

Assignment 4

Spectral Graph Theory

February 24, 2018

Assume the graphs are simple unless explicitly specified

Problem 1. Given 9 vertices, join all pairs of vertices by either a blue edge or a red edge. Show that there is always a red triangle or a blue quadrilateral.

Problem 2. How many Hamiltonian cycles does K_n contain? How many Hamiltonian cycles does $K_{m,n}$ contain? (Two Hamiltonian cycles are different when their edge sets are different)

Problem 3. How many subgraphs does K_n have? When will K_n have an Eulerian cycle?

Problem 4. How many simple graphs are possible having 5 vertices? How many distinct simple graphs are possible having 4 vertices? (Distinct means non-isomorphic)

Problem 5. Is 22222111 a possible degree sequence of a graph?

Problem 6. Give an example of a graph which has ($n > 2$)

1. Has an Euler cycle as well as Hamiltonian cycle.
2. Has an Euler cycle but not Hamiltonian cycle.
3. Has a Hamiltonian cycle but not Euler cycle.
4. Has neither an Euler cycle nor a Hamiltonian cycle.

Problem 7. Let G be a graph with n nodes. For each pair of vertices u, v it is given that $\deg(u) + \deg(v) \geq n$. Show that G is connected.

Problem 8. Let G be a graph in which each node has degree k . Show that it has a cycle of length at least $k + 1$.

Problem 9. Let G be a graph with 10 nodes and 38 edges, show that K_4 is a subgraph of G .