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In[1]:= (* find F1 and F0, numerators of rho^2 and log(rho^2) for M11
use expansion of F(k),E(k) about k=1
call log(rho1^2) =logm. rho1^2=|x-x0|^2
m3=rho1^2/rho2^2*)

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In[2]:= f = 2 * log2 - logm / 2 + a1 * m3 + a2 * m3 ^ 2 +
a3 * m3 ^ 3 - (b1 * m3 + b2 * m3 ^ 2 + b3 * m3 ^ 3 + b4 * m3 ^ 4) * logm;
e = 1 + c1 * m3 + c2 * m3 ^ 2 + c3 * m3 ^ 3 - (d1 * m3 + d2 * m3 ^ 2 + d3 * m3 ^ 3 + d4 * m3 ^ 4) * logm;

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eth = Series[m3 ^ 2 * e / m3, {m3, 0, 4}] / m3 ^ 2;

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c = Sqrt[a + b];
i10 = Series[4 * f / c, {m3, 0, 4}];
i11 = Series[m3 ^ 2 * 4 / b * (a * f / c - c * e), {alf, 0, 6}] / m3 ^ 2;
i30 = Series[m3 * 4 * eth / c ^ 3, {m3, 0, 5}] / m3;
i31 = Series[m3 * 4 / b * (a * eth / c ^ 3 - f / c), {m3, 0, 6}] / m3;
i32 = Series[m3 * (4 / b ^ 2) * (a ^ 2 * eth / c ^ 3 - 2 * a * f / c + c * e), {m3, 0, 6}] / m3;

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m11 = Simplify[Series[m3 * x * (i11 + (x ^ 2 + x0 ^ 2) * i31 - x * x0 * (i30 + i32)), {m3, 0, 2}] / m3;
m12 = Simplify[Series[m3 * x * (xi * (x * i31 - x0 * i30)), {m3, 0, 2}] / m3;
m21 = Simplify[Series[m3 * x * xi (x * i30 - x0 * i31), {m3, 0, 2}] / m3;
m22 = Simplify[Series[m3 * x * (i10 + xi ^ 2 * i30), {m3, 0, 2}] / m3;

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In[15]:= term=m22;
term=Simplify[term/.{d1→1/4,d2→3/32,d3→15/256,b1→1/8,b2→9/128,c1→(4*log2-1)/4,c2→(24*log2-1)/4,c3→(16*log2-1)/4,c4→(16*log2-1)/4}];
term=Simplify[term/.{b→2*x*x0,a→x0^2+x^2+xi^2}]

```

$$\begin{aligned}
\text{Out[17]} = & \frac{4 x x i^2}{(x^2 + 2 x x0 + x0^2 + x i^2)^{3/2} m3} + \\
& \frac{x (4 (4 \log 2 - \log m) x x0 + (-1 + 4 \log 2 - \log m) x i^2 + (8 \log 2 - 2 \log m) (x^2 + x0^2 + x i^2))}{(x^2 + 2 x x0 + x0^2 + x i^2)^{3/2}} + \\
& \frac{(x (16 (-2 + 4 \log 2 - \log m) x x0 + (-13 + 24 \log 2 - 6 \log m) x i^2 + 8 (-2 + 4 \log 2 - \log m) (x^2 + x0^2 + x i^2))}{m3} \Big/ \left(16 (x^2 + 2 x x0 + x0^2 + x i^2)^{3/2} \right) + O[m3]^2
\end{aligned}$$

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In[18]:= mcoeff=Simplify[Coefficient[term*m3^2,m3]]
msqden=Simplify[Denominator[mcoeff]]
mnum=Simplify[Numerator[mcoeff]/(4)]
(* Lead = 4
This terms is multiplied by rho2^2/rho^2 so denominator is 1/rho2 whole term is Lead*
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$$\text{Out[18]} = \frac{4 x x i^2}{(x^2 + 2 x x 0 + x 0^2 + x i^2)^{3/2}}$$

$$\text{Out[19]} = (x^2 + 2 x x 0 + x 0^2 + x i^2)^{3/2}$$

$$\text{Out[20]} = x x i^2$$

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In[21]:= small1=xi^2;
fact1=x;
Simplify[mnum-small1*fact1];
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In[24]:= logcoeff=Simplify[Coefficient[term,logm]/.m3->0]
lognum=Simplify[Numerator[logcoeff]/(-1)]
(* Lead = -1 *)
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$$\text{Out[24]} = -\frac{x (2 x^2 + 4 x x 0 + 2 x 0^2 + 3 x i^2)}{(x^2 + 2 x x 0 + x 0^2 + x i^2)^{3/2}}$$

$$\text{Out[25]} = x (2 x^2 + 4 x x 0 + 2 x 0^2 + 3 x i^2)$$

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In[26]:= piece1=Simplify[lognum/.xi->0]
piece2=Simplify[xi^2*Coefficient[lognum,xi^2]]
Simplify[lognum-piece1-piece2]
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$$\text{Out[26]} = 2 x (x + x 0)^2$$

$$\text{Out[27]} = 3 x x i^2$$

$$\text{Out[28]} = 0$$

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In[29]:= fact1=Simplify[piece1]
FortranForm[Expand[%]]
fact2=3*x
small2=xi^2
Simplify[lognum-fact2*small2-fact1]
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$$\text{Out[29]} = 2 x (x + x 0)^2$$

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Out[30]//FortranForm=
"2*x**3 + 4*x**2*x0 + 2*x*x0**2"
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$$\text{Out[31]} = 3 x$$

$$\text{Out[32]} = x i^2$$

$$\text{Out[33]} = 0$$

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In[34]:= logmcoeff = Simplify[Coefficient[term, m3]];
logmcoeff = Simplify[Coefficient[%, logm]]
mnum = Simplify[Numerator[logmcoeff]/(-1)]
FortranForm[%];
```

$$\text{Out[35]} = - \frac{x (4 x^2 + 8 x x_0 + 4 x_0^2 + 7 x i^2)}{8 (x^2 + 2 x x_0 + x_0^2 + x i^2)^{3/2}}$$

$$\text{Out[36]} = x (4 x^2 + 8 x x_0 + 4 x_0^2 + 7 x i^2)$$