MATH 241 Homework 10

Due: Sunday 5/9 11:59pm to Moodle

Note that for Problem 28 and Problem 48, read textbook section 5.5 on exponential random variables before proceeding.

- Chapter 6 Problem 28

  The time that it takes to service a car is an exponential random variable with rate 1.

  (a) If A.J. brings his car in at time 0 and M.J. brings her car in at time t, what is the probability that M.J.'s car is ready before A.J.'s car? (Assume that service times are independent and service begins upon arrival of the car.)

  (b) If both cars are brought in at time 0, with work starting on M.J.'s car only when A.J.'s car has been completely serviced, what is the probability that M.J.'s car is ready before time 2?

- Chapter 6 Problem 29

  The gross weekly sales at a certain restaurant are a normal random variable with mean $2200 and standard deviation $230. What is the probability that

  (a) the total gross sales over the next 2 weeks exceeds $5000;

  (b) weekly sales exceed $2000 in at least 2 of the next 3 weeks?

  What independence assumptions have you made?

- Chapter 6 Problem 30

  Jill's bowling scores are approximately normally distributed with mean 170 and standard deviation 20, while Jack's scores are approximately normally distributed with mean 160 and standard deviation 15. If Jack and Jill each bowl one game, then assuming that their scores are independent random variables, approximate the probability that

  (a) Jack's score is higher;

  (b) the total of their scores is above 350.

- Chapter 6 Problem 39

  Two dice are rolled. Let X and Y denote, respectively, the largest and smallest values obtained. Compute the conditional mass function of Y given $X = i$, for $i = 1, \cdots, 6$. Are X and Y independent? Why?

- Chapter 6 Problem 41

  The joint density function of X and Y is given by

  $$f(x, y) = xe^{-x(y+1)}, x > 0, y > 0$$

  (a) Find the conditional density of X, given $Y = y$, and that of Y, given $X = x$.

  (b) Find the density function of $Z = XY$.

- Chapter 6 Problem 48

  If X_1, X_2, X_3, X_4, X_5 are independent and identically distributed exponential random variables with the parameter λ, compute

  (a) $P\{\min(X_1, \cdots, X_5) \leq a\}$;

  (b) $P\{\max(X_1, \cdots, X_5) \leq a\}$.

Optional: if you feel like more practice

These will not be graded, but you are welcome to discuss these with me during the office hour.

- Textbook Chapter 6 Problems: 31-38, 40, 42-47