MATH 241 Homework 1 Questions

Due: Sunday 2/28 11:59pm to Moodle

- Chapter 1 Problem 3

  Twenty workers are to be assigned to 20 different jobs, one to each job. How many different assignments are possible?

- Chapter 1 Problem 5

  For years, telephone area codes in the United States and Canada considered a sequence of three digits. The first digit was an integer between 2 and 9, the second digit was either 0 or 1, and the third digit was any integer from 1 to 9. How many area codes were possible? How many area codes starting with a 4 were possible?

- Chapter 1 Problem 7

  (a) In how many ways can 3 boys and 3 girls sit in a row?

  (b) In how many ways can 3 boys and 3 girls sit in a row if the boys and the girls are each to sit together?

  (c) In how many ways if only the boys must sit together?

  (d) In how many ways if no two people of the same sex are allowed to sit together?

- Chapter 1 Problem 8

  How many different letter arrangements can be made from letters

  (a) Fluke?

  (b) Propose?

  (c) Mississippi?

  (d) Arrange?

- Chapter 1 Problem 13

  Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

- Chapter 1 Problem 15

  A dance class consists of 22 students of which 10 are women and 12 are men. If 5 men and 5 women are to be chosen and then paired off, how many results are possible?

- Chapter 1 Problem 19

  From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if

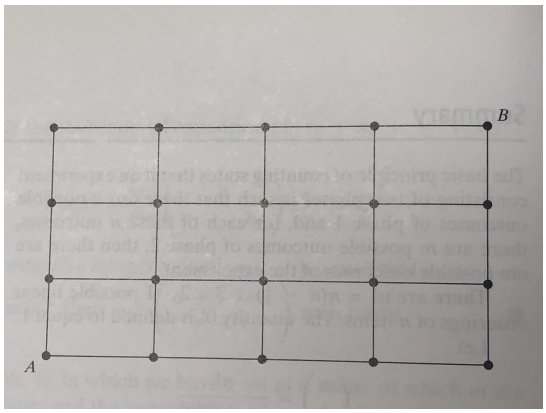
  (a) 2 of the men refuse to serve together?

  (b) 2 of the women refuse to serve together?

  (c) 1 man and 1 woman refuse to serve together?

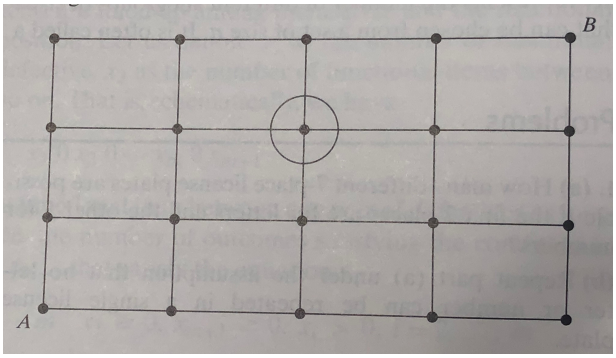
- Chapter 1 Problem 21

  Consider the grid of points shown here. Suppose that, starting at the point labeled A, you can go one step up or one step to the right at each move. This procedure is continued until the point labeled B is reached. How many different paths from A to B are possible? [Hint: Note that to reach B from A, you must take 4 steps to the right and 3 steps upward.]



- Chapter 1 Problem 22

  In Problem 21, how many different paths are there from A to B that go through the point circled in the following lattice?



- Chapter 1 Problem 24

  Expand $(3x^2 + y)^5$.

- Chapter 1 Problem 27

  If 12 people are to be divided into 3 committees of respective size 3, 4, and 5, how many divisions are possible?

- Chapter 1 Theoretical exercise 2

  Two experiments are to be performed. The first can result in any one of m possible outcomes. If the first experiment results in outcome i, then the second experiment can result in any of n_i possible outcomes, $i = 1, 2, \cdots, m$. What is the number of possible outcomes of the two experiments?

- Chapter 1 Theoretical exercise 8

  Prove that

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}$$

[Hint: Consider a group of n men and m women. How many groups of size r are possible?]

- Chapter 1 Theoretical exercise 9

  Use Theoretical Exercise 8 to prove that

$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$$

- Chapter 1 Theoretical exercise 13

  Show that, for $n > 0$,

$$\sum_{i=0}^{n} (-1)^i \binom{n}{i} = 0$$

  [Hint: Use the binomial theorem.]

Optional: if you feel like more practice
These will not be graded, but you are welcome to discuss these with me during the office hour.

- Textbook Chapter 1 Problems: 1, 2, 4, 6, 9, 10, 11, 12, 16, 17, 18, 20, 23, 30
- Textbook Chapter 1 Theoretical exercise: 10, 11