

```

In[1]:= (* find F1 and F0, numerators of rho^2 and log(rho^2) for M11
use expansion of F(k),E(k) about k=1
call log(rho1^2) =logm. rho1^2=|x-x0|^2
m3=rho1^2/rho2^2*)

```

```

In[2]:= f = 2 * log2 - logm/2 + a1 * m3 + a2 * m3^2 +
a3 * m3^3 - (b1 * m3 + b2 * m3^2 + b3 * m3^3 + b4 * m3^4) * logm;
e = 1 + c1 * m3 + c2 * m3^2 + c3 * m3^3 - (d1 * m3 + d2 * m3^2 + d3 * m3^3 + d4 * m3^4) * logm;

```

```

eth = Series[m3^2 * e / m3, {m3, 0, 4}] / m3^2;

```

```

c = Sqrt[a + b];
i10 = Series[4 * f / c, {m3, 0, 4}];
i11 = Series[m3^2 * 4 / b * (a * f / c - c * e), {alf, 0, 6}] / m3^2;
i30 = Series[m3 * 4 * eth / c^3, {m3, 0, 5}] / m3;
i31 = Series[m3 * 4 / b * (a * eth / c^3 - f / c), {m3, 0, 6}] / m3;
i32 = Series[m3 * (4 / b^2) * (a^2 * eth / c^3 - 2 * a * f / c + c * e), {m3, 0, 6}] / m3;

```

```

m11 =
Simplify[Series[m3^2 * x * (i11 + (x^2 + x0^2) * i31 - x * x0 * (i30 + i32)), {m3, 0, 4}] / m3^2;
m12 = Simplify[Series[m3 * x * (xi * (x * i31 - x0 * i30)), {m3, 0, 2}] / m3;
m21 = Simplify[Series[m3 * x * xi * (x * i30 - x0 * i31), {m3, 0, 2}] / m3;
m22 = Simplify[Series[m3 * x * (i10 + dely^2 * i30), {m3, 0, 2}] / m3;

```

```

In[15]:= term=m12;
term=Simplify[term/.{d1→1/4,d2→3/32,d3→15/256,b1→1/8,b2→9/128,c1→(4*log2-1)/4,c2→(24*log2-1)/4,c3→(16*log2-3)/8,c4→(16*log2-3)/8,d4→(16*log2-3)/8},{m3,0,4}];
term=Simplify[term/.{b→2*x*x0,a→x0^2+x^2+xi^2}]

```

$$\begin{aligned}
\text{Out[17]} = & \frac{2 x x i \left(x^2 - x 0^2 + x i^2\right)}{x 0 \left(x^2 + 2 x x 0 + x 0^2 + x i^2\right)^{3/2} m 3} - \\
& \frac{x i \left(-2 x x 0 \left(x 0 - 4 \log 2 \left(2 x + x 0\right) + \log m \left(2 x + x 0\right)\right) + \left(1 + 4 \log 2 - \log m\right) x \left(x^2 + x 0^2 + x i^2\right)\right)}{2 \left(x 0 \left(x^2 + 2 x x 0 + x 0^2 + x i^2\right)^{3/2}\right)} - \\
& \left(\left(x x i \left(16 \left(-2 + 4 \log 2 - \log m\right) x x 0 + 2 \left(-13 + 24 \log 2 - 6 \log m\right) x 0^2 + \right.\right.\right. \\
& \left.\left.\left(-3 + 8 \log 2 - 2 \log m\right) \left(x^2 + x 0^2 + x i^2\right)\right)\right) m 3\right) / \left(32 \left(x 0 \left(x^2 + 2 x x 0 + x 0^2 + x i^2\right)^{3/2}\right)\right) + O[m 3]^2
\end{aligned}$$

```

In[18]:= mcoeff=Simplify[Coefficient[term*m3^2,m3]]
msqden=Simplify[Denominator[mcoeff]]
mnum=Simplify[Numerator[mcoeff]/2]
(* Lead = 2/x0 *)

Out[18]= 
$$\frac{2 x x i \left(x^2 - x 0^2 + x i^2\right)}{x 0 \left(x^2 + 2 x x 0 + x 0^2 + x i^2\right)^{3/2}}$$


Out[19]= 
$$x 0 \left(x^2 + 2 x x 0 + x 0^2 + x i^2\right)^{3/2}$$


Out[20]= 
$$x x i \left(x^2 - x 0^2 + x i^2\right)$$


In[21]:=
piece1=Simplify[xi*Coefficient[mnum,xi]]
piece2=Simplify[xi^3*Coefficient[mnum,xi^3]]
Simplify[mnum-piece1-piece2]

Out[21]= 
$$x \left(x^2 - x 0^2\right) x i$$


Out[22]= 
$$x x i^3$$


Out[23]= 0

In[24]:=
small1=(x-x0)*xi
fact1=Simplify[piece1/small1]
FortranForm[Expand[%]]

Out[24]= 
$$(x - x 0) x i$$


Out[25]= 
$$x (x + x 0)$$


Out[26]//FortranForm=
"x**2 + x*x0"

In[27]:=
logcoeff=Simplify[Coefficient[term,logm]/.m3->0]
lognum=Numerator[logcoeff]
(* Lead = 1/(2*x0) *)

Out[27]= 
$$\frac{x x i \left(x^2 + 4 x x 0 + 3 x 0^2 + x i^2\right)}{2 x 0 \left(x^2 + 2 x x 0 + x 0^2 + x i^2\right)^{3/2}}$$


Out[28]= 
$$x x i \left(x^2 + 4 x x 0 + 3 x 0^2 + x i^2\right)$$


In[29]:=
piece1=Simplify[xi*Coefficient[lognum,xi]]
piece2=Simplify[xi^3*Coefficient[lognum,xi^3]]
Simplify[lognum-piece1-piece2]

Out[29]= 
$$x \left(x^2 + 4 x x 0 + 3 x 0^2\right) x i$$


Out[30]= 
$$x x i^3$$


Out[31]= 0

```

```

In[32]:= small1 = xi
         fact1=Simplify[piece1/xi]
         FortranForm[Expand[%]]

Out[32]= xi

Out[33]=  $x(x^2 + 4x x0 + 3x0^2)$ 

Out[34]//FortranForm=
         "x**3 + 4*x**2*x0 + 3*x*x0**2"

In[35]:= logmcoeff = Simplify[Coefficient[term, m3]];
         logmcoeff = Simplify[Coefficient[%, logm]]
         mnum = Simplify[Numerator[logmcoeff]];
         piece0 = Simplify[xi * Coefficient[mnum, xi]]
         small0 = xi
         fact0 = Simplify[piece0/xi]
         FortranForm[Expand[%]]
         Simplify[mnum - piece0]

Out[36]= 
$$\frac{x xi (x^2 + 8x x0 + 7x0^2 + xi^2)}{16 x0 (x^2 + 2x x0 + x0^2 + xi^2)^{3/2}}$$


Out[38]=  $x(x^2 + 8x x0 + 7x0^2) xi$ 

Out[39]= xi

Out[40]=  $x(x^2 + 8x x0 + 7x0^2)$ 

Out[41]//FortranForm=
         "x**3 + 8*x**2*x0 + 7*x*x0**2"

Out[42]=  $x xi^3$ 

```