MATH 241 Chapter 6 part 1 Live Exercises

1. Use joint cdf $F(x, y)$ to represent $P(x_1 < X \leq x_2, y_1 < Y \leq y_2)$. Show your reasoning.
   (a) $F(x_2, y_2) + F(x_1, y_1) - F(x_1, y_2) - F(x_2, y_1)$
   (b) $F(x_2, y_2) - F(x_1, y_1) - F(x_1, y_2) - F(x_2, y_1)$
   (c) $F(x_2, y_2) - F(x_1, y_1)$
   (d) none of the above
2. Draw two socks at random, without replacement, from a drawer full of twelve colored socks: 6 black, 4 white, 2 purple. Let B be the number of Black socks, W the number of White socks drawn. Find the pmf for white socks given no black socks were drawn.
3. Which of the following can be obtained if the joint pdf $f_{X,Y}(x, y)$ is known? Show your reasoning.
   (a) Joint cdf $F_{X,Y}(x, y)$
   (b) Marginal cdf's $F_X(x), F_Y(y)$.
   (c) Expected values $E[X], E[Y]$.
   (d) all above
4. Let X have a $\text{Bin}(n, p)$ distribution. What's the pmf of $Y = 2X$? Show your reasoning.
   (a) $f_Y(y) = \binom{2n}{y}(2p)^y(1-2p)^{2n-y}$ for any $y \in \{0, 2, 4, \ldots, 2n\}$
   (b) $f_Y(y) = \binom{2n}{y}p^y(1-p)^{2n-y}$ for any $y \in \{0, 1, 2, \ldots, 2n\}$
   (c) $f_Y(y) = \binom{n}{y/2}p^{\frac{y}{2}}(1-p)^{n-\frac{y}{2}}$ for any $y \in \{0, 2, 4, \ldots, 2n\}$
   (d) $f_Y(y) = \frac{1}{2}\binom{n}{y/2}p^{\frac{y}{2}}(1-p)^{n-\frac{y}{2}}$ for any $y \in \{0, 2, 4, \ldots, 2n\}$
5. Let X and Y have the following joint pdf

$$f(x, y) = \begin{cases} \frac{2}{9} & \text{for } x \geq 0, y \geq 0 \text{ and } x + y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

   Find the marginal pdf of Y.
6. Let $f(x, y) = cx^2y$ for $x^2 \leq y \leq 1$. Find:
   (a) c
   (b) $P[X \geq Y]$
   (c) $f_X(x)$ and $f_Y(y)$
7. Let X and Y be drawn uniformly from the triangle below, i.e., their joint pdf is

$$f(x, y) = \begin{cases} \frac{2}{9} & \text{for } x \geq 0, y \geq 0 \text{ and } x + y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

   Are they independent? Why? Show your work.