In[1]:= ```
(* find F1 and F0, numerators of rho^2 and log(rho^2) for M11
use expansion of F(k),E(k) about k=1
call log(rho1^2) =logm. rho1^2=|x-x0|^2
m3=rho1^2/rho2^2*)
```

In[2]:= ```
f = 2 * log2 - logm / 2 + a1 * m3 + a2 * m3 ^ 2 +
    a3 * m3 ^ 3 - (b1 * m3 + b2 * m3 ^ 2 + b3 * m3 ^ 3 + b4 * m3 ^ 4) * logm;
e = 1 + c1 * m3 + c2 * m3 ^ 2 + c3 * m3 ^ 3 - (d1 * m3 + d2 * m3 ^ 2 + d3 * m3 ^ 3 + d4 * m3 ^ 4) * logm;

eth = Series[m3 ^ 2 * e / m3, {m3, 0, 4}] / m3 ^ 2;

c = Sqrt[a + b];
i10 = Series[4 * f / c, {m3, 0, 4}];
i11 = Series[m3 ^ 2 * 4 / b * (a * f / c - c * e), {alf, 0, 6}] / m3 ^ 2;
i30 = Series[m3 * 4 * eth / c ^ 3, {m3, 0, 5}] / m3;
i31 = Series[m3 * 4 / b * (a * eth / c ^ 3 - f / c), {m3, 0, 6}] / m3;
i32 = Series[m3 * (4 / b ^ 2) * (a ^ 2 * eth / c ^ 3 - 2 * a * f / c + c * e), {m3, 0, 6}] / m3;

m11 = Simplify[Series[m3 * x * (i11 + (x ^ 2 + x0 ^ 2) * i31 - x * x0 * (i30 + i32)), {m3, 0, 2}]] / m3;
m12 = Simplify[Series[m3 * x * (xi * (x * i31 - x0 * i30)), {m3, 0, 2}]] / m3;
m21 = Simplify[Series[m3 * x * xi (x * i30 - x0 * i31), {m3, 0, 2}]] / m3;
m22 = Simplify[Series[m3 * x * (i10 + xi ^ 2 * i30), {m3, 0, 2}]] / m3;
```

In[15]:= ```
term=m21;
term=Simplify[term/.{d1→1/4,d2→3/32,d3→15/256,b1→1/8,b2→9/128,c1→(4*log2-1)/4,c2→(24*log2
term=Simplify[term/.{b→2*x*x0,a→x0^2+x^2+xi^2}]
```

Out[17]= 
$$-\frac{2\left(xi\left(-x^2 + x0^2 + xi^2\right)\right)}{\left(x^2 + 2\,x\,x0 + x0^2 + xi^2\right)^{3/2} m3} +$$

$$\frac{xi\left(\left(-1 + 12\,log2 - 3\,logm\right)x^2 + 4\left(4\,log2 - logm\right)x\,x0 + \left(1 + 4\,log2 - logm\right)\left(x0^2 + xi^2\right)\right)}{2\left(x^2 + 2\,x\,x0 + x0^2 + xi^2\right)^{3/2}} +$$

$$\frac{xi\left(\left(-29 + 56\,log2 - 14\,logm\right)x^2 + 16\left(-2 + 4\,log2 - logm\right)x\,x0 + \left(-3 + 8\,log2 - 2\,logm\right)\left(x0^2 + xi^2\right)\right)m3}{32\left(x^2 + 2\,x\,x0 + x0^2 + xi^2\right)^{3/2}} +$$

$$O[m3]^2$$

In[18]:= ```
mcoeff=Simplify[Coefficient[term*m3^2,m3]]
msqden=Simplify[Denominator[mcoeff]]
mnum=Simplify[Numerator[mcoeff]/(2)]
(* Lead = 2
This terms is multiplied by rho2^2/rho^2 so denominator is 1/rho2 whole term is Lead*
```

Out[18]= $-\dfrac{2\,xi\left(-x^2+x0^2+xi^2\right)}{\left(x^2+2\,x\,x0+x0^2+xi^2\right)^{3/2}}$

Out[19]= $\left(x^2+2\,x\,x0+x0^2+xi^2\right)^{3/2}$

Out[20]= $-xi\left(-x^2+x0^2+xi^2\right)$

In[21]:= ```
piece1=Simplify[xi*Coefficient[mnum,xi]]
piece2=Simplify[xi^3*Coefficient[mnum,xi^3]]
Simplify[mnum-piece1-piece2]
```

Out[21]= $\left(x^2-x0^2\right)xi$

Out[22]= $-xi^3$

Out[23]= 0

In[24]:= ```
small1=xi*(x-x0)
fact1=x+x0
small2=-xi^3
Simplify[mnum-small1*fact1-small2]
```

Out[24]= $(x-x0)\,xi$

Out[25]= $x+x0$

Out[26]= $-xi^3$

Out[27]= 0

In[28]:= ```
logcoeff=Simplify[Coefficient[term,logm]/.m3→0]
lognum=Simplify[Numerator[logcoeff]/(-1)]
(* Lead = -1/2 *)
```

Out[28]= $-\dfrac{xi\left(3\,x^2+4\,x\,x0+x0^2+xi^2\right)}{2\left(x^2+2\,x\,x0+x0^2+xi^2\right)^{3/2}}$

Out[29]= $xi\left(3\,x^2+4\,x\,x0+x0^2+xi^2\right)$

In[30]:= ```
piece1=Simplify[xi*Coefficient[lognum,xi]]
piece2=Simplify[xi^3*Coefficient[lognum,xi^3]]
Simplify[lognum-piece1-piece2]
```

Out[30]= $\left(3\,x^2+4\,x\,x0+x0^2\right)xi$

Out[31]= xi^3

Out[32]= 0

In[33]:= ```
small1=xi
fact1=Simplify[piece1/xi]
FortranForm[Expand[%]]
small2=xi^3
Simplify[lognum-fact1*small1-piece2]
```

Out[33]= xi

Out[34]= $3 x^2 + 4 x x0 + x0^2$

Out[35]//FortranForm=

"3*x**2 + 4*x*x0 + x0**2"

Out[36]= $xi^3$

Out[37]= 0

In[38]:= ```
logmcoeff = Simplify[Coefficient[term, m3]];
logmcoeff = Simplify[Coefficient[%, logm]]
mnum = Simplify[Numerator[logmcoeff]/(-1)]
(* Lead -1/16 *)
```

Out[39]= $-\dfrac{xi \left(7 x^2 + 8 x x0 + x0^2 + xi^2\right)}{16 \left(x^2 + 2 x x0 + x0^2 + xi^2\right)^{3/2}}$

Out[40]= $xi \left(7 x^2 + 8 x x0 + x0^2 + xi^2\right)$