MATH 241 Chapter 7 part 2 and Chapter 8 Live Exercises

1. If X is a Poisson random variable with parameter λ. Use the moment generating functions to obtain its mean and variance.
2. If X is an exponential random variable with parameter λ. Use the moment generating functions to obtain its mean and variance.
3. Show that if X and Y are independent normal random variables with respective parameters (μ_1, σ_1^2) and (μ_2, σ_2^2), then $X + Y$ is normal with mean $\mu_1 + \mu_2$ and variance $\sigma_1^2 + \sigma_2^2$. Note that for normal (μ, σ^2), the MGF is $e^{\{\frac{\sigma^2 t^2}{2} + \mu t\}}$. Textbook pages 339 and 340 for lists of MGFs of distributions.
4. A person has 100 light bulbs whose lifetimes are independent exponentials with mean 5 hours. If the bulbs are used one at a time, with a failed bulb being replaced immediately by a new one, approximate the probability that there is still a working bulb after 525 hours. Suppose that it takes a random time, uniformly distributed over $(0, .5)$, to replace a failed bulb. Approximate the probability that all bulbs have failed by time 550. Note that if $X \sim \text{Exponential}(\lambda)$, then $E[X] = \frac{1}{\lambda}$ and $Var(X) = \frac{1}{\lambda^2}$.