## A comparative study on surrogate models for SAEAs

Supplementary material

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## 1 Introduction

This document contains a supplementary material related to the paper "A comparative study on surrogate models for SAEAs", in which it is performed an investigative study to compare the metamodels Ordinary Kriging(OK), First-Order Universal Kriging (UK1), Second-Order Universal Kriging (UK2), Blind Kriging (BK) and Radial Basis Function (RBF), when embedded into a single-objective SAEA Framework (SAEA/F). For each metamodel, the SAEA/F was applied in a set of analytic functions, in which was observed the time spent to build the metamodel and the quality of the final solution found by SAEA/F, according to Tables 1 and 2. Table 1 presents the mean and the standard deviation for the percentage of improvement in the objective function of the best solution of the final iteration in relation to the best solution of the initial population. Table 2 presents the mean and the standard deviation for the time spent to build the metamodel.

Table 1: Mean of the percentage improvement in the value of objective function in relation to the best solution of the initial population.

		OK		UK1		UK2		ВК		RBF	
Function	n	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std
Ackley	2	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
Ü	5	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	90.3557	5.729'
	10	100.0000	0.0000	98.7834	2.6814	98.4777	3.4041	100.0000	0.0000	64.7121	11.4439
	15	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	91.6359	4.9756
	20	100.0000	0.0000	99.9962	0.0033	99.9698	0.0169	100.0000	0.0000	88.2180	3.7804
${\it Dixon-Price}$	2	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
	5	100.0000	0.0000	99.9881	0.0194	99.9909	0.0203	99.9904	0.0216	99.9777	0.0229
	10	99.9964	0.0053	99.9964	0.0053	99.9969	0.0056	99.9964	0.0053	99.9964	0.0053
	15	99.9997	0.0002	99.9996	0.0001	99.9996	0.0001	99.9996	0.0001	99.9997	0.0002
	20	99.9999	0.0000	99.9999	0.0000	99.9998	0.0000	99.9998	0.0000	99.9998	0.000
Ellipsoid	2	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
	5	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
	10	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
	15	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
	20	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
Griewank	2	94.9644	7.3095	96.5857	4.1862	99.6189	0.5354	97.5389	3.0623	72.3436	15.7630
	5	99.8709	0.1268	99.8031	0.2487	99.9815	0.0413	99.7385	0.2157	99.4244	0.2983
	10	99.8963	0.1516	99.9365	0.0594	99.9934	0.0147	99.9478	0.0454	99.8569	0.1514
	15	99.9962	0.0056	99.9976	0.0033	100.0000	0.0000	99.9984	0.0021	99.9842	0.0081
	20	99.9989	0.0015	100.0000	0.0000	100.0000	0.0000	99.9979	0.0047	99.7667	0.0710
Levy	2	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
J	5	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	98.1697	2.3712
	10	98.5221	1.7835	98.9473	1.5839	99.6594	0.7616	99.7415	0.5781	97.8159	1.1211
	15	97.0530	3.8378	100.0000	0.0000	100.0000	0.0000	99.4440	1.1431	98.3587	1.0275
	20	99.5091	0.1518	99.9833	0.0373	99.9742	0.0360	99.9154	0.1892	97.9579	1.4840
Perm $n, \beta$	2	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
	5	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
	10	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
	15	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
	20	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
Rastrigin	2	96.7601	7.2446	96.7601	7.2446	95.4043	10.2762	93.2719	10.1900	100.0000	0.0000
<b>y</b>	5	89.8688	4.3596	85.3022	7.0548	92.5885	2.1198	90.0934	6.1343	91.8485	4.3435
	10	91.5603	4.1201	91.0286	6.2375	74.6925	5.6449	93.0592	2.1180	82.6468	14.8880
	15	89.6333	2.3135	69.4725	3.7297	61.7373	6.1299	70.4244	7.6621	88.8467	6.5614
	20	84.6225	8.2187	63.8728	1.5434	51.5482	6.5888	64.2054	6.3586	87.8369	3.3274
Rosenbrock	2	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
1,000,000	5	99.4876	1.1456	99.0031	1.2391	99.6723	0.7328	100.0000	0.0000	99.9507	0.0968
	10	98.4653	1.0877	98.3385	1.1585	99.0832	0.6838	98.1624	1.2507	99.8404	0.1810
	15	99.1488	0.2755	98.9794	0.4799	99.1206	0.4879	99.0237	0.5181	99.3966	0.3473
	20	99.3816	0.0835	99.4315	0.0291	99.4114	0.0930	99.3523	0.1029	99.4905	0.1003
Styblinski-Tang	2	100.0587	0.0946	100.0587	0.0946	100.0587	0.0946	100.0587	0.0946	100.0587	0.0946
igotenesiei Tuneg	5	91.8003	11.2348	95.9944	8.9602	95.9944	8.9602	95.6843	9.6535	91.6772	11.4117
	10	87.0259	5.8436	93.8183	8.5030	96.5618	7.6903	83.3179	5.1655	73.0199	4.3948
	15	85.2936	9.7951	94.2889	6.3678	93.9511	10.5400	89.5117	11.6834	75.8299	6.4824
	20	88.6356	3.1147	93.0941	6.3940	72.7793	7.7189	86.2757	6.2603	84.8283	9.0133
Zakharov	2	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
Zuntuto	5	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000	100.0000	0.0000
	10	99.9816	0.0299	96.6839	1.7169	99.9798	0.0451	98.3437	1.1949	100.0000	0.0000
	15	85.5079	9.2265	85.9642	6.8519	96.6764	2.0882	83.6157	13.7491	97.1831	1.5357
	20	93.9260	9.3610	89.0425	15.8602	89.7224	15.3959	89.8602	14.3317	95.6673	6.0977

Table 2: Mean time (in seconds) spent each time the metamodel was built.

		Ol			K1			BK		RBF	
						UK2					
Function	n	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Ackley	2	0.1853	0.0640	0.1895	0.0363	0.1429	0.0413	1.6059	0.2211	0.0030	0.0008
	5	0.3704	0.0312	0.3690	0.0343	0.3165	0.0529	5.1376	0.6752	0.0018	0.0003
	10	0.8588	0.0477	1.0959	0.3061	0.7675	0.1126	6.7915	1.0418	0.0020	0.0004
	15	10.9447	0.6380	11.4520	0.4082	8.8239	0.8645	91.2870	11.9713	0.0034	0.0003
	20	94.8823	9.0157	59.3562	10.0771	18.8155	2.7591	1564.8168	140.2198	0.0069	0.0004
Dixon-Price	2	0.0916	0.0313	0.1206	0.0231	0.1156	0.0189	0.8246	0.1727	0.0020	0.0003
	5	0.1731	0.0473	0.2263	0.0576	0.2938	0.0085	1.7051	0.7325	0.0020	0.0005
	10	0.6580	0.0749	0.7888	0.1235	0.8972	0.2686	7.8019	1.2725	0.0019	0.0006
	15	8.3112	1.9072	9.7811	0.2684	9.0734	1.0477	115.9515	6.1893	0.0031	0.0009
	20	88.1172	3.7898	49.9669	7.8304	18.0735	4.3065	1056.8370	190.9416	0.0074	0.0009
Ellipsoid	2	0.1378	0.0222	0.1219	0.0149	0.0365	0.0016	0.6658	0.0642	0.0023	0.0006
	5	0.1621	0.0395	0.1668	0.0123	0.0685	0.0109	1.6551	0.9531	0.0020	0.0002
	10	0.6337	0.0435	1.0072	0.1263	0.2353	0.0374	7.6162	0.9778	0.0021	0.0003
	15	5.9984	0.6893	9.7038	0.9696	1.1833	0.8130	125.7142	26.6667	0.0043	0.0002
	20	57.6772	13.8426	46.8569	11.4844	31.7984	9.5932	1301.3887	175.2332	0.0061	0.0006
Griewank	2	0.1503	0.0342	0.1066	0.0013	0.0807	0.0376	1.0636	0.1505	0.0017	0.0003
	5	0.1730	0.0390	0.2375	0.0466	0.3442	0.0178	2.5235	0.8503	0.0019	0.0004
	10	0.7725	0.1211	0.9415	0.1250	1.0373	0.3133	9.9687	0.8231	0.0018	0.0003
	15	7.6425	0.6678	10.2436	1.3137	10.1252	2.5713	127.2574	34.9832	0.0031	0.0003
	20	70.1036	12.6678	58.6915	22.5808	59.4503	8.5663	1320.1691	329.2667	0.0060	0.0002
Levy	2	0.1434	0.0362	0.1243	0.0194	0.1190	0.0203	1.1282	0.2463	0.0028	0.0012
_	5	0.1671	0.0435	0.1938	0.0112	0.3241	0.0505	1.4984	0.0945	0.0019	0.0004
	10	0.6856	0.0557	0.8961	0.1285	0.8538	0.2690	8.2298	0.6599	0.0021	0.0003
	15	7.6708	0.6977	9.5258	1.5205	8.2180	0.6295	148.1594	18.5020	0.0036	0.0005
	20	72.3507	5.6833	51.1106	11.4200	15.2841	4.9718	1500.0091	450.5395	0.0078	0.0004
Perm $n, \beta$	2	0.1297	0.0120	0.0980	0.0256	0.1431	0.0207	0.9900	0.3626	0.0021	0.0010
	5	0.2953	0.0324	0.3553	0.0265	0.3793	0.0518	2.7260	1.2577	0.0020	0.0002
	10	0.7799	0.0355	0.8071	0.0378	0.9716	0.0442	8.8978	0.2014	0.0014	0.0005
	15	6.6113	0.2100	6.9262	0.1760	9.7526	0.1110	87.6333	5.8805	0.0035	0.0006
	20	76.2511	7.1718	6.4703	2.3791	9.6662	3.4859	182.1067	38.0025	0.0069	0.0004
Rastrigin	2	0.1355	0.0313	0.1191	0.0132	0.0684	0.0087	0.8687	0.1401	0.0020	0.0003
	5	0.1650	0.0382	0.1937	0.0477	0.4060	0.0702	1.8400	0.4749	0.0019	0.0003
	10	0.7705	0.0885	1.0730	0.0831	1.0360	0.0741	10.4501	1.1753	0.0022	0.0003
	15	9.1800	0.6215	12.0333	0.8557	11.4043	1.4011	156.1029	17.8677	0.0030	0.0003
	20	105.1134	8.3444	52.4638	20.2889	26.5260	14.4381	1108.6190	83.3914	0.0061	0.0005
Rosenbrock	2	0.1412	0.0361	0.1169	0.0345	0.1303	0.0165	1.0692	0.1908	0.0021	0.0005
	5	0.2643	0.0234	0.2569	0.0423	0.3734	0.1249	2.4130	0.4021	0.0020	0.0006
	10	0.5962	0.0445	0.9994	0.0406	0.8798	0.0671	9.7590	0.8209	0.0014	0.0004
	15	6.8911	0.3016	11.5860	0.3903	8.8577	0.7145	140.9204	17.7689	0.0033	0.0002
	20	79.6912	6.3612	44.1954	12.7430	22.6223	5.0309	877.1790	122.7184	0.0061	0.0003
Styblinski-Tang	2	0.1140	0.0230	0.1254	0.0317	0.0560	0.0051	0.7917	0.0515	0.0024	0.0006
<i>J</i> • <i>g</i>	5	0.0877	0.0072	0.1353	0.0323	0.3464	0.0350	1.2756	0.1687	0.0022	0.0003
	10	0.7563	0.0697	1.0216	0.1299	1.0385	0.0611	9.6773	2.0379	0.0023	0.0001
	15	7.0796	0.9799	10.9273	0.8324	9.3840	0.8604	146.7241	14.3419	0.0038	0.0004
	20	76.3130	7.6457	66.5185	10.2362	31.5641	9.9347	1262.0733	213.6312	0.0066	0.0004
Zakharov	2	0.1195	0.0198	0.1258	0.0077	0.1330	0.0151	1.2576	0.0258	0.0023	0.0003
	5	0.1875	0.0045	0.2147	0.0216	0.3138	0.0385	2.0506	0.2479	0.0021	0.0002
	10	0.7491	0.0286	0.7202	0.0239	1.0179	0.0641	7.5062	0.4530	0.0014	0.0002
	15	7.3769	0.2481	7.5446	0.4047	10.7905	0.7270	85.0442	4.6413	0.0032	0.0002
	20	83.2243	7.1576	59.4229	22.4984	23.5015	1.8889	828.3175	257.1672	0.0068	0.0005