MATH 241 Chapter 2 Live Exercises

1. Chapter 2 Problem 5

   A system is composed of 5 components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector $(x_1, x_2, x_3, x_4, x_5)$, where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed.

   (a) How many outcomes are in the sample space of this experiment?

   (b) Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. Specify all the outcomes in W.

   (c) Let A be the event that components 4 and 5 are both failed. How many outcomes are contained in the event A?

   (d) Write out all the outcomes in the event $A \cap W$.

2. What is the probability of drawing a jack or a red card from a well shuffled full deck?

3. Suppose that for a randomly selected student in a probability class,

   - $P(\text{live Eastern Time Zone at home}) = 63\%$, $P(\text{senior}) = 41\%$, $P(\text{brown eye color}) = 55\%$.

   - $P(\text{live Eastern Time Zone at home and senior}) = 31\%$,
     $P(\text{live Eastern Time Zone at home and brown eye color}) = 33\%$,
     $P(\text{senior and brown eye color}) = 24\%$

   - $P(\text{live Eastern Time Zone at home and senior and brown eye color}) = 18\%$

   Find the probability that a student is either live Eastern Time Zone at home, a senior or having brown eye color.

4. Two fair four-sided dice are rolled. Two events:
   $$A = \{\text{sum of two rolls is 5}\}$$
   $$B = \{\text{minimum roll is 2}\}$$

   (a) Compute $P(A)$ and $P(B)$

   (b) Compute $P(A \cup B)$

5. For a class of 30 students, (i.e. $n = 30$), $P(\text{no match}) = 29.4\%$. What is the chance of at least a tie in birthdays among these 30 students?

6. Randomly pair 4 keys {a, b, c, d} with 3 locks {a, b, c}. Compute $P(\text{at least one match})$.

7. In the game of bridge, the entire deck of 52 cards is dealt out to 4 players. What is the probability that one of the players receives all face cards?