```
In[44]:= (*Find F2,F1 and F0, numerators of 1/rho^4,1/rho^2, log(rho2) for Q11
use expansion of F(k) and E(k) about k=1
call log(rho^2)=logm, rho^2=|x-x0|^2
m3=rho^2 / rho2^2 where rho2^2=(x+x0)^2+(y-y0)^2

Version2: much simpler!
(1)when r(=x) is small, it is important to expand O(r)/O(rho2)
 together. so break up each numerator into pieces, each one of them is
 factored into "Small" times "fact". small is expanded to highest order,
fact to smaller order. In code,we then expand fact/rho2 or fact/(xrho2) first,
then multiplied by expansion of small.
(2)we dont need to expand "rest"! instead,
evaluate exactly in code using known values of x,x0,xi
(3)we no longer expand in d. that was a detour that lead to errors. high
 derivatives are large and need to be accurate to all powers in d.

For values of a1,a2,b1,b2,etc see Abramowitz and Stegun AND
https://functions.wolfram.com/EllipticIntegrals/EllipticK/introductions/
      CompleteEllipticIntegrals/05/

a0=Log[4], a1=(Log[4]-1)/4, a2=(6*Log[4]-7)*3/128
b1=1/8, b2=9/128, b3 not used
c1=(4log2-1)/4, c2=(24log2-13)/64 (not used), c3=3(5log2-3)/64 (not used)
d1=1/4, d2=3/32
*)

f = 2 * log2 - logm / 2 + a1 * m3 + a2 * m3 ^ 2 +
    a3 * m3 ^ 3 - (b1 * m3 + b2 * m3 ^ 2 + b3 * m3 ^ 3 + b4 * m3 ^ 4) * logm;
e = 1 + c1 * m3 + c2 * m3 ^ 2 + c3 * m3 ^ 3 - (d1 * m3 + d2 * m3 ^ 2 + d3 * m3 ^ 3 + d4 * m3 ^ 4) * logm;

eth = Series[m3 * e / m3, {m3, 0, 3}] / m3;
efh = Series[m3 ^ 2 * (2 * (1 + m3) * e / m3 ^ 2 - f / m3) / 3, {m3, 0, 3}] / m3 ^ 2;

c = Sqrt[a + b];
temp = Series[m3 ^ 2 * efh / c ^ 5, {m3, 0, 3}] / m3 ^ 2;
i50 = 4 * temp;
i51 = Series[m3 ^ 2 * 4 / b * (a * temp - eth / c ^ 3), {m3, 0, 3}] / m3 ^ 2;
i52 = Series[4 * m3 ^ 2 / b ^ 2 * (a ^ 2 * temp - 2 * a * eth / c ^ 3 + f / c), {m3, 0, 3}] / m3 ^ 2;
i53 = Series[4 * m3 ^ 2 / b ^ 3 * (a ^ 3 * temp - 3 * a ^ 2 * eth / c ^ 3 + 3 * a * f / c - c * e), {m3, 0, 3}] / m3 ^ 2;
```

In[54]:=
```
qu11 = Series[
    -m3^2*6*x*(x^3*i51 - x^2*x0*(i50 + 2*i52) + x*x0^2*(i53 + 2*i51) - x0^3*i52),
    {m3, 0, 3}] / m3^2;
qu12 = Series[-m3^2*6*x*xi*((x^2 + x0^2)*i51 - x*x0*(i50 + i52)), {m3, 0, 3}] / m3^2;
qu22 = Series[-m3^2*6*x*xi^2*(x*i51 - x0*i50), {m3, 0, 6}] / m3^2;
qv11 = Series[-m3^2*6*x*xi*(x0^2*i52 + x^2*i50 - 2*x*x0*i51), {m3, 0, 6}] / m3^2;
qv12 = Series[-m3^2*6*x*xi^2*(x*i50 - x0*i51), {m3, 0, 6}] / m3^2;
qv22 = Series[-m3^2 * 6 * x * xi^3 * i50, {m3, 0, 6}] / m3^2;
```

In[60]:=
```
term = qv22;
term = Simplify[term /. {d1 → 1/4, b1 → 1/8, d2 → 3/32, b2 → 9/128, c1 → (4*log2 - 1)/4}];
term = Simplify[term /. {b → 2*x*x0, a → x0^2 + x^2 + xi^2}]
```

Out[62]=
$$-\frac{16\,(x\,xi^3)}{(x^2+2\,x\,x0+x0^2+xi^2)^{5/2}\,m3^2}-\frac{12\,(x\,xi^3)}{(x^2+2\,x\,x0+x0^2+xi^2)^{5/2}\,m3}+\frac{(8+16\,a1-32\,c2-32\,log2+9\,logm)\,x\,xi^3}{2\,(x^2+2\,x\,x0+x0^2+xi^2)^{5/2}}-\frac{8\left(\left(-a2+2\,c2+2\,c3-\frac{15\,logm}{128}-2\,d3\,logm\right)x\,xi^3\right)m3}{(x^2+2\,x\,x0+x0^2+xi^2)^{5/2}}+O[m3]^2$$

In[63]:=
```
msqcoeff = Simplify[term*m3^2 /. m3 → 0]
mnum = Simplify[Numerator[msqcoeff] / (-16)]
(* Lead =
 -16  The whole term is multiplied by rho2^4/rho1^4 leaving 1/rho2 in denom

mnum is numerator without the lead. we now
  break up mnum into pieces consisting of O(r)*O(small) factors
we need to expand O(r)/O(r2) separately, to avoid errors when r2 is small
that term is then multiplied by remaining O(small) term*)
```

Out[63]=
$$-\frac{16\,x\,xi^3}{(x^2+2\,x\,x0+x0^2+xi^2)^{5/2}}$$

Out[64]= $x\,xi^3$

In[65]:=
```
piece1 = mnum
```

Out[65]= $x\,xi^3$

In[66]:=
```
mcoeff = Simplify[Coefficient[term * m3 ^ 2, m3]]
mnum = Simplify[Numerator[mcoeff] / (-12)]
(* Lead =
 -12  The whole term is multiplied by rho2^2/rho1^2 leaving 1/rho2^3 in denom*)
```

Out[66]= $-\frac{12\ x\ xi^3}{\left(x^2+2\ x\ x0+x0^2+xi^2\right)^{5/2}}$

Out[67]= $x\ xi^3$

In[68]:=
```
piece1 = mnum
```

Out[68]= $x\ xi^3$

In[69]:=
```
logcoeff=Simplify[Coefficient[term,logm]/.m3→0]
mnum=Simplify[Numerator[logcoeff]/(9)]
FortranForm[%]
(* Lead = 9/2  This terms is multiplied by log(rho1^2)-log(rho^2)  *)
```

Out[69]= $\frac{9\ x\ xi^3}{2\left(x^2+2\ x\ x0+x0^2+xi^2\right)^{5/2}}$

Out[70]= $x\ xi^3$

Out[71]//FortranForm=

```
"x*xi**3"
```

In[72]:=
```
piece1 = mnum
```

Out[72]= $x\ xi^3$

In[73]:=
```
rest = Simplify[term - msqcoeff / m3 ^ 2 - mcoeff / m3 - logcoeff * logm];
rest = Simplify[rest /. {a1 → (2 * log2 - 1) / 4, c2 → (24 * log2 - 13) / 64}];

rest0 = Simplify[rest /. m3 → 0];
rest0num = Simplify[Numerator[rest0]]
FortranForm[%]
Denominator[rest0]
```

Out[76]= $-3\ (-7+24\ \mathrm{log2})\ x\ xi^3$

Out[77]//FortranForm=

```
"-3*(-7 + 24*log2)*x*xi**3"
```

Out[78]= $4\left(x^2+2\ x\ x0+x0^2+xi^2\right)^{5/2}$

In[79]:=
```
rest1 = Simplify[Coefficient[rest, m3]];
(*rest1num=
 Simplify[Numerator[rest1]/.{a2→(6*Log[4]-7)*3/128,d3→15/256,c3→3*(5*Log[2]-3)/64}]*)
rest1num = Simplify[Numerator[rest1]];
rest1num =
 Simplify[rest1num /. {a2 → (12 * log2 - 7) * 3 / 128, d3 → 15 / 256, c3 → 3 * (5 * log2 - 3) / 64}]
FortranForm[%]
Denominator[rest1]
```

Out[81]= $-((-67+120\,\text{log2}-30\,\text{logm})\,x\,\text{xi}^3)$

Out[82]//FortranForm=

```
"-((-67 + 120*log2 - 30*logm)*x*xi**3)"
```

Out[83]= $16\left(x^2+2\,x\,\text{x0}+\text{x0}^2+\text{xi}^2\right)^{5/2}$