

# Risk-Efficient Bayesian Data Synthesis for Privacy Protection

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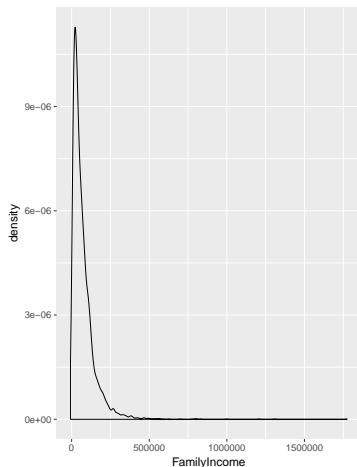
# Outline

- 1 The problem: CE income data
- 2 Synthetic data and probability of identification risk
- 3 Risk-adjusted synthesizer
- 4 “Whack-a-mole” risk pop-ups and our solution
- 5 Summary and references

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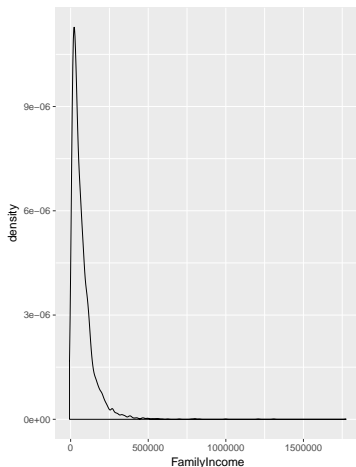
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# The CE data: highly skewed family income



- The Consumer Expenditure Surveys (CE).
- Family income: before tax for each consumer unit (CU).
- High risk records assumed to be in the tail, e.g., CUs with extremely high family income.
- What to do? Topcoding (statistical disclosure control).

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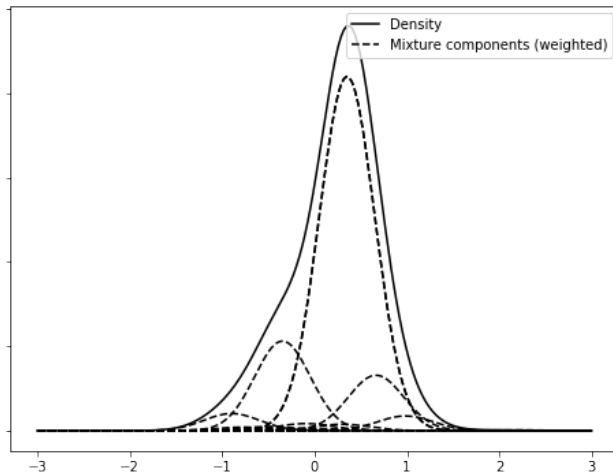
# Why synthetic data?

Rubin (1993) and Little (1993) proposed the synthetic data.

- Simulate records from statistical models that are estimated from the original confidential data.
- **Balance** of data **utility** and **disclosure risks**
  - preserve relationships of variables
  - low disclosure risks
- Allow data analysts to make valid inference for a wide class of analyses.

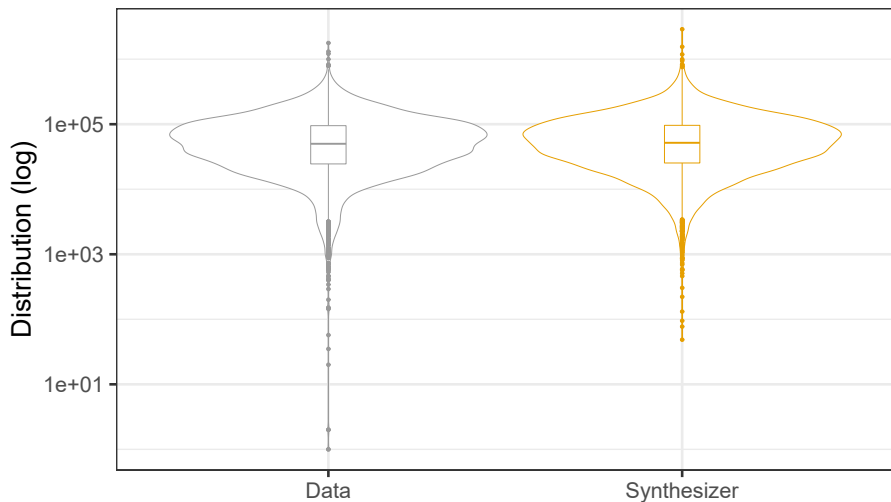
# Example of a flexible synthesizer

A two-level hierarchical parametric finite mixture synthesizer





# Synthesizer induces smoothing of data distribution



# Risk evaluation: Intruder's knowledge and behavior

Variable	Description
Gender	Gender of the reference person; 2 categories
Age	Age of the reference person; 5 categories
Region	Region of the CU; 4 categories
Education Level	Education level of the reference person; 8 categories
Urban	Urban status of the CU; 2 categories
Marital Status	Marital status of the reference person; 5 categories
Urban Type	Urban area type of the CU; 3 categories
Family Size	Size of the CU; 11 categories
Earners	Earners status of the reference person; 2 categories
Family Income	Imputed and reported income before tax of the CU;

- A known pattern of the un-synthesized categorical variables,  $\mathbf{X}_i^p \subseteq \mathbf{X}_i$ , e.g. (Gender, Age, Region).
- The true value of synthesized family income  $y_i$ .
- A name or identity of interest.

# Identification risks based on notion of isolation

- Define radius  $r$  of synthetic data  $y^*$  in pattern  $p$  around the truth  $y$ .
- Use percentage radius, e.g.  $r = 20\%$ .
  - e.g. For a CU  $i$  with \$50,000 family income, the interval / ball is: [\$40,000, \$60,000].
- Outside of radius  $\rightarrow$  isolation.
- Do this for each record  $y_i$ : all  $y_j^*$ 's in pattern  $p$ .
- Identification risk (IR) for each record is a **probability**.

# Risk as probability of identification disclosure

Formally, for CU  $i$  with pattern  $p$ :

$$\begin{aligned} IR_i &:= \Pr(\text{identification disclosure of } i) \\ &= \frac{\sum_{j \in M_{p,i}} \mathbb{I}(y_j^* \notin B(y_i, r))}{|M_{p,i}|} \times T_i. \end{aligned} \tag{1}$$

- $IR_i \in [0, 1]$ .
- Larger  $IR_i$  and closer to 1: higher risks.
- Smaller  $IR_i$  and closer to 0: lower risks.

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# A new risk-adjusted synthesizer

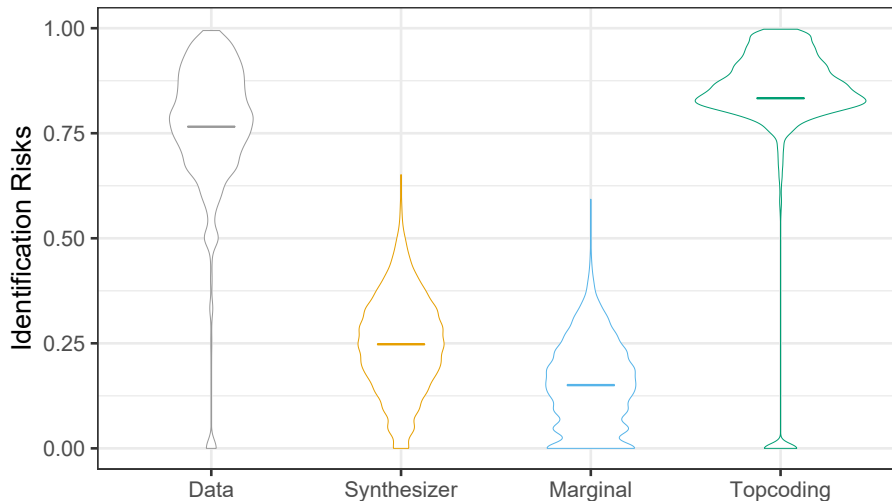
- Use weight  $\alpha_i \in [0, 1]$  for CU  $i$ .
- Evaluate  $IR_i^c$  in the **confidential data**.
- $\alpha_i = 1 - IR_i^c$ : higher  $IR_i^c \rightarrow$  higher risk  $\rightarrow$  lower  $\alpha_i$ .
- **Selectively downweight** to defeat the likelihood principle:

$$\left[ \prod_{i=1}^n p(y_i \mid \theta)^{\alpha_i} \right] p(\theta \mid \gamma). \quad (2)$$

- $\theta$ : model parameters
- $\gamma$ : model hyperparameters

- Surgical distortion: scalar  $\alpha$  vs vector  $\alpha_i$ .

# Violin plots of identification risks

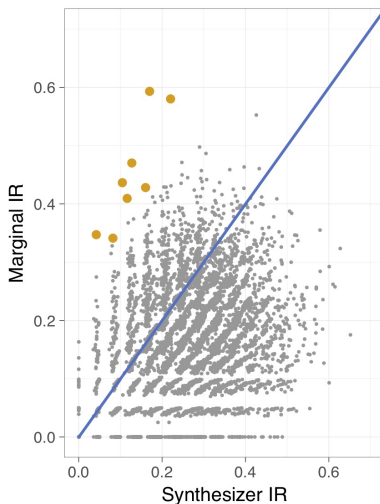


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# Whack-a-mole issue



- Risky record value shrinkage leaves moderate risk record exposed

# Pairwise weighting ties records together

- Marginal risk probability for CU  $i$  with pattern  $p$ :

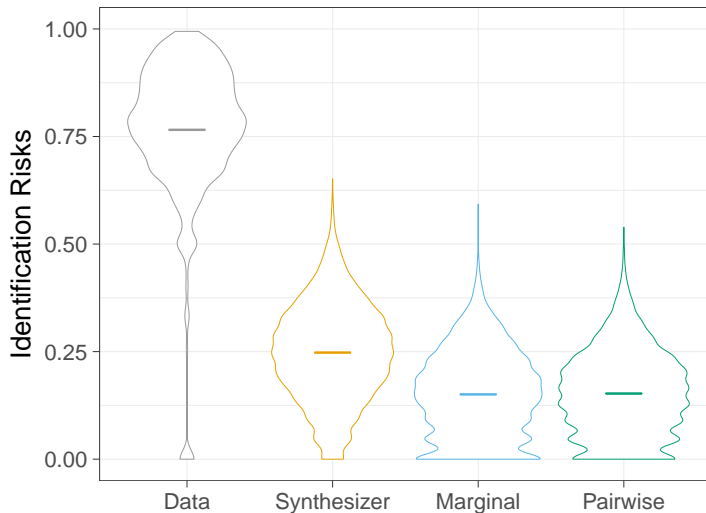
$$IR_i^c = \frac{\sum_{h \in M_{p,i}} \mathbb{I}(y_h \notin B(y_i, r))}{|M_{p,i}|}, \alpha_i = 1 - IR_i^c.$$

- **Pairwise** risk probability for pairs of CUs  $(i, j)$  in *same* pattern  $p$ :

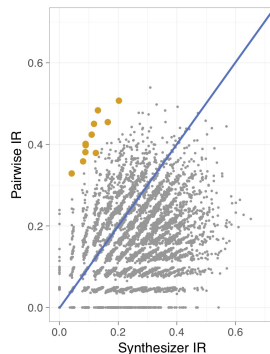
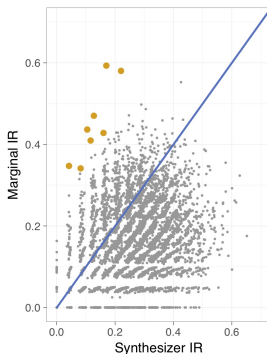
$$IR_{i,j}^c = \frac{\sum_{h \in M_{p,(i,j)}} \mathbb{I}(y_h \notin B(y_i, r) \cap y_h \notin B(y_j, r))}{|M_{p,(i,j)}|}, \alpha_{i,j} = 1 - IR_{i,j}^c.$$

- $(\tilde{\alpha}_i)$  within each pattern constructed as **dependent**.
- Leave moderate-risk records more covered in the synthetic data.
- Expected to **mitigate** the *whack-a-mole*.

# Pairwise distribution of risks is more compressed

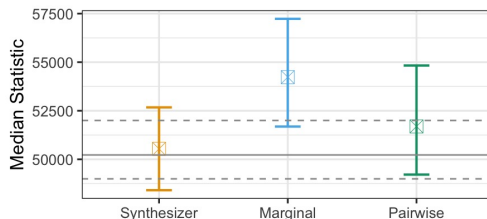
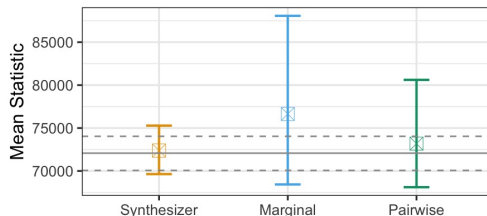


# Pairwise partially resolves whack-a-mole issues



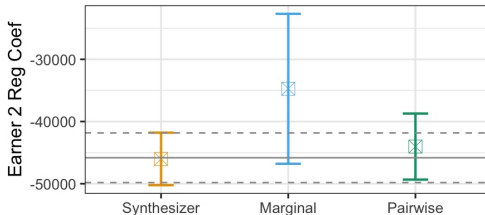
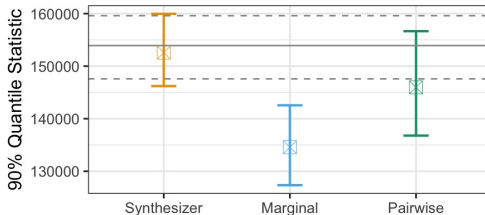
## Results of utility

Horizontal lines: mean (solid) and 95% confidence intervals (dashed) from confidential data.



## Results of utility cont'd

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# Summary

- A general framework to achieve desired utility-risk trade-off balance.
- Downweight scheme works for **any synthesizer** with high utility.
- Pairwise downweight is **more risk-efficient**: better control of the identification risks and little loss of utility.
- Local weight adjustments can improve utility preservation and little loss of privacy protection.
- The use of topcoding incorrectly assumes which records express high risks.



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# References

- An, D. and Little, R. J. A. (2007), Multiple imputation: an alternative to top coding for statistical disclosure control. *Journal of the Royal Statistical Society, Series A*, 170, 923-940.
- Hu, J., Savitsky, T. D. and Williams, M. R. (forthcoming), Risk-efficient Bayesian pseudo posterior data synthesis for privacy protection, *Journal of Survey Statistics and Methodology*.
- Little, R. J. A. (1993). Statistical analysis of masked data. *Journal of Official Statistics* 9, 407-426.
- Rubin, D. B. (1993). Discussion statistical disclosure limitation. *Journal of Official Statistics* 9, 461-468.

# Example of a flexible synthesizer

Two level hierarchical parametric **finite mixture** synthesizer:

$$y_i \mid \mathbf{X}_i, z_i, \mathbf{B}^*, \boldsymbol{\sigma}^* \sim \text{Normal}(y_i \mid \mathbf{x}_i' \boldsymbol{\beta}_{z_i}^*, \sigma_{z_i}^*), \quad (3)$$

$$z_i \mid \pi \sim \text{Multinomial}(1; \pi_1, \dots, \pi_K), \quad (4)$$

We induce **sparsity** in the number of **clusters** with,

$$(\pi_1, \dots, \pi_K) \sim \text{Dirichlet}\left(\frac{\alpha}{K}, \dots, \frac{\alpha}{K}\right), \quad (5)$$

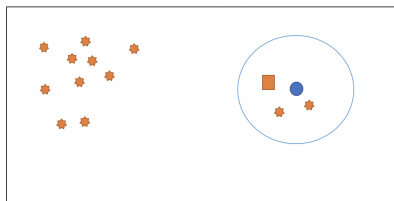
$$\alpha \sim \text{Gamma}(a_\alpha, b_\alpha). \quad (6)$$

Estimate the **location**, **scale**, **proportion**, and **number** of clusters

# Toy example: compute probability of identification

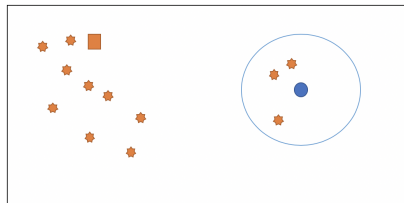
- Fewer synthetic values inside the interval / ball  $\rightarrow$  the intruder has a higher probability of guessing the record of the name they seek.

● Betty's true value     
 ■ Betty's synthetic value     
 ★ Other synthetic values



Scenario 1:

$$IR_i = \frac{10}{13} \times 1 = \frac{10}{13}.$$



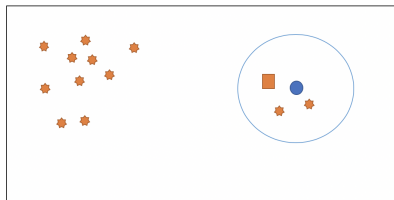
Scenario 2:

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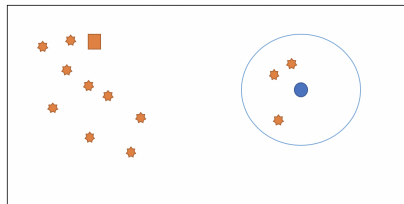
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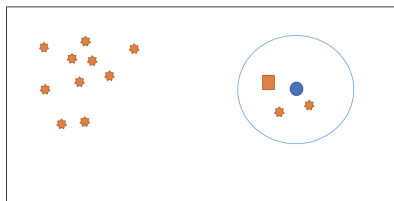
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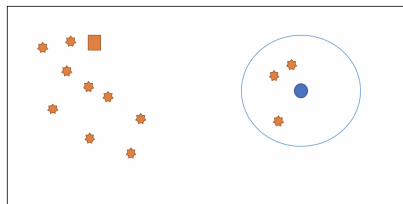
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$$\alpha_{i,j} = 1 - IR_{i,j}^c, \propto 1/IR_{i,j}^c \quad (8)$$

- Sum over all  $\alpha_{i,j}$  for  $j \neq i$ , and divide by  $|M_{p,i}| - 1$
- $(\tilde{\alpha}_i)$  within each pattern constructed as **dependent**.
- Reduce degree of shrinking of each high-risk record and leave moderate-risk records more covered in the synthetic data
- Expected to **mitigate** the *whack-a-mole*

$$\tilde{\alpha}_i = \frac{\sum_{j=1, j \neq i \in M_{p,i}} \alpha_{i,j}}{|M_{p,i}| - 1}. \quad (9)$$