MATH 241 Chapter 7 part 1 Live Exercises

1. What's the expected value of $X - Y$?
2. Suppose that n people throw their hats into the center of a room. The hats are mixed up, and each person randomly selects one. Find the expected number of people that select their own hat.
3. Let $X_1, ..., X_n$ be independent and identically distributed random variables having distribution function F and expected value μ. Such a sequence of random variables is said to constitute a sample from the distribution F. Then quantity

$$\bar{X} = \sum_{i=1}^{n} \frac{X_i}{n}$$

   is called the sample mean. Compute $E[\bar{X}]$.
4. Suppose Z_1 and Z_2 are two standard normal random variables. Let

$$X = Z_1 + Z_2,\ Y = Z_1 - Z_2$$

   Find $Cov(X, Y)$.
5. Let $X_1, \ldots, X_n$ be independent random variables having the same variance σ^2, and

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

   Find $Var(\bar{X})$.
6. Find the value of b in $Y = a + bX$ such that $\rho(X, Y) = 1$. What about $\rho(X, Y) = -1$?
7. Type i light bulbs function for a random amount of time having mean μ_i and standard deviation σ_i, $i = 1, 2$. A light bulb randomly chosen from a bin of bulbs is a type 1 bulb with probability p and a type 2 bulb with probability $1 - p$. Let X denote the lifetime of this bulb. Find (a) $E[X]$ (b) $Var(X)$