

## ass5prob2

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### Solution

A graph can be broken into finite number of completely disconnected components(trees). These trees can have different 2-edge-components(2-edge-component means those components which cannot be disconnected by removing of single vertex and its edges).

Now the newly added edge had two vertices (a and b) which can have following cases.

#### case 1.

if both vertices belong to different trees. then this edge will create a bridge and number of bridge increases by 1. Also we will need to unite the two trees which can be done by joining root of smaller tree to larger tree.(union by rank or size)

#### case 2.

if both vertices belong to the same tree and same 2-edge-component. in that case no effect will be seen.

#### case 3.

if both vertices belong to the same tree but different 2-edge-component.in this case number of bridges will decrease because consider an edge between x and y (x and y can be same as a and b) such that every path from a to b has to pass through this edge. this means this edge acted as a bridge.but now we added an edge between a and b therefore this bridge is unactivated. there can be multiple such pair of x and y.

now since a and b are part of same tree there must be a LCA of them. therefore adding edge between a and b creates a cycle from a to LCA to b to a. so we need to compress this cycle to single 2-edge-component.this is possible by making every vertex of this cycle connect to LCA.And one bridge is subtracted for every edge passed in the cycle.

Time complexity of this algo will be  $O(q \log n)$  where q is number of query.