CS576 Multimedia System Design

Assignment#2

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Programming assignment:

Instructions for compiling and running the program

Program Name: imageReader

To compile the program type "*javac imageReader.java*" in command prompt

To run the program type "*java imageReader <input_img_name> <number_of_coefficients>*"

Here:

- input_img_name is the name of the input image file
- number_of_coefficients is the integral number that represents the number of coefficients to use for decoding.
    - 262144, 131072, 16384
    - For progressive analysis, set number_of_coefficients = -1

For example: To perform DCT and DWT on input image (Lenna.rgb) with 131072 coefficients use the following command:

java imageReader Lenna.rgb 131072

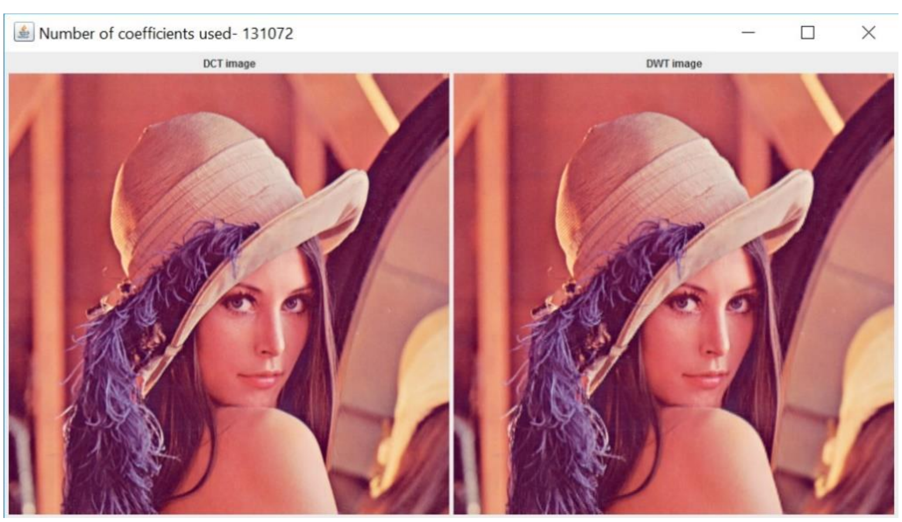
Output:

Program will produce 2 images one for DCT and one for DWT in the same panel. Left panel in the frame is DCT and right panel in the frame is DWT image.

For progressive analysis, new images get replaced on the same JFrame, and there is a delay of 5 milliseconds.

Input: java imageReader Lenna.rgb 131072

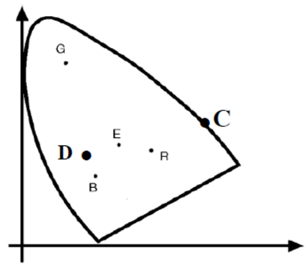
Output for the above command:



Theory Questions

Question 1: Color theory

One of the uses of chromaticity diagrams is to find the gamut of colors given the primaries. It can also be used to find dominant and complementary colors – Dominant color of a given color D (or dominant wavelength in a color D) is defined as the spectral color which can be mixed with white light in order to reproduce the desired D color. Complementary colors are those which when mixed in some proportion create the color white. Using these definitions and the understanding of the chromaticity diagram that you have, answer the following



- **Show in the plot alongside the dominant wavelength of color D.**

  Answer:
  The Dominant wavelength of color D in the given image above is found out by extending a line from white point E passing through D and touching the point ‘w’ as shown below. The point of intersection (i.e. w) nearer to the color D is the dominant wavelength.

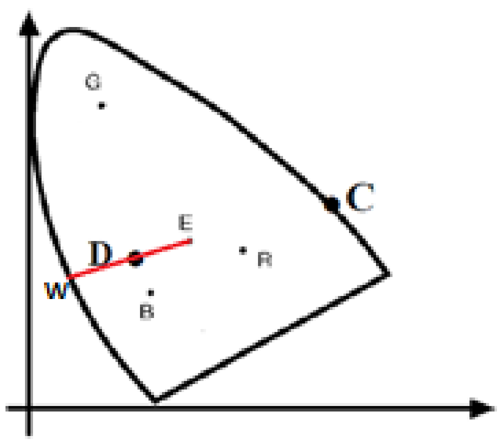


Figure 1

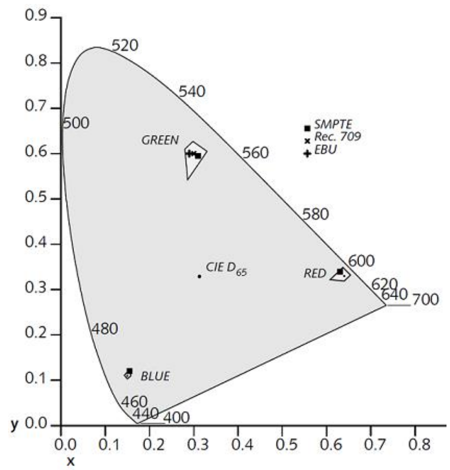


Figure 2 [1]

Thus, comparing Figure 1 with Figure 2, the point ‘w’ lies approximately between 480 – 490 nm.

- **Do all colors have a dominant wavelength? Explain your reasoning**

Answer:
All colors do not have a dominant wavelength. Only pure spectral colors like red, green, blue, etc. has dominant wavelength. The point of intersection of non-spectral color like purple may hit the flat base part, so, for non-spectral colors hue/dominant wavelength cannot be assigned.

- **Show in the plot the spectral color which is complimentary to color C.**

Answer:
The complimentary color of C is obtained by drawing a line from C passing through E (white light) towards the opposite end of C, where it touches point 'w' is the complimentary color to color C. Thus, comparing Figure 3 with Figure 2 above the wavelength ranges approximately between 480-490nm.

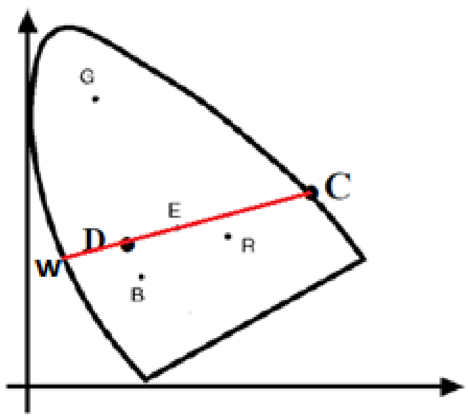


Figure 3

- **Given the placements of R, G, B as primaries around the equiluminous point E, can you find out all points which have a value of R=0.5, explaining your reasoning.** ***Remember that the chromaticity space is non-linear as explained in class, meaning that although E is defined by equal contributions of R, G and B, it may not necessarily be at the centroid of R, G and B.***

Answer:
We know that at point E, R=G=B =1/3.
At B, B =1 & R=G=0 and at G, B=R=0 & G=1 and at R, R=1 & G=B=0.
Dropping a perpendicular line from B through E to line RG, divides the line RG into two equal parts qualitatively at point 'X'. Similarly, dropping a perpendicular line from G through E to line RB divides the line RB into two equal parts qualitatively at point 'Y'. Thus, at point 'X' value of R=G=0.5 & B=0 and at point 'Y' value of R=B=0.5 & G=0.
As shown below in Figure 4 joining the point 'X' and 'Y' becomes the locus with R=0.5.

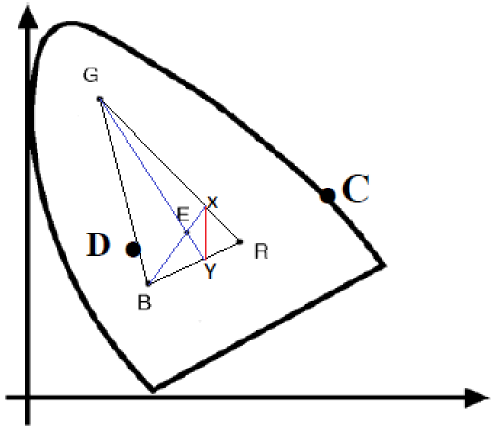


Figure 4

- **Take the R = 0.5 locus above, how does this line map in the RGB space. Explain the mapping from the chromaticity diagram to the RGB space.**

Answer:
When the above XY line from figure 4 is mapped to a RGB space, we get a plane (highlighted in color red) in RGB space as shown below in figure 5.
The locus R=0.5, corresponds to the plane because, all the points in the plane have R=0.5 and G and B varying between [0,1]

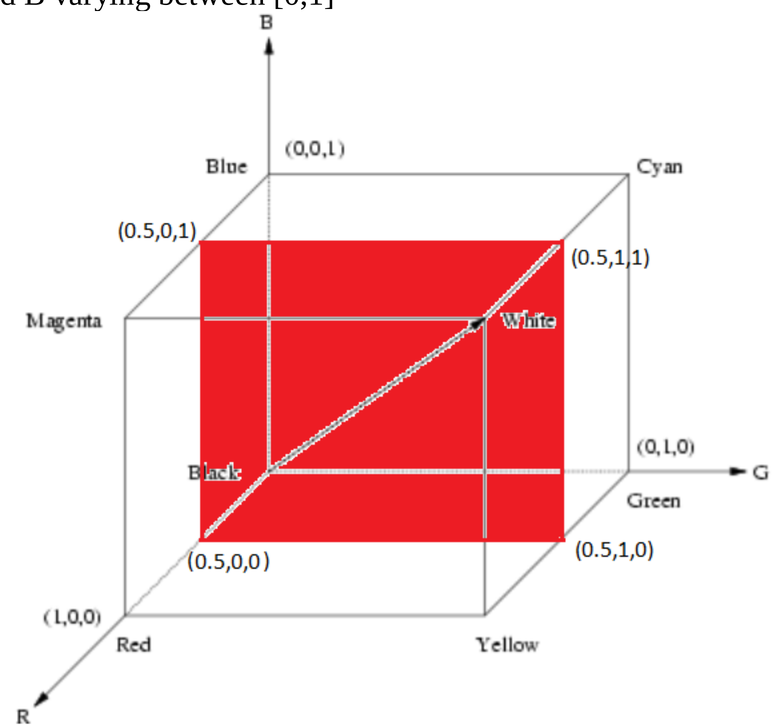


Figure 5

***Extra-Credit* Can you find the locus of points on the chromaticity diagram with R, G, B and E fixed as shown, for which R=0.75, this might not be as easy as you think!**

<u>Answer</u>:

From the above Figure 4, we know that point X represents R=0.5 & G=0.5 and point Y represents R=0.5 & B=0.5.

Since line RX and RY represent R=0.5, dividing these lines qualitatively into two equal parts gives R=0.25 with respect to triangle XBR, and R=0.75 with respect to triangle RGB.

So, in triangle XBR in Figure 4, dropping the perpendicular line from B to line RX divides the line RX qualitatively into two equal parts at point P and dropping the perpendicular line from X to line RY divides the line RY qualitatively into two equal parts at point Q, as shown below in Figure 6. Thus w.r.t triangle RGB at point P, value of R is 0.75 & value of G is 0.25 and at point Q, value of R is 0.75 & value of B is 0.25.

Joining the line PQ represents the locus of points in the chromatic diagram with R=0.75 (represented by red line in the diagram below).

To verify the point, extending the lines G=0.5 and G=0 meet at one point represented by G', similarly extending the line R=0 and R=0.5 meet at one point, same for B=0 and B=0.5.

Thus, line PQ represents the locus where R=0.75.

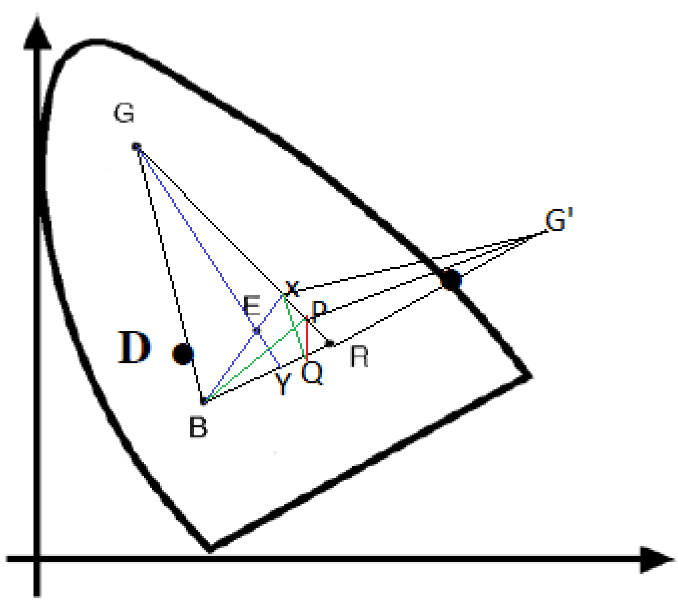


Figure 6

Question 2: Generic Compression Problem

The following sequence of real numbers has been obtained sampling a signal: 5.8, 6.2, 6.2, 7.2, 7.3, 7.3, 6.5, 6.8, 6.8, 6.8, 5.5, 5.0, 5.2, 5.2, 5.8, 6.2, 6.2, 6.2, 5.9, 6.3, 5.2, 4.2, 2.8, 2.8, 2.3, 2.9, 1.8, 2.5, 2.5, 3.3, 4.1, 4.9 This signal is then quantized using the interval [0,8] and dividing it into 32 uniformly distributed levels

- **What does the quantized sequence look like? For ease of computation, assume that you placed the level 0 at 0.25, the level 1 at 0.5P, level 2 at 0.75, level 3 at 1.0 and so on. This should simplify your calculations. Round off any fractional value to the nearest integral levels**

Answer:
By taking level 0 at 0.25, level 1 at 0.5, level 2 at 0.75 so on till level 31 at 8 we get the quantized sequence as follows:
22, 24, 24, 28, 28, 28, 25, 26, 26, 26, 21, 19, 20, 20, 22, 24, 24, 24, 23, 24, 20, 16, 10, 10, 8, 11, 6, 9, 9, 12, 15, 19

- **How many bits do you need to transmit it?**

Answer:
There are 32 samples, each need 5 bits.
Total bits needed to transmit above signal is 32 x 5 = 160 bits.

- **If you need to encode the quantized output using DPCM. Compute the successive differences between the values – what is the maximum and minimum value for the difference? Assuming that this is your range, how many bits are required to encode the sequence now? Ignore the first value**

Answer:
If we quantize the output using DPCM, the quantized sequence will be:
2, 0, 4, 0, 0, -3, 1, 0, 0, -5, -2, 1, 0, 2, 2, 0, 0, -1, 1, -4, -4, -6, 0, -2, 3, -5, 3, 0, 3, 3, 4.

Maximum value for the difference is: 4
Minimum value for the difference is: -6

Assuming the range to be from [-6, 4]
As there are 11 levels, it requires 4 bits to encode each level. Therefore, number of bits required to encode the sequence - ignoring the first value is 31 x 4 = 124 bits.

- **What is the compression ratio you have achieved?**

Answer:
Compression ratio is given by: uncompressed bits/ compressed bits = 31x5/31x4 = 1.25:1. This obtains space saving of 20%.

- **Instead of transmitting the differences, you use Huffman coded values for the differences. How many bits do you need now to encode the sequence?**

Answer:

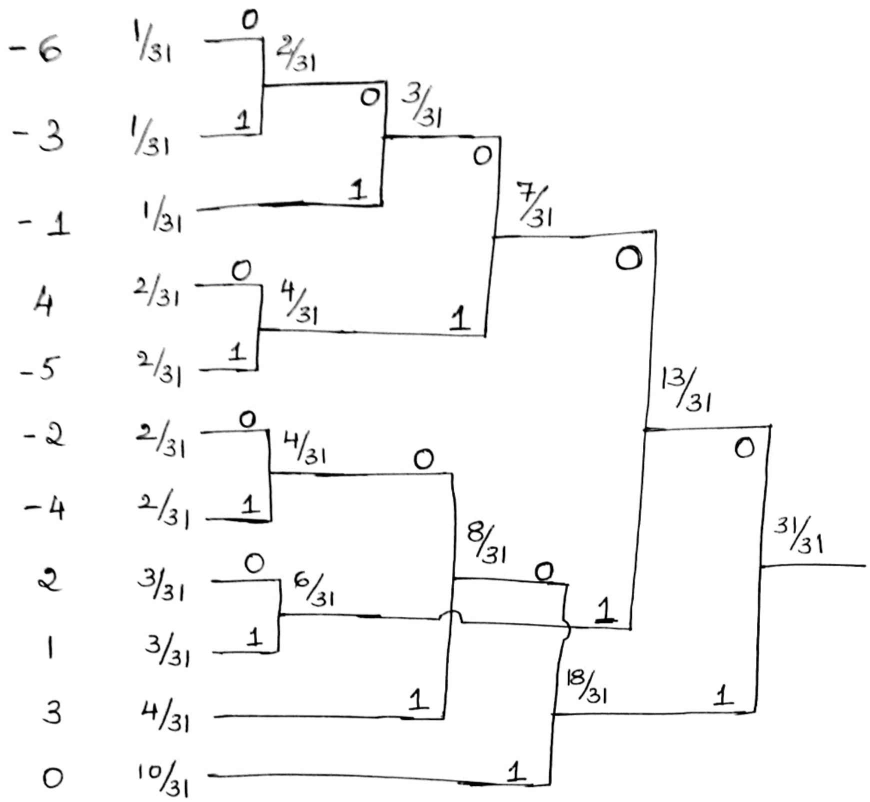


Figure 7

Differences	Probability	Huffman code
-6	1/31	00000
-3	1/31	10000
-1	1/31	1000
4	2/31	0100
-5	2/31	1100
-2	2/31	0001
-4	2/31	1001
2	3/31	010
1	3/31	110
3	4/31	100
0	10/31	11

Sequence of differences is: 2, 0, 4, 0, 0, -3, 1, 0, 0, -5, -2, 1, 0, 2, 2, 0, 0, -1, 1, -4, -4, -6, 0, -2, 3, -5, 3, 0, 3, 3, 4.
Number of bits required to encode this sequence is: 2x5 + 9x4 + 10x3 + 10x2 = 96 bits

- **What is the compression ratio you have achieved now?**

Answer:
Compression ratio is given by: uncompressed bits/compressed bits = 31x5/96 = 1.61:1. This results in space saving of 38%.

Question 3: Arithmetic Compression
Consider two symbols, A and B, with the probability of occurrence of 0.8 and 0.2, respectively. For the purpose of arithmetic compression, you organize a symbol stream of A's and B's in groups of three and give each unit of three symbols an arithmetic code (Assume that each symbol occurrence is independent of previous symbol occurrences).

- **How many types three symbol units are there and what are their probabilities?**

<u>Answer</u>:
There are 8 three symbol units as shown in Figure 8 and their probabilities are:
AAA: 0.512
AAB: 0.128
ABA: 0.128
ABB: 0.032
BAA: 0.128
BAB: 0.032
BBA: 0.032
BBB: 0.008

- **Show the arrangement of these three symbol units on the unit interval [0, 1] and determine the arithmetic code for each**

<u>Answer</u>:
Below is the representation of three symbol units on a unit interval [0,1] and their arithmetic code as follows:
AAA - 100
AAB - 101
ABA - 11000
ABB - 11001
BAA - 1110
BAB - 11110
BBA - 111110
BBB – 1111111

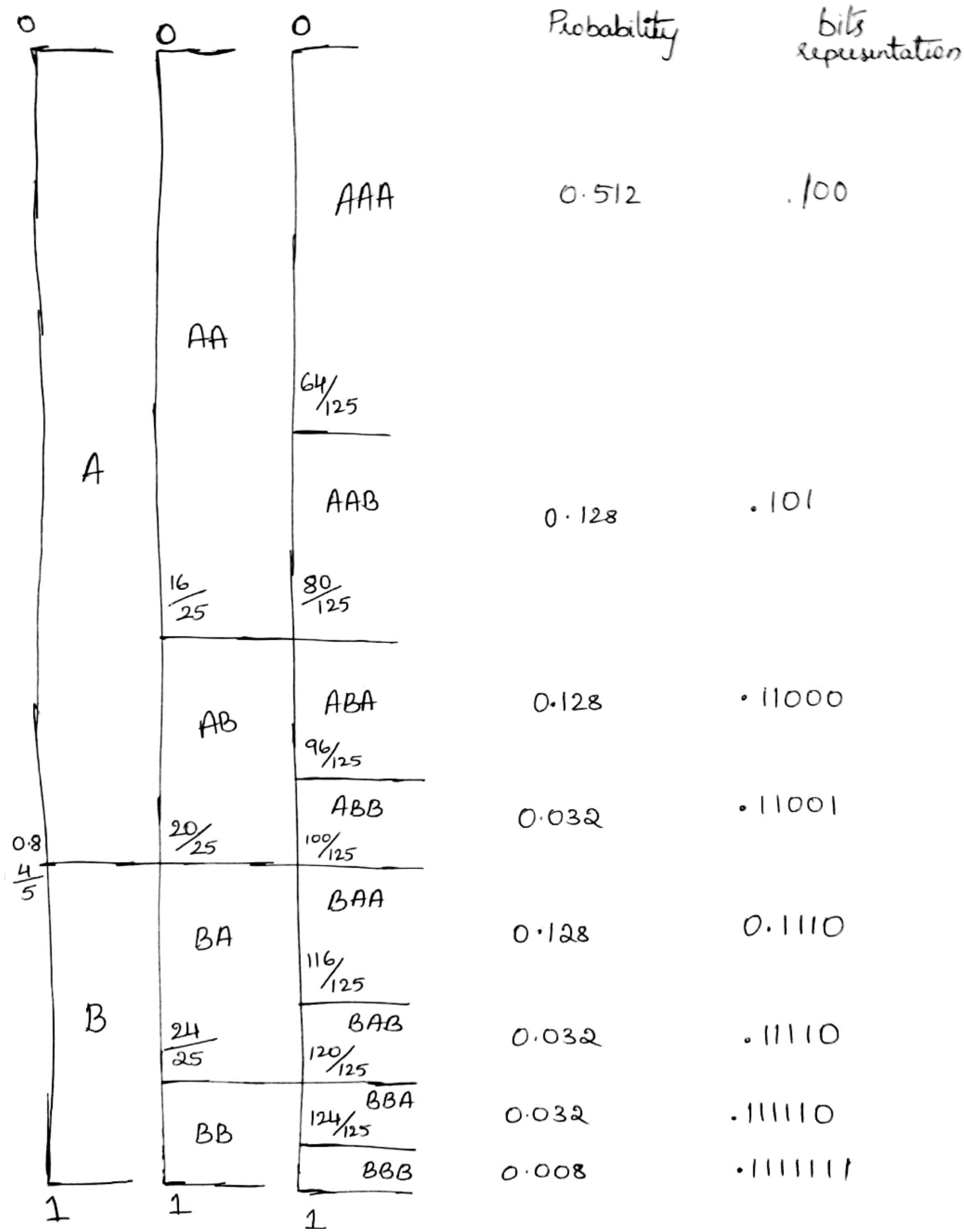

0
0
0
Probability
bits representation
AAA
0.512
.100
AA
64/125
A
AAB
0.128
.101
16/25
80/125
ABA
0.128
•11000
AB
96/125
ABB
0.032
•11001
0.8
4/5
20/25
100/125
BAA
0.128
0.1110
BA
116/125
B
BAB
0.032
•11110
24/25
120/125
BBA
124/125
0.032
•111110
BB
BBB
0.008
•1111111
1
1
1

Figure 8

- **What is the average code word length? Is it optimum?**

Answer:

Average code word length = $\sum_{i=1}^{N} P(i) * l(i)$, where N is number of symbols, P(i) is probability of the symbol and $l(i)$ is length of binary representation

Average code word length
= 0.512x3 + 0.128x3 + 0.128x5 + 0.032x5 + 0.128x4 + 0.032x5 + 0.032x6 + 0.008x7
= 1.536 +0.384+0.640+0.16+0.512+0.16+0.192+0.056
= 3.64

Entropy = $-\sum_{i=1}^{N} P(i) * log_2 P(i)$
= -[0.512x(-0.966) + 0.128x(-2.966) + 0.128x(-2.966) + 0.032x(-4.966) + 0.128x(-2.966) + 0.128x(-2.966) + 0.032x(-4.966) + 0.008x(-6.966)]
= -[(-0.495) + (-0.379) + (-0.379) + (-0.159) + (-0.379) + (-0.379) + (-0.159) + (-0.0557)]
= 2.384

Average code word length is 3.64 whereas Entropy is 2.384, hence it is not optimum.

- **How many bits are required to code the message "ABABBAABBAAABBB"**

Answer:

ABA – 11000
BBA – 111110
ABB – 11001
AAA – 100
BBB – 1111111

Number of bits required to code the "**ABABBAABBAAABBB**" message of 3 symbols is: 26 bits

- **How could you do better than the above code length?**

Answer:

Let us use 2 symbols each to encode the sequence

Code	Probability	Bits
AA	0.64	10
AB	0.16	110
BA	0.16	1110
BB	0.04	11111

Average code word length$\sum_{i=1}^{N} P(i) * l(i)$, where N is 4
= 0.6 * 2 + 0.16 * 3 + 0.16 * 4 + 0.04 * 5
= 1.2 + 0.48 + 0.64+0.2
= 2.52

Average code length when 3 symbols are taken = 3.64 whereas average code length when 2 symbols are taken = 2.52 which is approximately 30% better than 3 symbols. Hence taking 2 symbols at a time is better.

Remark: I also found that average code length for 2 is better than 4 symbols

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Question 4: Entropy *(20 points)*
Consider a communication system that gives out only two symbols X and Y. Assume that the parameterization followed by the probabilities are $P(X) = x^2$ and $P(Y) = (1-x^2)$.

- **Write down the entropy function and plot it as a function of *x*.**

Answer:
Entropy is: $H = -\sum_{i=1}^{N} P(i) * log_2 P(i)$
$= -[P(X) * \log_2 P(X) + P(Y) * \log_2 P(Y)]$
$= -[x^2 \log_2 x^2 + (1-x^2) \log_2(1-x^2)]$

Plot of entropy as a function of x

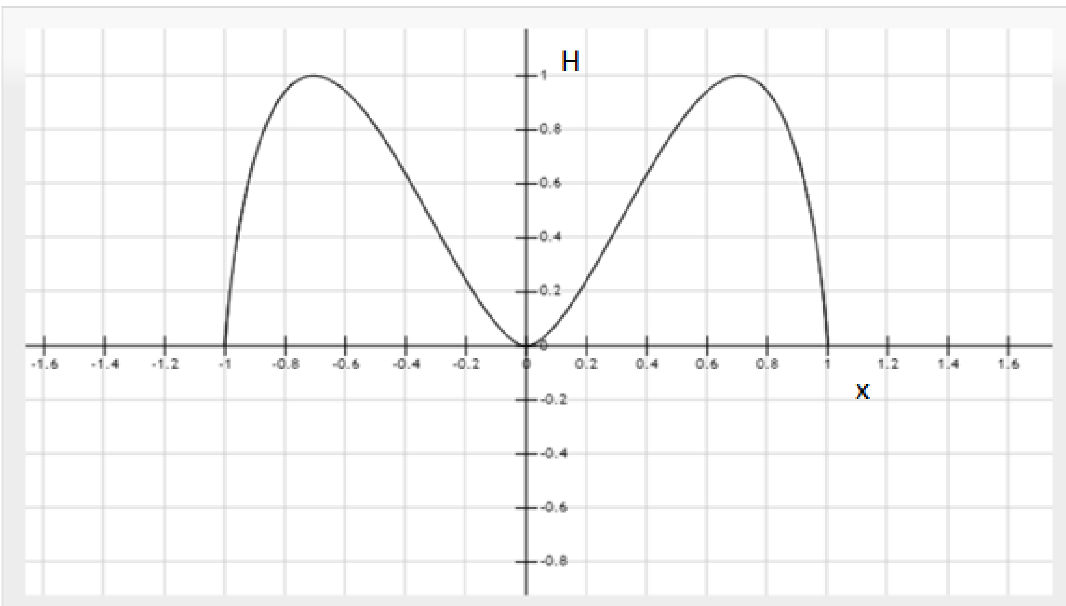

Figure 9

- **From your plot, for what value of *x* does the Entropy become a minimum?**

Answer:
Entropy becomes minimum for x = 0

- **Although the plot visually gives you the value of *x* for which the entropy in minimum, can you now mathematically find out the value(s) for which the entropy is a minimum?**

Answer:
Mathematical way to find out the value for which the entropy becomes minimum is by differentiating and equating to 0

From the Figure 10 below, solving equation (1) gives $x = 0, +1/\sqrt{2}$ and $-1/\sqrt{2}$

To get the minimum point, $dH/dx = 0$ and $d^2H/dx^2 > 0$. When x = 0 in equation (2), we find that $d^2H/dx^2 > 0$.

Now substituting x=0,

$H = -[x^2 \log_2 x^2 + (1-x^2) \log_2(1-x^2)]$

$= -[(0)^2 \log_2(0)^2 + (1-(0)^2) \log_2(1-(0)^2)]$

$= 0$

Hence, entropy becomes minimum at x=0.

$$H = -\left[x^2 \log_2 x^2 + (1-x^2)\log_2(1-x^2)\right]$$

$$-\frac{dH}{dx} = 2x \cdot \log_2 x^2 + \cancel{x^2} \cdot \frac{2x}{\cancel{x^2}} + (-2x)\cdot \log_2(1-x^2) + \cancel{(1-x^2)} \cdot \frac{(-2x)}{\cancel{(1-x^2)}}$$

$$= 2x\left(\log_2 x^2\right) + \cancel{2x} + (-2x)\cdot\log_2(1-x^2) - \cancel{2x}$$

$$= 2x\left(\log_2 x^2 - \log_2(1-x^2)\right)$$

$$= 2x \cdot \log_2\left(\frac{x^2}{1-x^2}\right) \quad \text{——} \quad (1)$$

$$-\frac{d^2H}{dx^2} = 2 \cdot \log_2\left(\frac{x^2}{1-x^2}\right) + 2x \cdot \frac{(1-x^2)}{x^2} \cdot \frac{2x(1-x^2) - x^2(-2x)}{(1-x^2)^2}$$

$$= 2 \log_2\left(\frac{x^2}{1-x^2}\right) + \frac{\cancel{2x(1-x^2)}}{\cancel{x^2}}\left[\frac{2x(1-x^2)}{\cancel{(1-x^2)^2}}\right] + \frac{2x\cancel{(1-x^2)}}{\cancel{x^2}}\left[\frac{\cancel{x^2}(-2x)}{\cancel{(1-x^2)}}\right]$$

$$= 2 \cdot \log_2\left(\frac{x^2}{1-x^2}\right) + 4(1-x^2) + (+4x^2)$$

$$= 2 \cdot \log_2\left(\frac{x^2}{1-x^2}\right) + 4$$

$$\frac{d^2H}{dx^2} = -\left[2 \cdot \log_2\left(\frac{x^2}{1-x^2}\right) + 4\right] \quad \text{——} \quad (2)$$

Solving equation (1), we get

$$2x \cdot \log_2\left(\frac{x^2}{1-x^2}\right) = 0$$

$$x = 0, \pm 1/\sqrt{2}$$

To find maximum and minimum value of x

Substituting 0 to equation (2)

$\frac{d^2H}{dx^2} > 0$, hence entropy becomes minimum at $x = 0$

Substituting $\pm 1/\sqrt{2}$ to equation (2) above,

gives $\dfrac{d^2H}{dx^2} < 0$, hence entropy becomes maximum at $x = \pm 1/\sqrt{2}$

Figure 10

- **Can you do the same for the maximum, that is can you find out value(s) of *x* for which the value is a maximum?**

Answer:

To get the maximum, $dH/dx = 0$ and $d^2H/dx^2 < 0$.
Now, substituting $x = +1/\sqrt{2}$ or $-1/\sqrt{2}$ to equation (2) in figure 10, we find that $d^2H/dx^2 = -4 < 0$.
Hence $x = +1/\sqrt{2}$ or $-1/\sqrt{2}$ results in the maximum value.

Now substituting, $x = +1/\sqrt{2}$ or $-1/\sqrt{2}$

$H = -[x^2 \log_2 x^2 + (1-x^2) \log_2(1-x^2)]$

$= -[(1/\sqrt{2})^2 \log_2(1/\sqrt{2})^2 + (1-(1/\sqrt{2})^2) \log_2(1-(1/\sqrt{2})^2)]$

$= -[(0.5)(-1) + (0.5)(-1)]$

$= -[-1] = 1$

Hence entropy becomes maximum when $x = +/- 1/\sqrt{2}$

References

[1] Poynton, Charles A. "A Guided Tour of Colour Space." Advanced Television and Electronic Imaging Conference, New Foundation for Video Technology: The SMPTE. SMPTE, 1995.