Investigation of SAEAs' metamodel samples for computationally expensive optimization problems

Supplementary material

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1 Introduction

This document contains a supplementary material related to the paper "Investigation of SAEAs' metamodel samples for computationally expensive optimization problems", in which it is performed an investigative study to compare five different strategies to define the metamodel sample in a SAEA Framework (SAEA/F). Each strategy (S1, S2, S3, S4 and S5) was incorporated into the SAEA/F, which was used to solve a set of analytical functions of single-objective optimization problems presented in Table 1. The Table 2 and Figure 1 show results related to each dimension $n \in \{2, 5, 10, 15, 20\}$. Table 2 presents mean values and standard deviation of the objective function regarding the best solution found. The results in this table provide an estimate to the accuracy of the metamodel. Figure 1 shows the convergence curves obtained by the mean improvement over the best solution from the initial population in function to the percentage of the budget used for function evaluations.

Table 1: Analytic functions used in the computational experiment.

Function	**Characteristics**	**Definition**
Ackley	*Multimodal*	$y_1(\mathbf{x}) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + \exp(1)$ $x_i \in [-32.768, 32.768]$
Dixon-Price	*Multimodal (valley-shaped)*	$y_2(\mathbf{x}) = (x_1 - 1)^2 + \sum_{i=1}^{n} i(2x_i^2 - x_{i-1})^2$ $x_i \in [-10, 10]$
Ellipsoid	*Unimodal*	$y_3(\mathbf{x}) = \sum_{i=1}^{n} i \cdot x_i^2$ $x_i \in [-5.12, 5.12]$
Griewank	*Multimodal*	$y_4(\mathbf{x}) = 1 + \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)$ $x_i \in [-600, 600]$
Levy	*Multimodal*	$y_5(\mathbf{x}) = \sin^2(\pi w_i) + \sum_{i=1}^{n-1}(w_i - 1)^2[1 + 10\sin^2(\pi w_i + 1)] + (w_n - 1)^2[1 + \sin^2(2\pi w_n)]$ $x_i \in [-10, 10]$, $w_i = 1 + (x_i - 1)/4$, $i = i, \ldots, n$
Rastrigin	*Multimodal*	$y_6(\mathbf{x}) = 10n + \sum_{i=1}^{n}[x_i^2 - 10\cos(2\pi x_i)]$ $x_i \in [-5.12, 5.12]$
Rosenbrock	*Multimodal (narrow valley)*	$y_7(\mathbf{x}) = \sum_{i=1}^{n}[100(x_{i+1} - x_i^2)^2 + (1 - x_i^2)^2]$ $x_i \in [-2.048, 2.048]$
Styblinski-Tang	*Multimodal*	$y_8(\mathbf{x}) = \frac{1}{2}\sum_{i=1}^{n}(x_i^4 - 16x_i^2 + 5x_i)$ $x_i \in [-5, 5]$
Zakharov	*Multimodal (plate-shaped)*	$y_9(\mathbf{x}) = \sum_{i=1}^{n} x_i^2 + \left(\sum_{i=1}^{n} 0.5ix_i\right)^2 + \left(\sum_{i=1}^{n} 0.5ix_i\right)^4$ $x_i \in [-5, 10]$

Table 2: Mean and standard deviation of the value of objective function of the best solution returned by the SAEA/F with each strategy over all test functions and number of variables.

	S1		S2		S3		S4		S5	
n	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Ackley										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3628	0.7562	0.0000	0.0000
10	0.0578	0.2583	0.1157	0.5176	0.1155	0.3555	0.8350	0.9105	0.0578	0.2583
15	0.2011	0.4911	0.0949	0.2905	0.6069	0.7441	2.1377	1.7248	0.2023	0.3805
20	0.0000	0.0000	0.0413	0.1735	0.0000	0.0000	0.3624	0.5654	0.0000	0.0000
Dixon-Price										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000
5	0.0337	0.1490	0.0894	0.2215	0.0670	0.2051	0.3031	0.3161	0.0000	0.0000
10	0.6667	0.0000	0.6933	0.0642	0.6687	0.0090	6.1594	20.2895	0.6001	0.2052
15	0.9298	0.4992	1.9161	3.9310	0.8989	0.4015	5.2942	10.4357	1.3914	2.7785
20	0.8459	0.2969	2.3633	5.1687	0.7556	0.2049	3.5767	5.7895	0.6890	0.0807
Ellipsoid										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0013	0.0053	0.0000	0.0000
15	0.0005	0.0020	0.0005	0.0019	0.0359	0.1595	0.2609	1.1474	0.0004	0.0015
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0087	0.0209	0.0000	0.0000
Griewank										
2	0.0094	0.0073	0.0322	0.0164	0.0096	0.0070	0.0381	0.0321	0.0044	0.0045
5	0.0658	0.0342	0.0897	0.0611	0.0704	0.0394	0.1200	0.0707	0.0658	0.0241
10	0.0383	0.0255	0.0364	0.0298	0.0676	0.0570	0.0827	0.0493	0.0417	0.0267
15	0.0292	0.0252	0.0296	0.0423	0.0379	0.0577	0.0783	0.1047	0.0304	0.0506
20	0.0099	0.0111	0.0076	0.0060	0.0072	0.0129	0.0163	0.0127	0.0082	0.0107
Levy										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2288	0.5658	0.0000	0.0000
10	0.0361	0.1037	0.0361	0.1037	0.0134	0.0328	0.8612	1.6802	0.0272	0.1025
15	0.1402	0.3082	0.7554	0.7978	0.3569	0.6764	1.1044	0.8479	0.1226	0.1754
20	0.1133	0.2497	0.2358	0.3013	0.1449	0.2403	1.1808	1.7168	0.0861	0.2215
Rastrigin										
2	0.0041	0.0103	0.0015	0.0033	0.0006	0.0017	0.5472	0.6018	0.0000	0.0000
5	1.9402	1.8110	4.1291	2.9809	2.4376	1.7226	4.4773	2.4900	3.3829	1.9741
10	11.6908	6.1999	13.0678	6.1244	10.5119	4.3065	11.8995	3.7340	9.1600	3.3185
15	19.8868	8.7546	23.6615	10.4629	19.5420	5.2859	24.5625	10.8525	18.0880	5.9917
20	21.7914	7.1368	30.2937	15.0465	24.0986	16.9873	36.4161	17.1783	23.2262	9.2057
Rosenbrock										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0310	0.0607	0.0000	0.0000
5	0.2169	0.4576	0.5136	1.1364	0.0013	0.0013	2.1565	1.4777	0.0004	0.0010
10	7.8640	3.9833	8.9284	5.8097	6.9615	0.8459	12.1593	13.4468	4.7810	2.2030
15	13.0383	3.8129	17.2529	11.7964	13.7343	3.1542	21.0908	17.5326	12.6438	1.6122
20	17.5239	1.1078	20.1859	11.8879	17.2072	1.5400	17.9645	1.4129	17.4264	1.3101
Styblinski-Tang										
2	0.0004	0.0000	0.0004	0.0000	0.0004	0.0000	0.0004	0.0000	0.0004	0.0000
5	1.4128	4.3512	2.1196	5.1790	1.4128	4.3512	7.7743	8.5500	0.0009	0.0000
10	29.6854	17.0999	36.7537	18.5741	28.9786	11.6710	30.3991	16.7049	22.6515	12.4530
15	50.2482	17.4350	50.9234	24.8786	42.5931	20.4015	53.5489	23.6295	41.0455	15.8099
20	61.4912	26.0567	70.6977	23.3832	69.9732	19.1732	69.2664	29.3325	51.5956	22.5978
Zakharov										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0045	0.0129	0.0000	0.0000
10	4.1863	3.5674	5.7906	10.5196	1.5220	6.5757	0.6039	1.2889	0.4376	0.6401
15	8.7745	8.2888	8.7970	10.9481	5.8560	9.4002	5.6822	6.9749	6.4023	4.9395
20	75.0373	23.7355	26.0338	12.2657	62.1064	29.0018	60.9728	18.8453	64.7967	22.8925

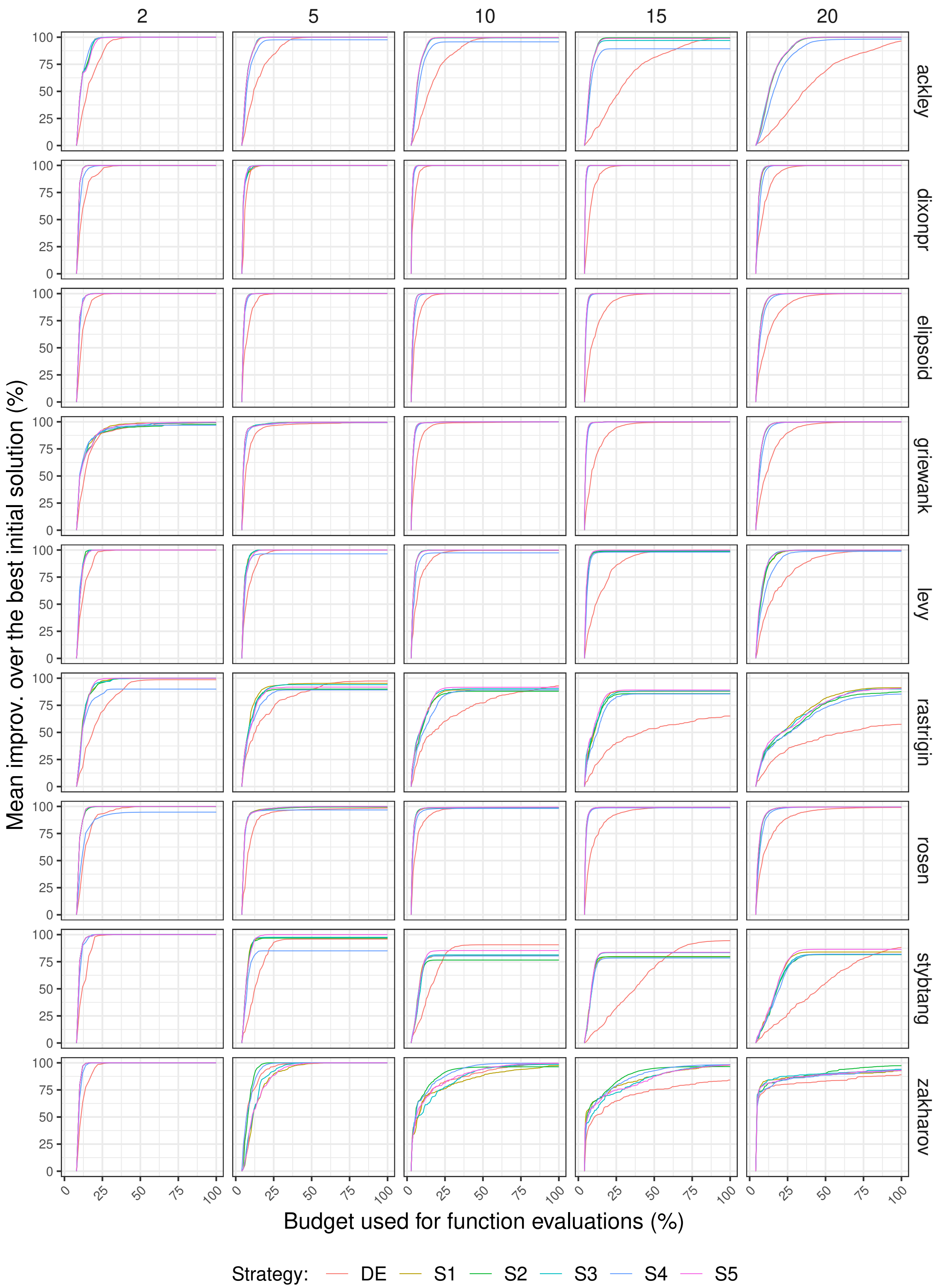

Figure 1: Convergence curves calculated as the mean improvement over the best solution of the initial population in function of the percentage of the budget used for function evaluations. Plots are discretized by function (vertical) and number of variables (horizontal).