Problem Statement 1:

A test is conducted which is consisting of 20 MCQs (multiple choices questions) with every MCQ having its four options out of which only one is correct. Determine the probability that a person undertaking that test has answered exactly 5 questions wrong.

Solution 1: Let X be the random variable representing number of questions answered wrong in a given MCQs test

$X \sim Bin(20, 0.75)$, therefore the probability is given by

$$P(X = 5) = \binom{n}{x} * p^x * (1-p)^{n-x}$$

$$= \binom{20}{5} * 0.75^5 * (1-0.75)^{20-5}$$

$$= 15504 * 0.2373 * (0.25)^{15} = 0.00000342$$

Problem Statement 2:

A die marked A to E is rolled 50 times. Find the probability of getting a "D" exactly 5 times.

Solution 2: Let X be the random variable representing number of questions answered wrong in a given MCQs test

$X \sim Bin(50, 0.2)$, therefore since n is large, the distribution is approximated to the normal distribution $X \sim N(10, 8)$, (*with continuity correction*)

$$P(X = 5) = P\left(Z = \frac{x-\mu}{\sigma}\right)$$

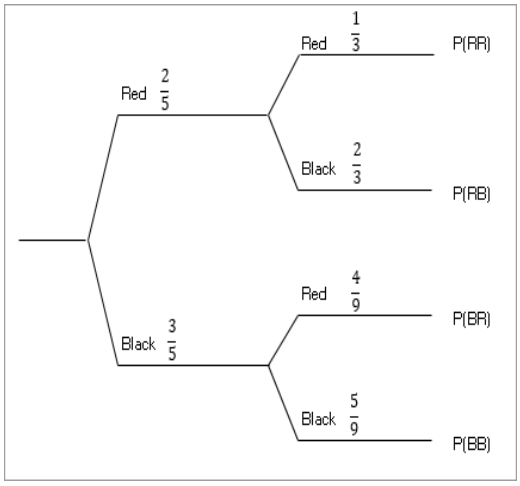
$$= P\left(\frac{4.5-10}{\sqrt{8}} \leq Z \leq \frac{5.5-10}{\sqrt{8}}\right) = 0.0299$$

Problem Statement 3:

Two balls are drawn at random in succession without replacement from an urn containing 4 red balls and 6 black balls. Find the probabilities of all the possible outcomes.

Solution 3:

These are independent events, we can come up with a tree diagram to represent the scenario



Outcome 1: $P(two\ red\ balls) = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$

Outcome 2: $P(red\ ball\ first\ and\ black\ ball\ second)$

$$= \frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$$

Outcome 3: $P(black\ ball\ first\ and\ red\ ball\ second)$

$$= \frac{3}{5} \times \frac{4}{9} = \frac{4}{15}$$

Outcome 4: $P(two\ black\ balls) = \frac{3}{5} \times \frac{5}{9} = \frac{1}{3}$