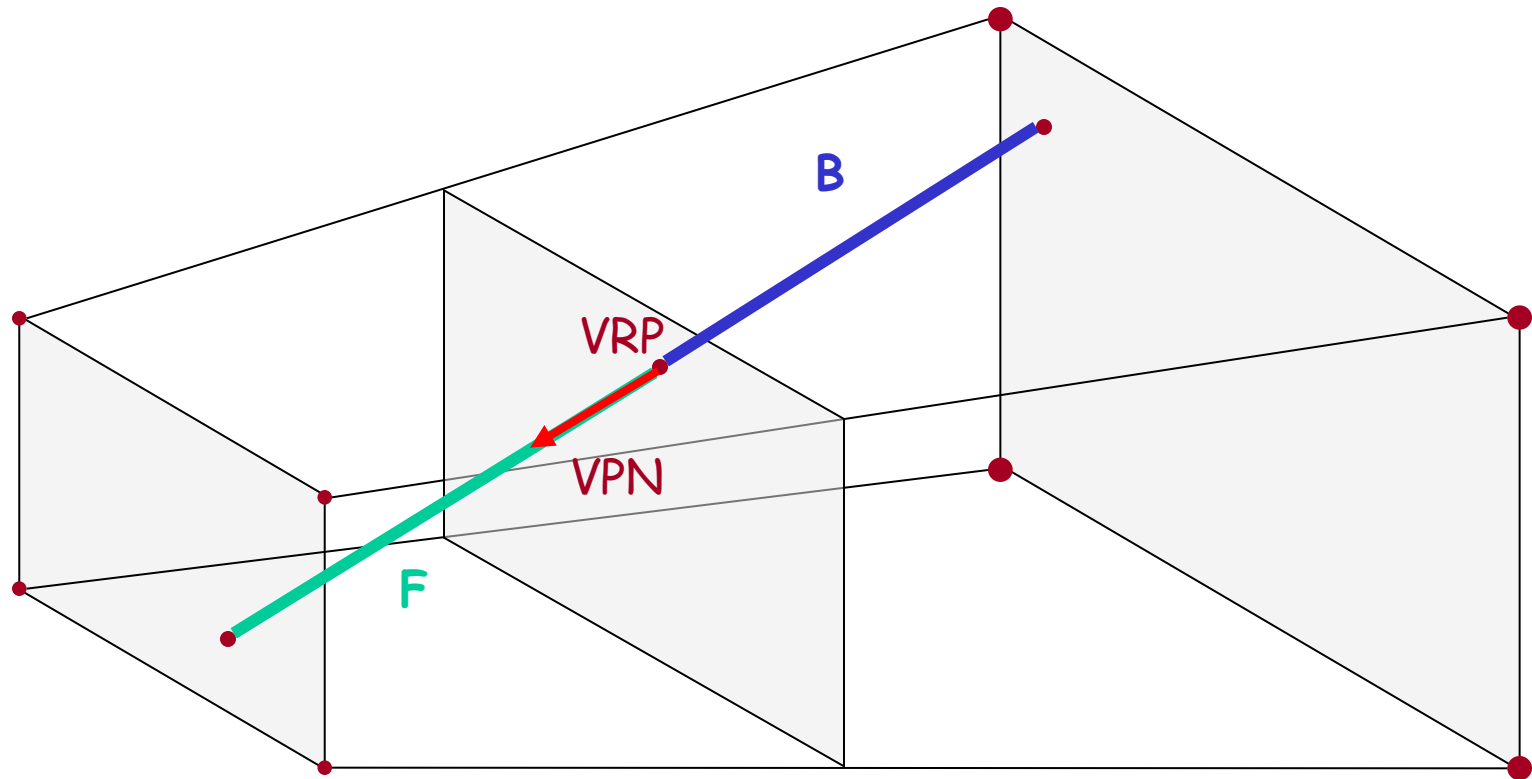




# Specificarea planelor de decupare



# Specificarea planelor de decupare

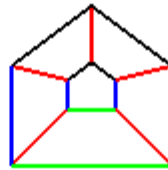


# Proiectii geometrice planare

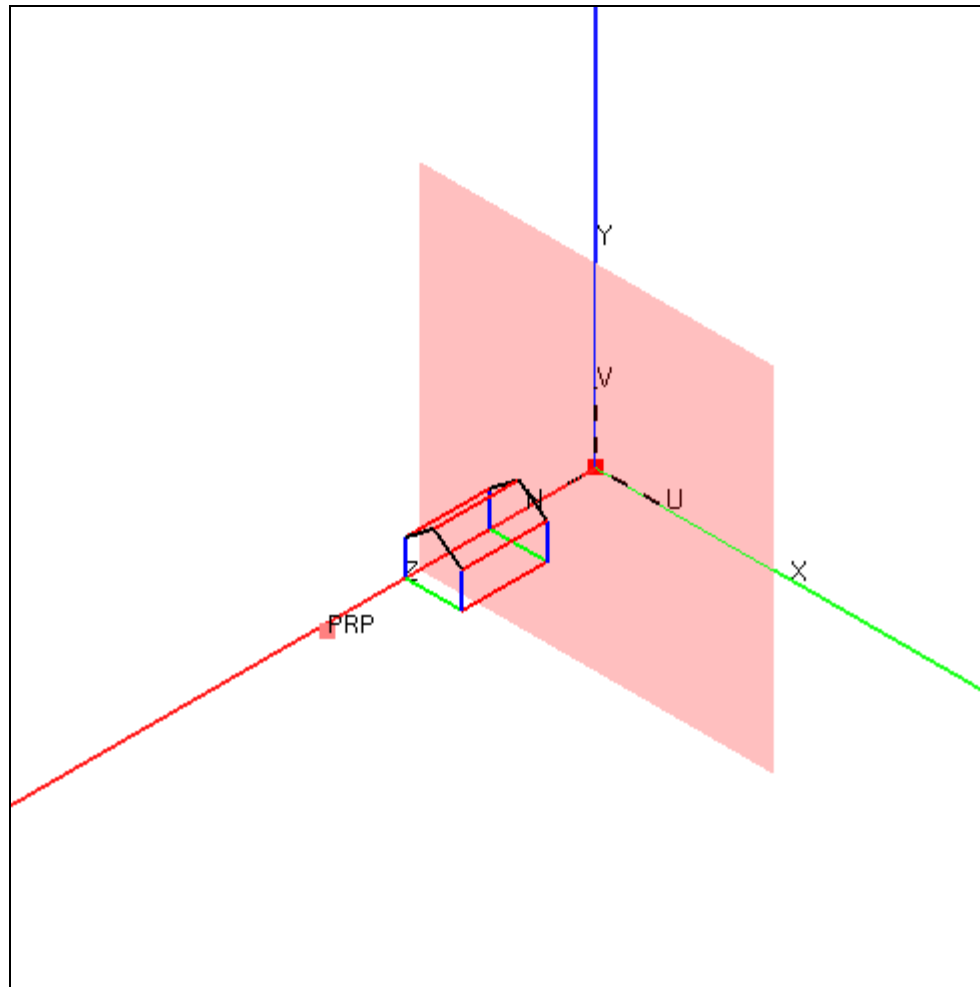
- Exemple PGP1

VRP (WCS)	(0,0,0)
VPN (WCS)	(0,0,1)
VUP (WCS)	(0,1,0)
PRP (VRC)	(8,6,84)
Fereastra (VRC) = (um,uM,vm,vM)	(-50,50,-50,50)
Tip proiectie	perspectiva

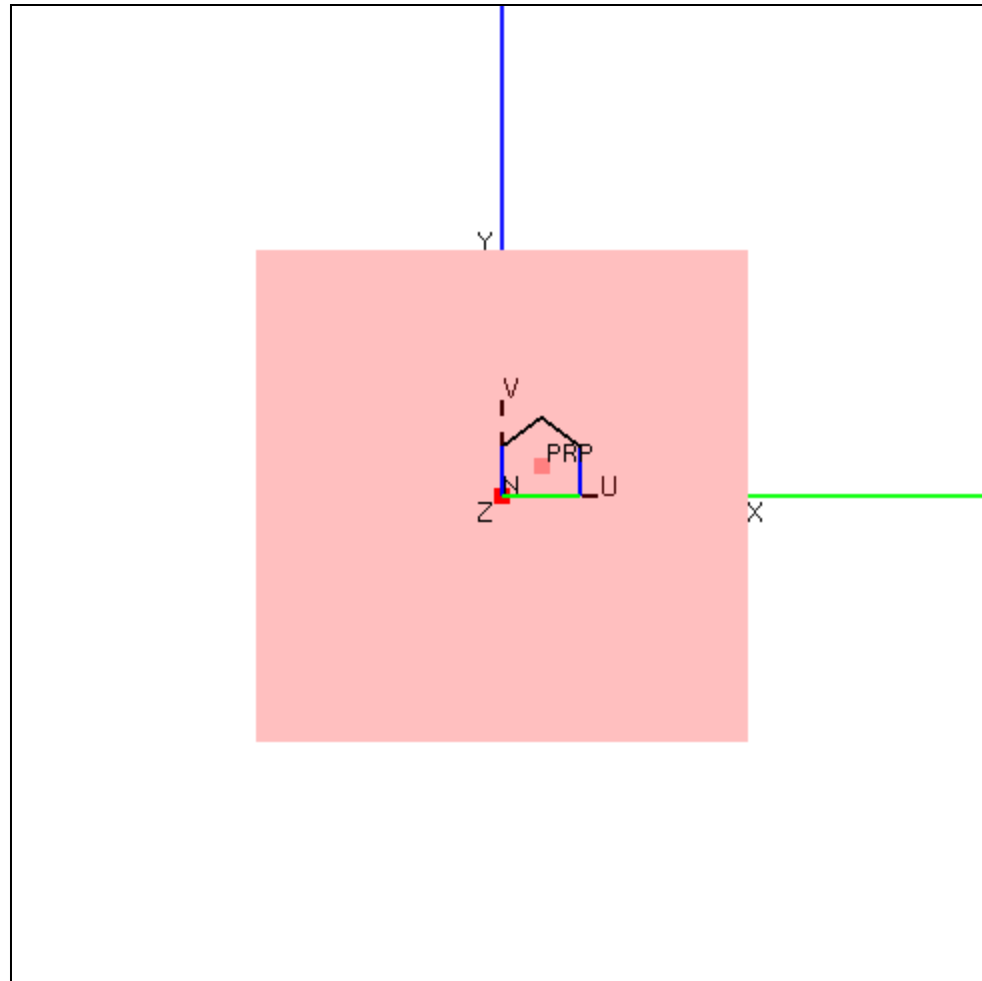
# PGP1



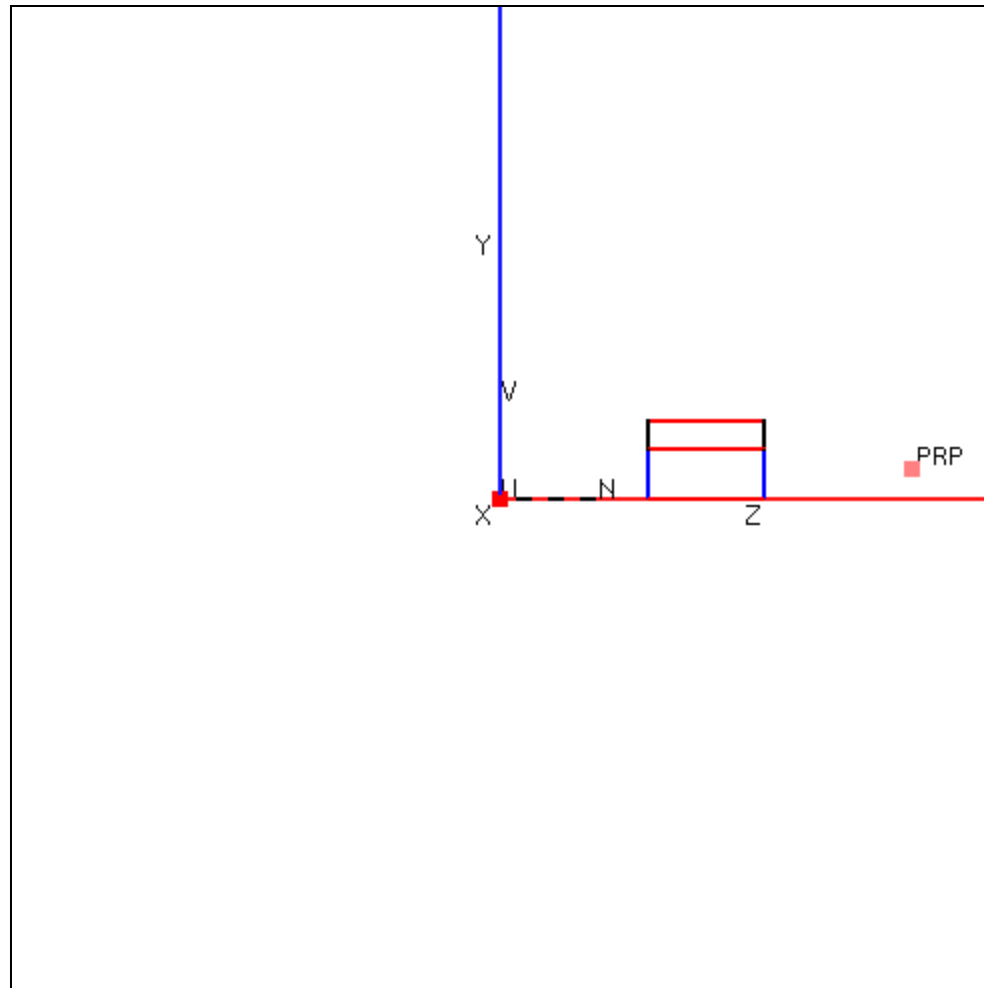
# PGP1



# PGP1



# PGP1



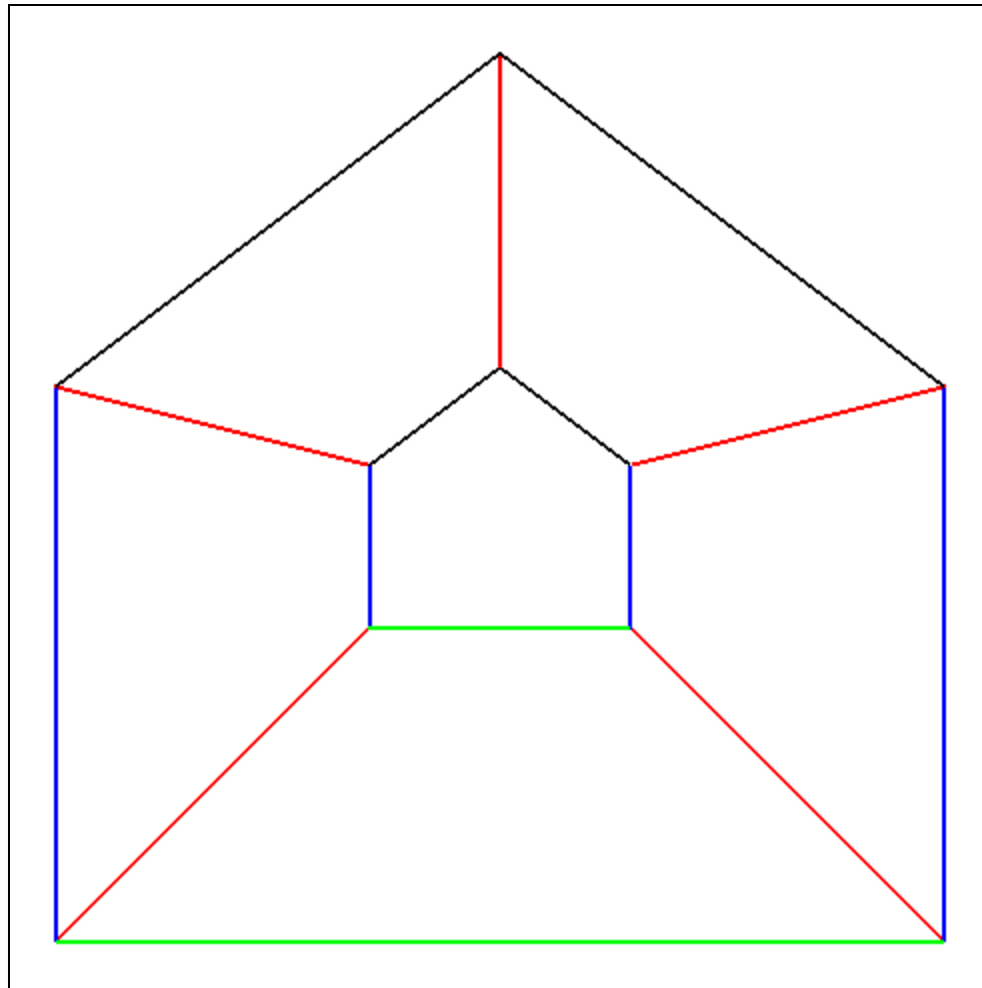
# Proiectii geometrice planare

- Exemple PGP2

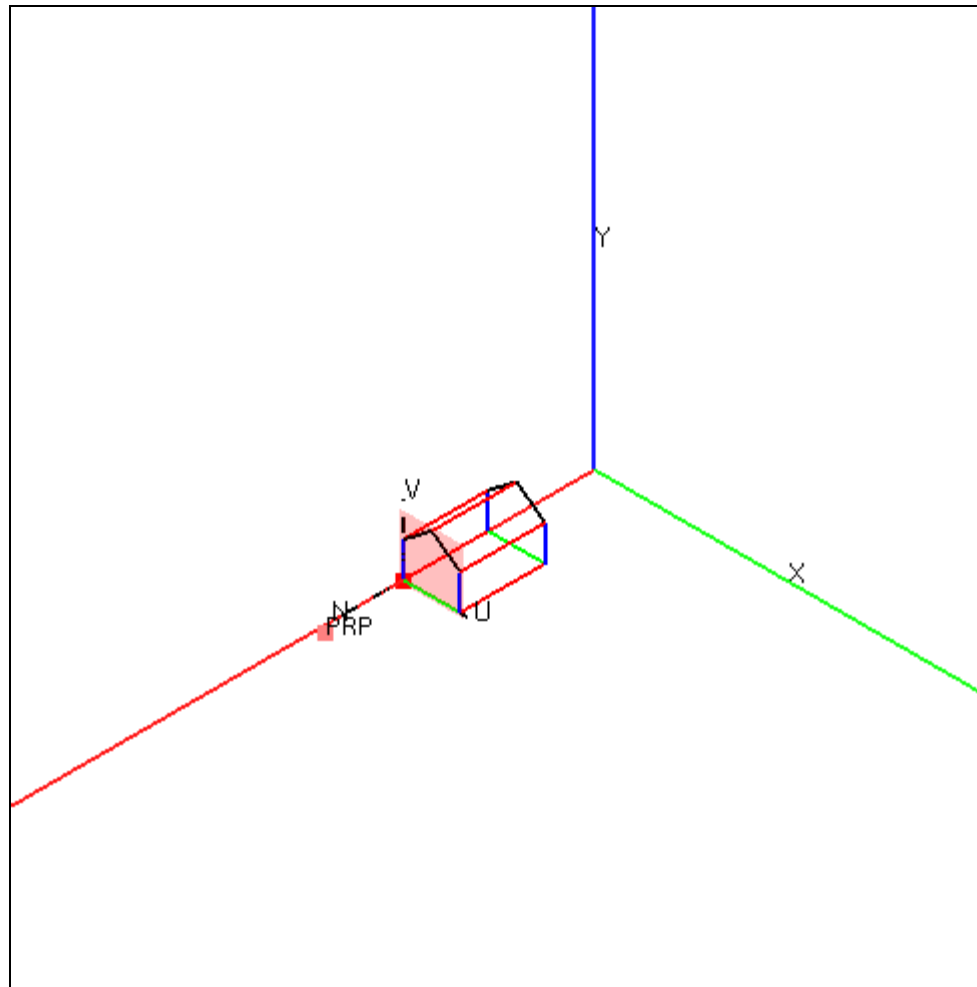
VRP (WCS)	(0,0,54)
VPN (WCS)	(0,0,1)
VUP (WCS)	(0,1,0)
PRP (VRC)	(8,6,30)
Fereastra (VRC) = (um,uM,vm,vM)	(-1,17,-1,17)
Tip proiectie	perspectiva



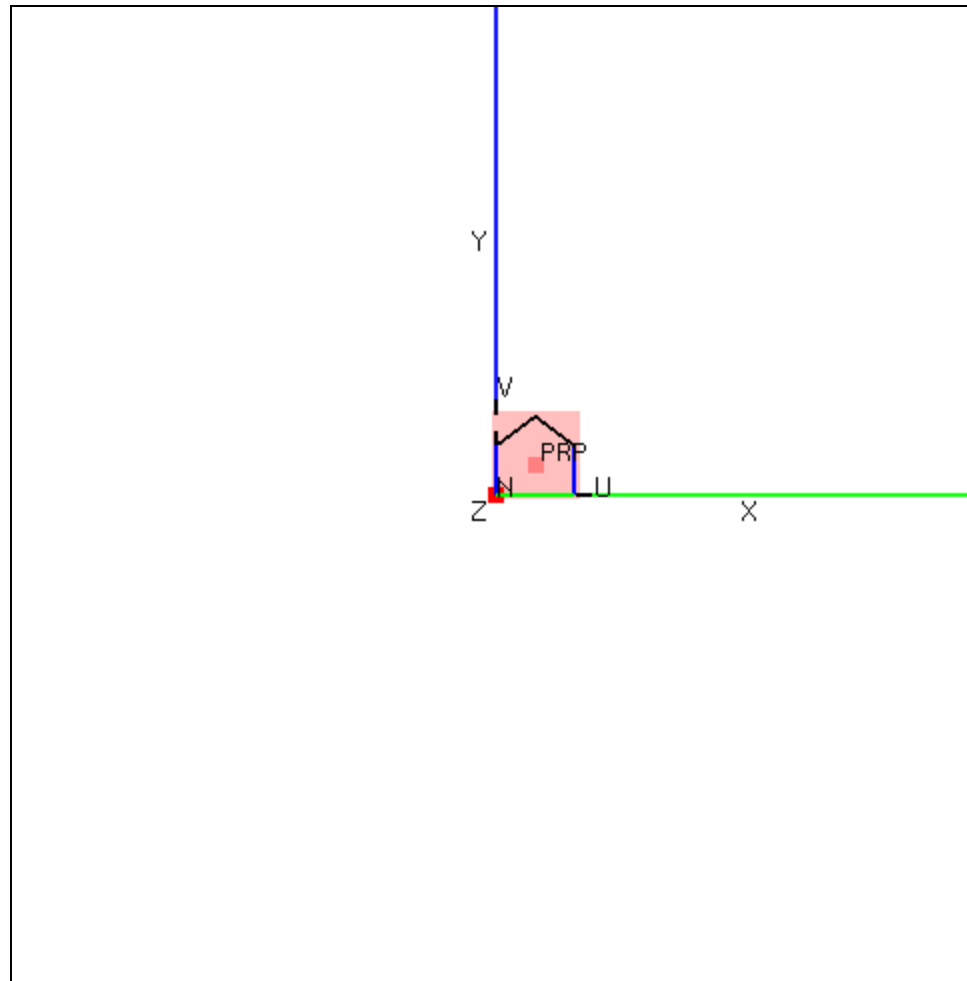
# PGP2



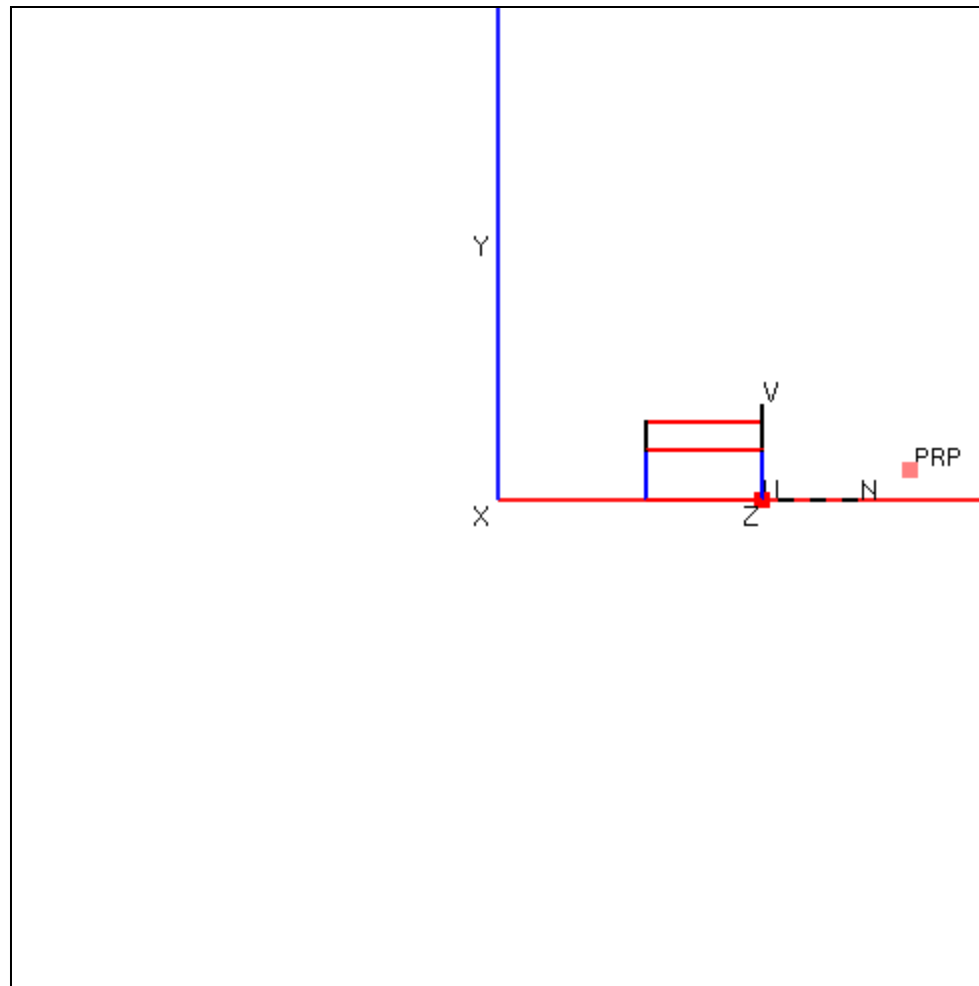
# PGP2



# PGP2



# PGP2

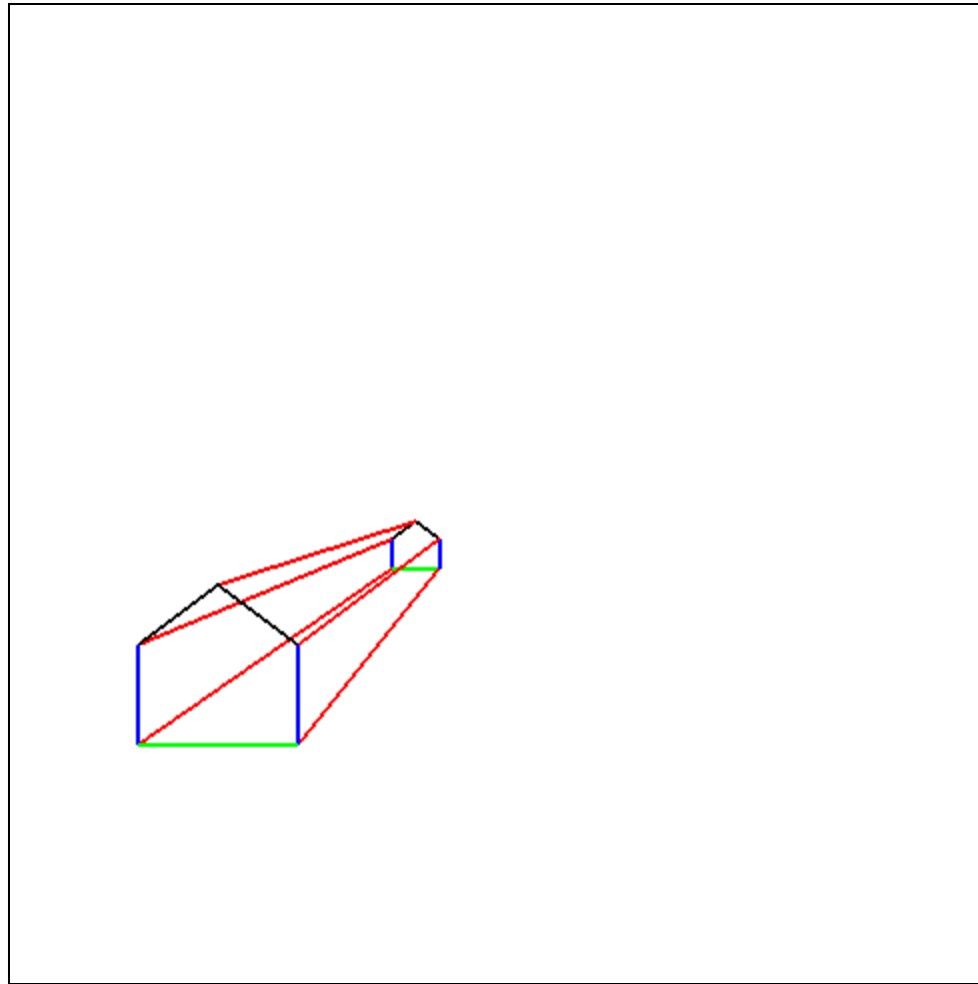


# Proiectii geometrice planare

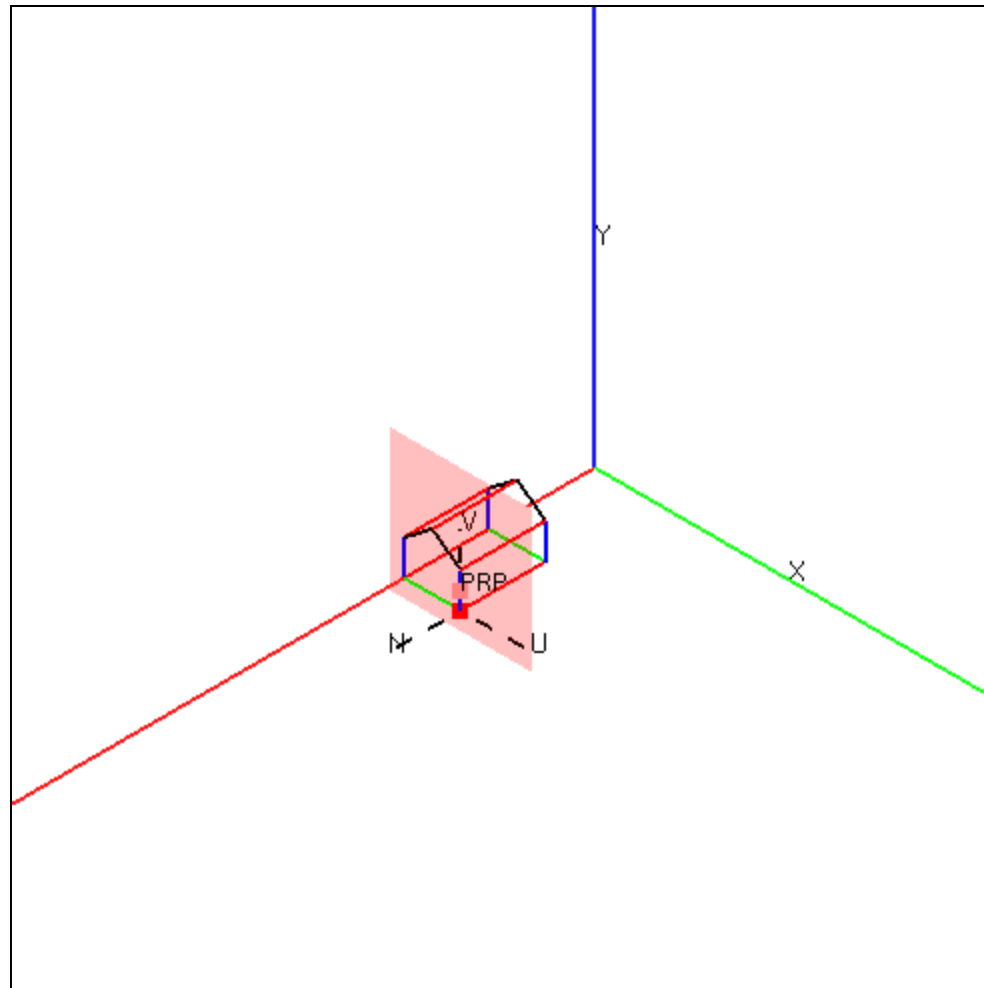
- Exemple PGP3

VRP (WCS)	(16,0,54)
VPN (WCS)	(0,0,1)
VUP (WCS)	(0,1,0)
PRP (VRC)	(20,25,20)
Fereastra (VRC) = (um,uM,vm,vM)	(-20,20,-5,35)
Tip proiectie	perspectiva

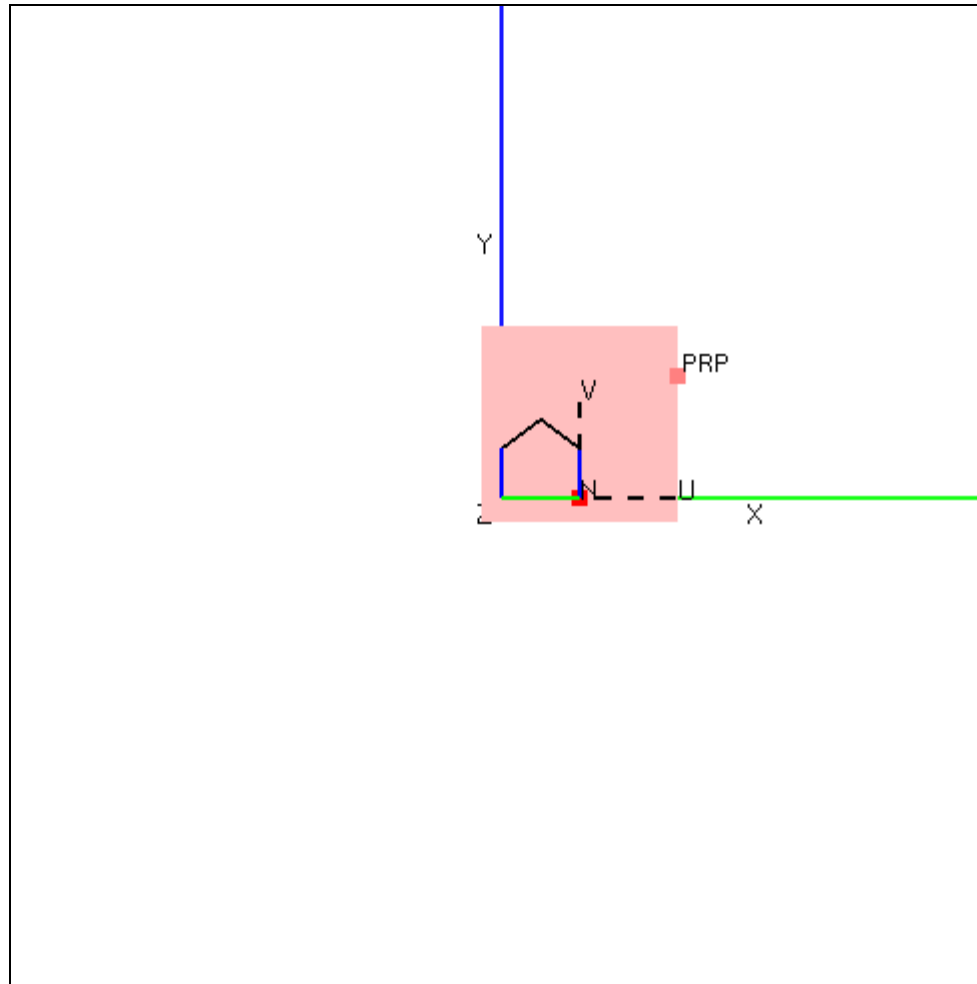
# PGP3



# PGP3

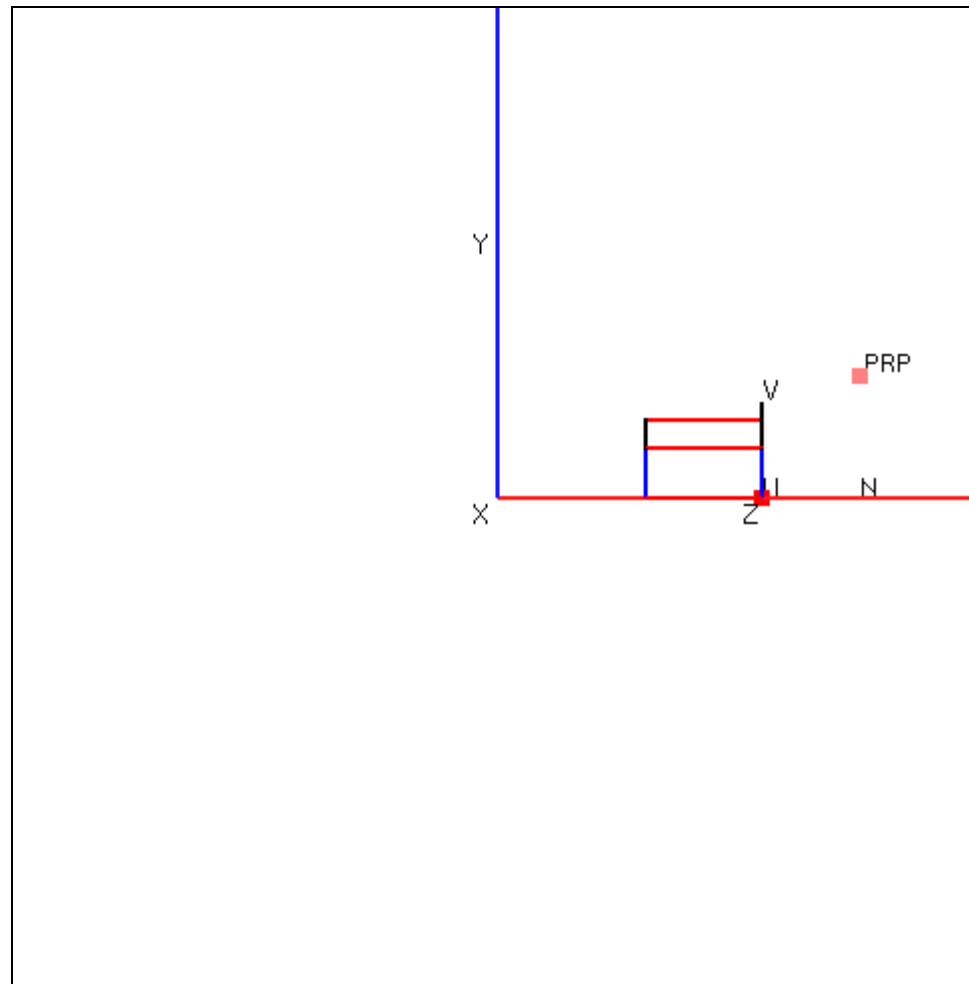


# PGP3





# PGP3

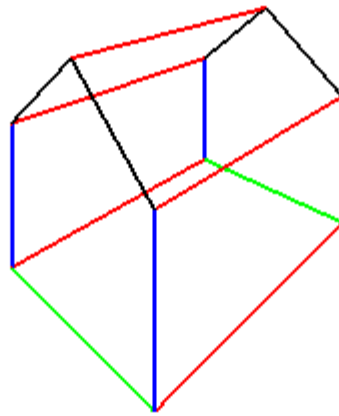


# Proiectii geometrice planare

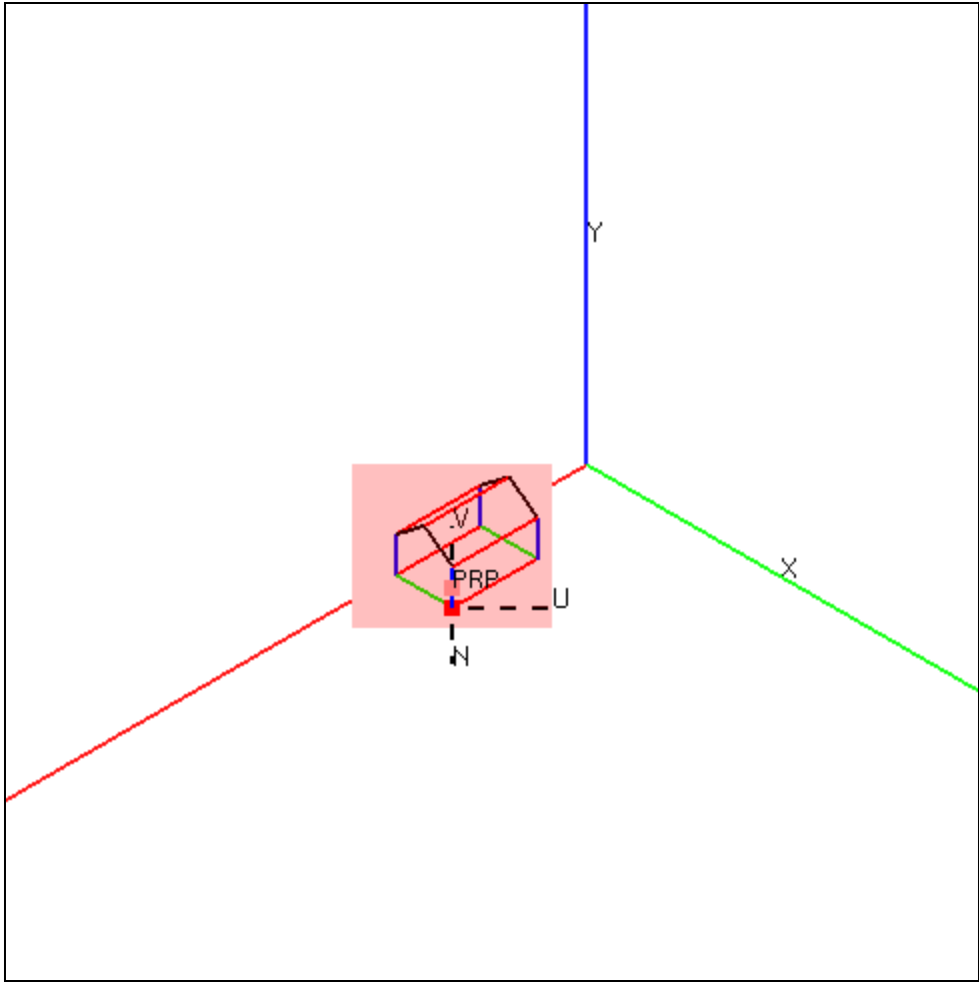
- Exemple PGP4

VRP (WCS)	(16,0,54)
VPN (WCS)	(1,0,1)
VUP (WCS)	(0,1,0)
PRP (VRC)	$(0,25,20\sqrt{2})$
Fereastra (VRC) = (um,uM,vm,vM)	(-20,20,-5,35)
Tip proiectie	perspectiva

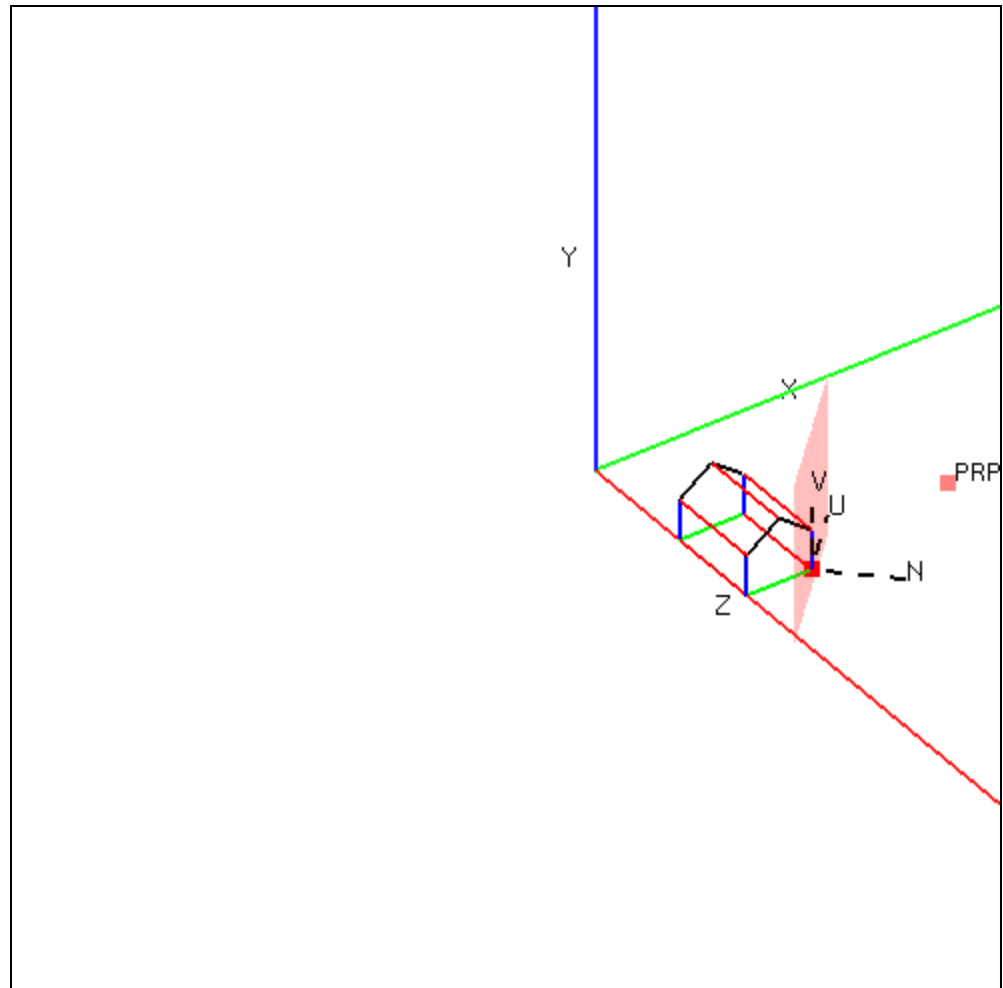
# PGP4



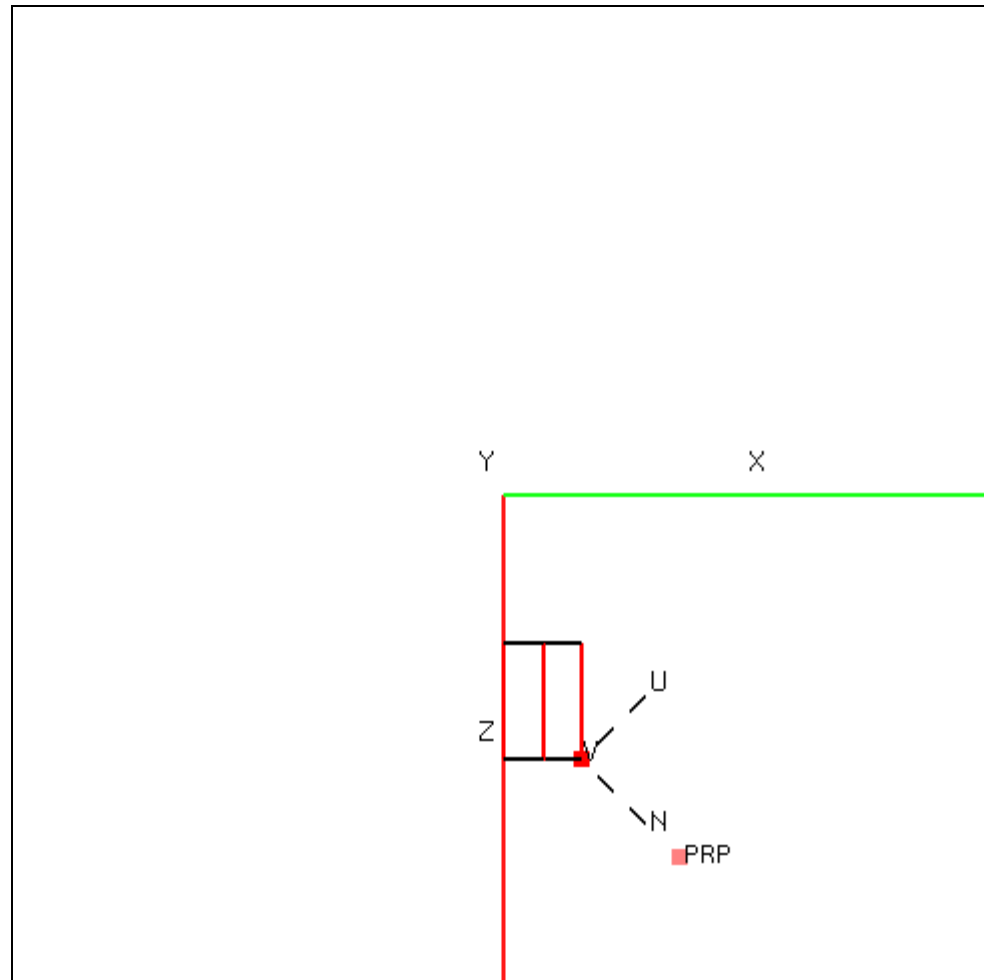
# PGP4



# PGP4



# PGP4

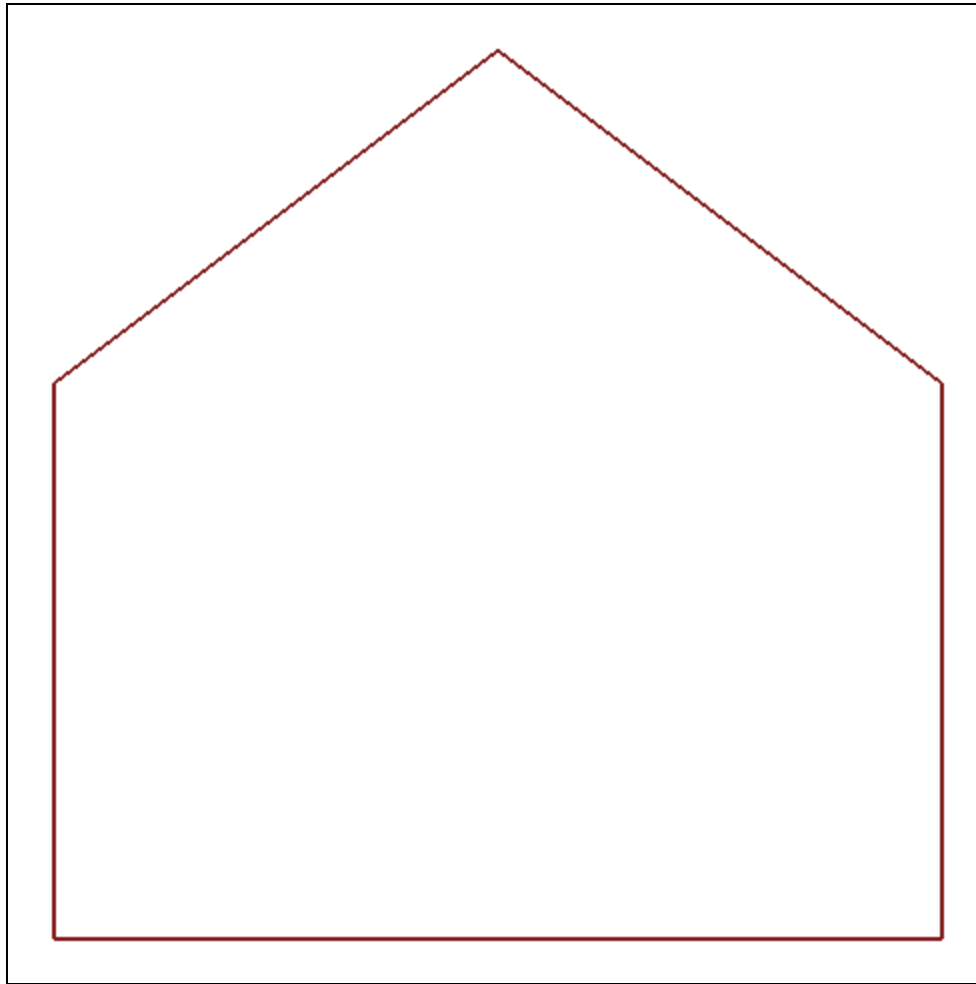


# Proiectii geometrice planare

- Exemple PGP5

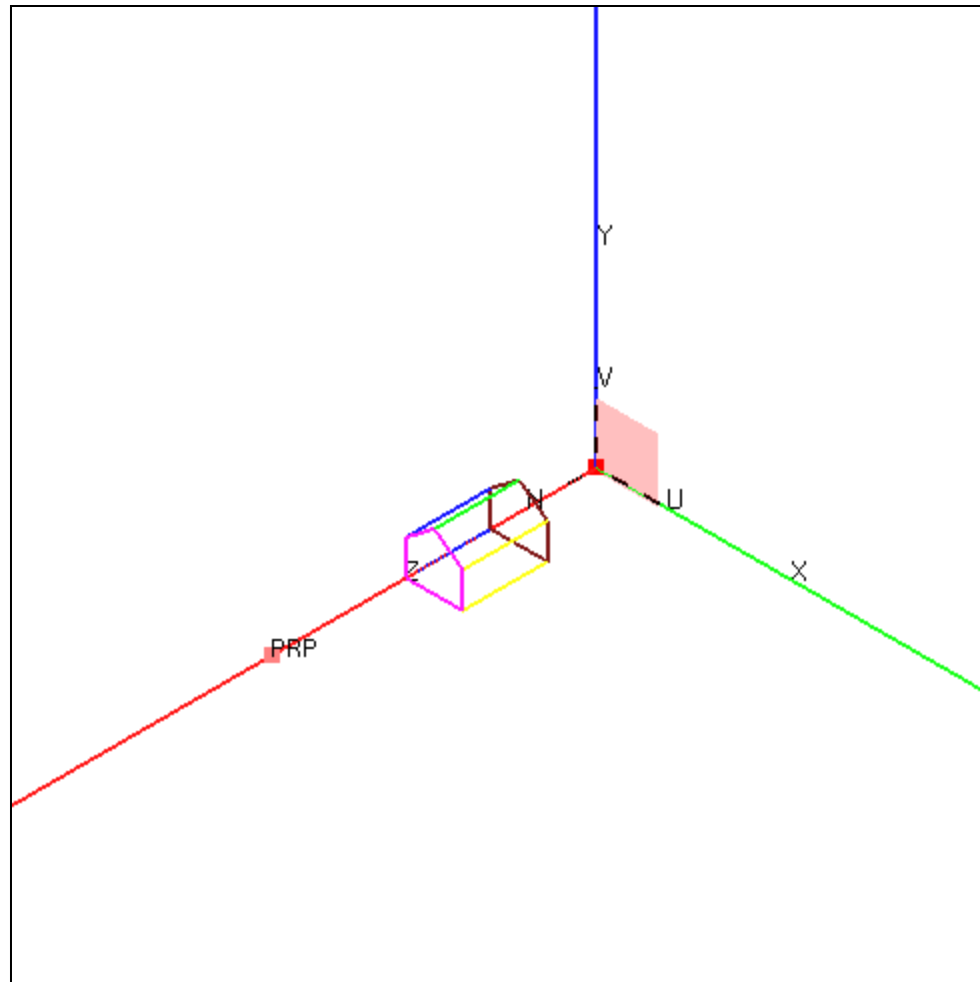
VRP (WCS)	(0,0,0)
VPN (WCS)	(0,0,1)
VUP (WCS)	(0,1,0)
PRP (VRC)	(8,8,100)
Fereastra (VRC) = (um,uM,vm,vM)	(-1,17,-1,17)
Tip proiectie	paralela

# PGP5

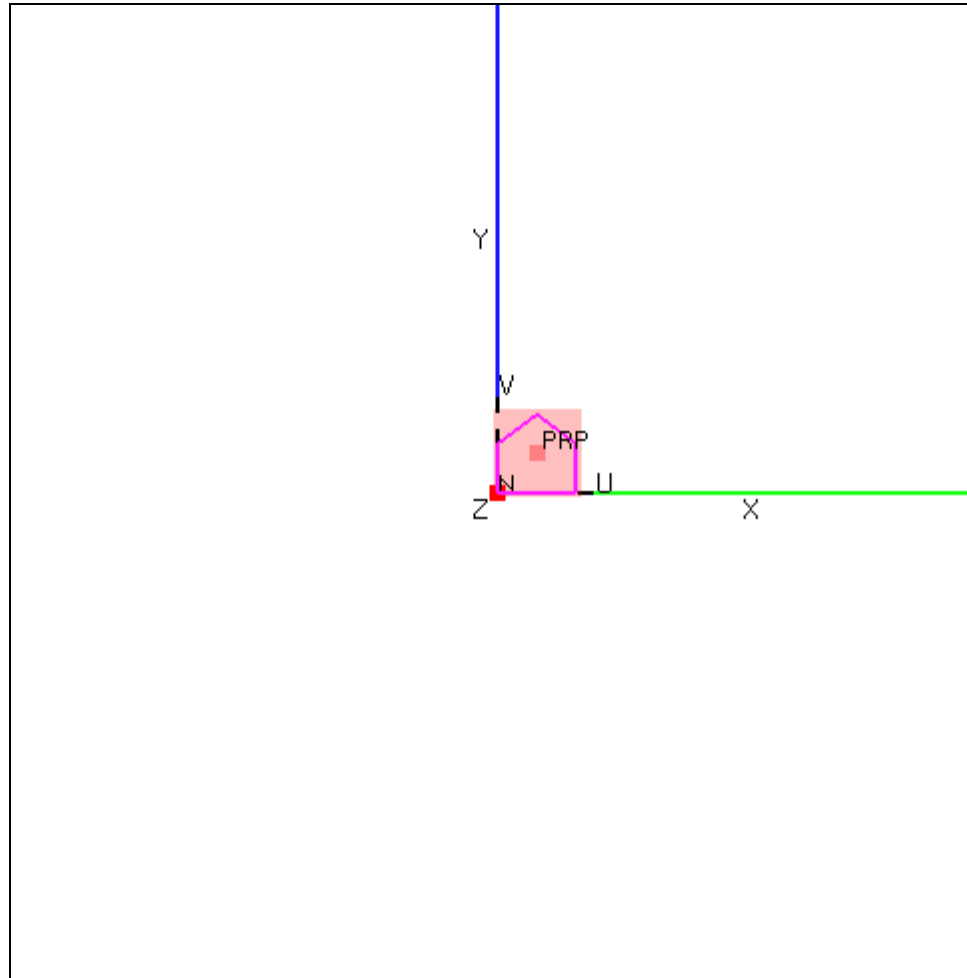




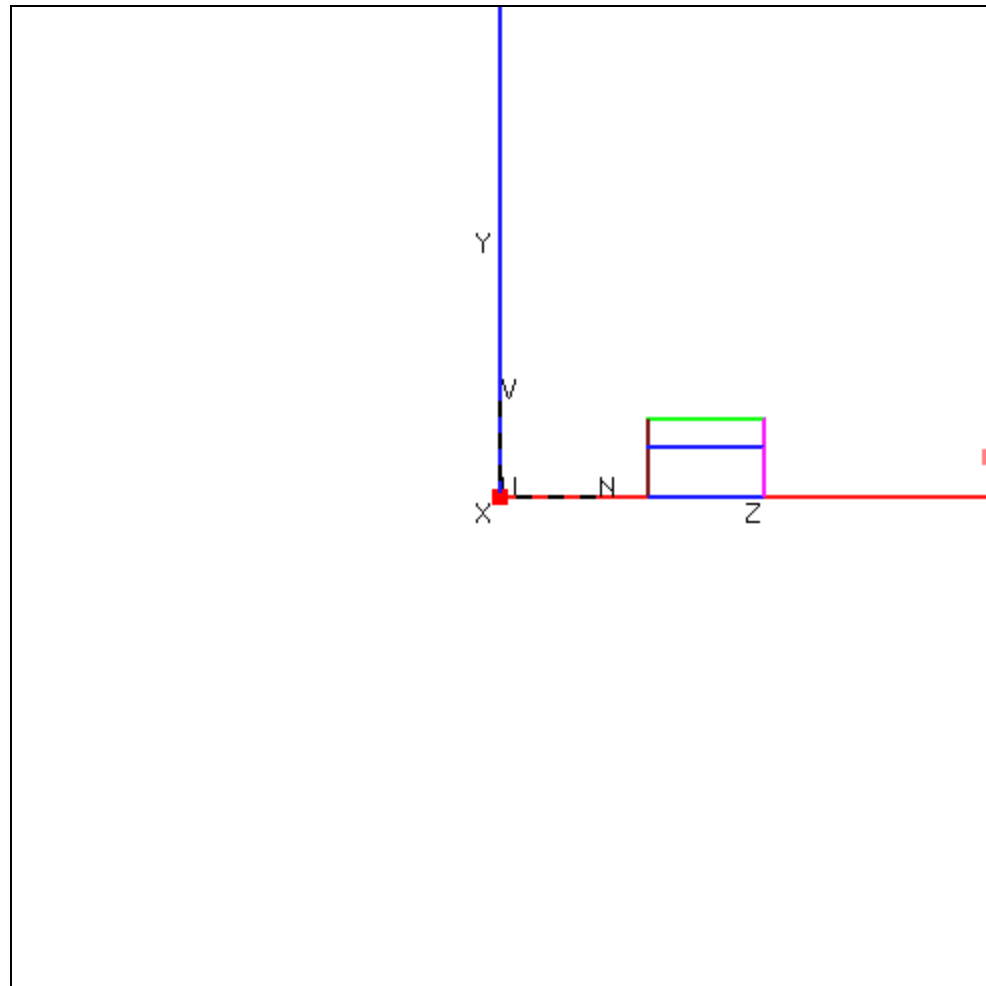
# PGP5



# PGP5



# PGP5

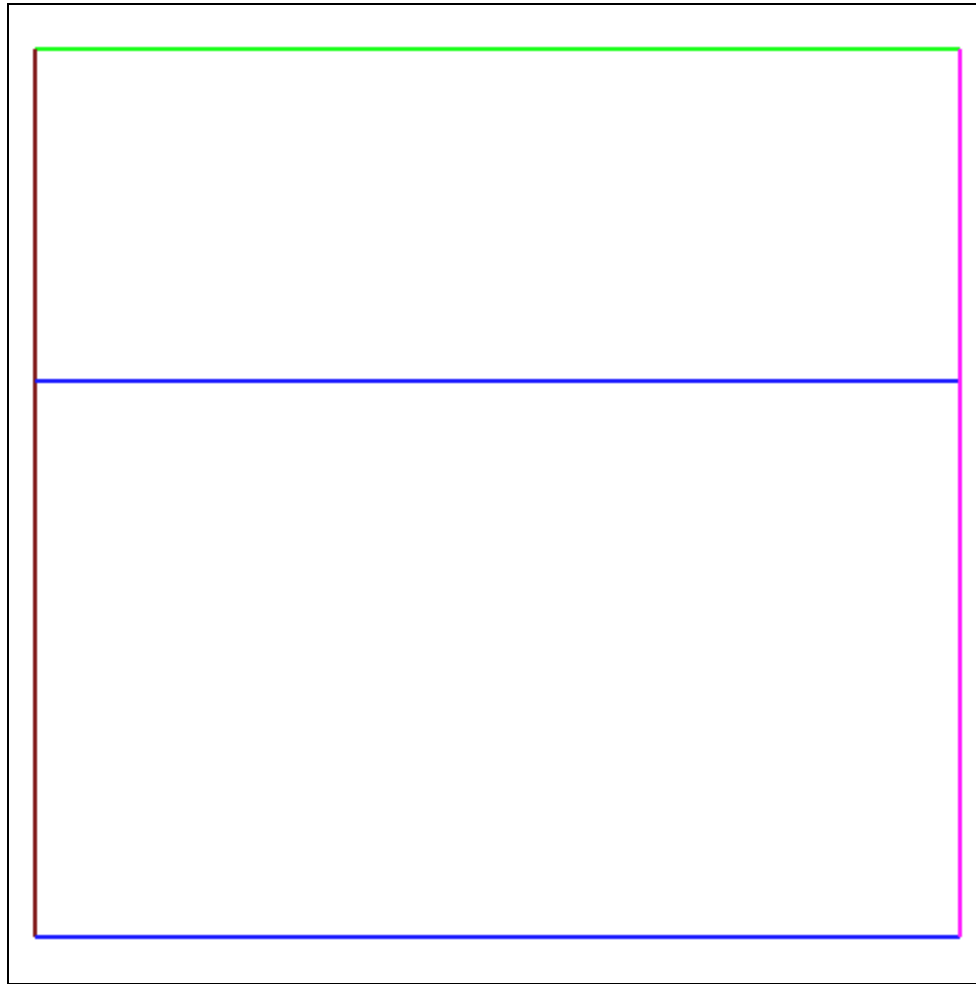


# Proiectii geometrice planare

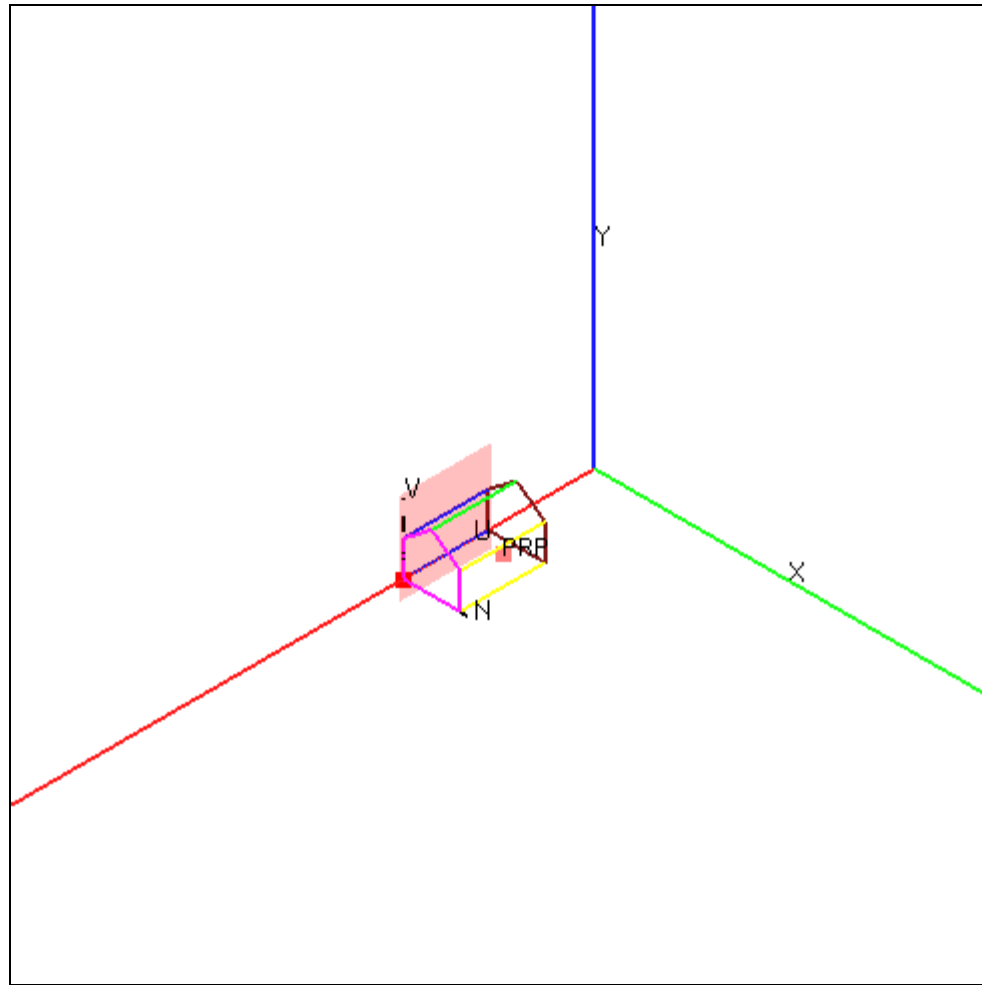
- Exemple PGP6

VRP (WCS)	(0,0,54)
VPN (WCS)	(1,0,0)
VUP (WCS)	(0,1,0)
PRP (VRC)	(12,8,16)
Fereastra (VRC) = (um,uM,vm,vM)	(-1,25,-5,21)
Tip proiectie	paralela

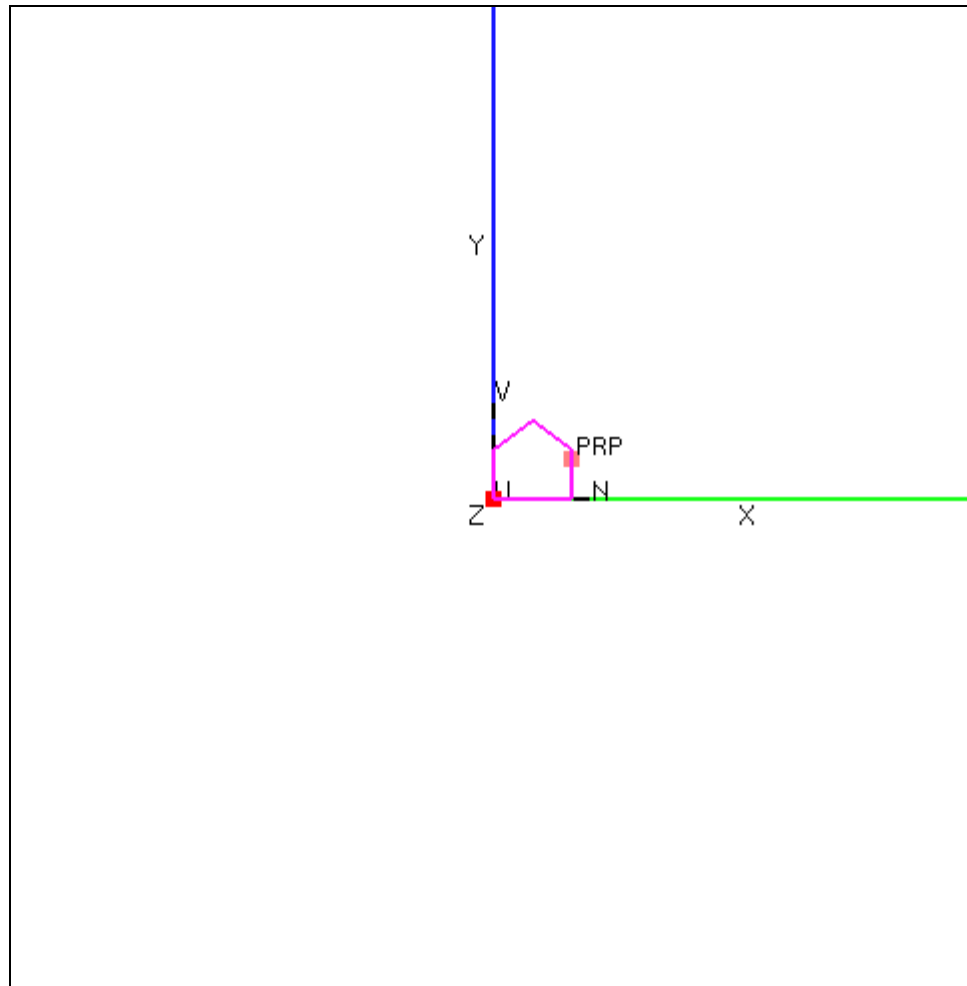
# PGP6



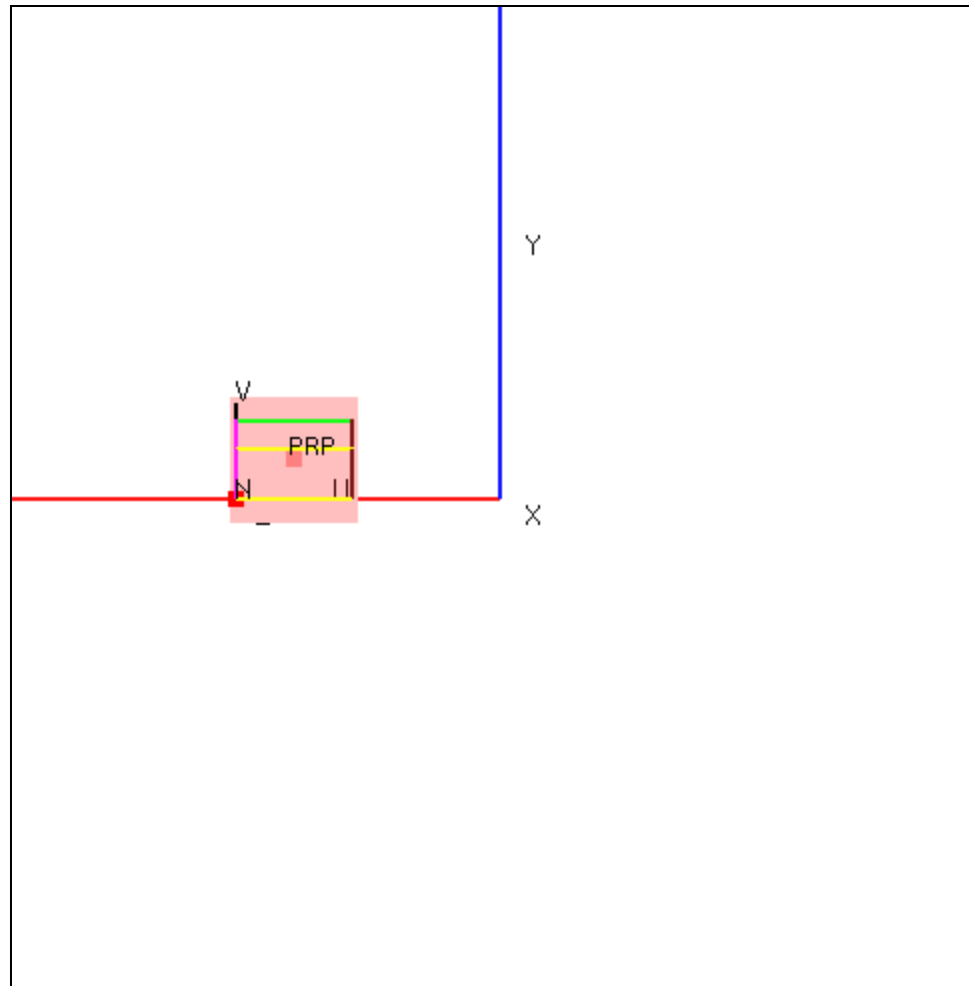
# PGP6



# PGP6



# PGP6



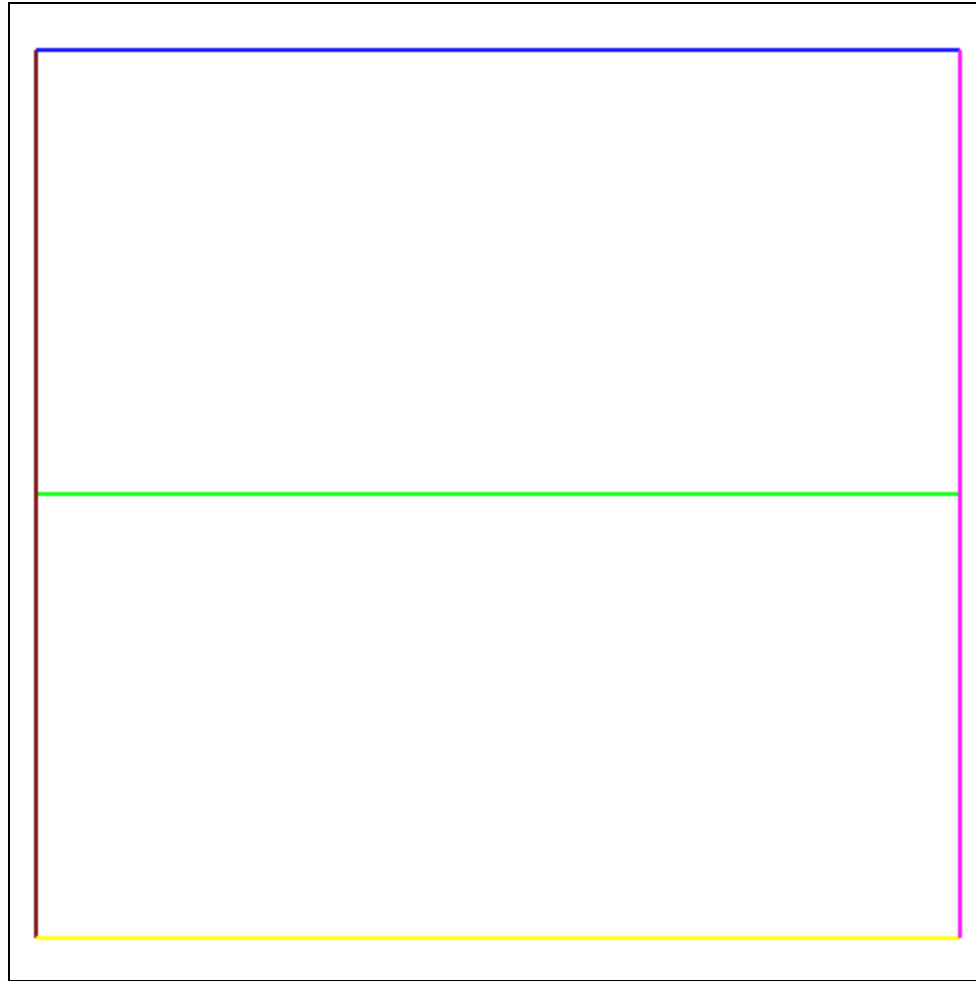


# Proiectii geometrice planare

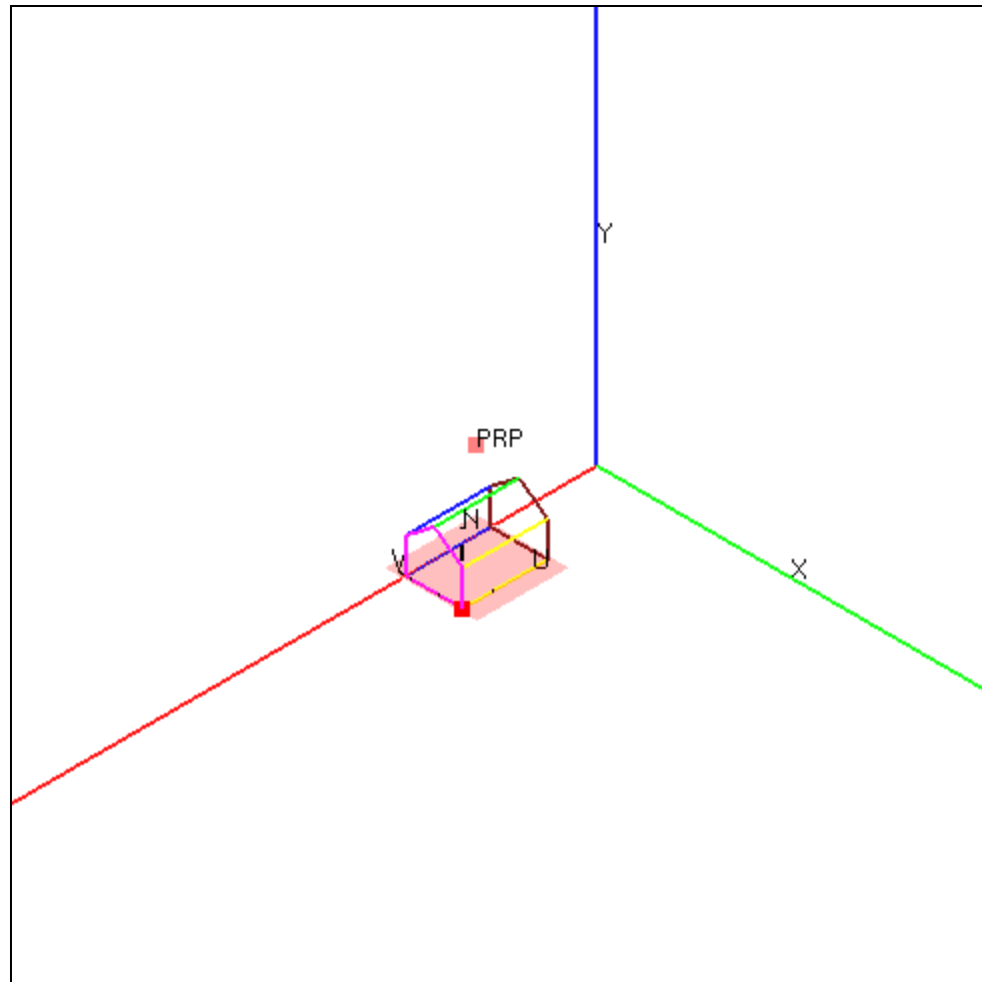
- Exemple PGP7

VRP (WCS)	(16,0,54)
VPN (WCS)	(0,1,0)
VUP (WCS)	(-1,0,0)
PRP (VRC)	(12,8,30)
Fereastra (VRC) = (um,uM,vm,vM)	(-1,25,-5,21)
Tip proiectie	paralela

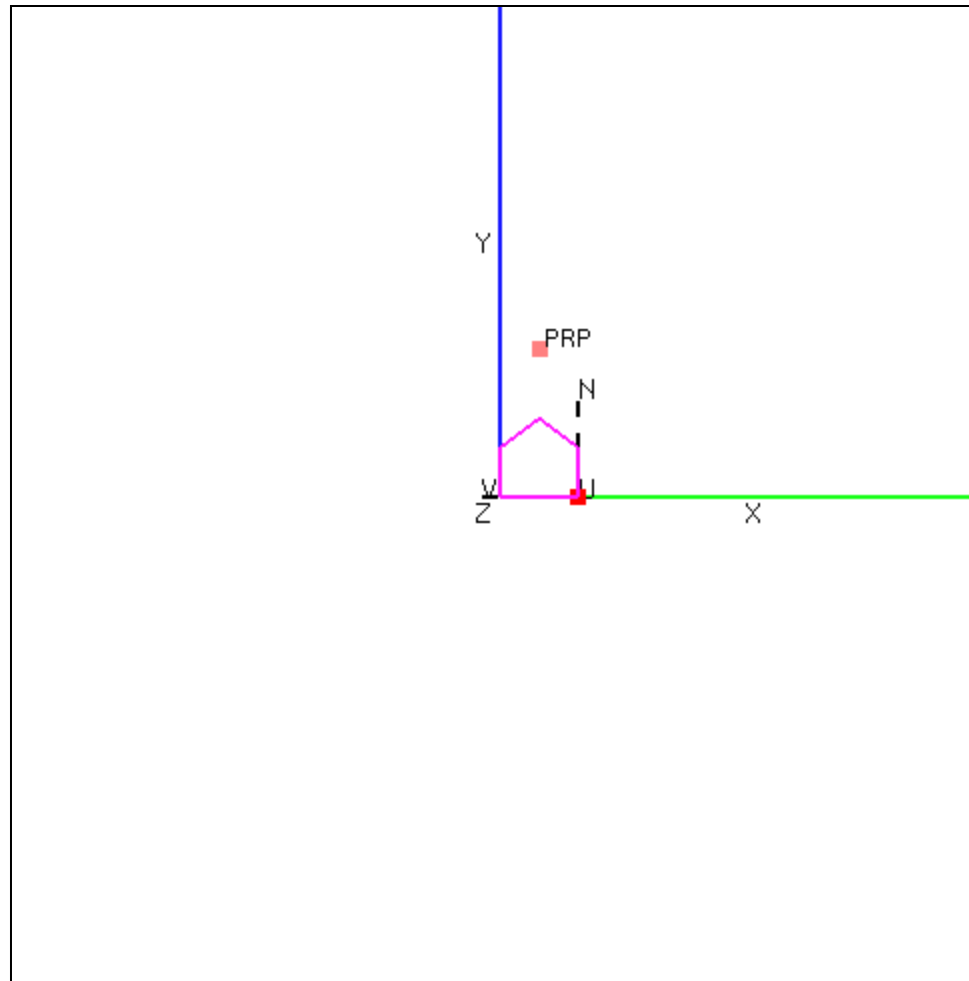
# PGP7



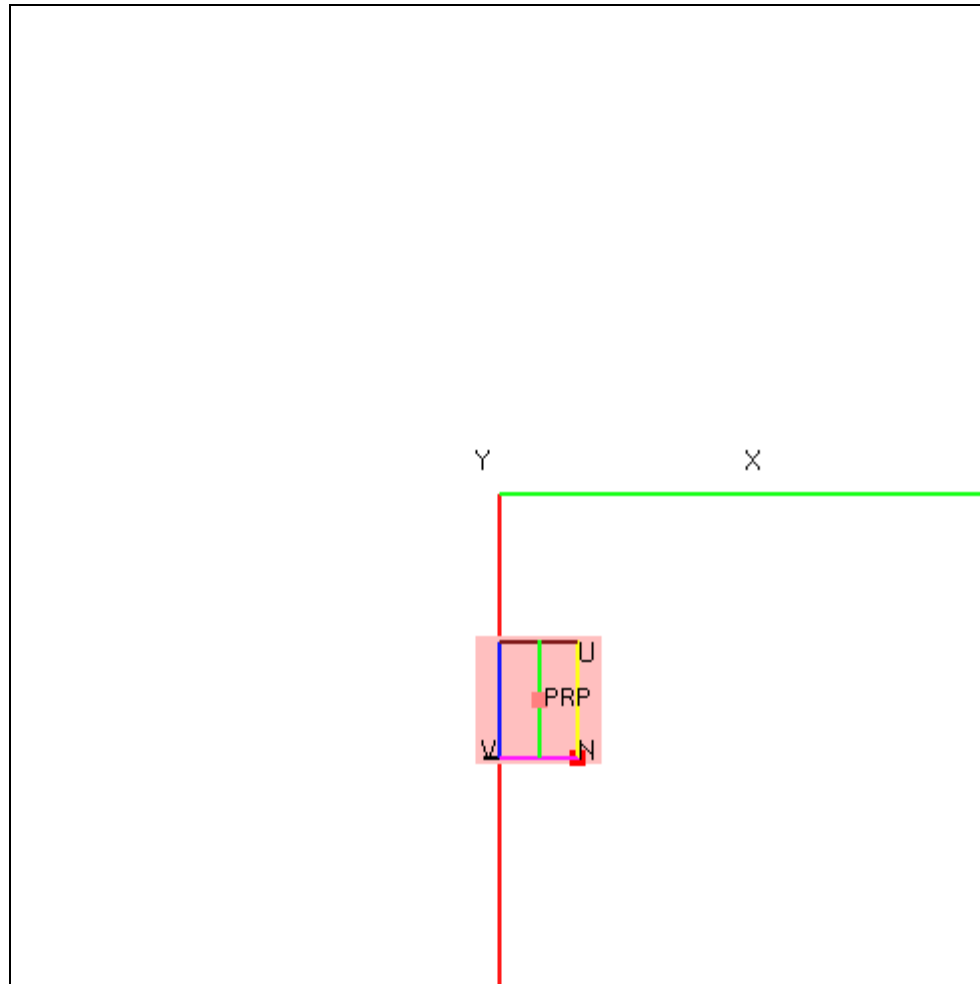
# PGP7



# PGP7



# PGP7

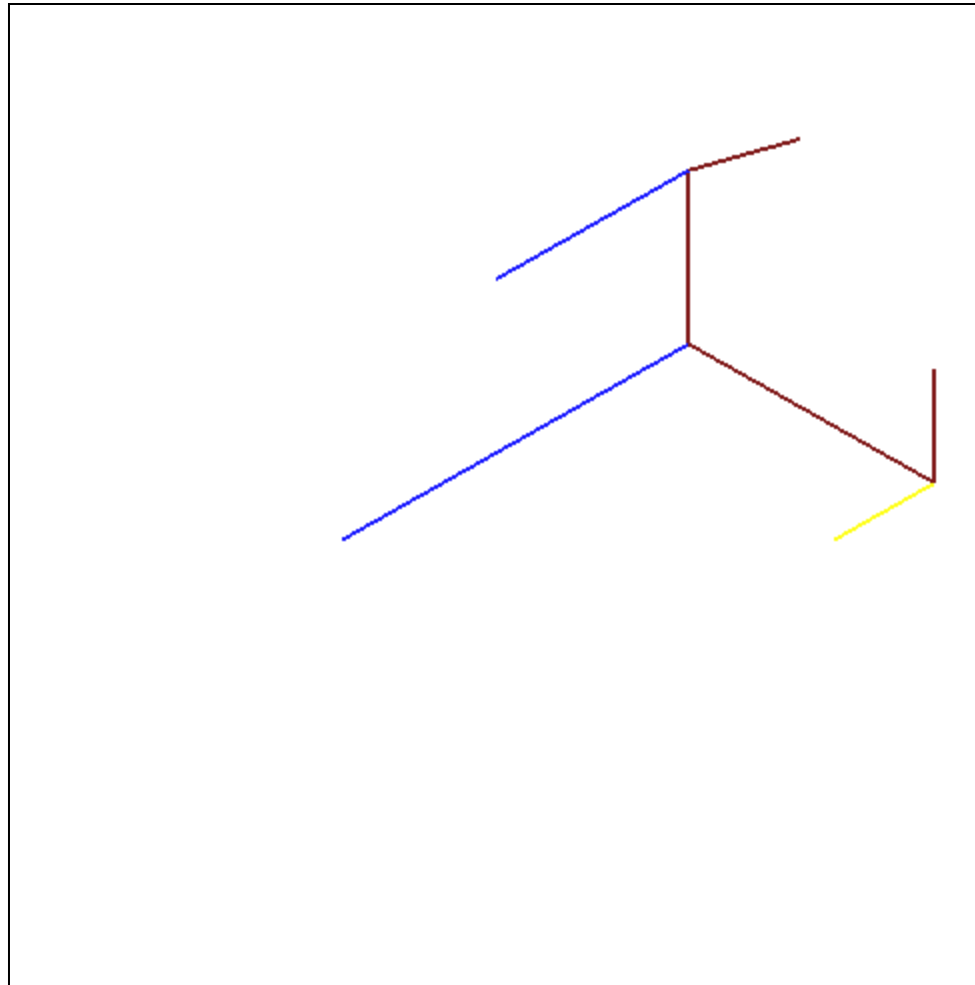


# Proiectii geometrice planare

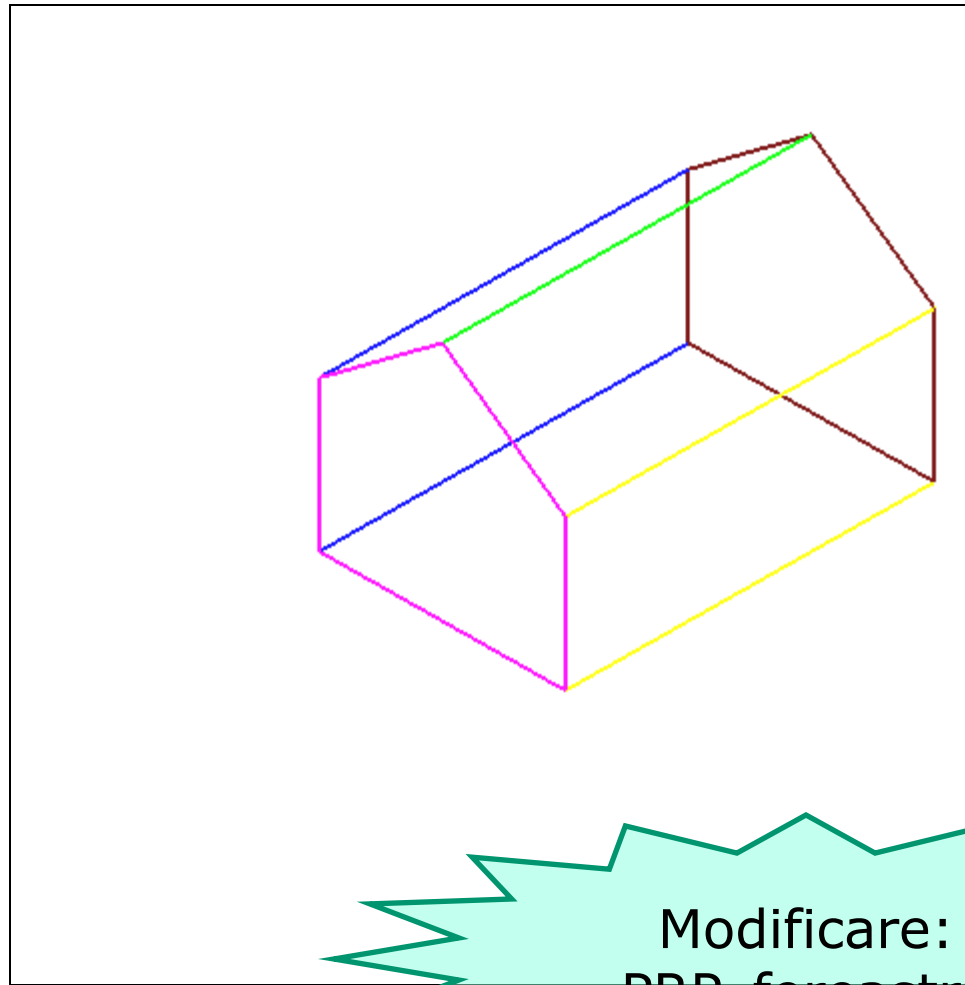
- Exemple PGP8

VRP (WCS)	(8,8,42)
VPN (WCS)	(1,1,1)
VUP (WCS)	(0,1,0)
PRP (VRC)	(0,0,10)
Fereastra (VRC) = (um,uM,vm,vM)	(-20,20,-20,20)
Tip proiectie	paralela

# PGP8



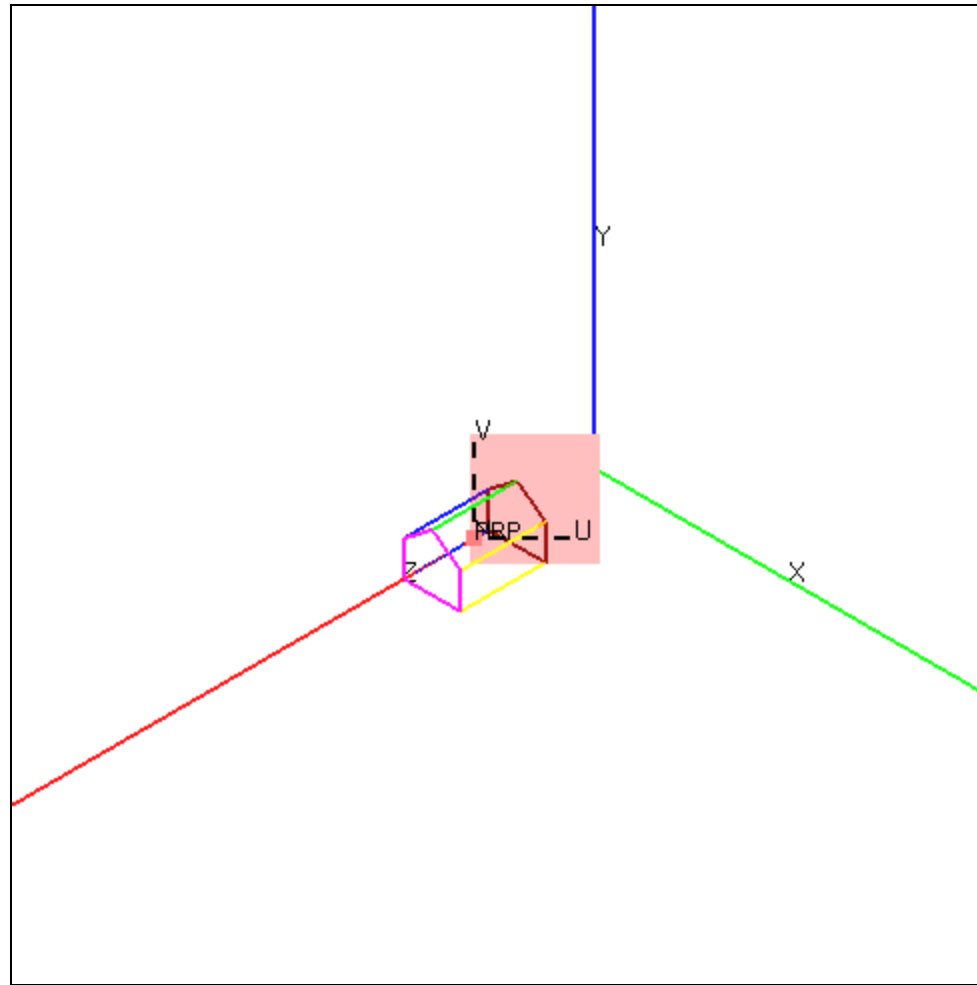
# PGP8



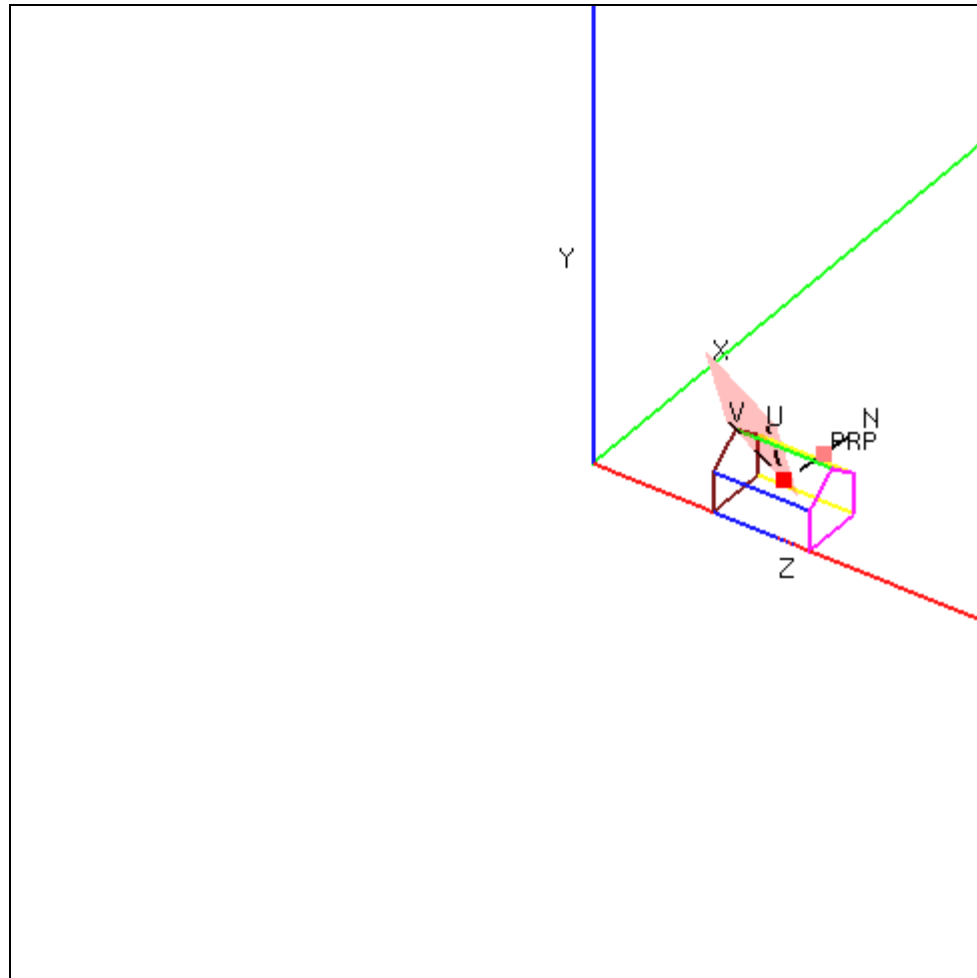
Modificare:  
PRP, fereastra



# PGP8



# PGP8

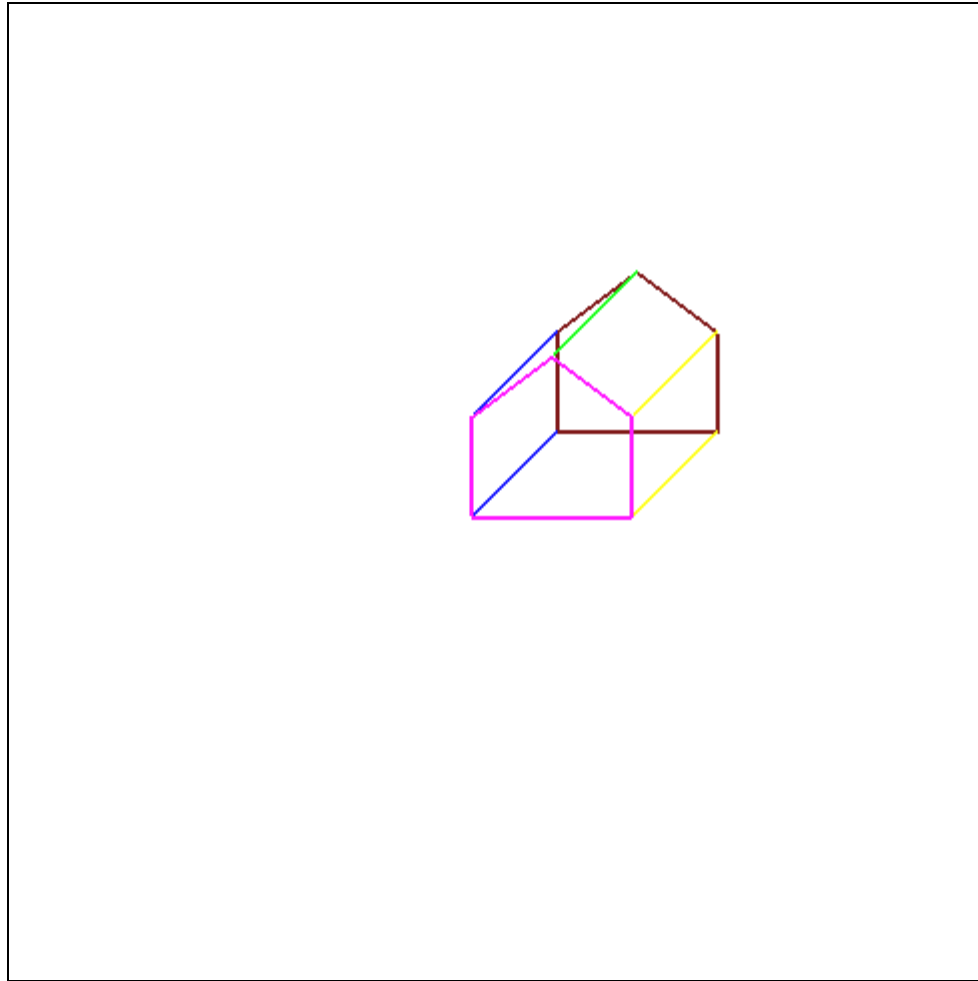


# Proiectii geometrice planare

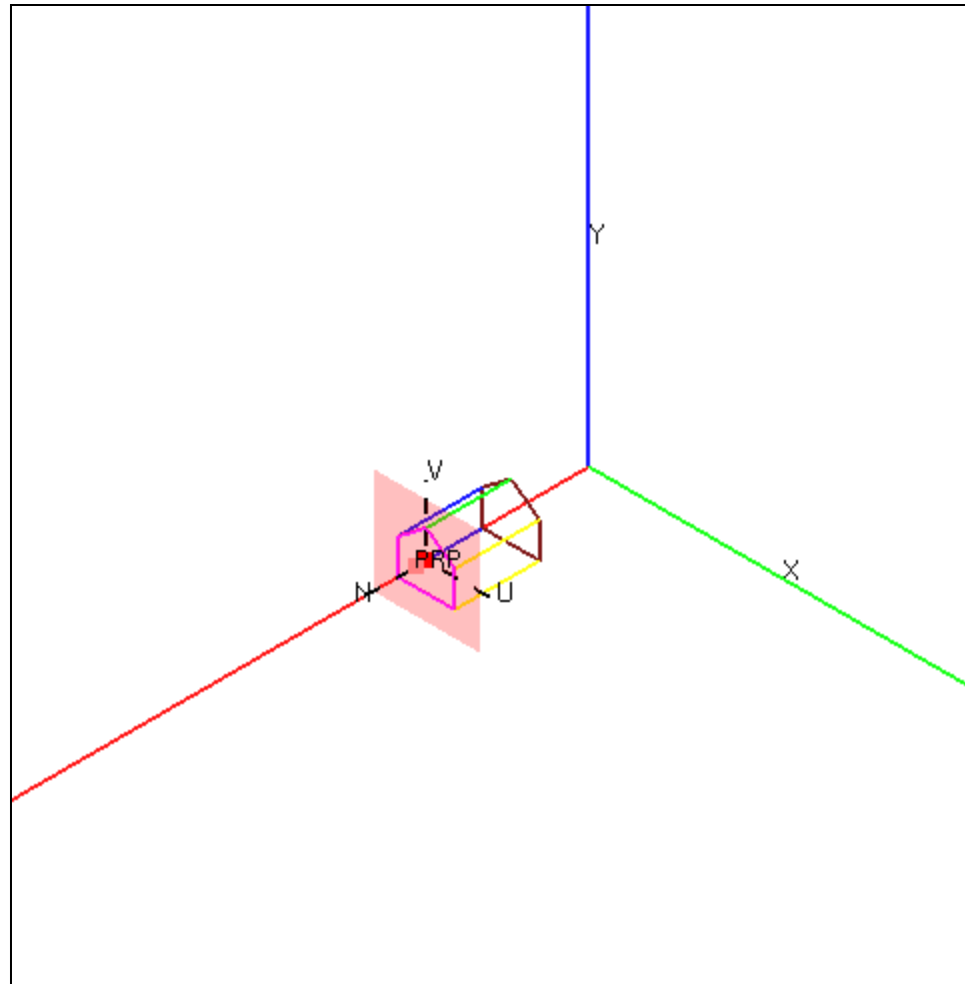
- Exemple PGP9

VRP (WCS)	(8,8,54)
VPN (WCS)	(0,0,1)
VUP (WCS)	(0,1,0)
PRP (VRC)	$(\cos \alpha, \sin \alpha, 1)$
Fereastra (VRC) = (um,uM,vm,vM)	(-15,15,-15,15)
Tip proiectie	paralela

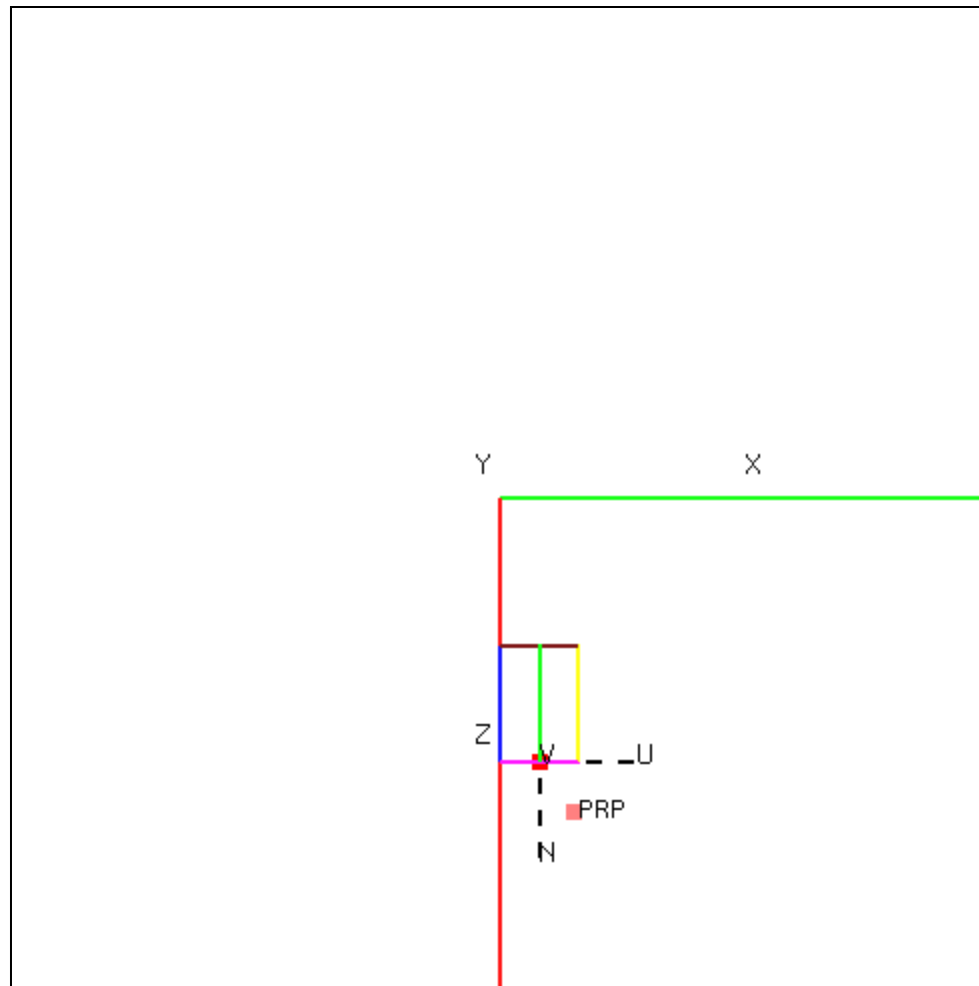
# PGP9



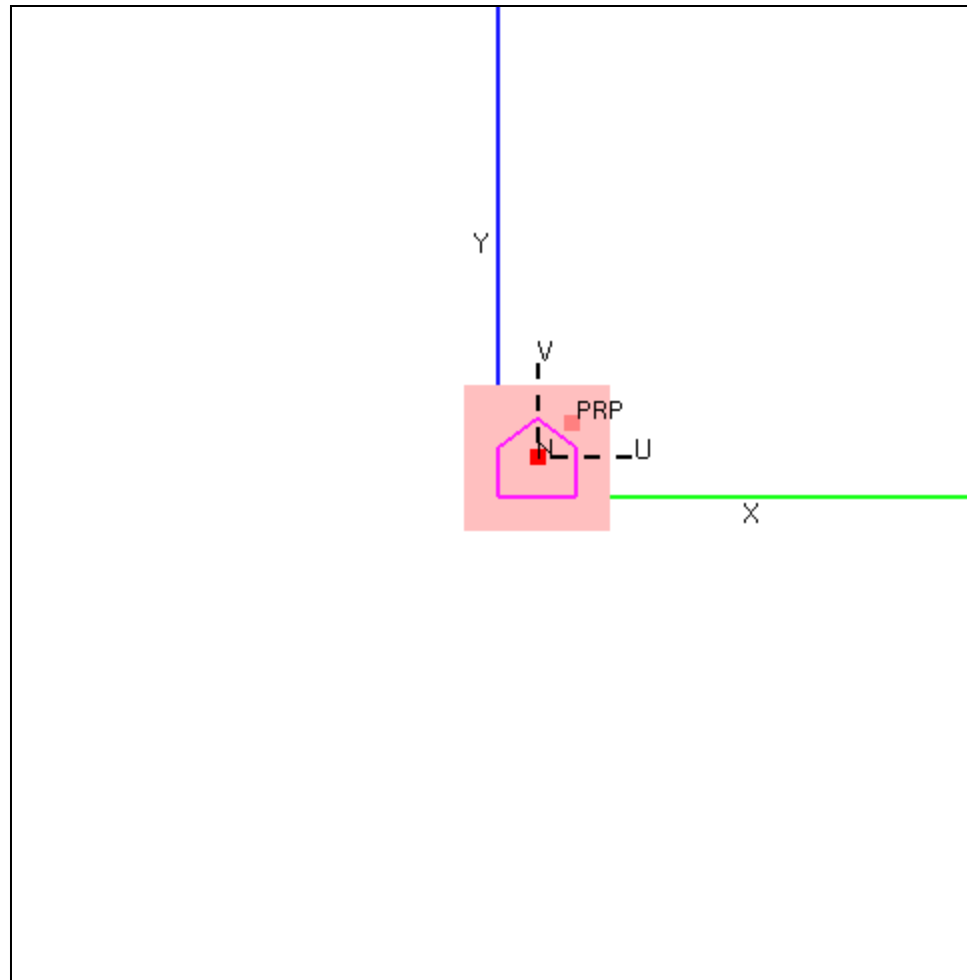
# PGP9



# PGP9



# PGP9



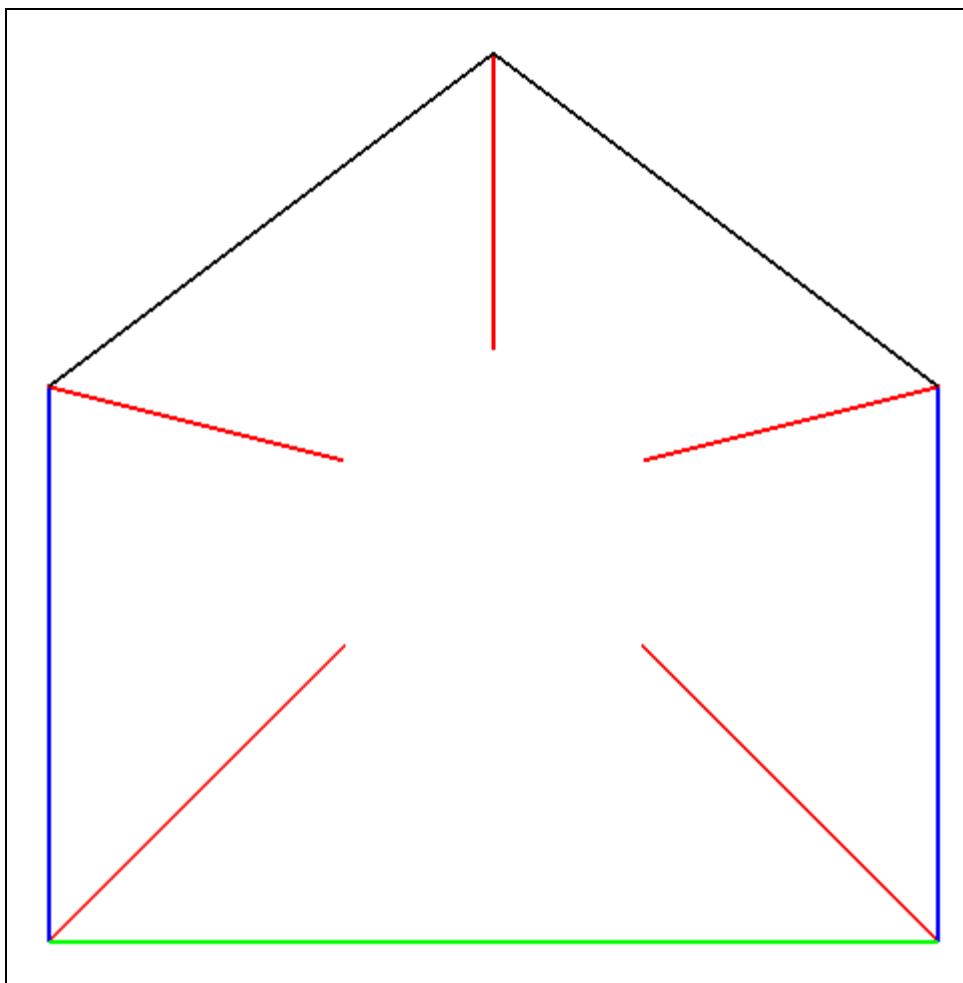
# Proiectii geometrice planare

- Exemple PGP10

VRP (WCS)	(0,0,54)
VPN (WCS)	(0,0,1)
VUP (WCS)	(0,1,0)
PRP (VRC)	(8,6,30)
Fereastra (VRC) = (um,uM,vm,vM)	(-1,17,-1,17)
Tip proiectie	perspectiva
F (VRC)	1
B (VRC)	-23



# PGP10



# Proiectii geometrice planare

- Rezumat
  - definitie
  - clasificare : paralele/perspectiva, perspectiva 1/2/3 pfa, paralele ortografice/oblice, etc.
  - exemple
  - specificarea unei proiectii :
    - VRP, VPN, VUP, CW, umax, umin, vmax, vmin, PRP (COP, DOP), vol. vizualizare, B, F

# Proiectii geometrice planare

- Descriere matematica
  - proiectia perspectiva
    - dat planul PL ( $||$  ( $Ox,Oy$ )) la distanta  $d$  de origine (deci avand ecuatia  $z = d$ ) si  $P(x,y,z)$  care sunt coordonatele punctului  $P_1(x_1,y_1,z_1)$ , obtinut prin proiectia punctului  $P$  pe PL ? Consideram ca  $COP = O(0,0,0)$ .

# Proiectii geometrice planare

- Descriere matematica
  - proiectia perspectiva
    - $M_{\text{per}}$  si omogenizare =>  
 $P1(x/(z/d), y/(z/d), d, 1)$

$$P1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \cdot P$$

# Proiectii geometrice planare

- Descriere matematica
  - proiectia perspectiva
    - dat planul PL (=  $(Ox,Oy)$ ), deci avand ecuatia  $z = 0$ ) si  $P(x,y,z)$  care sunt coordonatele punctului  $P_1(x_1,y_1,z_1)$ , obtinut prin proiectia punctului P pe PL ? Consideram ca COP(0,0,-d).

# Proiectii geometrice planare

- Descriere matematica
  - proiectia perspectiva
    - $M'_{\text{per}}$  si omogenizare =>  
 $P1(x/(z/d + 1), y/(z/d + 1), 0, 1)$

$$P1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{pmatrix} \cdot P$$

# Proiectii geometrice planare

- Descriere matematica
  - proiectia paralela ortografica
    - dat planul PL ( $= (Ox, Oy)$ ), deci avand ecuatia  $z = 0$ ) si  $P(x, y, z)$  care sunt coordonatele punctului  $P_1(x_1, y_1, z_1)$ , obtinut prin proiectia punctului P pe PL ? Consideram ca  $DOP = Oz$ .

# Proiectii geometrice planare

- Descriere matematica
  - proiectia paralela ortografica
    - $M_{\text{ort}} \Rightarrow$   
 $P1(x, y, 0, 1)$

$$\lim_{d \rightarrow \infty} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



# Proiectii geometrice planare

- Descriere matematica
  - proiectia generala
    - dat punctul  $P(x, y, z)$ , planul PL ( $|| (Ox,Oy), z = z_1$ ), COP (dat prin distanta  $Q$  la punctul  $(0,0,z_1)$  si prin vectorul  $d(dx, dy, dz)$  de lungime 1) care sunt coordonatele punctului  $P_1(x_1,y_1,z_1)$ , obtinut prin proiectia punctului  $P$  pe PL ?

# Proiectii geometrice planare

- Descriere matematica
  - proiectia generala
    - $M_{\text{general}}$  si omogenizare

$$\begin{pmatrix} 1 & 0 & -\frac{dx}{dz} & z_1 \cdot \frac{dx}{dz} \\ 0 & 1 & -\frac{dy}{dz} & z_1 \cdot \frac{dy}{dz} \\ 0 & 0 & -\frac{z_1}{Q \cdot dz} & \frac{z_1^2}{Q \cdot dz} + z_1 \\ 0 & 0 & -\frac{1}{Q \cdot dz} & \frac{z_1}{Q \cdot dz} + 1 \end{pmatrix}$$

# Proiectii geometrice planare

- Descriere matematica
  - particularizand  $M_{\text{general}}$  se obtin diverse proiectii

# Proiectii geometrice planare

	$z_1$	$Q$	$(dx, dy, dz)$
$M_{ort}$	0	$\infty$	$(0, 0, -1)$
$M_{per}$	$d$	$d$	$(0, 0, -1)$
$M'_{per}$	0	$d$	$(0, 0, -1)$
cavaliera	0	$\infty$	$(\cos t, \sin t, -1)$
cabinet	0	$\infty$	$((\cos t)/2, (\sin t)/2, -1)$

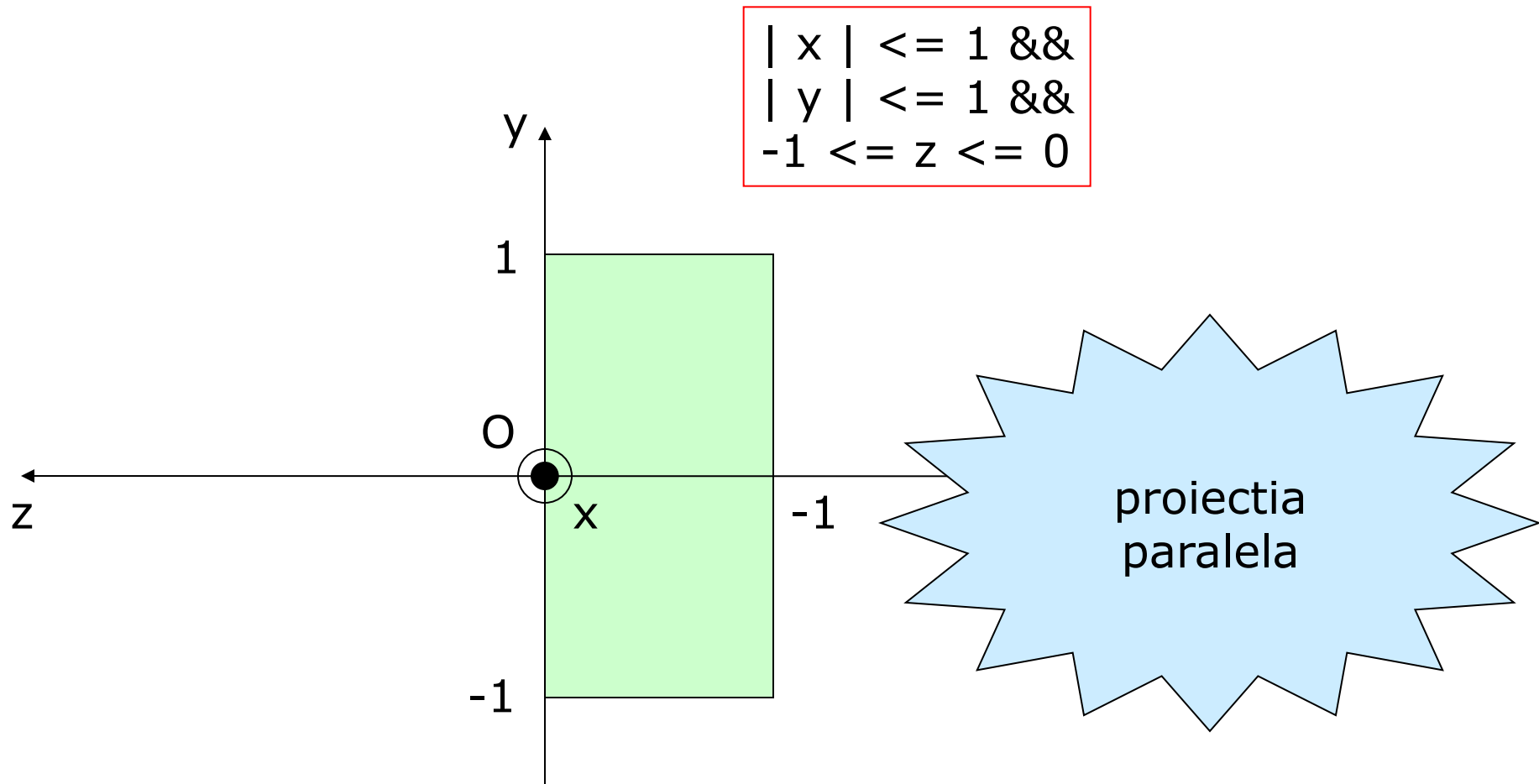
# Proiectii geometrice planare

- Rezumat
  - definitie
  - clasificare
  - exemple
  - specificarea unei proiectii
  - descriere matematica

# Proiectii geometrice planare

- Implementare
  - transformarea de normalizare
    - vol.de vizualizare arbitrare sunt transformate in vol.de viz. canonice
    - deoarece pentru aceste volume alg.de decupare sunt mai eficienti
  - transformarea vol.de viz.canonic pentru proiectia persp. in cel canonic pentru proiectia paralela

# Transformarea de normalizare volume de vizualizare canonice



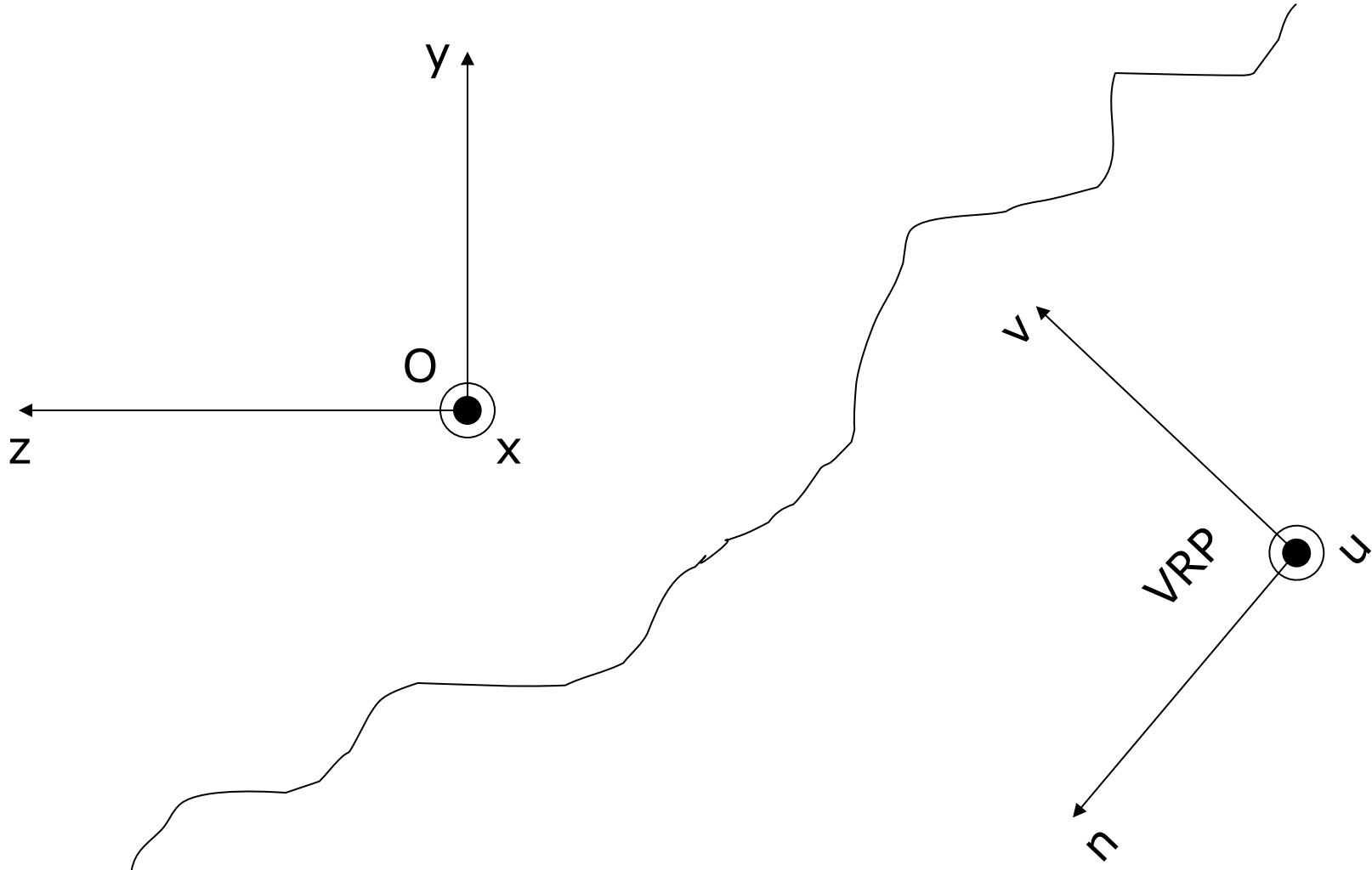
## Transformarea de normalizare pentru o proiectie paralela

- Translatie VRP -> originea O
- Rotatie a.i. (VRP,u,v,n) sa coincida cu (O,x,y,z)
- Transformare de forfecare a.i. DOP || Oz
- Translatie si scalare in vol.de viz.canonic pentru pr.paralela

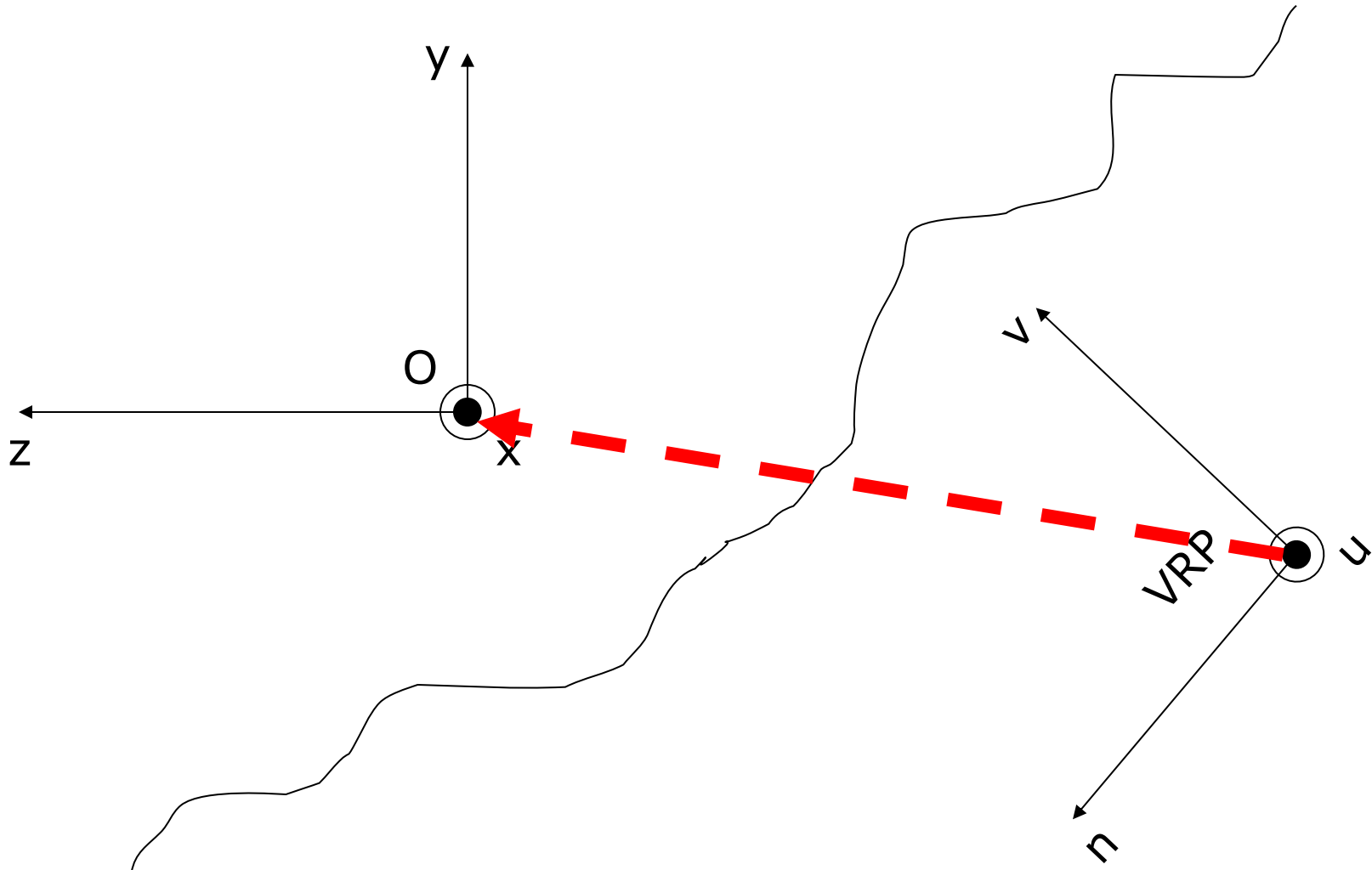
$$N_{par} = S_{par} \bullet T_{par} \bullet SH_{par} \bullet R \bullet T(-VRP)$$



# Transformarea de normalizare pentru o proiectie paralela

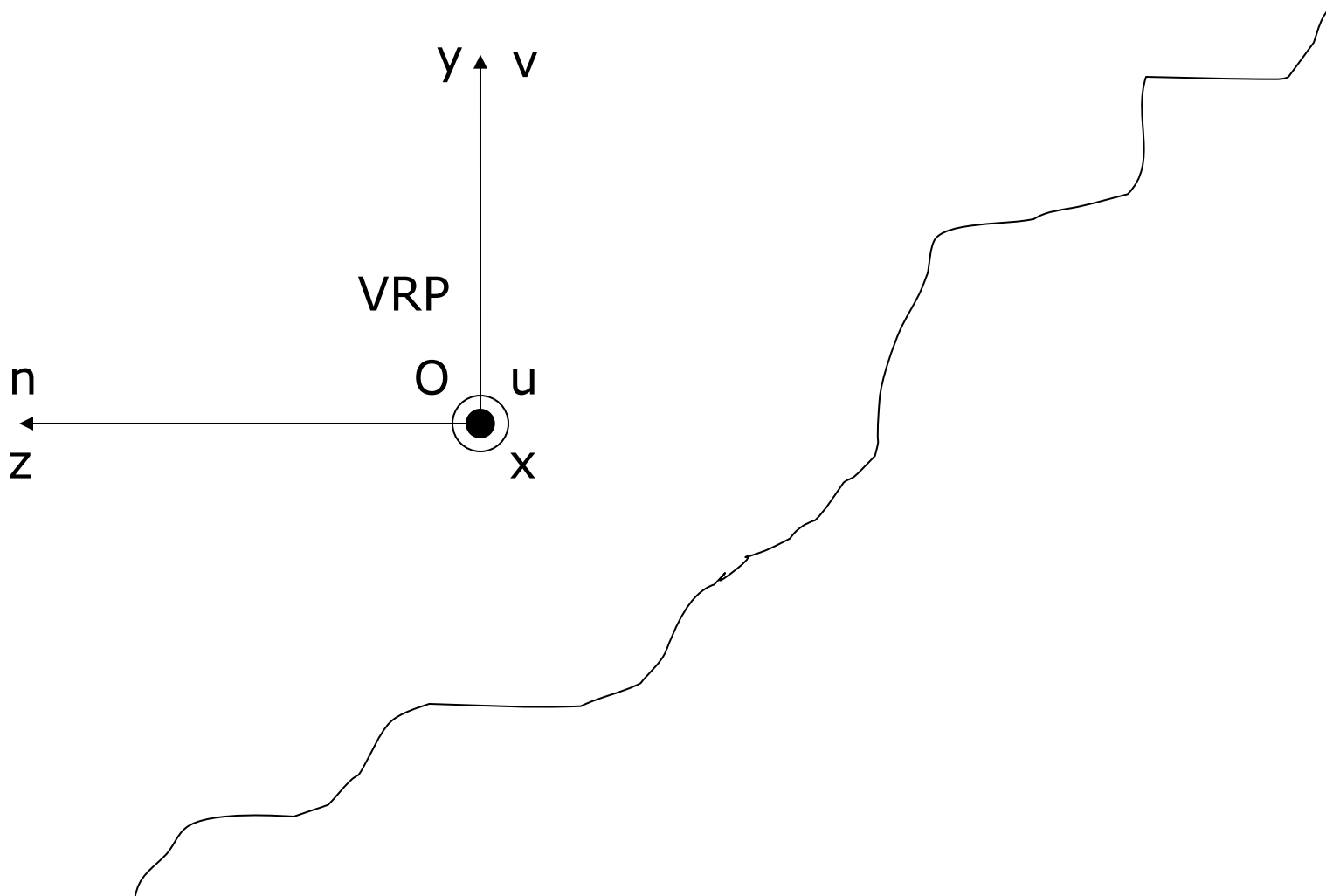


# Transformarea de normalizare pentru o proiectie paralela

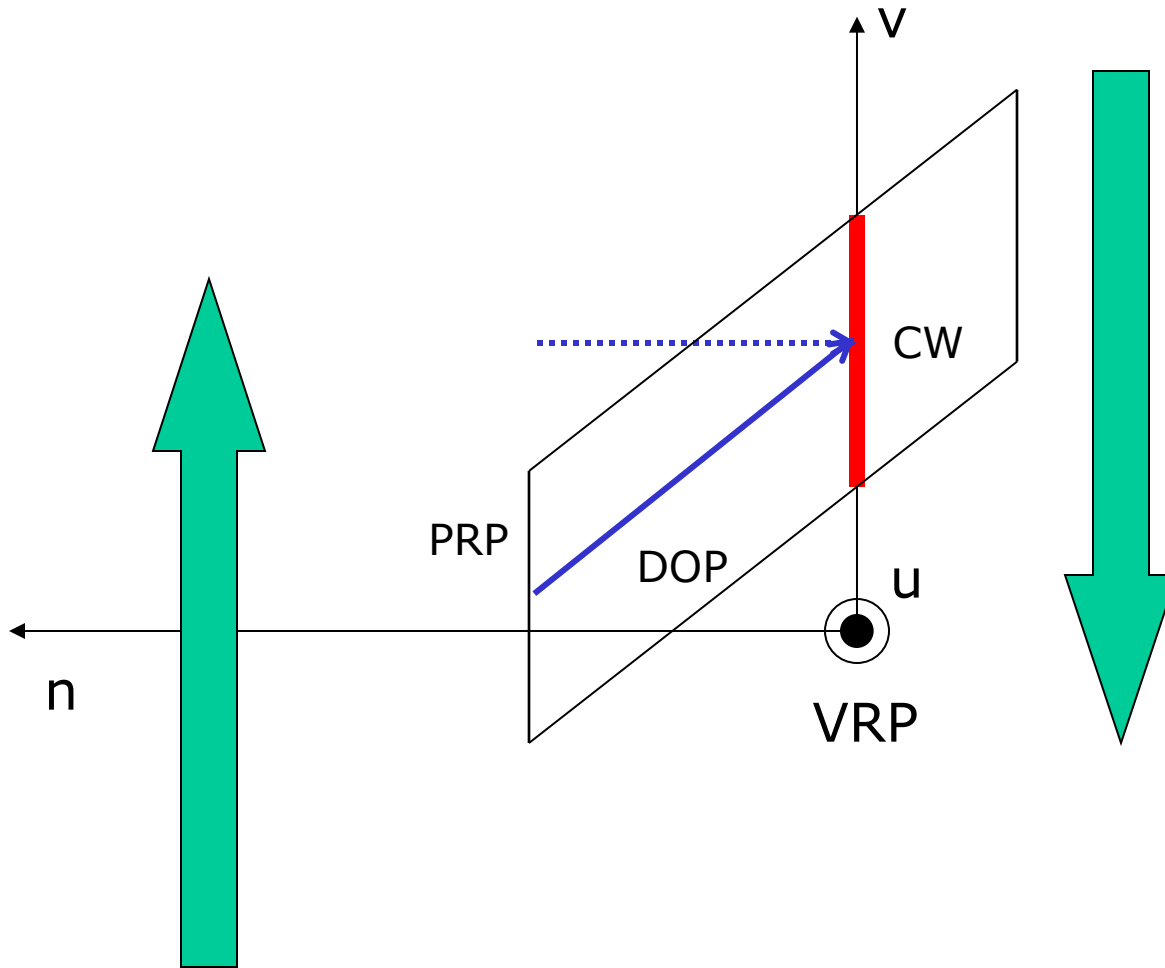


The diagram shows a 2D coordinate system with a horizontal  $x$ -axis and a vertical  $y$ -axis. A point  $O$  is marked at the origin. A vector  $u$  points from  $O$  into the first quadrant, and a vector  $v$  points from  $O$  into the second quadrant. A dashed red arc is centered at  $O$ , spanning from the negative  $x$ -axis to the vector  $u$ . A red arrow points from the arc towards the vector  $u$ . A wavy line representing a path starts from the bottom left and moves towards the top right, passing near the origin. A label  $VRP$  is placed near the origin, and a label  $n$  is placed near the wavy line.

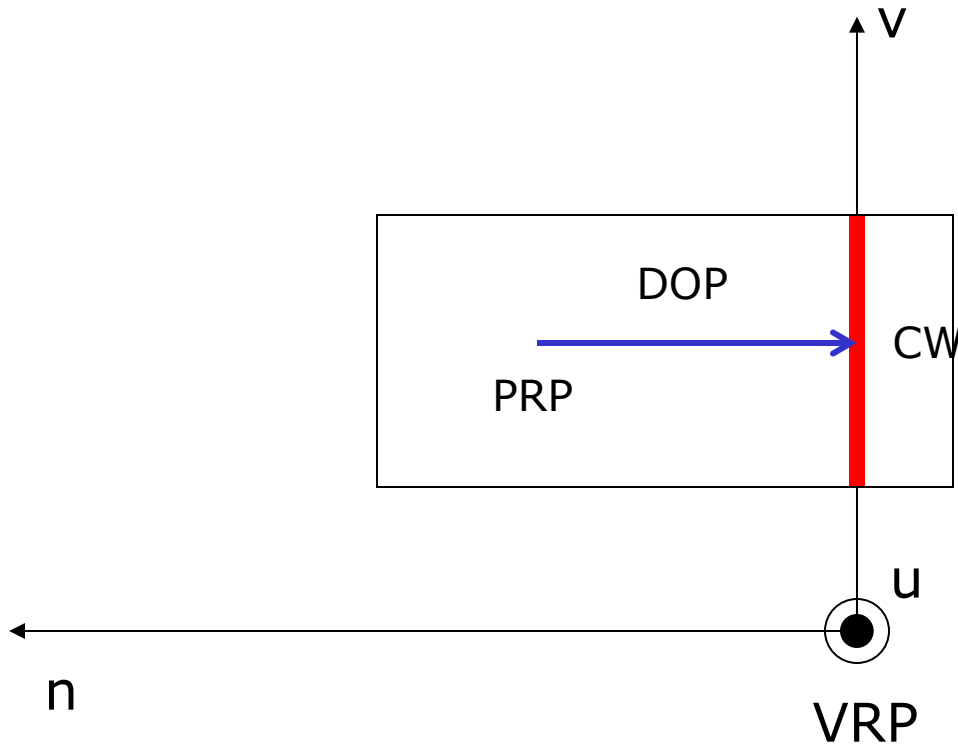
# Transformarea de normalizare pentru o proiectie paralela



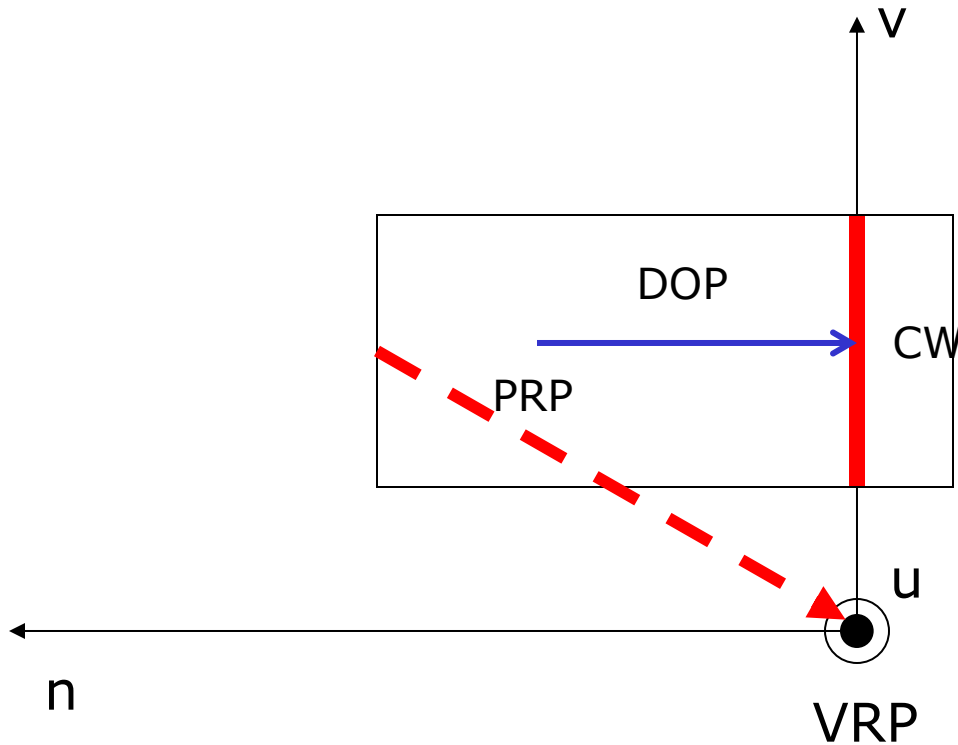
# Transformarea de normalizare pentru o proiectie paralela



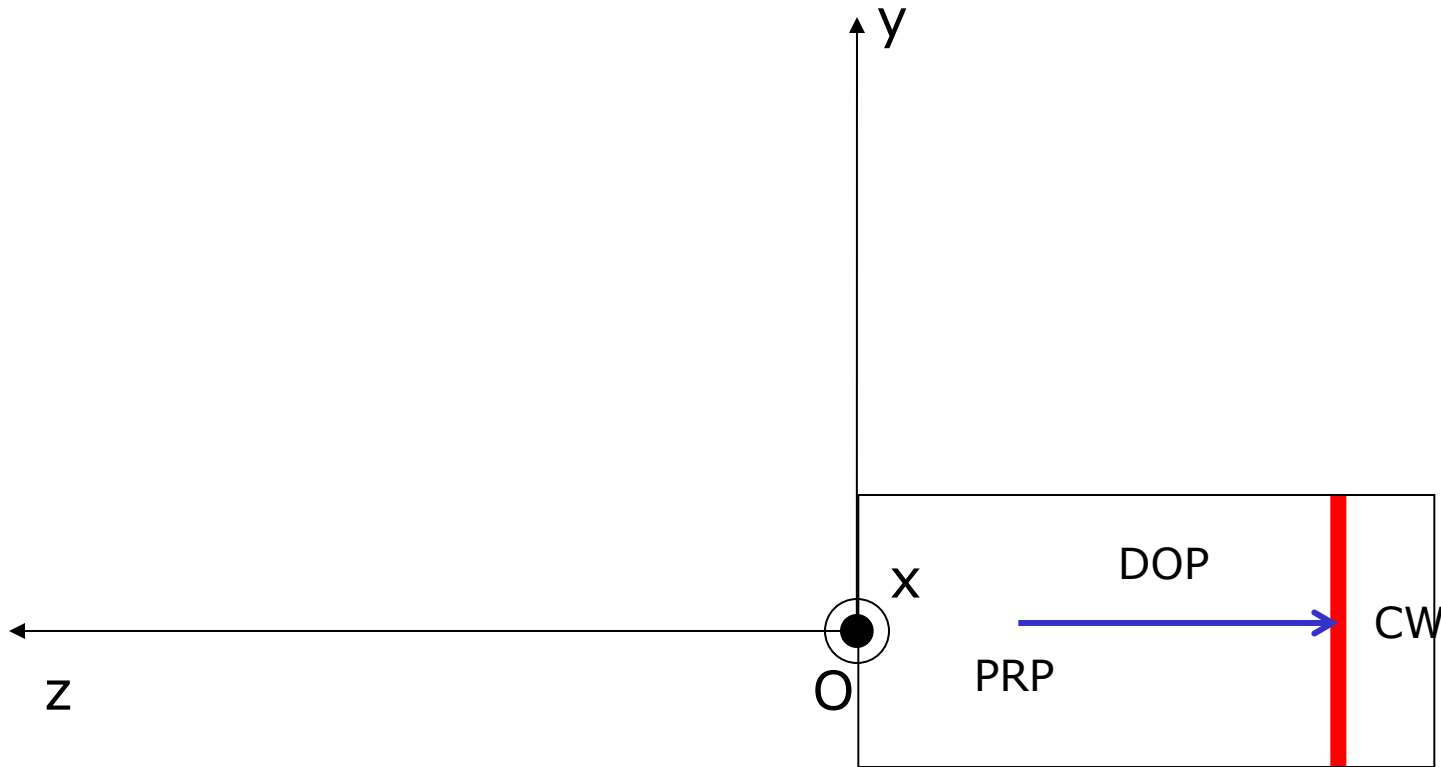
# Transformarea de normalizare pentru o proiectie paralela



# Transformarea de normalizare pentru o proiectie paralela

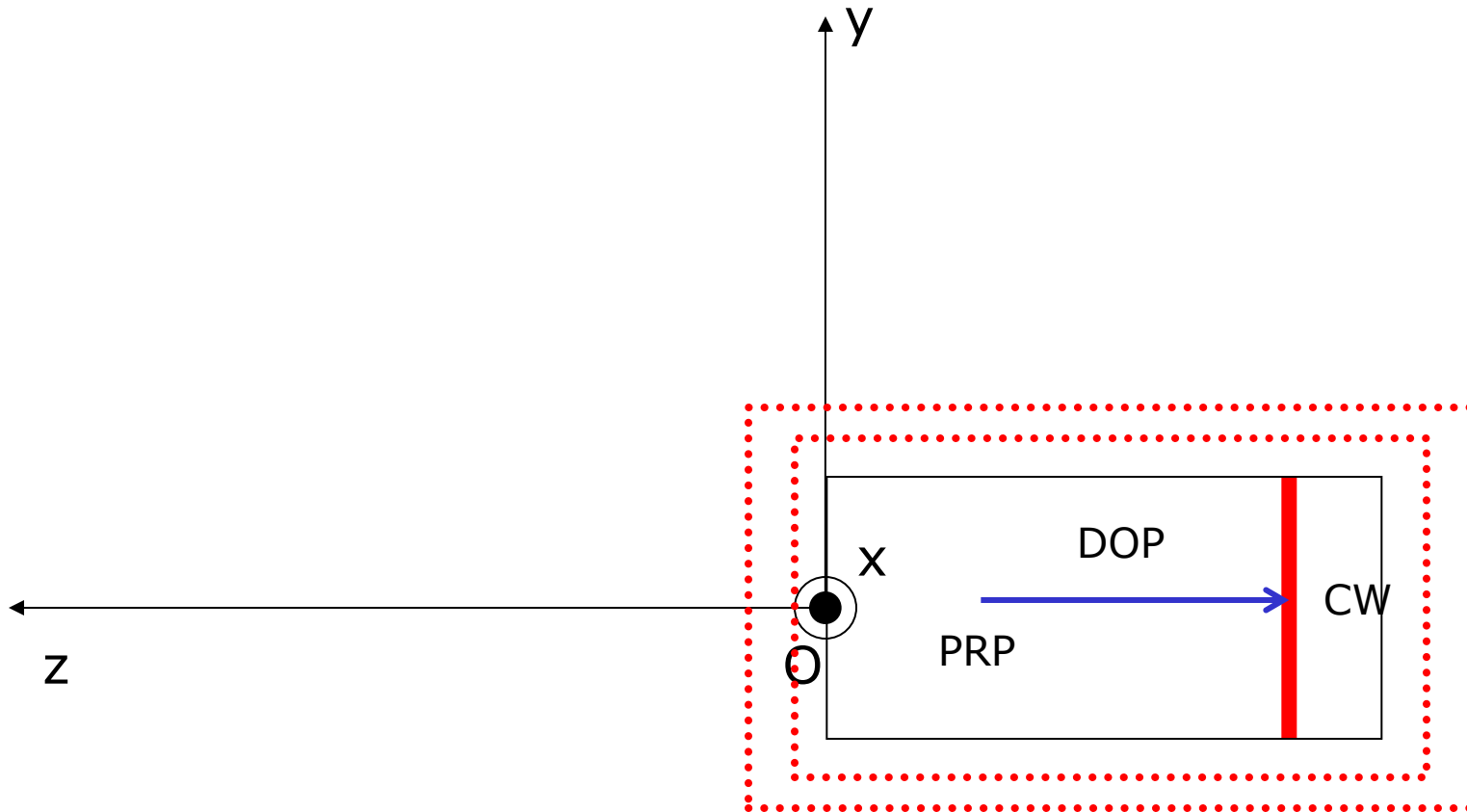


# Transformarea de normalizare pentru o proiectie paralela

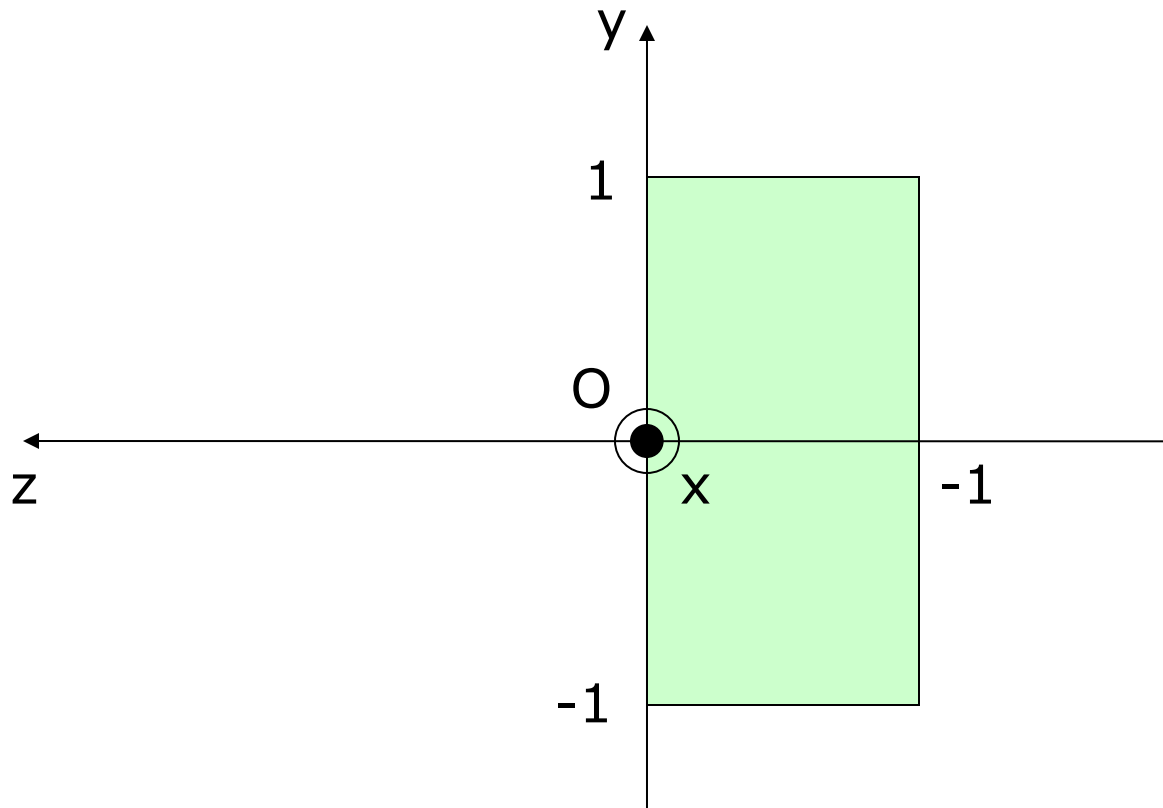




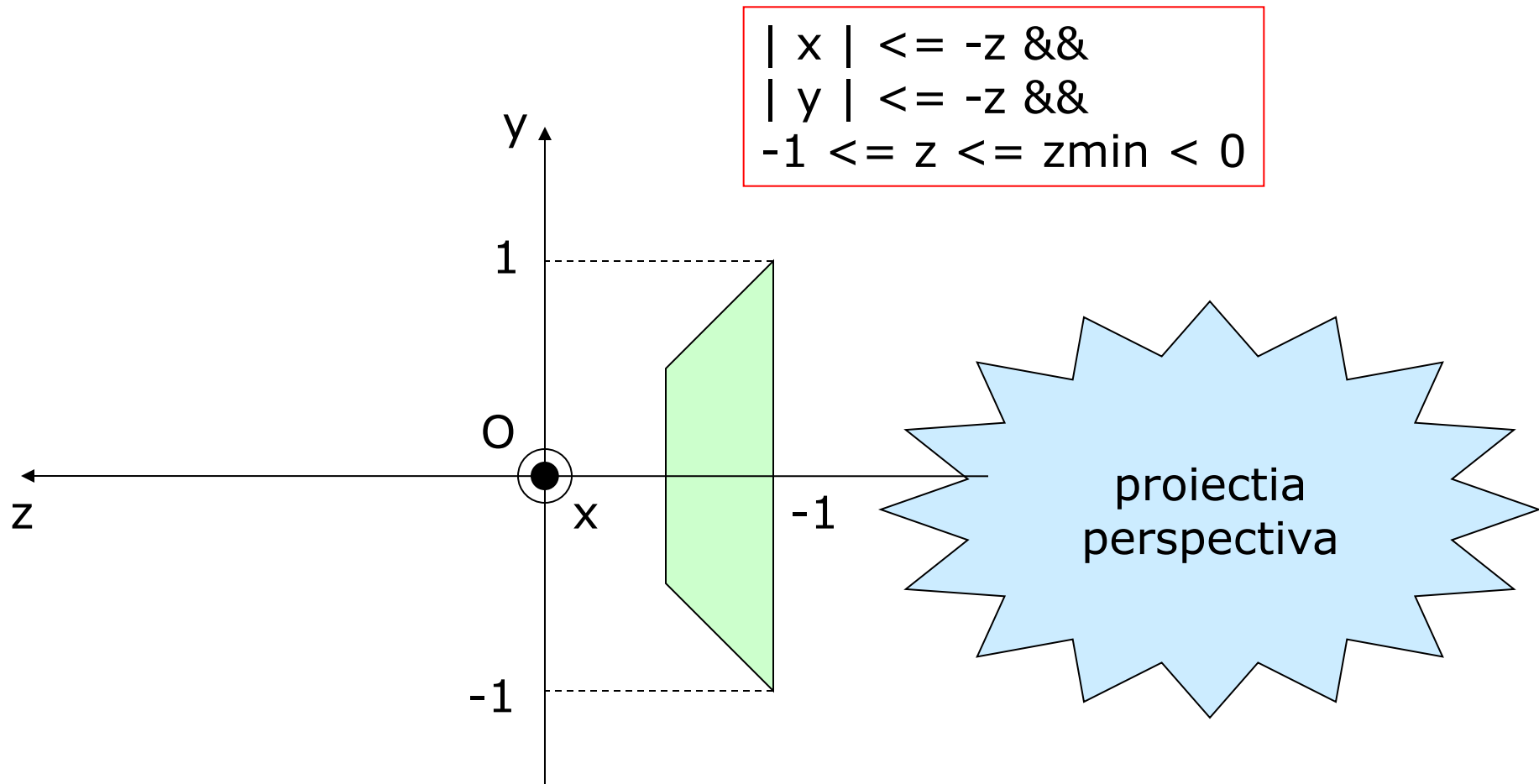
# Transformarea de normalizare pentru o proiectie paralela



# Transformarea de normalizare pentru o proiectie paralela



# Transformarea de normalizare volume de vizualizare canonice

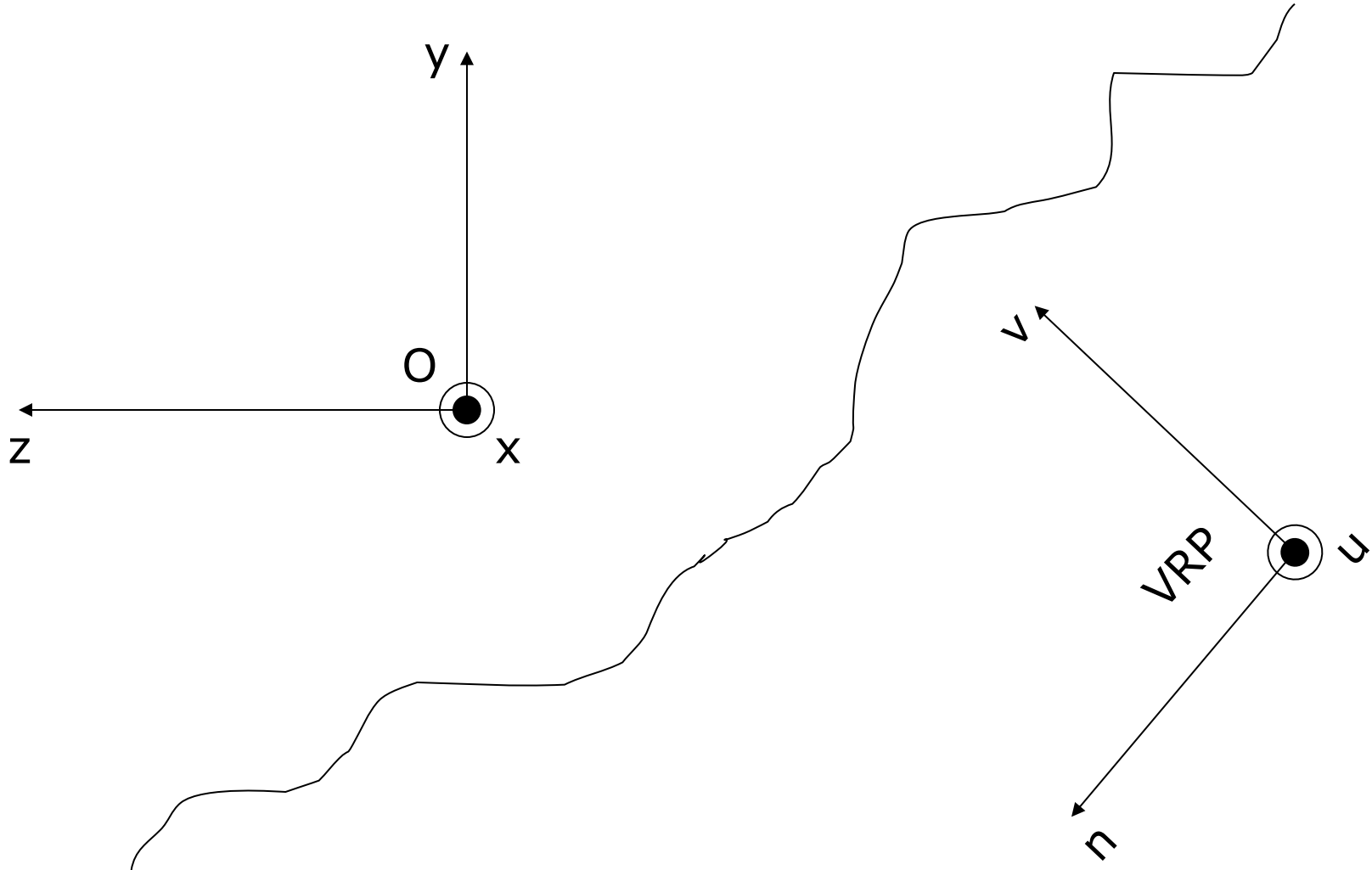


## Transformarea de normalizare pentru o proiectie perspectiva

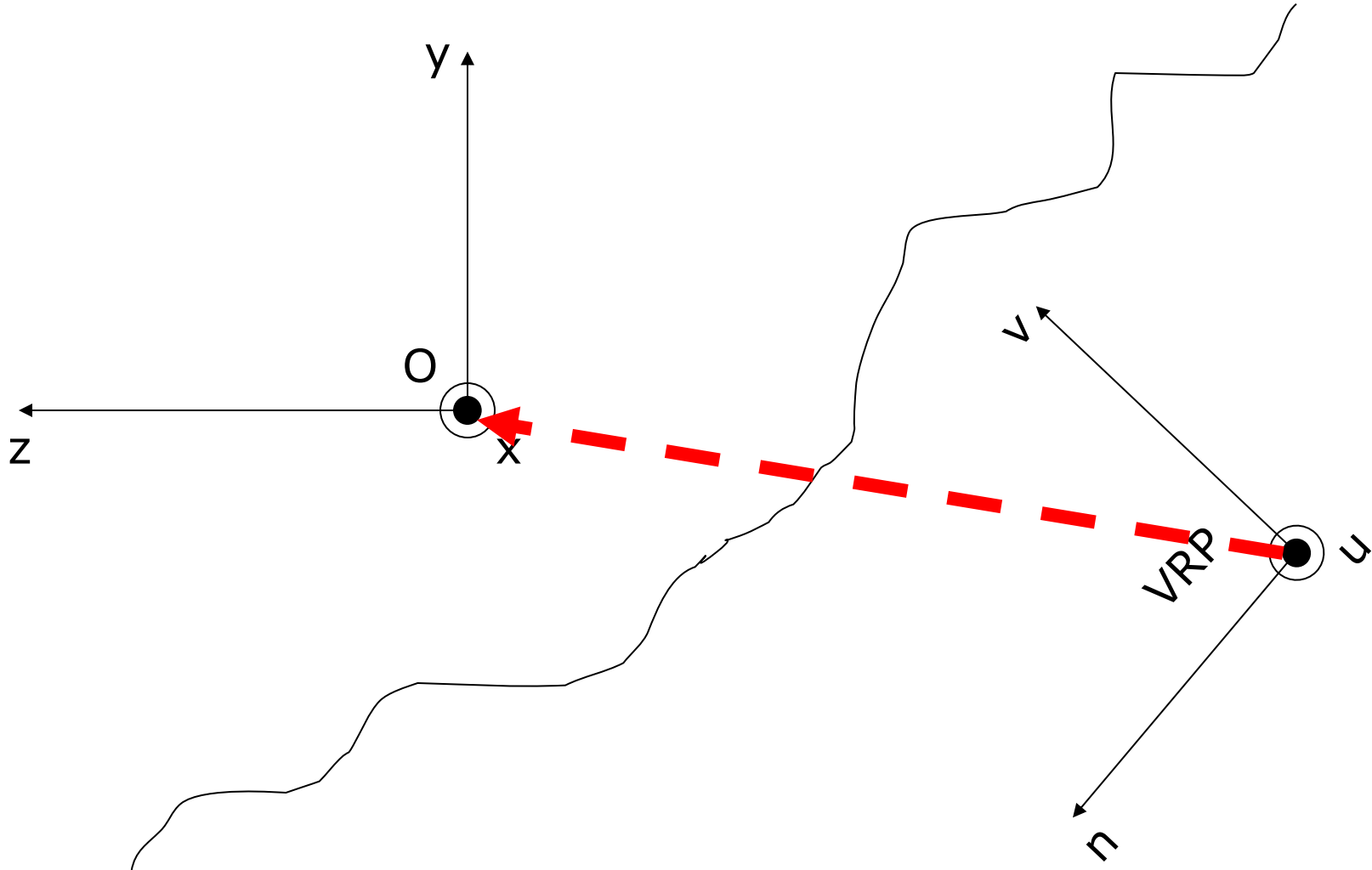
- Translatie VRP -> originea O
- Rotatie a.i. (VRP,u,v,n) sa coincida cu (O,x,y,z)
- Translatie a.i. PRP(COP) -> origine
- Transformare de forfecare
- Scalare in vol.de viz.canonic pentru pr.perspectiva

$$N_{per} = S_{per} \bullet SH_{par} \bullet T(-PRP) \bullet R \bullet T(-VRP)$$

# Transformarea de normalizare pentru o proiectie perspectiva

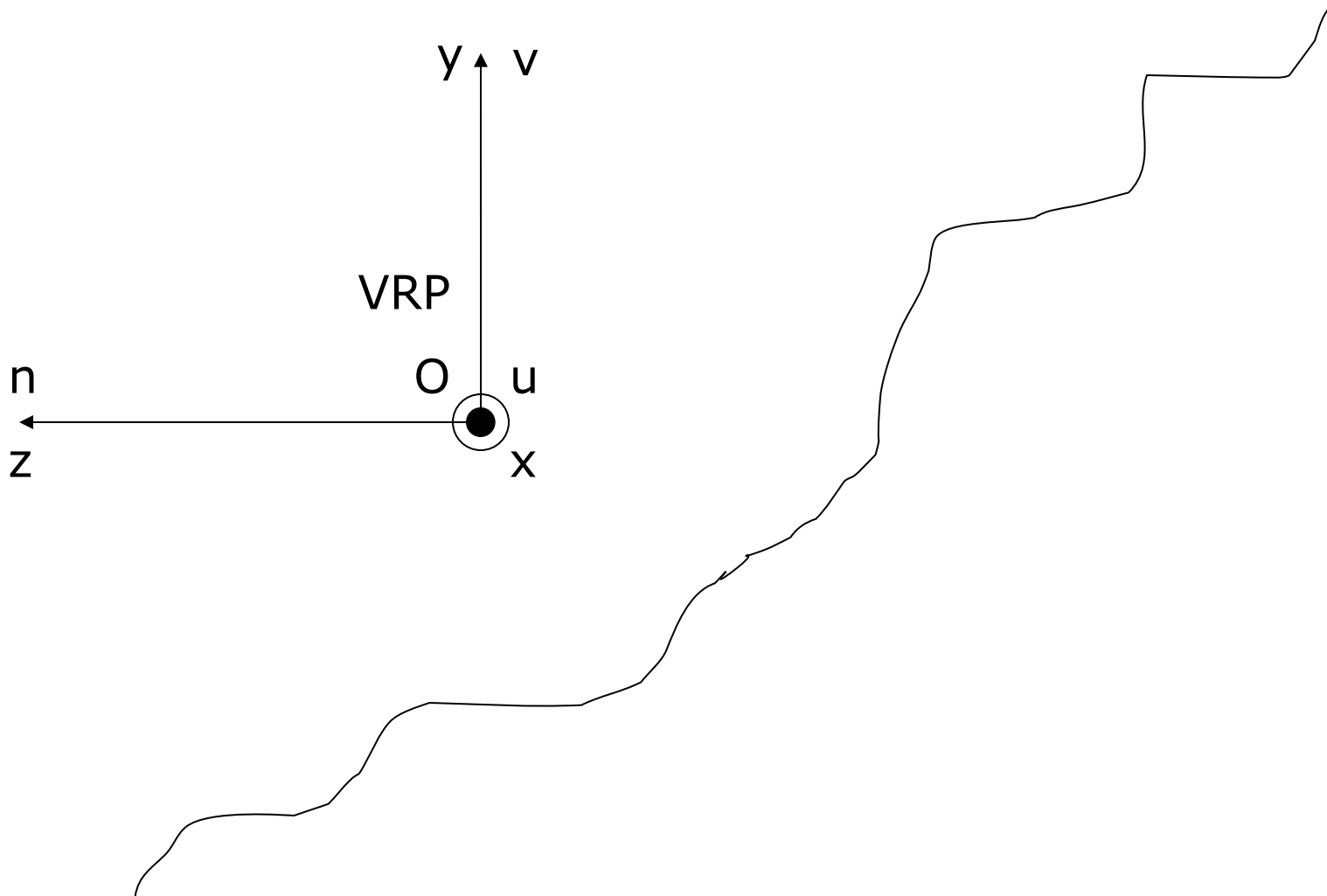


# Transformarea de normalizare pentru o proiectie perspectiva



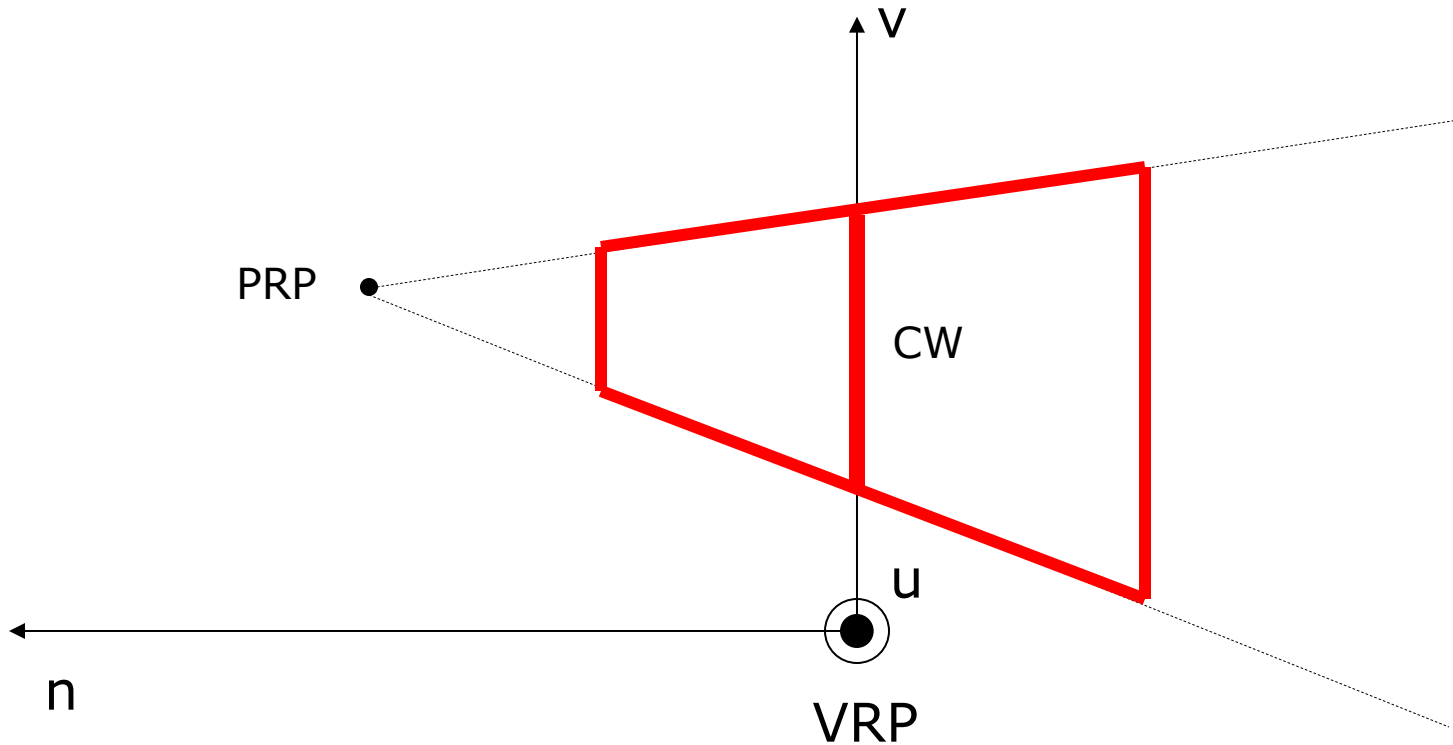
A diagram illustrating the relationship between a ship's heading and its velocity vector. A coordinate system is shown with a horizontal axis labeled  $x$  and a vertical axis labeled  $y$ . The origin is marked with a black dot and labeled  $O$ . A line representing the ship's heading is labeled  $VRP$  (Virtual Range Point) and is oriented at an angle  $\psi$  relative to the  $x$ -axis. A red dashed arc indicates the heading angle  $\psi$ . A red arrow labeled  $v$  represents the ship's velocity vector, which is oriented at an angle  $\beta$  relative to the heading line. A wavy line represents the ship's path, which is oriented at an angle  $\alpha$  relative to the  $x$ -axis.

# Transformarea de normalizare pentru o proiectie perspectiva

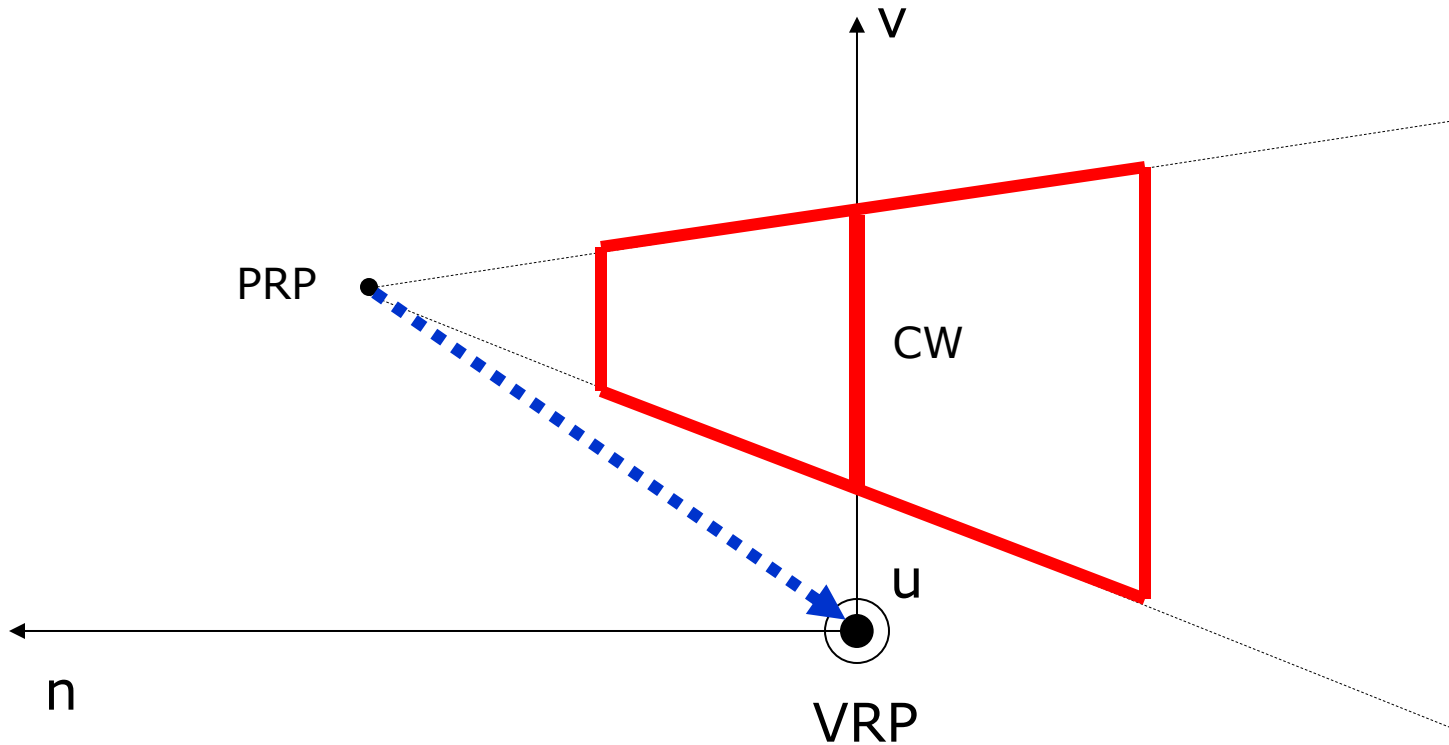




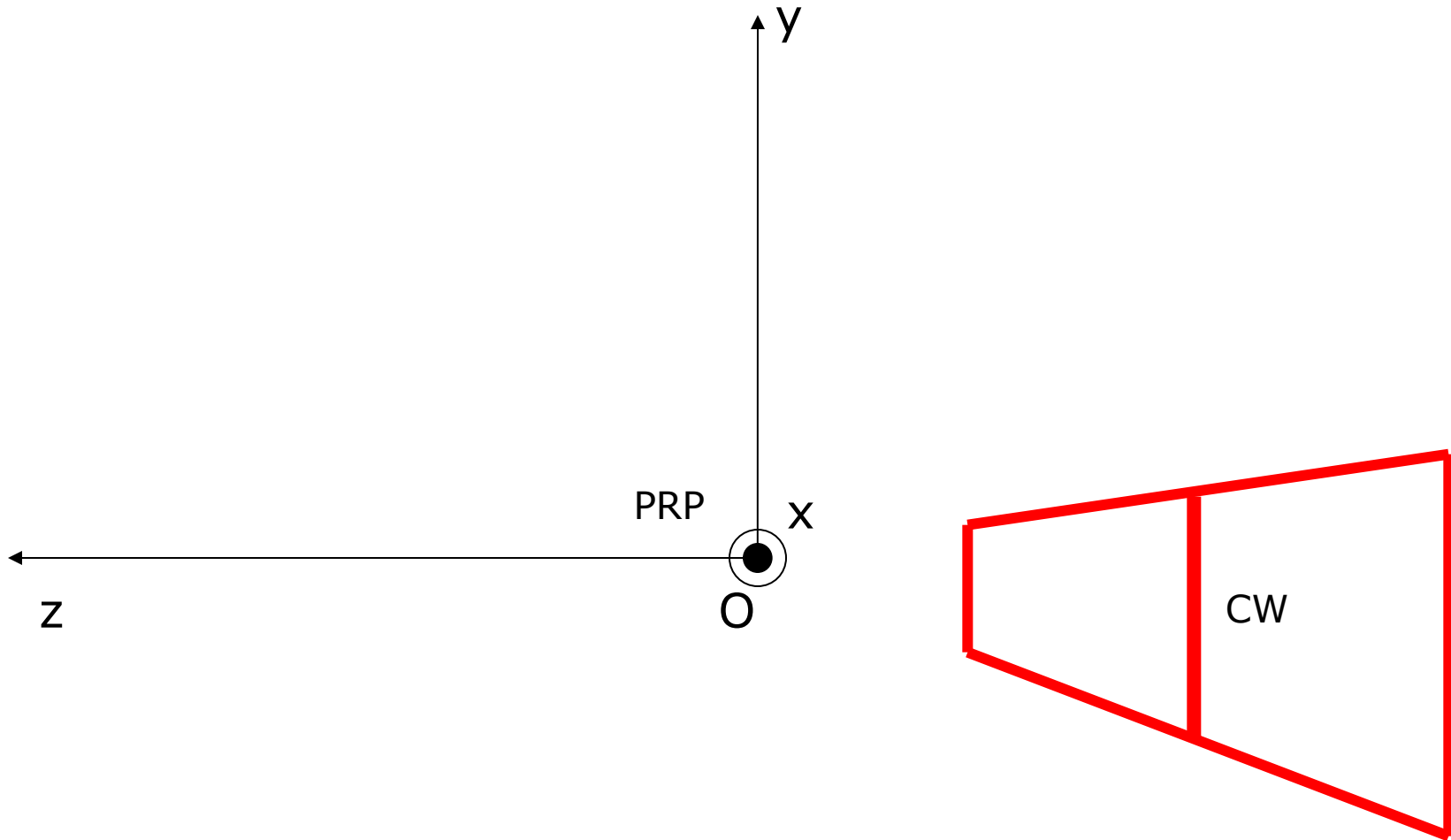
# Transformarea de normalizare pentru o proiectie perspectiva



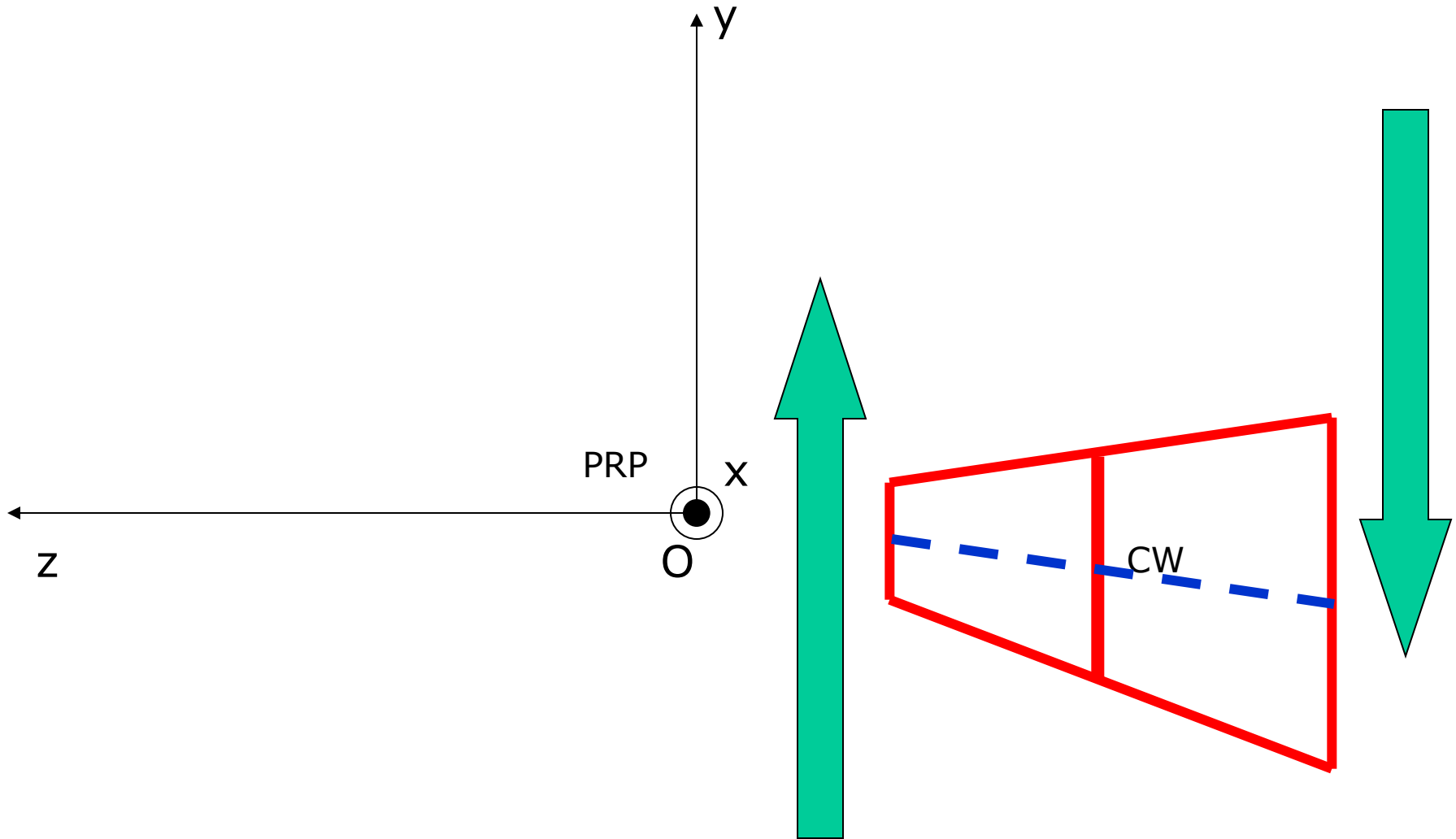
# Transformarea de normalizare pentru o proiectie perspectiva



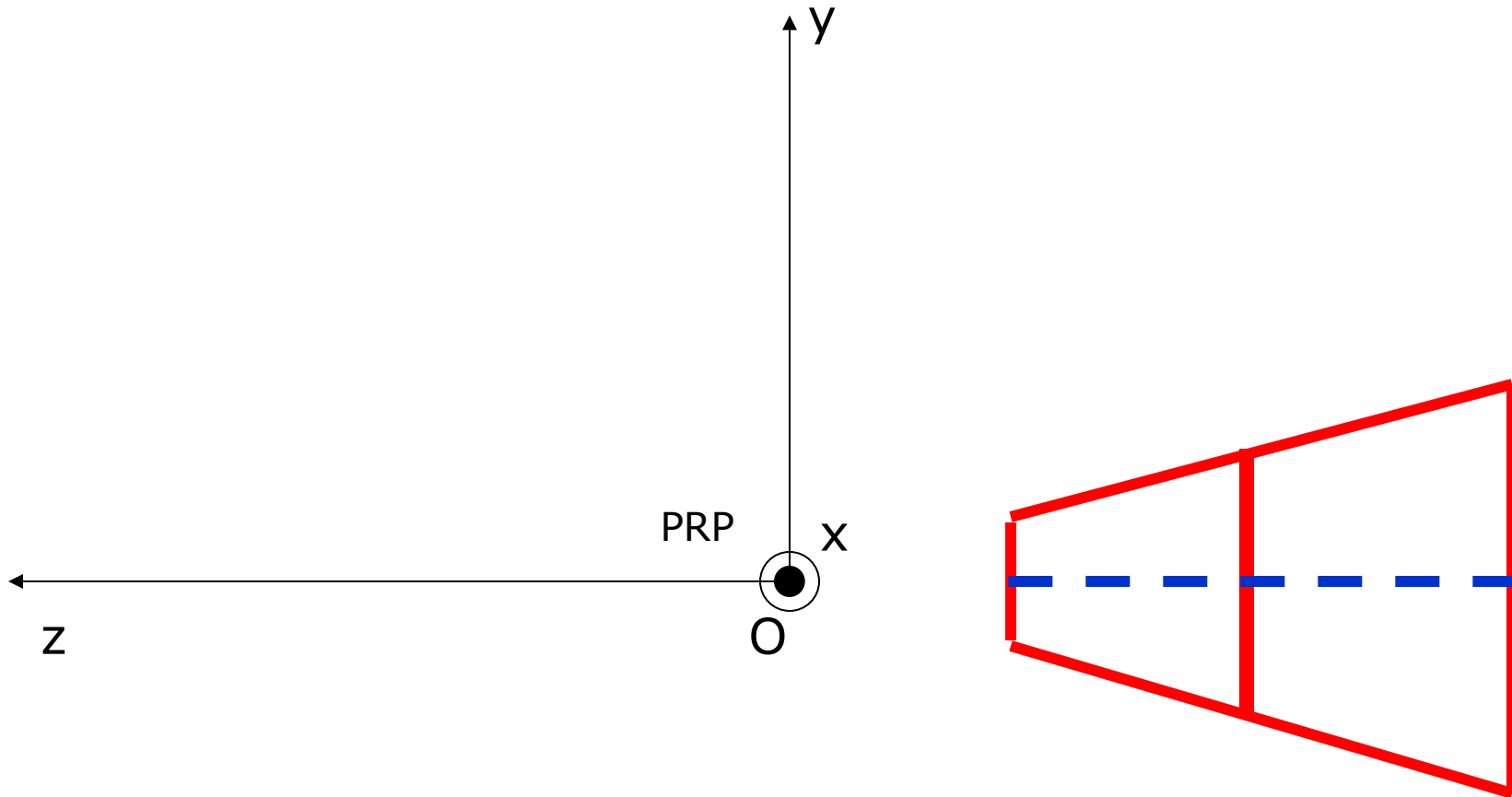
# Transformarea de normalizare pentru o proiectie perspectiva



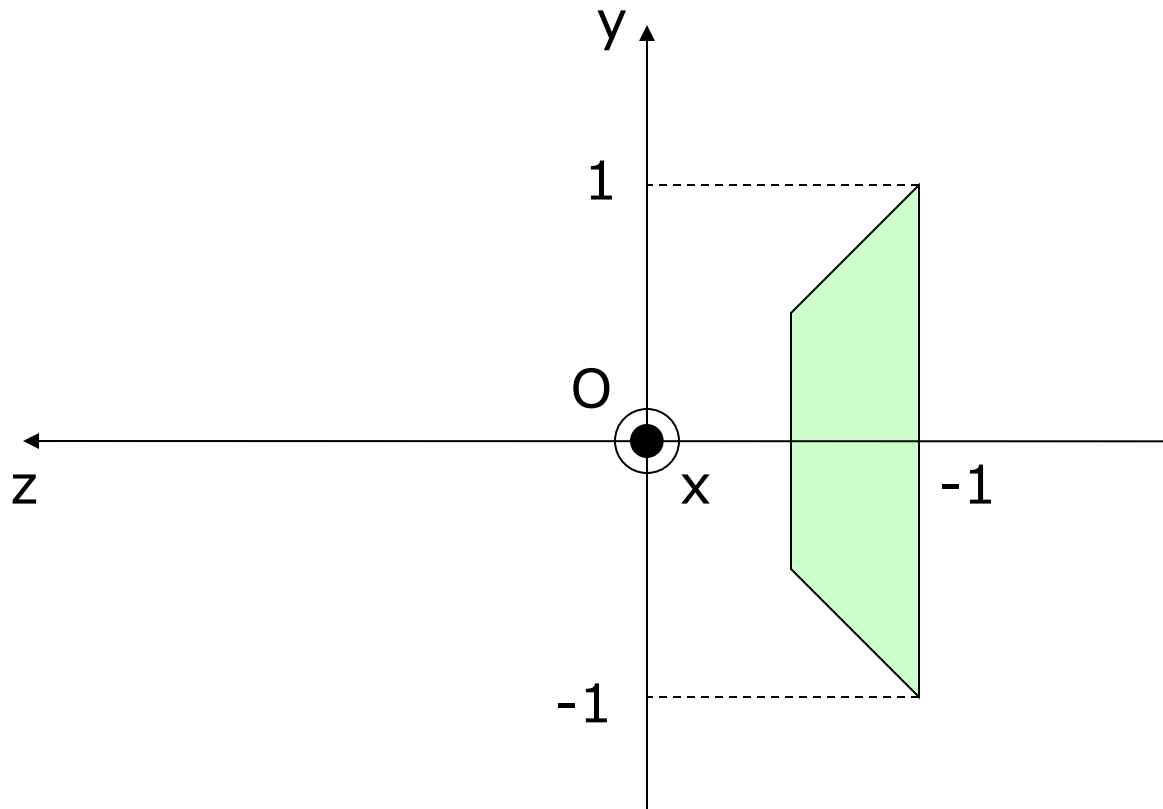
# Transformarea de normalizare pentru o proiectie perspectiva



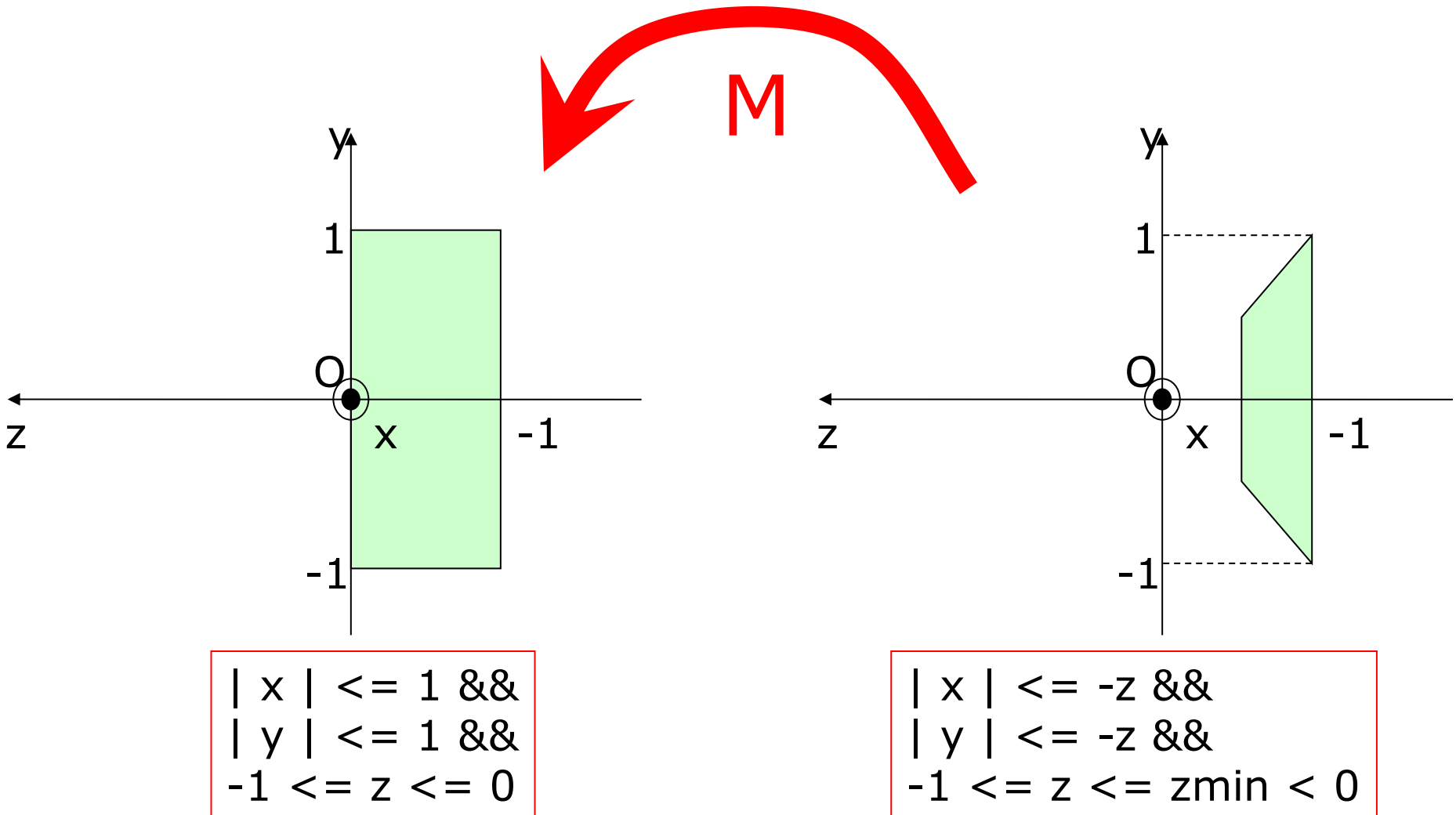
# Transformarea de normalizare pentru o proiectie perspectiva



# Transformarea de normalizare pentru o proiectie perspectiva



# Transformarea v.v.can.pr.perspectiva in v.v.can.pr.paralela



# Transformarea v.v.can.pr.perspectiva in v.v.can.pr.paralela

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{1 + z_{min}} & \frac{-z_{min}}{1 + z_{min}} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$



# Proiectii geometrice planare

- Rezumat
  - definitie
  - clasificare
  - exemple
  - specificarea unei proiectii
  - descriere matematica
  - implementare : transformarea de normalizare, vol.de viz.can.

# Proiectii geometrice planare

- Implementare

- volum de vizualizare canonic pentru proiectia paralela
- specificare viewport 3D :  
 $[x_{v.min}, x_{v.max}] \times [y_{v.min}, y_{v.max}] \times [z_{v.min}, z_{v.max}]$  inclus in  $[0,1]^3$
- mapare fata  $z = 1$  cu zona patratica c.m.mare care poate fi inscrisa in ecran
- afisarea punctelor facand abstractie de coordonata  $z$

# Proiectii geometrice planare

- Implementare

- translatie a.i. coltul  $(-1,-1,-1)$  al vol.de viz.can.pentru pr.par.sa ajunga in origine
- scalare a v.v.c.p.p. a.i. sa aiba dimensiunile viewport-ului 3D
- translatie a.i. v.v.c.p.p. modificat sa coincida cu viewport-ul 3D

# Proiectii geometrice planare

- Rezumat
  - definitie
  - clasificare
  - exemple
  - specificarea unei proiectii
  - descriere matematica
  - implementare
    - transformarea de normalizare
    - transformarea de viewport

# Vizualizare 3D

- Rezumat
  - proiectii geometrice planare
    - definitie
    - clasificare
    - exemple
    - specificarea unei proiectii
    - descriere matematica
    - implementare
      - transformarea de normalizare
      - transformarea de viewport

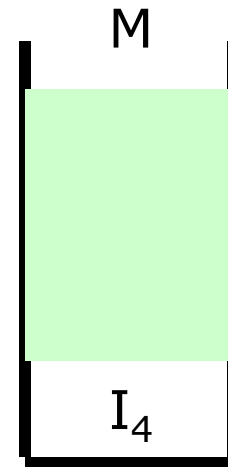
# Vizualizare 3D

- Realizarea vizualizarii 3D in OpenGL
- problema
  - cum se realizeaza corespondenta intre un punct `glVertex3D(x, y, z)` specificat intr-un program care utilizeaza OpenGL si punctul care se afiseaza pe ecran ?

# Vizualizare 3D OpenGL

eye  
coordinates

$$\begin{pmatrix} x_e \\ y_e \\ z_e \\ w_e \end{pmatrix}$$



object  
coordinates

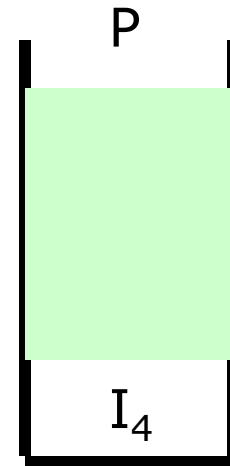
$$\begin{pmatrix} x_o \\ y_o \\ z_o \\ w_o \end{pmatrix}$$

model-view matrix stack

# Vizualizare 3D OpenGL

clip  
coordinates

$$\begin{pmatrix} x_c \\ y_c \\ z_c \\ w_c \end{pmatrix}$$



projection matrix stack

eye  
coordinates

$$\begin{pmatrix} x_e \\ y_e \\ z_e \\ w_e \end{pmatrix}$$





# Vizualizare 3D OpenGL

normalized device  
coordinates

$$\begin{pmatrix} x_d \\ y_d \\ z_d \end{pmatrix} = \begin{pmatrix} x_c / w_c \\ y_c / w_c \\ z_c / w_c \end{pmatrix}$$

clip  
coordinates

perspective  
division

# Vizualizare 3D OpenGL

window  
coordinates

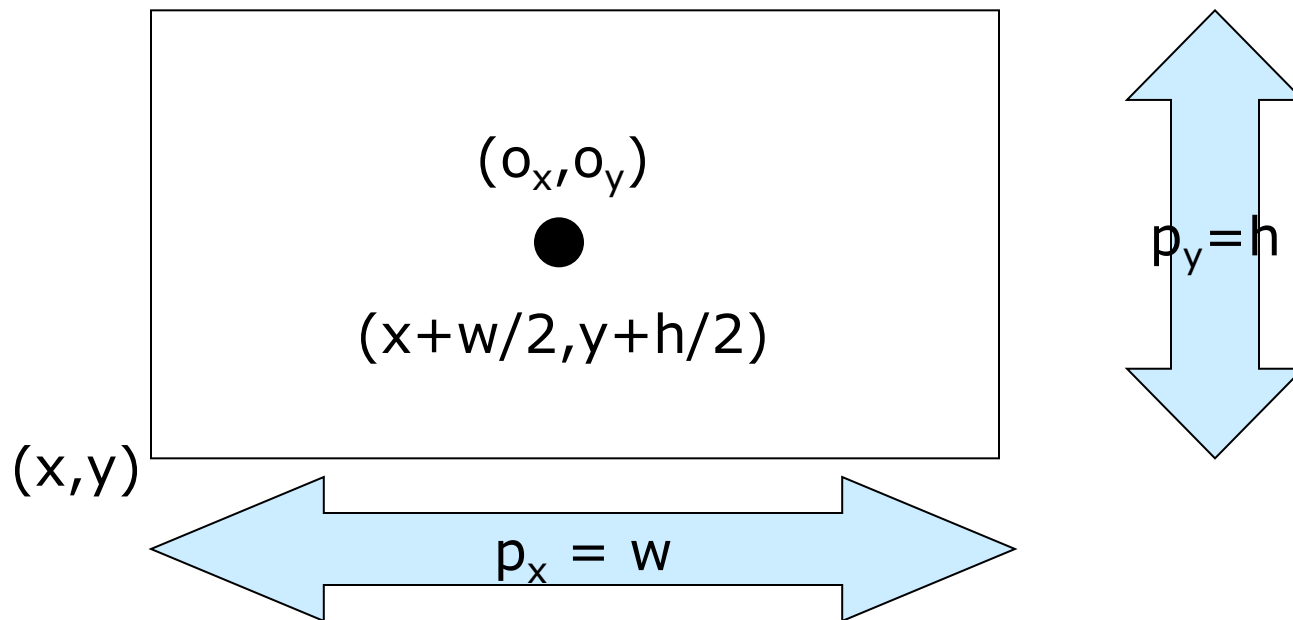
normalized device  
coordinates

$$\begin{pmatrix} x_w \\ y_w \\ z_w \end{pmatrix} = \begin{pmatrix} o_x + (p_x / 2)x_d \\ o_y + (p_y / 2)y_d \\ [(f - n) / 2]z_d + (n + f) / 2 \end{pmatrix}$$

viewport  
transformation

# Vizualizare 3D OpenGL

- `glDepthRange(n, f)`
- `glViewport(x, y, w, h)`



# Vizualizare 3D OpenGL

- Matricile de modelare/proiectie
  - selectare stiva; matricea curenta C este cea din varful stivei
    - `glMatrixMode(GL_MODELVIEW)`
    - `glMatrixMode(GL_PROJECTION)`
  - modificarea matricii curente
    - `glLoadMatrix{fd}(T m[16])`
      - este specificata o matrice 4x4 data pe coloane si care inlocuieste matricea curenta C

# Vizualizare 3D OpenGL

- Matricile de modelare/proiectie
  - modificarea matricii curente
    - `glMultMatrix{fd}(T m[16])`
      - este specificata o matrice M de 4x4 data pe coloane
      - $C' = C \times M$
      - C' inlocuieste matricea curenta C
    - variante
      - `glLoadTransposeMatrix(...)`
      - `glMultTransposeMatrix(...)`

# Vizualizare 3D OpenGL

- Matricile de modelare/proiectie
  - modificarea matricii curente
    - `glLoadIdentity()`
      - matricea  $I_4$  inlocuieste matricea curenta  $C$
    - `glRotate{fd}(T t, T x, T y, T z)`
      - calculeaza  $M$  matricea de rotatie cu unghiul  $t$  in jurul vectorului  $(x,y,z)$
      - inlocuieste  $C$  cu  $C' = C \times M$

# Vizualizare 3D OpenGL

- Matricile de modelare/proiectie
  - `glTranslate{fd}(T x, T y, T z)`
    - calculeaza M matricea de translatie  $T(x,y,z)$
    - inlocuieste C cu  $C' = C \times M$
  - `glScale{fd}(T x, T y, T z)`
    - calculeaza M matricea de scalare  $S(x,y,z)$
    - inlocuieste C cu  $C' = C \times M$

# Vizualizare 3D OpenGL

- proiectie perspectiva

- `glFrustum(l, r, b, t, n, f)`

- coordonatele  $(l, b, -n)$  si  $(r, t, -n)$  specifica 2 puncte din planul de decupare anterior care sunt mapate in colturile din stanga jos si dreapta sus ale ferestrei (presupunand ca observatorul este plasat in punctul  $(0,0,0)$ ). Parametrul  $f$  denota distanta dintre observator si planul de decupare posterior. Apar erori in unul din cazurile :  $n, f \leq 0, l = r, b = t, n = f$ .
    - se calculeaza  $M$  matricea corespunzatoare
    - inlocuieste  $C$  cu  $C' = C \times M$



# Vizualizare 3D OpenGL

- proiectie paralela

- `glOrtho(l, r, b, t, n, f)`

- coordonatele  $(l, b, -n)$  si  $(r, t, -n)$  specifica 2 puncte din planul de decupare anterior care sunt mapate in colturile din stanga jos si dreapta sus ale ferestrei (presupunand ca observatorul este plasat in punctul  $(0,0,0)$ ). Parametrul  $f$  denota distanta dintre observator si planul de decupare posterior. Apar erori in unul din cazurile :  $l = r, b = t, n = f$ .
    - se calculeaza  $M$  matricea corespunzatoare
    - inlocuieste  $C$  cu  $C' = C \times M$

# Vizualizare 3D OpenGL

- Stivele de matrici

- `glPushMatrix()`

- daca stiva  $S = CX$  atunci dupa executia comenzii  $S = CCX$

- eroare `GL_STACKOVERFLOW`

- `glPopMatrix()`

- daca stiva  $S = C'CX$  atunci dupa executia comenzii  $S = CX$

- eroare `GL_STACKUNDERFLOW`

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  - OpenGL