Investigation of SAEAs' metamodel samples for computationally expensive optimization problems

Supplementary material

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1 Introduction

This document contains a supplementary material related to the paper "Investigation of SAEAs' metamodel samples for computationally expensive optimization problems", in which it is performed an investigative study to compare five different strategies to define the metamodel sample in a SAEA Framework (SAEA/F). Each strategy (S1, S2, S3, S4 and S5) was incorporated into the SAEA/F, which was used to solve a set of analytical functions of single-objective optimization problems presented in Table 1. The Table 2 and Figure 1 show results related to each dimension $n \in \{2, 5, 10, 15, 20\}$. Table 2 presents mean values and standard deviation of the objective function regarding the best solution found. The results in this table provide an estimate to the accuracy of the metamodel. Figure 1 shows the convergence curves obtained by the mean improvement over the best solution from the initial population in function to the percentage of the budget used for function evaluations.

Table 1: Analytic functions used in the computational experiment.

Function	Characteristics	Definition
Ackley	*Multimodal*	$\mathrm{y}_1(\mathbf{x}) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right)$ $+20+\exp(1)$ $x_i \in [-32.768, 32.768]$
Dixon-Price	*Multimodal (valley-shaped)*	$\mathrm{y}_2(\mathbf{x}) = (x_1-1)^2 + \sum_{i=1}^{n} i(2x_i^2 - x_{i-1})^2$ $x_i \in [-10, 10]$
Ellipsoid	*Unimodal*	$\mathrm{y}_3(\mathbf{x}) = \sum_{i=1}^{n} i \cdot x_i^2$ $x_i \in [-5.12, 5.12]$
Griewank	*Multimodal*	$\mathrm{y}_4(\mathbf{x}) = 1 + \sum_{i=1}^{n}\frac{x_i^2}{4000} - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)$ $x_i \in [-600, 600]$
Levy	*Multimodal*	$\mathrm{y}_5(\mathbf{x}) = \sin^2(\pi w_i) + \sum_{i=1}^{n-1}(w_i-1)^2[1+10\sin^2(\pi w_i+1)]$ $+(w_n-1)^2[1+\sin^2(2\pi w_n)]$ $x_i \in [-10, 10]$, $w_i = 1 + (x_i-1)/4$, $i = i, \ldots, n$
Rastrigin	*Multimodal*	$\mathrm{y}_6(\mathbf{x}) = 10n + \sum_{i=1}^{n}[x_i^2 - 10\cos(2\pi x_i)]$ $x_i \in [-5.12, 5.12]$
Rosenbrock	*Multimodal (narrow valley)*	$\mathrm{y}_7(\mathbf{x}) = \sum_{i=1}^{n}[100(x_{i+1} - x_i^2)^2 + (1-x_i^2)^2]$ $x_i \in [-2.048, 2.048]$
Styblinski-Tang	*Multimodal*	$\mathrm{y}_8(\mathbf{x}) = \frac{1}{2}\sum_{i=1}^{n}(x_i^4 - 16x_i^2 + 5x_i)$ $x_i \in [-5, 5]$
Zakharov	*Multimodal (plate-shaped)*	$\mathrm{y}_9(\mathbf{x}) = \sum_{i=1}^{n} x_i^2 + \left(\sum_{i=1}^{n} 0.5ix_i\right)^2 + \left(\sum_{i=1}^{n} 0.5ix_i\right)^4$ $x_i \in [-5, 10]$

Table 2: Mean and standard deviation of the value of objective function of the best solution returned by the SAEA/F with each strategy over all test functions and number of variables.

	S1		S2		S3		S4		S5	
n	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Ackley										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0076	0.0087	0.0062	0.0056	0.0000	0.0000	2.4133	3.4438	0.0000	0.0000
10	0.0007	0.0016	0.0000	0.0000	0.0000	0.0000	2.6159	1.4369	0.0000	0.0000
15	0.0000	0.0000	1.2157	1.2920	0.2681	0.5995	4.2353	2.7920	0.1863	0.4165
20	0.0004	0.0002	0.2313	0.5165	0.2312	0.5165	0.6078	0.7555	0.0002	0.0001
Dixon-Price										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0080	0.0127	0.1350	0.2972	0.0033	0.0039	0.0493	0.1037	0.0000	0.0000
10	0.6670	0.0004	0.6667	0.0000	0.6667	0.0001	12.1250	15.2769	0.6669	0.0003
15	0.6864	0.0442	7.6668	15.4835	2.0154	3.0160	32.0741	40.2240	0.6779	0.0252
20	0.6729	0.0136	0.7385	0.1607	0.8064	0.2650	1.4871	1.1184	0.9172	0.3535
Ellipsoid										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0004	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0041	0.0057	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0003	0.0003	0.0000	0.0000
Griewank										
2	0.0216	0.0173	0.0099	0.0080	0.0206	0.0146	0.1114	0.0507	0.0015	0.0033
5	0.0555	0.0250	0.0641	0.0496	0.0730	0.0214	0.1685	0.0991	0.0286	0.0136
10	0.0418	0.0130	0.0506	0.0500	0.0374	0.0119	0.1978	0.1489	0.0459	0.0141
15	0.0121	0.0128	0.0089	0.0102	0.0138	0.0112	0.0906	0.0630	0.0093	0.0121
20	0.0084	0.0130	0.0163	0.0116	0.0113	0.0075	0.1016	0.0778	0.0138	0.0150
Levy										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0909	0.2032	0.0000	0.0000
10	0.0000	0.0000	0.0194	0.0394	0.1817	0.4064	0.5082	0.3879	0.0358	0.0490
15	0.5989	0.8976	1.2721	1.5658	0.6540	1.1309	3.5720	2.4289	0.7806	0.6818
20	0.1088	0.1970	0.8343	0.6960	0.1996	0.2354	0.5089	0.4576	0.2713	0.1824
Rastrigin										
2	0.0031	0.0027	0.0059	0.0074	0.0014	0.0020	0.1990	0.4450	0.0000	0.0000
5	0.8339	0.3967	1.9923	1.2177	1.2565	0.8065	3.5819	3.8918	1.7909	0.8324
10	9.3820	2.6980	7.7607	5.7329	8.5775	4.6843	11.5965	5.4096	7.7607	4.6348
15	12.1394	3.9409	21.4959	14.6673	14.9447	2.7874	20.2977	8.6915	9.3526	3.1932
20	72.3344	14.5614	49.5358	15.8760	75.6271	12.1143	93.5123	14.4965	52.8388	35.8797
Rosenbrock										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0385	0.0807	0.0000	0.0000
5	0.4373	0.5723	0.1578	0.1013	0.3176	0.3020	2.9221	1.1900	0.0001	0.0001
10	5.8288	2.6084	17.4753	29.1538	3.4505	2.3721	8.2455	0.9336	3.4052	1.7535
15	21.9348	18.8322	25.1590	18.2390	11.6821	3.5415	29.6975	24.1109	11.9564	3.0124
20	16.5439	1.0381	17.1021	1.6884	16.7211	1.0300	16.6233	2.6515	16.6263	1.9826
Styblinski-Tang										
2	0.0004	0.0000	0.0004	0.0000	0.0004	0.0000	0.0004	0.0000	0.0004	0.0000
5	0.0007	0.0003	0.0061	0.0146	0.0003	0.0025	0.0009	0.0000	0.0009	0.0000
10	22.6178	16.1187	16.9623	18.4320	22.6170	12.6443	25.4443	11.8276	14.1350	9.9962
15	31.0981	11.8276	45.2349	18.4320	28.2708	17.3139	36.9017	29.3383	42.4075	19.9923
20	48.0613	23.6553	48.0613	21.4394	36.7520	25.6806	53.7255	27.1903	59.3707	27.1925
Zakharov										
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.0112	0.0222	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	6.3676	3.1426	0.0099	0.0208	7.2788	10.3134	0.0101	0.0161	1.2494	0.8893
15	12.6189	9.1983	2.9641	6.0064	11.4033	7.3112	2.3401	2.3815	3.2856	3.3893
20	101.2524	27.4286	75.1562	21.9232	89.0697	16.8538	82.1671	21.2687	47.3504	7.0426

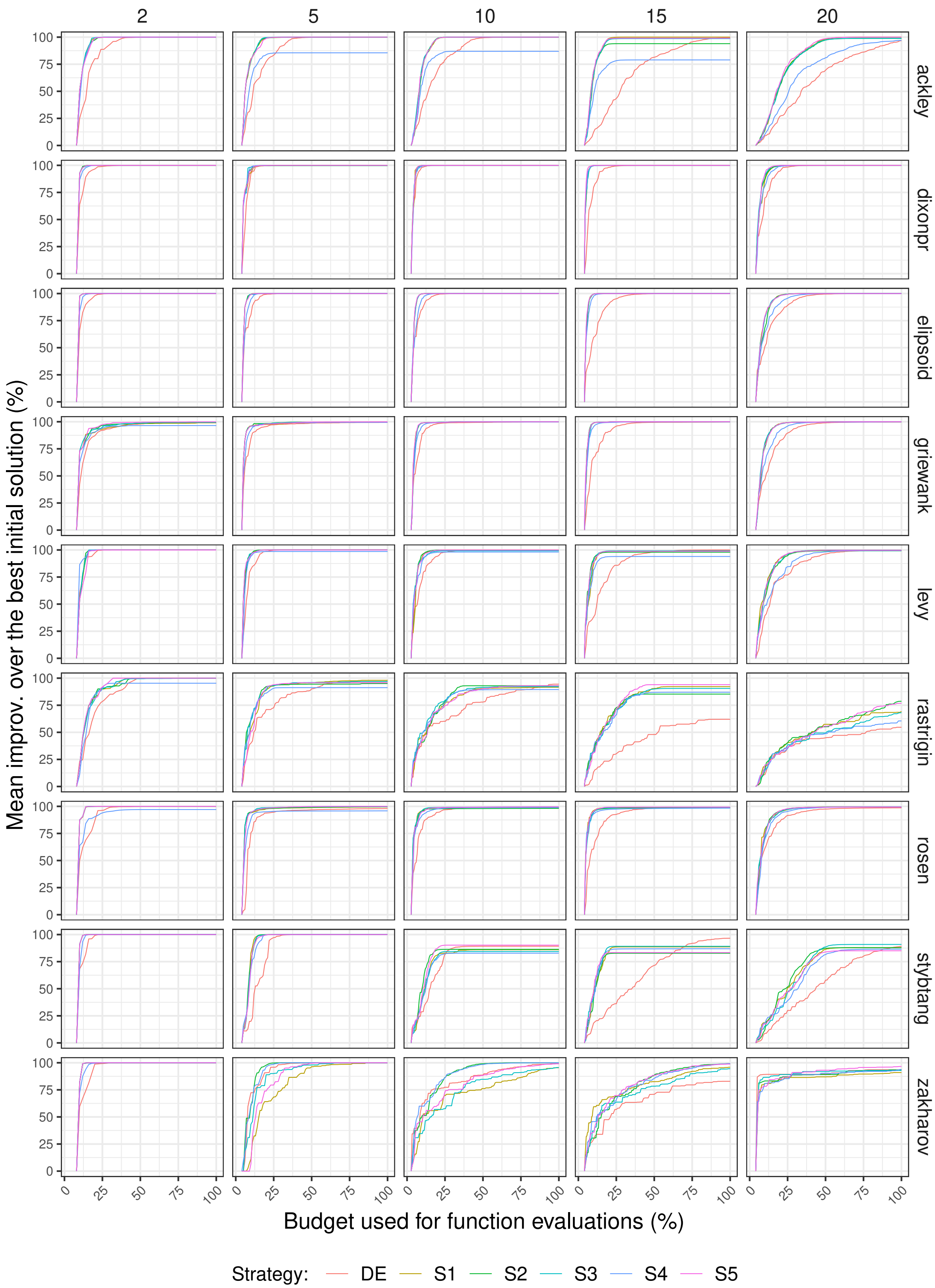

Figure 1: Convergence curves calculated as the mean improvement over the best solution of the initial population in function of the percentage of the budget used for function evaluations. Plots are discretized by function (vertical) and number of variables (horizontal).