Risk-Efficient Bayesian Data Synthesis for Privacy Protection

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Outline

1. The problem: CE income data
2. Synthetic data and probability of identification risk
3. Risk-adjusted synthesizer
4. “Whack-a-mole” risk pop-ups and our solution
5. Summary and references

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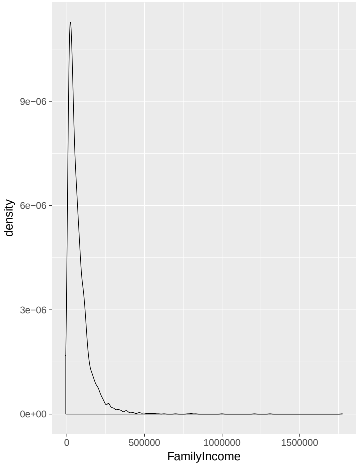
The CE data: highly skewed family income



- The Consumer Expenditure Surveys (CE).
- Family income: before tax for each consumer unit (CU).
- High risk records assumed to be in the tail, e.g., CUs with extremely high family income.
- What to do? Topcoding (statistical disclosure control).

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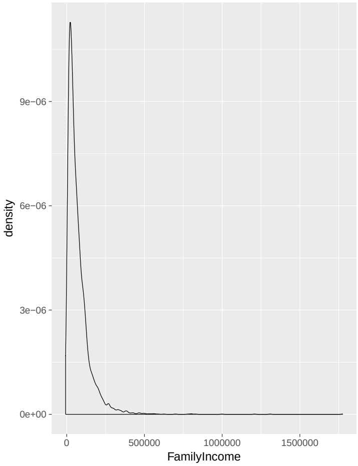

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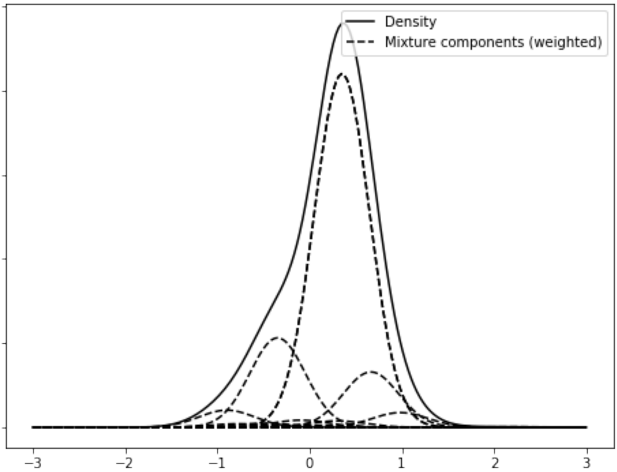
Why synthetic data?

Rubin (1993) and Little (1993) proposed the synthetic data.

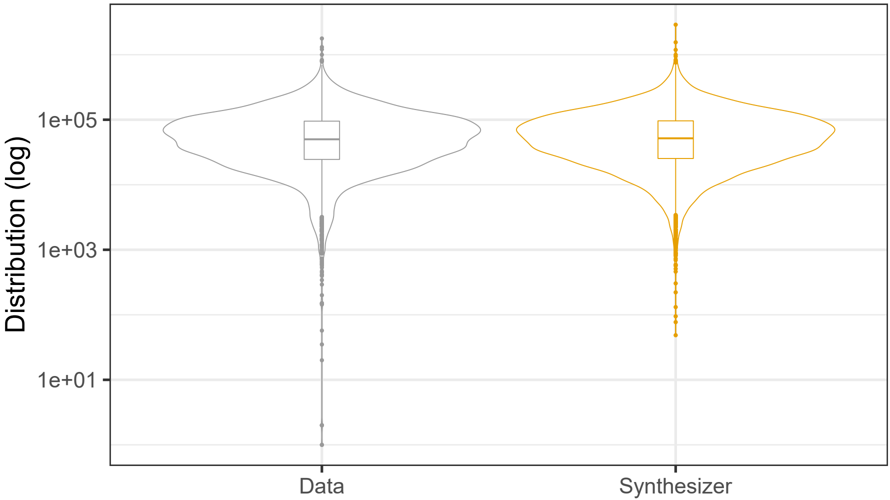
- Simulate records from statistical models that are estimated from the original confidential data.
- Balance of data utility and disclosure risks
  - preserve relationships of variables
  - low disclosure risks
- Allow data analysts to make valid inference for a wide class of analyses.

Example of a flexible synthesizer

A two-level hierarchical parametric finite mixture synthesizer



Synthesizer induces smoothing of data distribution



Risk evaluation: Intruder's knowledge and behavior

Variable	Description
Gender	Gender of the reference person; 2 categories
Age	Age of the reference person; 5 categories
Region	Region of the CU; 4 categories
Education Level	Education level of the reference person; 8 categories
Urban	Urban status of the CU; 2 categories
Marital Status	Marital status of the reference person; 5 categories
Urban Type	Urban area type of the CU; 3 categories
Family Size	Size of the CU; 11 categories
Earner	Earner status of the reference person; 2 categories
Family Income	Imputed and reported income before tax of the CU;

- A known pattern of the un-synthesized categorical variables, $\mathbf{X}_i^p \subseteq \mathbf{X}_i$, e.g. (Gender, Age, Region).
- The true value of synthesized family income y_i.
- A name or identity of interest.

Identification risks based on notion of isolation

- Define radius r of synthetic data y^* in pattern p around the truth y.
- Use percentage radius, e.g. $r = 20\%$.
  - e.g. For a CU i with $50,000 family income, the interval / ball is: [$40,000, $60,000].
- Outside of radius $\rightarrow$ isolation.
- Do this for each record y_i: all y_j^*'s in pattern p.
- Identification risk (IR) for each record is a probability.

Risk as probability of identification disclosure

Formally, for CU i with pattern p:

$$\begin{aligned} IR_i &:= \text{Pr}\,(\text{identification disclosure of } i) \\ &= \frac{\sum_{j \in M_{p,i}} \mathbb{I}\left(y_j^* \notin B(y_i, r)\right)}{|M_{p,i}|} \times T_i. \end{aligned} \quad (1)$$

- $IR_i \in [0, 1]$.
- Larger IR_i and closer to 1: higher risks.
- Smaller IR_i and closer to 0: lower risks.

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A new risk-adjusted synthesizer

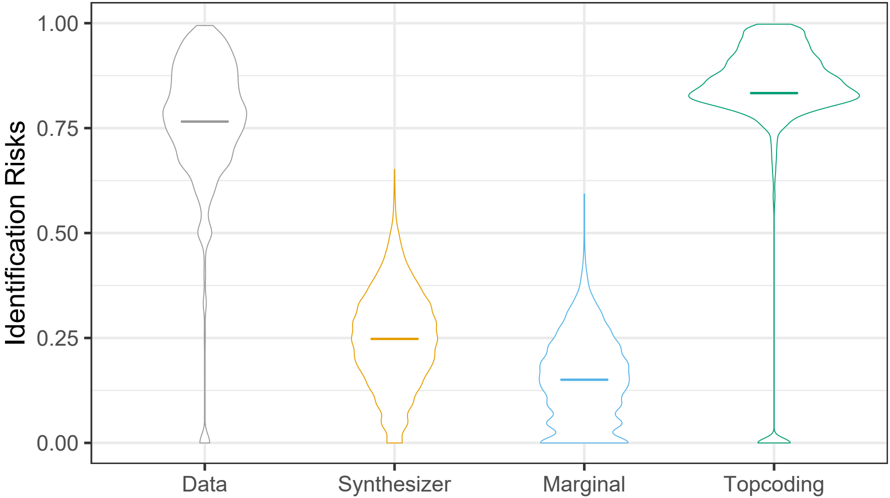
- Use weight $\alpha_i \in [0, 1]$ for CU i.
- Evaluate IR_i^c in the confidential data.
- $\alpha_i = 1 - IR_i^c$: higher $IR_i^c \rightarrow$ higher risk $\rightarrow$ lower α_i.
- Selectively downweight to defeat the likelihood principle:

$$\left[\prod_{i=1}^{n} p\left(y_i \mid \boldsymbol{\theta}\right)^{\alpha_i}\right] p\left(\boldsymbol{\theta} \mid \gamma\right). \tag{2}$$

  - $\boldsymbol{\theta}$: model parameters
  - γ: model hyperparameters

- Surgical distortion: scalar α vs vector α_i.

Violin plots of identification risks



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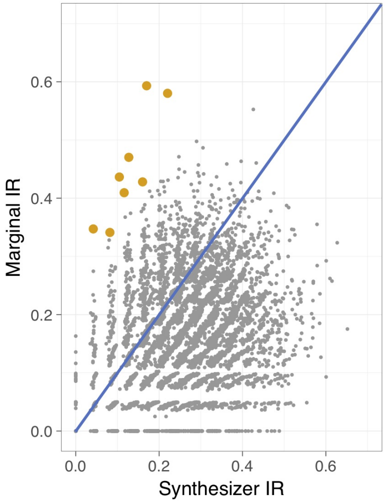
Whack-a-mole issue



- Risky record value shrinkage leaves moderate risk record exposed

Pairwise weighting ties records together

- Marginal risk probability for CU i with pattern p:

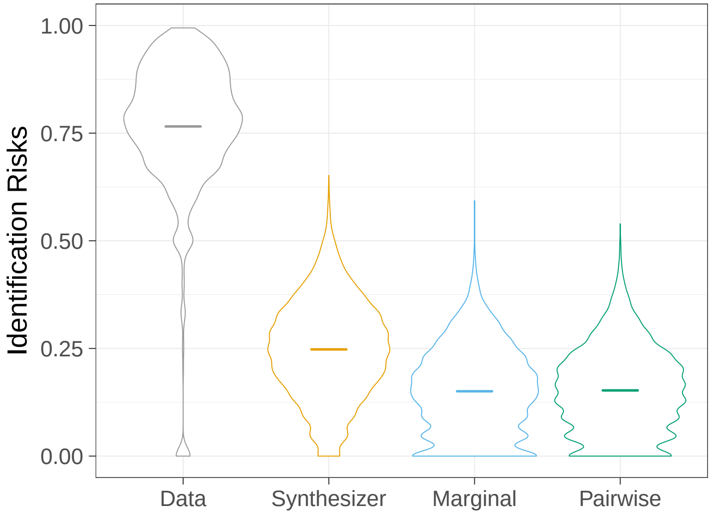
$$IR_i^c = \frac{\sum_{h \in M_{p,i}} \mathbb{I}\left(y_h \notin B(y_i, r)\right)}{|M_{p,i}|}, \; \alpha_i = 1 - IR_i^c.$$

- Pairwise risk probability for pairs of CUs (i, j) in *same* pattern p:

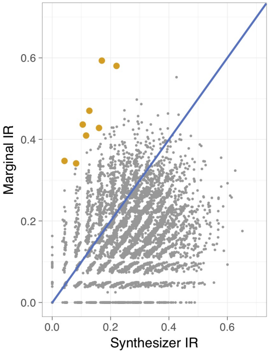
$$IR_{i,j}^c = \frac{\sum_{h \in M_{p,(i,j)}} \mathbb{I}\left(y_h \notin B(y_i, r) \cap y_h \notin B(y_j, r)\right)}{|M_{p,(i,j)}|}, \; \alpha_{i,j} = 1 - IR_{i,j}^c.$$

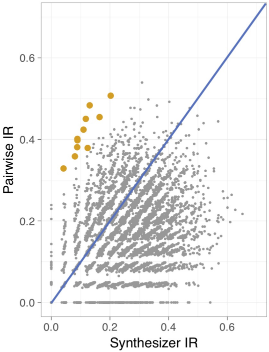
- $(\tilde{\alpha}_i)$ within each pattern constructed as dependent.
- Leave moderate-risk records more covered in the synthetic data.
- Expected to mitigate the *whack-a-mole*.

Pairwise distribution of risks is more compressed



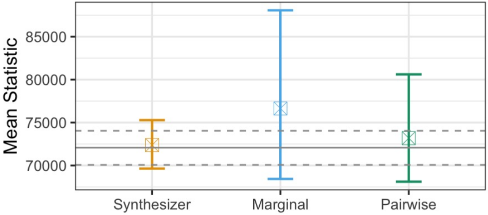
Pairwise partially resolves whack-a-mole issues

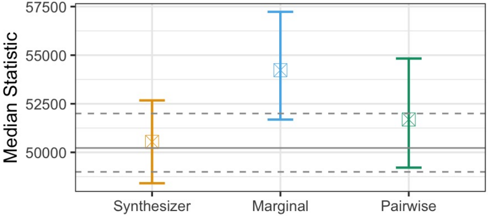




Results of utility

Horizontal lines: mean (solid) and 95% confidence intervals (dashed) from confidential data.





Results of utility cont’d

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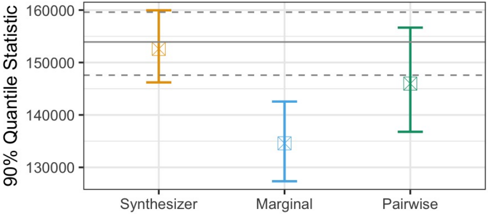

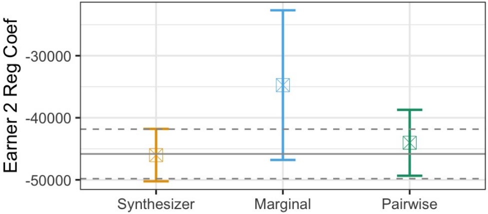

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Summary

- A general framework to achieve desired utility-risk trade-off balance.
- Downweight scheme works for any synthesizer with high utility.
- Pairwise downweight is more risk-efficient: better control of the identification risks and little loss of utility.
- Local weight adjustments can improve utility preservation and little loss of privacy protection.
- The use of topcoding incorrectly assumes which records express high risks.

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References

- An, D. and Little, R. J. A. (2007), Multiple imputation: an alternative to top coding for statistical disclosure control. *Journal of the Royal Statistical Society, Series A*, 170, 923-940.
- Hu, J., Savitsky, T. D. and Williams, M. R. (forthcoming), Risk-efficient Bayesian pseudo posterior data synthesis for privacy protection, *Journal of Survey Statistics and Methodology*.
- Little, R. J. A. (1993). Statistical analysis of masked data. *Journal of Official Statistics* 9, 407-426.
- Rubin, D. B. (1993). Discussion statistical disclosure limitation. *Journal of Official Statistics* 9, 461-468.

Example of a flexible synthesizer

Two level hierarchical parametric finite mixture synthesizer:

$$
\begin{aligned}
y_i \mid \mathbf{X}_i, z_i, \mathbf{B}^*, \boldsymbol{\sigma}^* &\sim \text{Normal}(y_i \mid \mathbf{x}_i^{'} \boldsymbol{\beta}_{z_i}^*, \sigma_{z_i}^*), && (3)\\
z_i \mid \pi &\sim \text{Multinomial}(1; \pi_1, \cdots, \pi_K), && (4)
\end{aligned}
$$

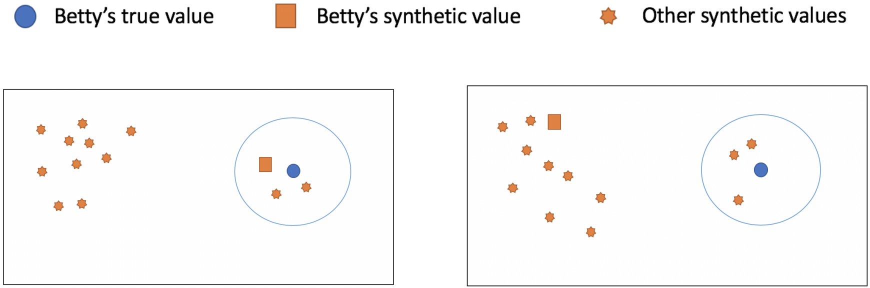
We induce sparsity in the number of clusters with,

$$
\begin{aligned}
(\pi_1, \ldots, \pi_K) &\sim \text{Dirichlet}\left(\frac{\alpha}{K}, \ldots, \frac{\alpha}{K}\right), && (5)\\
\alpha &\sim \text{Gamma}(a_\alpha, b_\alpha). && (6)
\end{aligned}
$$

Estimate the location, scale, proportion, and number of clusters

Toy example: compute probability of identification

- Fewer synthetic values inside the interval / ball $\rightarrow$ the intruder has a higher probability of guessing the record of the name they seek.



Scenario 1:
$IR_i = \frac{10}{13} \times 1 = \frac{10}{13}$.

Scenario 2:
$IR_i = \frac{10}{13} \times 0 = 0$.

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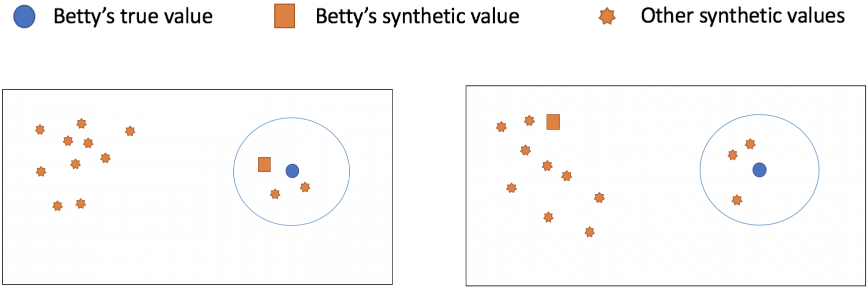

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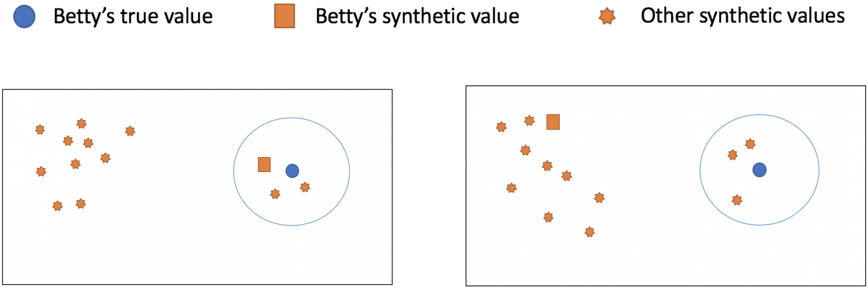

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$$\alpha_{i,j} = 1 - IR^c_{i,j}, \;\; \propto 1/IR^c_{i,j} \quad (8)$$

- Sum over all $\alpha_{i,j}$ for $j \neq i$, and divide by $|M_{p,i}| - 1$
- $(\tilde{\alpha}_i)$ within each pattern constructed as dependent.
- Reduce degree of shrinking of each high-risk record and leave moderate-risk records more covered in the synthetic data
- Expected to mitigate the *whack-a-mole*

$$\tilde{\alpha}_i = \frac{\sum_{j=1, j \neq i \in M_{p,i}} \alpha_{i,j}}{|M_{p,i}| - 1}. \quad (9)$$