MATH 241 Homework 7

Due: Sunday 4/18 11:59pm to Moodle

- Chapter 5 Problem 1

  Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1-x^2) & \text{if } -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

  (a) What is the value of c?

  (b) What is the cumulative distribution function of X?

- Chapter 5 Problem 2

  A system consisting of one original unit plus a spare can function for a random amount of time X. If the density of X is given (in units of months) by

$$f(x) = \begin{cases} Cxe^{-x/2} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

  what is the probability that the system functions for at least 5 months?

- Chapter 5 Problem 6

  Compute $E[X]$ if X has a density function given by

  (a)

$$f(x) = \begin{cases} \frac{1}{4}xe^{-x/2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

  (b)

$$f(x) = \begin{cases} c(1-x^2) & \text{if } -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

  (c)

$$f(x) = \begin{cases} \frac{5}{x^2} & x > 5 \\ 0 & x \leq 5 \end{cases}$$

- Chapter 5 Problem 10

  Trains headed for destination A arrive at the train station at 15-minute intervals starting at 7 A.M., whereas trains headed for destination B arrive at 15-minute intervals starting at 7:05 A.M.

(a) If a certain passenger arrives at the station at a time uniformly distributed between 7 and 8 A.M. and then gets on the first train that arrives, what proportion of time does he or she go to destination A?

(b) What if the passenger arrives at a time uniformly distributed between 7:10 and 8:10 A.M.?

- Chapter 5 Problem 14

  Let X be a uniform(0, 1) random variable. Compute $E[X^n]$ by using Proposition 2.1, and check the result by using the definition of expectation.

  Proposition 2.1 is on textbook page 181: If X is a continuous random variable with probability density function $f(x)$, then, for any real-valued function g,

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)dx$$

- Chapter 5 Theoretical exercise 2

  Show that

$$E[Y] = \int_0^{\infty} P\{Y > y\}dy - \int_0^{\infty} P\{Y < -y\}dy$$

  Hint: Show that

$$\int_0^{\infty} P\{Y < -y\}dy = -\int_{-\infty}^{0} x f_Y(x)dx$$

$$\int_0^{\infty} P\{Y > y\}dy = \int_0^{\infty} x f_Y(x)dx$$

- Chapter 5 Theoretical exercise 4

  Prove Corollary 2.1:

$$E[aX + b] = aE[X] + b$$

- Chapter 5 Theoretical exercise 7

  The standard deviation of X, denoted $SD(X)$, is given by

$$SD(X) = \sqrt{Var(X)}$$

  Find $SD(aX + b)$ if X has variance σ^2.

Optional: if you feel like more practice

These will not be graded, but you are welcome to discuss these with me during the office hour.

- Textbook Chapter 5 Problems: 3-5, 7-8, 11-13
- Textbook Chapter 5 Theoretical exercise: 1