MATH 241 Chapter 6 part 2 Live Exercises

1. Suppose $X, Y \overset{\text{ind}}{\sim} \text{Exp}(\lambda)$. What distribution does $X+Y$ have? Note that the pdf of exponential of $X \sim \text{Exp}(\lambda)$ is: $f(x) = \lambda e^{-\lambda x}, x > 0$.
2. Suppose $X \sim \text{N}(\mu_1, \sigma_1^2)$, and $Y \sim \text{N}(\mu_2, \sigma_2^2)$ independently. What distribution does $X - Y$ have? Hint: find the distribution of $W = -Y$ first.
3. If X and Y are independent Poisson random variables with respective parameters λ_1 and λ_2, calculate the conditional distribution of X given that $X + Y = n$.
4. Suppose the joint density of X and Y is given by

$$f(x,y) = \begin{cases} \frac{e^{-x/y}e^{-y}}{y} & \text{for } 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

   Find $P\{X > 1 \mid Y = y\}$.