```
In[4]:= (*Find F2,F1 and F0, numerators of 1/rho^4,1/rho^2, log(rho2) for Q11
use expansion of F(k) and E(k) about k=1
call log(rho^2)=logm, rho^2=|x-x0|^2
m3=rho^2 / rho2^2 where rho2^2=(x+x0)^2+(y-y0)^2

Version2: much simpler!
(1)when r(=x) is small, it is important to expand O(r)/O(rho2)
 together. so break up each numerator into pieces, each one of them is
 factored into "Small" times "fact". small is expanded to highest order,
fact to smaller order. In code,we then expand fact/rho2 or fact/(xrho2) first,
then multiplied by expansion of small.
(2)we dont need to expand "rest"! instead,
evaluate exactly in code using known values of x,x0,xi
(3)we no longer expand in d. that was a detour that lead to errors. high
 derivatives are large and need to be accurate to all powers in d.

For values of a1,a2,b1,b2,etc see Abramowitz and Stegun AND
https://functions.wolfram.com/EllipticIntegrals/EllipticK/introductions/
      CompleteEllipticIntegrals/05/

a0=Log[4], a1=(Log[4]-1)/4, a2=(6*Log[4]-7)*3/128
b1=1/8, b2=9/128, b3 not used
c1=(4log2-1)/4, c2=(24log2-13)/64 (not used), c3=3(5log2-3)/64 (not used)
d1=1/4, d2=3/32
*)

f = 2 * log2 - logm / 2 + a1 * m3 + a2 * m3 ^ 2 +
    a3 * m3 ^ 3 - (b1 * m3 + b2 * m3 ^ 2 + b3 * m3 ^ 3 + b4 * m3 ^ 4) * logm;
e = 1 + c1 * m3 + c2 * m3 ^ 2 + c3 * m3 ^ 3 - (d1 * m3 + d2 * m3 ^ 2 + d3 * m3 ^ 3 + d4 * m3 ^ 4) * logm;

eth = Series[m3 * e / m3, {m3, 0, 3}] / m3;
efh = Series[m3 ^ 2 * (2 * (1 + m3) * e / m3 ^ 2 - f / m3) / 3, {m3, 0, 3}] / m3 ^ 2;

c = Sqrt[a + b];
temp = Series[m3 ^ 2 * efh / c ^ 5, {m3, 0, 3}] / m3 ^ 2;
i50 = 4 * temp;
i51 = Series[m3 ^ 2 * 4 / b * (a * temp - eth / c ^ 3), {m3, 0, 3}] / m3 ^ 2;
i52 = Series[4 * m3 ^ 2 / b ^ 2 * (a ^ 2 * temp - 2 * a * eth / c ^ 3 + f / c), {m3, 0, 3}] / m3 ^ 2;
i53 = Series[4 * m3 ^ 2 / b ^ 3 * (a ^ 3 * temp - 3 * a ^ 2 * eth / c ^ 3 + 3 * a * f / c - c * e), {m3, 0, 3}] / m3 ^ 2;
```

In[14]:=
```
qu11 = Series[
    -m3^2*6*x*(x^3*i51 - x^2*x0*(i50 + 2*i52) + x*x0^2*(i53 + 2*i51) - x0^3*i52),
    {m3, 0, 3}]/m3^2;
qu12 = Series[-m3^2*6*x*xi*((x^2 + x0^2)*i51 - x*x0*(i50 + i52)), {m3, 0, 3}]/m3^2;
qu22 = Series[-m3^2*6*x*xi^2*(x*i51 - x0*i50), {m3, 0, 6}]/m3^2;
qv11 = Series[-m3^2*6*x*xi*(x0^2*i52 + x^2*i50 - 2*x*x0*i51), {m3, 0, 6}]/m3^2;
qv12 = Series[-m3^2*6*x*xi^2*(x*i50 - x0*i51), {m3, 0, 6}]/m3^2;
qv22 = Series[-m3^2 * 6 * x * xi^3 * i50, {m3, 0, 6}] / m3^2;
```

In[20]:=
```
term = qu22;
term = Simplify[term /. {d1 → 1/4, d2 → 3/32, d3 → 15/256,
      b1 → 1/8, b2 → 9/128, c1 → (4*log2 - 1)/4, c2 → (24*log2 - 13)/64,
      c3 → 3*(5*log2 - 3)/64, a1 → (2*log2 - 1)/4, a2 → (12*log2 - 7)*3/128}];
term = Simplify[term /. {b → 2*x*x0, a → x0^2 + x^2 + xi^2}]
```

Out[22]=
$$-\frac{8\left(x\,xi^2\left(x^2-x0^2+xi^2\right)\right)}{\left(x0\left(x^2+2\,x\,x0+x0^2+xi^2\right)^{5/2}\right)m3^2}+\frac{6\,x\,xi^2\left(x^2+4\,x\,x0+3\,x0^2+xi^2\right)}{x0\left(x^2+2\,x\,x0+x0^2+xi^2\right)^{5/2}\,m3}+$$
$$\left(3\,x\,xi^2\left(16\,(-1+4\,\text{log2}-\text{logm})\,x\,x0+2\,(-7+24\,\text{log2}-6\,\text{logm})\,x0^2+\right.\right.$$
$$\left.\left.(-1+8\,\text{log2}-2\,\text{logm})\left(x^2+x0^2+xi^2\right)\right)\right)\Big/\left(8\,x0\left(x^2+2\,x\,x0+x0^2+xi^2\right)^{5/2}\right)+$$
$$\left(x\,xi^2\left(12\,(-13+24\,\text{log2}-6\,\text{logm})\,x\,x0+2\,(-67+120\,\text{log2}-30\,\text{logm})\,x0^2+\right.\right.$$
$$\left.\left.(-11+24\,\text{log2}-6\,\text{logm})\left(x^2+x0^2+xi^2\right)\right)m3\right)\Big/\left(32\,x0\left(x^2+2\,x\,x0+x0^2+xi^2\right)^{5/2}\right)+O[m3]^2$$

In[23]:=
```
msqcoeff = Simplify[term*m3^2 /. m3 → 0]
mnum = Simplify[Numerator[msqcoeff]/(-8)]
(* Lead =
 -8/x0  The whole term is multiplied by rho2^4/rho1^4 leaving 1/rho2 in denom

mnum is numerator without the lead. we now
  break up mnum into pieces consisting of O(r)*O(small) factors
we need to expand O(r)/O(r2) separately, to avoid errors when r2 is small
that term is then multiplied by remaining O(small) term*)
```

Out[23]=
$$-\frac{8\,x\,xi^2\left(x^2-x0^2+xi^2\right)}{x0\left(x^2+2\,x\,x0+x0^2+xi^2\right)^{5/2}}$$

Out[24]= $x\,xi^2\left(x^2-x0^2+xi^2\right)$

In[25]:=
```
piece1 = x*(x + x0)*(x - x0)*xi^2
piece2 = Simplify[mnum - piece1]
```

Out[25]= $x\,(x-x0)\,(x+x0)\,xi^2$

Out[26]= $x\,xi^4$

In[27]:=
```
small1 = xi ^ 2 * (x - x0)
fact1 = x * (x + x0)
Expand[%]
```

Out[27]= $(x - x0)\, xi^2$

Out[28]= $x\,(x + x0)$

Out[29]= $x^2 + x\, x0$

In[30]:=
```
small2 = xi ^ 4
fact2 = x
Simplify[mnum - small1 * fact1 - small2 * fact2]
```

Out[30]= xi^4

Out[31]= x

Out[32]= 0

In[33]:=
```
mcoeff = Simplify[Coefficient[term * m3 ^ 2, m3]]
mnum = Simplify[Numerator[mcoeff] / (6)]
(* Lead =
 6/x0  The whole term is multiplied by rho2^2/rho1^2 leaving 1/rho2^3 in denom*)
```

Out[33]= $$\frac{6\, x\, xi^2 \left(x^2 + 4\, x\, x0 + 3\, x0^2 + xi^2\right)}{x0 \left(x^2 + 2\, x\, x0 + x0^2 + xi^2\right)^{5/2}}$$

Out[34]= $x\, xi^2 \left(x^2 + 4\, x\, x0 + 3\, x0^2 + xi^2\right)$

In[35]:=
```
piece2 = xi ^ 4 * x
piece1 = Simplify[mnum - piece2]
```

Out[35]= $x\, xi^4$

Out[36]= $x \left(x^2 + 4\, x\, x0 + 3\, x0^2\right) xi^2$

In[37]:=
```
small1 = xi ^ 2
fact1 = Simplify[piece1 / small1]
Expand[%]
FortranForm[%];
```

Out[37]= xi^2

Out[38]= $x \left(x^2 + 4\, x\, x0 + 3\, x0^2\right)$

Out[39]= $x^3 + 4\, x^2\, x0 + 3\, x\, x0^2$

In[41]:=
```
small2 = xi ^ 4
fact2 = Simplify[piece2 / small2]
Simplify[mnum - small1 * fact1 - small2 * fact2]
```

Out[41]= xi^4

Out[42]= x

Out[43]= 0

In[44]:=
```
logcoeff=Simplify[Coefficient[term,logm]/.m3→0]
mnum=Simplify[Numerator[logcoeff]/(-3)]
FortranForm[%];
(* Lead = -3/(4*x0)  This terms is multiplied by log(rho1^2/dsq)-log(rho^2/d^2)  *)
```

Out[44]= $-\frac{3\ x\ xi^2\ (x^2 + 8\ x\ x0 + 7\ x0^2 + xi^2)}{4\ x0\ (x^2 + 2\ x\ x0 + x0^2 + xi^2)^{5/2}}$

Out[45]= $x\ xi^2\ (x^2 + 8\ x\ x0 + 7\ x0^2 + xi^2)$

In[47]:=
```
piece2 = xi ^ 4 * x
piece1 = Simplify[mnum - piece2]
```

Out[47]= $x\ xi^4$

Out[48]= $x\ (x^2 + 8\ x\ x0 + 7\ x0^2)\ xi^2$

In[49]:=
```
small1 = xi ^ 2
fact1 = Simplify[piece1 / small1]
Expand[%]
```

Out[49]= xi^2

Out[50]= $x\ (x^2 + 8\ x\ x0 + 7\ x0^2)$

Out[51]= $x^3 + 8\ x^2\ x0 + 7\ x\ x0^2$

In[52]:=
```
small2 = xi ^ 4
fact2 = Simplify[piece2 / small2]
Simplify[mnum - small1 * fact1 - small2 * fact2]
```

Out[52]= xi^4

Out[53]= x

Out[54]= 0

In[55]:=
```
logmcoeff = Simplify[Coefficient[term, m3]];
logmcoeff = Simplify[Coefficient[%, logm]]
mnum = Simplify[Numerator[logmcoeff]]
denom = Simplify[Denominator[logmcoeff]]
mnum0 = Simplify[mnum /. xi → 0]
Simplify[Coefficient[mnum, xi]]
Simplify[Coefficient[mnum, xi^2]]
Simplify[Coefficient[mnum, xi^3]]
Simplify[Coefficient[mnum, xi^4]]
Simplify[Coefficient[mnum, xi^5]]
Simplify[Coefficient[mnum, xi^6]]
```

Out[56]= $-\frac{3\,x\,xi^2\left(x^2+12\,x\,x0+11\,x0^2+xi^2\right)}{16\,x0\left(x^2+2\,x\,x0+x0^2+xi^2\right)^{5/2}}$

Out[57]= $-3\,x\,xi^2\left(x^2+12\,x\,x0+11\,x0^2+xi^2\right)$

Out[58]= $16\,x0\left(x^2+2\,x\,x0+x0^2+xi^2\right)^{5/2}$

Out[59]= 0

Out[60]= 0

Out[61]= $-3\,x\left(x^2+12\,x\,x0+11\,x0^2\right)$

Out[62]= 0

Out[63]= $-3\,x$

Out[64]= 0

Out[65]= 0

In[91]:=
```
rest =
  Simplify[term - msqcoeff / m3 ^ 2 - mcoeff / m3 - logcoeff * logm - mnum0 * m3 * logm / denom];
```

```
rest0 = Simplify[rest /. m3 → 0];
rest0num = Simplify[Numerator[rest0]]
Denominator[rest0]
Simplify[rest0num /. xi → 0]
Simplify[Coefficient[rest0num, xi]]
Simplify[Coefficient[rest0num, xi ^ 2]]
Simplify[Coefficient[rest0num, xi ^ 3]]
Simplify[Coefficient[rest0num, xi ^ 4]]
Simplify[Coefficient[rest0num, xi ^ 5]]
Simplify[Coefficient[rest0num, xi ^ 6]]
```

Out[93]= $3\, x\, xi^2 \left((-1 + 8\, \log 2)\, x^2 + 16\, (-1 + 4\, \log 2)\, x\, x0 + (-15 + 56\, \log 2)\, x0^2 + (-1 + 8\, \log 2)\, xi^2\right)$

Out[94]= $8\, x0 \left(x^2 + 2\, x\, x0 + x0^2 + xi^2\right)^{5/2}$

Out[95]= 0

Out[96]= 0

Out[97]= $3\, x\, (x + x0)\, ((-1 + 8\, \log 2)\, x + (-15 + 56\, \log 2)\, x0)$

Out[98]= 0

Out[99]= $3\, (-1 + 8\, \log 2)\, x$

Out[100]=
0

Out[101]=
0

In[81]:=
```
rest1 = Simplify[Coefficient[rest, m3]];
rest1num = Simplify[Numerator[rest1]]
Denominator[rest1]
Simplify[rest1num /. xi → 0]
Simplify[Coefficient[rest1num, xi]]
Simplify[Coefficient[rest1num, xi^2]]
Simplify[Coefficient[rest1num, xi^3]]
Simplify[Coefficient[rest1num, xi^4]]
Simplify[Coefficient[rest1num, xi^5]]
Simplify[Coefficient[rest1num, xi^6]]
```

Out[82]= $x\,xi^2 \left(12 (-13 + 24\,\text{log2} - 6\,\text{logm})\, x\, x0 + 2 (-67 + 120\,\text{log2} - 30\,\text{logm})\, x0^2 + (-11 + 24\,\text{log2} - 6\,\text{logm}) \left(x^2 + x0^2 + xi^2\right)\right)$

Out[83]= $32\, x0 \left(x^2 + 2\, x\, x0 + x0^2 + xi^2\right)^{5/2}$

Out[84]= 0

Out[85]= 0

Out[86]= $x (x + x0) ((-11 + 24\,\text{log2} - 6\,\text{logm})\, x + (-145 + 264\,\text{log2} - 66\,\text{logm})\, x0)$

Out[87]= 0

Out[88]= $(-11 + 24\,\text{log2} - 6\,\text{logm})\, x$

Out[89]= 0

Out[90]= 0