A Probability Plus Bayes Course

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The Traditional Probability/Stats Course

- Calculus-based (math and science majors)
- 1st semester, Probability, 2nd semester Statistics
- Same course has been taught for 40+ years
- My 1970's textbook (Mendenhall and Scheaffer) is still popular

Rethinking Statistics Curriculum

- George Cobb (2015) argued for drastic changes in entire undergraduate Statistics curriculum
- Think about the foundations of our statistics curriculum
- Proposed five broad imperatives that should guide our effort

Cobb's Five Guiding Imperatives

- Flatten prerequisites
- Seek Depth
- Embrace Computation
- Exploit Context
- Teach through Research

Imperative 1: Flatten Prerequisites

- In applied statistics, common to use "just-in-time" approach
- Prerequisites limit course choices, limit enrollments
- Focus on what are important skills and concepts
- Encourages creative thinking about courses

Imperative 2: Seek Depth

- Strip away the technical stuff, to reveal what is fundamental
- Central ideas must be made simple and approachable
- Cobb suggests it is important to stress that "all probabilities are conditional" and all discrete probabilities reduce to "#A/#Ω"

Imperative 3: Embrace Computation

- Use computer-based exploration early and often in the curriculum
- More than a new course in statistics computing or a "big data" unit

Imperative 4: Exploit Context

- Standard type (interpretation, motivation, direction)
- Recognizing abstract structure from a verbal description
- Use context as the primary vehicle, instead of mathematics

Imperative 5: Teach Through Research

- In the Humanities, students are engaged with original sources (they read Austen or hear Bach)
- Our job is to help student them use data to answer a question that matters
- How early can we rely on research-based learning?
- Students need individual research experiences

Why Bayes in the First Prob/Stats Course?

- More intuitive than frequentist in learning inference
- Introduce modern Bayesian applications
- Abundance of Bayesian instructional material
- Lot of Bayesian computation resources

Teaching Bayes: Flatten Prerequisites

- Don't need a deep background in probability
- Don't need to know traditional inference
- Start with discrete priors
- Can avoid integration in continuous case by use of simulation of posteriors and predictive distributions

Teaching Bayes: Seek Depth

- Go beyond the basic prior to posterior calculations
- Prior elicitation
- Explore sensitivity of inference with respect to model assumptions
- Predictive model checking, model comparison
- Hierarchical modeling

Teaching Bayes: Embrace Computation

Choice of computational method should be guided by Bayesian learning goals:

- How do I construct a prior?
- How are the prior and data combined in the posterior?
- How do I implement posterior inference by simulation?
- How do I use the predictive distribution to check the Bayesian model?
- How do I implement Bayes in regression?

Our Suggested Path in Bayesian Computation

1. Bayes with discrete models
2. Conjugate models for proportion and mean
3. Introduce MCMC through Metropolis and Gibbs Sampling
4. Use JAGS for regression & hierarchical models

Teach Conjugate Priors?

- Students need some experience deriving posterior and predictive densities
- Provides intuition (how the prior and data combine)
- Helpful in understanding how hierarchical priors work
- Can extend by use of mixtures

Why Not Use Stan (Hamiltonian Sampling)?

- It is too much of a black box for Bayesian computing
  (Example on next slide.)
- Students can learn much about MCMC through Metropolis and Gibbs Sampling
- Easy transition from our MCMC material to Stan software

Example of Stan Code from `brms` Package:

```
fit <- brm(data = DeathHeartAttackManhattan,
           family = binomial,
           Deaths | trials(Cases) ~ 1 + (1 | Hospital),
           refresh = 0)
```

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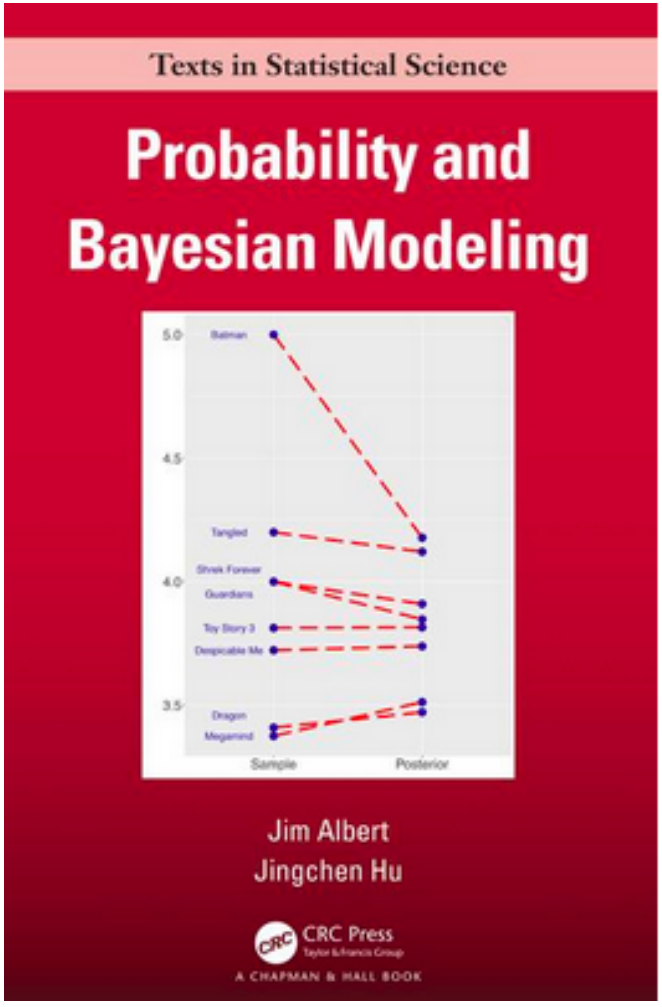
Teaching Bayes: Teach Through Research

- Bayesian thinking is best learned by doing
- Student project
- Done this for a Bayesian intro stats class
- Gelman and Nolan (2017) Bayesian activities
- Specifying subjective probabilities

Probability and Bayesian Modeling



- Coauthor: Monika Hu
- Bookdown version: `bitly.com/ProbBayes`
- R scripts and solutions available

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TOC of *Probability and Bayesian Modeling*

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Summing Up

- Agree with Cobb – we need an overhaul in our Statistics undergraduate teaching
- Good opportunity for a probability/Bayes text & course
- Need the right blend of mathematics, computation, and application
- Bayes fits well within the "Five Imperatives" for new statistics curriculum

References

- Cobb, G. (2015), "Mere Renovation is Too Little Too Late: We Need to Rethink our Undergraduate Curriculum from the Ground Up," *The American Statistician*.
- Albert, J. (2017), "A Bayesian Redesign of the First Probability/Statistics Course", arXiv:2007.04180.
- Albert, J. and Hu, J. (2020), "Bayesian Computing in the Undergraduate Statistics Curriculum", arXiv:2002.09716.

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