 Jingchen (Monika) Hu

MATH 241 Homework 9

Due: Sunday 5/2 11:59pm to Moodle

- Chapter 6 Problem 2

  Suppose that 3 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let X_i equal 1 if the ith ball selected is white, and let it equal 0 otherwise. Give the joint probability mass function of

  (a) X_1, X_2;

  (b) X_1, X_2, X_3.

- Chapter 6 Problem 7

  Consider a sequence of independent Bernoulli trials, each of which is a success with probability p. Let X_1 be the number of failures preceding the first success, and let X_2 be the number of failures between the first two successes. Find the joint mass function of X_1 and X_2.

- Chapter 6 Problem 8

  The joint probability density function of X and Y is given by

  $$f(x, y) = c(y^2 - x^2)e^{-y}, -y \leq x \leq y, 0 < y < \infty$$

  (a) Find c.

  (b) Find the marginal densities of X and Y.

  (c) Find $E[X]$.

- Chapter 6 Problem 9

  The joint probability density function of X and Y is given by

  $$f(x, y) = \frac{6}{7}(x^2 + \frac{xy}{2}), 0 < x < 1, 0 < y < 2$$

  (a) Verify that this is indeed a joint density function.

  (b) Compute the density function of X.

  (c) Find $P\{X > Y\}$.

  (d) Find $P\{Y > \frac{1}{2} \mid X < \frac{1}{2}\}$.

  (e) Find $E[X]$.

  (f) Find $E[Y]$.

- Chapter 6 Problem 15

  The random vector (X, Y) is said to be uniformly distributed over a region R in the plane if, for some constant c, its joint density is

$$f(x,y) = \begin{cases} c, & \text{if } (x,y) \in R \\ 0, & \text{otherwise} \end{cases}$$

(a) Show that $1/c =$ area of region R.

Suppose that (X, Y) is uniformly distributed over the square centered at (0, 0) and with sides of length 2.

(b) Show that X and Y are independent, with each being distributed uniformly over (-1, 1).

(c) What is the probability that (X, Y) lies int he circle of radius 1 centered at the origin? That is, find $P\{X^2 + Y^2 \leq 1\}$.

- Chapter 6 Problem 18

  Two points are selected randomly on a line of length L so as to be on opposite sides of the midpoint of the line. [In other words, the two points X and Y are independent random variables such that X is uniformly distributed over (0, $L/2$) and Y is uniformly distributed over $(L/2, L)$.] Find the probability that the distance between the two points is greater than $L/3$.

- Chapter 6 Problem 20

  The joint density of X and Y is given by

$$f(x,y) = \begin{cases} xe^{-(x+y)}, & x > 0, y > 0 \\ 0, & \text{otherwise} \end{cases}$$

  Are X and Y independent? If, instead, $f(x,y)$ were given by

$$f(x,y) = \begin{cases} 2, & 0 < x < y, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

  would X and Y be independent?

Optional: if you feel like more practice

These will not be graded, but you are welcome to discuss these with me during the office hour.

- Textbook Chapter 6 Problems: 1, 4-6, 10-14, 17, 19, 21-23