

MATH 241 Homework 5

Due: Sunday 3/28 11:59pm to Moodle

- Chapter 4 Problem 35

A box contains 5 red and 5 blue marbles. Two marbles are withdrawn randomly. If they are the same color, then you win \$1.10; if they are different colors, then you will -\$1.00. (That is, you lose \$1.00.) Calculate

- (a) the expected value of the amount you win;
- (b) the variance of the amount you win.

- Chapter 4 Problem 40

On a multiple-choice exam with 3 possible answers for each of the 5 questions, what is the probability that a student will get 4 or more correct answers just by guessing?

- Chapter 4 Problem 44

A satellite system consists of n components and functions on any given day if at least k of the n components function on that day. On a rainy day, each of the components independently functions with probability p_1 , whereas on a dry day, each independently functions with probability p_2 . If the probability of rain tomorrow is α , what is the probability that the satellite system will function?

- Chapter 4 Problem 49

When coin 1 is flipped, it lands on heads with probability 0.4; when coin 2 is flipped, it lands on heads with probability 0.7. One of these coins is randomly chosen and flipped 10 times.

- (a) What is the probability that the coin lands on heads on exactly 7 of the 10 flips?
- (b) Given that the first of these 10 flips lands heads, what is the conditional probability that exactly 7 of the 10 flips land on heads?

- Chapter 4 Theoretical exercise 7

Let X be a random variable having expected value μ and variance σ^2 . Find the expected value and variance of

$$Y = \frac{X - \mu}{\sigma}$$

- Chapter 4 Theoretical exercise 10. [Hint: use $\binom{n+1}{k+1} = \binom{n}{k} \frac{n+1}{k+1}$]

Let X be a binomial random variable with parameters n and p . Show that

$$E \left[\frac{1}{X+1} \right] = \frac{1 - (1-p)^{n+1}}{(n+1)p}$$

Optional: if you feel like more practice

These will not be graded, but you are welcome to discuss these with me during the office hour.

- Textbook Chapter 4 Problems: 37, 39, 41-42

- Textbook Chapter 4 Theoretical exercise: 8, 11, 13, 14