In[303]:=
```
f = Log[1 + u^2];
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[304]=

$$2\,\text{ArcTan}[u] + u\left(-2 + \text{Log}\left[1 + u^2\right]\right)$$

Out[305]=

$$\frac{2\,u}{1 + u^2}$$

Out[306]=

$$\frac{4\,u\left(-3 + u^2\right)}{\left(1 + u^2\right)^3}$$

In[307]:=
```
f = u * Log[1 + u^2];
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[308]=

$$\frac{1}{2}\left(-u^2 + \left(1 + u^2\right)\text{Log}\left[1 + u^2\right]\right)$$

Out[309]=

$$\frac{2\,u^2}{1 + u^2} + \text{Log}\left[1 + u^2\right]$$

Out[310]=

$$-\frac{2\left(-3 + 6\,u^2 + u^4\right)}{\left(1 + u^2\right)^3}$$

In[311]:=
```
f = u^2 * Log[1 + u^2];
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[312]=

$$\frac{1}{9}\left(6\,u - 2\,u^3 - 6\,\text{ArcTan}[u] + 3\,u^3\,\text{Log}\left[1 + u^2\right]\right)$$

Out[313]=

$$2\,u\left(\frac{u^2}{1 + u^2} + \text{Log}\left[1 + u^2\right]\right)$$

Out[314]=

$$\frac{4\,u\left(6 + 3\,u^2 + u^4\right)}{\left(1 + u^2\right)^3}$$

In[315]:=
```
f = u^3 * Log[1 + u^2];
Simplify[Integrate[f, u]]
FortranForm[%];
Simplify[D[f, u]]
FortranForm[%];
Simplify[D[f, {u, 3}]]
FortranForm[%];
```

Out[316]=

$$\frac{1}{8}\left(-u^2\left(-2+u^2\right)+2\left(-1+u^4\right)\text{Log}\left[1+u^2\right]\right)$$

Out[318]=

$$\frac{2\,u^4}{1+u^2}+3\,u^2\,\text{Log}\left[1+u^2\right]$$

Out[320]=

$$\frac{54\,u^2+60\,u^4+22\,u^6+6\left(1+u^2\right)^3\,\text{Log}\left[1+u^2\right]}{\left(1+u^2\right)^3}$$

In[322]:=
```
f = 1 / (1 + u^2);
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[323]=

$$\text{ArcTan}[u]$$

Out[324]=

$$-\frac{2\,u}{\left(1+u^2\right)^2}$$

Out[325]=

$$-\frac{24\,u\left(-1+u^2\right)}{\left(1+u^2\right)^4}$$

In[326]:=
```
f = u / (1 + u ^ 2);
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[327]=
$$\frac{1}{2}\,\text{Log}\left[1+u^2\right]$$

Out[328]=
$$\frac{1-u^2}{\left(1+u^2\right)^2}$$

Out[329]=
$$-\frac{6\left(1-6\,u^2+u^4\right)}{\left(1+u^2\right)^4}$$

In[330]:=
```
f = u ^ 2 / (1 + u ^ 2);
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[331]=
$$u-\text{ArcTan}[u]$$

Out[332]=
$$\frac{2\,u}{\left(1+u^2\right)^2}$$

Out[333]=
$$\frac{24\,u\left(-1+u^2\right)}{\left(1+u^2\right)^4}$$

In[334]:=
```
f = u ^ 3 / (1 + u ^ 2);
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[335]=
$$\frac{1}{2}\left(u^2-\text{Log}\left[1+u^2\right]\right)$$

Out[336]=
$$\frac{u^2\left(3+u^2\right)}{\left(1+u^2\right)^2}$$

Out[337]=
$$\frac{6\left(1-6\,u^2+u^4\right)}{\left(1+u^2\right)^4}$$

In[338]:=
```
f = u^4/(1 + u^2);
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[339]=
$$\frac{1}{3}\, u \left(-3 + u^2\right) + \text{ArcTan}[u]$$

Out[340]=
$$\frac{2\, u^3 \left(2 + u^2\right)}{\left(1 + u^2\right)^2}$$

Out[341]=
$$-\frac{24\, u \left(-1 + u^2\right)}{\left(1 + u^2\right)^4}$$

In[342]:=
```
f = 1/(1 + u^2)^2;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[343]=
$$\frac{1}{2}\left(\frac{u}{1 + u^2} + \text{ArcTan}[u]\right)$$

Out[344]=
$$-\frac{4\, u}{\left(1 + u^2\right)^3}$$

Out[345]=
$$\frac{24\, u \left(3 - 5\, u^2\right)}{\left(1 + u^2\right)^5}$$

In[346]:=
```
f = u/(1 + u^2)^2;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[347]=
$$-\frac{1}{2\left(1 + u^2\right)}$$

Out[348]=
$$\frac{1 - 3\, u^2}{\left(1 + u^2\right)^3}$$

Out[349]=
$$-\frac{12\left(1 - 10\, u^2 + 5\, u^4\right)}{\left(1 + u^2\right)^5}$$

In[350]:=
```
f = u^2/(1+u^2)^2;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[351]=
$$\frac{1}{2}\left(-\frac{u}{1+u^2}+\text{ArcTan}[u]\right)$$

Out[352]=
$$-\frac{2u\left(-1+u^2\right)}{\left(1+u^2\right)^3}$$

Out[353]=
$$-\frac{24u\left(2-5u^2+u^4\right)}{\left(1+u^2\right)^5}$$

In[354]:=
```
f = u^3/(1+u^2)^2;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[355]=
$$\frac{1}{2}\left(\frac{1}{1+u^2}+\text{Log}\left[1+u^2\right]\right)$$

Out[356]=
$$-\frac{u^2\left(-3+u^2\right)}{\left(1+u^2\right)^3}$$

Out[357]=
$$-\frac{6\left(-1+15u^2-15u^4+u^6\right)}{\left(1+u^2\right)^5}$$

In[358]:=
```
f = u^4/(1+u^2)^2;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[359]=
$$u+\frac{u}{2+2u^2}-\frac{3\,\text{ArcTan}[u]}{2}$$

Out[360]=
$$\frac{4u^3}{\left(1+u^2\right)^3}$$

Out[361]=
$$\frac{24\left(u-5u^3+2u^5\right)}{\left(1+u^2\right)^5}$$

In[362]:=
```
f = u^5/(1+u^2)^2;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[363]=
$$\frac{1}{2}\left(u^2 - \frac{1}{1+u^2} - 2\,\text{Log}\left[1+u^2\right]\right)$$

Out[364]=
$$\frac{u^4\left(5+u^2\right)}{\left(1+u^2\right)^3}$$

Out[365]=
$$\frac{12\,u^2\left(5-10\,u^2+u^4\right)}{\left(1+u^2\right)^5}$$

In[366]:=
```
f = u^6/(1+u^2)^2;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[367]=
$$\frac{1}{6}\,u\left(-12+2\,u^2-\frac{3}{1+u^2}\right)+\frac{5\,\text{ArcTan}[u]}{2}$$

Out[368]=
$$\frac{2\,u^5\left(3+u^2\right)}{\left(1+u^2\right)^3}$$

Out[369]=
$$\frac{24\,u^3\left(5-3\,u^2\right)}{\left(1+u^2\right)^5}$$

In[370]:=
```
f = u^3/(1+u^2)^3;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[371]=
$$-\frac{1+2\,u^2}{4\left(1+u^2\right)^2}$$

Out[372]=
$$-\frac{3\,u^2\left(-1+u^2\right)}{\left(1+u^2\right)^4}$$

Out[373]=
$$-\frac{6\left(-1+24\,u^2-45\,u^4+10\,u^6\right)}{\left(1+u^2\right)^6}$$

In[374]:=
```
f = u^4/(1+u^2)^3;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[375]=
$$\frac{1}{8}\left(-\frac{u\left(3+5\,u^2\right)}{\left(1+u^2\right)^2}+3\,\text{ArcTan}[u]\right)$$

Out[376]=
$$-\frac{2\,u^3\left(-2+u^2\right)}{\left(1+u^2\right)^4}$$

Out[377]=
$$-\frac{24\,u\left(-1+9\,u^2-9\,u^4+u^6\right)}{\left(1+u^2\right)^6}$$

In[378]:=
```
f = u^5/(1+u^2)^3;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[379]=
$$\frac{1}{4}\left(\frac{3+4\,u^2}{\left(1+u^2\right)^2}+2\,\text{Log}\left[1+u^2\right]\right)$$

Out[380]=
$$-\frac{u^4\left(-5+u^2\right)}{\left(1+u^2\right)^4}$$

Out[381]=
$$-\frac{6\,u^2\left(-10+45\,u^2-24\,u^4+u^6\right)}{\left(1+u^2\right)^6}$$

In[382]:=
```
f = u^6/(1+u^2)^3;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[383]=

$$\frac{1}{8}\left(\frac{u\left(15+25\,u^2+8\,u^4\right)}{\left(1+u^2\right)^2}-15\,\text{ArcTan}[u]\right)$$

Out[384]=

$$\frac{6\,u^5}{\left(1+u^2\right)^4}$$

Out[385]=

$$\frac{24\,u^3\left(5-12\,u^2+3\,u^4\right)}{\left(1+u^2\right)^6}$$

In[386]:=
```
f = u^7/(1+u^2)^3;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[387]=

$$\frac{1}{4}\left(2\,u^2+\frac{-5-6\,u^2}{\left(1+u^2\right)^2}-6\,\text{Log}\left[1+u^2\right]\right)$$

Out[388]=

$$\frac{u^6\left(7+u^2\right)}{\left(1+u^2\right)^4}$$

Out[389]=

$$\frac{6\,u^4\left(35-42\,u^2+3\,u^4\right)}{\left(1+u^2\right)^6}$$

In[390]:=
```
f = u^8/(1+u^2)^3;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[391]=
$$\frac{1}{24}\left(u\left(-72+8\,u^2+\frac{6}{\left(1+u^2\right)^2}-\frac{39}{1+u^2}\right)+105\,\text{ArcTan}[u]\right)$$

Out[392]=
$$\frac{2\,u^7\left(4+u^2\right)}{\left(1+u^2\right)^4}$$

Out[393]=
$$-\frac{48\,u^5\left(-7+3\,u^2\right)}{\left(1+u^2\right)^6}$$

In[394]:=
```
f = u^6/(1+u^2)^4;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[395]=
$$-\frac{u\left(15+40\,u^2+33\,u^4\right)}{48\left(1+u^2\right)^3}+\frac{5\,\text{ArcTan}[u]}{16}$$

Out[396]=
$$-\frac{2\,u^5\left(-3+u^2\right)}{\left(1+u^2\right)^5}$$

Out[397]=
$$-\frac{24\,u^3\left(-5+21\,u^2-13\,u^4+u^6\right)}{\left(1+u^2\right)^7}$$

In[398]:=
```
f = u^7/(1+u^2)^4;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[399]=
$$\frac{11 + 27\,u^2 + 18\,u^4 + 6\,(1+u^2)^3\,\text{Log}[1+u^2]}{12\,(1+u^2)^3}$$

Out[400]=
$$-\frac{u^6\,(-7+u^2)}{(1+u^2)^5}$$

Out[401]=
$$-\frac{6\,u^4\,(-35+91\,u^2-33\,u^4+u^6)}{(1+u^2)^7}$$

In[402]:=
```
f = u^8/(1+u^2)^4;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[403]=
$$\frac{1}{48}\left(\frac{u\,(105+280\,u^2+231\,u^4+48\,u^6)}{(1+u^2)^3} - 105\,\text{ArcTan}[u]\right)$$

Out[404]=
$$\frac{8\,u^7}{(1+u^2)^5}$$

Out[405]=
$$\frac{48\,u^5\,(7-11\,u^2+2\,u^4)}{(1+u^2)^7}$$

In[406]:=
```
f = u^9/(1+u^2)^4;
Simplify[Integrate[f, u]]
Simplify[D[f, u]]
Simplify[D[f, {u, 3}]]
```

Out[407]=

$$\frac{-13-27\,u^2-9\,u^4+9\,u^6+3\,u^8}{6\,\left(1+u^2\right)^3}-2\,\text{Log}\left[1+u^2\right]$$

Out[408]=

$$\frac{u^8\,\left(9+u^2\right)}{\left(1+u^2\right)^5}$$

Out[409]=

$$\frac{24\,u^6\,\left(21-18\,u^2+u^4\right)}{\left(1+u^2\right)^7}$$

In[410]:=
```
f = u^10/(1+u^2)^4;
Simplify[Integrate[f, u]]
FortranForm[%];
Simplify[D[f, u]]
FortranForm[%];
Simplify[D[f, {u, 3}]]
FortranForm[%];
```

Out[411]=

$$\frac{1}{48}\left(\frac{u\,\left(-315-840\,u^2-693\,u^4-144\,u^6+16\,u^8\right)}{\left(1+u^2\right)^3}+315\,\text{ArcTan}[u]\right)$$

Out[413]=

$$\frac{2\,u^9\,\left(5+u^2\right)}{\left(1+u^2\right)^5}$$

Out[415]=

$$-\frac{240\,u^7\,\left(-3+u^2\right)}{\left(1+u^2\right)^7}$$

In[417]:=
```
f = u^9/(1+u^2)^5;
Simplify[Integrate[f, u]]
FortranForm[%];
Simplify[D[f, u]]
FortranForm[%];
Simplify[D[f, {u, 3}]]
FortranForm[%];
```

Out[418]=

$$\frac{25 + 88\,u^2 + 108\,u^4 + 48\,u^6 + 12\,(1+u^2)^4\,\text{Log}[1+u^2]}{24\,(1+u^2)^4}$$

Out[420]=

$$-\frac{u^8\,(-9+u^2)}{(1+u^2)^6}$$

Out[422]=

$$-\frac{6\,u^6\,(-84+153\,u^2-42\,u^4+u^6)}{(1+u^2)^8}$$

In[424]:=
```
f = u^10/(1+u^2)^5;
Simplify[Integrate[f, u]]
FortranForm[%];
Simplify[D[f, u]]
FortranForm[%];
Simplify[D[f, {u, 3}]]
FortranForm[%];
```

Out[425]=

$$\frac{1}{128}\left(\frac{u\,(315+1155\,u^2+1533\,u^4+837\,u^6+128\,u^8)}{(1+u^2)^4} - 315\,\text{ArcTan}[u]\right)$$

Out[427]=

$$\frac{10\,u^9}{(1+u^2)^6}$$

Out[429]=

$$\frac{120\,u^7\,(6-7\,u^2+u^4)}{(1+u^2)^8}$$

In[431]:=
```
f = u^11/(1+u^2)^5;
Simplify[Integrate[f, u]]
FortranForm[%];
Simplify[D[f, u]]
FortranForm[%];
Simplify[D[f, {u, 3}]]
FortranForm[%];
```

Out[432]=

$$-\frac{77+248\,u^2+252\,u^4+48\,u^6-48\,u^8-12\,u^{10}}{24\,(1+u^2)^4}-\frac{5}{2}\,\text{Log}[1+u^2]$$

Out[434]=

$$\frac{u^{10}\,(11+u^2)}{(1+u^2)^6}$$

Out[436]=

$$\frac{30\,u^8\,(33-22\,u^2+u^4)}{(1+u^2)^8}$$

In[438]:=
```
f = u^12/(1+u^2)^5;
Simplify[Integrate[f, u]]
FortranForm[%];
Simplify[D[f, u]]
FortranForm[%];
Simplify[D[f, {u, 3}]]
FortranForm[%];
```

Out[439]=

$$\frac{1}{384}\left(\frac{u\,(-3465-12\,705\,u^2-16\,863\,u^4-9207\,u^6-1408\,u^8+128\,u^{10})}{(1+u^2)^4}+3465\,\text{ArcTan}[u]\right)$$

Out[441]=

$$\frac{2\,u^{11}\,(6+u^2)}{(1+u^2)^6}$$

Out[443]=

$$\frac{120\,u^9\,(11-3\,u^2)}{(1+u^2)^8}$$