In[1]:=
```
(* find F1 and F0, numerators of rho^2 and log(rho^2) for M11
use expansion of F(k),E(k) about k=1
call log(rho1^2) =logm. rho1^2=|x-x0|^2
m3=rho1^2/rho2^2*)
```

In[2]:=
```
f = 2 * log2 - logm / 2 + a1 * m3 + a2 * m3 ^ 2 +
    a3 * m3 ^ 3 - (b1 * m3 + b2 * m3 ^ 2 + b3 * m3 ^ 3 + b4 * m3 ^ 4) * logm;
e = 1 + c1 * m3 + c2 * m3 ^ 2 + c3 * m3 ^ 3 - (d1 * m3 + d2 * m3 ^ 2 + d3 * m3 ^ 3 + d4 * m3 ^ 4) * logm;

eth = Series[m3 ^ 2 * e / m3, {m3, 0, 4}] / m3 ^ 2;

c = Sqrt[a + b];
i10 = Series[4 * f / c, {m3, 0, 4}];
i11 = Series[m3 ^ 2 * 4 / b * (a * f / c - c * e), {alf, 0, 6}] / m3 ^ 2;
i30 = Series[m3 * 4 * eth / c ^ 3, {m3, 0, 5}] / m3;
i31 = Series[m3 * 4 / b * (a * eth / c ^ 3 - f / c), {m3, 0, 6}] / m3;
i32 = Series[m3 * (4 / b ^ 2) * (a ^ 2 * eth / c ^ 3 - 2 * a * f / c + c * e), {m3, 0, 6}] / m3;

m11 = Simplify[Series[m3 * x * (i11 + (x ^ 2 + x0 ^ 2) * i31 - x * x0 * (i30 + i32)), {m3, 0, 2}]] / m3;
m12 = Simplify[Series[m3 * x * (xi * (x * i31 - x0 * i30)), {m3, 0, 2}]] / m3;
m21 = Simplify[Series[m3 * x * xi (x * i30 - x0 * i31), {m3, 0, 2}]] / m3;
m22 = Simplify[Series[m3 * x * (i10 + xi ^ 2 * i30), {m3, 0, 2}]] / m3;
```

In[15]:=
```
term=m11;
term=Simplify[term/.{d1→1/4,d2→3/32,d3→15/256,b1→1/8,b2→9/128,c1→(4*log2-1)/4,c2→(24*log2
term=Simplify[term/.{b→2*x*x0,a→x0^2+x^2+xi^2}]
```

Out[17]=
$$\frac{x^4 - 2\,x^2\,x0^2 + x0^4 - xi^4}{x0\left(x^2 + 2\,x\,x0 + x0^2 + xi^2\right)^{3/2}\,m3} + \Big((-13 + 20\,\log2 - 5\,\log m)\,x^4 + 8\,(-6 + 4\,\log2 - \log m)\,x^3\,x0 + \left(x0^2 + xi^2\right)\left((-13 + 20\,\log2 - 5\,\log m)\,x0^2 + (-11 + 28\,\log2 - 7\,\log m)\,xi^2\right) + 2\,x^2\left((-35 + 12\,\log2 - 3\,\log m)\,x0^2 + 6\,(-2 + 4\,\log2 - \log m)\,xi^2\right) + 8\,x\left((-6 + 4\,\log2 - \log m)\,x0^3 + 2\,(-3 + 4\,\log2 - \log m)\,x0\,xi^2\right)\Big) \Big/ \left(4\,x0\left(x^2 + 2\,x\,x0 + x0^2 + xi^2\right)^{3/2}\right) + \Big(\big((3 - 104\,\log2 + 26\,\log m)\,x^4 - 32\,(-4 + 20\,\log2 - 5\,\log m)\,x^3\,x0 - \left(x0^2 + xi^2\right)\left((-3 + 104\,\log2 - 26\,\log m)\,x0^2 + (3 + 88\,\log2 - 22\,\log m)\,xi^2\right) + x^2\left((250 - 1072\,\log2 + 268\,\log m)\,x0^2 + 48\,(-4\,\log2 + \log m)\,xi^2\right) - 32\,x\left((-4 + 20\,\log2 - 5\,\log m)\,x0^3 + 2\,(-1 + 8\,\log2 - 2\,\log m)\,x0\,xi^2\right)\big)\,m3\Big) \Big/ \left(64\,x0\left(x^2 + 2\,x\,x0 + x0^2 + xi^2\right)^{3/2}\right) + O[m3]^2$$

In[18]:=
```
mcoeff=Simplify[Coefficient[term*m3^2,m3]]
msqden=Simplify[Denominator[mcoeff]]
mnum=Numerator[mcoeff]
(* Lead = 1/x0
This terms is multiplied by rho2^2/rho^2 so denominator is 1/rho2 whole term is Lead*
```

Out[18]=
$$\frac{x^4 - 2\,x^2\,x0^2 + x0^4 - xi^4}{x0\left(x^2 + 2\,x\,x0 + x0^2 + xi^2\right)^{3/2}}$$

Out[19]=
$$x0\left(x^2 + 2\,x\,x0 + x0^2 + xi^2\right)^{3/2}$$

Out[20]=
$$x^4 - 2\,x^2\,x0^2 + x0^4 - xi^4$$

In[21]:=
```
piece1=Simplify[mnum/.xi->0]
piece2=Simplify[mnum-piece1]
```

Out[21]=
$$\left(x^2 - x0^2\right)^2$$

Out[22]=
$$-xi^4$$

In[23]:=
```
small1=(x-x0)^2
fact1=(x+x0)^2
FortranForm[Expand[%]]
small2=-xi^4
Simplify[mnum-small2-small1*fact1]
```

Out[23]=
$$(x - x0)^2$$

Out[24]=
$$(x + x0)^2$$

Out[25]//FortranForm=
```
"x**2 + 2*x*x0 + x0**2"
```

Out[26]=
$$-xi^4$$

Out[27]=
$$0$$

In[28]:=
```
logcoeff=Simplify[Coefficient[term,logm]/.m3→0]
lognum=Numerator[-logcoeff]
(* Lead = -1/(4*x0) *)
```

Out[28]=
$$\frac{-5\,x^4 - 8\,x^3\,x0 - 5\,x0^4 - 12\,x0^2\,xi^2 - 7\,xi^4 - 6\,x^2\left(x0^2 + 2\,xi^2\right) - 8\,x\left(x0^3 + 2\,x0\,xi^2\right)}{4\,x0\left(x^2 + 2\,x\,x0 + x0^2 + xi^2\right)^{3/2}}$$

Out[29]=
$$5\,x^4 + 8\,x^3\,x0 + 5\,x0^4 + 12\,x0^2\,xi^2 + 7\,xi^4 + 6\,x^2\left(x0^2 + 2\,xi^2\right) + 8\,x\left(x0^3 + 2\,x0\,xi^2\right)$$

In[30]:=
```
piece1=Simplify[lognum/.xi->0]
FortranForm[Expand[%]]
piece2=Simplify[xi^2*Coefficient[lognum,xi^2]]
FortranForm[Expand[%/xi^2]]
piece3=Simplify[xi^4*Coefficient[lognum,xi^4]]
FortranForm[Expand[%]]
Simplify[lognum-piece1-piece3-piece2]
```

Out[30]= $(x + x0)^2 \left(5\, x^2 - 2\, x\, x0 + 5\, x0^2\right)$

Out[31]//FortranForm=
```
"5*x**4 + 8*x**3*x0 + 6*x**2*x0**2 + 8*x*x0**3 + 5*x0**4"
```

Out[32]= $4 \left(3\, x^2 + 4\, x\, x0 + 3\, x0^2\right) xi^2$

Out[33]//FortranForm=
```
"12*x**2 + 16*x*x0 + 12*x0**2"
```

Out[34]= $7\, xi^4$

Out[35]//FortranForm=
```
"7*xi**4"
```

Out[36]= 0

In[37]:=
```
logmcoeff = Simplify[Coefficient[term, m3]];
logmcoeff = Simplify[Coefficient[%, logm]]
mnum = Simplify[Numerator[logmcoeff]];
piece0 = Simplify[mnum /. xi → 0]
FortranForm[Expand[%]]
piece2 = Simplify[xi^2 * Coefficient[mnum - piece, xi^2]]
Simplify[mnum - piece0 - piece2]
```

Out[38]= $\dfrac{13\, x^4 + 80\, x^3\, x0 + 13\, x0^4 + 24\, x0^2\, xi^2 + 11\, xi^4 + 2\, x^2 \left(67\, x0^2 + 12\, xi^2\right) + 16\, x \left(5\, x0^3 + 4\, x0\, xi^2\right)}{32\, x0 \left(x^2 + 2\, x\, x0 + x0^2 + xi^2\right)^{3/2}}$

Out[40]= $(x + x0)^2 \left(13\, x^2 + 54\, x\, x0 + 13\, x0^2\right)$

Out[41]//FortranForm=
```
"13*x**4 + 80*x**3*x0 + 134*x**2*x0**2 + 80*x*x0**3 + 13*x0**4"
```

Out[42]= $8 \left(3\, x^2 + 8\, x\, x0 + 3\, x0^2\right) xi^2$

Out[43]= $11\, xi^4$