DISCRETE

Book: "Kenneth Rosen"

1. Logic
2. Combinatorics
3. Set Theory: Relations, functions, cryptography. (Posets, Lattices & BA)
4. Graph Theory.

Logic or Propositional Logic

1. Theory Proposition? or and Nor
2. Logical connectives : $\vee, \wedge, \sim, \Rightarrow, \Leftrightarrow, \oplus, \uparrow, \downarrow$
   (operators)
   - i) Truth table.
   - ii) Properties of $\wedge, \vee, \sim \Rightarrow$ Boolean algebra.
     $(+)\ (\cdot)\ (')$
   - (iii) Functional completeness.
3. Eg $\Rightarrow$ $\{\wedge, \vee, \sim\}$ $\Rightarrow$ func. comp.
   $\{\vee\}$ $\Rightarrow$ not fun/comp
   - (iv) minimum function complete
   - (v) Simplifications.

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3. Tautology, contradiction, contingency. {0 or 1 means may true or false}
= 1 {all values are 1} = 0. {all values are 0}

Satisfied, {may be Tautology or contigency}
unsatisfied {contradiction}

i) Theorems.

4. Implication ($\Rightarrow$), Bycondition ($\Leftrightarrow$), Direct, converse, Inverse, CP

i) Forms of $\Rightarrow$ & $\Leftrightarrow$
ii) Properties

5. Arguments and rules of Inference. : i) checking validity of argument.
ii) Premises $\Rightarrow$ conclusion : valid : ?
(iii) Rules of Inference. {9 standard rules}
+
(iv) Venn diagram. Predicate argument. 4 = 13.

6. Predicate logic:
i) Given a predicate $\rightarrow$ True or false ?

$\boxed{\forall \quad \exists}$ $\rightarrow$ quantifiers.

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7. Properties of predicate logic
   i) Properties of θ, $\exists$, $\forall$

8. {English to logic
   {. logic to English.

①

Proposition: Something which is True or False but not both

any statement end with ?, ! are not logical sentences.

This statement is false.: Negative self recursion.
{ logical setence}

Today is monday : Not logical because this statement is not valid for other days.

Note: If truth value changes regularly, then sentence is not logical sentence.

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10 is even → logical
11 is even → logical
x is even → Not logical ⇏ proposition.
↓
value of x is not specified. ⟹ Proposition function (predicate)

$P(x)$ = even {Predicate}

Predicate → $P(x)$ → Proposition function.
{Not a proposition}

Note: In most cases 'is' represent the sentence is proposition.

If we can say T or F then surely it is logical sentence.

② $\overset{+}{\vee}$, $\overset{\cdot}{\wedge}$, $\overset{\prime}{\sim}$ or $p \to \neg p$, $\Rightarrow$ $p \to q$, $\Leftrightarrow$ $p \leftrightarrow q$, $p \rightleftharpoons q$, $\oplus$ $p \veebar q$, $\uparrow$ → Nand, $\downarrow$ → ~~And~~ · Nor

⇓
Truth values are same

P or q $\longrightarrow$ Inclusive

P or q but not bot $\longrightarrow$ Exclusive.

p: 10 is even
q: 5+3 is odd

$P \vee q$: 10 is even $\vee$ 8 is odd
T $\vee$ F
$\Rightarrow$ T

<u>Precedence order</u>

$() \geq {}' \geq \wedge \geq \vee \geq \Rightarrow \geq \Leftrightarrow$

Eg. $\underbrace{\Big(\big(\overbrace{(P \vee q)}^{3} \Rightarrow \overbrace{(r \wedge s)}^{2}\big)^{4} \Leftrightarrow \overbrace{(t')}^{1}\Big)}_{5}$

<u>Truth Table</u>

p	q	$P \vee q$	$P . q$	$P \Rightarrow q$	$P \Leftrightarrow q$	$P \oplus q$
0	0	0	0	1	1	0
0	1	1	0	1	0	1
1	0	1	0	$\boxed{0}$	0	1
1	1	1	1	1	1	0

P	P'	
$x > 0$	$x \leq 0$	→ compliment
T	F	
F	F	⇒ It is not compliment

when $x = 0$

p	q	$p \uparrow q$	$p \downarrow q$
0	0	1	1
0	1	1	0
1	0	1	0
1	1	0	0

complement → opposite but not complete opposite.

raining	⇒	umbrella	
1		0	⇒ 0
0	⇒	1	→ 1
0	⇒	0	→ 1
1	⇒	1	→ 1

Properties of Boolean algebra ($\vee, \wedge, \sim$)

1. commutative : 6 closure
2. Associative ⇒ $p+(q+r) = (p+q)+r$
   $p\cdot(q\cdot r) = (p\cdot q)\cdot r$
3. Distributive → $p\cdot(q+r) = p\cdot q + p\cdot r$
   $p + q\cdot r = (p+q)(p+r)$
4. Identity
5. compliment

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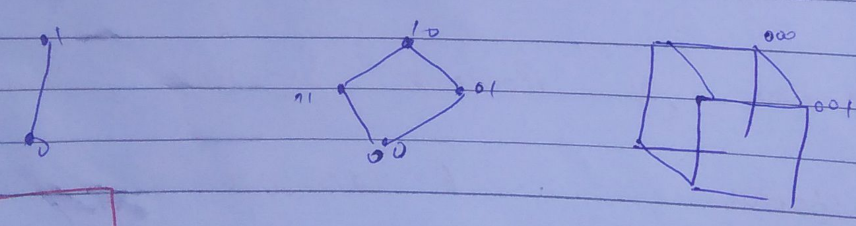
1 object
2 binary operator
1 unary operator
Satisfy properties. } Boolean algebra.

If no of elements are in 2^n eg $\{2, 4, 8, 16 \dots\}$ then algebra may or may not finite boolean algebra.

But if no. of elements are not in 2^n, then surely algebra is not boolean algebra.

If no. of element

2   4   8



$$\boxed{\begin{array}{l} P, q \in S \\ p + q \in S \\ p \cdot q \in S \\ p' \; q' \in S \end{array}} \longrightarrow \text{Boolean algebra.}$$

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$A, B \in S$
$A \cup B \in S$
$A \cap B \in S$
$A^c \longrightarrow \in S$
$B^c \longrightarrow S$

$\Longrightarrow$ Boolean algebra.

Set Theory Properties

Commutative

$p \cup q = q \cup p$, $p \wedge q = q \wedge p$

Associative

$A \cup (B \cup C) = (A \cup B) \cup C$

Distributive

$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

Note: Both Set theory and Boolean

Identity : $\exists 0 \;\; \forall p \;\; p+0 = p = 0+p$

$\exists 1 \;\; \forall p \;\; p.1 = p = 1.p$

{0, 1 should be unique.}

$\boxed{A \cup \phi = \phi \cup A}$ or $\boxed{A \cup A = A \cup A}$

It is correct but we cannot consider it because A is not unique.

Compliment : $\forall p \;\; \exists p' \;\; p + p' = 1$

$p . p' = 0$

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1. Double compement : $\forall_p \ (p')' = p$ or $p' = q \Rightarrow q' = p$

2. Demorgan's $\rightarrow (p+q)' = p'.q'$, $(p.q)' = p' + q'$

3 Domination $\longrightarrow$ $p.0 = 0 = 0.p$
$p+1 = 1 = 1+p$

4. IDempotent $\longrightarrow$ $p+p = p$
$p.p = p \rightarrow$ No power.

5. Law of absorption

IDempotent

$p^3 + 2p^2 + p$

$\Rightarrow p^2 + 2p + p$

$\Rightarrow p + p + p$

$= p$

So 'p' is largest element we can make.

Law of absorption

$\underbrace{p + (p.q)}_{\text{p in common}} = p$ (Something should be common)

$p.(q+p) = p$

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6.

$p + p'q = p + q$ {remove p'}

$p + pq = p$ {remove whole term}

$\underline{p + q'pr} + qp'r = p + qp'r$
$= p + qr$

** <u>Conversions</u>

1) $p + q \longrightarrow q' \Longrightarrow p$
$\quad\quad\quad\ \longrightarrow p' \Longrightarrow q$

2) $p \Longrightarrow q \equiv p' + q$

~~$P \oplus q \equiv$~~

3) $p \Longleftrightarrow q \equiv pq + p'q'$

4) $p \oplus q \equiv pq' + p'q$ { $p \oplus q \equiv (p \Leftrightarrow q)'$
$= (pq + p'q')'$
$= (pq)' \cdot (p'q')'$
$= (p'+q') \cdot (p+q)$
$= p'q + p'q$

5) $p \uparrow q \equiv (p \wedge q)' = p' + q'$

6) $p \downarrow q \equiv (p \vee q)' = p' q'$

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Simplification

1.

$(p \Longleftrightarrow q) \wedge (q \Longrightarrow r) \Longrightarrow (r \Longrightarrow p)$

$(pq + p'q') \wedge (q' + r) \Longrightarrow (r' + p)$

$(pq + p'q') \wedge ((q' + r)' + (r' + p))$

$[(p' + q)(q' + r)]' + r' + p$

$\Rightarrow \quad pq' + qr' + r' + p$

$\Rightarrow \quad p + \cancel{q} + r' \quad \Longrightarrow$ contigency.

2. I $p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

p' $(q \wedge r)$

II $p \Rightarrow (q \wedge r) \Rrightarrow (p \Rightarrow q) \wedge (p \Rightarrow r)$

An I $p' + q + r$ $\qquad (p' + q) + p' + r$

$\Rightarrow \quad p' + q + r = p' + q + r$

LHS $\qquad$ RHS

II $p' + qr$ $\qquad (p' + q) \wedge (p' + r)$

$\Rightarrow \quad p' + qr$

LHS = RHS

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Hence both statements are valid

Functionally complete: Every exp should be converted into another exp using the $\{\vee, \wedge, \neg\}$ set.

$\{\vee, \wedge, \neg\} \Rightarrow$ FC ✓, MFC ✗ {removing any operators, should not be FC}

$\{\vee, \wedge, \Rightarrow\} \not\Rightarrow$ FC ✗, MFC ✗ (minimum)

$\{\vee, \neg\} \Rightarrow$ FC ✓, MFC ✓

$\{\vee, \wedge\} \not\equiv$ Because we cannot get $\neg$ using $\vee, \wedge$

$\{\wedge, \neg\} \equiv$ FC ✓, MFC ✓

$\{\Rightarrow, \neg\} \equiv$ FC ✓, MFC ✓

$\{\uparrow\} \equiv$ FC ✓, MFC ✓, SMFC ✓ $\{\neg p = p \uparrow p\}$

$\{\downarrow\} \equiv$ smallest FC ✓, MFC ✓, SMFC ✓ $\{p \wedge q \equiv (p \uparrow q) \uparrow (p \uparrow q)$, $p \vee q = (p \uparrow p) \uparrow (q \uparrow q)\}$

{ ∨ } ≡ → FC ✗
→ MFC ✓

{ ∩ } → FC ✗

{ ч } → FC ✗

Theorem :

1. T ⟹ SAT. (Tautology)
2. CT ⟹ SAT (contingency)

{ Sat means there is one row which is 1 }

3. SAT ⟹ T or CT

4. C ⟺ USAT (contradiction)

$$\boxed{\begin{array}{l} \text{SAT} \Longleftrightarrow \text{T} \quad ✗ \\ \text{CT} \Longleftrightarrow \text{SAT} \quad ✗ \end{array}}$$

5. P is sat & P' is also Sat
⟹ P is CT.

P is sat & P' is unsat
⟹ P is C

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Note:

$T' = C$
$C' = T$
$CT' = CT$
$(SAT)' =$ may or may not unsat
$(unsat)' =$ Sat

Normal form

1) PDNF (Principle Disjunctive Normal Form)

2) PCNF (Principle conjuctive Normal Form)

Any function whose range is 0, 1 is called boolean function or well formed formular (WFF)

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PDNF & PCNF

$X \equiv Y$ iff $PDNF(X) = PDNF(Y)$

$\Updownarrow$

iff $PCNF(X) = PCNF(Y)$

Que

$P \Rightarrow (q' \Leftrightarrow r)$ what is PDNF?

Disjunction → means +
Conjunction → means ·

PDNF: (Product of literals) + (Product of literals) + (--)
(all literals (p, q, r) should be present)

PCNF = (Sum of literals) · (Sum of literals) · (--)

$$\boxed{\begin{array}{l} P \cdot q' + (r + s') \rightarrow DNF \\ P \cdot q' + r \cdot q + s \cdot p \rightarrow PDNF \end{array}}$$

$\boxed{\text{PDNF} \rightarrow \text{minterm Summation}}$

$\boxed{\text{PCNF} \rightarrow \text{maxterm product}}$

Ans

p	q	r	$q' \Rightarrow r$	$p \Rightarrow (q' \Leftrightarrow r)$
0	0	0	0	1
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

q'	r	
1	0	0
0	1	1

PDNF → add all min terms corresponding to 1

$= p'q'r' + p'q'r + p'qr' + p'qr + pq'r + pqr'$

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PCNF → all max terms.

= $pq'r' + pqr$

Properties of Normal form

1. For a given exp^n X, PDNF, CNF are unique

2. $X \equiv Y$ iff PDNF(X) = PDNF(Y)

3. PDNF in n variables has x max terms.
$\rightarrow 2^n - x$

no(PDNA + no(PCNF) = 2^n { n → no. of variables }

4. In n variables. maxterm → 2^n
minterms 2^n

5. No of distinct boolean functions = 2^{2^n}

6. $P(\text{Tautology}) = \dfrac{1}{2^{2^n}}$

$P(\text{comp}) = 1/2^{2^n}$
~~Contingency~~ CT = $2^{2^n} - 2$.

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Note only ∃ move forward and ∀ move backward.

\# Tautolog = 1
\# contra = 1
\# CT = $2^{2^n} - 2$

7. For a given ASSIGNMENT of T. table values of p, q, r etc (n variables)
How many max terms will evaluate to 1
= 1
\# min terms evaluate to 0 = $2^n - 1$
\# max terms evaluate to 0 = 1
\# max terms evaluate to 1 = $2^n - 1$

Implication, Bi condition

→ Antecedent
$P \Rightarrow q$ → consequent

1. P implies q	$P \Leftrightarrow q$
2. If P, then q	P biconditional to q
3. q, if P. {$q \Leftarrow p$}	P if and only if q
4. q follows from P.	P iff q
5. From p follow q	
6. If & only if P, q	P is necessary and sufficient for q
$P \Rightarrow q$	
7. P is sufficient for q	Eg p is nec for q & p is suff for q
$P \Rightarrow q$	$(q \Rightarrow p) \wedge (p \Rightarrow q)$
	Being connected and Acyclic is

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8. q is necessary for p

$q \Longleftarrow p$

necessary and sufficient for q
CAA ⇐ True

Q. p is neccessary but not sufficient for q

A. P is neccessary for q but P is not sufficient for q

$P \Longrightarrow q$

**
when ≡ if
but ≡ and
whenever ≡ if
neverthless ≡ and
unless ≡ OR

Q. Lions & tigers attack if hunger

A. Hungry ⇒ Attack.