Assignment 4

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Q1.

we assume that the graph neither contains red triangle nor blue quadrilateral.now choose a vertex(say v_1) has 8 edges out of which 4 or more of them cannot be red(i.e. at least 5 are blue). Because say v_1 to v_a, v_b, v_c, v_d are red, then line joining every pair of v_a, v_b, v_c, v_d must be blue(otherwise a red triangle will form).But this means v_a, v_b, v_c, v_d make blue quadrilateral which contradicts.Hence 4 or more edges from v_1 cannot be red.

now 2 cases arises.

case1. number of blue edges from v_1 are 5,6 or 7. pick a red edge vertex from v_1 say v_i. now, since v_i also has 5 blue edges there must exist two vertices v_p and v_q such that edges from v_1 and v_1 to above two vertices are blue. hence v_1,v_i,v_p,v_q form a blue quadrilateral. hence we arrive at contradiction.

case2. number of blue edges from v_1 is 8. In this case pick any vertex v_j.now, since v_j also has 5 blue edges there must exist two vertices v_p and v_q such that edges from v_1 and v_1 to above two vertices are blue. hence v_1,v_j,v_p,v_q form a blue quadrilateral. hence we arrive at contradiction.

hence our assumption was wrong.so there must be either a red triangle or blue quadrilateral.

Q2.

1).

this problem is same as circular permutatation of n objects which is $n - 1!$. but since 1-2-3.....-n and n-(n-1)-.....-2-1 is same we divide by 2.so answer is $(n - 1)!/2$.

2).

Firstly for hamiltonion cycle to be possible m should be equal to n.so,let $m = n = k$. now pick any vertex.it has m options on the opposite side of biparte.now the next vertex m-1 options on the opposite side of biparte. next one again m-1, after that m-2,again m-2 and so on. hence total possible is $m!m - 1!$.

alternatively. we have m! choices on each side of biparte. but we want distinct hamiltonion but we are repeating m times every case. hence total cases are $m!m!/m = m!m-1!$.

Q3.

1).

first let us find no. of graphs with m vertices(call it P_m). now every member of P_m is formed by removing zero or more edges of K_m.now number of ways to remove j edges from K_m is $\binom{\binom{m}{2}}{j}$. hence $P_m = \sum_{j=0}^{m} \binom{\binom{m}{2}}{j}$. or $P_m = 2^{\binom{m}{2}}$.

Now K_n can be broken into subgraphs with 0,1,2...n vertices. For m no. of vertices there is P_m graphs and to select m vertices out of n vertices no. of ways are $\binom{n}{m}$.

Hence total subgraph of K_n is $\sum_{m=0}^{n} \binom{n}{m} 2^{\binom{m}{2}}$

2).

If any vertex of a graph has odd degree it cannot have eulerian cycle. Because if we start our eulerian cycle from the odd degree vertex then we will never be able to come back without repeating an edge and if we start from somewhere else we will hit dead end at the odd degree vertex. So for a graph to have euler cycle we need degree of every vertex to be even. Since K_n has degee of n-1 for all vertices hence n must be odd.

Q4.

1).

The answer is same as P_5 in previous question which is $2^{10} = 1024$.

2).

Here we will have to count the cases.

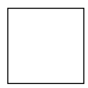
no. of distinct graphs with zero edges=1
no. of distinct graphs with one edge=1
no. of distinct graphs with two edges=2
no. of distinct graphs with three edges=3
no. of distinct graphs with four edges=2
no. of distinct graphs with five edges=2
Hence total distinct graphs are 11.

Q5.

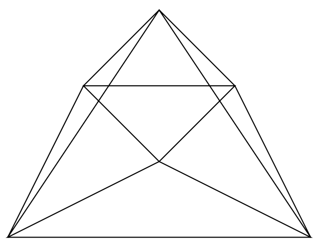
Since every edge is made of 2 vertices, while counting the degree sequence the edge is taken twice once for each vertex. Hence sum of degrees of a graph cannot be odd.Hence given degree sequence is **not** possible for a graph.

Q6.

1).



2).



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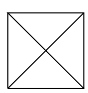

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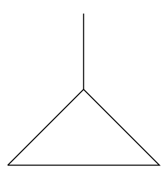

Q7.

Let the graph be disconnected. This means there are two vertices V_1 and V_2 such that there exist no path between them.This means V_1 and V_2 don't have a common connected vertex.Therefore the sum of degrees of V_1 and V_2 cannot be greater than n-2 which is a contradiction to the given statement of the question. Hence the graph must be connected

Q8

In the graph let the longest path be P.let this path start at V_a.Now V_a has k neighbours and all of them must be included in P otherwise P won't be the longest path.Let V_b be the most distant neighbour of V_a as we traverse P.

Which means there must exist a cycle which starts from V_a covers all neighbours ending at V_a via V_b. And this cycle has length at least k+1.

Q9.

Consider K_10. It contains a total of $\binom{10}{4} = 210$ K_4s.Now 38 edges means we have 7 less edges than K_10.

If we remove 1 edge we effectively destroy $\binom{8}{2}$ K_4s.Therefore we can destroy a maximum of $\binom{8}{2} * 7 = 196$ K_4s which is less than 210. Hence there will exist at least 1 K_4 as a subgraph.