Overview of synthetic data

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Statistical Data Privacy

Outline

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What are synthetic data?

- To provide privacy protection of individuals in datasets
- Usually created by simulating variables of records from statistical models estimated on the confidential data
- Objective: preserve data integrity (e.g., important characteristics in the confidential data, such as means and correlations of variables)

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- Usually created by simulating variables of records from statistical models estimated on the confidential data
- Objective: preserve data integrity (e.g., important characteristics in the confidential data, such as means and correlations of variables)

- To do so, we start with developing suitable statistical models for the confidential data

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- Moreover, they can provide some levels of privacy protection, as compared to releasing the confidential data

What we will do in this lecture

- We will go over important aspects of any synthetic data approach
  - Two flavors of synthetic data: partial synthesis and full synthesis
  - Two general approaches to synthetic data creation: sequential synthesis and joint synthesis
  - Two aspects of evaluation: utility and disclosure risks

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  - Two general approaches to synthetic data creation: sequential synthesis and joint synthesis
  - Two aspects of evaluation: utility and disclosure risks
- Our course focuses on Bayesian synthesis models (Lectures 3 and 4)
- There also exist non-Bayesian data synthesis models (e.g., the `synthpop` R package)

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Partial synthesis

- Proposed by Little (1993)
- Some variables in the collected dataset, such as sensitive variables and key identifiers, are synthesized
- The resulting synthetic data contain sensitive variables with synthesized values while other variables remain unchanged

Full synthesis

- Proposed by Rubin (1993)
- A synthetic population is first simulated
- Then a synthetic sample is selected from the synthetic population
- The resulting synthetic data have every variable synthesized, contain no records from the confidential data, and it may even have a different sample size than the confidential data if needed

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- Then a synthetic sample is selected from the synthetic population
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- One can also create fully synthetic data following the partial synthesis approach, i.e., only working on the sample
  - This approach is actually more widely used when creating fully synthetic data

Comparisons

- The choice depends on data disseminator's protection goals
- Assuming a quality synthesis
  - Utility: higher in partially synthetic data
  - Disclosure risks: higher in partially synthetic data
- Utility-risk trade-off

Example: SynLBD

SynLBD variable description. Taken from Table 1 in Kinney et al. (2011) with some modifications.

Name	Type	Notation	Action
ID	Identifier		Created
County	Categorical	x_1	Not released
SIC	Categorical	x_2	Not to synthesize
Firstyear	Categorical	y_1	To synthesize
Lastyear	Categorical	y_2	To synthesize
Year	Categorical		Created
Multiunit	Categorical	y_3	To synthesize
Employment	Continuous	y_4	To synthesize
Payroll	Continuous	y_5	To synthesize

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Partial synthesis or full synthesis?

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The joint distribution of synthesized variables

- This classification is based on what strategy is used to estimate the joint distribution of the variables to be synthesized
- Variables to be synthesized: $\{y_1, y_2, y_3\}$
- Insensitive variables: $\{x_1, x_2\}$
- The joint distribution of synthesized variables

$$f(y_1, y_2, y_3 \mid x_1, x_2) \tag{1}$$

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- How to estimate this joint distribution? Two general approaches

Sequential synthesis overview

- This approach specifies a sequence of univariate synthesis models for the sensitive variables
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- Variables to be synthesized: $\{y_1, y_2, \cdots, y_{p_1}\}$
- Insensitive variables: $\{x_1, x_2, \cdots, x_{p_2}\}$
- The joint model can be expressed in a sequence of univariate model as:

$$
\begin{aligned}
f(y_1, \cdots, y_{p_1} \mid x_1, \cdots, x_{p_2}) = \quad & f(y_1 \mid x_1, \cdots, x_{p_2}) \times \\
& f(y_2 \mid y_1, x_1, \cdots, x_{p_2}) \times \\
& \cdots \\
& f(y_{(p_1-1)} \mid y_1, \cdots, y_{(p_1-2)}, x_1, \cdots, x_{p_2}) \\
& f(y_{p_1} \mid y_1, \cdots, y_{(p_1-1)}, x_1, \cdots, x_{p_2})
\end{aligned}
$$

Sequential synthesis procedure

1. Specify a synthesis model for $y_1 \mid x_1, \cdots, x_{p_2}$. Estimate this model on the confidential data, and generate synthetic y_1^* using confidential $(x_1, \cdots, x_{p_2})$.
2. Specify a synthesis model for $y_2 \mid y_1, x_1, \cdots, x_{p_2}$. Estimate this model on the confidential data, and generate synthetic y_2^* using synthetic y_1^* from step 1 and confidential $(x_1, \cdots, x_{p_2})$.
3. Repeat Step 2 for each of the variables of $\{y_3, \cdots, y_{(p_1-1)}\}$.
4. Specify a synthesis model for $y_{p_1} \mid y_1, \cdots, y_{(p_1-1)}, x_1, \cdots, x_{p_2}$. Estimate this model on the confidential data, and generate synthetic $y_{p_1}^*$ using synthetic $(y_1^*, y_2^*, \cdots, y_{(p_1-1)}^*)$ from previous steps and confidential $(x_1, \cdots, x_{p_2})$.

Joint synthesis

- The joint distribution: $f(y_1, \cdots, y_{p_1} \mid x_1, \cdots, x_{p_2})$
- Directly specify a joint model for these sensitive variables
- For example, if $\{y_1, y_2, \cdots, y_{p_1}\}$ are all continuous (and marginally normal after transformation), we can use a multivariate normal distribution:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{p_1} \end{bmatrix} \sim \mathrm{MVN}_{p_1}\left(\begin{bmatrix} \mu_1 \\ \vdots \\ \mu_{p_1} \end{bmatrix} \Sigma\right), \tag{2}$$

  - MVN_{p_1} stands for multivariate normal distribution of dimension p_1
  - $\mu_1, \cdots, \mu_{p_1}$ are the mean parameters (conditional on $x_1, \cdots, x_{p_2}$)
  - Σ is covariance matrix

Joint synthesis cont'd

- If sensitive variables are all categorical. . .
- A well research model is the Dirichlet Process mixture of products of multinomials (DPMPM) (Hu, Reiter, and Wang (2014)); we will introduce it in Lecture 4
- Joint synthesis model estimation is usually more challenging than sequential synthesis

Joint synthesis cont'd

- If sensitive variables are all categorical. . .
- A well research model is the Dirichlet Process mixture of products of multinomials (DPMPM) (Hu, Reiter, and Wang (2014)); we will introduce it in Lecture 4
- Joint synthesis model estimation is usually more challenging than sequential synthesis
- Bayesian networks are good approaches (Young, Graham, and Penny (2009), Kaur et al. (2021)); this could be an interesting project

Example: SynLBD

- Details are in Kinney et al. (2011) Section 3
- The SynLBD uses the sequential synthesis approach

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The sequential synthesis procedure for the SynLBD follows the workflow below:

1. Synthesize `Firstyear` using the Dirichlet-multinomial approach and draw from the following estimated model to obtain y_1^*
$$f(y_1 \mid x_1, x_2). \tag{3}$$
2. Synthesize `Lastyear` using the Dirichlet-multinomial approach and approximate a draw from the following estimated model to obtain y_2^*
$$f(y_2 \mid y_1, x_1, x_2). \tag{4}$$

Example: SynLBD

3. Synthesize `Multiunit` using the Dirichlet-multinomial approach and approximate a draw from the following estimated model to obtain y_3^*
$$f(y_3 \mid y_2, y_1, x_1, x_2). \tag{5}$$
4. Synthesize `Employment` using the normal approach and approximate a draw from the following estimated model to obtain $y_4^{(t)*}$
$$f(y_4^{(t)} \mid y_4^{(t-1)}, y_3, y_2, y_1, x_1, x_2). \tag{6}$$
5. Synthesize `Payroll` using the normal approach and approximate a draw from the following estimated model to obtain $y_5^{(t)*}$
$$f(y_5^{(t)} \mid y_4^{(t)}, y_5^{(t-1)}, y_3, y_2, y_1, x_1, x_2). \tag{7}$$

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Utility evaluation

- Two general types of utility: global and analysis-specific

1. Global utility: Evaluate the closeness between the confidential data distribution and the synthetic data distribution
2. Analysis-specific utility: Evaluate whether synthetic data users can obtain statistical inferences on the synthetic data that are similar to those obtained on the confidential data

Utility evaluation

- Two general types of utility: global and analysis-specific

1. Global utility: Evaluate the closeness between the confidential data distribution and the synthetic data distribution
2. Analysis-specific utility: Evaluate whether synthetic data users can obtain statistical inferences on the synthetic data that are similar to those obtained on the confidential data

- How to capture the uncertainty in the synthetic data generation process? Create multiple synthetic datasets

Example: SynLBD

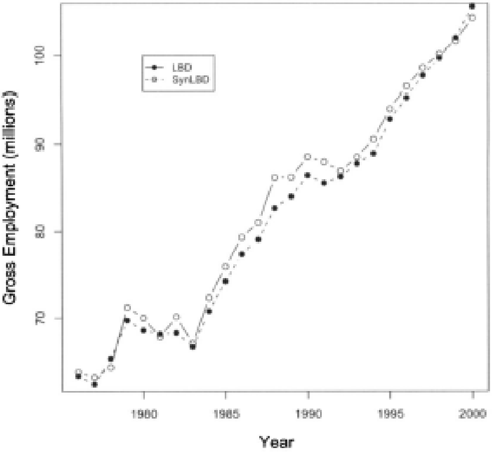


Figure 1. *Gross employment level by year, LBD versus Synthetic.*

Global utility or analysis-specific utility?

Example: SynLBD

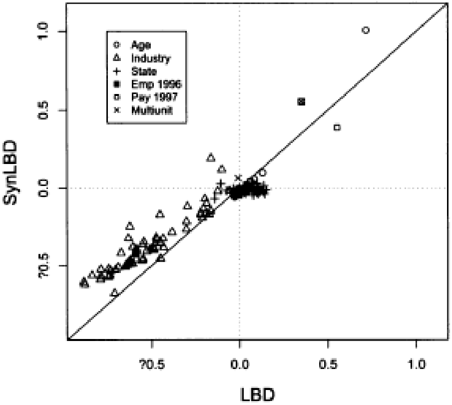


Figure 11. *Regression coefficients, LBD versus Synthetic.*

Global utility or analysis-specific utility?

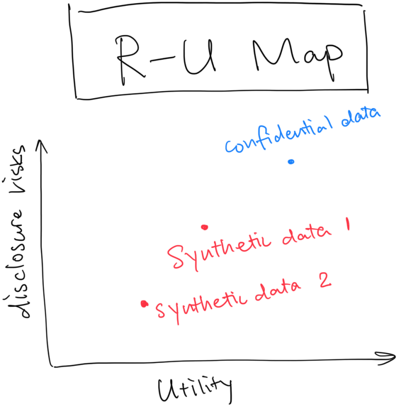
Disclosure risks evaluation

- Assuming the intruder has access to external data, two common disclosure risks: identification and attribute (Hu (2019))

1. Identification disclosure: The intruder correctly identifies records of interest in the released synthetic data
2. Attribute disclosure: The intruder correctly infers the true confidential values of the synthetic records using information from the released synthetic data

Utility-risk trade-off

- Ideally, the released synthetic data have high utility and low disclosure risks
- However this is usually not the case, due to the utility-risk trade-off (Duncan, Keller-McNulty, and Stokes (2001))



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  - Partial synthesis and full synthesis
  - Sequential synthesis and joint synthesis
  - Evaluations of synthetic data: utility and disclosure risks
  - The example of SynLBD

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  - Submission on Moodle and prepare to discuss next time

Summary

- Synthetic data
  - Partial synthesis and full synthesis
  - Sequential synthesis and joint synthesis
  - Evaluations of synthetic data: utility and disclosure risks
  - The example of SynLBD
- Homework 1: Read Ros, Olsson, and Hu (2020) and answer a few questions regarding the discussed aspects of synthetic data
  - Submission on Moodle and prepare to discuss next time
- Lecture 2: Introduction to Bayesian modeling
  - Chapter 7 of Albert and Hu (2019): https://bayesball.github.io/BOOK/proportion.html

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